Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.3-Tangent/106-4.3.7-d-trig-^m-a+b-c-tan-^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [499]. This is test number [106].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (499)	0.00 (0)
Mathematica	99.60 (497)	0.40 (2)
Maple	82.57 (412)	17.43 (87)
Fricas	81.56 (407)	18.44 (92)
Giac	62.12 (310)	37.88 (189)
Mupad	56.71 (283)	43.29 (216)
Maxima	53.91 (269)	46.09 (230)
Sympy	19.04 (95)	80.96 (404)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

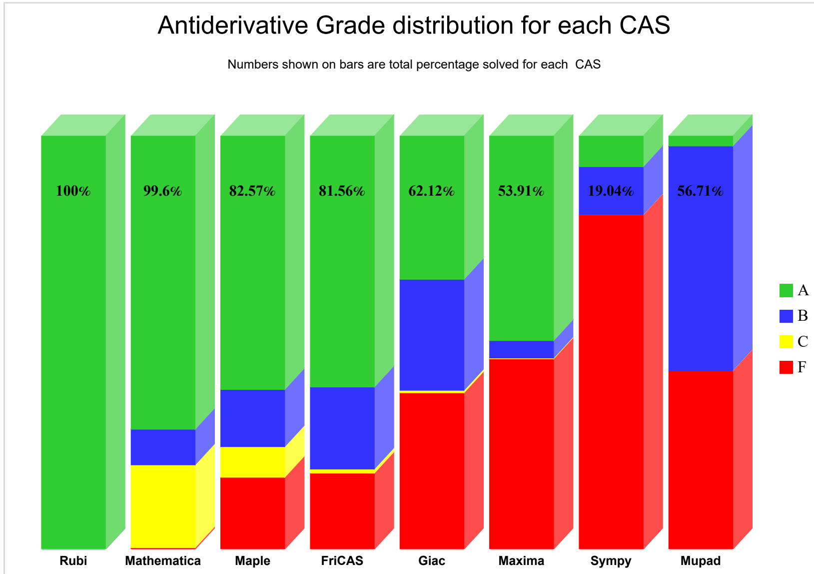
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

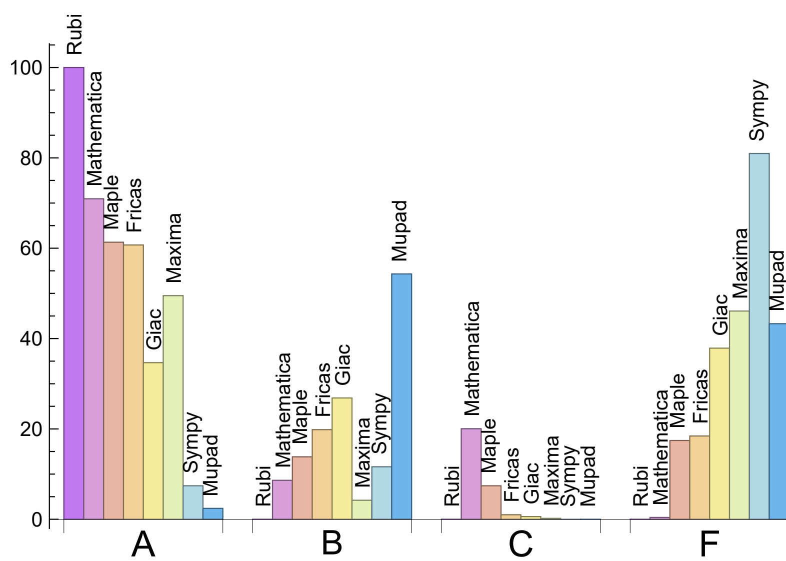
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	70.94	8.62	20.04	0.40
Maple	61.32	13.83	7.41	17.43
Fricas	60.72	19.84	1.00	18.44
Maxima	49.50	4.21	0.20	46.09
Giac	34.67	26.85	0.60	37.88
Sympy	7.41	11.62	0.00	80.96
Mupad	N/A	54.31	0.00	43.29

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	87	100.00 %	0.00 %	0.00 %
Fricas	92	82.61 %	10.87 %	6.52 %
Giac	189	78.84 %	1.59 %	19.58 %
Maxima	230	70.00 %	6.52 %	23.48 %
Sympy	404	76.73 %	20.54 %	2.72 %
Mupad	216	95.83 %	4.17 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

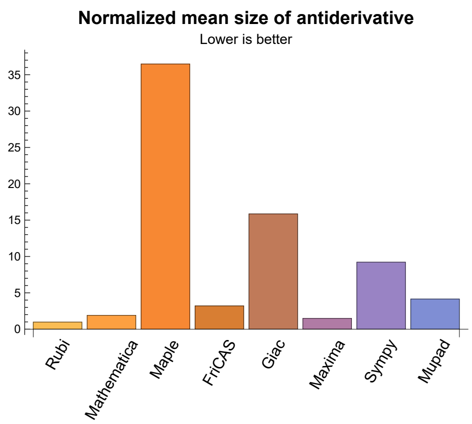
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

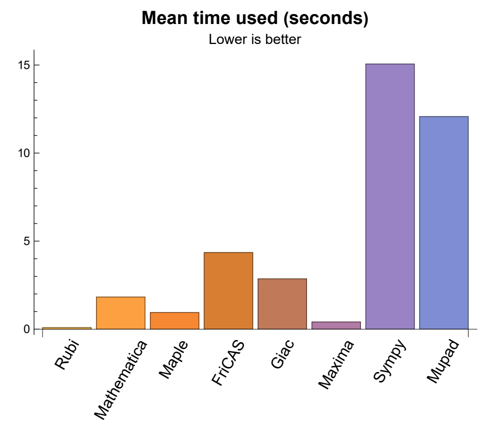
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	109.13	0.98	90.00	1.00
Mathematica	1.83	201.02	1.91	94.00	1.00
Maple	0.95	6052.27	36.48	117.50	1.20
Maxima	0.41	130.50	1.49	86.00	1.13
Fricas	4.35	422.91	3.21	225.00	2.81
Sympy	15.06	1049.93	9.22	131.00	1.92
Giac	2.86	1518.41	15.86	168.50	1.47
Mupad	12.07	508.58	4.15	108.00	1.31

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
5
10
15
20
25
30
35
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{174, 178, 420, 424, 486, 487, 488, 489, 490, 494, 495, 499\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {97, 108, 109, 131, 136, 144, 145, 148, 152, 153, 154, 157, 158, 159, 160, 164, 165, 170, 171, 173, 176, 304, 305, 318, 330, 331, 332, 342, 343, 344, 345, 354, 355, 356, 357, 368, 369, 370, 371, 372, 422, 437, 438, 475, 481, 484, 485, 496, 497, 498}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

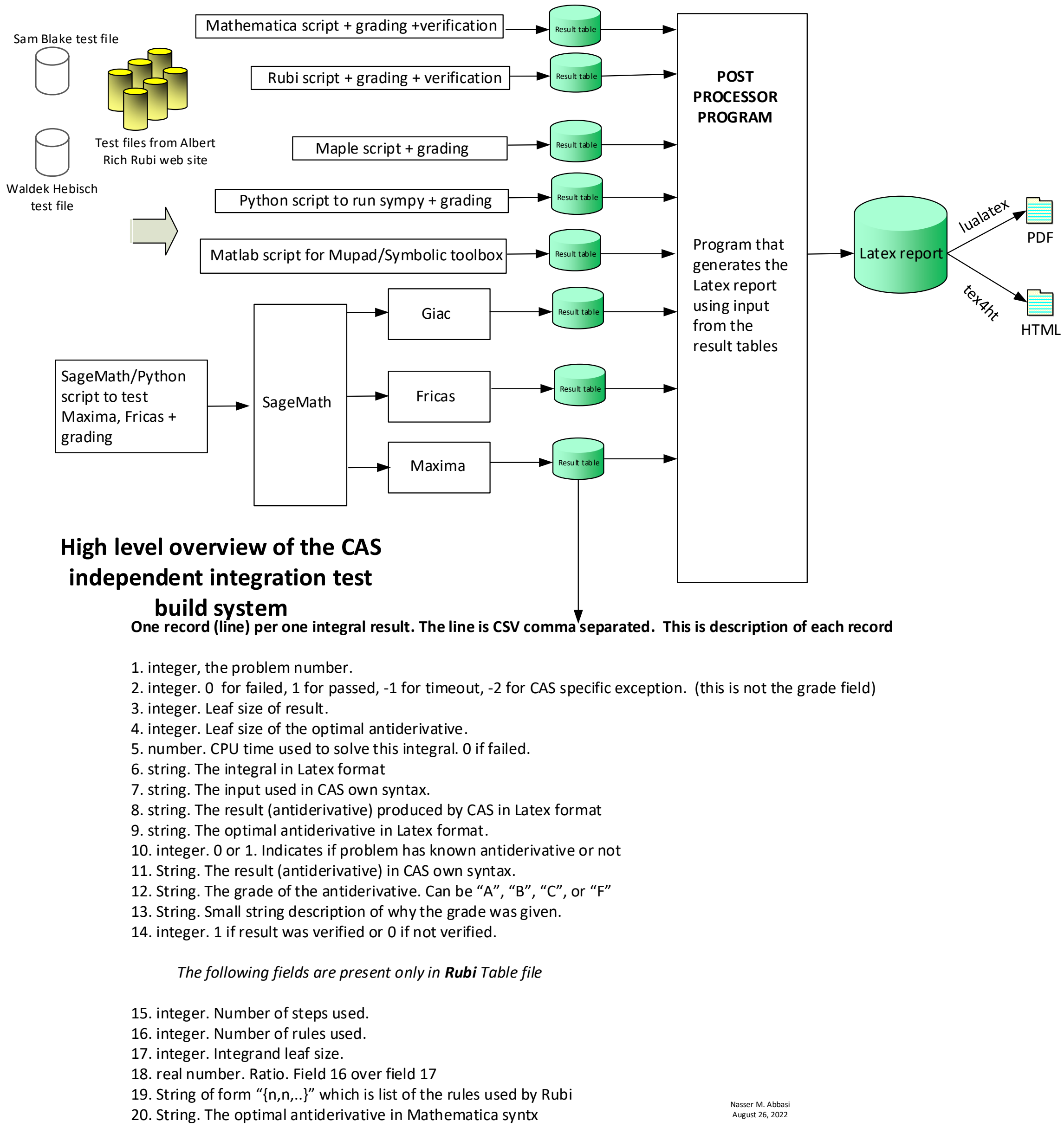
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 100, 104, 105, 106, 109, 112, 116, 117, 118, 121, 124, 125, 126, 127, 128, 129, 130, 133, 137, 138, 139, 140, 141, 142, 149, 150, 151, 154, 155, 156, 161, 162, 163, 166, 167, 168, 169, 172, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 206, 207, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 292, 293, 294, 295, 296, 297, 298, 302, 306, 307, 308, 309, 310, 311, 315, 319, 320, 321, 322, 323, 324, 325, 329, 334, 341, 353, 359, 360, 361, 362, 363, 364, 365, 366, 377, 380, 381, 382, 383, 384, 385, 389, 390, 391, 393, 394, 395, 396, 397, 398, 400, 401, 403, 404, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 440, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 482, 483, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499 }

B grade: { 33, 34, 47, 48, 57, 58, 59, 60, 95, 96, 97, 107, 108, 119, 120, 131, 132, 143, 144, 145, 153, 157, 158, 160, 176, 204, 205, 249, 287, 288, 290, 291, 368, 369, 370, 371, 372, 422, 439, 450, 451, 475, 497 }

C grade: { 8, 10, 11, 12, 16, 17, 18, 98, 99, 101, 102, 103, 110, 111, 113, 114, 115, 122, 123, 134, 135, 136, 146, 147, 148, 152, 159, 164, 165, 170, 171, 173, 195, 196, 197, 208, 210, 270, 299, 300, 301, 303, 304, 305, 312, 313, 314, 316, 317, 318, 326, 327, 328, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 374, 375, 376, 378, 379, 386, 387, 388, 392, 399, 402, 405, 407, 437, 438, 449, 481, 484, 485, 496, 498 }

F grade: { 367, 373 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 116, 117, 124, 125, 126, 127, 130, 136, 137, 138, 139, 140, 141, 142, 148, 149, 150, 151, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 339, 340, 341, 342, 346, 347, 348, 354, 355, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 390, 396, 397, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 44, 45, 50, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 108, 109, 112, 118, 119, 120, 121, 128, 129, 131, 132, 133, 143, 144, 145, 146, 147, 279, 296, 297, 298, 299, 300, 301, 302, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 319, 323, 324, 325, 336, 337, 338, 349, 350, 351, 352, 353, 389, 393, 394, 400, 401, 403, 404, 447 }

C grade: { 29, 98, 99, 101, 102, 103, 110, 111, 113, 114, 115, 122, 123, 134, 135, 167, 168, 169, 303, 304, 305, 316, 317, 318, 330, 331, 332, 343, 344, 345, 387, 388, 392, 399, 477, 478, 479 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 391, 395, 398, 402, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 474, 475, 476, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 101, 102, 103, 104, 105, 106, 113, 114, 115, 116, 117, 118, 125, 126, 127, 129, 130, 137, 138, 139, 141, 142, 149, 150, 151, 167, 168, 169, 174, 178, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 262, 264, 273, 274, 276, 278, 279, 281, 283, 284, 285, 286, 287, 288, 289, 291, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 456, 457, 458, 459, 460, 461, 468, 469, 470, 471, 472, 473, 477, 478, 479, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 128, 140, 179, 180, 238, 239, 242, 257, 258, 259, 261, 263, 265, 266, 267, 269, 270, 271, 272, 277, 282 }

C grade: { 290 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 80, 81, 82, 83, 84, 85, 95, 96, 97, 98, 99, 100, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 131, 132, 133, 134, 135, 136, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 260, 268, 275, 280, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 450, 451, 452, 453, 454, 455, 462, 463, 464, 465, 466, 467, 474, 475, 476, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 68, 69, 70, 74, 75, 76, 80, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 108, 109, 112, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 128, 129, 130, 132, 133, 136, 137, 138, 139, 140, 141, 142, 149, 150, 167, 168, 169, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 230, 231, 233, 234, 235, 236, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 277, 278, 282, 283, 284, 285, 286, 289, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 338, 339, 341, 342, 343, 344, 345, 353, 357, 358, 374, 375, 376, 377, 380, 381, 382, 383, 384, 389, 390, 391, 393, 394, 395, 396, 397, 398, 420, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 461, 462, 463, 464, 465, 472, 473, 477, 478, 479, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 33, 34, 35, 47, 48, 60, 65, 66, 67, 71, 72, 73, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 101, 102, 103, 110, 111, 113, 122, 123, 131, 134, 135, 143, 144, 145, 148, 224, 228, 229, 232, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 256, 274, 276, 279, 280, 281, 287, 288, 333, 334, 335, 336, 337, 340, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 385, 386, 400, 401, 402, 403, 404, 405, 458, 459, 466, 467, 468, 469, 470, 471 }

C grade: { 290, 291, 292, 378, 379 }

F grade: { 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 392, 399, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 474, 475, 476, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.6 Sympy

A grade: { 39, 51, 179, 180, 181, 185, 191, 192, 193, 194, 207, 251, 252, 253, 260, 268, 275, 278, 280, 289, 335, 348, 374, 375, 376, 377, 380, 381, 382, 383, 384, 420, 433, 486, 488, 489, 494 }

B grade: { 64, 76, 88, 182, 183, 184, 186, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 237, 238, 239, 243, 244, 245, 246, 250, 254, 255, 256 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 216, 223, 229, 235, 236, 240, 241, 242, 247, 248, 249, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 279, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 378, 379, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 487, 490, 491, 492, 493, 495, 496, 497, 498, 499 }

2.1.7 Giac

A grade: { 3, 7, 8, 9, 10, 11, 12, 16, 17, 18, 31, 32, 40, 41, 42, 44, 45, 52, 53, 54, 57, 61, 62, 63, 64, 65, 66, 67, 70, 74, 75, 76, 77, 78, 79, 82, 86, 87, 88, 89, 90, 91, 105, 142, 174, 178, 182, 183, 184, 188, 201, 208, 214, 217, 218, 219, 220, 221, 222, 223, 227, 230, 231, 232, 233, 234, 235, 236, 240, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 258, 259, 260, 261, 263, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 289, 293, 294, 295, 306, 307, 308, 320, 321, 322, 333, 334, 335, 346, 347, 348, 378, 379, 380, 385, 386, 389, 390, 393, 394, 396, 397, 400, 401, 420, 424, 425, 426, 427, 428, 431, 432, 433, 437, 438, 439, 440, 444, 445, 446, 450, 451, 452, 453, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 1, 2, 4, 5, 6, 13, 14, 15, 30, 33, 34, 35, 36, 37, 38, 39, 43, 46, 47, 48, 49, 50, 51, 55, 56, 58, 59, 60, 68, 69, 71, 72, 73, 80, 81, 83, 84, 85, 92, 93, 94, 106, 116, 117, 118, 121, 128, 129, 130, 133, 140, 141, 145, 179, 180, 181, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 215, 216, 224, 225, 226, 228, 229, 237, 238, 239, 241, 242, 250, 251, 252, 253, 262, 264, 265, 266, 270, 271, 272, 283, 284, 285, 286, 288, 296, 323, 374, 375, 376, 377, 381, 382, 383, 384, 403, 404, 429, 430, 434, 435, 436, 441, 447, 448, 449, 454, 455, 467 }

C grade: { 290, 291, 292 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 120, 122, 123, 124, 125, 126, 127, 131, 132, 134, 135, 136, 137, 138, 139, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 297, 298, 299, 300, 301, 302, 303, 304, 305, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 324, 325, 326, 327, 328, 329, 330, 331, 332, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 391, 392, 395, 398, 399, 402, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 442, 443, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 491, 492, 493, 496, 497, 498 }

2.1.8 Mupad

A grade: { 174, 178, 420, 424, 486, 487, 488, 489, 490, 494, 495, 499 }

B grade: { 4, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 124, 125, 126, 127, 137, 138, 149, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 268, 269, 273, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 306, 307, 308, 309, 310, 311, 320, 321, 322, 323, 324, 325, 329, 333, 334, 335, 336, 337, 338, 346, 347, 348, 349, 350, 351, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 493 }

C grade: { }

F grade: $\{$ 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 257, 259, 267, 270, 271, 272, 274, 282, 299, 300, 301, 302, 303, 304, 305, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 330, 331, 332, 339, 340, 341, 342, 343, 344, 345, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 474, 475, 476, 477, 478, 480, 481, 482, 483, 484, 485, 491, 492, 496, 497, 498 $\}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	56	58	50	79	0	757	-1
N.S.	1	1.00	0.57	0.59	0.51	0.81	0.00	7.72	-0.01
time (sec)	N/A	0.027	0.257	0.148	0.519	4.101	0.000	1.095	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	48	36	56	0	278	-1
N.S.	1	1.00	0.77	0.79	0.59	0.92	0.00	4.56	-0.02
time (sec)	N/A	0.020	0.078	0.059	0.506	3.369	0.000	0.671	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	20	41	0	25	-1
N.S.	1	1.00	1.00	1.16	0.62	1.28	0.00	0.78	-0.03
time (sec)	N/A	0.012	0.029	0.037	0.511	3.499	0.000	0.478	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	47	35	54	0	87	34
N.S.	1	1.00	1.26	1.52	1.13	1.74	0.00	2.81	1.10
time (sec)	N/A	0.012	0.060	0.045	0.515	2.085	0.000	0.536	11.444

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	56	64	49	75	0	190	-1
N.S.	1	1.00	0.85	0.97	0.74	1.14	0.00	2.88	-0.02
time (sec)	N/A	0.020	0.249	0.041	0.529	1.186	0.000	0.549	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	68	74	70	89	0	254	-1
N.S.	1	1.00	0.70	0.76	0.72	0.92	0.00	2.62	-0.01
time (sec)	N/A	0.031	0.179	0.050	0.519	1.785	0.000	0.583	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	199	263	188	0	0	305	-1
N.S.	1	1.00	0.55	0.72	0.52	0.00	0.00	0.84	-0.00
time (sec)	N/A	0.108	0.554	0.068	0.518	0.000	0.000	0.655	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	54	236	148	0	0	264	-1
N.S.	1	1.00	0.19	0.83	0.52	0.00	0.00	0.92	-0.00
time (sec)	N/A	0.085	0.061	0.027	0.517	0.000	0.000	0.519	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	161	205	140	0	0	203	-1
N.S.	1	1.00	0.63	0.80	0.55	0.00	0.00	0.80	-0.00
time (sec)	N/A	0.080	0.170	0.029	0.517	0.000	0.000	0.480	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	43	211	133	0	0	263	-1
N.S.	1	1.00	0.17	0.83	0.52	0.00	0.00	1.03	-0.00
time (sec)	N/A	0.080	0.024	0.035	0.509	0.000	0.000	0.555	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	45	233	173	0	0	293	-1
N.S.	1	1.00	0.15	0.78	0.58	0.00	0.00	0.98	-0.00
time (sec)	N/A	0.092	0.049	0.031	0.514	0.000	0.000	0.681	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	45	272	182	0	0	321	-1
N.S.	1	1.00	0.12	0.75	0.50	0.00	0.00	0.88	-0.00
time (sec)	N/A	0.106	0.039	0.032	0.548	0.000	0.000	0.925	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	86	84	85	103	0	1023	-1
N.S.	1	1.00	0.47	0.46	0.47	0.57	0.00	5.62	-0.01
time (sec)	N/A	0.047	0.502	0.062	0.510	2.907	0.000	4.749	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	64	57	67	0	1079	-1
N.S.	1	1.00	0.60	0.58	0.52	0.61	0.00	9.81	-0.01
time (sec)	N/A	0.031	0.516	0.033	0.504	2.137	0.000	2.414	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	42	28	40	0	250	-1
N.S.	1	1.00	0.82	0.84	0.56	0.80	0.00	5.00	-0.02
time (sec)	N/A	0.016	0.066	0.030	0.495	1.962	0.000	0.565	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	29	42	0	48	-1
N.S.	1	1.00	0.84	0.78	0.57	0.82	0.00	0.94	-0.02
time (sec)	N/A	0.016	0.035	0.034	0.503	2.026	0.000	0.563	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	45	63	54	67	0	131	-1
N.S.	1	1.00	0.38	0.53	0.45	0.56	0.00	1.10	-0.01
time (sec)	N/A	0.033	0.034	0.038	0.498	2.471	0.000	0.691	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	45	83	76	89	0	196	-1
N.S.	1	1.00	0.25	0.45	0.42	0.49	0.00	1.07	-0.01
time (sec)	N/A	0.048	0.025	0.049	0.533	1.452	0.000	1.222	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.076	0.094	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.049	0.071	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.028	0.081	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.033	0.079	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	60	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.049	0.079	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.050	0.079	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.034	0.109	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.033	0.138	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.029	0.129	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.024	0.138	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	12884	0	24	0	0	-1
N.S.	1	1.00	1.00	402.62	0.00	0.75	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.017	4.987	0.000	1.809	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	104	92	66	68	0	58071	92
N.S.	1	1.00	1.49	1.31	0.94	0.97	0.00	829.59	1.31
time (sec)	N/A	0.043	0.052	0.167	0.278	6.138	0.000	15.346	12.228

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	72	72	47	49	0	76	68
N.S.	1	1.00	1.50	1.50	0.98	1.02	0.00	1.58	1.42
time (sec)	N/A	0.035	0.041	0.112	0.291	4.265	0.000	0.613	12.010

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	46	52	34	33	0	41	39
N.S.	1	1.00	1.64	1.86	1.21	1.18	0.00	1.46	1.39
time (sec)	N/A	0.019	0.041	0.085	0.286	3.980	0.000	0.576	11.785

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	51	34	43	61	0	62	37
N.S.	1	1.00	2.04	1.36	1.72	2.44	0.00	2.48	1.48
time (sec)	N/A	0.020	0.022	0.115	0.290	5.097	0.000	0.589	11.563

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	123	68	81	133	0	185	95
N.S.	1	1.00	2.41	1.33	1.59	2.61	0.00	3.63	1.86
time (sec)	N/A	0.040	0.038	0.169	0.286	3.324	0.000	0.627	12.096

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	131	102	108	191	0	259	138
N.S.	1	1.00	1.66	1.29	1.37	2.42	0.00	3.28	1.75
time (sec)	N/A	0.049	5.450	0.178	0.298	2.239	0.000	0.610	11.975

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	89	122	119	96	0	7897	105
N.S.	1	1.00	0.87	1.20	1.17	0.94	0.00	77.42	1.03
time (sec)	N/A	0.087	0.251	0.152	0.521	3.122	0.000	2.430	11.890

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	58	102	88	77	0	4350	79
N.S.	1	1.00	0.78	1.38	1.19	1.04	0.00	58.78	1.07
time (sec)	N/A	0.050	0.251	0.109	0.497	4.010	0.000	1.915	11.429

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	81	55	58	0	395	41
N.S.	1	1.00	0.93	1.76	1.20	1.26	0.00	8.59	0.89
time (sec)	N/A	0.034	0.161	0.108	0.487	2.764	0.000	0.614	11.316

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	29	25	22	20	252	21
N.S.	1	1.00	1.47	1.53	1.32	1.16	1.05	13.26	1.11
time (sec)	N/A	0.010	0.010	0.019	0.494	2.072	0.057	0.540	11.276

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	26	40	0	26	26
N.S.	1	1.00	1.00	0.96	1.08	1.67	0.00	1.08	1.08
time (sec)	N/A	0.021	0.020	0.139	0.284	1.368	0.000	0.542	11.270

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	60	54	43	71	0	53	41
N.S.	1	1.00	1.43	1.29	1.02	1.69	0.00	1.26	0.98
time (sec)	N/A	0.029	0.053	0.125	0.302	1.438	0.000	0.599	11.486

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	106	83	63	98	0	79	59
N.S.	1	1.00	1.66	1.30	0.98	1.53	0.00	1.23	0.92
time (sec)	N/A	0.039	0.041	0.137	0.284	1.016	0.000	0.618	11.411

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	97	185	109	110	0	207699	183
N.S.	1	1.00	0.91	1.73	1.02	1.03	0.00	1941.11	1.71
time (sec)	N/A	0.080	0.490	0.161	0.299	2.527	0.000	266.311	12.648

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	155	84	84	0	144	128
N.S.	1	1.00	0.90	1.94	1.05	1.05	0.00	1.80	1.60
time (sec)	N/A	0.058	0.366	0.125	0.293	2.545	0.000	1.167	15.396

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	125	77	62	0	94	126
N.S.	1	1.00	0.89	2.31	1.43	1.15	0.00	1.74	2.33
time (sec)	N/A	0.033	0.208	0.135	0.293	2.110	0.000	0.802	14.122

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	66	97	72	93	0	149	86
N.S.	1	1.00	1.27	1.87	1.38	1.79	0.00	2.87	1.65
time (sec)	N/A	0.037	0.123	0.144	0.292	3.231	0.000	0.790	12.644

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	231	85	117	178	0	254	188
N.S.	1	1.00	2.82	1.04	1.43	2.17	0.00	3.10	2.29
time (sec)	N/A	0.078	5.458	0.184	0.297	3.452	0.000	0.837	12.609

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	447	145	171	298	0	421	243
N.S.	1	1.00	3.63	1.18	1.39	2.42	0.00	3.42	1.98
time (sec)	N/A	0.091	6.134	0.192	0.280	3.483	0.000	0.859	12.062

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	96	199	137	126	0	13574	128
N.S.	1	1.00	0.79	1.63	1.12	1.03	0.00	111.26	1.05
time (sec)	N/A	0.094	0.997	0.118	0.490	1.333	0.000	18.185	12.330

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	168	92	99	0	1411	114
N.S.	1	1.00	0.84	1.98	1.08	1.16	0.00	16.60	1.34
time (sec)	N/A	0.075	0.503	0.110	0.497	1.600	0.000	1.064	11.796

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	73	59	63	53	68	382	76
N.S.	1	1.00	1.59	1.28	1.37	1.15	1.48	8.30	1.65
time (sec)	N/A	0.023	0.394	0.030	0.507	2.169	0.115	0.907	11.897

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	48	44	75	0	44	67
N.S.	1	1.00	0.96	1.04	0.96	1.63	0.00	0.96	1.46
time (sec)	N/A	0.038	0.363	0.110	0.285	1.158	0.000	0.812	11.856

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	59	81	70	98	0	84	69
N.S.	1	1.00	0.84	1.16	1.00	1.40	0.00	1.20	0.99
time (sec)	N/A	0.050	0.333	0.151	0.299	1.090	0.000	0.806	11.820

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	136	93	145	0	128	90
N.S.	1	1.00	0.95	1.46	1.00	1.56	0.00	1.38	0.97
time (sec)	N/A	0.062	0.568	0.139	0.304	1.002	0.000	0.838	12.192

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	177	144	0	304	0	377	643
N.S.	1	1.00	1.51	1.23	0.00	2.60	0.00	3.22	5.50
time (sec)	N/A	0.125	2.039	0.345	0.000	0.828	0.000	0.706	14.414

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	149	87	0	214	0	180	382
N.S.	1	1.00	1.77	1.04	0.00	2.55	0.00	2.14	4.55
time (sec)	N/A	0.087	0.448	0.266	0.000	0.795	0.000	0.685	13.369

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	121	61	0	164	0	81	112
N.S.	1	1.00	2.02	1.02	0.00	2.73	0.00	1.35	1.87
time (sec)	N/A	0.039	0.176	0.214	0.000	0.825	0.000	0.710	11.790

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	144	70	0	192	0	113	91
N.S.	1	1.00	2.40	1.17	0.00	3.20	0.00	1.88	1.52
time (sec)	N/A	0.048	0.145	0.242	0.000	1.165	0.000	0.916	11.861

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	195	117	0	345	0	214	591
N.S.	1	1.00	2.19	1.31	0.00	3.88	0.00	2.40	6.64
time (sec)	N/A	0.080	0.424	0.315	0.000	1.030	0.000	0.765	13.390

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	326	185	0	658	0	377	740
N.S.	1	1.00	2.51	1.42	0.00	5.06	0.00	2.90	5.69
time (sec)	N/A	0.125	6.180	0.390	0.000	1.061	0.000	0.731	14.718

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	140	185	313	539	0	292	2500
N.S.	1	1.00	0.79	1.04	1.76	3.03	0.00	1.64	14.04
time (sec)	N/A	0.201	0.387	0.375	0.506	0.808	0.000	1.043	17.016

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	99	131	189	399	0	190	2500
N.S.	1	1.00	0.77	1.02	1.47	3.09	0.00	1.47	19.38
time (sec)	N/A	0.105	0.188	0.311	0.507	2.284	0.000	0.737	15.739

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	83	97	288	0	113	190
N.S.	1	1.00	0.84	1.01	1.18	3.51	0.00	1.38	2.32
time (sec)	N/A	0.067	0.120	0.310	0.492	3.994	0.000	0.717	12.749

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	50	50	190	240	68	948
N.S.	1	1.00	0.98	1.00	1.00	3.80	4.80	1.36	18.96
time (sec)	N/A	0.051	0.043	0.133	0.499	3.086	1.282	0.644	11.687

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	44	44	273	0	62	40
N.S.	1	1.00	1.00	0.92	0.92	5.69	0.00	1.29	0.83
time (sec)	N/A	0.042	0.086	0.191	0.506	3.081	0.000	0.694	10.985

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	67	71	395	0	97	67
N.S.	1	1.00	0.96	0.88	0.93	5.20	0.00	1.28	0.88
time (sec)	N/A	0.062	0.219	0.250	0.519	3.800	0.000	0.682	11.053

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	103	99	108	571	0	151	115
N.S.	1	1.00	0.98	0.94	1.03	5.44	0.00	1.44	1.10
time (sec)	N/A	0.082	0.551	0.299	0.500	4.451	0.000	0.717	11.393

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	215	197	0	609	0	561	1049
N.S.	1	1.00	1.05	0.97	0.00	2.99	0.00	2.75	5.14
time (sec)	N/A	0.224	2.510	0.481	0.000	5.253	0.000	0.851	15.486

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	182	152	0	470	0	368	737
N.S.	1	1.00	1.37	1.14	0.00	3.53	0.00	2.77	5.54
time (sec)	N/A	0.134	2.177	0.397	0.000	6.686	0.000	0.846	14.720

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	146	103	0	319	0	153	436
N.S.	1	1.00	1.45	1.02	0.00	3.16	0.00	1.51	4.32
time (sec)	N/A	0.051	0.544	0.342	0.000	7.361	0.000	0.836	13.856

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	184	128	0	488	0	270	1140
N.S.	1	1.00	1.67	1.16	0.00	4.44	0.00	2.45	10.36
time (sec)	N/A	0.087	0.598	0.411	0.000	8.648	0.000	0.701	13.721

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	218	156	0	700	0	417	917
N.S.	1	1.00	1.48	1.06	0.00	4.76	0.00	2.84	6.24
time (sec)	N/A	0.126	4.133	0.384	0.000	4.518	0.000	0.719	12.373

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	392	232	0	1090	0	551	1113
N.S.	1	1.00	1.87	1.10	0.00	5.19	0.00	2.62	5.30
time (sec)	N/A	0.195	6.248	0.444	0.000	5.283	0.000	0.767	12.314

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	136	159	320	729	0	266	2500
N.S.	1	1.00	0.69	0.81	1.63	3.72	0.00	1.36	12.76
time (sec)	N/A	0.183	1.083	0.414	0.500	4.495	0.000	0.804	16.143

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	111	120	191	590	0	195	2500
N.S.	1	1.00	0.80	0.87	1.38	4.28	0.00	1.41	18.12
time (sec)	N/A	0.107	1.166	0.337	0.501	3.654	0.000	0.783	14.937

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	93	118	406	2125	127	2489
N.S.	1	1.00	0.91	0.96	1.22	4.19	21.91	1.31	25.66
time (sec)	N/A	0.059	0.745	0.215	0.518	2.585	15.702	0.584	13.532

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	69	77	395	0	93	70
N.S.	1	1.00	1.01	0.84	0.94	4.82	0.00	1.13	0.85
time (sec)	N/A	0.050	0.457	0.223	0.518	2.283	0.000	0.694	11.499

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	100	120	615	0	142	108
N.S.	1	1.00	0.97	0.86	1.03	5.30	0.00	1.22	0.93
time (sec)	N/A	0.104	0.659	0.313	0.515	3.070	0.000	0.727	11.370

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	151	144	167	889	0	212	178
N.S.	1	1.00	0.83	0.79	0.92	4.88	0.00	1.16	0.98
time (sec)	N/A	0.159	1.273	0.360	0.503	3.300	0.000	0.788	12.366

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	278	282	0	1040	0	885	1536
N.S.	1	1.00	1.05	1.07	0.00	3.94	0.00	3.35	5.82
time (sec)	N/A	0.298	3.686	0.658	0.000	4.290	0.000	1.193	16.328

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	230	196	0	795	0	563	1154
N.S.	1	1.00	1.28	1.09	0.00	4.42	0.00	3.13	6.41
time (sec)	N/A	0.177	3.713	0.523	0.000	3.754	0.000	1.154	15.454

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	170	132	0	574	0	223	780
N.S.	1	1.00	1.23	0.96	0.00	4.16	0.00	1.62	5.65
time (sec)	N/A	0.075	1.115	0.387	0.000	2.843	0.000	1.085	14.875

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	247	183	0	1078	0	534	1844
N.S.	1	1.00	1.49	1.10	0.00	6.49	0.00	3.22	11.11
time (sec)	N/A	0.161	2.111	0.530	0.000	1.384	0.000	0.984	16.268

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	286	206	0	1457	0	622	1652
N.S.	1	1.00	1.40	1.00	0.00	7.11	0.00	3.03	8.06
time (sec)	N/A	0.212	5.617	0.529	0.000	1.759	0.000	1.029	13.219

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	468	265	0	1741	0	922	1357
N.S.	1	1.00	1.81	1.02	0.00	6.72	0.00	3.56	5.24
time (sec)	N/A	0.265	6.378	0.491	0.000	1.669	0.000	1.081	12.824

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	194	209	470	1223	0	399	2500
N.S.	1	1.00	0.78	0.84	1.88	4.89	0.00	1.60	10.00
time (sec)	N/A	0.230	0.611	0.545	0.520	2.969	0.000	1.185	16.390

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	164	172	354	1106	0	282	2500
N.S.	1	1.00	0.85	0.89	1.83	5.73	0.00	1.46	12.95
time (sec)	N/A	0.172	1.671	0.461	0.513	2.755	0.000	1.124	16.487

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	142	233	766	8964	213	2500
N.S.	1	1.00	0.92	0.95	1.55	5.11	59.76	1.42	16.67
time (sec)	N/A	0.114	1.342	0.325	0.506	3.401	78.799	0.688	14.931

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	144	83	111	583	0	109	102
N.S.	1	1.00	1.29	0.74	0.99	5.21	0.00	0.97	0.91
time (sec)	N/A	0.059	0.685	0.268	0.496	4.359	0.000	1.057	11.221

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	146	123	165	891	0	175	147
N.S.	1	1.00	0.95	0.80	1.07	5.79	0.00	1.14	0.95
time (sec)	N/A	0.142	1.209	0.376	0.506	5.954	0.000	1.034	12.283

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	346	175	220	1239	0	263	199
N.S.	1	1.00	1.50	0.76	0.95	5.36	0.00	1.14	0.86
time (sec)	N/A	0.206	1.123	0.401	0.514	7.486	0.000	1.951	13.310

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	208	7044	215	396	0	2695	-1
N.S.	1	1.00	1.29	43.75	1.34	2.46	0.00	16.74	-0.01
time (sec)	N/A	0.110	2.233	1.393	0.503	3.385	0.000	1.403	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	170	4296	140	293	0	498	-1
N.S.	1	1.00	1.50	38.02	1.24	2.59	0.00	4.41	-0.01
time (sec)	N/A	0.067	0.735	0.349	0.518	6.200	0.000	0.924	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	140	144	103	217	0	142	-1
N.S.	1	1.00	1.94	2.00	1.43	3.01	0.00	1.97	-0.01
time (sec)	N/A	0.039	0.426	0.135	0.533	2.715	0.000	0.871	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	300	721	0	546	0	0	-1
N.S.	1	1.00	3.57	8.58	0.00	6.50	0.00	0.00	-0.01
time (sec)	N/A	0.058	4.676	0.382	0.000	2.571	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	586	2075	0	905	0	0	-1
N.S.	1	1.00	4.61	16.34	0.00	7.13	0.00	0.00	-0.01
time (sec)	N/A	0.100	2.532	0.556	0.000	5.378	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	1049	5378	0	1345	0	0	-1
N.S.	1	1.00	5.61	28.76	0.00	7.19	0.00	0.00	-0.01
time (sec)	N/A	0.153	6.464	0.310	0.000	7.021	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	330	2498	0	2154	0	0	-1
N.S.	1	1.00	1.75	13.22	0.00	11.40	0.00	0.00	-0.01
time (sec)	N/A	0.160	2.558	0.648	0.000	16.916	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	273	1342	0	1929	0	0	-1
N.S.	1	1.00	2.13	10.48	0.00	15.07	0.00	0.00	-0.01
time (sec)	N/A	0.088	3.990	0.289	0.000	2.892	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	108	167	0	432	0	0	-1
N.S.	1	1.00	1.27	1.96	0.00	5.08	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.493	0.076	0.000	1.250	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	156	1213	50	355	0	0	-1
N.S.	1	1.00	2.36	18.38	0.76	5.38	0.00	0.00	-0.02
time (sec)	N/A	0.052	2.175	0.586	0.286	1.346	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	204	2441	81	465	0	0	-1
N.S.	1	1.00	2.04	24.41	0.81	4.65	0.00	0.00	-0.01
time (sec)	N/A	0.062	4.552	0.471	0.293	3.187	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	287	3769	140	623	0	0	-1
N.S.	1	1.00	2.04	26.73	0.99	4.42	0.00	0.00	-0.01
time (sec)	N/A	0.090	3.574	0.372	0.289	2.974	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	233	2399	350	448	0	0	-1
N.S.	1	1.00	1.03	10.57	1.54	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.146	5.805	0.608	0.517	2.652	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	188	1104	314	327	0	326	-1
N.S.	1	1.00	1.01	5.94	1.69	1.76	0.00	1.75	-0.01
time (sec)	N/A	0.105	2.006	0.284	0.520	3.194	0.000	1.835	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	170	359	186	286	0	277	-1
N.S.	1	1.00	1.50	3.18	1.65	2.53	0.00	2.45	-0.01
time (sec)	N/A	0.054	1.430	0.111	0.519	4.685	0.000	1.419	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	492	1248	0	799	0	0	-1
N.S.	1	1.00	3.87	9.83	0.00	6.29	0.00	0.00	-0.01
time (sec)	N/A	0.098	5.231	0.265	0.000	4.475	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	1012	2904	0	1062	0	0	-1
N.S.	1	1.00	6.06	17.39	0.00	6.36	0.00	0.00	-0.01
time (sec)	N/A	0.136	6.639	0.293	0.000	4.623	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	415	6194	0	1449	0	0	-1
N.S.	1	1.00	1.86	27.78	0.00	6.50	0.00	0.00	-0.00
time (sec)	N/A	0.234	4.736	0.329	0.000	4.116	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	278	2630	0	2261	0	0	-1
N.S.	1	1.00	1.25	11.85	0.00	10.18	0.00	0.00	-0.00
time (sec)	N/A	0.220	5.209	0.643	0.000	160.745	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	324	2261	0	2025	0	0	-1
N.S.	1	1.00	1.96	13.70	0.00	12.27	0.00	0.00	-0.01
time (sec)	N/A	0.137	5.740	0.354	0.000	27.080	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	142	327	0	567	0	0	-1
N.S.	1	1.00	1.14	2.62	0.00	4.54	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.880	0.064	0.000	4.521	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	220	1355	78	415	0	0	-1
N.S.	1	1.00	2.20	13.55	0.78	4.15	0.00	0.00	-0.01
time (sec)	N/A	0.065	2.822	0.304	0.291	1.884	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	177	4594	187	531	0	0	-1
N.S.	1	1.00	1.09	28.36	1.15	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.094	1.965	0.398	0.285	2.738	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	213	6988	216	695	0	0	-1
N.S.	1	1.00	1.09	35.65	1.10	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.117	2.230	0.480	0.300	5.953	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	112	169	226	129	0	1376	-1
N.S.	1	1.00	0.78	1.17	1.57	0.90	0.00	9.56	-0.01
time (sec)	N/A	0.098	42.118	0.418	0.307	2.250	0.000	1.340	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	104	112	79	0	205	-1
N.S.	1	1.00	0.84	1.18	1.27	0.90	0.00	2.33	-0.01
time (sec)	N/A	0.065	1.538	0.293	0.358	2.932	0.000	1.201	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	52	78	37	48	0	88	-1
N.S.	1	1.00	1.41	2.11	1.00	1.30	0.00	2.38	-0.03
time (sec)	N/A	0.032	0.615	0.219	0.296	2.528	0.000	1.120	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	221	351	0	142	0	0	-1
N.S.	1	1.00	5.26	8.36	0.00	3.38	0.00	0.00	-0.02
time (sec)	N/A	0.047	2.397	0.323	0.000	5.024	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	367	2801	0	302	0	0	-1
N.S.	1	1.00	4.03	30.78	0.00	3.32	0.00	0.00	-0.01
time (sec)	N/A	0.082	3.194	0.379	0.000	2.388	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	273	6334	0	461	0	906	-1
N.S.	1	1.00	1.91	44.29	0.00	3.22	0.00	6.34	-0.01
time (sec)	N/A	0.107	4.267	0.344	0.000	4.561	0.000	1.178	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	314	1169	0	817	0	0	-1
N.S.	1	1.00	2.15	8.01	0.00	5.60	0.00	0.00	-0.01
time (sec)	N/A	0.115	4.365	0.399	0.000	5.696	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	270	795	0	723	0	0	-1
N.S.	1	1.00	2.90	8.55	0.00	7.77	0.00	0.00	-0.01
time (sec)	N/A	0.071	3.344	0.296	0.000	2.745	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	67	0	131	0	0	40
N.S.	1	1.00	1.00	1.46	0.00	2.85	0.00	0.00	0.87
time (sec)	N/A	0.023	0.089	0.083	0.000	6.726	0.000	0.000	12.364

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	49	57	32	53	0	0	36
N.S.	1	1.00	1.63	1.90	1.07	1.77	0.00	0.00	1.20
time (sec)	N/A	0.047	0.280	0.281	0.277	7.864	0.000	0.000	12.858

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	86	93	96	0	0	145
N.S.	1	1.00	0.92	1.16	1.26	1.30	0.00	0.00	1.96
time (sec)	N/A	0.062	0.454	0.305	0.274	5.265	0.000	0.000	18.816

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	90	148	185	149	0	0	761
N.S.	1	1.00	0.73	1.20	1.50	1.21	0.00	0.00	6.19
time (sec)	N/A	0.092	1.967	0.349	0.293	10.248	0.000	0.000	22.108

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	186	67748	407	240	0	3432	-1
N.S.	1	1.00	0.93	340.44	2.05	1.21	0.00	17.25	-0.01
time (sec)	N/A	0.128	1.995	5.385	0.376	3.227	0.000	2.321	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	106	14991	226	164	0	1181	-1
N.S.	1	1.00	0.81	114.44	1.73	1.25	0.00	9.02	-0.01
time (sec)	N/A	0.089	1.207	1.444	0.289	4.045	0.000	1.774	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	72	103	87	109	0	178	-1
N.S.	1	1.00	0.95	1.36	1.14	1.43	0.00	2.34	-0.01
time (sec)	N/A	0.040	1.689	0.115	0.295	5.510	0.000	1.601	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	1007	3491	0	373	0	0	-1
N.S.	1	1.00	11.99	41.56	0.00	4.44	0.00	0.00	-0.01
time (sec)	N/A	0.065	6.740	0.364	0.000	3.657	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	304	5633	0	479	0	0	-1
N.S.	1	1.00	2.39	44.35	0.00	3.77	0.00	0.00	-0.01
time (sec)	N/A	0.105	3.652	0.384	0.000	2.799	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	345	10582	0	735	0	1318	-1
N.S.	1	1.00	1.84	56.59	0.00	3.93	0.00	7.05	-0.01
time (sec)	N/A	0.159	4.858	0.339	0.000	1.715	0.000	1.576	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	325	6028	0	1081	0	0	-1
N.S.	1	1.00	1.74	32.24	0.00	5.78	0.00	0.00	-0.01
time (sec)	N/A	0.154	3.588	2.846	0.000	101.424	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	282	1615	0	941	0	0	-1
N.S.	1	1.00	2.10	12.05	0.00	7.02	0.00	0.00	-0.01
time (sec)	N/A	0.108	2.890	0.404	0.000	6.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	214	102	0	324	0	0	-1
N.S.	1	1.00	2.52	1.20	0.00	3.81	0.00	0.00	-0.01
time (sec)	N/A	0.038	7.329	0.066	0.000	0.857	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	74	109	62	96	0	0	2978
N.S.	1	1.00	1.19	1.76	1.00	1.55	0.00	0.00	48.03
time (sec)	N/A	0.058	0.873	0.395	0.279	1.118	0.000	0.000	18.300

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	119	170	151	163	0	0	2500
N.S.	1	1.00	1.04	1.49	1.32	1.43	0.00	0.00	21.93
time (sec)	N/A	0.077	0.997	0.327	0.301	4.042	0.000	0.000	35.944

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	135	264	273	242	0	0	-1
N.S.	1	1.00	0.79	1.54	1.60	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.499	0.418	0.295	86.677	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	294	391	558	379	0	5506	-1
N.S.	1	1.00	1.19	1.58	2.25	1.53	0.00	22.20	-0.00
time (sec)	N/A	0.162	2.422	1.950	0.303	5.591	0.000	4.375	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	205	262	322	278	0	1511	-1
N.S.	1	1.00	1.22	1.56	1.92	1.65	0.00	8.99	-0.01
time (sec)	N/A	0.108	1.557	0.808	0.290	6.221	0.000	2.812	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	124	147	141	209	0	223	-1
N.S.	1	1.00	1.05	1.25	1.19	1.77	0.00	1.89	-0.01
time (sec)	N/A	0.052	1.307	0.117	0.280	2.674	0.000	2.558	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	300	27448	0	720	0	0	-1
N.S.	1	1.00	2.21	201.82	0.00	5.29	0.00	0.00	-0.01
time (sec)	N/A	0.102	4.600	3.794	0.000	3.044	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	380	38486	0	919	0	0	-1
N.S.	1	1.00	2.15	217.44	0.00	5.19	0.00	0.00	-0.01
time (sec)	N/A	0.142	2.693	5.309	0.000	4.203	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F	B	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	1132	49917	0	1073	0	1728	-1
N.S.	1	1.00	4.78	210.62	0.00	4.53	0.00	7.29	-0.00
time (sec)	N/A	0.217	6.604	6.609	0.000	4.097	0.000	2.197	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	378	7943	0	0	0	0	-1
N.S.	1	1.00	1.54	32.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	3.749	2.512	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	309	2511	0	0	0	0	-1
N.S.	1	1.00	1.71	13.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	2.916	0.378	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	163	0	581	0	0	-1
N.S.	1	1.00	9.93	1.22	0.00	4.34	0.00	0.00	-0.01
time (sec)	N/A	0.077	8.172	0.072	0.000	5.091	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	133	153	91	164	0	0	324
N.S.	1	1.00	1.37	1.58	0.94	1.69	0.00	0.00	3.34
time (sec)	N/A	0.065	0.702	0.429	0.287	13.553	0.000	0.000	27.443

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	140	245	209	250	0	0	-1
N.S.	1	1.00	0.96	1.68	1.43	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.782	0.552	0.287	95.735	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	174	371	361	0	0	0	-1
N.S.	1	1.00	0.79	1.69	1.65	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.152	1.540	0.367	0.295	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	292	0	0	0	0	0	-1
N.S.	1	1.00	3.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	1.368	0.217	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	275	0	0	0	0	0	-1
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	1.614	0.273	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	283	0	0	0	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.152	5.016	0.803	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	184	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	2.703	0.700	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	80	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.641	0.009	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	1215	0	0	0	0	0	-1
N.S.	1	1.00	13.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	14.072	0.211	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	252	0	0	0	0	0	-1
N.S.	1	1.00	2.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	15.054	0.218	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	3698	0	0	0	0	0	-1
N.S.	1	1.00	44.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	15.667	0.731	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	192	0	0	0	0	0	-1
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.350	0.011	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.432	0.213	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.937	0.220	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	176	141	0	0	0	0	0	-1
N.S.	1	0.98	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.711	0.230	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	295	0	0	0	0	0	-1
N.S.	1	1.00	3.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.316	0.223	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	517	0	0	0	0	0	-1
N.S.	1	1.00	8.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.075	1.672	0.642	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.039	0.028	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	10285	39	55	0	0	-1
N.S.	1	1.00	0.94	311.67	1.18	1.67	0.00	0.00	-0.03
time (sec)	N/A	0.063	0.040	6.617	0.306	1.441	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	34276	77	110	0	0	-1
N.S.	1	1.00	0.86	496.75	1.12	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.112	1.801	0.309	1.061	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	89	70269	114	188	0	0	-1
N.S.	1	1.00	0.86	675.66	1.10	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.197	3.079	0.321	0.761	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	506	0	0	0	0	0	-1
N.S.	1	1.00	5.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	1.817	0.573	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	284	0	0	0	0	0	-1
N.S.	1	1.00	3.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.898	0.013	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.156	0.176	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	1399	0	0	0	0	0	-1
N.S.	1	1.00	15.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	14.764	0.199	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	1.733	0.268	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.363	0.251	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	2033	0	0	0	0	0	-1
N.S.	1	1.00	18.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	15.125	0.273	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.343	0.228	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.086	1.747	0.288	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	43	157	56	68	519	53
N.S.	1	1.00	0.71	0.66	2.42	0.86	1.05	7.98	0.82
time (sec)	N/A	0.024	0.133	0.043	0.493	4.694	0.204	1.200	11.525

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	38	35	102	43	54	297	36
N.S.	1	1.00	0.76	0.70	2.04	0.86	1.08	5.94	0.72
time (sec)	N/A	0.021	0.072	0.029	0.494	2.742	0.150	0.732	11.468

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	59	29	37	133	24
N.S.	1	1.00	0.81	0.78	1.84	0.91	1.16	4.16	0.75
time (sec)	N/A	0.019	0.030	0.023	0.491	5.562	0.090	0.613	11.566

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	38	36	40	87	37	26
N.S.	1	1.00	0.84	1.23	1.16	1.29	2.81	1.19	0.84
time (sec)	N/A	0.018	0.019	0.061	0.493	4.085	0.283	0.556	11.830

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	58	67	84	248	51	35
N.S.	1	1.00	0.65	1.05	1.22	1.53	4.51	0.93	0.64
time (sec)	N/A	0.026	0.032	0.068	0.501	3.783	0.423	0.619	11.896

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	46	78	90	120	454	61	51
N.S.	1	1.00	0.58	0.99	1.14	1.52	5.75	0.77	0.65
time (sec)	N/A	0.037	0.029	0.076	0.494	3.015	0.558	0.646	11.932

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	79	105	71	116	1719	68
N.S.	1	1.00	0.85	1.07	1.42	0.96	1.57	23.23	0.92
time (sec)	N/A	0.038	0.181	0.083	0.282	1.428	0.178	3.706	11.712

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	65	57	74	54	88	1071	57
N.S.	1	1.00	1.23	1.08	1.40	1.02	1.66	20.21	1.08
time (sec)	N/A	0.030	0.117	0.047	0.292	1.217	0.117	1.448	11.704

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	40	35	39	38	60	500	37
N.S.	1	1.00	1.18	1.03	1.15	1.12	1.76	14.71	1.09
time (sec)	N/A	0.016	0.053	0.032	0.302	1.560	0.082	0.733	11.747

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	34	25	33	49	58	34	36
N.S.	1	1.00	1.31	0.96	1.27	1.88	2.23	1.31	1.38
time (sec)	N/A	0.020	0.031	0.133	0.271	3.055	0.209	0.648	11.658

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	56	37	33	66	100	165	54
N.S.	1	1.00	1.65	1.09	0.97	1.94	2.94	4.85	1.59
time (sec)	N/A	0.023	0.114	0.126	0.275	2.410	0.736	0.784	11.618

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	58	55	91	124	265	74
N.S.	1	1.00	1.06	1.09	1.04	1.72	2.34	5.00	1.40
time (sec)	N/A	0.032	0.158	0.132	0.281	3.158	1.804	0.956	11.649

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	129	91	77	73	109	1087	70
N.S.	1	1.00	1.61	1.14	0.96	0.91	1.36	13.59	0.88
time (sec)	N/A	0.040	0.038	0.058	0.494	2.917	0.211	3.158	11.566

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	97	69	61	57	82	633	53
N.S.	1	1.00	1.62	1.15	1.02	0.95	1.37	10.55	0.88
time (sec)	N/A	0.033	0.025	0.040	0.487	2.802	0.152	1.380	11.635

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	65	47	44	40	54	289	37
N.S.	1	1.00	1.62	1.18	1.10	1.00	1.35	7.22	0.92
time (sec)	N/A	0.029	0.021	0.029	0.485	1.935	0.095	0.737	11.540

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	29	25	22	20	252	21
N.S.	1	1.00	1.47	1.53	1.32	1.16	1.05	13.26	1.11
time (sec)	N/A	0.010	0.006	0.000	0.565	1.953	0.056	0.548	11.435

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	34	31	29	31	46	46	21
N.S.	1	1.00	1.62	1.48	1.38	1.48	2.19	2.19	1.00
time (sec)	N/A	0.019	0.012	0.089	0.509	1.579	0.429	0.671	11.470

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	65	47	49	52	70	106	40
N.S.	1	1.00	1.67	1.21	1.26	1.33	1.79	2.72	1.03
time (sec)	N/A	0.027	0.031	0.114	0.497	1.299	0.954	0.781	11.735

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	69	67	65	68	97	168	57
N.S.	1	1.00	1.13	1.10	1.07	1.11	1.59	2.75	0.93
time (sec)	N/A	0.034	0.036	0.130	0.490	1.637	2.739	1.028	11.937

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	143	170	112	206	3813	113
N.S.	1	1.00	0.85	1.36	1.62	1.07	1.96	36.31	1.08
time (sec)	N/A	0.080	0.235	0.105	0.286	0.966	0.273	10.395	11.587

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	110	133	90	160	2600	97
N.S.	1	1.00	0.88	1.34	1.62	1.10	1.95	31.71	1.18
time (sec)	N/A	0.065	0.194	0.072	0.270	0.905	0.192	3.777	11.520

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	67	86	67	112	1510	68
N.S.	1	1.00	0.87	1.08	1.39	1.08	1.81	24.35	1.10
time (sec)	N/A	0.042	0.151	0.049	0.280	0.711	0.124	1.619	11.893

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	65	50	62	73	97	91	62
N.S.	1	1.00	1.27	0.98	1.22	1.43	1.90	1.78	1.22
time (sec)	N/A	0.042	0.091	0.149	0.269	0.666	0.600	0.899	11.952

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	53	54	100	131	167	68
N.S.	1	1.00	0.91	0.95	0.96	1.79	2.34	2.98	1.21
time (sec)	N/A	0.056	0.174	0.161	0.276	0.981	1.804	1.146	12.094

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	73	64	105	172	315	91
N.S.	1	1.00	0.80	0.96	0.84	1.38	2.26	4.14	1.20
time (sec)	N/A	0.059	0.204	0.154	0.273	2.171	5.313	1.565	12.176

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	243	173	124	120	212	2619	155
N.S.	1	1.00	2.15	1.53	1.10	1.06	1.88	23.18	1.37
time (sec)	N/A	0.060	0.062	0.085	0.479	2.565	0.345	7.269	12.093

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	190	133	102	98	165	1684	127
N.S.	1	1.00	2.09	1.46	1.12	1.08	1.81	18.51	1.40
time (sec)	N/A	0.060	0.045	0.063	0.492	3.103	0.221	2.446	12.130

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	137	97	80	76	117	937	100
N.S.	1	1.00	1.99	1.41	1.16	1.10	1.70	13.58	1.45
time (sec)	N/A	0.050	0.043	0.046	0.492	2.372	0.153	1.270	11.674

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	73	59	63	53	68	382	76
N.S.	1	1.00	1.59	1.28	1.37	1.15	1.48	8.30	1.65
time (sec)	N/A	0.023	0.380	0.000	0.490	4.126	0.099	0.638	12.019

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	66	53	49	53	73	49	70
N.S.	1	1.00	1.74	1.39	1.29	1.39	1.92	1.29	1.84
time (sec)	N/A	0.043	0.079	0.113	0.476	3.423	0.979	0.985	12.020

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	71	60	60	63	90	122	58
N.S.	1	1.00	1.61	1.36	1.36	1.43	2.05	2.77	1.32
time (sec)	N/A	0.049	0.851	0.130	0.501	5.886	2.695	1.264	11.607

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	104	91	82	85	134	222	76
N.S.	1	1.00	1.53	1.34	1.21	1.25	1.97	3.26	1.12
time (sec)	N/A	0.053	0.075	0.150	0.481	4.203	6.548	1.762	11.518

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	67	79	96	338	341	74
N.S.	1	1.00	0.90	0.94	1.11	1.35	4.76	4.80	1.04
time (sec)	N/A	0.067	0.112	0.108	0.270	2.511	8.996	1.594	11.856

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	52	55	68	230	189	54
N.S.	1	1.00	0.82	1.04	1.10	1.36	4.60	3.78	1.08
time (sec)	N/A	0.059	0.023	0.089	0.275	2.345	1.934	0.841	11.779

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	48	31	40	133	122	66
N.S.	1	1.00	1.03	1.33	0.86	1.11	3.69	3.39	1.83
time (sec)	N/A	0.046	0.029	0.061	0.275	4.850	1.135	0.647	11.852

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	71	51	76	388	59	68
N.S.	1	1.00	0.89	1.11	0.80	1.19	6.06	0.92	1.06
time (sec)	N/A	0.062	0.040	0.212	0.268	4.728	4.211	0.716	11.712

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	63	117	71	136	733	263	89
N.S.	1	1.00	0.71	1.31	0.80	1.53	8.24	2.96	1.00
time (sec)	N/A	0.081	0.183	0.240	0.270	5.225	16.261	0.911	11.601

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	83	167	100	172	0	408	118
N.S.	1	1.00	0.72	1.45	0.87	1.50	0.00	3.55	1.03
time (sec)	N/A	0.099	0.243	0.257	0.276	3.357	0.000	1.166	11.782

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	92	88	87	290	595	118	1310
N.S.	1	1.00	1.08	1.04	1.02	3.41	7.00	1.39	15.41
time (sec)	N/A	0.127	0.570	0.187	0.490	4.947	19.885	2.120	11.997

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	70	65	68	230	427	86	1212
N.S.	1	1.00	1.11	1.03	1.08	3.65	6.78	1.37	19.24
time (sec)	N/A	0.072	0.204	0.167	0.497	3.315	3.714	1.066	12.183

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	50	49	189	252	67	135
N.S.	1	1.00	0.98	1.00	0.98	3.78	5.04	1.34	2.70
time (sec)	N/A	0.052	0.019	0.139	0.496	2.597	1.242	0.746	11.568

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	50	50	190	240	68	948
N.S.	1	1.00	0.98	1.00	1.00	3.80	4.80	1.36	18.96
time (sec)	N/A	0.051	0.035	0.000	0.489	2.780	1.225	0.553	12.297

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	68	68	68	257	522	86	438
N.S.	1	1.00	1.06	1.06	1.06	4.02	8.16	1.34	6.84
time (sec)	N/A	0.074	0.181	0.274	0.511	1.883	8.013	0.829	11.774

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	92	88	90	324	775	118	484
N.S.	1	1.00	1.10	1.05	1.07	3.86	9.23	1.40	5.76
time (sec)	N/A	0.115	0.476	0.313	0.509	2.790	87.965	1.002	12.200

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	121	111	117	370	0	164	524
N.S.	1	1.00	1.07	0.98	1.04	3.27	0.00	1.45	4.64
time (sec)	N/A	0.165	1.278	0.344	0.491	4.313	0.000	1.128	13.742

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	73	83	131	193	1542	399	90
N.S.	1	1.00	0.81	0.92	1.46	2.14	17.13	4.43	1.00
time (sec)	N/A	0.085	0.492	0.145	0.279	2.719	32.771	1.745	11.592

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	74	90	103	910	295	270
N.S.	1	1.00	0.88	1.07	1.30	1.49	13.19	4.28	3.91
time (sec)	N/A	0.067	0.387	0.099	0.280	3.354	14.365	0.945	11.601

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	78	90	103	796	307	195
N.S.	1	1.00	0.88	1.20	1.38	1.58	12.25	4.72	3.00
time (sec)	N/A	0.048	0.438	0.103	0.280	2.016	14.270	0.734	11.641

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	90	115	127	205	2341	152	104
N.S.	1	1.00	0.87	1.12	1.23	1.99	22.73	1.48	1.01
time (sec)	N/A	0.084	1.420	0.296	0.292	1.191	104.087	1.005	11.733

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	98	161	192	304	3526	684	144
N.S.	1	1.00	0.74	1.22	1.45	2.30	26.71	5.18	1.09
time (sec)	N/A	0.109	0.584	0.330	0.289	1.481	129.011	1.110	11.883

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	121	217	242	360	0	685	191
N.S.	1	1.00	0.75	1.35	1.50	2.24	0.00	4.25	1.19
time (sec)	N/A	0.130	0.713	0.371	0.304	1.298	0.000	1.114	12.369

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	118	104	140	492	2859	149	2581
N.S.	1	1.00	0.91	0.80	1.08	3.78	21.99	1.15	19.85
time (sec)	N/A	0.132	0.874	0.266	0.495	1.575	43.524	2.212	13.010

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	90	118	397	2157	127	2358
N.S.	1	1.00	0.99	0.95	1.24	4.18	22.71	1.34	24.82
time (sec)	N/A	0.082	0.550	0.224	0.507	1.867	15.291	1.151	13.519

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	83	101	409	2113	112	2136
N.S.	1	1.00	0.97	0.92	1.12	4.54	23.48	1.24	23.73
time (sec)	N/A	0.071	0.394	0.214	0.489	2.544	15.319	0.853	13.035

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	93	118	406	2125	127	2489
N.S.	1	1.00	0.91	0.96	1.22	4.19	21.91	1.31	25.66
time (sec)	N/A	0.054	0.714	0.000	0.502	2.956	14.926	0.593	13.523

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	117	106	156	525	3245	171	2674
N.S.	1	1.00	0.91	0.83	1.22	4.10	25.35	1.34	20.89
time (sec)	N/A	0.136	2.137	0.383	0.497	3.616	141.132	1.146	14.234

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	137	126	199	620	0	179	2000
N.S.	1	1.00	0.81	0.75	1.18	3.67	0.00	1.06	11.83
time (sec)	N/A	0.188	2.408	0.419	0.500	4.789	0.000	0.974	15.751

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	165	152	246	698	0	225	2500
N.S.	1	1.00	0.76	0.70	1.13	3.20	0.00	1.03	11.47
time (sec)	N/A	0.225	4.705	0.467	0.510	2.171	0.000	1.118	16.004

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	117	193	214	3315	469	577
N.S.	1	1.00	0.90	1.08	1.79	1.98	30.69	4.34	5.34
time (sec)	N/A	0.106	0.772	0.168	0.279	3.035	73.268	2.036	12.518

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	87	101	198	220	2819	506	532
N.S.	1	1.00	0.90	1.04	2.04	2.27	29.06	5.22	5.48
time (sec)	N/A	0.081	0.524	0.167	0.283	1.662	74.582	1.282	12.515

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	108	196	214	2846	650	375
N.S.	1	1.00	0.88	1.16	2.11	2.30	30.60	6.99	4.03
time (sec)	N/A	0.062	0.507	0.171	0.288	1.688	75.063	1.046	12.462

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	126	168	255	434	0	267	181
N.S.	1	1.00	0.85	1.14	1.72	2.93	0.00	1.80	1.22
time (sec)	N/A	0.115	1.213	0.388	0.298	1.471	0.000	1.294	12.593

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	144	217	352	561	0	906	229
N.S.	1	1.00	0.80	1.20	1.94	3.10	0.00	5.01	1.27
time (sec)	N/A	0.149	1.291	0.421	0.314	1.995	0.000	1.383	13.465

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	178	272	424	628	0	1497	269
N.S.	1	1.00	0.85	1.30	2.02	2.99	0.00	7.13	1.28
time (sec)	N/A	0.173	1.760	0.460	0.299	4.444	0.000	1.364	13.514

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	142	142	235	767	8974	215	2500
N.S.	1	1.00	0.93	0.93	1.54	5.01	58.65	1.41	16.34
time (sec)	N/A	0.158	1.592	0.353	0.503	3.722	77.531	2.495	15.523

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	136	132	218	773	8957	199	2500
N.S.	1	1.00	0.94	0.91	1.50	5.33	61.77	1.37	17.24
time (sec)	N/A	0.122	1.388	0.340	0.516	4.026	75.290	1.457	15.486

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	139	137	219	783	9051	200	2500
N.S.	1	1.00	0.97	0.95	1.52	5.44	62.85	1.39	17.36
time (sec)	N/A	0.109	1.405	0.332	0.496	3.711	76.243	1.123	15.488

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	142	233	766	8964	213	2500
N.S.	1	1.00	0.92	0.95	1.55	5.11	59.76	1.42	16.67
time (sec)	N/A	0.107	1.324	0.085	0.501	4.655	75.983	0.648	15.345

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	174	153	282	911	0	231	915
N.S.	1	1.00	0.92	0.81	1.49	4.82	0.00	1.22	4.84
time (sec)	N/A	0.205	1.550	0.516	0.509	2.798	0.000	1.232	15.047

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	184	173	340	1038	0	261	986
N.S.	1	1.00	0.77	0.72	1.42	4.32	0.00	1.09	4.11
time (sec)	N/A	0.250	3.323	0.545	0.501	2.958	0.000	1.260	15.359

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	949	199	405	1148	0	307	2500
N.S.	1	1.00	3.20	0.67	1.36	3.87	0.00	1.03	8.42
time (sec)	N/A	0.318	6.250	0.595	0.503	4.391	0.000	1.456	16.126

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	137	173	162	134	209	2209	164
N.S.	1	1.00	1.19	1.50	1.41	1.17	1.82	19.21	1.43
time (sec)	N/A	0.049	1.249	0.073	0.486	4.243	0.231	2.268	11.384

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	102	108	104	90	126	1027	115
N.S.	1	1.00	1.32	1.40	1.35	1.17	1.64	13.34	1.49
time (sec)	N/A	0.034	0.624	0.046	0.512	3.602	0.154	0.924	11.475

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	73	59	58	51	68	359	76
N.S.	1	1.00	1.59	1.28	1.26	1.11	1.48	7.80	1.65
time (sec)	N/A	0.022	0.384	0.029	0.491	1.754	0.101	0.647	11.420

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	29	23	21	20	231	21
N.S.	1	1.00	1.47	1.53	1.21	1.11	1.05	12.16	1.11
time (sec)	N/A	0.009	0.006	0.019	0.504	3.341	0.054	0.533	11.438

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	50	48	182	240	65	948
N.S.	1	1.00	0.98	1.00	0.96	3.64	4.80	1.30	18.96
time (sec)	N/A	0.048	0.054	0.155	0.563	3.659	1.249	0.587	11.904

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	93	114	390	2125	122	2489
N.S.	1	1.00	0.91	0.96	1.18	4.02	21.91	1.26	25.66
time (sec)	N/A	0.070	0.705	0.230	0.510	2.311	10.581	0.583	13.150

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	142	227	742	8964	205	2500
N.S.	1	1.00	0.92	0.95	1.51	4.95	59.76	1.37	16.67
time (sec)	N/A	0.103	1.321	0.357	0.521	2.890	51.355	0.659	13.979

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	32	56	860	56	0	48	-1
N.S.	1	1.00	0.59	1.04	15.93	1.04	0.00	0.89	-0.02
time (sec)	N/A	0.069	0.055	0.120	0.854	4.790	0.000	0.440	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	29	276	18	0	29	19
N.S.	1	1.00	0.67	0.97	9.20	0.60	0.00	0.97	0.63
time (sec)	N/A	0.056	0.018	0.059	0.544	1.774	0.000	0.420	11.643

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	39	295	47	0	40	-1
N.S.	1	1.00	0.67	1.08	8.19	1.31	0.00	1.11	-0.03
time (sec)	N/A	0.060	0.035	0.062	0.559	2.796	0.000	0.474	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	10	10	10	10
N.S.	1	1.00	1.00	1.10	0.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.029	0.007	0.040	0.000	4.000	0.319	0.429	11.626

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	23	38	63	0	24	12
N.S.	1	1.00	1.25	0.96	1.58	2.62	0.00	1.00	0.50
time (sec)	N/A	0.050	0.015	0.206	0.556	3.208	0.000	0.430	0.177

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	17	16	0	32	25
N.S.	1	1.00	1.00	1.21	1.21	1.14	0.00	2.29	1.79
time (sec)	N/A	0.055	0.011	0.128	0.509	2.546	0.000	0.480	11.979

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	38	51	303	58	0	42	37
N.S.	1	1.00	0.84	1.13	6.73	1.29	0.00	0.93	0.82
time (sec)	N/A	0.060	0.057	0.162	0.559	2.402	0.000	0.426	12.037

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	22	25	29	24	0	59	40
N.S.	1	1.00	0.65	0.74	0.85	0.71	0.00	1.74	1.18
time (sec)	N/A	0.059	0.016	0.138	0.528	2.307	0.000	0.434	11.846

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	34	65	90	0	66	41
N.S.	1	1.00	0.86	0.94	1.81	2.50	0.00	1.83	1.14
time (sec)	N/A	0.025	0.022	0.154	0.555	4.398	0.000	0.622	11.859

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	29	559	29	0	72	19
N.S.	1	1.00	0.69	0.91	17.47	0.91	0.00	2.25	0.59
time (sec)	N/A	0.070	0.032	0.064	0.579	2.416	0.000	0.424	12.014

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	34	57	934	57	0	49	-1
N.S.	1	1.00	0.58	0.97	15.83	0.97	0.00	0.83	-0.02
time (sec)	N/A	0.078	0.047	0.064	0.865	2.025	0.000	0.447	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	12	12	12	11
N.S.	1	1.00	1.00	0.93	0.00	0.86	0.86	0.86	0.79
time (sec)	N/A	0.033	0.011	0.037	0.000	2.136	1.080	0.440	0.176

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	32	134	49	0	42	33
N.S.	1	1.00	0.92	0.86	3.62	1.32	0.00	1.14	0.89
time (sec)	N/A	0.056	0.025	0.133	0.550	2.531	0.000	0.413	11.666

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	54	134	53	0	62	-1
N.S.	1	1.00	0.82	1.64	4.06	1.61	0.00	1.88	-0.03
time (sec)	N/A	0.070	0.019	0.172	0.536	3.392	0.000	0.449	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	43	60	556	72	0	2125	-1
N.S.	1	1.00	0.63	0.88	8.18	1.06	0.00	31.25	-0.01
time (sec)	N/A	0.027	0.047	0.100	0.533	3.823	0.000	1.903	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	65	86	1769	91	0	7084	-1
N.S.	1	1.00	0.66	0.88	18.05	0.93	0.00	72.29	-0.01
time (sec)	N/A	0.034	0.147	0.108	0.848	2.671	0.000	3.631	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	17	26	37	17	0	27	22
N.S.	1	1.00	0.68	1.04	1.48	0.68	0.00	1.08	0.88
time (sec)	N/A	0.060	0.018	0.056	0.313	2.418	0.000	0.443	0.288

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	38	42	64	0	40	-1
N.S.	1	1.00	1.58	1.23	1.35	2.06	0.00	1.29	-0.03
time (sec)	N/A	0.064	0.027	0.062	0.551	3.908	0.000	0.466	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	12	14	12	11
N.S.	1	1.00	1.00	1.08	0.00	1.00	1.17	1.00	0.92
time (sec)	N/A	0.031	0.009	0.040	0.000	2.617	0.333	0.401	11.800

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	42	66	0	34	31
N.S.	1	1.00	0.91	0.83	1.20	1.89	0.00	0.97	0.89
time (sec)	N/A	0.053	0.023	0.143	0.555	2.635	0.000	0.425	0.140

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	24	128	33	0	47	40
N.S.	1	1.00	0.71	0.77	4.13	1.06	0.00	1.52	1.29
time (sec)	N/A	0.065	0.020	0.139	0.553	3.230	0.000	0.450	11.818

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	23	29	38	43	36	26	29
N.S.	1	1.00	0.77	0.97	1.27	1.43	1.20	0.87	0.97
time (sec)	N/A	0.065	0.018	0.052	0.297	1.973	1.965	0.411	11.755

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	56	14	39	0	16	28
N.S.	1	1.00	0.78	2.43	0.61	1.70	0.00	0.70	1.22
time (sec)	N/A	0.067	0.012	0.047	0.553	3.951	0.000	0.443	11.650

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	35	15	12	23
N.S.	1	1.00	1.00	0.93	0.00	2.50	1.07	0.86	1.64
time (sec)	N/A	0.033	0.008	0.036	0.000	1.462	1.550	0.407	0.147

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	38	48	94	0	53	46
N.S.	1	1.00	0.89	0.72	0.91	1.77	0.00	1.00	0.87
time (sec)	N/A	0.063	0.039	0.152	0.549	3.389	0.000	0.427	11.686

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	31	31	225	52	0	55	-1
N.S.	1	1.00	0.52	0.52	3.75	0.87	0.00	0.92	-0.02
time (sec)	N/A	0.082	0.039	0.136	0.559	1.473	0.000	0.468	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	25	13	38	0	47	55
N.S.	1	1.00	1.00	1.04	0.54	1.58	0.00	1.96	2.29
time (sec)	N/A	0.023	0.030	0.088	0.542	3.208	0.000	0.666	12.074

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	57	26	70	0	691	35
N.S.	1	1.00	0.69	0.98	0.45	1.21	0.00	11.91	0.60
time (sec)	N/A	0.024	0.042	0.085	0.537	3.677	0.000	1.907	11.707

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	59	88	39	94	0	1429	47
N.S.	1	1.00	0.67	1.00	0.44	1.07	0.00	16.24	0.53
time (sec)	N/A	0.031	0.129	0.097	0.545	1.538	0.000	1.868	0.198

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	69	119	50	118	0	2455	161
N.S.	1	1.00	0.58	1.01	0.42	1.00	0.00	20.81	1.36
time (sec)	N/A	0.038	0.156	0.096	0.548	3.187	0.000	2.867	11.785

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	52	19	18	72	0	29	18
N.S.	1	1.00	2.36	0.86	0.82	3.27	0.00	1.32	0.82
time (sec)	N/A	0.010	0.048	0.059	0.507	2.130	0.000	0.416	0.109

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	44	4	3	60	0	16	3
N.S.	1	1.00	14.67	1.33	1.00	20.00	0.00	5.33	1.00
time (sec)	N/A	0.008	0.007	0.042	0.510	1.877	0.000	0.458	0.059

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	12	11	9
N.S.	1	1.00	1.00	1.09	1.00	1.00	1.09	1.00	0.82
time (sec)	N/A	0.009	0.006	0.029	0.298	2.720	0.163	0.415	0.028

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	72	32	20	73	0	29	31
N.S.	1	1.00	2.06	0.91	0.57	2.09	0.00	0.83	0.89
time (sec)	N/A	0.016	0.046	0.068	0.572	2.049	0.000	0.397	11.590

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	17	17	19	0	16	16
N.S.	1	1.00	2.88	1.06	1.06	1.19	0.00	1.00	1.00
time (sec)	N/A	0.013	0.006	0.062	0.624	2.918	0.000	0.420	0.111

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	12	0	12	13
N.S.	1	1.00	1.00	1.08	0.00	0.92	0.00	0.92	1.00
time (sec)	N/A	0.015	0.006	0.055	0.000	2.111	0.000	0.407	11.978

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	109	157	0	334	0	143	157
N.S.	1	1.00	0.93	1.34	0.00	2.85	0.00	1.22	1.34
time (sec)	N/A	0.105	0.931	0.080	0.000	3.273	0.000	0.486	20.970

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	82	112	0	266	0	96	76
N.S.	1	1.00	0.93	1.27	0.00	3.02	0.00	1.09	0.86
time (sec)	N/A	0.076	0.242	0.064	0.000	2.324	0.000	0.481	14.379

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	59	91	0	224	0	60	54
N.S.	1	1.00	0.95	1.47	0.00	3.61	0.00	0.97	0.87
time (sec)	N/A	0.050	0.035	0.054	0.000	3.267	0.000	0.450	12.512

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	615	0	398	0	207	83
N.S.	1	1.00	0.97	8.31	0.00	5.38	0.00	2.80	1.12
time (sec)	N/A	0.071	0.042	0.711	0.000	3.133	0.000	1.005	0.285

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	2135	0	624	0	0	238
N.S.	1	1.00	1.00	18.57	0.00	5.43	0.00	0.00	2.07
time (sec)	N/A	0.104	0.254	0.474	0.000	2.301	0.000	0.000	11.870

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	5676	0	765	0	0	542
N.S.	1	1.00	0.85	34.82	0.00	4.69	0.00	0.00	3.33
time (sec)	N/A	0.152	0.883	0.348	0.000	2.758	0.000	0.000	12.018

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	420	429	0	864	0	0	-1
N.S.	1	1.00	1.89	1.93	0.00	3.89	0.00	0.00	-0.00
time (sec)	N/A	0.227	5.945	0.064	0.000	5.765	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	767	307	0	705	0	0	-1
N.S.	1	1.00	4.54	1.82	0.00	4.17	0.00	0.00	-0.01
time (sec)	N/A	0.147	6.168	0.059	0.000	5.021	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	251	223	0	569	0	0	-1
N.S.	1	1.00	2.04	1.81	0.00	4.63	0.00	0.00	-0.01
time (sec)	N/A	0.092	4.009	0.061	0.000	3.660	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	108	167	0	432	0	0	-1
N.S.	1	1.00	1.27	1.96	0.00	5.08	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.236	0.065	0.000	2.812	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	64	2233	0	273	0	0	-1
N.S.	1	1.00	0.85	29.77	0.00	3.64	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.177	0.760	0.000	3.756	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	241	4518	0	329	0	0	-1
N.S.	1	1.00	2.06	38.62	0.00	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.108	4.359	0.362	0.000	3.762	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	321	6894	0	395	0	0	-1
N.S.	1	1.00	1.92	41.28	0.00	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.155	8.806	0.410	0.000	5.254	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	139	263	0	430	0	196	233
N.S.	1	1.00	0.96	1.81	0.00	2.97	0.00	1.35	1.61
time (sec)	N/A	0.123	0.924	0.063	0.000	4.505	0.000	0.629	40.837

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	218	0	348	0	150	156
N.S.	1	1.00	0.97	1.88	0.00	3.00	0.00	1.29	1.34
time (sec)	N/A	0.098	0.593	0.055	0.000	4.355	0.000	0.573	22.575

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	80	197	0	267	0	114	91
N.S.	1	1.00	0.89	2.19	0.00	2.97	0.00	1.27	1.01
time (sec)	N/A	0.069	0.276	0.049	0.000	1.937	0.000	0.520	14.989

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	1765	0	619	0	0	546
N.S.	1	1.00	0.95	18.58	0.00	6.52	0.00	0.00	5.75
time (sec)	N/A	0.092	0.173	0.297	0.000	5.919	0.000	0.000	12.030

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	109	2011	0	616	0	0	447
N.S.	1	1.00	0.94	17.34	0.00	5.31	0.00	0.00	3.85
time (sec)	N/A	0.121	0.225	0.408	0.000	1.809	0.000	0.000	12.026

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	140	5224	0	784	0	0	578
N.S.	1	1.00	0.87	32.45	0.00	4.87	0.00	0.00	3.59
time (sec)	N/A	0.155	0.939	0.297	0.000	3.366	0.000	0.000	12.253

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	908	667	0	1101	0	0	-1
N.S.	1	1.00	3.09	2.27	0.00	3.74	0.00	0.00	-0.00
time (sec)	N/A	0.302	6.361	0.059	0.000	7.254	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	442	519	0	899	0	0	-1
N.S.	1	1.00	1.97	2.32	0.00	4.01	0.00	0.00	-0.00
time (sec)	N/A	0.238	5.164	0.056	0.000	7.474	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	771	409	0	742	0	0	-1
N.S.	1	1.00	4.48	2.38	0.00	4.31	0.00	0.00	-0.01
time (sec)	N/A	0.168	6.216	0.057	0.000	4.549	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	142	327	0	567	0	0	-1
N.S.	1	1.00	1.14	2.62	0.00	4.54	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.431	0.000	0.000	4.552	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	256	3333	0	756	0	0	-1
N.S.	1	1.00	2.25	29.24	0.00	6.63	0.00	0.00	-0.01
time (sec)	N/A	0.095	4.402	0.503	0.000	4.439	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	78	6591	0	326	0	0	-1
N.S.	1	1.00	0.68	57.31	0.00	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.240	0.369	0.000	3.131	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	10026	0	405	0	0	-1
N.S.	1	1.00	0.85	60.76	0.00	2.45	0.00	0.00	-0.01
time (sec)	N/A	0.167	4.673	0.428	0.000	3.971	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	169	578	0	703	0	0	-1
N.S.	1	1.00	0.99	3.40	0.00	4.14	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.699	0.129	0.000	5.099	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	87	103	0	326	0	114	97
N.S.	1	1.00	0.92	1.08	0.00	3.43	0.00	1.20	1.02
time (sec)	N/A	0.097	1.632	0.065	0.000	3.324	0.000	0.501	12.877

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	56	0	258	0	62	56
N.S.	1	1.00	0.97	0.88	0.00	4.03	0.00	0.97	0.88
time (sec)	N/A	0.071	0.197	0.062	0.000	3.385	0.000	0.461	12.328

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	0	193	0	35	35
N.S.	1	1.00	1.00	0.85	0.00	4.71	0.00	0.85	0.85
time (sec)	N/A	0.043	0.021	0.089	0.000	4.169	0.000	0.446	12.344

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	496	0	462	0	208	232
N.S.	1	1.00	0.97	6.70	0.00	6.24	0.00	2.81	3.14
time (sec)	N/A	0.074	0.057	0.390	0.000	3.937	0.000	1.602	12.027

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	135	3601	0	729	0	0	830
N.S.	1	1.00	1.16	31.04	0.00	6.28	0.00	0.00	7.16
time (sec)	N/A	0.113	0.519	0.445	0.000	3.304	0.000	0.000	0.437

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	162	7641	0	893	0	0	1215
N.S.	1	1.00	0.98	46.03	0.00	5.38	0.00	0.00	7.32
time (sec)	N/A	0.150	1.278	0.362	0.000	4.262	0.000	0.000	12.139

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	768	248	0	851	0	0	-1
N.S.	1	1.00	4.34	1.40	0.00	4.81	0.00	0.00	-0.01
time (sec)	N/A	0.146	6.226	0.071	0.000	5.956	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	271	157	0	677	0	0	-1
N.S.	1	1.00	2.17	1.26	0.00	5.42	0.00	0.00	-0.01
time (sec)	N/A	0.090	4.835	0.066	0.000	5.569	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	149	100	0	501	0	0	-1
N.S.	1	1.00	1.73	1.16	0.00	5.83	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.526	0.060	0.000	4.835	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	67	0	131	0	0	40
N.S.	1	1.00	1.00	1.46	0.00	2.85	0.00	0.00	0.87
time (sec)	N/A	0.022	0.047	0.061	0.000	2.843	0.000	0.000	12.687

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	212	1195	0	305	0	0	-1
N.S.	1	1.00	2.72	15.32	0.00	3.91	0.00	0.00	-0.01
time (sec)	N/A	0.078	8.171	0.414	0.000	3.221	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	263	2433	0	377	0	0	-1
N.S.	1	1.00	2.19	20.28	0.00	3.14	0.00	0.00	-0.01
time (sec)	N/A	0.108	6.092	0.513	0.000	3.323	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	1374	3741	0	457	0	0	-1
N.S.	1	1.00	8.08	22.01	0.00	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.156	12.243	0.415	0.000	4.789	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	84	130	0	448	0	99	112
N.S.	1	1.00	0.86	1.33	0.00	4.57	0.00	1.01	1.14
time (sec)	N/A	0.113	0.259	0.072	0.000	3.252	0.000	0.565	13.732

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	87	0	372	0	76	90
N.S.	1	1.00	1.03	1.19	0.00	5.10	0.00	1.04	1.23
time (sec)	N/A	0.085	0.245	0.060	0.000	2.975	0.000	0.514	13.100

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	56	66	0	346	56	69	85
N.S.	1	1.00	0.81	0.96	0.00	5.01	0.81	1.00	1.23
time (sec)	N/A	0.056	0.052	0.050	0.000	3.302	11.085	0.488	13.074

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	91	32888	0	952	0	0	1922
N.S.	1	1.00	0.86	310.26	0.00	8.98	0.00	0.00	18.13
time (sec)	N/A	0.098	0.092	0.967	0.000	3.463	0.000	0.000	12.640

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	115	54353	0	1300	0	0	2483
N.S.	1	1.00	0.73	346.20	0.00	8.28	0.00	0.00	15.82
time (sec)	N/A	0.164	0.301	1.413	0.000	4.753	0.000	0.000	12.538

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	142	79934	0	1574	0	0	2118
N.S.	1	1.00	0.66	371.79	0.00	7.32	0.00	0.00	9.85
time (sec)	N/A	0.222	0.804	2.101	0.000	4.441	0.000	0.000	13.133

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	327	270	0	1253	0	0	-1
N.S.	1	1.00	1.80	1.48	0.00	6.88	0.00	0.00	-0.01
time (sec)	N/A	0.159	5.914	0.065	0.000	3.732	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	250	182	0	1016	0	0	-1
N.S.	1	1.00	2.03	1.48	0.00	8.26	0.00	0.00	-0.01
time (sec)	N/A	0.103	1.953	0.059	0.000	5.480	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	154	126	0	299	0	0	-1
N.S.	1	1.00	1.90	1.56	0.00	3.69	0.00	0.00	-0.01
time (sec)	N/A	0.070	2.218	0.057	0.000	1.736	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	214	102	0	324	0	0	-1
N.S.	1	1.00	2.52	1.20	0.00	3.81	0.00	0.00	-0.01
time (sec)	N/A	0.035	4.224	0.000	0.000	3.254	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	882	1305	0	493	0	0	-1
N.S.	1	1.00	6.89	10.20	0.00	3.85	0.00	0.00	-0.01
time (sec)	N/A	0.117	10.010	0.438	0.000	3.210	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	1398	2577	0	603	0	0	-1
N.S.	1	1.00	7.60	14.01	0.00	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.169	11.817	0.367	0.000	4.615	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	1994	3925	0	713	0	0	-1
N.S.	1	1.00	7.91	15.58	0.00	2.83	0.00	0.00	-0.00
time (sec)	N/A	0.233	15.448	0.434	0.000	3.238	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	91	155	0	628	0	137	148
N.S.	1	1.00	0.79	1.35	0.00	5.46	0.00	1.19	1.29
time (sec)	N/A	0.132	0.316	0.073	0.000	4.613	0.000	0.612	16.029

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	84	110	0	592	0	116	138
N.S.	1	1.00	0.82	1.07	0.00	5.75	0.00	1.13	1.34
time (sec)	N/A	0.098	0.201	0.059	0.000	4.026	0.000	0.581	15.720

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	58	89	0	564	83	105	131
N.S.	1	1.00	0.59	0.90	0.00	5.70	0.84	1.06	1.32
time (sec)	N/A	0.075	0.094	0.051	0.000	4.147	14.454	0.513	15.696

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	94	331597	0	1697	0	0	2788
N.S.	1	1.00	0.64	2255.76	0.00	11.54	0.00	0.00	18.97
time (sec)	N/A	0.145	0.254	47.964	0.000	2.616	0.000	0.000	12.456

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	138	531573	0	2147	0	0	2500
N.S.	1	1.00	0.67	2580.45	0.00	10.42	0.00	0.00	12.14
time (sec)	N/A	0.230	0.396	76.059	0.000	3.284	0.000	0.000	13.100

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	165	790286	0	2501	0	0	2500
N.S.	1	1.00	0.61	2905.46	0.00	9.19	0.00	0.00	9.19
time (sec)	N/A	0.285	1.313	107.504	0.000	3.504	0.000	0.000	14.006

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	295	381	0	1772	0	0	-1
N.S.	1	1.00	1.73	2.23	0.00	10.36	0.00	0.00	-0.01
time (sec)	N/A	0.167	2.959	0.101	0.000	7.271	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	260	295	0	518	0	0	-1
N.S.	1	1.00	1.98	2.25	0.00	3.95	0.00	0.00	-0.01
time (sec)	N/A	0.105	6.038	0.097	0.000	1.598	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	365	212	0	549	0	0	-1
N.S.	1	1.00	2.85	1.66	0.00	4.29	0.00	0.00	-0.01
time (sec)	N/A	0.098	7.130	0.093	0.000	2.771	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	163	0	581	0	0	-1
N.S.	1	1.00	9.93	1.22	0.00	4.34	0.00	0.00	-0.01
time (sec)	N/A	0.070	7.025	0.100	0.000	1.367	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	1890	0	0	781	0	0	-1
N.S.	1	1.00	10.16	0.00	0.00	4.20	0.00	0.00	-0.01
time (sec)	N/A	0.180	12.080	0.326	0.000	3.575	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	871	0	0	909	0	0	-1
N.S.	1	1.00	3.50	0.00	0.00	3.65	0.00	0.00	-0.00
time (sec)	N/A	0.240	16.393	0.335	0.000	1.615	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	441	0	0	1055	0	0	-1
N.S.	1	1.00	1.35	0.00	0.00	3.23	0.00	0.00	-0.00
time (sec)	N/A	0.315	14.485	0.365	0.000	2.649	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	74	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.070	0.296	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	101	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.211	0.340	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	106	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.593	0.280	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	73	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.086	0.246	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.065	0.178	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	98	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.121	0.255	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	142	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.472	0.288	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	172	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	1.732	0.278	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	2.324	0.270	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1896	0	0	0	0	0	-1
N.S.	1	1.00	22.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	14.712	0.247	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1992	0	0	0	0	0	-1
N.S.	1	1.00	24.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	13.924	0.232	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	192	0	0	0	0	0	-1
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.303	0.002	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	1989	0	0	0	0	0	-1
N.S.	1	1.00	25.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	13.850	0.269	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1887	0	0	0	0	0	-1
N.S.	1	1.00	22.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	6.528	0.254	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	3.041	0.275	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	224	246	260	225	301	5949	310
N.S.	1	1.00	0.88	0.96	1.02	0.88	1.18	23.33	1.22
time (sec)	N/A	0.096	0.634	0.164	0.512	2.736	0.488	29.923	11.849

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	160	153	183	148	194	3125	174
N.S.	1	1.00	0.95	0.91	1.09	0.88	1.15	18.60	1.04
time (sec)	N/A	0.069	0.298	0.113	0.517	1.784	0.295	8.067	11.600

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	107	85	83	85	94	1065	117
N.S.	1	1.00	1.20	0.96	0.93	0.96	1.06	11.97	1.31
time (sec)	N/A	0.041	0.324	0.075	0.511	1.565	0.144	1.616	11.598

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	36	36	36	37	251	34
N.S.	1	1.00	0.94	1.12	1.12	1.12	1.16	7.84	1.06
time (sec)	N/A	0.014	0.054	0.043	0.313	2.035	0.067	0.634	11.530

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	278	293	291	4817	0	334	342
N.S.	1	1.00	1.09	1.14	1.14	18.82	0.00	1.30	1.34
time (sec)	N/A	0.253	0.412	0.368	0.512	7.747	0.000	0.683	12.653

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	504	411	502	11554	0	605	988
N.S.	1	1.00	0.90	0.74	0.90	20.71	0.00	1.08	1.77
time (sec)	N/A	0.484	6.060	0.930	0.520	7.505	0.000	0.751	12.519

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	43	36	33	48	34	34	41
N.S.	1	1.00	1.16	0.97	0.89	1.30	0.92	0.92	1.11
time (sec)	N/A	0.038	0.018	0.073	0.504	2.001	0.058	0.425	11.618

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	196	313	265	225	386	7741	271
N.S.	1	1.00	0.91	1.45	1.23	1.04	1.79	35.84	1.25
time (sec)	N/A	0.082	3.210	0.193	0.512	2.686	0.974	48.793	11.586

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	128	187	167	145	224	3499	180
N.S.	1	1.00	0.89	1.30	1.16	1.01	1.56	24.30	1.25
time (sec)	N/A	0.055	0.674	0.115	0.514	3.670	0.498	15.356	11.662

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	75	97	91	81	116	1181	109
N.S.	1	1.00	0.91	1.18	1.11	0.99	1.41	14.40	1.33
time (sec)	N/A	0.034	0.378	0.066	0.517	2.310	0.239	1.853	11.471

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	43	34	32	32	590	31
N.S.	1	1.00	1.26	1.23	0.97	0.91	0.91	16.86	0.89
time (sec)	N/A	0.017	0.015	0.035	0.529	2.921	0.082	0.893	11.590

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	228	292	261	1541	0	354	2500
N.S.	1	1.00	0.75	0.97	0.86	5.10	0.00	1.17	8.28
time (sec)	N/A	0.204	0.366	0.316	0.512	3.565	0.000	0.939	15.043

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	598	356	394	4291	0	517	2500
N.S.	1	1.00	0.92	0.55	0.61	6.62	0.00	0.80	3.86
time (sec)	N/A	0.446	6.183	1.037	0.523	4.570	0.000	1.078	15.685

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	219	531	0	0	0	0	-1
N.S.	1	1.00	0.34	0.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	10.192	0.157	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	106	123	0	0	0	0	-1
N.S.	1	1.00	0.30	0.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	10.264	0.255	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	145	181	0	555	0	107	-1
N.S.	1	1.00	1.41	1.76	0.00	5.39	0.00	1.04	-0.01
time (sec)	N/A	0.138	2.556	0.139	0.000	4.706	0.000	0.453	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	86	139	0	475	0	89	-1
N.S.	1	1.00	0.96	1.54	0.00	5.28	0.00	0.99	-0.01
time (sec)	N/A	0.078	0.030	0.087	0.000	2.630	0.000	0.450	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	1021	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	10.01	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.047	0.152	0.000	3.068	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	643	404	537	0	0	0	0	-1
N.S.	1	1.00	0.63	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	12.482	0.102	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	189	374	0	758	0	176	-1
N.S.	1	1.00	1.28	2.53	0.00	5.12	0.00	1.19	-0.01
time (sec)	N/A	0.203	4.216	0.110	0.000	4.187	0.000	0.464	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	166	313	0	593	0	138	-1
N.S.	1	1.00	1.32	2.48	0.00	4.71	0.00	1.10	-0.01
time (sec)	N/A	0.127	3.044	0.086	0.000	2.908	0.000	0.530	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	190	0	0	1269	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	8.19	0.00	0.00	-0.01
time (sec)	N/A	0.162	2.045	0.139	0.000	36.689	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	91	0	483	0	75	-1
N.S.	1	1.00	1.00	1.23	0.00	6.53	0.00	1.01	-0.01
time (sec)	N/A	0.078	0.037	0.110	0.000	1.993	0.000	0.428	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	65	0	150	0	46	-1
N.S.	1	1.00	1.00	1.59	0.00	3.66	0.00	1.12	-0.02
time (sec)	N/A	0.043	0.012	0.195	0.000	2.351	0.000	0.462	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	475	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	6.79	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.034	0.149	0.000	3.920	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	122	179	0	0	0	0	-1
N.S.	1	1.00	0.42	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	1.472	0.138	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	267	0	292	0	103	-1
N.S.	1	1.00	0.94	3.76	0.00	4.11	0.00	1.45	-0.01
time (sec)	N/A	0.102	0.209	0.918	0.000	3.178	0.000	0.423	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	248	0	319	0	119	-1
N.S.	1	1.00	0.99	3.35	0.00	4.31	0.00	1.61	-0.01
time (sec)	N/A	0.073	0.196	0.087	0.000	2.675	0.000	0.436	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	108	0	0	954	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	7.88	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.410	0.128	0.000	4.250	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	654	0	556	0	597	-1
N.S.	1	1.00	0.95	6.00	0.00	5.10	0.00	5.48	-0.01
time (sec)	N/A	0.145	0.515	0.944	0.000	2.242	0.000	0.443	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	113	602	0	599	0	618	-1
N.S.	1	1.00	0.97	5.15	0.00	5.12	0.00	5.28	-0.01
time (sec)	N/A	0.118	0.529	0.092	0.000	2.967	0.000	0.454	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	149	0	0	1749	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	9.56	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.963	0.134	0.000	4.901	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	151	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	0.947	0.427	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	304	0	0	0	0	0	-1
N.S.	1	1.00	2.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.416	0.293	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	265	0	0	0	0	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.857	4.870	0.286	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	381	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.075	4.055	0.372	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	76	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.060	0.287	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.058	0.195	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.029	0.001	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.040	0.254	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.059	0.266	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.068	0.282	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.048	0.231	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.042	0.162	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.013	0.219	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	59	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.043	0.256	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.039	2.420	0.357	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.106	0.370	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	265	0	0	0	0	0	-1
N.S.	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	1.729	0.418	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.102	0.355	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	5.956	0.398	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	93	102	95	95	0	98	147
N.S.	1	1.00	1.33	1.46	1.36	1.36	0.00	1.40	2.10
time (sec)	N/A	0.035	0.051	0.237	0.281	2.894	0.000	0.614	14.406

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	67	62	76	0	64	79
N.S.	1	1.00	1.14	1.60	1.48	1.81	0.00	1.52	1.88
time (sec)	N/A	0.023	0.025	0.139	0.290	2.517	0.000	0.595	12.294

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	47	39	46	44	0	48	32
N.S.	1	1.00	1.68	1.39	1.64	1.57	0.00	1.71	1.14
time (sec)	N/A	0.022	0.021	0.155	0.294	3.124	0.000	0.587	11.872

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	44	36	29	30	0	36	47
N.S.	1	1.00	1.38	1.12	0.91	0.94	0.00	1.12	1.47
time (sec)	N/A	0.023	0.013	0.189	0.283	2.288	0.000	0.601	12.095

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	72	47	47	0	2147	71
N.S.	1	1.00	0.96	1.33	0.87	0.87	0.00	39.76	1.31
time (sec)	N/A	0.032	0.130	0.227	0.280	2.503	0.000	13.194	12.069

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	92	64	63	0	29589	95
N.S.	1	1.00	0.99	1.21	0.84	0.83	0.00	389.33	1.25
time (sec)	N/A	0.041	0.199	0.232	0.298	2.780	0.000	18.658	11.998

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	75	94	56	74	0	70	56
N.S.	1	1.00	1.10	1.38	0.82	1.09	0.00	1.03	0.82
time (sec)	N/A	0.034	0.181	0.210	0.289	2.456	0.000	0.634	12.013

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	53	66	39	56	0	48	40
N.S.	1	1.00	1.15	1.43	0.85	1.22	0.00	1.04	0.87
time (sec)	N/A	0.027	0.104	0.193	0.288	2.261	0.000	0.587	11.937

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	33	25	37	36	25	25
N.S.	1	1.00	1.00	1.18	0.89	1.32	1.29	0.89	0.89
time (sec)	N/A	0.019	0.011	0.209	0.280	2.339	0.958	0.578	11.862

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	54	39	30	0	169	32
N.S.	1	1.00	0.97	1.64	1.18	0.91	0.00	5.12	0.97
time (sec)	N/A	0.024	0.045	0.157	0.491	2.852	0.000	0.627	11.942

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	46	81	69	49	0	2010	67
N.S.	1	1.00	0.75	1.33	1.13	0.80	0.00	32.95	1.10
time (sec)	N/A	0.031	0.095	0.193	0.507	4.047	0.000	1.883	12.064

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	74	102	97	66	0	3757	93
N.S.	1	1.00	0.85	1.17	1.11	0.76	0.00	43.18	1.07
time (sec)	N/A	0.035	0.149	0.242	0.515	3.689	0.000	2.263	12.540

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	875	199	156	137	0	167	269
N.S.	1	1.00	6.84	1.55	1.22	1.07	0.00	1.30	2.10
time (sec)	N/A	0.104	7.421	0.321	0.299	3.286	0.000	0.780	15.643

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	347	146	119	116	0	120	177
N.S.	1	1.00	3.61	1.52	1.24	1.21	0.00	1.25	1.84
time (sec)	N/A	0.056	4.476	0.198	0.297	2.660	0.000	0.743	14.568

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	146	100	105	106	0	104	148
N.S.	1	1.00	2.35	1.61	1.69	1.71	0.00	1.68	2.39
time (sec)	N/A	0.059	0.486	0.189	0.288	1.764	0.000	0.781	14.313

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	71	76	72	79	0	96	136
N.S.	1	1.00	1.27	1.36	1.29	1.41	0.00	1.71	2.43
time (sec)	N/A	0.042	0.263	0.203	0.295	2.426	0.000	0.855	14.043

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	70	89	56	71	0	2946	119
N.S.	1	1.00	1.23	1.56	0.98	1.25	0.00	51.68	2.09
time (sec)	N/A	0.043	0.187	0.237	0.291	2.421	0.000	118.776	12.195

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	90	153	81	95	0	0	160
N.S.	1	1.00	1.05	1.78	0.94	1.10	0.00	0.00	1.86
time (sec)	N/A	0.054	0.445	0.282	0.301	2.578	0.000	0.000	12.269

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	116	183	104	117	0	0	188
N.S.	1	1.00	1.02	1.61	0.91	1.03	0.00	0.00	1.65
time (sec)	N/A	0.067	0.365	0.288	0.290	2.916	0.000	0.000	12.396

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	106	157	85	114	0	118	80
N.S.	1	1.00	1.10	1.64	0.89	1.19	0.00	1.23	0.83
time (sec)	N/A	0.055	0.261	0.272	0.295	1.534	0.000	0.832	12.223

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	83	111	66	94	0	80	60
N.S.	1	1.00	1.12	1.50	0.89	1.27	0.00	1.08	0.81
time (sec)	N/A	0.045	0.358	0.240	0.305	1.256	0.000	0.800	12.281

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	57	42	69	0	42	40
N.S.	1	1.00	1.00	1.16	0.86	1.41	0.00	0.86	0.82
time (sec)	N/A	0.035	0.097	0.211	0.288	1.598	0.000	0.769	12.134

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	111	66	69	0	594	91
N.S.	1	1.00	1.00	2.02	1.20	1.25	0.00	10.80	1.65
time (sec)	N/A	0.052	0.274	0.194	0.516	2.304	0.000	0.855	12.250

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	65	122	97	75	0	3916	93
N.S.	1	1.00	0.75	1.40	1.11	0.86	0.00	45.01	1.07
time (sec)	N/A	0.068	0.214	0.234	0.506	2.094	0.000	21.828	12.239

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	87	166	131	98	0	4487	126
N.S.	1	1.00	0.71	1.36	1.07	0.80	0.00	36.78	1.03
time (sec)	N/A	0.087	0.241	0.254	0.534	1.817	0.000	22.433	13.237

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	207	129	0	292	0	131	268
N.S.	1	1.00	2.30	1.43	0.00	3.24	0.00	1.46	2.98
time (sec)	N/A	0.097	0.879	0.408	0.000	2.836	0.000	0.648	13.761

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	136	75	0	169	0	88	67
N.S.	1	1.00	2.31	1.27	0.00	2.86	0.00	1.49	1.14
time (sec)	N/A	0.055	0.180	0.302	0.000	1.875	0.000	0.635	12.668

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	122	0	47	32
N.S.	1	1.00	1.00	0.90	0.00	3.05	0.00	1.18	0.80
time (sec)	N/A	0.031	0.035	0.245	0.000	2.406	0.000	0.619	12.571

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	89	61	0	182	0	73	61
N.S.	1	1.00	1.48	1.02	0.00	3.03	0.00	1.22	1.02
time (sec)	N/A	0.054	0.161	0.319	0.000	2.561	0.000	0.642	12.321

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	115	98	0	276	0	161	251
N.S.	1	1.00	1.31	1.11	0.00	3.14	0.00	1.83	2.85
time (sec)	N/A	0.084	0.324	0.355	0.000	2.056	0.000	0.638	15.436

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	148	165	0	395	0	319	1493
N.S.	1	1.00	1.17	1.31	0.00	3.13	0.00	2.53	11.85
time (sec)	N/A	0.102	1.172	0.428	0.000	2.382	0.000	0.660	15.077

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	103	123	110	425	0	151	136
N.S.	1	1.00	0.95	1.14	1.02	3.94	0.00	1.40	1.26
time (sec)	N/A	0.074	0.589	0.397	0.516	3.998	0.000	0.667	12.293

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	74	74	69	339	0	96	90
N.S.	1	1.00	0.96	0.96	0.90	4.40	0.00	1.25	1.17
time (sec)	N/A	0.061	0.228	0.354	0.522	2.925	0.000	0.653	12.229

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	44	45	267	0	62	44
N.S.	1	1.00	1.00	0.85	0.87	5.13	0.00	1.19	0.85
time (sec)	N/A	0.045	0.102	0.290	0.549	2.995	0.000	0.639	12.396

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	23	205	0	40	24
N.S.	1	1.00	1.00	0.75	0.72	6.41	0.00	1.25	0.75
time (sec)	N/A	0.038	0.042	0.308	0.553	2.562	0.000	0.627	12.503

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	85	95	290	0	110	254
N.S.	1	1.00	0.94	1.02	1.14	3.49	0.00	1.33	3.06
time (sec)	N/A	0.075	0.124	0.397	0.521	2.305	0.000	0.642	13.894

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	113	130	185	401	0	183	2500
N.S.	1	1.00	0.88	1.01	1.43	3.11	0.00	1.42	19.38
time (sec)	N/A	0.109	0.282	0.390	0.522	3.046	0.000	0.636	15.867

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	254	175	0	635	0	245	2500
N.S.	1	1.00	1.52	1.05	0.00	3.80	0.00	1.47	14.97
time (sec)	N/A	0.177	2.558	0.520	0.000	2.702	0.000	0.783	15.237

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	191	124	0	407	0	153	946
N.S.	1	1.00	1.75	1.14	0.00	3.73	0.00	1.40	8.68
time (sec)	N/A	0.093	0.545	0.428	0.000	2.542	0.000	0.740	14.371

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	80	0	266	0	91	187
N.S.	1	1.00	0.95	1.01	0.00	3.37	0.00	1.15	2.37
time (sec)	N/A	0.051	0.165	0.287	0.000	2.903	0.000	0.732	12.937

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	92	102	0	337	0	112	239
N.S.	1	1.00	0.98	1.09	0.00	3.59	0.00	1.19	2.54
time (sec)	N/A	0.055	0.166	0.422	0.000	2.975	0.000	0.690	12.777

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	124	118	0	451	0	152	269
N.S.	1	1.00	1.09	1.04	0.00	3.96	0.00	1.33	2.36
time (sec)	N/A	0.116	0.729	0.477	0.000	3.779	0.000	0.749	15.552

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	147	164	0	600	0	329	1690
N.S.	1	1.00	1.03	1.15	0.00	4.20	0.00	2.30	11.82
time (sec)	N/A	0.145	1.020	0.509	0.000	3.323	0.000	0.770	16.012

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	135	137	137	597	0	180	167
N.S.	1	1.00	1.06	1.08	1.08	4.70	0.00	1.42	1.31
time (sec)	N/A	0.097	0.501	0.427	0.530	3.150	0.000	0.758	12.175

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	97	100	479	0	128	119
N.S.	1	1.00	1.00	0.93	0.96	4.61	0.00	1.23	1.14
time (sec)	N/A	0.091	0.424	0.379	0.531	2.541	0.000	0.731	12.414

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	83	69	69	367	0	92	65
N.S.	1	1.00	1.08	0.90	0.90	4.77	0.00	1.19	0.84
time (sec)	N/A	0.052	0.208	0.349	0.557	3.757	0.000	0.711	12.192

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	55	53	327	0	70	54
N.S.	1	1.00	0.95	0.83	0.80	4.95	0.00	1.06	0.82
time (sec)	N/A	0.041	0.194	0.281	0.526	2.703	0.000	0.702	12.140

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	116	128	209	614	0	211	2500
N.S.	1	1.00	0.78	0.86	1.41	4.15	0.00	1.43	16.89
time (sec)	N/A	0.127	0.809	0.430	0.525	3.922	0.000	0.755	16.295

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	148	173	355	801	0	269	2500
N.S.	1	1.00	0.70	0.82	1.67	3.78	0.00	1.27	11.79
time (sec)	N/A	0.198	1.356	0.513	0.519	2.141	0.000	0.794	17.275

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	81	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.129	0.353	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	2033	0	0	0	0	0	-1
N.S.	1	1.00	18.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	14.719	0.357	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	89	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.108	0.319	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	122	70270	112	113	0	0	-1
N.S.	1	1.00	1.23	709.80	1.13	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.094	1.411	7.580	0.342	3.209	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	87	34277	75	80	0	0	-1
N.S.	1	1.00	1.34	527.34	1.15	1.23	0.00	0.00	-0.02
time (sec)	N/A	0.073	1.361	5.004	0.330	2.521	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	10286	37	53	0	0	31
N.S.	1	1.00	1.00	331.81	1.19	1.71	0.00	0.00	1.00
time (sec)	N/A	0.060	0.016	4.157	0.315	2.275	0.000	0.000	13.291

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.032	0.003	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	1060	0	0	0	0	0	-1
N.S.	1	1.00	17.38	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	3.424	0.354	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.087	0.263	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	70	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.051	0.187	0.000	0.000	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	482	0	0	0	0	0	-1
N.S.	1	1.00	6.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	2.295	0.226	0.000	0.000	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	1552	0	0	0	0	0	-1
N.S.	1	1.00	18.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	4.277	0.436	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.038	1.754	0.431	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	2.974	0.345	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	1.231	0.255	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	1.776	0.296	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	3.093	0.520	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	165	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.137	1.432	0.359	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	122	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	2.418	0.352	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	76
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.01
time (sec)	N/A	0.054	0.090	0.305	0.000	0.000	0.000	0.000	14.046

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.571	0.250	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	2.636	0.461	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	299	0	0	0	0	0	-1
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	1.300	0.393	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	292	0	0	0	0	0	-1
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	2.401	0.408	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	319	0	0	0	0	0	-1
N.S.	1	1.00	3.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	1.457	0.352	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	2.938	0.417	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [406] had the largest ratio of [29]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	14	0.214
2	A	3	3	1.00	14	0.214
3	A	2	2	1.00	14	0.143
4	A	2	2	1.00	14	0.143
5	A	3	3	1.00	14	0.214
6	A	4	3	1.00	14	0.214
7	A	16	10	1.00	14	0.714
8	A	14	10	1.00	14	0.714
9	A	13	10	1.00	14	0.714
10	A	13	10	1.00	14	0.714
11	A	14	10	1.00	14	0.714
12	A	16	10	1.00	14	0.714
13	A	7	3	1.00	14	0.214
14	A	5	3	1.00	14	0.214
15	A	3	3	1.00	14	0.214
16	A	3	3	1.00	14	0.214
17	A	5	3	1.00	14	0.214
18	A	7	3	1.00	14	0.214
19	A	3	3	1.00	14	0.214
20	A	3	3	1.00	14	0.214
21	A	3	3	1.00	14	0.214
22	A	3	3	1.00	14	0.214
23	A	3	3	1.00	14	0.214
24	A	3	3	1.00	14	0.214
25	A	3	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	3	1.00	12	0.250
27	A	3	3	1.00	12	0.250
28	A	3	3	1.00	12	0.250
29	A	2	2	1.00	14	0.143
30	A	3	2	1.00	21	0.095
31	A	3	2	1.00	21	0.095
32	A	3	2	1.00	19	0.105
33	A	3	3	1.00	19	0.158
34	A	4	4	1.00	21	0.190
35	A	5	5	1.00	21	0.238
36	A	6	6	1.00	21	0.286
37	A	5	5	1.00	21	0.238
38	A	4	4	1.00	21	0.190
39	A	3	2	1.00	12	0.167
40	A	3	2	1.00	21	0.095
41	A	3	2	1.00	21	0.095
42	A	3	2	1.00	21	0.095
43	A	3	2	1.00	23	0.087
44	A	3	2	1.00	23	0.087
45	A	3	2	1.00	21	0.095
46	A	4	3	1.00	21	0.143
47	A	5	5	1.00	23	0.217
48	A	6	5	1.00	23	0.217
49	A	6	5	1.00	23	0.217
50	A	5	5	1.00	23	0.217
51	A	4	3	1.00	14	0.214
52	A	3	2	1.00	23	0.087
53	A	3	2	1.00	23	0.087
54	A	3	2	1.00	23	0.087
55	A	4	3	1.00	23	0.130
56	A	4	4	1.00	23	0.174
57	A	3	3	1.00	21	0.143
58	A	4	4	1.00	21	0.190
59	A	5	5	1.00	23	0.217
60	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	7	1.00	23	0.304
62	A	6	6	1.00	23	0.261
63	A	5	5	1.00	23	0.217
64	A	3	3	1.00	14	0.214
65	A	3	3	1.00	23	0.130
66	A	4	4	1.00	23	0.174
67	A	4	3	1.00	23	0.130
68	A	6	5	1.00	23	0.217
69	A	5	4	1.00	23	0.174
70	A	4	4	1.00	21	0.190
71	A	5	5	1.00	21	0.238
72	A	6	6	1.00	23	0.261
73	A	7	6	1.00	23	0.261
74	A	7	6	1.00	23	0.261
75	A	6	6	1.00	23	0.261
76	A	5	5	1.00	14	0.357
77	A	4	4	1.00	23	0.174
78	A	5	4	1.00	23	0.174
79	A	6	5	1.00	23	0.217
80	A	7	6	1.00	23	0.261
81	A	6	5	1.00	23	0.217
82	A	5	4	1.00	21	0.190
83	A	6	6	1.00	21	0.286
84	A	7	6	1.00	23	0.261
85	A	8	6	1.00	23	0.261
86	A	8	6	1.00	23	0.261
87	A	7	6	1.00	23	0.261
88	A	6	6	1.00	14	0.429
89	A	5	4	1.00	23	0.174
90	A	6	5	1.00	23	0.217
91	A	7	6	1.00	23	0.261
92	A	6	6	1.00	25	0.240
93	A	5	5	1.00	25	0.200
94	A	4	4	1.00	23	0.174
95	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
96	A	7	7	1.00	25	0.280
97	A	8	8	1.00	25	0.320
98	A	8	8	1.00	25	0.320
99	A	7	7	1.00	25	0.280
100	A	6	6	1.00	16	0.375
101	A	4	4	1.00	25	0.160
102	A	5	5	1.00	25	0.200
103	A	6	6	1.00	25	0.240
104	A	7	7	1.00	25	0.280
105	A	6	6	1.00	25	0.240
106	A	5	5	1.00	23	0.217
107	A	7	7	1.00	23	0.304
108	A	8	8	1.00	25	0.320
109	A	9	9	1.00	25	0.360
110	A	9	9	1.00	25	0.360
111	A	8	8	1.00	25	0.320
112	A	7	7	1.00	16	0.438
113	A	5	5	1.00	25	0.200
114	A	6	6	1.00	25	0.240
115	A	7	7	1.00	25	0.280
116	A	4	4	1.00	25	0.160
117	A	3	3	1.00	25	0.120
118	A	2	2	1.00	23	0.087
119	A	3	3	1.00	23	0.130
120	A	5	5	1.00	25	0.200
121	A	6	6	1.00	25	0.240
122	A	6	6	1.00	25	0.240
123	A	5	5	1.00	25	0.200
124	A	3	3	1.00	16	0.188
125	A	2	2	1.00	25	0.080
126	A	3	3	1.00	25	0.120
127	A	4	4	1.00	25	0.160
128	A	5	5	1.00	25	0.200
129	A	4	4	1.00	25	0.160
130	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	4	1.00	23	0.174
132	A	6	6	1.00	25	0.240
133	A	7	6	1.00	25	0.240
134	A	7	6	1.00	25	0.240
135	A	6	6	1.00	25	0.240
136	A	4	4	1.00	16	0.250
137	A	3	3	1.00	25	0.120
138	A	4	4	1.00	25	0.160
139	A	5	5	1.00	25	0.200
140	A	6	6	1.00	25	0.240
141	A	5	5	1.00	25	0.200
142	A	4	4	1.00	23	0.174
143	A	6	6	1.00	23	0.261
144	A	7	6	1.00	25	0.240
145	A	8	6	1.00	25	0.240
146	A	8	6	1.00	25	0.240
147	A	7	6	1.00	25	0.240
148	A	6	6	1.00	16	0.375
149	A	4	4	1.00	25	0.160
150	A	5	5	1.00	25	0.200
151	A	6	6	1.00	25	0.240
152	A	3	3	1.00	23	0.130
153	A	3	3	1.00	25	0.120
154	A	5	5	1.00	23	0.217
155	A	4	4	1.00	23	0.174
156	A	3	3	1.00	21	0.143
157	A	3	3	1.00	21	0.143
158	A	3	3	1.00	23	0.130
159	A	3	3	1.00	23	0.130
160	A	3	3	1.00	14	0.214
161	A	3	3	1.00	23	0.130
162	A	4	4	1.00	23	0.174
163	A	5	5	0.98	23	0.217
164	A	3	3	1.00	25	0.120
165	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	3	1.00	14	0.214
167	A	3	3	1.00	23	0.130
168	A	4	3	1.00	23	0.130
169	A	4	3	1.00	23	0.130
170	A	3	3	1.00	23	0.130
171	A	3	3	1.00	21	0.143
172	A	3	3	1.00	21	0.143
173	A	3	3	1.00	23	0.130
174	A	0	0	0.00	0	0.000
175	A	3	3	1.00	23	0.130
176	A	4	4	1.00	25	0.160
177	A	3	3	1.00	25	0.120
178	A	0	0	0.00	0	0.000
179	A	4	3	1.00	14	0.214
180	A	4	3	1.00	14	0.214
181	A	4	3	1.00	14	0.214
182	A	4	4	1.00	14	0.286
183	A	5	4	1.00	14	0.286
184	A	6	4	1.00	14	0.286
185	A	4	3	1.00	21	0.143
186	A	3	3	1.00	21	0.143
187	A	2	2	1.00	19	0.105
188	A	3	2	1.00	19	0.105
189	A	3	3	1.00	21	0.143
190	A	4	4	1.00	21	0.190
191	A	5	3	1.00	21	0.143
192	A	4	3	1.00	21	0.143
193	A	3	3	1.00	21	0.143
194	A	3	2	1.00	12	0.167
195	A	2	2	1.00	21	0.095
196	A	4	4	1.00	21	0.190
197	A	5	4	1.00	21	0.190
198	A	4	3	1.00	23	0.130
199	A	4	3	1.00	23	0.130
200	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	3	1.00	21	0.143
202	A	4	3	1.00	23	0.130
203	A	4	3	1.00	23	0.130
204	A	4	3	1.00	23	0.130
205	A	4	3	1.00	23	0.130
206	A	4	3	1.00	23	0.130
207	A	4	3	1.00	14	0.214
208	A	4	3	1.00	23	0.130
209	A	4	3	1.00	23	0.130
210	A	4	3	1.00	23	0.130
211	A	4	3	1.00	23	0.130
212	A	4	3	1.00	23	0.130
213	A	5	4	1.00	21	0.190
214	A	4	3	1.00	21	0.143
215	A	4	3	1.00	23	0.130
216	A	4	3	1.00	23	0.130
217	A	6	6	1.00	23	0.261
218	A	5	5	1.00	23	0.217
219	A	4	4	1.00	23	0.174
220	A	3	3	1.00	14	0.214
221	A	5	5	1.00	23	0.217
222	A	6	6	1.00	23	0.261
223	A	7	6	1.00	23	0.261
224	A	4	3	1.00	23	0.130
225	A	4	3	1.00	23	0.130
226	A	4	3	1.00	21	0.143
227	A	4	3	1.00	21	0.143
228	A	4	3	1.00	23	0.130
229	A	4	3	1.00	23	0.130
230	A	6	6	1.00	23	0.261
231	A	5	5	1.00	23	0.217
232	A	5	5	1.00	23	0.217
233	A	5	5	1.00	14	0.357
234	A	6	6	1.00	23	0.261
235	A	7	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	8	6	1.00	23	0.261
237	A	4	3	1.00	23	0.130
238	A	4	3	1.00	23	0.130
239	A	4	3	1.00	21	0.143
240	A	4	3	1.00	21	0.143
241	A	4	3	1.00	23	0.130
242	A	4	3	1.00	23	0.130
243	A	6	6	1.00	23	0.261
244	A	6	6	1.00	23	0.261
245	A	6	6	1.00	23	0.261
246	A	6	6	1.00	14	0.429
247	A	7	7	1.00	23	0.304
248	A	8	7	1.00	23	0.304
249	A	9	7	1.00	23	0.304
250	A	4	3	1.00	14	0.214
251	A	4	3	1.00	14	0.214
252	A	4	3	1.00	14	0.214
253	A	3	2	1.00	12	0.167
254	A	3	3	1.00	14	0.214
255	A	5	5	1.00	14	0.357
256	A	6	6	1.00	14	0.429
257	A	5	4	1.00	17	0.235
258	A	4	3	1.00	17	0.176
259	A	4	4	1.00	17	0.235
260	A	3	3	1.00	15	0.200
261	A	4	4	1.00	15	0.267
262	A	4	4	1.00	17	0.235
263	A	5	5	1.00	17	0.294
264	A	4	3	1.00	17	0.176
265	A	4	4	1.00	16	0.250
266	A	4	3	1.00	17	0.176
267	A	5	5	1.00	17	0.294
268	A	3	3	1.00	15	0.200
269	A	5	5	1.00	15	0.333
270	A	5	5	1.00	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	5	5	1.00	16	0.312
272	A	6	5	1.00	16	0.312
273	A	4	3	1.00	17	0.176
274	A	5	5	1.00	17	0.294
275	A	3	3	1.00	15	0.200
276	A	5	5	1.00	15	0.333
277	A	5	4	1.00	17	0.235
278	A	4	3	1.00	17	0.176
279	A	4	4	1.00	17	0.235
280	A	3	3	1.00	15	0.200
281	A	6	5	1.00	15	0.333
282	A	5	4	1.00	17	0.235
283	A	3	3	1.00	16	0.188
284	A	4	4	1.00	16	0.250
285	A	5	4	1.00	16	0.250
286	A	6	4	1.00	16	0.250
287	A	4	4	1.00	10	0.400
288	A	3	3	1.00	10	0.300
289	A	3	3	1.00	10	0.300
290	A	5	5	1.00	12	0.417
291	A	4	4	1.00	12	0.333
292	A	3	3	1.00	12	0.250
293	A	7	6	1.00	25	0.240
294	A	6	6	1.00	25	0.240
295	A	5	5	1.00	23	0.217
296	A	7	5	1.00	23	0.217
297	A	8	6	1.00	25	0.240
298	A	9	7	1.00	25	0.280
299	A	9	8	1.00	25	0.320
300	A	8	8	1.00	25	0.320
301	A	7	7	1.00	25	0.280
302	A	6	6	1.00	16	0.375
303	A	5	5	1.00	25	0.200
304	A	6	6	1.00	25	0.240
305	A	7	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
306	A	8	6	1.00	25	0.240
307	A	7	6	1.00	25	0.240
308	A	6	5	1.00	23	0.217
309	A	8	6	1.00	23	0.261
310	A	8	6	1.00	25	0.240
311	A	9	7	1.00	25	0.280
312	A	10	8	1.00	25	0.320
313	A	9	8	1.00	25	0.320
314	A	8	8	1.00	25	0.320
315	A	7	7	1.00	16	0.438
316	A	7	7	1.00	25	0.280
317	A	6	6	1.00	25	0.240
318	A	7	6	1.00	25	0.240
319	A	8	8	1.00	16	0.500
320	A	6	5	1.00	25	0.200
321	A	5	5	1.00	25	0.200
322	A	4	4	1.00	23	0.174
323	A	7	5	1.00	23	0.217
324	A	8	6	1.00	25	0.240
325	A	9	7	1.00	25	0.280
326	A	8	8	1.00	25	0.320
327	A	7	7	1.00	25	0.280
328	A	6	6	1.00	25	0.240
329	A	3	3	1.00	16	0.188
330	A	5	5	1.00	25	0.200
331	A	6	6	1.00	25	0.240
332	A	7	6	1.00	25	0.240
333	A	6	5	1.00	25	0.200
334	A	5	5	1.00	25	0.200
335	A	5	5	1.00	23	0.217
336	A	8	6	1.00	23	0.261
337	A	9	7	1.00	25	0.280
338	A	10	8	1.00	25	0.320
339	A	8	8	1.00	25	0.320
340	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
341	A	4	4	1.00	25	0.160
342	A	4	4	1.00	16	0.250
343	A	6	6	1.00	25	0.240
344	A	7	6	1.00	25	0.240
345	A	8	6	1.00	25	0.240
346	A	6	5	1.00	25	0.200
347	A	6	6	1.00	25	0.240
348	A	6	5	1.00	23	0.217
349	A	9	7	1.00	23	0.304
350	A	10	7	1.00	25	0.280
351	A	11	8	1.00	25	0.320
352	A	8	8	1.00	25	0.320
353	A	6	6	1.00	25	0.240
354	A	6	6	1.00	25	0.240
355	A	6	6	1.00	16	0.375
356	A	7	7	1.00	25	0.280
357	A	8	7	1.00	25	0.280
358	A	9	7	1.00	25	0.280
359	A	4	4	1.00	23	0.174
360	A	3	3	1.00	25	0.120
361	A	5	4	1.00	23	0.174
362	A	4	4	1.00	23	0.174
363	A	3	3	1.00	21	0.143
364	A	5	5	1.00	21	0.238
365	A	6	6	1.00	23	0.261
366	A	7	7	1.00	23	0.304
367	A	3	3	1.00	23	0.130
368	A	3	3	1.00	23	0.130
369	A	3	3	1.00	23	0.130
370	A	3	3	1.00	14	0.214
371	A	3	3	1.00	23	0.130
372	A	3	3	1.00	23	0.130
373	A	3	3	1.00	23	0.130
374	A	6	5	1.00	14	0.357
375	A	6	5	1.00	14	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	6	5	1.00	14	0.357
377	A	3	2	1.00	12	0.167
378	A	14	12	1.00	14	0.857
379	A	21	13	1.00	14	0.929
380	A	7	6	1.00	8	0.750
381	A	4	3	1.00	14	0.214
382	A	4	3	1.00	14	0.214
383	A	4	3	1.00	14	0.214
384	A	4	2	1.00	12	0.167
385	A	13	9	1.00	14	0.643
386	A	23	10	1.00	14	0.714
387	A	8	7	1.00	16	0.438
388	A	4	4	1.00	16	0.250
389	A	8	7	1.00	17	0.412
390	A	8	7	1.00	15	0.467
391	A	11	10	1.00	15	0.667
392	A	12	9	1.00	17	0.529
393	A	9	7	1.00	17	0.412
394	A	9	8	1.00	15	0.533
395	A	13	12	1.00	15	0.800
396	A	7	6	1.00	17	0.353
397	A	4	4	1.00	15	0.267
398	A	9	8	1.00	15	0.533
399	A	4	4	1.00	17	0.235
400	A	6	6	1.00	17	0.353
401	A	6	6	1.00	15	0.400
402	A	12	11	1.00	15	0.733
403	A	7	6	1.00	17	0.353
404	A	7	7	1.00	15	0.467
405	A	14	12	1.00	15	0.800
406	A	9	5	1.00	29	0.172
407	A	7	4	1.00	27	0.148
408	A	14	7	1.00	29	0.241
409	A	15	7	1.00	29	0.241
410	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	4	4	1.00	23	0.174
412	A	3	3	1.00	14	0.214
413	A	4	4	1.00	23	0.174
414	A	4	4	1.00	23	0.174
415	A	4	4	1.00	23	0.174
416	A	4	4	1.00	23	0.174
417	A	4	4	1.00	21	0.190
418	A	4	4	1.00	21	0.190
419	A	4	4	1.00	23	0.174
420	A	0	0	0.00	0	0.000
421	A	4	4	1.00	23	0.174
422	A	4	4	1.00	25	0.160
423	A	4	4	1.00	25	0.160
424	A	0	0	0.00	0	0.000
425	A	4	4	1.00	21	0.190
426	A	3	3	1.00	19	0.158
427	A	3	3	1.00	19	0.158
428	A	2	1	1.00	21	0.048
429	A	3	2	1.00	21	0.095
430	A	3	2	1.00	21	0.095
431	A	3	2	1.00	21	0.095
432	A	3	2	1.00	21	0.095
433	A	2	1	1.00	21	0.048
434	A	3	3	1.00	21	0.143
435	A	4	4	1.00	21	0.190
436	A	5	4	1.00	21	0.190
437	A	5	5	1.00	23	0.217
438	A	4	4	1.00	21	0.190
439	A	5	4	1.00	21	0.190
440	A	4	3	1.00	23	0.130
441	A	3	2	1.00	23	0.087
442	A	3	2	1.00	23	0.087
443	A	3	2	1.00	23	0.087
444	A	3	2	1.00	23	0.087
445	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	3	2	1.00	23	0.087
447	A	5	4	1.00	23	0.174
448	A	4	4	1.00	23	0.174
449	A	5	5	1.00	23	0.217
450	A	5	5	1.00	23	0.217
451	A	4	4	1.00	23	0.174
452	A	2	2	1.00	21	0.095
453	A	3	3	1.00	21	0.143
454	A	4	3	1.00	23	0.130
455	A	4	3	1.00	23	0.130
456	A	4	3	1.00	23	0.130
457	A	4	3	1.00	23	0.130
458	A	3	3	1.00	23	0.130
459	A	2	2	1.00	23	0.087
460	A	5	5	1.00	23	0.217
461	A	6	6	1.00	23	0.261
462	A	6	6	1.00	23	0.261
463	A	5	5	1.00	23	0.217
464	A	3	3	1.00	23	0.130
465	A	3	3	1.00	21	0.143
466	A	5	4	1.00	21	0.190
467	A	5	4	1.00	23	0.174
468	A	5	4	1.00	23	0.174
469	A	5	4	1.00	23	0.174
470	A	3	3	1.00	23	0.130
471	A	3	3	1.00	23	0.130
472	A	6	6	1.00	23	0.261
473	A	7	6	1.00	23	0.261
474	A	2	2	1.00	23	0.087
475	A	3	3	1.00	25	0.120
476	A	2	2	1.00	25	0.080
477	A	4	3	1.00	23	0.130
478	A	4	3	1.00	23	0.130
479	A	3	3	1.00	23	0.130
480	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	3	3	1.00	23	0.130
482	A	2	2	1.00	23	0.087
483	A	2	2	1.00	21	0.095
484	A	2	2	1.00	21	0.095
485	A	2	2	1.00	23	0.087
486	A	0	0	0.00	0	0.000
487	A	0	0	0.00	0	0.000
488	A	0	0	0.00	0	0.000
489	A	0	0	0.00	0	0.000
490	A	0	0	0.00	0	0.000
491	A	9	6	1.00	25	0.240
492	A	7	6	1.00	25	0.240
493	A	3	3	1.00	25	0.120
494	A	0	0	0.00	0	0.000
495	A	0	0	0.00	0	0.000
496	A	4	4	1.00	23	0.174
497	A	4	4	1.00	25	0.160
498	A	4	4	1.00	25	0.160
499	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx$

Optimal. Leaf size=98

$$-\frac{b^2\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}-\frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}+\frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f}$$

```
[Out] -b^2*cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^2)^(1/2)/f-1/2*b^2*(b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)/f+1/4*b^2*(b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 3556}

$$-\frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}+\frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f}-\frac{b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\,\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((b^2*Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f) - (b^2*Tan
[e + f*x]*Sqrt[b*Tan[e + f*x]^2])/(2*f) + (b^2*Tan[e + f*x]^3*Sqrt[b*Tan[e
+ f*x]^2])/(4*f)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^2(e+fx)\right)^{5/2} dx = \left(b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan^5(e+fx)\,dx$$
$$= \frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f} - \left(b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan$$
$$= -\frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f} + \frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f} + \Big(b^2 c$$
$$= -\frac{b^2\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f} - \frac{b^2\tan(e+fx)\sqrt{b\tan^2(e}{2f}$$

Mathematica [A]
time = 0.26, size = 56, normalized size = 0.57

$$-\frac{\cot(e+fx)\left(-1+2\cot^2(e+fx)+4\cot^4(e+fx)\log(\cos(e+fx))\right)\left(b\tan^2(e+fx)\right)^{5/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/4*(Cot[e + f*x]*(-1 + 2*Cot[e + f*x]^2 + 4*Cot[e + f*x]^4*Log[Cos[e + f*
x]])*(b*Tan[e + f*x]^2)^(5/2))/f
```

Maple [A]
time = 0.15, size = 58, normalized size = 0.59

method	result
derivativedivides	$\frac{(b(\tan^2(fx+e)))^{\frac{5}{2}}(\tan^4(fx+e)-2(\tan^2(fx+e))+2\ln(1+\tan^2(fx+e)))}{4f\tan(fx+e)^5}$
default	$\frac{(b(\tan^2(fx+e)))^{\frac{5}{2}}(\tan^4(fx+e)-2(\tan^2(fx+e))+2\ln(1+\tan^2(fx+e)))}{4f\tan(fx+e)^5}$
risch	$\frac{b^2(e^{2i(fx+e)}+1)\sqrt{-\frac{b(e^{2i(fx+e)}-1)^2}{(e^{2i(fx+e)}+1)^2}}\,x}{e^{2i(fx+e)}-1} - \frac{2b^2(e^{2i(fx+e)}+1)\sqrt{-\frac{b(e^{2i(fx+e)}-1)^2}{(e^{2i(fx+e)}+1)^2}}\,(fx+e)}{(e^{2i(fx+e)}-1)f} - \frac{4ib^2\sqrt{-\frac{b(e^{2i(}{(e^{2i(}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(b*tan(f*x+e)^2)^(5/2)*(tan(f*x+e)^4-2*tan(f*x+e)^2+2*ln(1+tan(f*x+e)
^2))/tan(f*x+e)^5
```

Maxima [A]
time = 0.52, size = 50, normalized size = 0.51

$$\frac{b^{\frac{5}{2}}\tan\left(fx+e\right)^{4}-2\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{2}+2\,b^{\frac{5}{2}}\log\left(\tan\left(fx+e\right)^{2}+1\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b^(5/2)*tan(f*x + e)^4 - 2*b^(5/2)*tan(f*x + e)^2 + 2*b^(5/2)*log(tan(
f*x + e)^2 + 1))/f
```

Fricas [A]
time = 4.10, size = 79, normalized size = 0.81

$$\frac{\left(b^{2}\tan\left(fx+e\right)^{4}-2\,b^{2}\tan\left(fx+e\right)^{2}-2\,b^{2}\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)-3\,b^{2}\right)\sqrt{b\tan\left(fx+e\right)^{2}}}{4\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*tan(f*x + e)^4 - 2*b^2*tan(f*x + e)^2 - 2*b^2*log(1/(tan(f*x + e)^
2 + 1)) - 3*b^2)*sqrt(b*tan(f*x + e)^2)/(f*tan(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 757 vs. 2(95) = 190.
time = 1.09, size = 757, normalized size = 7.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*sgn(tan(f*x +
```

```
 e))*tan(f*x)^4*tan(e)^4 + 3*b^2*sgn(tan(f*x + e))*tan(f*x)^4*tan(e)^4 - 8*
b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*sgn(tan(f*x + e))*tan
(f*x)^3*tan(e)^3 + 2*b^2*sgn(tan(f*x + e))*tan(f*x)^4*tan(e)^2 - 8*b^2*sgn(
tan(f*x + e))*tan(f*x)^3*tan(e)^3 + 2*b^2*sgn(tan(f*x + e))*tan(f*x)^2*tan(
e)^4 + 12*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2
*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*sgn(tan(f*x
 + e))*tan(f*x)^2*tan(e)^2 - b^2*sgn(tan(f*x + e))*tan(f*x)^4 - 8*b^2*sgn(t
an(f*x + e))*tan(f*x)^3*tan(e) + 4*b^2*sgn(tan(f*x + e))*tan(f*x)^2*tan(e)^
2 - 8*b^2*sgn(tan(f*x + e))*tan(f*x)*tan(e)^3 - b^2*sgn(tan(f*x + e))*tan(e
)^4 - 8*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*sgn(tan(f*x +
 e))*tan(f*x)*tan(e) + 2*b^2*sgn(tan(f*x + e))*tan(f*x)^2 - 8*b^2*sgn(tan(f
*x + e))*tan(f*x)*tan(e) + 2*b^2*sgn(tan(f*x + e))*tan(e)^2 + 2*b^2*log(4*(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*sgn(tan(f*x + e)) + 3*b^2*sgn(ta
n(f*x + e)))*sqrt(b)/(f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*f
*tan(f*x)^2*tan(e)^2 - 4*f*tan(f*x)*tan(e) + f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan(e+f\,x)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int((b*tan(e + f*x)^2)^(5/2), x)
```

3.2 $\int (b\tan^2(e+fx))^{3/2}\, dx$

Optimal. Leaf size=61

$$\frac{b\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}+\frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}$$

```
[Out] b*cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^2)^(1/2)/f+1/2*b*(b*tan(f*x+e)^2)
^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 3556}

$$\frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}+\frac{b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\,\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f + (b*Tan[e + f*
x]*Sqrt[b*Tan[e + f*x]^2])/(2*f)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^2(e+fx)\right)^{3/2}\,dx = \left(b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan^3(e+fx)\,dx$$
$$= \frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f} - \left(b\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan(e$$
$$= \frac{b\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f} + \frac{b\tan(e+fx)\sqrt{b\tan^2(e+}}{2f}$$

Mathematica [A]
time = 0.08, size = 47, normalized size = 0.77

$$\frac{\cot^3(e+fx)\left(b\tan^2(e+fx)\right)^{3/2}\left(2\log(\cos(e+fx))+\tan^2(e+fx)\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cot[e + f*x]^3*(b*Tan[e + f*x]^2)^(3/2)*(2*Log[Cos[e + f*x]] + Tan[e + f*x
]^2))/(2*f)
```

Maple [A]
time = 0.06, size = 48, normalized size = 0.79

method	result
derivativedivides	$-\frac{\left(b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}\left(-\left(\tan^2(fx+e)\right)+\ln\left(1+\tan^2(fx+e)\right)\right)}{2f\tan(fx+e)^3}$
default	$-\frac{\left(b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}\left(-\left(\tan^2(fx+e)\right)+\ln\left(1+\tan^2(fx+e)\right)\right)}{2f\tan(fx+e)^3}$
risch	$-\frac{b\left(e^{2i(fx+e)}+1\right)\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,x}{e^{2i(fx+e)}-1}+\frac{2b\left(e^{2i(fx+e)}+1\right)\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,(fx+e)}{\left(e^{2i(fx+e)}-1\right)f}+\frac{2ib\sqrt{-\frac{b\left(e^{2i(f}\right.}{\left(e^{2i(f}\right.}}}{\left(e^{2i(fx+e)}-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(b*tan(f*x+e)^2)^(3/2)*(-tan(f*x+e)^2+ln(1+tan(f*x+e)^2))/tan(f*x+e)
^3
```

Maxima [A]
time = 0.51, size = 36, normalized size = 0.59

$$\frac{b^{\frac{3}{2}}\tan\left(fx+e\right)^2 - b^{\frac{3}{2}}\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^(3/2)*tan(f*x + e)^2 - b^(3/2)*log(tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 3.37, size = 56, normalized size = 0.92

$$\frac{\left(b\tan\left(fx+e\right)^{2}+b\log\left(\frac{1}{\tan(fx+e)^{2}+1}\right)+b\right)\sqrt{b\tan\left(fx+e\right)^{2}}}{2\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*tan(f*x + e)^2 + b*log(1/(tan(f*x + e)^2 + 1)) + b)*sqrt(b*tan(f*x +
 e)^2)/(f*tan(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(60) = 120.
time = 0.67, size = 278, normalized size = 4.56

$$\frac{\left(\log\left(\frac{4\left(\tan(fx)^{4}\tan(e)^{2}-2\tan(fx)^{3}\tan(e)+\tan(fx)^{2}\tan(e)^{2}+\tan(fx)^{2}-2\tan(fx)\tan(e)+1\right)}{\tan(e)^{2}+1}\right)\tan\left(fx\right)^{2}\tan\left(e\right)^{2}+\tan\left(fx\right)^{2}\tan\left(e\right)^{2}-2\log\left(\frac{4\left(\tan(fx)^{4}\tan(e)^{2}-2\tan(fx)^{3}\tan(e)+\tan(fx)^{2}\tan(e)^{2}+\tan(fx)^{2}-2\tan(fx)\tan(e)+1\right)}{\tan(e)^{2}+1}\right)\tan\left(fx\right)\tan\left(e\right)+\tan\left(fx\right)^{2}+\tan\left(e\right)^{2}+\log\left(\frac{4\left(\tan(fx)^{4}\tan(e)^{2}-2\tan(fx)^{3}\tan(e)+\tan(fx)^{2}\tan(e)^{2}+\tan(fx)^{2}-2\tan(fx)\tan(e)+1\right)}{\tan(e)^{2}+1}\right)+1\right)b^{\frac{3}{2}}\mathrm{sgn}(\tan\left(fx+e\right))}{2\left(f\tan\left(fx\right)^{2}\tan\left(e\right)^{2}-2\,f\tan\left(fx\right)\tan\left(e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2
+ tan(f*x)^2*tan(e)^2 - 2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)*tan(e) + tan(f*x)^2 + tan(e)^2 + log(4*(tan(f*x)^4*tan(e)^2 - 2*t
an(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1
)/(tan(e)^2 + 1)) + 1)*b^(3/2)*sgn(tan(f*x + e))/(f*tan(f*x)^2*tan(e)^2 - 2
*f*tan(f*x)*tan(e) + f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\tan(e+f\,x)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^(3/2),x)
[Out] int((b*tan(e + f*x)^2)^(3/2), x)
```

3.3 $\int \sqrt{b \tan^2(e + fx)}\, dx$

Optimal. Leaf size=32

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}$$

```
[Out] -cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3739, 3556}

$$-\frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)}\,\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b\tan^2(e+fx)}\,dx &= \left(\cot(e+fx)\sqrt{b\tan^2(e+fx)}\right)\int \tan(e+fx)\,dx\\
&= -\frac{\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^2],x]
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f)
```

Maple [A]
time = 0.04, size = 37, normalized size = 1.16

method	result
derivativedivides	$\frac{\sqrt{b\left(\tan^2\left(fx+e\right)\right)}\,\ln(1+\tan^2(fx+e))}{2f\tan(fx+e)}$
default	$\frac{\sqrt{b\left(\tan^2\left(fx+e\right)\right)}\,\ln(1+\tan^2(fx+e))}{2f\tan(fx+e)}$
risch	$\frac{\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,\left(e^{2i(fx+e)}+1\right)x}{e^{2i(fx+e)}-1} - \frac{2\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,\left(e^{2i(fx+e)}+1\right)(fx+e)}{\left(e^{2i(fx+e)}-1\right)f} - \frac{i\sqrt{-\frac{b\left(e^{2i(fx+e)}-\right.}{\left(e^{2i(fx+e)}+\right.}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/f*(b*tan(f*x+e)^2)^(1/2)/tan(f*x+e)*ln(1+tan(f*x+e)^2)
```

Maxima [A]
time = 0.51, size = 20, normalized size = 0.62

$$\frac{\sqrt{b}\,\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b)*log(tan(f*x + e)^2 + 1)/f
```

Fricas [A]
time = 3.50, size = 41, normalized size = 1.28

$$-\frac{\sqrt{b\tan\left(fx+e\right)^2}\,\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(f*x + e)^2)*log(1/(tan(f*x + e)^2 + 1))/(f*tan(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**2), x)
```

Giac [A]
time = 0.48, size = 25, normalized size = 0.78

$$-\frac{\sqrt{b}\,\log\left(\left|\cos\left(fx+e\right)\right|\right)\operatorname{sgn}(\tan\left(fx+e\right))}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b)*log(abs(cos(f*x + e)))*sgn(tan(f*x + e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{b\tan\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int((b*tan(e + f*x)^2)^(1/2), x)
```

3.4 $\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{b\tan^2(e+fx)}}$$

```
[Out] ln(sin(f*x+e))*tan(f*x+e)/f/(b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3739, 3556}

$$\frac{\tan(e+fx)\log(\sin(e+fx))}{f\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^2],x]
```

```
[Out] (Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^2])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx &= \frac{\tan(e+fx)\int\cot(e+fx)\,dx}{\sqrt{b\tan^2(e+fx)}}\\ &= \frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{b\tan^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 39, normalized size = 1.26

$$\frac{(\log(\cos(e+fx))+\log(\tan(e+fx)))\tan(e+fx)}{f\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^2],x]
[Out] ((Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*
x]^2])
```

Maple [A]
time = 0.04, size = 47, normalized size = 1.52

method	result
derivativedivides	$\frac{\tan(fx+e)(2\ln(\tan(fx+e))-\ln(1+\tan^2(fx+e)))}{2f\sqrt{b(\tan^2(fx+e))}}$
default	$\frac{\tan(fx+e)(2\ln(\tan(fx+e))-\ln(1+\tan^2(fx+e)))}{2f\sqrt{b(\tan^2(fx+e))}}$
risch	$\frac{(e^{2i(fx+e)}-1)x}{\sqrt{-\frac{b(e^{2i(fx+e)}-1)^2}{(e^{2i(fx+e)}+1)^2}}\,(e^{2i(fx+e)}+1)}-\frac{2(e^{2i(fx+e)}-1)(fx+e)}{\sqrt{-\frac{b(e^{2i(fx+e)}-1)^2}{(e^{2i(fx+e)}+1)^2}}\,(e^{2i(fx+e)}+1)f}-\frac{i(e^{2i(fx+e)}-1)\ln(e^{2i(fx+}}{\sqrt{-\frac{b(e^{2i(fx+e)}-1)^2}{(e^{2i(fx+e)}+1)^2}}\,(e^{2i(f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/f*tan(f*x+e)*(2*ln(tan(f*x+e))-ln(1+tan(f*x+e)^2))/(b*tan(f*x+e)^2)^(1/
2)
```

Maxima [A]
time = 0.51, size = 35, normalized size = 1.13

$$-\frac{\frac{\log\left(\tan(fx+e)^2+1\right)}{\sqrt{b}}-\frac{2\log(\tan(fx+e))}{\sqrt{b}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*(log(tan(f*x + e)^2 + 1)/sqrt(b) - 2*log(tan(f*x + e))/sqrt(b))/f
```

Fricas [A]
time = 2.08, size = 54, normalized size = 1.74

$$\frac{\sqrt{b\tan(fx+e)^2}\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)}{2bf\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))/(b*f*ta
n(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(32) = 64$.
time = 0.54, size = 87, normalized size = 2.81

$$\frac{\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{\sqrt{b}\,\mathrm{sgn}(\tan(fx+e))}-\frac{2\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{\sqrt{b}\,\mathrm{sgn}(\tan(fx+e))}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(sqrt(b)*sgn(tan(f*x
 + e))) - 2*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(sqrt(b)*s
gn(tan(f*x + e))))/f
```

Mupad [B]
time = 11.44, size = 34, normalized size = 1.10

$$\frac{\mathrm{atan}\left(\frac{\sqrt{-b}\,\tan(e+f\,x)}{\sqrt{b\tan\left(e+f\,x\right)^2}}\right)}{\sqrt{-b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] atan(((-b)^(1/2)*tan(e + f*x))/(b*tan(e + f*x)^2)^(1/2))/((-b)^(1/2)*f)
```

3.5 $\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}}-\frac{\log(\sin(e+fx))\tan(e+fx)}{bf\sqrt{b\tan^2(e+fx)}}$$

```
[Out] -1/2*cot(f*x+e)/b/f/(b*tan(f*x+e)^2)^(1/2)-ln(sin(f*x+e))*tan(f*x+e)/b/f/(b
*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 3556}

$$-\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}}-\frac{\tan(e+fx)\log(\sin(e+fx))}{bf\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] -1/2*Cot[e + f*x]/(b*f*Sqrt[b*Tan[e + f*x]^2]) - (Log[Sin[e + f*x]]*Tan[e +
 f*x])/(b*f*Sqrt[b*Tan[e + f*x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\tan(e+fx)\int \cot^3(e+fx)\,dx}{b\sqrt{b\tan^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}} - \frac{\tan(e+fx)\int \cot(e+fx)\,dx}{b\sqrt{b\tan^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}} - \frac{\log(\sin(e+fx))\tan(e+fx)}{bf\sqrt{b\tan^2(e+fx)}}$$

Mathematica [A]
time = 0.25, size = 56, normalized size = 0.85

$$-\frac{\left(\cot^2(e+fx)+2\log(\cos(e+fx))+2\log(\tan(e+fx))\right)\tan^3(e+fx)}{2f\left(b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] -1/2*((Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Log[Tan[e + f*x]])*Tan[e +
f*x]^3)/(f*(b*Tan[e + f*x]^2)^(3/2))
```

Maple [A]
time = 0.04, size = 64, normalized size = 0.97

method	result
derivativedivides	$-\frac{\tan(fx+e)\left(2\ln(\tan(fx+e))(\tan^2(fx+e))-\ln(1+\tan^2(fx+e))(\tan^2(fx+e))+1\right)}{2f\left(b(\tan^2(fx+e))\right)^{\frac{3}{2}}}$
default	$-\frac{\tan(fx+e)\left(2\ln(\tan(fx+e))(\tan^2(fx+e))-\ln(1+\tan^2(fx+e))(\tan^2(fx+e))+1\right)}{2f\left(b(\tan^2(fx+e))\right)^{\frac{3}{2}}}$
risch	$-\frac{(e^{2i(fx+e)}-1)x}{b(e^{2i(fx+e)}+1)\sqrt{-\frac{b(e^{2i(fx+e)}-1)^2}{(e^{2i(fx+e)}+1)^2}}}+\frac{2(e^{2i(fx+e)}-1)(fx+e)}{b(e^{2i(fx+e)}+1)\sqrt{-\frac{b(e^{2i(fx+e)}-1)^2}{(e^{2i(fx+e)}+1)^2}}\,f}-\frac{2i}{b(e^{2i(fx+e)}-1)(e^{2i(fx+e}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*tan(f*x+e)*(2*ln(tan(f*x+e))*tan(f*x+e)^2-ln(1+tan(f*x+e)^2)*tan(f*x
+e)^2+1)/(b*tan(f*x+e)^2)^(3/2)
```

Maxima [A]
time = 0.53, size = 49, normalized size = 0.74

$$\frac{\frac{\log\left(\tan(fx+e)^2+1\right)}{b^{\frac{3}{2}}}-\frac{2\log(\tan(fx+e))}{b^{\frac{3}{2}}}-\frac{1}{b^{\frac{3}{2}}\tan(fx+e)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(tan(f*x + e)^2 + 1)/b^(3/2) - 2*log(tan(f*x + e))/b^(3/2) - 1/(b^(
3/2)*tan(f*x + e)^2))/f
```

Fricas [A]
time = 1.19, size = 75, normalized size = 1.14

$$-\frac{\sqrt{b\tan\left(fx+e\right)^{2}}\left(\log\left(\frac{\tan(fx+e)^{2}}{\tan(fx+e)^{2}+1}\right)\tan\left(fx+e\right)^{2}+\tan\left(fx+e\right)^{2}+1\right)}{2\,b^{2}f\tan\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(f*x + e)^2)*(log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f
*x + e)^2 + tan(f*x + e)^2 + 1)/(b^2*f*tan(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(65) = 130.
time = 0.55, size = 190, normalized size = 2.88

$$-\frac{\frac{4\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{\sqrt{b}\,\mathrm{sgn}(\tan(fx+e))}-\frac{8\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{\sqrt{b}\,\mathrm{sgn}(\tan(fx+e))}-\frac{\left(\sqrt{b}+\frac{4\sqrt{b}\,(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{b(\cos(fx+e)-1)\mathrm{sgn}(\tan(fx+e))}-\frac{\cos(fx+e)-1}{\sqrt{b}\,(\cos(fx+e)+1)\mathrm{sgn}(\tan(fx+e))}}{8\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(sqrt(b)*sgn(tan(
f*x + e))) - 8*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(sqrt(b
)*sgn(tan(f*x + e))) - (sqrt(b) + 4*sqrt(b)*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1))*(cos(f*x + e) + 1)/(b*(cos(f*x + e) - 1)*sgn(tan(f*x + e))) - (cos(
f*x + e) - 1)/(sqrt(b)*(cos(f*x + e) + 1)*sgn(tan(f*x + e))))/(b*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^2)^(3/2), x)
```

3.6 $\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}}-\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\log(\sin(e+fx))\tan(e+fx)}{b^2f\sqrt{b\tan^2(e+fx)}}$$

```
[Out] 1/2*cot(f*x+e)/b^2/f/(b*tan(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^3/b^2/f/(b*tan(f
*x+e)^2)^(1/2)+ln(sin(f*x+e))*tan(f*x+e)/b^2/f/(b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 3556}

$$-\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\tan(e+fx)\log(\sin(e+fx))}{b^2f\sqrt{b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] Cot[e + f*x]/(2*b^2*f*Sqrt[b*Tan[e + f*x]^2]) - Cot[e + f*x]^3/(4*b^2*f*Sqr
t[b*Tan[e + f*x]^2]) + (Log[Sin[e + f*x]]*Tan[e + f*x])/(b^2*f*Sqrt[b*Tan[e
 + f*x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\tan(e+fx)\int \cot^5(e+fx)\,dx}{b^2\sqrt{b\tan^2(e+fx)}}$$
$$= -\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}} - \frac{\tan(e+fx)\int \cot^3(e+fx)\,dx}{b^2\sqrt{b\tan^2(e+fx)}}$$
$$= \frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}} + \frac{\tan(e+fx)\int \cot(e+fx)\,d}{b^2\sqrt{b\tan^2(e+fx)}}$$
$$= \frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}} + \frac{\log(\sin(e+fx))\tan(e+fx}{b^2f\sqrt{b\tan^2(e+fx)}}$$

Mathematica [A]
time = 0.18, size = 68, normalized size = 0.70

$$\frac{\left(2\cot^2(e+fx)-\cot^4(e+fx)+4\log(\cos(e+fx))+4\log(\tan(e+fx))\right)\tan^5(e+fx)}{4f\left(b\tan^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((2*Cot[e + f*x]^2 - Cot[e + f*x]^4 + 4*Log[Cos[e + f*x]] + 4*Log[Tan[e + f
*x]])*Tan[e + f*x]^5)/(4*f*(b*Tan[e + f*x]^2)^(5/2))
```

Maple [A]
time = 0.05, size = 74, normalized size = 0.76

method	result
derivativedivides	$\frac{\tan(fx+e)\left(4\ln(\tan(fx+e))\left(\tan^4(fx+e)\right)-2\ln\left(1+\tan^2(fx+e)\right)\left(\tan^4(fx+e)\right)+2\left(\tan^2(fx+e)\right)-1\right)}{4f\left(b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}$
default	$\frac{\tan(fx+e)\left(4\ln(\tan(fx+e))\left(\tan^4(fx+e)\right)-2\ln\left(1+\tan^2(fx+e)\right)\left(\tan^4(fx+e)\right)+2\left(\tan^2(fx+e)\right)-1\right)}{4f\left(b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}$
risch	$\frac{\left(e^{2i(fx+e)}-1\right)x}{b^2\left(e^{2i(fx+e)}+1\right)\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}} - \frac{2\left(e^{2i(fx+e)}-1\right)(fx+e)}{b^2\left(e^{2i(fx+e)}+1\right)\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,f} + \frac{4i\left(e^{6i(fx+e)}-\right.}{b^2\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx\cdot\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

[Out] 1/4/f*tan(f*x+e)*(4*ln(tan(f*x+e))*tan(f*x+e)^4-2*ln(1+tan(f*x+e)^2)*tan(f*x+e)^4+2*tan(f*x+e)^2-1)/(b*tan(f*x+e)^2)^(5/2)

Maxima [A]
time = 0.52, size = 70, normalized size = 0.72

$$-\frac{\frac{2\,\log\left(\tan\left(fx+e\right)^2+1\right)}{b^{\frac{5}{2}}}-\frac{4\,\log\left(\tan\left(fx+e\right)\right)}{b^{\frac{5}{2}}}-\frac{2\,\sqrt{b}\,\tan\left(fx+e\right)^2-\sqrt{b}}{b^3\,\tan\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(tan(f*x + e)^2 + 1)/b^(5/2) - 4*log(tan(f*x + e))/b^(5/2) - (2*sqrt(b)*tan(f*x + e)^2 - sqrt(b))/(b^3*tan(f*x + e)^4))/f
```

Fricas [A]
time = 1.78, size = 89, normalized size = 0.92

$$\frac{\left(2\,\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^4+3\,\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2-1\right)\sqrt{b\tan\left(fx+e\right)^2}}{4\,b^3 f\tan\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + 3*tan(f*x + e)^4 + 2*tan(f*x + e)^2 - 1)*sqrt(b*tan(f*x + e)^2)/(b^3*f*tan(f*x + e)^5
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(94) = 188.
time = 0.58, size = 254, normalized size = 2.62

$$\frac{\frac{32\,\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{b^{\frac{5}{2}}\,\mathrm{sgn}(\tan(fx+e))}-\frac{64\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{b^{\frac{5}{2}}\,\mathrm{sgn}(\tan(fx+e))}-\frac{\left(\sqrt{b}+\frac{12\,\sqrt{b}\,(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,\sqrt{b}\,(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{b^3(\cos(fx+e)-1)^2\mathrm{sgn}(\tan(fx+e))}-\frac{\frac{12\,b^{\frac{7}{2}}\,(\cos(fx+e)-1)\mathrm{sgn}(\tan(fx+e))}{\cos(fx+e)+1}+\frac{b^{\frac{7}{2}}\,(\cos(fx+e)-1)^2\mathrm{sgn}(\tan(fx+e))}{(\cos(fx+e)+1)^2}}{b^6}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(32*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(b^(5/2)*sgn(tan
(f*x + e))) - 64*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(b^(5
/2)*sgn(tan(f*x + e))) - (sqrt(b) + 12*sqrt(b)*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 48*sqrt(b)*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x
+ e) + 1)^2/(b^3*(cos(f*x + e) - 1)^2*sgn(tan(f*x + e))) - (12*b^(7/2)*(cos
(f*x + e) - 1)*sgn(tan(f*x + e))/(cos(f*x + e) + 1) + b^(7/2)*(cos(f*x + e)
 - 1)^2*sgn(tan(f*x + e))/(cos(f*x + e) + 1)^2)/b^6)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^2)^(5/2), x)
```

3.7 $\int (b\tan^3(e+fx))^{5/2}\,dx$

Optimal. Leaf size=364

$$-\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f}-\frac{b^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{b^2\text{ArcTan}\left(1+\right.}{}$$

```
[Out] -2*b^2*cot(f*x+e)*(b*tan(f*x+e)^3)^(1/2)/f+1/2*b^2*arctan(-1+2^(1/2)*tan(f*
x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/2*b^2*arcta
n(1+2^(1/2)*tan(f*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(
3/2)-1/4*b^2*ln(1-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^3)^(1/
2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/4*b^2*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x
+e))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+2/5*b^2*(b*tan(f*x+e
)^3)^(1/2)*tan(f*x+e)/f-2/9*b^2*(b*tan(f*x+e)^3)^(1/2)*tan(f*x+e)^3/f+2/13*
b^2*(b*tan(f*x+e)^3)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.11, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{b^2\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}-\frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f}+\frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f}+\frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f}-\frac{b^2\sqrt{b\tan^3(e+fx)}\log\left(\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}+\frac{b^2\sqrt{b\tan^3(e+fx)}\log\left(\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}-\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(5/2),x]
```

```
[Out] (-2*b^2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/f - (b^2*ArcTan[1 - Sqrt[2]*Sq
rt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) +
(b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2
]*f*Tan[e + f*x]^(3/2)) - (b^2*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e +
 f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) + (b^2*Log[
1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*S
qrt[2]*f*Tan[e + f*x]^(3/2)) + (2*b^2*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/
(5*f) - (2*b^2*Tan[e + f*x]^3*Sqrt[b*Tan[e + f*x]^3])/(9*f) + (2*b^2*Tan[e
+ f*x]^5*Sqrt[b*Tan[e + f*x]^3])/(13*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^3(e+fx)\right)^{5/2}\,dx = \frac{\left(b^2\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{15}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f} - \frac{\left(b^2\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{11}{2}}(e+fx)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f} + \frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f} +$$

$$= \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f} + 2b^2$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - 2b^2$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - 2b^2$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - 2b^2$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - 2b^2$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - 2b^2$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{b^2\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e\right.}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+}$$

$$= -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{b^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}$$

<u>**Mathematica**</u> [A]

time = 0.55, size = 199, normalized size = 0.55

$$\frac{b\left(b\tan^3(e+fx)\right)^{3/2}\left(-1170\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)+1170\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)-585\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)+585\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-4680\sqrt{\tan(e+fx)}+936\tan^{\frac{5}{2}}(e+fx)-520\tan^{\frac{9}{2}}(e+fx)+360\tan^{\frac{13}{2}}(e+fx)\right)}{2340f\tan^{\frac{9}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(5/2),x]
```

```
[Out] (b*(b*Tan[e + f*x]^3)^(3/2)*(-1170*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e +
f*x]]] + 1170*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - 585*Sqrt[2]*
Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] + 585*Sqrt[2]*Log[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 4680*Sqrt[Tan[e + f*x]] + 936*Ta
n[e + f*x]^(5/2) - 520*Tan[e + f*x]^(9/2) + 360*Tan[e + f*x]^(13/2)))/(2340
*f*Tan[e + f*x]^(9/2))
```

Maple [A]
time = 0.07, size = 263, normalized size = 0.72

method	result
derivativedivides	$\frac{\left(b\left(\tan^{3}(fx+e)\right)\right)^{\frac{5}{2}}\left(360\left(b\tan(fx+e)\right)^{\frac{13}{2}}-520b^{2}\left(b\tan(fx+e)\right)^{\frac{9}{2}}+585b^{6}\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(fx+e)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(j\right.}}{b\tan(fx+e)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(j\right.}}\right.\right.}{}$
default	$\frac{\left(b\left(\tan^{3}(fx+e)\right)\right)^{\frac{5}{2}}\left(360\left(b\tan(fx+e)\right)^{\frac{13}{2}}-520b^{2}\left(b\tan(fx+e)\right)^{\frac{9}{2}}+585b^{6}\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(fx+e)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(j\right.}}{b\tan(fx+e)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(j\right.}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2340/f*(b*tan(f*x+e)^3)^(5/2)*(360*(b*tan(f*x+e))^(13/2)-520*b^2*(b*tan(f
*x+e))^(9/2)+585*b^6*(b^2)^(1/4)*2^(1/2)*ln((b*tan(f*x+e)+(b^2)^(1/4)*(b*ta
n(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(f*x+e)-(b^2)^(1/4)*(b*tan(f*x+e
))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*b^6*(b^2)^(1/4)*2^(1/2)*arctan((2^(1/2)
*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+1170*b^6*(b^2)^(1/4)*2^(1/2
)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+936*b^4*(b
*tan(f*x+e))^(5/2)-4680*b^6*(b*tan(f*x+e))^(1/2))/tan(f*x+e)^5/(b*tan(f*x+e
))^(5/2)/b^4
```

Maxima [A]
time = 0.52, size = 188, normalized size = 0.52

$$\frac{360\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{\frac{13}{2}}-520\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{\frac{9}{2}}+936\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{\frac{5}{2}}+585\left(2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)+\sqrt{2}\sqrt{b}\log\left(\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)-\sqrt{2}\sqrt{b}\log\left(-\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)\right)b^{2}-4680\,b^{\frac{5}{2}}\sqrt{\tan\left(fx+e\right)}}{2340\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(360*b^(5/2)*tan(f*x + e)^(13/2) - 520*b^(5/2)*tan(f*x + e)^(9/2) +
936*b^(5/2)*tan(f*x + e)^(5/2) + 585*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*
(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(
```

```
sqrt(2) - 2*sqrt(tan(f*x + e)))) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(f*x
 + e)) + tan(f*x + e) + 1) - sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(f*x + e)
) + tan(f*x + e) + 1))*b^2 - 4680*b^(5/2)*sqrt(tan(f*x + e)))/f
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{3}\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**(5/2), x)
```

Giac [**A**]
time = 0.65, size = 305, normalized size = 0.84

$$\frac{1}{2340}\left(\frac{1170\sqrt{2}\,b\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{f}+\frac{1170\sqrt{2}\,b\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan\left(fx+e\right)}\right)}{2\sqrt{|b|}}\right)}{f}+\frac{585\sqrt{2}\,b\sqrt{|b|}\log\left(b\tan\left(fx+e\right)+\sqrt{2}\sqrt{b\tan\left(fx+e\right)}\sqrt{|b|}+|b|\right)}{f}-\frac{585\sqrt{2}\,b\sqrt{|b|}\log\left(b\tan\left(fx+e\right)-\sqrt{2}\sqrt{b\tan\left(fx+e\right)}\sqrt{|b|}+|b|\right)}{f}+\frac{8\left(45\sqrt{b\tan\left(fx+e\right)}\,b^{66}f^{12}\tan\left(fx+e\right)^{6}-65\sqrt{b\tan\left(fx+e\right)}\,b^{66}f^{12}\tan\left(fx+e\right)^{4}+117\sqrt{b\tan\left(fx+e\right)}\,b^{66}f^{12}\tan\left(fx+e\right)^{2}-585\sqrt{b\tan\left(fx+e\right)}\,b^{66}f^{12}\right)}{b^{65}f^{13}}\right)b\,\mathrm{sgn}(\tan\left(fx+e\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2340*(1170*sqrt(2)*b*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)
) + 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/f + 1170*sqrt(2)*b*sqrt(abs(b))*a
rctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x + e)))/sqrt(abs
(b)))/f + 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*ta
n(f*x + e))*sqrt(abs(b)) + abs(b))/f - 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan
(f*x + e) - sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/f + 8*(45*s
qrt(b*tan(f*x + e))*b^66*f^12*tan(f*x + e)^6 - 65*sqrt(b*tan(f*x + e))*b^66
*f^12*tan(f*x + e)^4 + 117*sqrt(b*tan(f*x + e))*b^66*f^12*tan(f*x + e)^2 -
585*sqrt(b*tan(f*x + e))*b^66*f^12)/(b^65*f^13))*b*sgn(tan(f*x + e))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^3\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^3)^(5/2),x)
```

```
[Out] int((b*tan(e + f*x)^3)^(5/2), x)
```

3.8 $\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=286

$$-\frac{2b\sqrt{b\tan^3(e+fx)}}{3f}-\frac{b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}+\frac{b\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(e+fx)}\right.}{\sqrt{2}\,f\tan^{\frac{3}{2}}}$$

```
[Out] -2/3*b*(b*tan(f*x+e)^3)^(1/2)/f+1/2*b*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2))*(
b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/2*b*arctan(1+2^(1/2)*tan
(f*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/4*b*ln(1
-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(
f*x+e)^(3/2)-1/4*b*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^
3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+2/7*b*(b*tan(f*x+e)^3)^(1/2)*tan(f*x+e)
^2/f
```

Rubi [A]
time = 0.08, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}+\frac{b\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(e+fx)}+1\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}-\frac{2b\sqrt{b\tan^3(e+fx)}}{3f}+\frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f}+\frac{b\sqrt{b\tan^3(e+fx)}\,\log\left(\tan(e+fx)-\sqrt{2}\,\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}-\frac{b\sqrt{b\tan^3(e+fx)}\,\log\left(\tan(e+fx)+\sqrt{2}\,\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Tan[e + f*x]^3])/(3*f) - (b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*
x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) + (b*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x
]^(3/2)) + (b*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan
[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (b*Log[1 + Sqrt[2]*Sqrt[Ta
n[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*
x]^(3/2)) + (2*b*Tan[e + f*x]^2*Sqrt[b*Tan[e + f*x]^3])/(7*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx = \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{9}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} - \frac{\left(b\sqrt{b\tan^3(e+fx)}\right)\int \tan^{\frac{5}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(b\sqrt{b\tan^3(e+f\right.}}{\tan}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(b\sqrt{b\tan^3(e+f\right.}}{}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(2b\sqrt{b\tan^3(e+\right.}}{}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} - \frac{\left(b\sqrt{b\tan^3(e+f\right.}}{}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f} + \frac{\left(b\sqrt{b\tan^3(e+f\right.}}{}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} + \frac{b\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\sqrt{b\tan^3}}{2\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}$$

$$= -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} - \frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 54, normalized size = 0.19

$$\frac{2b\sqrt{b\tan^3(e+fx)}\left(-7+7\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+3\tan^2(e+fx)\right)}{21f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Tan[e + f*x]^3]*(-7 + 7*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e +
 f*x]^2] + 3*Tan[e + f*x]^2))/(21*f)
```

Maple [A]
time = 0.03, size = 236, normalized size = 0.83

method	result
derivativedivides	$\frac{\left(b\left(\tan^{3}(fx+e)\right)\right)^{\frac{3}{2}}\left(24(b\tan(fx+e))^{\frac{7}{2}}\left(b^{2}\right)^{\frac{1}{4}}+21b^{4}\sqrt{2}\,\ln\left(-\frac{\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^{2}}\cdots}{b\tan(fx+e)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^{2}}\cdots}\right.\right.}{8\cdots}$
default	$\frac{\left(b\left(\tan^{3}(fx+e)\right)\right)^{\frac{3}{2}}\left(24(b\tan(fx+e))^{\frac{7}{2}}\left(b^{2}\right)^{\frac{1}{4}}+21b^{4}\sqrt{2}\,\ln\left(-\frac{\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^{2}}\cdots}{b\tan(fx+e)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^{2}}\cdots}\right.\right.}{8\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84/f*(b*tan(f*x+e)^3)^(3/2)*(24*(b*tan(f*x+e))^(7/2)*(b^2)^(1/4)+21*b^4*2
^(1/2)*ln(-((b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/
2))/(b*tan(f*x+e)+(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42
*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))
+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/
4))-56*b^2*(b*tan(f*x+e))^(3/2)*(b^2)^(1/4))/tan(f*x+e)^3/(b*tan(f*x+e))^(3
/2)/b^2/(b^2)^(1/4)
```

Maxima [A]
time = 0.52, size = 148, normalized size = 0.52

$$\frac{24\,b^{\frac{3}{2}}\tan(fx+e)^{\frac{7}{2}}-56\,b^{\frac{3}{2}}\tan(fx+e)^{\frac{3}{2}}+21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(fx+e)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(fx+e)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(fx+e)}+\tan(fx+e)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(fx+e)}+\tan(fx+e)+1\right)\right)b^{\frac{3}{2}}}{84f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(24*b^(3/2)*tan(f*x + e)^(7/2) - 56*b^(3/2)*tan(f*x + e)^(3/2) + 21*(2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2)*log(sqrt(2)*
sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(f*x
+ e)) + tan(f*x + e) + 1))*b^(3/2))/f
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^3\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**(3/2), x)
```

Giac [A]
time = 0.52, size = 264, normalized size = 0.92

$$\frac{1}{84}\,b\left(\frac{42\,\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}+2\,\sqrt{b\tan\left(fx+e\right)}\right)}{2\,\sqrt{|b|}}\right)}{bf}+\frac{42\,\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}-2\,\sqrt{b\tan\left(fx+e\right)}\right)}{2\,\sqrt{|b|}}\right)}{bf}-\frac{21\,\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\sqrt{|b|}+|b|\right)}{bf}+\frac{21\,\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\sqrt{|b|}+|b|\right)}{bf}+\frac{8\left(3\,\sqrt{b\tan\left(fx+e\right)}\,b^{21}f^{6}\tan\left(fx+e\right)^{3}-7\,\sqrt{b\tan\left(fx+e\right)}\,b^{21}f^{6}\tan\left(fx+e\right)\right)}{b^{21}f^{7}}\right)\mathrm{sgn}\left(\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/84*b*(42*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b*f) + 42*sqrt(2)*abs(b)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b))
)/(b*f) - 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f
*x + e))*sqrt(abs(b)) + abs(b))/(b*f) + 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(f
*x + e) - sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b*f) + 8*(3*
sqrt(b*tan(f*x + e))*b^21*f^6*tan(f*x + e)^3 - 7*sqrt(b*tan(f*x + e))*b^21*
f^6*tan(f*x + e))/(b^21*f^7))*sgn(tan(f*x + e))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^3\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^3)^(3/2),x)
```

```
[Out] int((b*tan(e + f*x)^3)^(3/2), x)
```

3.9 $\int \sqrt{b\tan^3(e+fx)}\,dx$

Optimal. Leaf size=255

$$\frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f}+\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}-\frac{\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{}\right.}{\sqrt{}}$$

```
[Out] 2*cot(f*x+e)*(b*tan(f*x+e)^3)^(1/2)/f-1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2
))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)-1/2*arctan(1+2^(1/2)*t
an(f*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)+1/4*ln(1
-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^3)^(1/2)/f*2^(1/2)/tan(
f*x+e)^(3/2)-1/4*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*(b*tan(f*x+e)^3)
^(1/2)/f*2^(1/2)/tan(f*x+e)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}-\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(e+fx)}+1\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}+\frac{\sqrt{b\tan^3(e+fx)}\log\left(\tan(e+fx)-\sqrt{2}\,\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}-\frac{\sqrt{b\tan^3(e+fx)}\log\left(\tan(e+fx)+\sqrt{2}\,\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}+\frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] (2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^3])/f + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[e
+ f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (ArcTan[1
 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Sqrt[b*Tan[e + f*x]^3])/(Sqrt[2]*f*Tan[e + f
*x]^(3/2)) + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan
[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]^(3/2)) - (Log[1 + Sqrt[2]*Sqrt[Tan[
e + f*x]] + Tan[e + f*x]]*Sqrt[b*Tan[e + f*x]^3])/(2*Sqrt[2]*f*Tan[e + f*x]
^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{b\tan^3(e+fx)}\,dx = \frac{\sqrt{b\tan^3(e+fx)}\int \tan^{\frac{3}{2}}(e+fx)\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\int \frac{1}{\sqrt{\tan(e+fx)}}\,dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx, x, \text{ta}\right.}{f\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\left(2\sqrt{b\tan^3(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\right.}{f\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(e}\right.}{f\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{\sqrt{b\tan^3(e+fx)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \right.}{2f\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)}{2\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}$$

$$= \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+}}{\sqrt{2}\,f\tan^{\frac{3}{2}}(e+fx)}$$

Mathematica [A]
time = 0.17, size = 161, normalized size = 0.63

$$\frac{\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)+8\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{4f\tan^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*Sqrt[Tan[e + f*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + T
an[e + f*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] +
 8*Sqrt[Tan[e + f*x]])*Sqrt[b*Tan[e + f*x]^3])/(4*f*Tan[e + f*x]^(3/2))
```

Maple [A]
time = 0.03, size = 205, normalized size = 0.80

method	result
derivativedivides	$-\frac{\sqrt{b\left(\tan^{3}\left(fx+e\right)\right)}\,\left(\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan\left(fx+e\right)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{b^{2}}}{b\tan\left(fx+e\right)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\cdots\right.}{4f\tan\cdots}$
default	$-\frac{\sqrt{b\left(\tan^{3}\left(fx+e\right)\right)}\,\left(\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan\left(fx+e\right)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{b^{2}}}{b\tan\left(fx+e\right)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(fx+e\right)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\cdots\right.}{4f\tan\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(b*tan(f*x+e)^3)^(1/2)*((b^2)^(1/4)*2^(1/2)*ln((b*tan(f*x+e)+(b^2)^(
1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(f*x+e)-(b^2)^(1/4)*(b
*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*(b^2)^(1/4)*2^(1/2)*arctan((2^(1
/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*(b^2)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))-8*(b*tan(f*x+e
))^(1/2))/tan(f*x+e)/(b*tan(f*x+e))^(1/2)
```

Maxima [A]
time = 0.52, size = 140, normalized size = 0.55

$$-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)+\sqrt{2}\sqrt{b}\log\left(\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)-\sqrt{2}\sqrt{b}\log\left(-\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan\left(fx+e\right)+1\right)-8\sqrt{b}\sqrt{\tan\left(fx+e\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))
) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e))))
 + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - sqr
t(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - 8*sqrt(b
)*sqrt(tan(f*x + e)))/f
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan^3{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**3), x)
```

Giac [**A**]
time = 0.48, size = 203, normalized size = 0.80

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{f}+\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{f}+\frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan(fx+e)+\sqrt{2}\sqrt{b\tan(fx+e)}\sqrt{|b|}+|b|\right)}{f}-\frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan(fx+e)-\sqrt{2}\sqrt{b\tan(fx+e)}\sqrt{|b|}+|b|\right)}{f}-\frac{8\sqrt{b\tan(fx+e)}}{f}\right)\mathrm{sgn}(\tan(fx+e))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*s
qrt(b*tan(f*x + e)))/sqrt(abs(b)))/f + 2*sqrt(2)*sqrt(abs(b))*arctan(-1/2*s
qrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/f + sq
rt(2)*sqrt(abs(b))*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(a
bs(b)) + abs(b))/f - sqrt(2)*sqrt(abs(b))*log(b*tan(f*x + e) - sqrt(2)*sqrt
(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/f - 8*sqrt(b*tan(f*x + e))/f)*sgn(t
an(f*x + e))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^3)^(1/2),x)
```

```
[Out] int((b*tan(e + f*x)^3)^(1/2), x)
```

3.10 $\int \frac{1}{\sqrt{b \tan^3(e+fx)}}\,dx$

Optimal. Leaf size=255

$$-\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}}+\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}$$

```
[Out] -2*tan(f*x+e)/f/(b*tan(f*x+e)^3)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/
2))*tan(f*x+e)^(3/2)/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)-1/2*arctan(1+2^(1/2)*
tan(f*x+e)^(1/2))*tan(f*x+e)^(3/2)/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)-1/4*ln(
1-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(3/2)/f*2^(1/2)/(b*tan(f*
x+e)^3)^(1/2)+1/4*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(3/2
)/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\tan^{\frac{3}{2}}(e+fx)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{\frac{3}{2}}(e+fx)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}-\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{\frac{3}{2}}(e+fx)\log\left(\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}f\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{\frac{3}{2}}(e+fx)\log\left(\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}f\sqrt{b\tan^3(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] (-2*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan
[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*f*Sqrt[b*Tan[e + f*x]^3]) - (ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*f*Sqrt[b*Tan
[e + f*x]^3]) - (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e +
 f*x]^(3/2))/(2*Sqrt[2]*f*Sqrt[b*Tan[e + f*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[T
an[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*f*Sqrt[b*Tan[e
+ f*x]^3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx = \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{3}{2}}(e+fx)}\,dx}{\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int \sqrt{\tan(e+fx)}\,dx}{\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\left(2\tan^{\frac{3}{2}}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(e+fx)}\right)}{f\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(e+fx)}\right)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(e+fx)}\right)}{2f\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)\tan^{\frac{3}{2}}(e+f}{2\sqrt{2}f\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{-1}(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.02, size = 43, normalized size = 0.17

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(e+fx)\right)\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^3],x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*Sqrt[
b*Tan[e + f*x]^3])
```

Maple [A]
time = 0.04, size = 211, normalized size = 0.83

method	result
derivativedivides	$-\frac{\tan(fx+e)\left(\sqrt{2}\,\sqrt{b\tan(fx+e)}\,\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}{b\tan(fx+e)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\right.}{}$
default	$-\frac{\tan(fx+e)\left(\sqrt{2}\,\sqrt{b\tan(fx+e)}\,\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}{b\tan(fx+e)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*tan(f*x+e)*(2^(1/2)*(b*tan(f*x+e))^(1/2)*ln(-((b^2)^(1/4)*(b*tan(f*x
+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2))/(b*tan(f*x+e)+(b^2)^(1/4)*(b*t
an(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*2^(1/2)*(b*tan(f*x+e))^(1/2)*arcta
n((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*2^(1/2)*(b*tan(
f*x+e))^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4)
)+8*(b^2)^(1/4))/(b*tan(f*x+e)^3)^(1/2)/(b^2)^(1/4)
```

Maxima [A]
time = 0.51, size = 133, normalized size = 0.52

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(fx+e)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(fx+e)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(fx+e)}+\tan(fx+e)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(fx+e)}+\tan(fx+e)+1\right)}{\sqrt{b}}+\frac{8}{\sqrt{b}\sqrt{\tan(fx+e)}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2)*log(
```

```
sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(
tan(f*x + e)) + tan(f*x + e) + 1))/sqrt(b) + 8/(sqrt(b)*sqrt(tan(f*x + e)))
)/f
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \tan^{3}{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**3), x)
```

Giac [**A**]
time = 0.55, size = 263, normalized size = 1.03

$$-\frac{1}{4}\,b^{2}\left(\frac{2\,\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}+2\,\sqrt{b\tan\left(fx+e\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{4}f\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{2\,\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}-2\,\sqrt{b\tan\left(fx+e\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{4}f\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}-\frac{\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\sqrt{|b|}+|b|\right)}{b^{4}f\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\sqrt{|b|}+|b|\right)}{b^{4}f\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{8}{\sqrt{b\tan\left(fx+e\right)}\,b^{2}f\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b^2*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b^4*f*sgn(tan(f*x + e))) + 2*sqrt(2
)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x
 + e)))/sqrt(abs(b)))/(b^4*f*sgn(tan(f*x + e))) - sqrt(2)*abs(b)^(3/2)*log(
b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b^4*f
*sgn(tan(f*x + e))) + sqrt(2)*abs(b)^(3/2)*log(b*tan(f*x + e) - sqrt(2)*sqr
t(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b^4*f*sgn(tan(f*x + e))) + 8/(sqr
t(b*tan(f*x + e))*b^2*f*sgn(tan(f*x + e))))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^3)^(1/2),x)
[Out] int(1/(b*tan(e + f*x)^3)^(1/2), x)
```

3.11 $\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=298

$$\frac{2}{3bf\sqrt{b\tan^3(e+fx)}}-\frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}}-\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}\,bf\sqrt{b\tan^3(e+fx)}}+\frac{\mathrm{ArcTan}\left(1\right.}{}$$

```
[Out] 2/3/b/f/(b*tan(f*x+e)^3)^(1/2)-2/7*cot(f*x+e)^2/b/f/(b*tan(f*x+e)^3)^(1/2)+
1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2))*tan(f*x+e)^(3/2)/b/f*2^(1/2)/(b*tan
(f*x+e)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(f*x+e)^(1/2))*tan(f*x+e)^(3/2)/b/
f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)-1/4*ln(1-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+
e))*tan(f*x+e)^(3/2)/b/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)+1/4*ln(1+2^(1/2)*ta
n(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(3/2)/b/f*2^(1/2)/(b*tan(f*x+e)^3)^(1
/2)
```

Rubi [A]
time = 0.09, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\tan^{\frac{3}{2}}(e+fx)\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(e+fx)}\right)}{\sqrt{2}\,bf\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{\frac{3}{2}}(e+fx)\mathrm{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}\,bf\sqrt{b\tan^3(e+fx)}}+\frac{2}{3bf\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{\frac{3}{2}}(e+fx)\log\left(\tan(e+fx)-\sqrt{2}\,\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}\,bf\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{\frac{3}{2}}(e+fx)\log\left(\tan(e+fx)+\sqrt{2}\,\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}\,bf\sqrt{b\tan^3(e+fx)}}-\frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(-3/2),x]
```

```
[Out] 2/(3*b*f*Sqrt[b*Tan[e + f*x]^3]) - (2*Cot[e + f*x]^2)/(7*b*f*Sqrt[b*Tan[e +
 f*x]^3]) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sq
rt[2]*b*f*Sqrt[b*Tan[e + f*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]
*Tan[e + f*x]^(3/2))/(Sqrt[2]*b*f*Sqrt[b*Tan[e + f*x]^3]) - (Log[1 - Sqrt[2
]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b*f*Sqr
t[b*Tan[e + f*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*
Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b*f*Sqrt[b*Tan[e + f*x]^3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{9}{2}}(e+fx)}\,dx}{b\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{5}{2}}(e+fx)}\,dx}{b\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\sqrt{\tan(e+fx}}}{b\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1-}\right.}{bf\sqrt{b\tan^3(e}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\left(2\tan^{\frac{3}{2}}(e+fx)\right)\text{Subst}\left(\int \frac{}{1+}\right.}{bf\sqrt{b\tan^3(}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx\right.}{bf\sqrt{b\tan^3(e+}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}}\right.}{2bf\sqrt{b\tan^3}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right.}{2\sqrt{2}\,bf\sqrt{b\,t}}$$

$$= \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(e+fx}\right.}{\sqrt{2}\,bf\sqrt{b\tan^3(e}}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 45, normalized size = 0.15

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\tan^2(e+fx)\right)\tan(e+fx)}{7f\left(b\tan^3(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Tan[e + f*x]^3)^(-3/2),x]

[Out] (-2*Hypergeometric2F1[-7/4, 1, -3/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(7*f*(b*Tan[e + f*x]^3)^(3/2))

Maple [A]
time = 0.03, size = 233, normalized size = 0.78

method	result
derivativedivides	$\frac{\tan(fx+e)\left(42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\tan(fx+e)\right)^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}+\left(b^2\right)^{\frac{1}{4}}}{\left(b^2\right)^{\frac{1}{4}}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\tan(fx+e\right.\right.}{}$
default	$\frac{\tan(fx+e)\left(42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\tan(fx+e)\right)^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}+\left(b^2\right)^{\frac{1}{4}}}{\left(b^2\right)^{\frac{1}{4}}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\tan(fx+e\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*tan(f*x+e)^3)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/84/f*tan(f*x+e)/b^4*(42*(b^2)^(1/4)*2^(1/2)*(b*tan(f*x+e))^(7/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+42*(b^2)^(1/4)*2^(1/2)*(b*tan(f*x+e))^(7/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+21*(b^2)^(1/4)*2^(1/2)*(b*tan(f*x+e))^(7/2)*ln((b*tan(f*x+e)+(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(f*x+e)-(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+56*b^4*tan(f*x+e)^2-24*b^4)/(b*tan(f*x+e)^3)^(3/2)

Maxima [A]
time = 0.51, size = 173, normalized size = 0.58

$$\frac{\frac{21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(fx+e\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(fx+e\right)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan(fx+e)+1\right)-\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(fx+e\right)}+\tan(fx+e)+1\right)\right)}{b^{\frac{3}{2}}}+\frac{8\left(21\sqrt{\tan\left(fx+e\right)}+\frac{7}{\tan(fx+e)^{\frac{3}{2}}}-\frac{3}{\tan(fx+e)^{\frac{7}{2}}}\right)}{b^{\frac{3}{2}}}-\frac{168\sqrt{\tan\left(fx+e\right)}}{b^{\frac{3}{2}}}}{84\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="maxima")

[Out] 1/84*(21*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) + sqrt(2)*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - sqrt(2)*log(-sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1))/b^(3/2) + 8*(21*sqrt(tan(f*x + e)) + 7/tan(f*x + e)^(3/2) - 3/tan(f*x + e)^(7/2))/b^(3/2) - 168*sqrt(tan(f*x + e))/b^(3/2))/f

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^{3}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**(-3/2), x)
```

Giac [A]
time = 0.68, size = 293, normalized size = 0.98

$$\frac{1}{84}\,b^{4}\left(\frac{42\,\sqrt{2}\,\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}+2\,\sqrt{b\tan\left(fx+e\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{6}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{42\,\sqrt{2}\,\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}-2\,\sqrt{b\tan\left(fx+e\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{6}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{21\,\sqrt{2}\,\sqrt{|b|}\log\left(b\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\sqrt{|b|}+|b|\right)}{b^{6}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)}-\frac{21\,\sqrt{2}\,\sqrt{|b|}\log\left(b\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\sqrt{|b|}+|b|\right)}{b^{6}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{8\left(7\,b^{2}\tan\left(fx+e\right)^{2}-3\,b^{2}\right)}{\sqrt{b\tan\left(fx+e\right)}\,b^{7}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)\tan\left(fx+e\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/84*b^4*(42*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
+ 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b^6*f*sgn(tan(f*x + e))) + 42*sqrt
(2)*sqrt(abs(b))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f
*x + e)))/sqrt(abs(b)))/(b^6*f*sgn(tan(f*x + e))) + 21*sqrt(2)*sqrt(abs(b))
*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(
b^6*f*sgn(tan(f*x + e))) - 21*sqrt(2)*sqrt(abs(b))*log(b*tan(f*x + e) - sqr
t(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b^6*f*sgn(tan(f*x + e)))
+ 8*(7*b^2*tan(f*x + e)^2 - 3*b^2)/(sqrt(b*tan(f*x + e))*b^7*f*sgn(tan(f*x
+ e))*tan(f*x + e)^3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\tan\left(e+f\,x\right)}^3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^3)^(3/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^3)^(3/2), x)
```

3.12 $\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=364

$$-\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}}+\frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}}-\frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}}+\frac{2\tan(e+fx)}{b^2f\sqrt{b\tan^3(e+fx)}}-\frac{\text{ArcTan}}{}$$

```
[Out] -2/5*cot(f*x+e)/b^2/f/(b*tan(f*x+e)^3)^(1/2)+2/9*cot(f*x+e)^3/b^2/f/(b*tan(
f*x+e)^3)^(1/2)-2/13*cot(f*x+e)^5/b^2/f/(b*tan(f*x+e)^3)^(1/2)+2*tan(f*x+e)
/b^2/f/(b*tan(f*x+e)^3)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2))*tan(f
*x+e)^(3/2)/b^2/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(f
*x+e)^(1/2))*tan(f*x+e)^(3/2)/b^2/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)+1/4*ln(1
-2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(3/2)/b^2/f*2^(1/2)/(b*tan
(f*x+e)^3)^(1/2)-1/4*ln(1+2^(1/2)*tan(f*x+e)^(1/2)+tan(f*x+e))*tan(f*x+e)^(
3/2)/b^2/f*2^(1/2)/(b*tan(f*x+e)^3)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\tan^{\frac{3}{2}}(e+fx)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{\frac{3}{2}}(e+fx)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}}+\frac{2\tan(e+fx)}{b^2f\sqrt{b\tan^3(e+fx)}}+\frac{\tan^{\frac{3}{2}}(e+fx)\log\left(\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}}-\frac{\tan^{\frac{3}{2}}(e+fx)\log\left(\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}}-\frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}}+\frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}}-\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^(-5/2),x]
```

```
[Out] (-2*Cot[e + f*x])/(5*b^2*f*Sqrt[b*Tan[e + f*x]^3]) + (2*Cot[e + f*x]^3)/(9*
b^2*f*Sqrt[b*Tan[e + f*x]^3]) - (2*Cot[e + f*x]^5)/(13*b^2*f*Sqrt[b*Tan[e +
 f*x]^3]) + (2*Tan[e + f*x])/(b^2*f*Sqrt[b*Tan[e + f*x]^3]) - (ArcTan[1 - S
qrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(Sqrt[2]*b^2*f*Sqrt[b*Tan[e
+ f*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]*Tan[e + f*x]^(3/2))/(S
qrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3]) + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]
+ Tan[e + f*x]]*Tan[e + f*x]^(3/2))/(2*Sqrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3]
) - (Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]*Tan[e + f*x]^(3/2))
/(2*Sqrt[2]*b^2*f*Sqrt[b*Tan[e + f*x]^3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{15}{2}}(e+fx)}\,dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} - \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{11}{2}}(e+fx)}\,dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$= \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} + \frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{7}{2}}(e+fx)}}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} -$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} +$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} +$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} +$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} +$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} +$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} +$$

$$= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} + \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.04, size = 45, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{13}{4},1;-\frac{9}{4};-\tan^2(e+fx)\right)\tan(e+fx)}{13f\left(b\tan^3(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-13/4, 1, -9/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(13*f*
(b*Tan[e + f*x]^3)^(5/2))
```

Maple [A]
time = 0.03, size = 272, normalized size = 0.75

method	result
derivativedivides	$\frac{\tan(fx+e)\left(585\sqrt{2}\,(b\tan(fx+e))^{\frac{13}{2}}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}{b\tan(fx+e)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\,\mathrm{ta}\right.}{}$
default	$\frac{\tan(fx+e)\left(585\sqrt{2}\,(b\tan(fx+e))^{\frac{13}{2}}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}{b\tan(fx+e)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\,\mathrm{ta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2340/f*tan(f*x+e)/b^6*(585*2^(1/2)*(b*tan(f*x+e))^(13/2)*ln(-((b^2)^(1/4)
*(b*tan(f*x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2))/(b*tan(f*x+e)+(b^2)
^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*2^(1/2)*(b*tan(f*x+e
))^(13/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+11
70*2^(1/2)*(b*tan(f*x+e))^(13/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)
^(1/4))/(b^2)^(1/4))+4680*(b^2)^(1/4)*b^6*tan(f*x+e)^6-936*b^6*(b^2)^(1/4)*
tan(f*x+e)^4+520*b^6*(b^2)^(1/4)*tan(f*x+e)^2-360*b^6*(b^2)^(1/4))/(b*tan(f
*x+e)^3)^(5/2)/(b^2)^(1/4)
```

Maxima [A]
time = 0.55, size = 182, normalized size = 0.50

$$\frac{\frac{585\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(fx+e)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(fx+e)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(fx+e)}+\tan(fx+e)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(fx+e)}+\tan(fx+e)+1\right)\right)}{b^{\frac{5}{2}}}+\frac{8\left(\frac{585\sqrt{b}}{\sqrt{\tan(fx+e)}}-\frac{117\sqrt{b}}{\tan(fx+e)^{\frac{5}{2}}}+\frac{65\sqrt{b}}{\tan(fx+e)^{\frac{9}{2}}}-\frac{45\sqrt{b}}{\tan(fx+e)^{\frac{13}{2}}}\right)}{b^3}}{2340\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(585*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e))))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2
```

```
)*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)
*sqrt(tan(f*x + e)) + tan(f*x + e) + 1))/b^(5/2) + 8*(585*sqrt(b)/sqrt(tan(
f*x + e)) - 117*sqrt(b)/tan(f*x + e)^(5/2) + 65*sqrt(b)/tan(f*x + e)^(9/2)
- 45*sqrt(b)/tan(f*x + e)^(13/2))/b^3)/f
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^{3}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**3)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**3)**(-5/2), x)
```

Giac [A]
time = 0.92, size = 321, normalized size = 0.88

$$\frac{1}{2340}\,b^{6}\left(\frac{1170\,\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}+2\,\sqrt{b\tan\left(fx+e\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{10}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{1170\,\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}-2\,\sqrt{b\tan\left(fx+e\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{10}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)}-\frac{585\,\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\sqrt{|b|}+|b|\right)}{b^{10}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{585\,\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{b\tan\left(fx+e\right)}\,\sqrt{|b|}+|b|\right)}{b^{10}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{8\left(585\,b^{6}\tan\left(fx+e\right)^{6}-117\,b^{6}\tan\left(fx+e\right)^{4}+65\,b^{6}\tan\left(fx+e\right)^{2}-45\,b^{6}\right)}{\sqrt{b\tan\left(fx+e\right)}\,b^{14}f\mathrm{sgn}\left(\tan\left(fx+e\right)\right)\tan\left(fx+e\right)^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2340*b^6*(1170*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(
b)) + 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b^10*f*sgn(tan(f*x + e))) + 11
70*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(
b*tan(f*x + e)))/sqrt(abs(b)))/(b^10*f*sgn(tan(f*x + e))) - 585*sqrt(2)*abs
(b)^(3/2)*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) +
abs(b))/(b^10*f*sgn(tan(f*x + e))) + 585*sqrt(2)*abs(b)^(3/2)*log(b*tan(f*x
 + e) - sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b^10*f*sgn(tan
(f*x + e))) + 8*(585*b^6*tan(f*x + e)^6 - 117*b^6*tan(f*x + e)^4 + 65*b^6*t
an(f*x + e)^2 - 45*b^6)/(sqrt(b*tan(f*x + e))*b^14*f*sgn(tan(f*x + e))*tan(
f*x + e)^6))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^3)^(5/2),x)
[Out] int(1/(b*tan(e + f*x)^3)^(5/2), x)
```

3.13 $\int \left(b\tan^4(e+fx)\right)^{5/2} \, dx$

Optimal. Leaf size=182

$$\frac{b^2\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-b^2x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}-\frac{b^2\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f}+b^2 \mathrm{t}$$

```
[Out] b^2*cot(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f-b^2*x*cot(f*x+e)^2*(b*tan(f*x+e)^4)
^(1/2)-1/3*b^2*(b*tan(f*x+e)^4)^(1/2)*tan(f*x+e)/f+1/5*b^2*(b*tan(f*x+e)^4)
^(1/2)*tan(f*x+e)^3/f-1/7*b^2*(b*tan(f*x+e)^4)^(1/2)*tan(f*x+e)^5/f+1/9*b^2
*(b*tan(f*x+e)^4)^(1/2)*tan(f*x+e)^7/f
```

Rubi [A]

time = 0.05, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{b^2\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f}+\frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f}-\frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f}+\frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}-b^2x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}+\frac{b^2\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(5/2),x]
```

```
[Out] (b^2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - b^2*x*Cot[e + f*x]^2*Sqrt[b*T
an[e + f*x]^4] - (b^2*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/(3*f) + (b^2*Tan
[e + f*x]^3*Sqrt[b*Tan[e + f*x]^4])/(5*f) - (b^2*Tan[e + f*x]^5*Sqrt[b*Tan[
e + f*x]^4])/(7*f) + (b^2*Tan[e + f*x]^7*Sqrt[b*Tan[e + f*x]^4])/(9*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx = \left(b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^{10}(e+fx)\,dx$$

$$= \frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f} - \left(b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^8$$

$$= -\frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f} + \frac{b^2\tan^7(e+fx)\sqrt{b\tan^4(e+fx)}}{9f} + \left(b^2 \mathrm{c}\right.$$

$$= \frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \frac{b^2\tan^5(e+fx)\sqrt{b\tan^4(e+fx)}}{7f} + \frac{b^2\tan^7}{}$$

$$= -\frac{b^2\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b^2\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \frac{b^2\tan^5}{}$$

$$= \frac{b^2\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - \frac{b^2\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b^2\tan^3(e}{}$$

$$= \frac{b^2\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - b^2x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)} - \frac{b^2\tan(e}{}$$

Mathematica [A]
time = 0.50, size = 86, normalized size = 0.47

$$\frac{\cot(e+fx)\left(35-45\cot^2(e+fx)+63\cot^4(e+fx)-105\cot^6(e+fx)+315\cot^8(e+fx)-315\mathrm{ArcTan}(\tan(e+fx))\cot^9(e+fx)\right)\left(b\tan^4(e+fx)\right)^{5/2}}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(5/2),x]
```

```
[Out] (Cot[e + f*x]*(35 - 45*Cot[e + f*x]^2 + 63*Cot[e + f*x]^4 - 105*Cot[e + f*x
]^6 + 315*Cot[e + f*x]^8 - 315*ArcTan[Tan[e + f*x]]*Cot[e + f*x]^9)*(b*Tan[
e + f*x]^4)^(5/2))/(315*f)
```

Maple [A]
time = 0.06, size = 84, normalized size = 0.46

method	result
derivativedivides	$-\frac{\left(b\left(\tan^4(fx+e)\right)\right)^{\frac{5}{2}}\left(-35\left(\tan^9(fx+e)\right)+45\left(\tan^7(fx+e)\right)-63\left(\tan^5(fx+e)\right)+105\left(\tan^3(fx+e)\right)+315\arctan(\tan(fx+e)\right.}{315f\tan(fx+e)^{10}}$
default	$-\frac{\left(b\left(\tan^4(fx+e)\right)\right)^{\frac{5}{2}}\left(-35\left(\tan^9(fx+e)\right)+45\left(\tan^7(fx+e)\right)-63\left(\tan^5(fx+e)\right)+105\left(\tan^3(fx+e)\right)+315\arctan(\tan(fx+e)\right.}{315f\tan(fx+e)^{10}}$

risch	$\frac{b^2(e^{2i(fx+e)}+1)^2\sqrt{\frac{b(e^{2i(fx+e)}-1)^4}{(e^{2i(fx+e)}+1)^4}}\,x}{(e^{2i(fx+e)}-1)^2} - \frac{2ib^2\sqrt{\frac{b(e^{2i(fx+e)}-1)^4}{(e^{2i(fx+e)}+1)^4}}\,(1575\,e^{16i(fx+e)}+6300\,e^{14i(fx+e)}+21000\,e^{12i}}{315(}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*tan(f*x+e)^4)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/315/f*(b*tan(f*x+e)^4)^(5/2)*(-35*tan(f*x+e)^9+45*tan(f*x+e)^7-63*tan(f*x+e)^5+105*tan(f*x+e)^3+315*arctan(tan(f*x+e))-315*tan(f*x+e))/tan(f*x+e)^10

Maxima [A]
time = 0.51, size = 85, normalized size = 0.47

$$\frac{35\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{9}-45\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{7}+63\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{5}-105\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{3}-315\left(fx+e\right)b^{\frac{5}{2}}+315\,b^{\frac{5}{2}}\tan\left(fx+e\right)}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="maxima")

[Out] 1/315*(35*b^(5/2)*tan(f*x + e)^9 - 45*b^(5/2)*tan(f*x + e)^7 + 63*b^(5/2)*tan(f*x + e)^5 - 105*b^(5/2)*tan(f*x + e)^3 - 315*(f*x + e)*b^(5/2) + 315*b^(5/2)*tan(f*x + e))/f

Fricas [A]
time = 2.91, size = 103, normalized size = 0.57

$$\frac{\left(35\,b^{2}\tan\left(fx+e\right)^{9}-45\,b^{2}\tan\left(fx+e\right)^{7}+63\,b^{2}\tan\left(fx+e\right)^{5}-105\,b^{2}\tan\left(fx+e\right)^{3}-315\,b^{2}fx+315\,b^{2}\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{4}}}{315\,f\tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="fricas")

[Out] 1/315*(35*b^2*tan(f*x + e)^9 - 45*b^2*tan(f*x + e)^7 + 63*b^2*tan(f*x + e)^5 - 105*b^2*tan(f*x + e)^3 - 315*b^2*f*x + 315*b^2*tan(f*x + e))*sqrt(b*tan(f*x + e)^4)/(f*tan(f*x + e)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^{4}\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*tan(f*x+e)**4)**(5/2),x)

```
[Out] Integral((b*tan(e + f*x)**4)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(174) = 348$.
time = 4.75, size = 1023, normalized size = 5.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/315*(315*b^2*f*x*tan(f*x)^9*tan(e)^9 - 2835*b^2*f*x*tan(f*x)^8*tan(e)^8
+ 315*b^2*tan(f*x)^9*tan(e)^8 + 315*b^2*tan(f*x)^8*tan(e)^9 + 11340*b^2*f*x
*tan(f*x)^7*tan(e)^7 - 105*b^2*tan(f*x)^9*tan(e)^6 - 2835*b^2*tan(f*x)^8*ta
n(e)^7 - 2835*b^2*tan(f*x)^7*tan(e)^8 - 105*b^2*tan(f*x)^6*tan(e)^9 - 26460
*b^2*f*x*tan(f*x)^6*tan(e)^6 + 63*b^2*tan(f*x)^9*tan(e)^4 + 945*b^2*tan(f*x
)^8*tan(e)^5 + 11340*b^2*tan(f*x)^7*tan(e)^6 + 11340*b^2*tan(f*x)^6*tan(e)^
7 + 945*b^2*tan(f*x)^5*tan(e)^8 + 63*b^2*tan(f*x)^4*tan(e)^9 + 39690*b^2*f*
x*tan(f*x)^5*tan(e)^5 - 45*b^2*tan(f*x)^9*tan(e)^2 - 567*b^2*tan(f*x)^8*tan
(e)^3 - 3780*b^2*tan(f*x)^7*tan(e)^4 - 26460*b^2*tan(f*x)^6*tan(e)^5 - 2646
0*b^2*tan(f*x)^5*tan(e)^6 - 3780*b^2*tan(f*x)^4*tan(e)^7 - 567*b^2*tan(f*x)
^3*tan(e)^8 - 45*b^2*tan(f*x)^2*tan(e)^9 - 39690*b^2*f*x*tan(f*x)^4*tan(e)^
4 + 35*b^2*tan(f*x)^9 + 405*b^2*tan(f*x)^8*tan(e) + 2268*b^2*tan(f*x)^7*tan
(e)^2 + 8820*b^2*tan(f*x)^6*tan(e)^3 + 39690*b^2*tan(f*x)^5*tan(e)^4 + 3969
0*b^2*tan(f*x)^4*tan(e)^5 + 8820*b^2*tan(f*x)^3*tan(e)^6 + 2268*b^2*tan(f*x
)^2*tan(e)^7 + 405*b^2*tan(f*x)*tan(e)^8 + 35*b^2*tan(e)^9 + 26460*b^2*f*x*
tan(f*x)^3*tan(e)^3 - 45*b^2*tan(f*x)^7 - 567*b^2*tan(f*x)^6*tan(e) - 3780*
b^2*tan(f*x)^5*tan(e)^2 - 26460*b^2*tan(f*x)^4*tan(e)^3 - 26460*b^2*tan(f*x
)^3*tan(e)^4 - 3780*b^2*tan(f*x)^2*tan(e)^5 - 567*b^2*tan(f*x)*tan(e)^6 - 4
5*b^2*tan(e)^7 - 11340*b^2*f*x*tan(f*x)^2*tan(e)^2 + 63*b^2*tan(f*x)^5 + 94
5*b^2*tan(f*x)^4*tan(e) + 11340*b^2*tan(f*x)^3*tan(e)^2 + 11340*b^2*tan(f*x
)^2*tan(e)^3 + 945*b^2*tan(f*x)*tan(e)^4 + 63*b^2*tan(e)^5 + 2835*b^2*f*x*t
an(f*x)*tan(e) - 105*b^2*tan(f*x)^3 - 2835*b^2*tan(f*x)^2*tan(e) - 2835*b^2
*tan(f*x)*tan(e)^2 - 105*b^2*tan(e)^3 - 315*b^2*f*x + 315*b^2*tan(f*x) + 31
5*b^2*tan(e))*sqrt(b)/(f*tan(f*x)^9*tan(e)^9 - 9*f*tan(f*x)^8*tan(e)^8 + 36
*f*tan(f*x)^7*tan(e)^7 - 84*f*tan(f*x)^6*tan(e)^6 + 126*f*tan(f*x)^5*tan(e)
^5 - 126*f*tan(f*x)^4*tan(e)^4 + 84*f*tan(f*x)^3*tan(e)^3 - 36*f*tan(f*x)^2
*tan(e)^2 + 9*f*tan(f*x)*tan(e) - f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan(e+f\,x)^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^4)^(5/2),x)
[Out] int((b*tan(e + f*x)^4)^(5/2), x)
```

3.14 $\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{b\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)} - \frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b\tan^3(e$$

```
[Out] b*cot(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f-b*x*cot(f*x+e)^2*(b*tan(f*x+e)^4)^(1/
2)-1/3*b*(b*tan(f*x+e)^4)^(1/2)*tan(f*x+e)/f+1/5*b*(b*tan(f*x+e)^4)^(1/2)*t
an(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = $\{3739, 3554, 8\}$

$$-\frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)} + \frac{b\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(3/2),x]
```

```
[Out] (b*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - b*x*Cot[e + f*x]^2*Sqrt[b*Tan[e
 + f*x]^4] - (b*Tan[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/(3*f) + (b*Tan[e + f*x
]^3*Sqrt[b*Tan[e + f*x]^4])/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx &= \left(b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^6(e+fx)\,dx\\ &= \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} - \left(b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^4\\ &= -\frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f} + \Big(b\cot^2\\ &= \frac{b\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - \frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f} + \frac{b\tan^3(e+}{}\\ &= \frac{b\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)} - \frac{b\tan(e-}{}\end{aligned}$$

Mathematica [A]
time = 0.52, size = 66, normalized size = 0.60

$$\frac{\cot(e+fx)\left(3-5\cot^2(e+fx)+15\cot^4(e+fx)-15\text{ArcTan}(\tan(e+fx))\cot^5(e+fx)\right)\left(b\tan^4(e+fx)\right)^{3/2}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(3/2),x]
```

```
[Out] (Cot[e + f*x]*(3 - 5*Cot[e + f*x]^2 + 15*Cot[e + f*x]^4 - 15*ArcTan[Tan[e +
 f*x]]*Cot[e + f*x]^5)*(b*Tan[e + f*x]^4)^(3/2))/(15*f)
```

Maple [A]
time = 0.03, size = 64, normalized size = 0.58

method	result
derivativedivides	$-\frac{\left(b\left(\tan^4(fx+e)\right)\right)^{\frac{3}{2}}\left(-3\left(\tan^5(fx+e)\right)+5\left(\tan^3(fx+e)\right)+15\arctan(\tan(fx+e))-15\tan(fx+e)\right)}{15f\tan(fx+e)^6}$
default	$-\frac{\left(b\left(\tan^4(fx+e)\right)\right)^{\frac{3}{2}}\left(-3\left(\tan^5(fx+e)\right)+5\left(\tan^3(fx+e)\right)+15\arctan(\tan(fx+e))-15\tan(fx+e)\right)}{15f\tan(fx+e)^6}$
risch	$\frac{b\left(e^{2i(fx+e)}+1\right)^2\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\,x}{\left(e^{2i(fx+e)}-1\right)^2} - \frac{2ib\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\left(45\,e^{8i(fx+e)}+90\,e^{6i(fx+e)}+140\,e^{4i(fx+e)}+70\,e^{2}\right.}{15\left(e^{2i(fx+e)}-1\right)^2\left(e^{2i(fx+e)}+1\right)^3 f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(b*tan(f*x+e)^4)^(3/2)*(-3*tan(f*x+e)^5+5*tan(f*x+e)^3+15*arctan(ta
n(f*x+e))-15*tan(f*x+e))/tan(f*x+e)^6
```

Maxima [A]
time = 0.50, size = 57, normalized size = 0.52

$$\frac{3\,b^{\frac{3}{2}}\tan\left(fx+e\right)^{5}-5\,b^{\frac{3}{2}}\tan\left(fx+e\right)^{3}-15\,(fx+e)b^{\frac{3}{2}}+15\,b^{\frac{3}{2}}\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^(3/2)*tan(f*x + e)^5 - 5*b^(3/2)*tan(f*x + e)^3 - 15*(f*x + e)*b^
(3/2) + 15*b^(3/2)*tan(f*x + e))/f
```

Fricas [A]
time = 2.14, size = 67, normalized size = 0.61

$$\frac{\left(3\,b\tan\left(fx+e\right)^{5}-5\,b\tan\left(fx+e\right)^{3}-15\,bfx+15\,b\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{4}}}{15\,f\tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 - 5*b*tan(f*x + e)^3 - 15*b*f*x + 15*b*tan(f*x + e
))*sqrt(b*tan(f*x + e)^4)/(f*tan(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^{4}\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(106) = 212$.
time = 2.41, size = 1079, normalized size = 9.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/60*(15*pi - 60*f*x*tan(f*x)^5*tan(e)^5 - 15*pi*sgn(2*tan(f*x)^2*tan(e) +
2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^5 - 15*pi*ta
n(f*x)^5*tan(e)^5 + 30*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*ta
n(f*x)^5*tan(e)^5 + 30*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*ta
n(f*x)^5*tan(e)^5 + 300*f*x*tan(f*x)^4*tan(e)^4 + 75*pi*sgn(2*tan(f*x)^2*ta
n(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^4 + 7
5*pi*tan(f*x)^4*tan(e)^4 - 150*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan
(e)))*tan(f*x)^4*tan(e)^4 - 150*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e)
 - 1))*tan(f*x)^4*tan(e)^4 - 60*tan(f*x)^5*tan(e)^4 - 60*tan(f*x)^4*tan(e)^
5 - 600*f*x*tan(f*x)^3*tan(e)^3 - 150*pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*
x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^3 + 20*tan(f*x)^5*ta
n(e)^2 - 150*pi*tan(f*x)^3*tan(e)^3 + 300*arctan((tan(f*x)*tan(e) - 1)/(tan
(f*x) + tan(e)))*tan(f*x)^3*tan(e)^3 + 300*arctan((tan(f*x) + tan(e))/(tan(
f*x)*tan(e) - 1))*tan(f*x)^3*tan(e)^3 + 300*tan(f*x)^4*tan(e)^3 + 300*tan(f
*x)^3*tan(e)^4 + 20*tan(f*x)^2*tan(e)^5 + 600*f*x*tan(f*x)^2*tan(e)^2 + 150
*pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*
tan(f*x)^2*tan(e)^2 - 12*tan(f*x)^5 - 100*tan(f*x)^4*tan(e) + 150*pi*tan(f*
x)^2*tan(e)^2 - 300*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*tan(f
*x)^2*tan(e)^2 - 300*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(
f*x)^2*tan(e)^2 - 600*tan(f*x)^3*tan(e)^2 - 600*tan(f*x)^2*tan(e)^3 - 100*t
an(f*x)*tan(e)^4 - 12*tan(e)^5 - 300*f*x*tan(f*x)*tan(e) - 75*pi*sgn(2*tan(
f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e
) + 20*tan(f*x)^3 - 75*pi*tan(f*x)*tan(e) + 150*arctan((tan(f*x)*tan(e) - 1
)/(tan(f*x) + tan(e)))*tan(f*x)*tan(e) + 150*arctan((tan(f*x) + tan(e))/(ta
n(f*x)*tan(e) - 1))*tan(f*x)*tan(e) + 300*tan(f*x)^2*tan(e) + 300*tan(f*x)*
tan(e)^2 + 20*tan(e)^3 + 60*f*x + 15*pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x
)*tan(e)^2 - 2*tan(f*x) - 2*tan(e)) - 30*arctan((tan(f*x)*tan(e) - 1)/(tan(
f*x) + tan(e))) - 30*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1)) - 60
*tan(f*x) - 60*tan(e))*b^(3/2)/(f*tan(f*x)^5*tan(e)^5 - 5*f*tan(f*x)^4*tan(
e)^4 + 10*f*tan(f*x)^3*tan(e)^3 - 10*f*tan(f*x)^2*tan(e)^2 + 5*f*tan(f*x)*t
an(e) - f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan(e+f\,x)^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^4)^(3/2),x)
[Out] int((b*tan(e + f*x)^4)^(3/2), x)
```

3.15 $\int \sqrt{b \tan^4(e+fx)}\, dx$

Optimal. Leaf size=50

$$\frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

```
[Out] cot(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f-x*cot(f*x+e)^2*(b*tan(f*x+e)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$\frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] (Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^4])/f - x*Cot[e + f*x]^2*Sqrt[b*Tan[e + f
*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{b\tan^4(e+fx)}\,dx = \left(\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int \tan^2(e+fx)\,dx$$
$$= \frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - \left(\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\right)\int 1\,dx$$
$$= \frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f} - x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.82

$$-\frac{\cot(e+fx)(-1+\text{ArcTan}(\tan(e+fx))\cot(e+fx))\sqrt{b\tan^4(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] -((Cot[e + f*x]*(-1 + ArcTan[Tan[e + f*x]]*Cot[e + f*x])*Sqrt[b*Tan[e + f*x
]^4])/f)
```

Maple [A]
time = 0.03, size = 42, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{\sqrt{b\left(\tan^4\left(fx+e\right)\right)}\,(-\tan(fx+e)+\arctan(\tan(fx+e)))}{f\tan(fx+e)^2}$	42
default	$-\frac{\sqrt{b\left(\tan^4\left(fx+e\right)\right)}\,(-\tan(fx+e)+\arctan(\tan(fx+e)))}{f\tan(fx+e)^2}$	42
risch	$\frac{\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\,\left(e^{2i(fx+e)}+1\right)^2 x}{\left(e^{2i(fx+e)}-1\right)^2} - \frac{2i\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\,\left(e^{2i(fx+e)}+1\right)}{\left(e^{2i(fx+e)}-1\right)^2 f}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(b*tan(f*x+e)^4)^(1/2)*(-tan(f*x+e)+arctan(tan(f*x+e)))/tan(f*x+e)^2
```

Maxima [A]
time = 0.49, size = 28, normalized size = 0.56

$$-\frac{(fx+e)\sqrt{b}-\sqrt{b}\,\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)*sqrt(b) - sqrt(b)*tan(f*x + e))/f
```

Fricas [A]
time = 1.96, size = 40, normalized size = 0.80

$$-\frac{\sqrt{b \tan\left(fx+e\right)^{4}}\,\left(fx-\tan\left(fx+e\right)\right)}{f \tan\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(f*x + e)^4)*(f*x - tan(f*x + e))/(f*tan(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan^{4}\left(e+fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(50) = 100.
time = 0.57, size = 250, normalized size = 5.00

$$\frac{\left(\pi-4\,fx\tan\left(fx\right)\tan\left(e\right)-\pi\operatorname{sgn}\left(2\,\tan\left(fx\right)^{2}\tan\left(e\right)+2\,\tan\left(fx\right)\tan\left(e\right)^{2}-2\,\tan\left(fx\right)-2\,\tan\left(e\right)\right)\tan\left(fx\right)\tan\left(e\right)-\pi\tan\left(fx\right)\tan\left(e\right)+2\,\arctan\left(\frac{\tan(fx)\tan(e)-1}{\tan(fx)+\tan(e)}\right)\tan\left(fx\right)\tan\left(e\right)+2\,\arctan\left(\frac{\tan(fx)+\tan(e)}{\tan(fx)\tan(e)-1}\right)\tan\left(fx\right)\tan\left(e\right)+4\,fx+\pi\operatorname{sgn}\left(2\,\tan\left(fx\right)^{2}\tan\left(e\right)+2\,\tan\left(fx\right)\tan\left(e\right)^{2}-2\,\tan\left(fx\right)-2\,\tan\left(e\right)\right)-2\,\arctan\left(\frac{\tan(fx)\tan(e)-1}{\tan(fx)+\tan(e)}\right)-2\,\arctan\left(\frac{\tan(fx)+\tan(e)}{\tan(fx)\tan(e)-1}\right)-4\,\tan\left(fx\right)-4\,\tan\left(e\right)\right)\sqrt{b}}{4\left(f\tan\left(fx\right)\tan\left(e\right)-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*f*x*tan(f*x)*tan(e) - pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*t
an(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e) - pi*tan(f*x)*tan(e) + 2*a
rctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*tan(f*x)*tan(e) + 2*arctan
((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e) + 4*f*x + pi*sg
n(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e)) - 2*ar
ctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e))) - 2*arctan((tan(f*x) + tan(
e))/(tan(f*x)*tan(e) - 1)) - 4*tan(f*x) - 4*tan(e))*sqrt(b)/(f*tan(f*x)*tan
(e) - f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\tan\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^4)^(1/2),x)
[Out] int((b*tan(e + f*x)^4)^(1/2), x)
```

3.16 $\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}-\frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}$$

```
[Out] -tan(f*x+e)/f/(b*tan(f*x+e)^4)^(1/2)-x*tan(f*x+e)^2/(b*tan(f*x+e)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}-\frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] -(Tan[e + f*x]/(f*Sqrt[b*Tan[e + f*x]^4])) - (x*Tan[e + f*x]^2)/Sqrt[b*Tan[
e + f*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = \frac{\tan^2(e+fx)\int \cot^2(e+fx)\,dx}{\sqrt{b\tan^4(e+fx)}}$$
$$= -\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int 1\,dx}{\sqrt{b\tan^4(e+fx)}}$$
$$= -\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}} - \frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 43, normalized size = 0.84

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^4],x]
```

```
[Out] -((Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*Sqrt[b
*Tan[e + f*x]^4]))
```

Maple [A]
time = 0.03, size = 40, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{\tan(fx+e)(\arctan(\tan(fx+e))\tan(fx+e)+1)}{f\sqrt{b\left(\tan^4(fx+e)\right)}}$	40
default	$-\frac{\tan(fx+e)(\arctan(\tan(fx+e))\tan(fx+e)+1)}{f\sqrt{b\left(\tan^4(fx+e)\right)}}$	40
risch	$\frac{\left(e^{2i(fx+e)}-1\right)^2 x}{\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\left(e^{2i(fx+e)}+1\right)^2} + \frac{2i\left(e^{2i(fx+e)}-1\right)}{\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\left(e^{2i(fx+e)}+1\right)^2 f}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*tan(f*x+e)*(arctan(tan(f*x+e))*tan(f*x+e)+1)/(b*tan(f*x+e)^4)^(1/2)
```

Maxima [A]
time = 0.50, size = 29, normalized size = 0.57

$$-\frac{\frac{fx+e}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)/sqrt(b) + 1/(sqrt(b)*tan(f*x + e)))/f
```

Fricas [A]
time = 2.03, size = 42, normalized size = 0.82

$$-\frac{\sqrt{b\tan\left(fx+e\right)^{4}}\left(fx\tan\left(fx+e\right)+1\right)}{bf\tan\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(f*x + e)^4)*(f*x*tan(f*x + e) + 1)/(b*f*tan(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\tan^{4}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**4), x)
```

Giac [A]
time = 0.56, size = 48, normalized size = 0.94

$$-\frac{\frac{2\,(fx+e)}{\sqrt{b}}-\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)/sqrt(b) - tan(1/2*f*x + 1/2*e)/sqrt(b) + 1/(sqrt(b)*tan(1
/2*f*x + 1/2*e)))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\tan\left(e+f\,x\right)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^4)^(1/2),x)
[Out] int(1/(b*tan(e + f*x)^4)^(1/2), x)
```

3.17 $\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$\frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}}-\frac{x\tan^2(e+fx)}{b\sqrt{b\tan^4(e+fx)}}$$

```
[Out] 1/3*cot(f*x+e)/b/f/(b*tan(f*x+e)^4)^(1/2)-1/5*cot(f*x+e)^3/b/f/(b*tan(f*x+e
)^4)^(1/2)-tan(f*x+e)/b/f/(b*tan(f*x+e)^4)^(1/2)-x*tan(f*x+e)^2/b/(b*tan(f*
x+e)^4)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}}-\frac{x\tan^2(e+fx)}{b\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}}+\frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(-3/2),x]
```

```
[Out] Cot[e + f*x]/(3*b*f*Sqrt[b*Tan[e + f*x]^4]) - Cot[e + f*x]^3/(5*b*f*Sqrt[b*
Tan[e + f*x]^4]) - Tan[e + f*x]/(b*f*Sqrt[b*Tan[e + f*x]^4]) - (x*Tan[e + f
*x]^2)/(b*Sqrt[b*Tan[e + f*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx = \frac{\tan^2(e+fx)\int \cot^6(e+fx)\,dx}{b\sqrt{b\tan^4(e+fx)}}$$
$$= -\frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int \cot^4(e+fx)\,dx}{b\sqrt{b\tan^4(e+fx)}}$$
$$= \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} + \frac{\tan^2(e+fx)\int \cot^2(e+fx)\,d}{b\sqrt{b\tan^4(e+fx)}}$$
$$= \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(}{b\sqrt{b}}$$
$$= \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}} - \frac{x\,\mathrm{ta}}{b\sqrt{b\,t}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 45, normalized size = 0.38

$$-\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{5f\left(b\tan^4(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(-3/2),x]
```

```
[Out] -1/5*(Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*(b
*Tan[e + f*x]^4)^(3/2))
```

Maple [A]
time = 0.04, size = 63, normalized size = 0.53

method	result	size
derivativedivides	$-\frac{\tan(fx+e)\left(15\arctan(\tan(fx+e))\left(\tan^5(fx+e)\right)+15\left(\tan^4(fx+e)\right)-5\left(\tan^2(fx+e)\right)+3\right)}{15f\left(b\left(\tan^4(fx+e)\right)\right)^{\frac{3}{2}}}$	63
default	$-\frac{\tan(fx+e)\left(15\arctan(\tan(fx+e))\left(\tan^5(fx+e)\right)+15\left(\tan^4(fx+e)\right)-5\left(\tan^2(fx+e)\right)+3\right)}{15f\left(b\left(\tan^4(fx+e)\right)\right)^{\frac{3}{2}}}$	63
risch	$\frac{\left(e^{2i(fx+e)}-1\right)^2 x}{b\left(e^{2i(fx+e)}+1\right)^2\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}}+\frac{2i\left(45\,e^{8i(fx+e)}-90\,e^{6i(fx+e)}+140\,e^{4i(fx+e)}-70\,e^{2i(fx+e)}+23\right)}{15b\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)^2\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\,f}$	174

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*tan(f*x+e)*(15*arctan(tan(f*x+e))*tan(f*x+e)^5+15*tan(f*x+e)^4-5*ta
n(f*x+e)^2+3)/(b*tan(f*x+e)^4)^(3/2)
```

Maxima [A]
time = 0.50, size = 54, normalized size = 0.45

$$-\frac{\frac{15\,(fx+e)}{b^{\frac{3}{2}}}+\frac{15\,\tan(fx+e)^4-5\,\tan(fx+e)^2+3}{b^{\frac{3}{2}}\,\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)/b^(3/2) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/(b
^(3/2)*tan(f*x + e)^5))/f
```

Fricas [A]
time = 2.47, size = 67, normalized size = 0.56

$$-\frac{\left(15\,fx\tan\left(fx+e\right)^{5}+15\,\tan\left(fx+e\right)^{4}-5\,\tan\left(fx+e\right)^{2}+3\right)\sqrt{b\tan\left(fx+e\right)^{4}}}{15\,b^{2}f\tan\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*f*x*tan(f*x + e)^5 + 15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)*sq
rt(b*tan(f*x + e)^4)/(b^2*f*tan(f*x + e)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\tan^{4}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**4)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(-3/2), x)
```

Giac [A]
time = 0.69, size = 131, normalized size = 1.10

$$-\frac{\frac{480\,(fx+e)}{\sqrt{b}}-\frac{3\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-35\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+330\,b^{\frac{9}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{b^{5}}+\frac{330\,\sqrt{b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-35\,\sqrt{b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,\sqrt{b}}{b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}}}{480\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*(480*(f*x + e)/sqrt(b) - (3*b^(9/2)*tan(1/2*f*x + 1/2*e)^5 - 35*b^(9
/2)*tan(1/2*f*x + 1/2*e)^3 + 330*b^(9/2)*tan(1/2*f*x + 1/2*e))/b^5 + (330*s
qrt(b)*tan(1/2*f*x + 1/2*e)^4 - 35*sqrt(b)*tan(1/2*f*x + 1/2*e)^2 + 3*sqrt(
b))/(b*tan(1/2*f*x + 1/2*e)^5))/(b*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^4)^(3/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^4)^(3/2), x)
```

3.18 $\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}}+\frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\tan(e}{b^2f\sqrt{b\tan}}$$

```
[Out] 1/3*cot(f*x+e)/b^2/f/(b*tan(f*x+e)^4)^(1/2)-1/5*cot(f*x+e)^3/b^2/f/(b*tan(f
*x+e)^4)^(1/2)+1/7*cot(f*x+e)^5/b^2/f/(b*tan(f*x+e)^4)^(1/2)-1/9*cot(f*x+e)
^7/b^2/f/(b*tan(f*x+e)^4)^(1/2)-tan(f*x+e)/b^2/f/(b*tan(f*x+e)^4)^(1/2)-x*t
an(f*x+e)^2/b^2/(b*tan(f*x+e)^4)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{\tan(e+fx)}{b^2f\sqrt{b\tan^4(e+fx)}}-\frac{x\tan^2(e+fx)}{b^2\sqrt{b\tan^4(e+fx)}}-\frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}}+\frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}}-\frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}}+\frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^(-5/2),x]
```

```
[Out] Cot[e + f*x]/(3*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Cot[e + f*x]^3/(5*b^2*f*Sqr
t[b*Tan[e + f*x]^4]) + Cot[e + f*x]^5/(7*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Co
t[e + f*x]^7/(9*b^2*f*Sqrt[b*Tan[e + f*x]^4]) - Tan[e + f*x]/(b^2*f*Sqrt[b*
Tan[e + f*x]^4]) - (x*Tan[e + f*x]^2)/(b^2*Sqrt[b*Tan[e + f*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx = \frac{\tan^2(e+fx)\int \cot^{10}(e+fx)\,dx}{b^2\sqrt{b\tan^4(e+fx)}}$$
$$= -\frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\tan^2(e+fx)\int \cot^8(e+fx)\,dx}{b^2\sqrt{b\tan^4(e+fx)}}$$
$$= \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\tan^2(e+fx)\int \cot^6(e+fx)}{b^2\sqrt{b\tan^4(e+fx)}}$$
$$= -\frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} -$$
$$= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{}{9b}$$
$$= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{}{9b}$$
$$= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}} - \frac{}{9b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 45, normalized size = 0.25

$$-\frac{{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{9f\left(b\tan^4(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^(-5/2),x]
```

```
[Out] -1/9*(Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*(b
*Tan[e + f*x]^4)^(5/2))
```

Maple [A]
time = 0.05, size = 83, normalized size = 0.45

method	result
derivativedivides	$-\frac{\tan(fx+e)(315\arctan(\tan(fx+e))(\tan^9(fx+e))+315(\tan^8(fx+e))-105(\tan^6(fx+e))+63(\tan^4(fx+e))-45(\tan^2(f}{315f(b(\tan^4(fx+e)))^{\frac{5}{2}}}$
default	$-\frac{\tan(fx+e)(315\arctan(\tan(fx+e))(\tan^9(fx+e))+315(\tan^8(fx+e))-105(\tan^6(fx+e))+63(\tan^4(fx+e))-45(\tan^2(f}{315f(b(\tan^4(fx+e)))^{\frac{5}{2}}}$
risch	$\frac{(e^{2i(fx+e)}-1)^2 x}{b^2(e^{2i(fx+e)}+1)^2\sqrt{\frac{b(e^{2i(fx+e)}-1)^4}{(e^{2i(fx+e)}+1)^4}}}+\frac{2i(1575\,e^{16i(fx+e)}-6300\,e^{14i(fx+e)}+21000\,e^{12i(fx+e)}-31500\,e^{10i(fx+e)}+39}{315b^2(e^{2i(fx+e)}-1)^7(e^{2i(fx+e)}-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315/f*tan(f*x+e)*(315*arctan(tan(f*x+e))*tan(f*x+e)^9+315*tan(f*x+e)^8-1
05*tan(f*x+e)^6+63*tan(f*x+e)^4-45*tan(f*x+e)^2+35)/(b*tan(f*x+e)^4)^(5/2)
```

Maxima [A]
time = 0.53, size = 76, normalized size = 0.42

$$-\frac{\frac{315\,(fx+e)}{b^{\frac{5}{2}}}+\frac{315\,\tan(fx+e)^8-105\,\tan(fx+e)^6+63\,\tan(fx+e)^4-45\,\tan(fx+e)^2+35}{b^{\frac{5}{2}}\tan(fx+e)^9}}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/315*(315*(f*x + e)/b^(5/2) + (315*tan(f*x + e)^8 - 105*tan(f*x + e)^6 +
63*tan(f*x + e)^4 - 45*tan(f*x + e)^2 + 35)/(b^(5/2)*tan(f*x + e)^9))/f
```

Fricas [A]
time = 1.45, size = 89, normalized size = 0.49

$$-\frac{\left(315\,fx\tan\left(fx+e\right)^9+315\,\tan\left(fx+e\right)^8-105\,\tan\left(fx+e\right)^6+63\,\tan\left(fx+e\right)^4-45\,\tan\left(fx+e\right)^2+35\right)\sqrt{b\tan\left(fx+e\right)^4}}{315\,b^3f\tan\left(fx+e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/315*(315*f*x*tan(f*x + e)^9 + 315*tan(f*x + e)^8 - 105*tan(f*x + e)^6 +
63*tan(f*x + e)^4 - 45*tan(f*x + e)^2 + 35)*sqrt(b*tan(f*x + e)^4)/(b^3*f*t
an(f*x + e)^11)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\tan^4\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**4)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**4)**(-5/2), x)
```

Giac [A]
time = 1.22, size = 196, normalized size = 1.07

$$-\frac{\frac{161280\,(fx+e)}{b^{\frac{5}{2}}}+\frac{121590\,\sqrt{b}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-18480\,\sqrt{b}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+3528\,\sqrt{b}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-495\,\sqrt{b}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+35\,\sqrt{b}}{b^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}}-\frac{35\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}-495\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+3528\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-18480\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+121590\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{b^{27}}}{161280\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/161280*(161280*(f*x + e)/b^(5/2) + (121590*sqrt(b)*tan(1/2*f*x + 1/2*e)^
8 - 18480*sqrt(b)*tan(1/2*f*x + 1/2*e)^6 + 3528*sqrt(b)*tan(1/2*f*x + 1/2*e
)^4 - 495*sqrt(b)*tan(1/2*f*x + 1/2*e)^2 + 35*sqrt(b))/(b^3*tan(1/2*f*x + 1
/2*e)^9) - (35*b^(49/2)*tan(1/2*f*x + 1/2*e)^9 - 495*b^(49/2)*tan(1/2*f*x +
 1/2*e)^7 + 3528*b^(49/2)*tan(1/2*f*x + 1/2*e)^5 - 18480*b^(49/2)*tan(1/2*f
*x + 1/2*e)^3 + 121590*b^(49/2)*tan(1/2*f*x + 1/2*e))/b^27)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\tan\left(e+f\,x\right)}^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^4)^(5/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^4)^(5/2), x)
```

3.19 $\int (b \tan^n(e+fx))^{5/2}\, dx$

Optimal. Leaf size=71

$$\frac{2b^2\, {}_2F_1\left(1,\frac{1}{4}(2+5n);\frac{1}{4}(6+5n);-\tan^2(e+fx)\right)\tan^{1+2n}(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+5n)}$$

```
[Out] 2*b^2*hypergeom([1, 1/2+5/4*n],[3/2+5/4*n],-tan(f*x+e)^2)*(b*tan(f*x+e)^n)^
(1/2)*tan(f*x+e)^(1+2*n)/f/(2+5*n)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2b^2\tan^{2n+1}(e+fx)\sqrt{b\tan^n(e+fx)}\, {}_2F_1\left(1,\frac{1}{4}(5n+2);\frac{1}{4}(5n+6);-\tan^2(e+fx)\right)}{f(5n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(5/2),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[1, (2 + 5*n)/4, (6 + 5*n)/4, -Tan[e + f*x]^2]*Tan[
e + f*x]^(1 + 2*n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 5*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (b\tan^n(e+fx))^{5/2}\,dx = \left(b^2\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\int \tan^{\frac{5n}{2}}(e+fx)\,dx$$
$$= \frac{\left(b^2\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\text{Subst}\left(\int \frac{x^{5n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2b^2\,{}_2F_1\left(1,\frac{1}{4}(2+5n);\frac{1}{4}(6+5n);-\tan^2(e+fx)\right)\tan^{1+2n}(e+fx)\sqrt{b\tan^n(e+}}{f(2+5n)}$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 0.87

$$\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2+5n);\frac{1}{4}(6+5n);-\tan^2(e+fx)\right)\tan(e+fx)\,(b\tan^n(e+fx))^{5/2}}{f(2+5n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(5/2),x]
[Out] (2*Hypergeometric2F1[1, (2 + 5*n)/4, (6 + 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^n)^(5/2))/(f*(2 + 5*n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (b(\tan^n(fx+e)))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(5/2),x)
[Out] int((b*tan(f*x+e)^n)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{n}\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(5/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^n\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^(5/2),x)
```

```
[Out] int((b*tan(e + f*x)^n)^(5/2), x)
```

3.20 $\int (b\tan^n(e+fx))^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{2b\,{}_2F_1\left(1,\frac{1}{4}(2+3n);\frac{3(2+n)}{4};-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+3n)}$$

```
[Out] 2*b*hypergeom([1, 1/2+3/4*n],[3/2+3/4*n],-tan(f*x+e)^2)*(b*tan(f*x+e)^n)^(1
/2)*tan(f*x+e)^(1+n)/f/(2+3*n)
```

Rubi [A]

time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2b\tan^{n+1}(e+fx)\sqrt{b\tan^n(e+fx)}\,{}_2F_1\left(1,\frac{1}{4}(3n+2);\frac{3(n+2)}{4};-\tan^2(e+fx)\right)}{f(3n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(3/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[1, (2 + 3*n)/4, (3*(2 + n))/4, -Tan[e + f*x]^2]*Tan[
e + f*x]^(1 + n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 3*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (b\tan^n(e+fx))^{3/2}\,dx = \left(b\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\int \tan^{\frac{3n}{2}}(e+fx)\,dx$$
$$= \frac{\left(b\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\text{Subst}\left(\int \frac{x^{3n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2b\,{}_2F_1\left(1,\frac{1}{4}(2+3n);\frac{3(2+n)}{4};-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+3n)}$$

Mathematica [**A**]
time = 0.05, size = 60, normalized size = 0.92

$$\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2+3n);\frac{3(2+n)}{4};-\tan^2(e+fx)\right)\tan(e+fx)\,(b\tan^n(e+fx))^{3/2}}{f(2+3n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(3/2),x]
[Out] (2*Hypergeometric2F1[1, (2 + 3*n)/4, (3*(2 + n))/4, -Tan[e + f*x]^2]*Tan[e
+ f*x]*(b*Tan[e + f*x]^n)^(3/2))/(f*(2 + 3*n))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int (b(\tan^n(fx+e)))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(3/2),x)
[Out] int((b*tan(f*x+e)^n)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tan^{n}\left(e + f x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(3/2),x)
[Out] Integral((b*tan(e + f*x)**n)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^n)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^n\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^(3/2),x)
[Out] int((b*tan(e + f*x)^n)^(3/2), x)
```

3.21 $\int \sqrt{b\tan^n(e+fx)}\,dx$

Optimal. Leaf size=56

$$\frac{2\,{}_2F_1\left(1,\frac{2+n}{4};\frac{6+n}{4};-\tan^2(e+fx)\right)\tan(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+n)}$$

```
[Out] 2*hypergeom([1, 1/2+1/4*n],[3/2+1/4*n],-tan(f*x+e)^2)*(b*tan(f*x+e)^n)^(1/2
)*tan(f*x+e)/f/(2+n)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\tan(e+fx)\sqrt{b\tan^n(e+fx)}\,{}_2F_1\left(1,\frac{n+2}{4};\frac{n+6}{4};-\tan^2(e+fx)\right)}{f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]^n],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + n)/4, (6 + n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{b\tan^n(e+fx)}\,dx = \left(\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\int \tan^{\frac{n}{2}}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\right)\text{Subst}\left(\int \frac{x^{n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{2\,{}_2F_1\left(1,\frac{2+n}{4};\frac{6+n}{4};-\tan^2(e+fx)\right)\tan(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+n)}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 1.00

$$\frac{2\,{}_2F_1\left(1,\frac{2+n}{4};\frac{6+n}{4};-\tan^2(e+fx)\right)\tan(e+fx)\sqrt{b\tan^n(e+fx)}}{f(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]^n],x]
[Out] (2*Hypergeometric2F1[1, (2 + n)/4, (6 + n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \sqrt{b\left(\tan^n(fx+e)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(1/2),x)
[Out] int((b*tan(f*x+e)^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e)^n), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan^{n}{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^(1/2),x)
```

```
[Out] int((b*tan(e + f*x)^n)^(1/2), x)
```

3.22 $\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\,{}_2F_1\left(1,\frac{2-n}{4};\frac{6-n}{4};-\tan^2(e+fx)\right)\tan(e+fx)}{f(2-n)\sqrt{b\tan^n(e+fx)}}$$

```
[Out] 2*hypergeom([1, 1/2-1/4*n],[3/2-1/4*n],-tan(f*x+e)^2)*tan(f*x+e)/f/(2-n)/(b
*tan(f*x+e)^n)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\tan(e+fx)\,{}_2F_1\left(1,\frac{2-n}{4};\frac{6-n}{4};-\tan^2(e+fx)\right)}{f(2-n)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[e + f*x]^n],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - n)/4, (6 - n)/4, -Tan[e + f*x]^2]*Tan[e + f*x]
)/(f*(2 - n)*Sqrt[b*Tan[e + f*x]^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx &= \frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{n}{2}}(e+fx)\,dx}{\sqrt{b\tan^n(e+fx)}} \\
&= \frac{\tan^{\frac{n}{2}}(e+fx)\text{Subst}\left(\int \frac{x^{-n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f\sqrt{b\tan^n(e+fx)}} \\
&= \frac{2\,{}_2F_1\left(1,\frac{2-n}{4};\frac{6-n}{4};-\tan^2(e+fx)\right)\tan(e+fx)}{f(2-n)\sqrt{b\tan^n(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.97

$$
-\frac{2\,{}_2F_1\left(1,\frac{2-n}{4};\frac{6-n}{4};-\tan^2(e+fx)\right)\tan(e+fx)}{f(-2+n)\sqrt{b\tan^n(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[e + f*x]^n],x]
[Out] (-2*Hypergeometric2F1[1, (2 - n)/4, (6 - n)/4, -Tan[e + f*x]^2]*Tan[e + f*x
])/(f*(-2 + n)*Sqrt[b*Tan[e + f*x]^n])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$
\int \frac{1}{\sqrt{b\left(\tan^n\left(fx+e\right)\right)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^n)^(1/2),x)
[Out] int(1/(b*tan(f*x+e)^n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*tan(f*x + e)^n), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \tan^{n}(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(e + f*x)**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(f*x + e)^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b \tan(e + f\,x)^{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^n)^(1/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^n)^(1/2), x)
```

3.23 $\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-3n);\frac{3(2-n)}{4};-\tan^2(e+fx)\right)\tan^{1-n}(e+fx)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

```
[Out] 2*hypergeom([1, 1/2-3/4*n],[3/2-3/4*n],-tan(f*x+e)^2)*tan(f*x+e)^(1-n)/b/f/
(2-3*n)/(b*tan(f*x+e)^n)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\tan^{1-n}(e+fx)\,{}_2F_1\left(1,\frac{1}{4}(2-3n);\frac{3(2-n)}{4};-\tan^2(e+fx)\right)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(-3/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 3*n)/4, (3*(2 - n))/4, -Tan[e + f*x]^2]*Tan[e
+ f*x]^(1 - n))/(b*f*(2 - 3*n)*Sqrt[b*Tan[e + f*x]^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
```

```
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx &= \frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{3n}{2}}(e+fx)\,dx}{b\sqrt{b\tan^n(e+fx)}}\\ &= \frac{\tan^{\frac{n}{2}}(e+fx)\mathrm{Subst}\left(\int \frac{x^{-3n/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{bf\sqrt{b\tan^n(e+fx)}}\\ &= \frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-3n);\frac{3(2-n)}{4};-\tan^2(e+fx)\right)\tan^{1-n}(e+fx)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.85

$$-\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-3n);-\frac{3}{4}(-2+n);-\tan^2(e+fx)\right)\tan(e+fx)}{f(-2+3n)\,(b\tan^n(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(-3/2),x]
[Out] (-2*Hypergeometric2F1[1, (2 - 3*n)/4, (-3*(-2 + n))/4, -Tan[e + f*x]^2]*Tan
[e + f*x])/(f*(-2 + 3*n)*(b*Tan[e + f*x]^n)^(3/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,(\tan^n(fx+e)))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^n)^(3/2),x)
[Out] int(1/(b*tan(f*x+e)^n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(-3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \tan^{n}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**n)**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^n\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^n)^(3/2),x)
```

```
[Out] int(1/(b*tan(e + f*x)^n)^(3/2), x)
```

3.24 $\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-5n);\frac{1}{4}(6-5n);-\tan^2(e+fx)\right)\tan^{1-2n}(e+fx)}{b^2f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

```
[Out] 2*hypergeom([1, 1/2-5/4*n],[3/2-5/4*n],-tan(f*x+e)^2)*tan(f*x+e)^(1-2*n)/b^
2/f/(2-5*n)/(b*tan(f*x+e)^n)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\tan^{1-2n}(e+fx)\,{}_2F_1\left(1,\frac{1}{4}(2-5n);\frac{1}{4}(6-5n);-\tan^2(e+fx)\right)}{b^2f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^(-5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 5*n)/4, (6 - 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
f*x]^(1 - 2*n))/(b^2*f*(2 - 5*n)*Sqrt[b*Tan[e + f*x]^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx = \frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{5n}{2}}(e+fx)\,dx}{b^2\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{\tan^{\frac{n}{2}}(e+fx)\text{Subst}\left(\int \frac{x^{-5n/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{b^2 f\sqrt{b\tan^n(e+fx)}}$$
$$= \frac{2\,{}_2F_1\left(1, \frac{1}{4}(2-5n); \frac{1}{4}(6-5n); -\tan^2(e+fx)\right)\tan^{1-2n}(e+fx)}{b^2 f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

Mathematica [A]
time = 0.05, size = 62, normalized size = 0.87

$$-\frac{2\,{}_2F_1\left(1, \frac{1}{4}(2-5n); \frac{1}{4}(6-5n); -\tan^2(e+fx)\right)\tan(e+fx)}{f(-2+5n)\,(b\tan^n(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^(-5/2),x]
[Out] (-2*Hypergeometric2F1[1, (2 - 5*n)/4, (6 - 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
 f*x])/(f*(-2 + 5*n)*(b*Tan[e + f*x]^n)^(5/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(b(\tan^n(fx+e)))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(f*x+e)^n)^(5/2),x)
[Out] int(1/(b*tan(f*x+e)^n)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\tan^n(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)**n)**(5/2),x)
[Out] Integral((b*tan(e + f*x)**n)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^n)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\tan(e+f\,x)^n)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(e + f*x)^n)^(5/2),x)
[Out] int(1/(b*tan(e + f*x)^n)^(5/2), x)
```

3.25 $\int (b\tan^n(e+fx))^p \, dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b\tan^n(e+fx))^p}{f(1+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f
*x+e)^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3740, 3557, 371}

$$\frac{\tan(e+fx)\,(b\tan^n(e+fx))^p\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^n)^p)/(f*(1 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (b\tan^n(e+fx))^p\,dx &= \left(\tan^{-np}(e+fx)\,(b\tan^n(e+fx))^p\right)\int \tan^{np}(e+fx)\,dx\\ &= \frac{\left(\tan^{-np}(e+fx)\,(b\tan^n(e+fx))^p\right)\mathrm{Subst}\left(\int \frac{x^{np}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b\tan^n(e+fx))^p}{f(1+np)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.97

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b\tan^n(e+fx))^p}{f+fnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^p,x]
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^n)^p)/(f + f*n*p)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^n\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^p,x)
[Out] int((b*tan(f*x+e)^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^n)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan^n(e+fx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**p,x)
[Out] Integral((b*tan(e + f*x)**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^n)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\tan(e+f\,x)^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^p,x)
[Out] int((b*tan(e + f*x)^n)^p, x)
```

3.26 $\int (b\tan^2(e+fx))^p \, dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+2p);\frac{1}{2}(3+2p);-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^2(e+fx)\right)^p}{f(1+2p)}$$

```
[Out] hypergeom([1, 1/2+p],[3/2+p],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f
/(1+2*p)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3739, 3557, 371}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p {}_2F_1\left(1,\frac{1}{2}(2p+1);\frac{1}{2}(2p+3);-\tan^2(e+fx)\right)}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 2*p)/2, (3 + 2*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^2(e+fx)\right)^p\,dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{2p}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{2p}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1, \frac{1}{2}(1+2p); \frac{1}{2}(3+2p); -\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^2(e+fx)\right)^p}{f(1+2p)}$$

Mathematica [**A**]
time = 0.03, size = 49, normalized size = 0.83

$$\frac{{}_2F_1\left(1, \frac{1}{2}+p; \frac{3}{2}+p; -\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^2(e+fx)\right)^p}{f+2fp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^2)^p,x]
[Out] (Hypergeometric2F1[1, 1/2 + p, 3/2 + p, -Tan[e + f*x]^2]*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f + 2*f*p)
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^2)^p,x)
[Out] int((b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{2}\left(e+fx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**2)**p,x)
[Out] Integral((b*tan(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^p,x)
[Out] int((b*tan(e + f*x)^2)^p, x)
```

3.27 $\int (b\tan^3(e+fx))^p \, dx$

Optimal. Leaf size=57

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+3p);\frac{3(1+p)}{2};-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^3(e+fx)\right)^p}{f(1+3p)}$$

```
[Out] hypergeom([1, 1/2+3/2*p],[3/2+3/2*p],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e
)^3)^p/f/(1+3*p)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3739, 3557, 371}

$$\frac{\tan(e+fx)\left(b\tan^3(e+fx)\right)^p {}_2F_1\left(1,\frac{1}{2}(3p+1);\frac{3(p+1)}{2};-\tan^2(e+fx)\right)}{f(3p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^3)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 3*p)/2, (3*(1 + p))/2, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^3)^p)/(f*(1 + 3*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^3(e+fx)\right)^p\,dx = \left(\tan^{-3p}(e+fx)\left(b\tan^3(e+fx)\right)^p\right)\int \tan^{3p}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-3p}(e+fx)\left(b\tan^3(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{3p}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+3p);\frac{3(1+p)}{2};-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^3(e+fx)\right)^p}{f(1+3p)}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.96

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+3p);\frac{3(1+p)}{2};-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^3(e+fx)\right)^p}{f+3fp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^3)^p,x]
[Out] (Hypergeometric2F1[1, (1 + 3*p)/2, (3*(1 + p))/2, -Tan[e + f*x]^2]*Tan[e +
f*x]*(b*Tan[e + f*x]^3)^p)/(f + 3*f*p)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^3(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^3)^p,x)
[Out] int((b*tan(f*x+e)^3)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^3)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^3)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{3}\left(e+fx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**3)**p,x)
[Out] Integral((b*tan(e + f*x)**3)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^3)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^3)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^3\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^3)^p,x)
[Out] int((b*tan(e + f*x)^3)^p, x)
```

3.28 $\int (b\tan^4(e+fx))^p \, dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+4p);\frac{1}{2}(3+4p);-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^4(e+fx)\right)^p}{f(1+4p)}$$

```
[Out] hypergeom([1, 1/2+2*p],[3/2+2*p],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^4)
^p/f/(1+4*p)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3739, 3557, 371}

$$\frac{\tan(e+fx)\left(b\tan^4(e+fx)\right)^p {}_2F_1\left(1,\frac{1}{2}(4p+1);\frac{1}{2}(4p+3);-\tan^2(e+fx)\right)}{f(4p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 4*p)/2, (3 + 4*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*Tan[e + f*x]^4)^p)/(f*(1 + 4*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^4(e+fx)\right)^p\,dx = \left(\tan^{-4p}(e+fx)\left(b\tan^4(e+fx)\right)^p\right)\int \tan^{4p}(e+fx)\,dx$$
$$= \frac{\left(\tan^{-4p}(e+fx)\left(b\tan^4(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x^{4p}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+4p);\frac{1}{2}(3+4p);-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^4(e+fx)\right)^p}{f(1+4p)}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.90

$$\frac{{}_2F_1\left(1,\frac{1}{2}+2p;\frac{3}{2}+2p;-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^4(e+fx)\right)^p}{f+4fp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^4)^p,x]
[Out] (Hypergeometric2F1[1, 1/2 + 2*p, 3/2 + 2*p, -Tan[e + f*x]^2]*Tan[e + f*x]*(
b*Tan[e + f*x]^4)^p)/(f + 4*f*p)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^4(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^4)^p,x)
[Out] int((b*tan(f*x+e)^4)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^4)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^4)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{4}\left(e+fx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**4)**p,x)
[Out] Integral((b*tan(e + f*x)**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^4)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^4\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^4)^p,x)
[Out] int((b*tan(e + f*x)^4)^p, x)
```

3.29 $\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx$

Optimal. Leaf size=32

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\,(b\tan^n(e+fx))^{\frac{1}{n}}}{f}$$

```
[Out] -cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^n)^(1/n)/f
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3740, 3556}

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\,(b\tan^n(e+fx))^{\frac{1}{n}}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*(b*Tan[e + f*x]^n)^n^(-1))/f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx &= \left(\cot(e+fx)\,(b\tan^n(e+fx))^{\frac{1}{n}}\right)\int \tan(e+fx)\,dx\\ &= -\frac{\cot(e+fx)\log(\cos(e+fx))\,(b\tan^n(e+fx))^{\frac{1}{n}}}{f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\left(b\tan^n(e+fx)\right)^{\frac{1}{n}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x]^n)^n^(-1),x]
[Out] -((Cot[e + f*x]*Log[Cos[e + f*x]]*(b*Tan[e + f*x]^n)^n^(-1))/f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.99, size = 12884, normalized size = 402.62

method	result	size
risch	Expression too large to display	12884

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e)^n)^(1/n),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n)^(1/n), x)
```

Fricas [A]
time = 1.81, size = 24, normalized size = 0.75

$$-\frac{b^{\left(\frac{1}{n}\right)}\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="fricas")
[Out] -1/2*b^(1/n)*log(1/(tan(f*x + e)^2 + 1))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^n\left(e+fx\right)\right)^{\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)**n)**(1/n),x)
```

```
[Out] Integral((b*tan(e + f*x)**n)**(1/n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^n)^(1/n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(b\tan(e+f\,x)^n\right)^{1/n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^n)^(1/n),x)
```

```
[Out] int((b*tan(e + f*x)^n)^(1/n), x)
```

3.30 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=70

$$-\frac{(a-3b)\cos(e+fx)}{f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-b)\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-3*b)*cos(f*x+e)/f+1/3*(2*a-3*b)*cos(f*x+e)^3/f-1/5*(a-b)*cos(f*x+e)^5/f
+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3745, 459}

$$-\frac{(a-b)\cos^5(e+fx)}{5f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - 3*b)*Cos[e + f*x])/f) + ((2*a - 3*b)*Cos[e + f*x]^3)/(3*f) - ((a -
b)*Cos[e + f*x]^5)/(5*f) + (b*Sec[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a-b+bx^2)}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b+\frac{a-b}{x^6}+\frac{-2a+3b}{x^4}+\frac{a-3b}{x^2}\right)\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{(a-3b)\cos(e+fx)}{f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-b)\cos^5(e}{5f}$$

Mathematica [A]
time = 0.05, size = 104, normalized size = 1.49

$$-\frac{5a\cos(e+fx)}{8f}+\frac{19b\cos(e+fx)}{8f}+\frac{5a\cos(3(e+fx))}{48f}-\frac{3b\cos(3(e+fx))}{16f}-\frac{a\cos(5(e+fx))}{80f}+\frac{b\cos(5(e+fx))}{80f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-5*a*Cos[e + f*x])/(8*f) + (19*b*Cos[e + f*x])/(8*f) + (5*a*Cos[3*(e + f*x
)])/(48*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (a*Cos[5*(e + f*x)])/(80*f) +
(b*Cos[5*(e + f*x)])/(80*f) + (b*Sec[e + f*x])/f
```

Maple [A]
time = 0.17, size = 92, normalized size = 1.31

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin^8(fx+e)}{\cos(fx+e)}+\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(f}{f}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin^8(fx+e)}{\cos(fx+e)}+\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(f}{f}$
risch	$-\frac{5\,e^{i(fx+e)}a}{16f}+\frac{19\,e^{i(fx+e)}b}{16f}-\frac{5\,e^{-i(fx+e)}a}{16f}+\frac{19\,e^{-i(fx+e)}b}{16f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}-\frac{\cos(5fx+5e)a}{80f}+\frac{\cos(5fx+}{80f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^8/
cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)
))
```

Maxima [A]
time = 0.28, size = 66, normalized size = 0.94

$$-\frac{3\,(a-b)\cos\left(fx+e\right)^{5}-5\,(2\,a-3\,b)\cos\left(fx+e\right)^{3}+15\,(a-3\,b)\cos\left(fx+e\right)-\frac{15\,b}{\cos(fx+e)}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(3*(a - b)*cos(f*x + e)^5 - 5*(2*a - 3*b)*cos(f*x + e)^3 + 15*(a - 3*
b)*cos(f*x + e) - 15*b/cos(f*x + e))/f
```

Fricas [A]
time = 6.14, size = 68, normalized size = 0.97

$$-\frac{3\,(a-b)\cos\left(fx+e\right)^{6}-5\,(2\,a-3\,b)\cos\left(fx+e\right)^{4}+15\,(a-3\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a - b)*cos(f*x + e)^6 - 5*(2*a - 3*b)*cos(f*x + e)^4 + 15*(a - 3*
b)*cos(f*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\sin^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58071 vs. 2(70) = 140.
time = 15.35, size = 58071, normalized size = 829.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/1920*(315*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2
*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*
```

```
tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2
+ 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^12*tan(1/2*e)^12 + 315*pi*b*sgn(tan(1/2*
f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/
2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)
*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2
*f*x)^12*tan(1/2*e)^12 - 315*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1
/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*
tan(1/2*f*x)^12*tan(1/2*e)^12 + 315*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 -
2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x
) - 1)*tan(1/2*f*x)^12*tan(1/2*e)^12 - 630*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*
e)^2 - tan(1/2*f*x)^2 - 4*tan(1/2*f*x)*tan(1/2*e) - tan(1/2*e)^2 + 1)*tan(1
/2*f*x)^12*tan(1/2*e)^12 + 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*ta
n(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)
*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*
x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^12*tan(1/2*e)^10 + 1
260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + ta
n(1/2*f*x)^2 - tan(1/2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*
e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(
1/2*f*x) - 1)*tan(1/2*f*x)^12*tan(1/2*e)^10 - 1260*pi*b*sgn(tan(1/2*f*x)^2*
tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2
+ 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/
2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^1
1*tan(1/2*e)^11 - 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x
)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(
1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + ta
n(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^11*tan(1/2*e)^11 + 1260*pi*b*
sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x
)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*
tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x)
- 1)*tan(1/2*f*x)^10*tan(1/2*e)^12 + 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e
)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 - 2*tan(1
/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 -
tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^10*tan(1/2
*e)^12 + 630*pi*b*tan(1/2*f*x)^12*tan(1/2*e)^12 - 630*b*arctan((tan(1/2*f*x
)*tan(1/2*e) + tan(1/2*f*x) - tan(1/2*e) + 1)/(tan(1/2*f*x)*tan(1/2*e) - ta
n(1/2*f*x) + tan(1/2*e) + 1))*tan(1/2*f*x)^12*tan(1/2*e)^12 - 630*b*arctan(
(tan(1/2*f*x)*tan(1/2*e) - tan(1/2*f*x) + tan(1/2*e) + 1)/(tan(1/2*f*x)*tan
(1/2*e) + tan(1/2*f*x) - tan(1/2*e) + 1))*tan(1/2*f*x)^12*tan(1/2*e)^12 + 6
30*b*arctan((tan(1/2*f*x)*tan(1/2*e) + tan(1/2*f*x) + tan(1/2*e) - 1)/(tan(
1/2*f*x)*tan(1/2*e) - tan(1/2*f*x) - tan(1/2*e) - 1))*tan(1/2*f*x)^12*tan(1
/2*e)^12 + 630*b*arctan((tan(1/2*f*x)*tan(1/2*e) - tan(1/2*f*x) - tan(1/2*e
) - 1)/(tan(1/2*f*x)*tan(1/2*e) + tan(1/2*f*x) + tan(1/2*e) - 1))*tan(1/2*f
*x)^12*tan(1/2*e)^12 - 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/
2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*t
an(1/2*f*x)^12*tan(1/2*e)^10 + 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 -
```

```
2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x
) - 1)*tan(1/2*f*x)^12*tan(1/2*e)^10 - 2520*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2
*e)^2 - tan(1/2*f*x)^2 - 4*tan(1/2*f*x)*tan(1/2*e) - tan(1/2*e)^2 + 1)*tan(
1/2*f*x)^12*tan(1/2*e)^10 + 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*t
an(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) -
 1)*tan(1/2*f*x)^11*tan(1/2*e)^11 - 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)
^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/
2*f*x) - 1)*tan(1/2*f*x)^11*tan(1/2*e)^11 + 2520*pi*b*sgn(tan(1/2*f*x)^2*ta
n(1/2*e)^2 - tan(1/2*f*x)^2 - 4*tan(1/2*f*x)*tan(1/2*e) - tan(1/2*e)^2 + 1)
*tan(1/2*f*x)^11*tan(1/2*e)^11 - 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2
+ 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f
*x) - 1)*tan(1/2*f*x)^10*tan(1/2*e)^12 + 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1
/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*t
an(1/2*f*x) - 1)*tan(1/2*f*x)^10*tan(1/2*e)^12 - 2520*pi*b*sgn(tan(1/2*f*x)
^2*tan(1/2*e)^2 - tan(1/2*f*x)^2 - 4*tan(1/2*f*x)*tan(1/2*e) - tan(1/2*e)^2
 + 1)*tan(1/2*f*x)^10*tan(1/2*e)^12 - 1024*a*tan(1/2*f*x)^12*tan(1/2*e)^12
+ 6144*b*tan(1/2*f*x)^12*tan(1/2*e)^12 + 1575*pi*b*sgn(tan(1/2*f*x)^2*tan(1
/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*t
an(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^
2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f...
```

Mupad [B]
time = 12.23, size = 92, normalized size = 1.31

$$-\frac{\frac{5\,a}{16}-\frac{35\,b}{16}+\frac{25\,a\,\cos\left(2\,e+2\,f\,x\right)}{96}-\frac{11\,a\,\cos\left(4\,e+4\,f\,x\right)}{240}+\frac{a\,\cos\left(6\,e+6\,f\,x\right)}{160}-\frac{35\,b\,\cos\left(2\,e+2\,f\,x\right)}{32}+\frac{7\,b\,\cos\left(4\,e+4\,f\,x\right)}{80}-\frac{b\,\cos\left(6\,e+6\,f\,x\right)}{160}}{f\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2),x)
```

```
[Out] -((5*a)/16 - (35*b)/16 + (25*a*cos(2*e + 2*f*x))/96 - (11*a*cos(4*e + 4*f*x
))/240 + (a*cos(6*e + 6*f*x))/160 - (35*b*cos(2*e + 2*f*x))/32 + (7*b*cos(4
*e + 4*f*x))/80 - (b*cos(6*e + 6*f*x))/160)/(f*cos(e + f*x))
```

3.31 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=48

$$-\frac{(a-2b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-2*b)*cos(f*x+e)/f+1/3*(a-b)*cos(f*x+e)^3/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3745, 459}

$$\frac{(a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - 2*b)*Cos[e + f*x])/f) + ((a - b)*Cos[e + f*x]^3)/(3*f) + (b*Sec[e +
 f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx&=\frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)}{x^4}\,dx,x,\sec(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int \left(b+\frac{-a+b}{x^4}+\frac{a-2b}{x^2}\right)\,dx,x,\sec(e+fx)\right)}{f}\\&=-\frac{(a-2b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 72, normalized size = 1.50

$$-\frac{3a\cos(e+fx)}{4f}+\frac{7b\cos(e+fx)}{4f}+\frac{a\cos(3(e+fx))}{12f}-\frac{b\cos(3(e+fx))}{12f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*a*Cos[e + f*x])/(4*f) + (7*b*Cos[e + f*x])/(4*f) + (a*Cos[3*(e + f*x)])
/(12*f) - (b*Cos[3*(e + f*x)])/(12*f) + (b*Sec[e + f*x])/f
```

Maple [**A**]
time = 0.11, size = 72, normalized size = 1.50

method	result
derivativedivides	$\frac{b\left(\frac{\sin^6(fx+e)}{\cos(fx+e)}+\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)\right)-\frac{a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}}{f}$
default	$\frac{b\left(\frac{\sin^6(fx+e)}{\cos(fx+e)}+\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)\right)-\frac{a\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}}{f}$
risch	$-\frac{3\,e^{i(fx+e)}a}{8f}+\frac{7\,e^{i(fx+e)}b}{8f}-\frac{3\,e^{-i(fx+e)}a}{8f}+\frac{7\,e^{-i(fx+e)}b}{8f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\cos(3fx+3e)a}{12f}-\frac{\cos(3fx+3}{12f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(sin(f*x+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x
+e))-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [**A**]
time = 0.29, size = 47, normalized size = 0.98

$$\frac{(a-b)\cos\left(fx+e\right)^3-3\left(a-2\,b\right)\cos\left(fx+e\right)+\frac{3\,b}{\cos(fx+e)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*((a - b)*cos(f*x + e)^3 - 3*(a - 2*b)*cos(f*x + e) + 3*b/cos(f*x + e))/
f
```

Fricas [**A**]
time = 4.26, size = 49, normalized size = 1.02

$$\frac{(a-b)\cos\left(fx+e\right)^4-3\left(a-2\,b\right)\cos\left(fx+e\right)^2+3\,b}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - b)*cos(f*x + e)^4 - 3*(a - 2*b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x
+ e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)\sin^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**3, x)
```

Giac [A]
time = 0.61, size = 76, normalized size = 1.58

$$\frac{b}{f\cos\left(fx+e\right)}+\frac{af^5\cos\left(fx+e\right)^3-bf^5\cos\left(fx+e\right)^3-3\,af^5\cos\left(fx+e\right)+6\,bf^5\cos\left(fx+e\right)}{3\,f^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] b/(f*cos(f*x + e)) + 1/3*(a*f^5*cos(f*x + e)^3 - b*f^5*cos(f*x + e)^3 - 3*a
*f^5*cos(f*x + e) + 6*b*f^5*cos(f*x + e))/f^6
```

Mupad [B]
time = 12.01, size = 68, normalized size = 1.42

$$-\frac{\frac{3\,a}{8}-\frac{15\,b}{8}+\frac{a\,\cos(2\,e+2\,f\,x)}{3}-\frac{a\,\cos(4\,e+4\,f\,x)}{24}-\frac{5\,b\,\cos(2\,e+2\,f\,x)}{6}+\frac{b\,\cos(4\,e+4\,f\,x)}{24}}{f\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2),x)
```

```
[Out] -((3*a)/8 - (15*b)/8 + (a*cos(2*e + 2*f*x))/3 - (a*cos(4*e + 4*f*x))/24 - (
5*b*cos(2*e + 2*f*x))/6 + (b*cos(4*e + 4*f*x))/24)/(f*cos(e + f*x))
```

3.32 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=28

$$-\frac{(a-b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(a-b)*cos(f*x+e)/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3745, 14}

$$\frac{b\sec(e+fx)}{f}-\frac{(a-b)\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Cos[e + f*x])/f) + (b*Sec[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{a-b+bx^2}{x^2}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(b+\frac{a-b}{x^2}\right)\,dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{(a-b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 1.64

$$-\frac{a\cos(e)\cos(fx)}{f}+\frac{b\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}+\frac{a\sin(e)\sin(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2),x]
[Out] -((a*Cos[e]*Cos[f*x])/f) + (b*Cos[e + f*x])/f + (b*Sec[e + f*x])/f + (a*Sin
[e]*Sin[f*x])/f
```

Maple [A]
time = 0.08, size = 52, normalized size = 1.86

method	result	size
derivativedivides	$\frac{b\left(\frac{\sin^4(fx+e)}{\cos(fx+e)}+(2+\sin^2(fx+e))\cos(fx+e)\right)-\cos(fx+e)a}{f}$	52
default	$\frac{b\left(\frac{\sin^4(fx+e)}{\cos(fx+e)}+(2+\sin^2(fx+e))\cos(fx+e)\right)-\cos(fx+e)a}{f}$	52
risch	$-\frac{e^{i(fx+e)}a}{2f}+\frac{e^{i(fx+e)}b}{2f}-\frac{e^{-i(fx+e)}a}{2f}+\frac{e^{-i(fx+e)}b}{2f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f*x+e)^2)*cos(f*x+e))-cos(f*x+e)*a)
```

Maxima [A]
time = 0.29, size = 34, normalized size = 1.21

$$\frac{b\left(\frac{1}{\cos(fx+e)}+\cos\left(fx+e\right)\right)-a\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
[Out] (b*(1/cos(f*x + e) + cos(f*x + e)) - a*cos(f*x + e))/f
```

Fricas [A]
time = 3.98, size = 33, normalized size = 1.18

$$-\frac{(a-b)\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -((a - b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^{2}\left(e + fx\right)\right)\sin\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x), x)
```

Giac [A]
time = 0.58, size = 41, normalized size = 1.46

$$b\left(\frac{\cos\left(fx + e\right)}{f} + \frac{1}{f\cos\left(fx + e\right)}\right) - \frac{a\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] b*(cos(f*x + e)/f + 1/(f*cos(f*x + e))) - a*cos(f*x + e)/f
```

Mupad [B]
time = 11.78, size = 39, normalized size = 1.39

$$\frac{\left(\cos\left(e + f\,x\right) + 1\right)\,\left(b - a\,\cos\left(e + f\,x\right) + b\,\cos\left(e + f\,x\right)\right)}{f\,\cos\left(e + f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2),x)
```

```
[Out] ((cos(e + f*x) + 1)*(b - a*cos(e + f*x) + b*cos(e + f*x)))/(f*cos(e + f*x))
```

3.33 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=25

$$-\frac{a\tanh^{-1}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -a*arctanh(cos(f*x+e))/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3745, 396, 213}

$$\frac{b\sec(e+fx)}{f}-\frac{a\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a-b+bx^2}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}$$
$$= \frac{b\sec(e+fx)}{f} + \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}$$
$$= -\frac{a\tanh^{-1}(\cos(e+fx))}{f} + \frac{b\sec(e+fx)}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.
time = 0.02, size = 51, normalized size = 2.04

$$-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} + \frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} + \frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2),x]
[Out] -((a*Log[Cos[e/2 + (f*x)/2]])/f) + (a*Log[Sin[e/2 + (f*x)/2]])/f + (b*Sec[e
 + f*x])/f
```

Maple [A]
time = 0.12, size = 34, normalized size = 1.36

method	result	size
derivativedivides	$\frac{\frac{b}{\cos(fx+e)}+a\ln(\csc(fx+e)-\cot(fx+e))}{f}$	34
default	$\frac{\frac{b}{\cos(fx+e)}+a\ln(\csc(fx+e)-\cot(fx+e))}{f}$	34
risch	$\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{a\ln\left(e^{i(fx+e)}-1\right)}{f}-\frac{a\ln\left(e^{i(fx+e)}+1\right)}{f}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(b/cos(f*x+e)+a*ln(csc(f*x+e)-cot(f*x+e)))
```

Maxima [A]
time = 0.29, size = 43, normalized size = 1.72

$$-\frac{a\log\left(\cos\left(fx+e\right)+1\right)-a\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,b}{\cos(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(cos(f*x + e) + 1) - a*log(cos(f*x + e) - 1) - 2*b/cos(f*x + e))
/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(27) = 54$.
time = 5.10, size = 61, normalized size = 2.44

$$-\frac{a\cos\left(fx+e\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-a\cos\left(fx+e\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-2\,b}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - a*cos(f*x + e)*log(-1/2*
cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\csc\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(27) = 54$.
time = 0.59, size = 62, normalized size = 2.48

$$\frac{a\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{4\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) + 4*b/((cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 1))/f
```

Mupad [B]
time = 11.56, size = 37, normalized size = 1.48

$$\frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{2\,b}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x),x)
[Out] (a*log(tan(e/2 + (f*x)/2)))/f - (2*b)/(f*(tan(e/2 + (f*x)/2)^2 - 1))
```

3.34 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+2b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -1/2*(a+2*b)*arctanh(cos(f*x+e))/f-1/2*a*cot(f*x+e)*csc(f*x+e)/f+b*sec(f*x+
e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3745, 466, 396, 213}

$$-\frac{(a+2b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/2*((a + 2*b)*ArcTanh[Cos[e + f*x]])/f - (a*Cot[e + f*x]*Csc[e + f*x])/(2
*f) + (b*Sec[e + f*x])/f
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a-b+bx^2)}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{a\cot(e+fx)\csc(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a-2bx^2}{-1+x^2}\,dx, x, \sec(e+fx\right.}{2f}$$
$$= -\frac{a\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(e+fx)}{f} + \frac{(a+2b)\text{Subst}(\int}{}$$
$$= -\frac{(a+2b)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{a\cot(e+fx)\csc(e+fx)}{2f} + \frac{b}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(51) = 102.
time = 0.04, size = 123, normalized size = 2.41

$$-\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} - \frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/8*(a*Csc[(e + f*x)/2]^2)/f - (a*Log[Cos[(e + f*x)/2]])/(2*f) - (b*Log[Co
s[(e + f*x)/2]])/f + (a*Log[Sin[(e + f*x)/2]])/(2*f) + (b*Log[Sin[(e + f*x)
/2]])/f + (a*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sec[e + f*x])/f
```

Maple [A]
time = 0.17, size = 68, normalized size = 1.33

method	result
derivativedivides	$\frac{b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
default	$\frac{b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$

risch	$\frac{a\,e^{5i(fx+e)}+2b\,e^{5i(fx+e)}+2a\,e^{3i(fx+e)}-4b\,e^{3i(fx+e)}+a\,e^{i(fx+e)}+2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2\left(e^{2i(fx+e)}+1\right)}-\frac{a\ln\left(e^{i(fx+e)}+1\right)}{2f}-\frac{\ln\left(e^{i(fx+e)}+1\right)b}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e)))+a*(-1/2*csc(f*x+e)*cot(f*x+
e)+1/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Maxima [A]
time = 0.29, size = 81, normalized size = 1.59

$$-\frac{(a+2\,b)\log\left(\cos\left(fx+e\right)+1\right)-(a+2\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left((a+2\,b)\cos(fx+e)^2-2\,b\right)}{\cos(fx+e)^3-\cos(fx+e)}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/4*((a + 2*b)*log(cos(f*x + e) + 1) - (a + 2*b)*log(cos(f*x + e) - 1) - 2
*((a + 2*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(51) = 102.
time = 3.32, size = 133, normalized size = 2.61

$$\frac{2\,(a+2\,b)\cos\left(fx+e\right)^2-\left((a+2\,b)\cos\left(fx+e\right)^3-(a+2\,b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)+\left((a+2\,b)\cos\left(fx+e\right)^3-(a+2\,b)\cos\left(fx+e\right)\right)\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-4\,b}{4\left(f\cos\left(fx+e\right)^3-f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a + 2*b)*cos(f*x + e)^2 - ((a + 2*b)*cos(f*x + e)^3 - (a + 2*b)*cos
(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 2*b)*cos(f*x + e)^3 - (a + 2
*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3 - f
*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(51) = 102.
time = 0.63, size = 185, normalized size = 3.63

$$\frac{2\,(a+2\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a+\frac{14\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{2\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+\frac{(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 2*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - a*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + (a + 14*b*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + (cos(
f*x + e) - 1)^2/(cos(f*x + e) + 1)^2))/f
```

Mupad [B]
time = 12.10, size = 95, normalized size = 1.86

$$\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}-\frac{\frac{a}{2}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a}{2}+8\,b\right)}{f\,\left(4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\right)}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(\frac{a}{2}+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^3,x)
```

```
[Out] (a*tan(e/2 + (f*x)/2)^2)/(8*f) - (a/2 - tan(e/2 + (f*x)/2)^2*(a/2 + 8*b))/(
f*(4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^4)) + (log(tan(e/2 + (f*x)
/2))*(a/2 + b))/f
```

3.35 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=79

$$-\frac{3(a+4b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a\cot^3(e+fx)\csc(e+fx)}{4f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -3/8*(a+4*b)*arctanh(cos(f*x+e))/f-1/8*(5*a+4*b)*cot(f*x+e)*csc(f*x+e)/f-1/
4*a*cot(f*x+e)^3*csc(f*x+e)/f+b*sec(f*x+e)/f
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3745, 466, 1171, 396, 213}

$$-\frac{3(a+4b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a\cot^3(e+fx)\csc(e+fx)}{4f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*(a + 4*b)*ArcTanh[Cos[e + f*x]])/(8*f) - ((5*a + 4*b)*Cot[e + f*x]*Csc[
e + f*x])/(8*f) - (a*Cot[e + f*x]^3*Csc[e + f*x])/(4*f) + (b*Sec[e + f*x])/
f
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-b+bx^2\right)}{\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{a\cot^3(e+fx)\csc(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{-a-4ax^2-4bx^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(e\right.}{4f}$$

$$= -\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a\cot^3(e+fx)\csc(e+fx}{4f}$$

$$= -\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a\cot^3(e+fx)\csc(e+fx}{4f}$$

$$= -\frac{3(a+4b)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{(5a+4b)\cot(e+fx)\csc(e+}{8f}$$

Mathematica [A]

time = 5.45, size = 131, normalized size = 1.66

$$\frac{-2(3a+4b)\csc^2\left(\frac{1}{2}(e+fx)\right)-a\csc^4\left(\frac{1}{2}(e+fx)\right)-24(a+4b)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+(6a+8b)\sec^2\left(\frac{1}{2}(e+fx)\right)+a\sec^4\left(\frac{1}{2}(e+fx)\right)+128b\sec(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)}{64f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-2*(3*a + 4*b)*Csc[(e + f*x)/2]^2 - a*Csc[(e + f*x)/2]^4 - 24*(a + 4*b)*(L
og[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]) + (6*a + 8*b)*Sec[(e + f*x)/2
]^2 + a*Sec[(e + f*x)/2]^4 + 128*b*Sec[e + f*x]*Sin[(e + f*x)/2]^2)/(64*f)
```

Maple [A]
time = 0.18, size = 102, normalized size = 1.29

method	result
derivativedivides	$\frac{b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+a\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(}{}\right.}{f}$
default	$\frac{b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+a\left(\left(-\frac{\left(\csc^3(fx+e)\right)}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(}{}\right.}{f}$
risch	$\frac{3a\,e^{9i(fx+e)}+12b\,e^{9i(fx+e)}-8a\,e^{7i(fx+e)}-32b\,e^{7i(fx+e)}-22a\,e^{5i(fx+e)}+40b\,e^{5i(fx+e)}-8a\,e^{3i(fx+e)}-32b\,e^{3i(fx+e)}+3a\,e}{4f\left(e^{2i(fx+e)}-1\right)^4\left(e^{2i(fx+e)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(-1/2/sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f
*x+e)))+a*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-
cot(f*x+e))))
```

Maxima [A]
time = 0.30, size = 108, normalized size = 1.37

$$-\frac{3\,(a+4\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+4\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+4\,b)\cos(fx+e)^4-5\,(a+4\,b)\cos(fx+e)^2+8\,b\right)}{\cos(fx+e)^5-2\,\cos(fx+e)^3+\cos(fx+e)}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a + 4*b)*log(cos(f*x + e) + 1) - 3*(a + 4*b)*log(cos(f*x + e) - 1
) - 2*(3*(a + 4*b)*cos(f*x + e)^4 - 5*(a + 4*b)*cos(f*x + e)^2 + 8*b)/(cos(
f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(79) = 158.
time = 2.24, size = 191, normalized size = 2.42

$$\frac{6\,(a+4\,b)\cos\left(fx+e\right)^4-10\,(a+4\,b)\cos\left(fx+e\right)^2-3\,\left((a+4\,b)\cos\left(fx+e\right)^5-2\,(a+4\,b)\cos\left(fx+e\right)^3+(a+4\,b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+3\,\left((a+4\,b)\cos\left(fx+e\right)^5-2\,(a+4\,b)\cos\left(fx+e\right)^3+(a+4\,b)\cos\left(fx+e\right)\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+16\,b}{16\,\left(f\cos\left(fx+e\right)^5-2\,f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a + 4*b)*cos(f*x + e)^4 - 10*(a + 4*b)*cos(f*x + e)^2 - 3*((a + 4*
b)*cos(f*x + e)^5 - 2*(a + 4*b)*cos(f*x + e)^3 + (a + 4*b)*cos(f*x + e))*lo
g(1/2*cos(f*x + e) + 1/2) + 3*((a + 4*b)*cos(f*x + e)^5 - 2*(a + 4*b)*cos(f
*x + e)^3 + (a + 4*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b)/(f
*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)\csc^5\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(79) = 158.
time = 0.61, size = 259, normalized size = 3.28

$$\frac{12\,(a+4\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{\left(a-\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{72\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}-\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{128\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(12*(a + 4*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - (a -
 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 18*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 72*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)
^2 - 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 128*b/((cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/f
```

Mupad [B]
time = 11.97, size = 138, normalized size = 1.75

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{a}{8}+\frac{b}{8}\right)}{f}-\frac{\left(-2\,a-34\,b\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\left(\frac{7\,a}{4}+2\,b\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\frac{a}{4}}{f\,\left(16\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-16\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\right)}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(\frac{3\,a}{8}+\frac{3\,b}{2}\right)}{f}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^5,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^2*(a/8 + b/8))/f - (a/4 + tan(e/2 + (f*x)/2)^2*((7*a)/4
 + 2*b) - tan(e/2 + (f*x)/2)^4*(2*a + 34*b))/(f*(16*tan(e/2 + (f*x)/2)^4 -
16*tan(e/2 + (f*x)/2)^6)) + (log(tan(e/2 + (f*x)/2))*((3*a)/8 + (3*b)/2))/f
 + (a*tan(e/2 + (f*x)/2)^4)/(64*f)
```

3.36 $\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=102

$$\frac{5}{16}(a-7b)x-\frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(13a-19b)\cos^3(e+fx)\sin(e+fx)}{24f}-\frac{(a-b)\cos^5(e+f\dots}{6f}$$

```
[Out] 5/16*(a-7*b)*x-1/16*(11*a-29*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(13*a-19*b)*co
s(f*x+e)^3*sin(f*x+e)/f-1/6*(a-b)*cos(f*x+e)^5*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3744, 466, 1828, 1171, 396, 209}

$$-\frac{(a-b)\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{(13a-19b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-29b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x(a-7b)+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (5*(a - 7*b)*x)/16 - ((11*a - 29*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((1
3*a - 19*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - ((a - b)*Cos[e + f*x]^5*S
in[e + f*x])/(6*f) + (b*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x^6(a+bx^2)}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cos^5(e+fx)\sin(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-a+b+6(a-b)x^2-6(a-\cdots}{(1+x^2)^3}\cdots\right.}{6\cdots}$$

$$= \frac{(13a-19b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{(a-b)\cos^5(e+fx)\sin(\cdots}{6f}$$

$$= -\frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-19b)\cos^3(e+f\cdots}{24f}$$

$$= -\frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-19b)\cos^3(e+f\cdots}{24f}$$

$$= \frac{5}{16}(a-7b)x - \frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-19\cdots}{\cdots}$$

Mathematica [A]
time = 0.25, size = 89, normalized size = 0.87

$$\frac{60ae-420be+60afx-420bfx+(-45a+141b)\sin(2(e+fx))+3(3a-5b)\sin(4(e+fx))-a\sin(6(e+fx))+b\sin(6(e+fx))+192b\tan(e+fx)}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
[Out] (60*a*e - 420*b*e + 60*a*f*x - 420*b*f*x + (-45*a + 141*b)*Sin[2*(e + f*x)]
 + 3*(3*a - 5*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + b*Sin[6*(e + f*x)]
 + 192*b*Tan[e + f*x])/(192*f)
```

Maple [A]
time = 0.15, size = 122, normalized size = 1.20

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin^9(fx+e)}{\cos(fx+e)}+\left(\sin^7(fx+e)+\frac{7\left(\sin^5(fx+e)\right)}{6}+\frac{35}{}\right.\right.}{f}$
default	$\frac{a\left(-\frac{\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin^9(fx+e)}{\cos(fx+e)}+\left(\sin^7(fx+e)+\frac{7\left(\sin^5(fx+e)\right)}{6}+\frac{35}{}\right.\right.}{f}$
risch	$\frac{5ax}{16}-\frac{35bx}{16}+\frac{15i\mathrm{e}^{2i(fx+e)}a}{128f}-\frac{47i\mathrm{e}^{2i(fx+e)}b}{128f}-\frac{15i\mathrm{e}^{-2i(fx+e)}a}{128f}+\frac{47i\mathrm{e}^{-2i(fx+e)}b}{128f}+\frac{2ib}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}-\frac{\sin(6}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/1
6*f*x+5/16*e)+b*(sin(f*x+e)^9/cos(f*x+e)+(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/
24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)-35/16*f*x-35/16*e))
```

Maxima [A]
time = 0.52, size = 119, normalized size = 1.17

$$\frac{15\,(fx+e)(a-7\,b)+48\,b\tan\left(fx+e\right)-\frac{3\,(11\,a-29\,b)\tan(fx+e)^5+8\,(5\,a-17\,b)\tan(fx+e)^3+3\,(5\,a-19\,b)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
[Out] 1/48*(15*(f*x + e)*(a - 7*b) + 48*b*tan(f*x + e) - (3*(11*a - 29*b)*tan(f*x
 + e)^5 + 8*(5*a - 17*b)*tan(f*x + e)^3 + 3*(5*a - 19*b)*tan(f*x + e))/(tan
(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 3.12, size = 96, normalized size = 0.94

$$\frac{15\,(a-7\,b)fx\cos\left(fx+e\right)-\left(8\,(a-b)\cos\left(fx+e\right)^{6}-2\,(13\,a-19\,b)\cos\left(fx+e\right)^{4}+3\,(11\,a-29\,b)\cos\left(fx+e\right)^{2}-48\,b\right)\sin\left(fx+e\right)}{48\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a - 7*b)*f*x*cos(f*x + e) - (8*(a - b)*cos(f*x + e)^6 - 2*(13*a -
 19*b)*cos(f*x + e)^4 + 3*(11*a - 29*b)*cos(f*x + e)^2 - 48*b)*sin(f*x + e)
)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{2}\left(e+fx\right)\right)\sin^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7897 vs. $2(101) = 202$.
time = 2.43, size = 7897, normalized size = 77.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/192*(21*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*
tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^7 + 60*a*f*x*t
an(f*x)^7*tan(e)^7 - 420*b*f*x*tan(f*x)^7*tan(e)^7 + 21*pi*b*sgn(-2*tan(f*x
)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)
^7 + 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*ta
n(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^5 - 21*pi*b*sgn(
2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 +
 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e)^6 + 63*pi*b*sgn(2*tan(f*x)^2*tan(
e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*t
an(e))*tan(f*x)^5*tan(e)^7 + 42*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(
e) - 1))*tan(f*x)^7*tan(e)^7 - 42*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*t
an(e) + 1))*tan(f*x)^7*tan(e)^7 + 180*a*f*x*tan(f*x)^7*tan(e)^5 - 1260*b*f*
x*tan(f*x)^7*tan(e)^5 + 63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e
```

)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^5 - 60*a*f*x*tan(f*x)^6*tan(
e)^6 + 420*b*f*x*tan(f*x)^6*tan(e)^6 - 21*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2
*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e)^6 + 180*a*f*x
*tan(f*x)^5*tan(e)^7 - 1260*b*f*x*tan(f*x)^5*tan(e)^7 + 63*pi*b*sgn(-2*tan(
f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan
(e)^7 + 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2
*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^3 - 63*pi*b*s
gn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^
2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e)^4 + 189*pi*b*sgn(2*tan(f*x)^2*
tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) -
2*tan(e))*tan(f*x)^5*tan(e)^5 + 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)
*tan(e) - 1))*tan(f*x)^7*tan(e)^5 - 126*b*arctan(-(tan(f*x) - tan(e))/(tan(
f*x)*tan(e) + 1))*tan(f*x)^7*tan(e)^5 - 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 -
2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))
*tan(f*x)^4*tan(e)^6 - 42*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1
))*tan(f*x)^6*tan(e)^6 + 42*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e)
+ 1))*tan(f*x)^6*tan(e)^6 + 60*a*tan(f*x)^7*tan(e)^6 - 420*b*tan(f*x)^7*tan
(e)^6 + 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2
*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^7 + 126*b*arc
tan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^5*tan(e)^7 - 126*b*
arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^5*tan(e)^7 + 60
*a*tan(f*x)^6*tan(e)^7 - 420*b*tan(f*x)^6*tan(e)^7 + 180*a*f*x*tan(f*x)^7*t
an(e)^3 - 1260*b*f*x*tan(f*x)^7*tan(e)^3 + 63*pi*b*sgn(-2*tan(f*x)^2*tan(e)
+ 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^3 - 180*a
*f*x*tan(f*x)^6*tan(e)^4 + 1260*b*f*x*tan(f*x)^6*tan(e)^4 - 63*pi*b*sgn(-2*
tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6
*tan(e)^4 + 540*a*f*x*tan(f*x)^5*tan(e)^5 - 3780*b*f*x*tan(f*x)^5*tan(e)^5
+ 189*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*
tan(e))*tan(f*x)^5*tan(e)^5 - 180*a*f*x*tan(f*x)^4*tan(e)^6 + 1260*b*f*x*ta
n(f*x)^4*tan(e)^6 - 63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2
+ 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^6 + 180*a*f*x*tan(f*x)^3*tan(e)^
7 - 1260*b*f*x*tan(f*x)^3*tan(e)^7 + 63*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*t
an(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^7 + 21*pi*b*sgn
(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2
+ 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e) - 63*pi*b*sgn(2*tan(f*x)^2*tan(e
)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*ta
n(e))*tan(f*x)^6*tan(e)^2 + 189*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*
tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5
*tan(e)^3 + 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x
)^7*tan(e)^3 - 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan
(f*x)^7*tan(e)^3 - 189*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^
2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^4
- 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^6*tan(e
)^4 + 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^6*t
an(e)^4 + 160*a*tan(f*x)^7*tan(e)^4 - 1120*b*tan(f*x)^7*tan(e)^4 + 189*pi*b

```
*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e
)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^5 + 378*b*arctan((tan(f*x) +
 tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^5*tan(e)^5 - 378*b*arctan(-(tan(f*
x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^5*tan(e)^5 + 120*a*tan(f*x)^6*
tan(e)^5 - 840*b*tan(f*x)^6*tan(e)^5 - 63*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 -
2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*
tan(f*x)^2*tan(e)^6 - 126*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1
))*tan(f*x)^4*tan(e)^6 + 126*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e)
 + 1))*tan(f*x)^4*tan(e)^6 + 120*a*tan(f*x)^5*t...
```

Mupad [B]
time = 11.89, size = 105, normalized size = 1.03

$$x\left(\frac{5\,a}{16}-\frac{35\,b}{16}\right)-\frac{\left(\frac{11\,a}{16}-\frac{29\,b}{16}\right)\,\tan(e+f\,x)^5+\left(\frac{5\,a}{6}-\frac{17\,b}{6}\right)\,\tan(e+f\,x)^3+\left(\frac{5\,a}{16}-\frac{19\,b}{16}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}+\frac{b\tan(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(a + b*tan(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 - (35*b)/16) - (tan(e + f*x)^3*((5*a)/6 - (17*b)/6) + tan(e + f
*x)^5*((11*a)/16 - (29*b)/16) + tan(e + f*x)*((5*a)/16 - (19*b)/16))/(f*(3*
tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (b*tan(e + f*x))
/f
```

3.37 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=74

$$\frac{3}{8}(a-5b)x-\frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{(a-b)\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] 3/8*(a-5*b)*x-1/8*(5*a-9*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*(a-b)*cos(f*x+e)^3*
sin(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3744, 466, 1171, 396, 209}

$$\frac{(a-b)\sin(e+fx)\cos^3(e+fx)}{4f}-\frac{(5a-9b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-5b)+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (3*(a - 5*b)*x)/8 - ((5*a - 9*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + ((a - b
)*Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (b*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)\cos^3(e+fx)\sin(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{a-b-4(a-b)x^2-4bx^4}{(1+x^2)^2}\,dx\right.}{4f}$$

$$= -\frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)\cos^3(e+fx)\sin(e}{4f}$$

$$= -\frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)\cos^3(e+fx)\sin(e}{4f}$$

$$= \frac{3}{8}(a-5b)x - \frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{(a-b)\cos^3(e}{}$$

Mathematica [A]
time = 0.25, size = 58, normalized size = 0.78

$$\frac{12(a-5b)(e+fx)-8(a-2b)\sin(2(e+fx))+(a-b)\sin(4(e+fx))+32b\tan(e+fx)}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (12*(a - 5*b)*(e + f*x) - 8*(a - 2*b)*Sin[2*(e + f*x)] + (a - b)*Sin[4*(e +
 f*x)] + 32*b*Tan[e + f*x])/(32*f)
```

Maple [A]
time = 0.11, size = 102, normalized size = 1.38

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin^7(fx+e)}{\cos(fx+e)}+\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(}{f}$
default	$\frac{a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin^7(fx+e)}{\cos(fx+e)}+\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(}{f}$
risch	$\frac{3ax}{8}-\frac{15bx}{8}+\frac{i\mathrm{e}^{2i(fx+e)}a}{8f}-\frac{i\mathrm{e}^{2i(fx+e)}b}{4f}-\frac{i\mathrm{e}^{-2i(fx+e)}a}{8f}+\frac{i\mathrm{e}^{-2i(fx+e)}b}{4f}+\frac{2ib}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}+\frac{\sin(4fx+4e)a}{32f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+b*(sin
(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*
x+e)-15/8*f*x-15/8*e))
```

Maxima [A]
time = 0.50, size = 88, normalized size = 1.19

$$\frac{3\,(fx+e)(a-5\,b)+8\,b\tan\left(fx+e\right)-\frac{(5\,a-9\,b)\tan(fx+e)^3+(3\,a-7\,b)\tan(fx+e)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(f*x + e)*(a - 5*b) + 8*b*tan(f*x + e) - ((5*a - 9*b)*tan(f*x + e)^3
 + (3*a - 7*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 4.01, size = 77, normalized size = 1.04

$$\frac{3\,(a-5\,b)fx\cos\left(fx+e\right)+\left(2\,(a-b)\cos\left(fx+e\right)^4-(5\,a-9\,b)\cos\left(fx+e\right)^2+8\,b\right)\sin\left(fx+e\right)}{8\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a - 5*b)*f*x*cos(f*x + e) + (2*(a - b)*cos(f*x + e)^4 - (5*a - 9*b)
*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)\sin^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4350 vs. 2(73) = 146.
time = 1.92, size = 4350, normalized size = 58.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*ta
n(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^5 + 24*a*f*x*tan
(f*x)^5*tan(e)^5 - 120*b*f*x*tan(f*x)^5*tan(e)^5 + 3*pi*b*sgn(-2*tan(f*x)^2
*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^5
+ 6*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*
x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^3 - 3*pi*b*sgn(2*tan
(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*ta
n(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^4 + 6*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 -
 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))
*tan(f*x)^3*tan(e)^5 + 6*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1)
)*tan(f*x)^5*tan(e)^5 - 6*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) +
1))*tan(f*x)^5*tan(e)^5 + 48*a*f*x*tan(f*x)^5*tan(e)^3 - 240*b*f*x*tan(f*x)
^5*tan(e)^3 + 6*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan
(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^3 - 24*a*f*x*tan(f*x)^4*tan(e)^4 + 120*
b*f*x*tan(f*x)^4*tan(e)^4 - 3*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*ta
n(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^4 + 48*a*f*x*tan(f*x)^3*t
an(e)^5 - 240*b*f*x*tan(f*x)^3*tan(e)^5 + 6*pi*b*sgn(-2*tan(f*x)^2*tan(e) +
 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^5 + 3*pi*b*
sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)
^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e) - 6*pi*b*sgn(2*tan(f*x)^2*tan
(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*
tan(e))*tan(f*x)^4*tan(e)^2 + 12*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2
*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^
3*tan(e)^3 + 12*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x
)^5*tan(e)^3 - 12*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(
```

```
f*x)^5*tan(e)^3 - 6*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*t
an(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^4 -
6*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^4*tan(e)^4 +
 6*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^4*tan(e)^4
 + 24*a*tan(f*x)^5*tan(e)^4 - 120*b*tan(f*x)^5*tan(e)^4 + 3*pi*b*sgn(2*tan(
f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan
(f*x) - 2*tan(e))*tan(f*x)*tan(e)^5 + 12*b*arctan((tan(f*x) + tan(e))/(tan(
f*x)*tan(e) - 1))*tan(f*x)^3*tan(e)^5 - 12*b*arctan(-(tan(f*x) - tan(e))/(t
an(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^5 + 24*a*tan(f*x)^4*tan(e)^5 - 120*b
*tan(f*x)^4*tan(e)^5 + 24*a*f*x*tan(f*x)^5*tan(e) - 120*b*f*x*tan(f*x)^5*ta
n(e) + 3*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) -
 2*tan(e))*tan(f*x)^5*tan(e) - 48*a*f*x*tan(f*x)^4*tan(e)^2 + 240*b*f*x*tan
(f*x)^4*tan(e)^2 - 6*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 +
2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^2 + 96*a*f*x*tan(f*x)^3*tan(e)^3 -
 480*b*f*x*tan(f*x)^3*tan(e)^3 + 12*pi*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f
*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^3 - 48*a*f*x*tan(f*
x)^2*tan(e)^4 + 240*b*f*x*tan(f*x)^2*tan(e)^4 - 6*pi*b*sgn(-2*tan(f*x)^2*ta
n(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^2*tan(e)^4 + 2
4*a*f*x*tan(f*x)*tan(e)^5 - 120*b*f*x*tan(f*x)*tan(e)^5 + 3*pi*b*sgn(-2*tan
(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(
e)^5 - 3*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*t
an(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4 + 6*pi*b*sgn(2*tan(f*x
)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*
x) - 2*tan(e))*tan(f*x)^3*tan(e) + 6*b*arctan((tan(f*x) + tan(e))/(tan(f*x)
*tan(e) - 1))*tan(f*x)^5*tan(e) - 6*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)
*tan(e) + 1))*tan(f*x)^5*tan(e) - 12*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sg
n(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f
*x)^2*tan(e)^2 - 12*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan
(f*x)^4*tan(e)^2 + 12*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*
tan(f*x)^4*tan(e)^2 + 40*a*tan(f*x)^5*tan(e)^2 - 200*b*tan(f*x)^5*tan(e)^2
+ 6*pi*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*
x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e)^3 + 24*b*arctan((tan(f
*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^3*tan(e)^3 - 24*b*arctan(-(ta
n(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^3*tan(e)^3 + 24*a*tan(f*x)
^4*tan(e)^3 - 120*b*tan(f*x)^4*tan(e)^3 - 3*pi*b*sgn(2*tan(f*x)^2*tan(e)^2
- 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e)
)*tan(e)^4 - 12*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x
)^2*tan(e)^4 + 12*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(
f*x)^2*tan(e)^4 + 24*a*tan(f*x)^3*tan(e)^4 - 120*b*tan(f*x)^3*tan(e)^4 + 6*
b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e)^5 - 6*b
*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)*tan(e)^5 + 40*
a*tan(f*x)^2*tan(e)^5 - 200*b*tan(f*x)^2*tan(e)...
```

Mupad [B]

time = 11.43, size = 79, normalized size = 1.07

$$x\left(\frac{3a}{8}-\frac{15b}{8}\right)-\frac{\left(\frac{5a}{8}-\frac{9b}{8}\right)\tan(e+fx)^3+\left(\frac{3a}{8}-\frac{7b}{8}\right)\tan(e+fx)}{f\left(\tan\left(e+fx\right)^4+2\tan\left(e+fx\right)^2+1\right)}+\frac{b\tan(e+fx)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2),x)
```

```
[Out] x*((3*a)/8 - (15*b)/8) - (tan(e + f*x)^3*((5*a)/8 - (9*b)/8) + tan(e + f*x)
*((3*a)/8 - (7*b)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b*tan(
e + f*x))/f
```

3.38 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}(a-3b)x-\frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

[Out] 1/2*(a-3*b)*x-1/2*(a-b)*cos(f*x+e)*sin(f*x+e)/f+b*tan(f*x+e)/f

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3744, 466, 396, 209}

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}x(a-3b)+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]

[Out] ((a - 3*b)*x)/2 - ((a - b)*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b*Tan[e + f*x])/f

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{\text{Subst}\left(\int\frac{x^2(a+bx^2)}{(1+x^2)^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f}-\frac{\text{Subst}\left(\int\frac{-a+b-2bx^2}{1+x^2}\,dx,x,\tan\right.}{2f}$$

$$=-\frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}+\frac{(a-3b)\text{Su}}{}$$

$$=\frac{1}{2}(a-3b)x-\frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

Mathematica [A]
time = 0.16, size = 43, normalized size = 0.93

$$\frac{2(a-3b)(e+fx)+(-a+b)\sin(2(e+fx))+4b\tan(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
[Out] (2*(a - 3*b)*(e + f*x) + (-a + b)*Sin[2*(e + f*x)] + 4*b*Tan[e + f*x])/(4*f
)
```

Maple [A]
time = 0.11, size = 81, normalized size = 1.76

method	result	size
derivativedivides	$\frac{b\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)+a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$	81
default	$\frac{b\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)+a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$	81
risch	$\frac{ax}{2}-\frac{3bx}{2}+\frac{i e^{2i(fx+e)}a}{8f}-\frac{i e^{2i(fx+e)}b}{8f}-\frac{i e^{-2i(fx+e)}a}{8f}+\frac{i e^{-2i(fx+e)}b}{8f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(sin(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/
2*f*x-3/2*e)+a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.49, size = 55, normalized size = 1.20

$$\frac{(fx+e)(a-3\,b)+2\,b\tan\left(fx+e\right)-\frac{(a-b)\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a - 3*b) + 2*b*tan(f*x + e) - (a - b)*tan(f*x + e)/(tan(f*x
 + e)^2 + 1))/f
```

Fricas [A]
time = 2.76, size = 58, normalized size = 1.26

$$\frac{(a-3\,b)fx\cos\left(fx+e\right)-\left((a-b)\cos\left(fx+e\right)^2-2\,b\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a - 3*b)*f*x*cos(f*x + e) - ((a - b)*cos(f*x + e)^2 - 2*b)*sin(f*x +
e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(45) = 90.
time = 0.61, size = 395, normalized size = 8.59

$$\frac{afx\tan\left(fx\right)^3\tan\left(e\right)^3-3\,bfx\tan\left(fx\right)^3\tan\left(e\right)^3+afx\tan\left(fx\right)^3\tan\left(e\right)-3\,bfx\tan\left(fx\right)^3\tan\left(e\right)-afx\tan\left(fx\right)^2\tan\left(e\right)^2+3\,bfx\tan\left(fx\right)^2\tan\left(e\right)^2+afx\tan\left(fx\right)\tan\left(e\right)^3-3\,bfx\tan\left(fx\right)\tan\left(e\right)^3+a\tan\left(fx\right)^3\tan\left(e\right)^2-3\,b\tan\left(fx\right)^3\tan\left(e\right)^2+a\tan\left(fx\right)^2\tan\left(e\right)^3-3\,b\tan\left(fx\right)^2\tan\left(e\right)^3-afx\tan\left(fx\right)^2+3\,bfx\tan\left(fx\right)^2+afx\tan\left(fx\right)\tan\left(e\right)-3\,bfx\tan\left(fx\right)\tan\left(e\right)-afx\tan\left(e\right)^2+3\,bfx\tan\left(e\right)^2-2\,b\tan\left(fx\right)^3-2\,a\tan\left(fx\right)^2\tan\left(e\right)-2\,a\tan\left(fx\right)\tan\left(e\right)^2-2\,b\tan\left(e\right)^3-afx+3\,bfx+a\tan\left(fx\right)-3\,b\tan\left(fx\right)+a\tan\left(e\right)-3\,b\tan\left(e\right)}{2\left(f\tan\left(fx\right)^3\tan\left(e\right)^3+f\tan\left(fx\right)^3\tan\left(e\right)-f\tan\left(fx\right)^2\tan\left(e\right)^2+f\tan\left(fx\right)\tan\left(e\right)^3-f\tan\left(fx\right)^2+f\tan\left(fx\right)\tan\left(e\right)-f\tan\left(e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*f*x*tan(f*x)^3*tan(e)^3 - 3*b*f*x*tan(f*x)^3*tan(e)^3 + a*f*x*tan(f*
x)^3*tan(e) - 3*b*f*x*tan(f*x)^3*tan(e) - a*f*x*tan(f*x)^2*tan(e)^2 + 3*b*f
*x*tan(f*x)^2*tan(e)^2 + a*f*x*tan(f*x)*tan(e)^3 - 3*b*f*x*tan(f*x)*tan(e)^
3 + a*tan(f*x)^3*tan(e)^2 - 3*b*tan(f*x)^3*tan(e)^2 + a*tan(f*x)^2*tan(e)^3
 - 3*b*tan(f*x)^2*tan(e)^3 - a*f*x*tan(f*x)^2 + 3*b*f*x*tan(f*x)^2 + a*f*x*
tan(f*x)*tan(e) - 3*b*f*x*tan(f*x)*tan(e) - a*f*x*tan(e)^2 + 3*b*f*x*tan(e)
^2 - 2*b*tan(f*x)^3 - 2*a*tan(f*x)^2*tan(e) - 2*a*tan(f*x)*tan(e)^2 - 2*b*t
an(e)^3 - a*f*x + 3*b*f*x + a*tan(f*x) - 3*b*tan(f*x) + a*tan(e) - 3*b*tan(
e))/(f*tan(f*x)^3*tan(e)^3 + f*tan(f*x)^3*tan(e) - f*tan(f*x)^2*tan(e)^2 +
f*tan(f*x)*tan(e)^3 - f*tan(f*x)^2 + f*tan(f*x)*tan(e) - f*tan(e)^2 - f)
```

Mupad [B]
time = 11.32, size = 41, normalized size = 0.89

$$\frac{b\tan(e+f\,x)-\sin\left(2\,e+2\,f\,x\right)\,\left(\frac{a}{4}-\frac{b}{4}\right)+f\,x\,\left(\frac{a}{2}-\frac{3\,b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (b*tan(e + f*x) - sin(2*e + 2*f*x)*(a/4 - b/4) + f*x*(a/2 - (3*b)/2))/f
```

3.39 $\int \left(a+b\tan^2(e+fx)\right) \, dx$

Optimal. Leaf size=19

$$ax-bx+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x-b*x+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3554, 8}

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - b*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^2(e+fx)\right) \, dx &= ax+b\int \tan^2(e+fx)\,dx\\
&= ax+\frac{b\tan(e+fx)}{f}-b\int 1\,dx\\
&= ax-bx+\frac{b\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.47

$$ax-\frac{b\text{ArcTan}(\tan(e+fx))}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[e + f*x]^2,x]
[Out] a*x - (b*ArcTan[Tan[e + f*x]])/f + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.53

method	result	size
norman	$(a-b)\,x+\frac{b\tan(fx+e)}{f}$	20
derivativedivides	$\frac{b\tan(fx+e)+(a-b)\arctan(\tan(fx+e))}{f}$	27
default	$ax+\frac{b\tan(fx+e)}{f}-\frac{b\arctan(\tan(fx+e))}{f}$	29
risch	$ax-bx+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] a*x+b*tan(f*x+e)/f-b/f*arctan(tan(f*x+e))
```

Maxima [A]
time = 0.49, size = 25, normalized size = 1.32

$$ax-\frac{(fx+e-\tan(fx+e))b}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="maxima")
[Out] a*x - (f*x + e - tan(f*x + e))*b/f
```

Fricas [A]
time = 2.07, size = 22, normalized size = 1.16

$$\frac{(a-b)fx+b\tan(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="fricas")
[Out] ((a - b)*f*x + b*tan(f*x + e))/f
```

Sympy [A]
time = 0.06, size = 20, normalized size = 1.05

$$ax+b\left(\begin{cases}-x+\frac{\tan(e+fx)}{f} & \text{for } f\neq 0\\ x\tan^2(e) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)**2,x)
```

```
[Out] a*x + b*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(20) = 40.
time = 0.54, size = 252, normalized size = 13.26

$$ax + \frac{\left(\pi - 4\,fx\tan\left(fx\right)\tan\left(e\right) - \pi\mathrm{sgn}\left(2\,\tan\left(fx\right)^{2}\tan\left(e\right) + 2\,\tan\left(fx\right)\tan\left(e\right)^{2} - 2\,\tan\left(fx\right) - 2\,\tan\left(e\right)\right)\tan\left(fx\right)\tan\left(e\right) - \pi\tan\left(fx\right)\tan\left(e\right) + 2\,\arctan\left(\frac{\tan(fx)\tan(e)-1}{\tan(fx)+\tan(e)}\right)\tan\left(fx\right)\tan\left(e\right) + 2\,\arctan\left(\frac{\tan(fx)+\tan(e)}{\tan(fx)\tan(e)-1}\right)\tan\left(fx\right)\tan\left(e\right) + 4\,fx + \pi\mathrm{sgn}\left(2\,\tan\left(fx\right)^{2}\tan\left(e\right) + 2\,\tan\left(fx\right)\tan\left(e\right)^{2} - 2\,\tan\left(fx\right) - 2\,\tan\left(e\right)\right) - 2\,\arctan\left(\frac{\tan(fx)\tan(e)-1}{\tan(fx)+\tan(e)}\right) - 2\,\arctan\left(\frac{\tan(fx)+\tan(e)}{\tan(fx)\tan(e)-1}\right) - 4\,\tan\left(fx\right) - 4\,\tan\left(e\right)\right)b}{4\left(f\tan\left(fx\right)\tan\left(e\right) - f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + 1/4*(pi - 4*f*x*tan(f*x)*tan(e) - pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(
f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e) - pi*tan(f*x)*tan(e)
 + 2*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*tan(f*x)*tan(e) + 2*
arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e) + 4*f*x +
 pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))
- 2*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e))) - 2*arctan((tan(f*x)
+ tan(e))/(tan(f*x)*tan(e) - 1)) - 4*tan(f*x) - 4*tan(e))*b/(f*tan(f*x)*tan
(e) - f)
```

Mupad [B]
time = 11.28, size = 21, normalized size = 1.11

$$\frac{b\tan\left(e+f\,x\right)+f\,x\,\left(a-b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + f*x*(a - b))/f
```

3.40 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=24

$$-\frac{a\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -a*cot(f*x+e)/f+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3744, 14}

$$\frac{b\tan(e+fx)}{f}-\frac{a\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx&=\frac{\text{Subst}\left(\int \frac{a+bx^2}{x^2}\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int \left(b+\frac{a}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\&=-\frac{a\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$-\frac{a\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.14, size = 23, normalized size = 0.96

method	result	size
derivativedivides	$\frac{b\tan(fx+e)-\cot(fx+e)a}{f}$	23
default	$\frac{b\tan(fx+e)-\cot(fx+e)a}{f}$	23
risch	$-\frac{2i\left(a\,\mathrm{e}^{2i(fx+e)}-b\,\mathrm{e}^{2i(fx+e)}+a+b\right)}{f\left(\mathrm{e}^{2i(fx+e)}-1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*tan(f*x+e)-cot(f*x+e)*a)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.08

$$\frac{b\tan\left(fx+e\right)-\frac{a}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (b*tan(f*x + e) - a/tan(f*x + e))/f
```

Fricas [A]
time = 1.37, size = 40, normalized size = 1.67

$$-\frac{(a+b)\cos\left(fx+e\right)^{2}-b}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -((a + b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)\csc^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**2, x)
```

Giac [**A**]
time = 0.54, size = 26, normalized size = 1.08

$$\frac{b\tan\left(fx + e\right) - \frac{a}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
[Out] (b*tan(f*x + e) - a/tan(f*x + e))/f
```

Mupad [**B**]
time = 11.27, size = 26, normalized size = 1.08

$$\frac{b\tan(e + f\,x)}{f} - \frac{a}{f\tan\left(e + f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^2,x)
[Out] (b*tan(e + f*x))/f - a/(f*tan(e + f*x))
```

3.41 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=42

$$-\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+b)*cot(f*x+e)/f-1/3*a*cot(f*x+e)^3/f+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3744, 459}

$$-\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
[Out] -(((a + b)*Cot[e + f*x])/f) - (a*Cot[e + f*x]^3)/(3*f) + (b*Tan[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+bx^2)}{x^4}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(b+\frac{a}{x^4}+\frac{a+b}{x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= -\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 1.43

$$-\frac{2a\cot(e+fx)}{3f}-\frac{b\cot(e+fx)}{f}-\frac{a\cot(e+fx)\csc^2(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-2*a*Cot[e + f*x])/(3*f) - (b*Cot[e + f*x])/f - (a*Cot[e + f*x]*Csc[e + f*
x]^2)/(3*f) + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.12, size = 54, normalized size = 1.29

method	result	size
derivativedivides	$\frac{b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)}{f}$	54
default	$\frac{b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)}{f}$	54
risch	$\frac{4i\left(3a\,e^{4i(fx+e)}-3b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}-a-3b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e))+a*(-2/3-1/3*csc(f*x+e)^2)*cot
(f*x+e))
```

Maxima [A]
time = 0.30, size = 43, normalized size = 1.02

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,(a+b)\tan(fx+e)^2+a}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*(a + b)*tan(f*x + e)^2 + a)/tan(f*x + e)^3)/f
```

Fricas [A]
time = 1.44, size = 71, normalized size = 1.69

$$-\frac{2\,(a+3\,b)\cos\left(fx+e\right)^4-3\,(a+3\,b)\cos\left(fx+e\right)^2+3\,b}{3\,\left(f\cos\left(fx+e\right)^3-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a + 3*b)*cos(f*x + e)^4 - 3*(a + 3*b)*cos(f*x + e)^2 + 3*b)/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)\csc^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**4, x)
```

Giac [A]
time = 0.60, size = 53, normalized size = 1.26

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,a\tan(fx+e)^2+3\,b\tan(fx+e)^2+a}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 3*b*tan(f*x + e)^2 + a)/tan(f
*x + e)^3)/f
```

Mupad [B]
time = 11.49, size = 41, normalized size = 0.98

$$\frac{b\tan(e+f\,x)}{f}-\frac{(a+b)\,\tan(e+f\,x)^2+\frac{a}{3}}{f\tan\left(e+f\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^4,x)
```

```
[Out] (b*tan(e + f*x))/f - (a/3 + tan(e + f*x)^2*(a + b))/(f*tan(e + f*x)^3)
```

3.42 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=64

$$-\frac{(a+2b)\cot(e+fx)}{f}-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{a\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(a+2*b)*cot(f*x+e)/f-1/3*(2*a+b)*cot(f*x+e)^3/f-1/5*a*cot(f*x+e)^5/f+b*tan
(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3744, 459}

$$-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}-\frac{a\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cot[e + f*x])/f) - ((2*a + b)*Cot[e + f*x]^3)/(3*f) - (a*Cot[e
 + f*x]^5)/(5*f) + (b*Tan[e + f*x])/f
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+bx^2)}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b+\frac{a}{x^6}+\frac{2a+b}{x^4}+\frac{a+2b}{x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a+2b)\cot(e+fx)}{f}-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{a\cot^5(e+fx)}{5f}$$

Mathematica [A]
time = 0.04, size = 106, normalized size = 1.66

$$-\frac{8a\cot(e+fx)}{15f}-\frac{5b\cot(e+fx)}{3f}-\frac{4a\cot(e+fx)\csc^2(e+fx)}{15f}-\frac{b\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{a\cot(e+fx)\csc^4(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
[Out] (-8*a*Cot[e + f*x])/(15*f) - (5*b*Cot[e + f*x])/(3*f) - (4*a*Cot[e + f*x]*C
sc[e + f*x]^2)/(15*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) - (a*Cot[e +
f*x]*Csc[e + f*x]^4)/(5*f) + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.14, size = 83, normalized size = 1.30

method	result	s
derivativedivides	$\frac{b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(fx+e)\right)}{5}-\frac{4\left(\csc^2(fx+e)\right)}{15}\right)\cot(fx+e)}{f}$	8
default	$\frac{b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(fx+e)\right)}{5}-\frac{4\left(\csc^2(fx+e)\right)}{15}\right)\cot(fx+e)}{f}$	8
risch	$-\frac{16i\left(10a\,e^{6i(fx+e)}-10b\,e^{6i(fx+e)}+5a\,e^{4i(fx+e)}+25b\,e^{4i(fx+e)}-4a\,e^{2i(fx+e)}-20b\,e^{2i(fx+e)}+a+5b\right)}{15f\left(e^{2i(fx+e)}-1\right)^5\left(e^{2i(fx+e)}+1\right)}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+
e))+a*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e))
```

Maxima [A]
time = 0.28, size = 63, normalized size = 0.98

$$\frac{15\,b\tan\left(fx+e\right)-\frac{15\,(a+2\,b)\tan(fx+e)^4+5\,(2\,a+b)\tan(fx+e)^2+3\,a}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*(a + 2*b)*tan(f*x + e)^4 + 5*(2*a + b)*tan(f*
x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

Fricas [A]
time = 1.02, size = 98, normalized size = 1.53

$$-\frac{8\,(a+5\,b)\cos\left(fx+e\right)^{6}-20\,(a+5\,b)\cos\left(fx+e\right)^{4}+15\,(a+5\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\,\left(f\cos\left(fx+e\right)^{5}-2\,f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a + 5*b)*cos(f*x + e)^6 - 20*(a + 5*b)*cos(f*x + e)^4 + 15*(a + 5
*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(
f*x + e))*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)\csc^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**6, x)
```

Giac [A]
time = 0.62, size = 79, normalized size = 1.23

$$\frac{15\,b\tan\left(fx+e\right)-\frac{15\,a\tan(fx+e)^{4}+30\,b\tan(fx+e)^{4}+10\,a\tan(fx+e)^{2}+5\,b\tan(fx+e)^{2}+3\,a}{\tan(fx+e)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 30*b*tan(f*x + e)^4 + 10*a
*tan(f*x + e)^2 + 5*b*tan(f*x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

Mupad [B]
time = 11.41, size = 59, normalized size = 0.92

$$\frac{b\tan(e+f\,x)}{f}-\frac{(a+2\,b)\,\tan(e+f\,x)^{4}+\left(\frac{2\,a}{3}+\frac{b}{3}\right)\,\tan(e+f\,x)^{2}+\frac{a}{5}}{f\tan\left(e+f\,x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)/sin(e + f*x)^6,x)
```

```
[Out] (b*tan(e + f*x))/f - (a/5 + tan(e + f*x)^2*((2*a)/3 + b/3) + tan(e + f*x)^4
*(a + 2*b))/(f*tan(e + f*x)^5)
```

3.43 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=107

$$-\frac{(a^2-6ab+6b^2)\cos(e+fx)}{f}+\frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f}-\frac{(a-b)^2\cos^5(e+fx)}{5f}+\frac{2(a-2b)b\sec(e+\ldots}{f}$$

```
[Out] -(a^2-6*a*b+6*b^2)*cos(f*x+e)/f+2/3*(a-2*b)*(a-b)*cos(f*x+e)^3/f-1/5*(a-b)^
2*cos(f*x+e)^5/f+2*(a-2*b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A]

time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3745, 459}

$$-\frac{(a^2-6ab+6b^2)\cos(e+fx)}{f}-\frac{(a-b)^2\cos^5(e+fx)}{5f}+\frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-2b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a^2 - 6*a*b + 6*b^2)*Cos[e + f*x])/f) + (2*(a - 2*b)*(a - b)*Cos[e + f*
x]^3)/(3*f) - ((a - b)^2*Cos[e + f*x]^5)/(5*f) + (2*(a - 2*b)*b*Sec[e + f*x
])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a-b+bx^2)^2}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(2(a-2b)b+\frac{(a-b)^2}{x^6}+\frac{2(a-2b)(-a+b)}{x^4}+\frac{a^2-6ab+6b^2}{x^2}+b^2x^2\right.\right.}{f}$$

$$= -\frac{(a^2-6ab+6b^2)\cos(e+fx)}{f}+\frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.49, size = 97, normalized size = 0.91

$$\frac{-30(5a^2-38ab+41b^2)\cos(e+fx)+5(5a-13b)(a-b)\cos(3(e+fx))-3(a-b)^2\cos(5(e+fx))+480(a-2b)b\sec(e+fx)+80b^2\sec^3(e+fx)}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-30*(5*a^2 - 38*a*b + 41*b^2)*Cos[e + f*x] + 5*(5*a - 13*b)*(a - b)*Cos[3*
(e + f*x)] - 3*(a - b)^2*Cos[5*(e + f*x)] + 480*(a - 2*b)*b*Sec[e + f*x] +
80*b^2*Sec[e + f*x]^3)/(240*f)
```

Maple [A]
time = 0.16, size = 185, normalized size = 1.73

method	result
derivativedivides	$b^2\left(\frac{\sin^{10}(fx+e)}{3\cos(fx+e)^3}-\frac{7\left(\sin^{10}(fx+e)\right)}{3\cos(fx+e)}-\frac{7\left(\frac{128}{35}+\sin^8(fx+e)+\frac{8\left(\sin^6(fx+e)\right)}{7}+\frac{48\left(\sin^4(fx+e)\right)}{35}+\frac{64\left(\sin^2(fx+e)\right)}{35}\right)\cos(fx+e)}{3}\right)+2a$
default	$b^2\left(\frac{\sin^{10}(fx+e)}{3\cos(fx+e)^3}-\frac{7\left(\sin^{10}(fx+e)\right)}{3\cos(fx+e)}-\frac{7\left(\frac{128}{35}+\sin^8(fx+e)+\frac{8\left(\sin^6(fx+e)\right)}{7}+\frac{48\left(\sin^4(fx+e)\right)}{35}+\frac{64\left(\sin^2(fx+e)\right)}{35}\right)\cos(fx+e)}{3}\right)+2a$
risch	$-\frac{e^{5i(fx+e)}a^2}{160f}+\frac{e^{5i(fx+e)}ab}{80f}-\frac{e^{5i(fx+e)}b^2}{160f}+\frac{5e^{3i(fx+e)}a^2}{96f}-\frac{3e^{3i(fx+e)}ab}{16f}+\frac{13e^{3i(fx+e)}b^2}{96f}-\frac{5e^{i(fx+e)}a^2}{16f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/3*sin(f*x+e)^10/cos(f*x+e)^3-7/3*sin(f*x+e)^10/cos(f*x+e)-7/3*(
128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*sin(f*x+e)^2)
*cos(f*x+e))+2*a*b*(sin(f*x+e)^8/cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+
```

```
e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e))-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)
^2)*cos(f*x+e))
```

Maxima [A]
time = 0.30, size = 109, normalized size = 1.02

$$-\frac{3\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{5}-10\left(a^{2}-3\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{3}+15\left(a^{2}-6\,ab+6\,b^{2}\right)\cos\left(fx+e\right)-\frac{5\left(6\left(ab-2\,b^{2}\right)\cos\left(fx+e\right)^{2}+b^{2}\right)}{\cos\left(fx+e\right)^{3}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 10*(a^2 - 3*a*b + 2*b^2)*cos(
f*x + e)^3 + 15*(a^2 - 6*a*b + 6*b^2)*cos(f*x + e) - 5*(6*(a*b - 2*b^2)*cos
(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Fricas [A]
time = 2.53, size = 110, normalized size = 1.03

$$-\frac{3\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{8}-10\left(a^{2}-3\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{6}+15\left(a^{2}-6\,ab+6\,b^{2}\right)\cos\left(fx+e\right)^{4}-30\left(ab-2\,b^{2}\right)\cos\left(fx+e\right)^{2}-5\,b^{2}}{15\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^8 - 10*(a^2 - 3*a*b + 2*b^2)*cos(
f*x + e)^6 + 15*(a^2 - 6*a*b + 6*b^2)*cos(f*x + e)^4 - 30*(a*b - 2*b^2)*cos
(f*x + e)^2 - 5*b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)^{2}\sin^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 207699 vs. 2(106) = 212.
time = 266.31, size = 207699, normalized size = 1941.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")

[Out] 1/3840*(1260*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 - 2205*pi*b^2*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 + 1260*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 - 2205*pi*b^2*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 - 1260*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 + 2205*pi*b^2*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 + 1260*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 - 2205*pi*b^2*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 - 2520*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - tan(1/2*f*x)^2 - 4*tan(1/2*f*x)*tan(1/2*e) - tan(1/2*e)^2 + 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 + 4410*pi*b^2*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - tan(1/2*f*x)^2 - 4*tan(1/2*f*x)*tan(1/2*e) - tan(1/2*e)^2 + 1)*tan(1/2*f*x)^16*tan(1/2*e)^16 + 2520*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^14 - 4410*pi*b^2*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^14 + 2520*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^14 - 4410*pi*b^2*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^16*tan(1/2*e)^14 - 15120*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e

```
)^2 + 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*ta
n(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*
x)^15*tan(1/2*e)^15 + 26460*pi*b^2*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(
1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*s
gn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)
^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^15*tan(1/2*e)^15 - 151
20*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + t
an(1/2*f*x)^2 - tan(1/2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2
*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan
(1/2*f*x) - 1)*tan(1/2*f*x)^15*tan(1/2*e)^15 + 26460*pi*b^2*sgn(tan(1/2*f*x
)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e
)^2 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*ta
n(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*tan(1/2*f*
x)^15*tan(1/2*e)^15 + 2520*pi*a*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1
/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*sg
n(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^
2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^14*tan(1/2*e)^16 - 4410
*pi*b^2*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan
(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e
)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1
/2*f*x) - 1)*tan(1/2*f*x)^14*tan(1/2*e)^16 + 2520*pi*a*b*sgn(tan(1/2*f*x)^2
*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2
 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1
/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan...
```

Mupad [B]
time = 12.65, size = 183, normalized size = 1.71

$$\frac{2\,a^2\cos\left(e+f\,x\right)^3}{3\,f}-\frac{6\,b^2\,\cos\left(e+f\,x\right)}{f}-\frac{a^2\,\cos\left(e+f\,x\right)}{f}-\frac{a^2\cos\left(e+f\,x\right)^5}{5\,f}-\frac{4\,b^2}{f\,\cos\left(e+f\,x\right)}+\frac{b^2}{3\,f\cos\left(e+f\,x\right)^3}+\frac{4\,b^2\cos\left(e+f\,x\right)^3}{3\,f}-\frac{b^2\cos\left(e+f\,x\right)^5}{5\,f}+\frac{6\,a\,b\,\cos\left(e+f\,x\right)}{f}+\frac{2\,a\,b}{f\,\cos\left(e+f\,x\right)}-\frac{2\,a\,b\cos\left(e+f\,x\right)^3}{f}+\frac{2\,a\,b\cos\left(e+f\,x\right)^5}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (2*a^2*cos(e + f*x)^3)/(3*f) - (6*b^2*cos(e + f*x))/f - (a^2*cos(e + f*x))/
f - (a^2*cos(e + f*x)^5)/(5*f) - (4*b^2)/(f*cos(e + f*x)) + b^2/(3*f*cos(e
+ f*x)^3) + (4*b^2*cos(e + f*x)^3)/(3*f) - (b^2*cos(e + f*x)^5)/(5*f) + (6*
a*b*cos(e + f*x))/f + (2*a*b)/(f*cos(e + f*x)) - (2*a*b*cos(e + f*x)^3)/f +
 (2*a*b*cos(e + f*x)^5)/(5*f)
```

3.44 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal. Leaf size=80

$$-\frac{(a-3b)(a-b)\cos(e+fx)}{f}+\frac{(a-b)^2\cos^3(e+fx)}{3f}+\frac{(2a-3b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a-3*b)*(a-b)*cos(f*x+e)/f+1/3*(a-b)^2*cos(f*x+e)^3/f+(2*a-3*b)*b*sec(f*x+
e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3745, 459}

$$\frac{(a-b)^2\cos^3(e+fx)}{3f}-\frac{(a-3b)(a-b)\cos(e+fx)}{f}+\frac{b(2a-3b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - 3*b)*(a - b)*Cos[e + f*x])/f) + ((a - b)^2*Cos[e + f*x]^3)/(3*f) +
((2*a - 3*b)*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)^2}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((2a-3b)b - \frac{(a-b)^2}{x^4} + \frac{(a-3b)(a-b)}{x^2} + b^2x^2\right)dx, x, \sec(e$$
$$= -\frac{(a-3b)(a-b)\cos(e+fx)}{f} + \frac{(a-b)^2\cos^3(e+fx)}{3f} + \frac{(2a-$$

Mathematica [A]
time = 0.37, size = 72, normalized size = 0.90

$$\frac{(-9a^2+42ab-33b^2)\cos(e+fx)+(a-b)^2\cos(3(e+fx))+4b\sec(e+fx)\left(6a-9b+b\sec^2(e+fx)\right)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((-9*a^2 + 42*a*b - 33*b^2)*Cos[e + f*x] + (a - b)^2*Cos[3*(e + f*x)] + 4*b
*Sec[e + f*x]*(6*a - 9*b + b*Sec[e + f*x]^2))/(12*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(76) = 152.
time = 0.12, size = 155, normalized size = 1.94

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^8(fx+e)}{3\cos(fx+e)^3}-\frac{5\left(\sin^8(fx+e)\right)}{3\cos(fx+e)}-\frac{5\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{3}\right)+2ab\left(\frac{\sin^6(fx+e)}{\cos(fx+e)}+\left(\right.\right.}{f}$
default	$\frac{b^2\left(\frac{\sin^8(fx+e)}{3\cos(fx+e)^3}-\frac{5\left(\sin^8(fx+e)\right)}{3\cos(fx+e)}-\frac{5\left(\frac{16}{5}+\sin^6(fx+e)+\frac{6\left(\sin^4(fx+e)\right)}{5}+\frac{8\left(\sin^2(fx+e)\right)}{5}\right)\cos(fx+e)}{3}\right)+2ab\left(\frac{\sin^6(fx+e)}{\cos(fx+e)}+\left(\right.\right.}{f}$
risch	$\frac{e^{3i(fx+e)}a^2}{24f}-\frac{e^{3i(fx+e)}ab}{12f}+\frac{e^{3i(fx+e)}b^2}{24f}-\frac{3\,e^{i(fx+e)}a^2}{8f}+\frac{7\,e^{i(fx+e)}ab}{4f}-\frac{11\,e^{i(fx+e)}b^2}{8f}-\frac{3\,e^{-i(fx+e)}a^2}{8f}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/3*sin(f*x+e)^8/cos(f*x+e)^3-5/3*sin(f*x+e)^8/cos(f*x+e)-5/3*(16
/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e))+2*a*b*(sin(f
```

```
*x+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))-1/3*a^2*
(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A]
time = 0.29, size = 84, normalized size = 1.05

$$\frac{(a^2 - 2\,ab + b^2)\cos{(fx+e)}^3 - 3\,(a^2 - 4\,ab + 3\,b^2)\cos{(fx+e)} + \frac{3\,(2\,ab - 3\,b^2)\cos(fx+e)^2 + b^2}{\cos(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - 3*(a^2 - 4*a*b + 3*b^2)*cos(f*x +
 e) + (3*(2*a*b - 3*b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Fricas [A]
time = 2.54, size = 84, normalized size = 1.05

$$\frac{(a^2 - 2\,ab + b^2)\cos{(fx+e)}^6 - 3\,(a^2 - 4\,ab + 3\,b^2)\cos{(fx+e)}^4 + 3\,(2\,ab - 3\,b^2)\cos{(fx+e)}^2 + b^2}{3\,f\cos{(fx+e)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^6 - 3*(a^2 - 4*a*b + 3*b^2)*cos(f*x +
 e)^4 + 3*(2*a*b - 3*b^2)*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2{(e+fx)}\right)^2\sin^3{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**3, x)
```

Giac [A]
time = 1.17, size = 144, normalized size = 1.80

$$\frac{6\,ab\cos{(fx+e)}^2 - 9\,b^2\cos{(fx+e)}^2 + b^2}{3\,f\cos{(fx+e)}^3} + \frac{a^2f^{11}\cos{(fx+e)}^3 - 2\,abf^{11}\cos{(fx+e)}^3 + b^2f^{11}\cos{(fx+e)}^3 - 3\,a^2f^{11}\cos{(fx+e)} + 12\,abf^{11}\cos{(fx+e)} - 9\,b^2f^{11}\cos{(fx+e)}}{3\,f^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*cos(f*x + e)^2 - 9*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
+ 1/3*(a^2*f^11*cos(f*x + e)^3 - 2*a*b*f^11*cos(f*x + e)^3 + b^2*f^11*cos(f
*x + e)^3 - 3*a^2*f^11*cos(f*x + e) + 12*a*b*f^11*cos(f*x + e) - 9*b^2*f^11
*cos(f*x + e))/f^12
```

Mupad [B]
time = 15.40, size = 128, normalized size = 1.60

$$-\frac{32\,a\,b+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(64\,a\,b-32\,a^2\right)+12\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(24\,a^2-96\,a\,b+96\,b^2\right)-4\,a^2-32\,b^2}{f\,\left(3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-9\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+9\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] -(32*a*b + tan(e/2 + (f*x)/2)^6*(64*a*b - 32*a^2) + 12*a^2*tan(e/2 + (f*x)/
2)^8 + tan(e/2 + (f*x)/2)^4*(24*a^2 - 96*a*b + 96*b^2) - 4*a^2 - 32*b^2)/(f
*(9*tan(e/2 + (f*x)/2)^4 - 9*tan(e/2 + (f*x)/2)^8 + 3*tan(e/2 + (f*x)/2)^12
 - 3))
```

3.45 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=54

$$-\frac{(a-b)^2\cos(e+fx)}{f}+\frac{2(a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(a-b)^2*cos(f*x+e)/f+2*(a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3745, 276}

$$-\frac{(a-b)^2\cos(e+fx)}{f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Cos[e + f*x])/f) + (2*(a - b)*b*Sec[e + f*x])/f + (b^2*Sec[e +
 f*x]^3)/(3*f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^2}{x^2}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(2(a-b)b+\frac{(a-b)^2}{x^2}+b^2x^2\right)\,dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{(a-b)^2\cos(e+fx)}{f}+\frac{2(a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 48, normalized size = 0.89

$$\frac{-3(a-b)^2\cos(e+fx)+b\sec(e+fx)\left(6a-6b+b\sec^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*(a - b)^2*Cos[e + f*x] + b*Sec[e + f*x]*(6*a - 6*b + b*Sec[e + f*x]^2))
/(3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(52) = 104.
time = 0.14, size = 125, normalized size = 2.31

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^6(fx+e)}{3\cos(fx+e)^3}-\frac{\sin^6(fx+e)}{\cos(fx+e)}-\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)\right)+2ab\left(\frac{\sin^4(fx+e)}{\cos(fx+e)}+\left(2+\sin^2(fx+e)\right)\cos(\right.}{f}$
default	$\frac{b^2\left(\frac{\sin^6(fx+e)}{3\cos(fx+e)^3}-\frac{\sin^6(fx+e)}{\cos(fx+e)}-\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4\left(\sin^2(fx+e)\right)}{3}\right)\cos(fx+e)\right)+2ab\left(\frac{\sin^4(fx+e)}{\cos(fx+e)}+\left(2+\sin^2(fx+e)\right)\cos(\right.}{f}$
risch	$-\frac{e^{i(fx+e)}a^2}{2f}+\frac{e^{i(fx+e)}ab}{f}-\frac{e^{i(fx+e)}b^2}{2f}-\frac{e^{-i(fx+e)}a^2}{2f}+\frac{e^{-i(fx+e)}ab}{f}-\frac{e^{-i(fx+e)}b^2}{2f}-\frac{4b\left(-3a\,e^{5i(fx+e)}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/3*sin(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f*
x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+2*a*b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(
f*x+e)^2)*cos(f*x+e))-a^2*cos(f*x+e))
```

Maxima [A]
time = 0.29, size = 77, normalized size = 1.43

$$\frac{6\,ab\left(\frac{1}{\cos(fx+e)}+\cos\left(fx+e\right)\right)-b^2\left(\frac{6\,\cos(fx+e)^2-1}{\cos(fx+e)^3}+3\,\cos\left(fx+e\right)\right)-3\,a^2\cos\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a*b*(1/cos(f*x + e) + cos(f*x + e)) - b^2*((6*cos(f*x + e)^2 - 1)/co
s(f*x + e)^3 + 3*cos(f*x + e)) - 3*a^2*cos(f*x + e))/f
```

Fricas [A]
time = 2.11, size = 62, normalized size = 1.15

$$-\frac{3\,(a^2-2\,ab+b^2)\cos\left(fx+e\right)^4-6\,(ab-b^2)\cos\left(fx+e\right)^2-b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 6*(a*b - b^2)*cos(f*x + e)^2 -
 b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^2\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x), x)
```

Giac [A]
time = 0.80, size = 94, normalized size = 1.74

$$-\frac{a^2f^3\cos\left(fx+e\right)-2\,abf^3\cos\left(fx+e\right)+b^2f^3\cos\left(fx+e\right)}{f^4}+\frac{6\,ab\cos\left(fx+e\right)^2-6\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -(a^2*f^3*cos(f*x + e) - 2*a*b*f^3*cos(f*x + e) + b^2*f^3*cos(f*x + e))/f^4
 + 1/3*(6*a*b*cos(f*x + e)^2 - 6*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^
3)
```

Mupad [B]
time = 14.12, size = 126, normalized size = 2.33

$$-\frac{8\,a\,b+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(8\,a\,b-6\,a^2)+2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(6\,a^2-16\,a\,b+\frac{32\,b^2}{3}\right)-2\,a^2-\frac{16\,b^2}{3}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] -(8*a*b + tan(e/2 + (f*x)/2)^4*(8*a*b - 6*a^2) + 2*a^2*tan(e/2 + (f*x)/2)^6
 + tan(e/2 + (f*x)/2)^2*(6*a^2 - 16*a*b + (32*b^2)/3) - 2*a^2 - (16*b^2)/3)
/(f*(2*tan(e/2 + (f*x)/2)^2 - 2*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8
 - 1))
```

3.46 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -a^2*arctanh(cos(f*x+e))/f+(2*a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3745, 398, 213}

$$-\frac{a^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[e + f*x]])/f) + ((2*a - b)*b*Sec[e + f*x])/f + (b^2*Sec[
e + f*x]^3)/(3*f)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-b+bx^2)^2}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{a^2}{-1+x^2}\right)\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{(2a-b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f} + \frac{a^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x,\right.}{f}$$
$$= -\frac{a^2\tanh^{-1}(\cos(e+fx))}{f} + \frac{(2a-b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+f}{3f}$$

Mathematica [A]
time = 0.12, size = 66, normalized size = 1.27

$$\frac{3a^2\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+3(2a-b)b\sec(e+fx)+b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
[Out] (3*a^2*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]) + 3*(2*a - b)*b*Sec
[e + f*x] + b^2*Sec[e + f*x]^3)/(3*f)
```

Maple [A]
time = 0.14, size = 97, normalized size = 1.87

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^4(fx+e)}{3\cos(fx+e)^3}-\frac{\sin^4(fx+e)}{3\cos(fx+e)}-\frac{\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}\right)+\frac{2ab}{\cos(fx+e)}+a^2\ln(\csc(fx+e)-\cot(fx+e))}{f}$
default	$\frac{b^2\left(\frac{\sin^4(fx+e)}{3\cos(fx+e)^3}-\frac{\sin^4(fx+e)}{3\cos(fx+e)}-\frac{\left(2+\sin^2(fx+e)\right)\cos(fx+e)}{3}\right)+\frac{2ab}{\cos(fx+e)}+a^2\ln(\csc(fx+e)-\cot(fx+e))}{f}$
risch	$-\frac{2b\left(-6a\,e^{5i(fx+e)}+3b\,e^{5i(fx+e)}-12a\,e^{3i(fx+e)}+2b\,e^{3i(fx+e)}-6a\,e^{i(fx+e)}+3b\,e^{i(fx+e)}\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}+\frac{a^2\ln\left(e^{i(fx+e)}-1\right)}{f}-\frac{a^2\,l}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(b^2*(1/3*sin(f*x+e)^4/cos(f*x+e)^3-1/3*sin(f*x+e)^4/cos(f*x+e)-1/3*(2+
sin(f*x+e)^2)*cos(f*x+e))+2*a*b/cos(f*x+e)+a^2*ln(csc(f*x+e)-cot(f*x+e)))
```

Maxima [**A**]
time = 0.29, size = 72, normalized size = 1.38

$$-\frac{3\,a^2\log\left(\cos\left(fx+e\right)+1\right)-3\,a^2\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(2\,ab-b^2\right)\cos(fx+e)^2+b^2\right)}{\cos(fx+e)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*log(cos(f*x + e) + 1) - 3*a^2*log(cos(f*x + e) - 1) - 2*(3*(2*a
*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Fricas [**A**]
time = 3.23, size = 93, normalized size = 1.79

$$-\frac{3\,a^2\cos\left(fx+e\right)^3\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-3\,a^2\cos\left(fx+e\right)^3\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-6\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2-2\,b^2}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3*a^2*cos(f*x + e)
^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*b - b^2)*cos(f*x + e)^2 - 2*b^2)/(
f*cos(f*x + e)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(e+fx\right)\right)^2\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(53) = 106.
time = 0.79, size = 149, normalized size = 2.87

$$\frac{3\,a^2\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{8\left(3\,ab-b^2+\frac{6\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{3\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) + 8*(3*a*b - b
^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3*b^2*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

Mupad [B]
time = 12.64, size = 86, normalized size = 1.65

$$\frac{a^2 \ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{4\,a\,b-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(8\,a\,b-4\,b^2\right)-\frac{4\,b^2}{3}+4\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x),x)
```

```
[Out] (a^2*log(tan(e/2 + (f*x)/2)))/f - (4*a*b - tan(e/2 + (f*x)/2)^2*(8*a*b - 4*
b^2) - (4*b^2)/3 + 4*a*b*tan(e/2 + (f*x)/2)^4)/(f*(tan(e/2 + (f*x)/2)^2 - 1
)^3)
```

3.47 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=82

$$-\frac{a(a+4b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{a(a+4b)\sec(e+fx)}{2f}-\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -1/2*a*(a+4*b)*arctanh(cos(f*x+e))/f+1/2*a*(a+4*b)*sec(f*x+e)/f-1/2*a^2*csc
(f*x+e)^2*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 474, 470, 327, 213}

$$-\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f}+\frac{a(a+4b)\sec(e+fx)}{2f}-\frac{a(a+4b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a*(a + 4*b)*ArcTanh[Cos[e + f*x]])/f + (a*(a + 4*b)*Sec[e + f*x])/(2*
f) - (a^2*Csc[e + f*x]^2*Sec[e + f*x])/(2*f) + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-b+bx^2\right)^2}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f} + \frac{\text{Subst}\left(\int \frac{x^2\left(a^2+4ab-2b^2+2b^2x^2\right)}{-1+x^2}\,dx,\right.}{2f}$$

$$= -\frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f} + \frac{b^2\sec^3(e+fx)}{3f} + \frac{(a(a+4b))\text{Su}}{}$$

$$= \frac{a(a+4b)\sec(e+fx)}{2f} - \frac{a^2\csc^2(e+fx)\sec(e+fx)}{2f} + \frac{b^2\sec^3(e}{3f}$$

$$= -\frac{a(a+4b)\tanh^{-1}(\cos(e+fx))}{2f} + \frac{a(a+4b)\sec(e+fx)}{2f} - \frac{a^2\text{cs}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 231 vs. 2(82) = 164.
time = 5.46, size = 231, normalized size = 2.82

$$\frac{\left(24ab+8b^2+24ab\cos(2(e+fx))-12ab\cos(3(e+fx))-b^2\cos(3(e+fx))-12a^2\cos^2(e+fx)\cot^2(e+fx)-3a^2\cos(3(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-12ab\cos(3(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+3a^2\cos(3(e+fx))\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+12ab\cos(3(e+fx))\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-3\cos(e+fx)\left(b(12a+b)+3a(a+4b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-3a(a+4b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\sec^3(e+fx)}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((24*a*b + 8*b^2 + 24*a*b*Cos[2*(e + f*x)] - 12*a*b*Cos[3*(e + f*x)] - b^2*
Cos[3*(e + f*x)] - 12*a^2*Cos[e + f*x]^2*Cot[e + f*x]^2 - 3*a^2*Cos[3*(e +
```

```
f*x)]*Log[Cos[(e + f*x)/2]] - 12*a*b*Cos[3*(e + f*x)]*Log[Cos[(e + f*x)/2]]
 + 3*a^2*Cos[3*(e + f*x)]*Log[Sin[(e + f*x)/2]] + 12*a*b*Cos[3*(e + f*x)]*L
og[Sin[(e + f*x)/2]] - 3*Cos[e + f*x]*(b*(12*a + b) + 3*a*(a + 4*b)*Log[Cos
[(e + f*x)/2]] - 3*a*(a + 4*b)*Log[Sin[(e + f*x)/2]]))*Sec[e + f*x]^3)/(24*
f)
```

Maple [A]
time = 0.18, size = 85, normalized size = 1.04

method	result
derivativedivides	$\frac{\frac{b^{2}}{3 \cos(fx+e)^{3}}+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+a^{2}\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
default	$\frac{\frac{b^{2}}{3 \cos(fx+e)^{3}}+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+a^{2}\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
risch	$\frac{3a^{2}e^{9i(fx+e)}+12ab\,e^{9i(fx+e)}+12a^{2}e^{7i(fx+e)}+8b^{2}e^{7i(fx+e)}+18a^{2}e^{5i(fx+e)}-24ab\,e^{5i(fx+e)}-16b^{2}e^{5i(fx+e)}+12a^{2}e^{3i(f}}{3f\left(e^{2i(fx+e)}-1\right)^{2}\left(e^{2i(fx+e)}+1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*b^2/cos(f*x+e)^3+2*a*b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e)))+a^
2*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Maxima [A]
time = 0.30, size = 117, normalized size = 1.43

$$-\frac{3\left(a^{2}+4\,ab\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^{2}+4\,ab\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(a^{2}+4\,ab\right)\cos(fx+e)^{4}-2\left(6\,ab-b^{2}\right)\cos(fx+e)^{2}-2\,b^{2}\right)}{\cos(fx+e)^{5}-\cos(fx+e)^{3}}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*(a^2 + 4*a*b)*log(cos(f*x + e) + 1) - 3*(a^2 + 4*a*b)*log(cos(f*x
+ e) - 1) - 2*(3*(a^2 + 4*a*b)*cos(f*x + e)^4 - 2*(6*a*b - b^2)*cos(f*x + e
)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(79) = 158$.
time = 3.45, size = 178, normalized size = 2.17

$$\frac{6\left(a^{2}+4\,ab\right)\cos\left(fx+e\right)^{4}-4\left(6\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}-4\,b^{2}-3\left(\left(a^{2}+4\,ab\right)\cos\left(fx+e\right)^{5}-\left(a^{2}+4\,ab\right)\cos\left(fx+e\right)^{3}\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+3\left(\left(a^{2}+4\,ab\right)\cos\left(fx+e\right)^{5}-\left(a^{2}+4\,ab\right)\cos\left(fx+e\right)^{3}\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{12\left(f\cos\left(fx+e\right)^{5}-f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + 4*a*b)*cos(f*x + e)^4 - 4*(6*a*b - b^2)*cos(f*x + e)^2 - 4*b
^2 - 3*((a^2 + 4*a*b)*cos(f*x + e)^5 - (a^2 + 4*a*b)*cos(f*x + e)^3)*log(1/
2*cos(f*x + e) + 1/2) + 3*((a^2 + 4*a*b)*cos(f*x + e)^5 - (a^2 + 4*a*b)*cos
(f*x + e)^3)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*x +
e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^2 \csc^3\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(79) = 158.
time = 0.84, size = 254, normalized size = 3.10

$$-\frac{\frac{3\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - 6\left(a^2+4\,ab\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - \frac{3\left(a^2-\frac{2\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1} - \frac{16\left(6\,ab+b^2+\frac{12\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{6\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{3\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 6*(a^2 + 4*a*b)*log(ab
s(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 3*(a^2 - 2*a^2*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos
(f*x + e) + 1)/(cos(f*x + e) - 1) - 16*(6*a*b + b^2 + 12*a*b*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 1)^3)/f
```

Mupad [B]
time = 12.61, size = 188, normalized size = 2.29

$$\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(\frac{a^2}{2}+2\,b\,a\right)}{f}+\frac{a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{8\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{3\,a^2}{2}+32\,b\,a\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\left(\frac{a^2}{2}+16\,a\,b+8\,b^2\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{3\,a^2}{2}+16\,a\,b+\frac{8\,b^2}{3}\right)+\frac{a^2}{2}}{f\,\left(-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+12\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-12\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^3,x)
```

```
[Out] (log(tan(e/2 + (f*x)/2))*(2*a*b + a^2/2))/f + (a^2*tan(e/2 + (f*x)/2)^2)/(8
*f) - (tan(e/2 + (f*x)/2)^4*(32*a*b + (3*a^2)/2) - tan(e/2 + (f*x)/2)^6*(16
*a*b + a^2/2 + 8*b^2) - tan(e/2 + (f*x)/2)^2*(16*a*b + (3*a^2)/2 + (8*b^2)/
3) + a^2/2)/(f*(4*tan(e/2 + (f*x)/2)^2 - 12*tan(e/2 + (f*x)/2)^4 + 12*tan(e
/2 + (f*x)/2)^6 - 4*tan(e/2 + (f*x)/2)^8))
```

3.48 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=123

$$-\frac{(3a^2+24ab+8b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f}+\frac{(a^2+8ab+4b^2)\sec(e+fx}{4f}$$

```
[Out] -1/8*(3*a^2+24*a*b+8*b^2)*arctanh(cos(f*x+e))/f-1/8*a*(a+8*b)*cot(f*x+e)*cs
c(f*x+e)/f+1/4*(a^2+8*a*b+4*b^2)*sec(f*x+e)/f-1/4*a^2*csc(f*x+e)^4*sec(f*x+
e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 474, 466, 1167, 213}

$$\frac{(a^2+8ab+4b^2)\sec(e+fx)}{4f}-\frac{(3a^2+24ab+8b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f}-\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/8*((3*a^2 + 24*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/f - (a*(a + 8*b)*Cot[
e + f*x]*Csc[e + f*x])/(8*f) + ((a^2 + 8*a*b + 4*b^2)*Sec[e + f*x])/(4*f) -
 (a^2*Csc[e + f*x]^4*Sec[e + f*x])/(4*f) + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
```

```
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{x^4\left(a-b+bx^2\right)^2}{\left(-1+x^2\right)^3}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=-\frac{a^2\csc^4(e+fx)\sec(e+fx)}{4f}+\frac{\text{Subst}\left(\int\frac{x^4\left(a^2+8ab-4b^2+4b^2x^2\right)}{\left(-1+x^2\right)^2}\,dx,\right.}{4f}$$

$$=-\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a^2\csc^4(e+fx)\sec(e+f}{4f}$$

$$=-\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a^2\csc^4(e+fx)\sec(e+f}{4f}$$

$$=-\frac{a(a+8b)\cot(e+fx)\csc(e+fx)}{8f}+\frac{\left(a^2+8ab+4b^2\right)\sec(e+}{4f}$$

$$=-\frac{\left(3a^2+24ab+8b^2\right)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{a(a+8b)\cot(e+f}{8f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 447 vs. 2(123) = 246.
time = 6.13, size = 447, normalized size = 3.63

$$\frac{\left(-3a^2-8ab\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f}-\frac{a^2\csc^4\left(\frac{1}{2}(e+fx)\right)}{64f}+\frac{\left(-3a^2-24ab-8b^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8f}+\frac{\left(3a^2+24ab+8b^2\right)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f}+\frac{\left(3a^2+8ab\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f}+\frac{a^2\sec^4\left(\frac{1}{2}(e+fx)\right)}{64f}+\frac{b^2}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{b^2\sin\left(\frac{1}{2}(e+fx)\right)}{6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{b^2\sin\left(\frac{1}{2}(e+fx)\right)}{6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{b^2}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{-12ab\sin\left(\frac{1}{2}(e+fx)\right)-7b^2\sin\left(\frac{1}{2}(e+fx)\right)}{6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{12ab\sin\left(\frac{1}{2}(e+fx)\right)+7b^2\sin\left(\frac{1}{2}(e+fx)\right)}{6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((-3*a^2 - 8*a*b)*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*Csc[(e + f*x)/2]^4)/(64
*f) + ((-3*a^2 - 24*a*b - 8*b^2)*Log[Cos[(e + f*x)/2]])/(8*f) + ((3*a^2 + 2
4*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*f) + ((3*a^2 + 8*a*b)*Sec[(e + f*x
)/2]^2)/(32*f) + (a^2*Sec[(e + f*x)/2]^4)/(64*f) + b^2/(12*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^2) + (b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^3) - (b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^3) + b^2/(12*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) + (
-12*a*b*Sin[(e + f*x)/2] - 7*b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])) + (12*a*b*Sin[(e + f*x)/2] + 7*b^2*Sin[(e + f*x)/2])/(6
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A]

time = 0.19, size = 145, normalized size = 1.18

method	result
derivativedivides	$\frac{b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\mathrm{c}}{2}\right.}{f}$
default	$\frac{b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\mathrm{c}}{2}\right.}{f}$
risch	$\underline{9a^2\mathrm{e}^{13i(fx+e)}+72ab\,\mathrm{e}^{13i(fx+e)}+24b^2\mathrm{e}^{13i(fx+e)}-6a^2\mathrm{e}^{11i(fx+e)}-48ab\,\mathrm{e}^{11i(fx+e)}-16b^2\mathrm{e}^{11i(fx+e)}-105a^2\mathrm{e}^{9i(fx+e)}-72}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/3/cos(f*x+e)^3+1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e)))+2*a*b*(-
1/2/sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e)))+a
^2*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-cot(f*x
+e))))
```

Maxima [A]

time = 0.28, size = 171, normalized size = 1.39

$$-\frac{3\,(3\,a^2+24\,ab+8\,b^2)\log\left(\cos\left(fx+e\right)+1\right)-3\,(3\,a^2+24\,ab+8\,b^2)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(3\,a^2+24\,ab+8\,b^2)\cos(fx+e)^6-5\,(3\,a^2+24\,ab+8\,b^2)\cos(fx+e)^4+8\,(6\,ab+b^2)\cos(fx+e)^2+8\,b^2\right)}{\cos(fx+e)^7-2\cos(fx+e)^5+\cos(fx+e)^3}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(3*a^2 + 24*a*b + 8*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 24*a*b
 + 8*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e
```

```
)^6 - 5*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(6*a*b + b^2)*cos(f*x +
 e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(120) = 240$.
time = 3.48, size = 298, normalized size = 2.42

$$\frac{6\,(3\,a^2+24\,ab+8\,b^2)\cos\left(fx+e\right)^6-10\,(3\,a^2+24\,ab+8\,b^2)\cos\left(fx+e\right)^4+16\,(6\,ab+b^2)\cos\left(fx+e\right)^2+16\,b^2-3\,\left((3\,a^2+24\,ab+8\,b^2)\cos\left(fx+e\right)^7-2\,(3\,a^2+24\,ab+8\,b^2)\cos\left(fx+e\right)^5+(3\,a^2+24\,ab+8\,b^2)\cos\left(fx+e\right)^3\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+3\,\left((3\,a^2+24\,ab+8\,b^2)\cos\left(fx+e\right)^7-2\,(3\,a^2+24\,ab+8\,b^2)\cos\left(fx+e\right)^5+(3\,a^2+24\,ab+8\,b^2)\cos\left(fx+e\right)^3\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{48\,\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 24*a*b + 8*b^
2)*cos(f*x + e)^4 + 16*(6*a*b + b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*a^2 +
24*a*b + 8*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^5
+ (3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2) + 3*
((3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 24*a*b + 8*b^2)*cos(f
*x + e)^5 + (3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x + e)
+ 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^2\csc^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(120) = 240$.
time = 0.86, size = 421, normalized size = 3.42

$$-\frac{\frac{24\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{3\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-12\,(3\,a^2+24\,ab+8\,b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{3\left(a^2-\frac{8\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{144\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{48\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}-\frac{256\left(3\,ab+2\,b^2+\frac{6\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{3\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{192\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/192*(24*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - 3*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
- 12*(3*a^2 + 24*a*b + 8*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) +
 1)) + 3*(a^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^2*(cos(f
```

```
*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1
)^2 - 256*(3*a*b + 2*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*
b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2 + 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

Mupad [B]
time = 12.06, size = 243, normalized size = 1.98

$$\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{64f}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(\frac{3a^2}{8}+3ab+b^2\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{5a^2}{4}+4ba\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(2a^2+68ab+64b^2\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{21a^2}{4}+76ab+\frac{128b^2}{3}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(\frac{23a^2}{4}+140ab+64b^2\right)+\frac{a^2}{4}}{f\left(-16\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+48\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8-48\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+16\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\right)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a^2}{8}+\frac{ba}{4}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^5,x)
```

```
[Out] (a^2*tan(e/2 + (f*x)/2)^4)/(64*f) + (log(tan(e/2 + (f*x)/2))*(3*a*b + (3*a^
2)/8 + b^2))/f - (tan(e/2 + (f*x)/2)^2*(4*a*b + (5*a^2)/4) - tan(e/2 + (f*x
)/2)^8*(68*a*b + 2*a^2 + 64*b^2) - tan(e/2 + (f*x)/2)^4*(76*a*b + (21*a^2)/
4 + (128*b^2)/3) + tan(e/2 + (f*x)/2)^6*(140*a*b + (23*a^2)/4 + 64*b^2) + a
^2/4)/(f*(16*tan(e/2 + (f*x)/2)^4 - 48*tan(e/2 + (f*x)/2)^6 + 48*tan(e/2 +
(f*x)/2)^8 - 16*tan(e/2 + (f*x)/2)^10)) + (tan(e/2 + (f*x)/2)^2*((a*b)/4 +
a^2/8))/f
```

3.49 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=122

$$\frac{1}{8}\left(3a^2-30ab+35b^2\right)x-\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{\left(a^2-10ab+13b^2\right)\tan(e+fx)}{4f}+\frac{(a-b)^2}{}$$

```
[Out] 1/8*(3*a^2-30*a*b+35*b^2)*x-1/8*(a-9*b)*(a-b)*cos(f*x+e)*sin(f*x+e)/f-1/4*(
a^2-10*a*b+13*b^2)*tan(f*x+e)/f+1/4*(a-b)^2*sin(f*x+e)^4*tan(f*x+e)/f+1/3*b
^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 474, 466, 1167, 209}

$$-\frac{\left(a^2-10ab+13b^2\right)\tan(e+fx)}{4f}+\frac{1}{8}x\left(3a^2-30ab+35b^2\right)+\frac{(a-b)^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{(a-9b)(a-b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 30*a*b + 35*b^2)*x)/8 - ((a - 9*b)*(a - b)*Cos[e + f*x]*Sin[e + f
*x])/(8*f) - ((a^2 - 10*a*b + 13*b^2)*Tan[e + f*x])/(4*f) + ((a - b)^2*Sin[
e + f*x]^4*Tan[e + f*x])/(4*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
```

```
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{x^4(a+bx^2)^2}{(1+x^2)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{(a-b)^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{\text{Subst}\left(\int\frac{x^4(a^2-10ab+5b^2-4b^2x}{(1+x^2)^2}\right.}{4f}$$

$$=-\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{(a-b)^2\sin^4(e+f}{4f}$$

$$=-\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{(a-b)^2\sin^4(e+f}{4f}$$

$$=-\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{(a^2-10ab+13b^2)}{4f}$$

$$=\frac{1}{8}\left(3a^2-30ab+35b^2\right)x-\frac{(a-9b)(a-b)\cos(e+fx)\sin(e+f}{8f}$$

Mathematica [A]
time = 1.00, size = 96, normalized size = 0.79

$$\frac{12(3a^2-30ab+35b^2)(e+fx)-24(a^2-4ab+3b^2)\sin(2(e+fx))+3(a-b)^2\sin(4(e+fx))+32b(6a-10b+b\sec^2(e+fx))\tan(e+fx)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (12*(3*a^2 - 30*a*b + 35*b^2)*(e + f*x) - 24*(a^2 - 4*a*b + 3*b^2)*Sin[2*(e
 + f*x)] + 3*(a - b)^2*Sin[4*(e + f*x)] + 32*b*(6*a - 10*b + b*Sec[e + f*x]
^2)*Tan[e + f*x])/(96*f)
```

Maple [A]
time = 0.12, size = 199, normalized size = 1.63

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^9(fx+e)}{3\cos(fx+e)^3}-\frac{2\left(\sin^9(fx+e)\right)}{\cos(fx+e)}-2\left(\sin^7(fx+e)+\frac{7\left(\sin^5(fx+e)\right)}{6}+\frac{35\left(\sin^3(fx+e)\right)}{24}+\frac{35\sin(fx+e)}{16}\right)\cos(fx+e)+\frac{35fx}{8}+\frac{35e}{8}}{}$
default	$\frac{b^2\left(\frac{\sin^9(fx+e)}{3\cos(fx+e)^3}-\frac{2\left(\sin^9(fx+e)\right)}{\cos(fx+e)}-2\left(\sin^7(fx+e)+\frac{7\left(\sin^5(fx+e)\right)}{6}+\frac{35\left(\sin^3(fx+e)\right)}{24}+\frac{35\sin(fx+e)}{16}\right)\cos(fx+e)+\frac{35fx}{8}+\frac{35e}{8}}{}$
risch	$\frac{3x\,a^2}{8}-\frac{15xab}{4}+\frac{35x\,b^2}{8}+\frac{i\mathrm{e}^{2i(fx+e)}a^2}{8f}-\frac{i\mathrm{e}^{-2i(fx+e)}a^2}{8f}-\frac{i\mathrm{e}^{4i(fx+e)}b^2}{64f}-\frac{4ib\left(-3a\,\mathrm{e}^{4i(fx+e)}+6b\,\mathrm{e}^{4i(fx+e)}-6a}{3f\left(\mathrm{e}^{2i(fx+}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/3*sin(f*x+e)^9/cos(f*x+e)^3-2*sin(f*x+e)^9/cos(f*x+e)-2*(sin(f*
x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/
8*f*x+35/8*e)+2*a*b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3
+15/8*sin(f*x+e))*cos(f*x+e)-15/8*f*x-15/8*e)+a^2*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A]
time = 0.49, size = 137, normalized size = 1.12

$$\frac{8\,b^2\tan\left(fx+e\right)^3+3\left(3\,a^2-30\,ab+35\,b^2\right)\left(fx+e\right)+24\left(2\,ab-3\,b^2\right)\tan\left(fx+e\right)-\frac{3\left(\left(5\,a^2-18\,ab+13\,b^2\right)\tan(fx+e)^3+\left(3\,a^2-14\,ab+11\,b^2\right)\tan(fx+e)\right)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 30*a*b + 35*b^2)*(f*x + e) + 24*(2*
a*b - 3*b^2)*tan(f*x + e) - 3*((5*a^2 - 18*a*b + 13*b^2)*tan(f*x + e)^3 + (
3*a^2 - 14*a*b + 11*b^2)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 +
 1))/f
```

Fricas [A]
time = 1.33, size = 126, normalized size = 1.03

$$\frac{3\left(3\,a^2-30\,ab+35\,b^2\right)fx\cos\left(fx+e\right)^3+\left(6\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^6-3\left(5\,a^2-18\,ab+13\,b^2\right)\cos\left(fx+e\right)^4+16\left(3\,ab-5\,b^2\right)\cos\left(fx+e\right)^2+8\,b^2\right)\sin\left(fx+e\right)}{24\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^2 - 30*a*b + 35*b^2)*f*x*cos(f*x + e)^3 + (6*(a^2 - 2*a*b + b^
2)*cos(f*x + e)^6 - 3*(5*a^2 - 18*a*b + 13*b^2)*cos(f*x + e)^4 + 16*(3*a*b
- 5*b^2)*cos(f*x + e)^2 + 8*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^2\sin^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13574 vs. 2(118) = 236.
time = 18.19, size = 13574, normalized size = 111.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(9*pi*a*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*
tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^7 - 15*pi*b^2*
sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)
^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^7 + 36*a^2*f*x*tan(f*x)^7*tan
(e)^7 - 360*a*b*f*x*tan(f*x)^7*tan(e)^7 + 420*b^2*f*x*tan(f*x)^7*tan(e)^7 +
 9*pi*a*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*t
an(e))*tan(f*x)^7*tan(e)^7 - 15*pi*b^2*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x
)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^7 + 18*pi*a*b*sgn(2*t
an(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*
tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^5 - 30*pi*b^2*sgn(2*tan(f*x)^2*tan(e
)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*ta
n(e))*tan(f*x)^7*tan(e)^5 - 27*pi*a*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2
*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^
6*tan(e)^6 + 45*pi*b^2*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan
(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e)^6 + 18
*pi*a*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x
)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^7 - 30*pi*b^2*sgn(2*t
an(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*
tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^7 + 18*a*b*arctan((tan(f*x) + tan(e)
```

)/(tan(f*x)*tan(e) - 1))*tan(f*x)^7*tan(e)^7 - 30*b^2*arctan((tan(f*x) + ta
n(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^7*tan(e)^7 - 18*a*b*arctan(-(tan(f*x)
 - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^7*tan(e)^7 + 30*b^2*arctan(-(tan
(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^7*tan(e)^7 + 72*a^2*f*x*tan
(f*x)^7*tan(e)^5 - 720*a*b*f*x*tan(f*x)^7*tan(e)^5 + 840*b^2*f*x*tan(f*x)^7
*tan(e)^5 + 18*pi*a*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*ta
n(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^5 - 30*pi*b^2*sgn(-2*tan(f*x)^2*tan(e)
 + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^5 - 108*a
^2*f*x*tan(f*x)^6*tan(e)^6 + 1080*a*b*f*x*tan(f*x)^6*tan(e)^6 - 1260*b^2*f*
x*tan(f*x)^6*tan(e)^6 - 27*pi*a*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan
(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e)^6 + 45*pi*b^2*sgn(-2*tan(f
*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(
e)^6 + 72*a^2*f*x*tan(f*x)^5*tan(e)^7 - 720*a*b*f*x*tan(f*x)^5*tan(e)^7 + 8
40*b^2*f*x*tan(f*x)^5*tan(e)^7 + 18*pi*a*b*sgn(-2*tan(f*x)^2*tan(e) + 2*tan
(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^7 - 30*pi*b^2*sgn
(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*
x)^5*tan(e)^7 + 9*pi*a*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*t
an(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^3 -
15*pi*b^2*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f
*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^7*tan(e)^3 - 54*pi*a*b*sgn(2
*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 +
2*tan(f*x) - 2*tan(e))*tan(f*x)^6*tan(e)^4 + 90*pi*b^2*sgn(2*tan(f*x)^2*tan
(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*
tan(e))*tan(f*x)^6*tan(e)^4 + 63*pi*a*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(
-2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x
)^5*tan(e)^5 - 105*pi*b^2*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*
tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^5*tan(e)^5 +
 36*a*b*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^7*tan(e)
^5 - 60*b^2*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^7*ta
n(e)^5 - 36*a*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)
^7*tan(e)^5 + 60*b^2*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan
(f*x)^7*tan(e)^5 - 54*pi*a*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)
^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^
6 + 90*pi*b^2*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*t
an(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^4*tan(e)^6 - 54*a*b*arct
an((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^6*tan(e)^6 + 90*b^2*
arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^6*tan(e)^6 + 54*
a*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^6*tan(e)^6
 - 90*b^2*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^6*tan(
e)^6 + 36*a^2*tan(f*x)^7*tan(e)^6 - 360*a*b*tan(f*x)^7*tan(e)^6 + 420*b^2*t
an(f*x)^7*tan(e)^6 + 9*pi*a*b*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x
)^2*tan(e) + 2*tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)
^7 - 15*pi*b^2*sgn(2*tan(f*x)^2*tan(e)^2 - 2)*sgn(-2*tan(f*x)^2*tan(e) + 2*
tan(f*x)*tan(e)^2 + 2*tan(f*x) - 2*tan(e))*tan(f*x)^3*tan(e)^7 + 36*a*b*arc
tan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^5*tan(e)^7 - 60*b^2

```
*arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)^5*tan(e)^7 - 36
*a*b*arctan(-(tan(f*x) - tan(e))/(tan(f*x)*tan(e) + 1))*tan(f*x)^5*tan(e)^7
 + 60*b^2*arctan(-(tan(f*x) - tan(e))/(tan(f*x)...
```

Mupad [B]
time = 12.33, size = 128, normalized size = 1.05

$$x\left(\frac{3a^2}{8}-\frac{15ab}{4}+\frac{35b^2}{8}\right)+\frac{\tan(e+fx)\,(2ab-3b^2)}{f}+\frac{b^2\tan(e+fx)^3}{3f}-\frac{\left(\frac{5a^2}{8}-\frac{9ab}{4}+\frac{13b^2}{8}\right)\tan(e+fx)^3+\left(\frac{3a^2}{8}-\frac{7ab}{4}+\frac{11b^2}{8}\right)\tan(e+fx)}{f\,\left(\tan\left(e+fx\right)^4+2\tan\left(e+fx\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] x*((3*a^2)/8 - (15*a*b)/4 + (35*b^2)/8) + (tan(e + f*x)*(2*a*b - 3*b^2))/f
+ (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*((3*a^2)/8 - (7*a*b)/4 + (11*b
^2)/8) + tan(e + f*x)^3*((5*a^2)/8 - (9*a*b)/4 + (13*b^2)/8))/(f*(2*tan(e +
 f*x)^2 + tan(e + f*x)^4 + 1))
```

3.50 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=85

$$\frac{1}{2}(a-5b)(a-b)x-\frac{(a-5b)(a-b)\tan(e+fx)}{2f}+\frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] 1/2*(a-5*b)*(a-b)*x-1/2*(a-5*b)*(a-b)*tan(f*x+e)/f+1/2*(a-b)^2*sin(f*x+e)^2
*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 474, 470, 327, 209}

$$-\frac{(a-5b)(a-b)\tan(e+fx)}{2f}+\frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}x(a-5b)(a-b)+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a - 5*b)*(a - b)*x)/2 - ((a - 5*b)*(a - b)*Tan[e + f*x])/(2*f) + ((a - b)
^2*Sin[e + f*x]^2*Tan[e + f*x])/(2*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{x^2(a^2-6ab+3b^2-2b^2x^2}{1+x^2}\right.}{2f}$$

$$= \frac{(a-b)^2\sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2\tan^3(e+fx)}{3f} - \frac{((a-5b}{}$$

$$= -\frac{(a-5b)(a-b)\tan(e+fx)}{2f} + \frac{(a-b)^2\sin^2(e+fx)\tan(e+f}{2f}$$

$$= \frac{1}{2}(a-5b)(a-b)x - \frac{(a-5b)(a-b)\tan(e+fx)}{2f} + \frac{(a-b)^2\sin}{}$$

Mathematica [A]
time = 0.50, size = 71, normalized size = 0.84

$$\frac{6(a^2-6ab+5b^2)(e+fx)-3(a-b)^2\sin(2(e+fx))+4b(6a-7b+b\sec^2(e+fx))\tan(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (6*(a^2 - 6*a*b + 5*b^2)*(e + f*x) - 3*(a - b)^2*Sin[2*(e + f*x)] + 4*b*(6*
a - 7*b + b*Sec[e + f*x]^2)*Tan[e + f*x])/(12*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(77) = 154$.

time = 0.11, size = 168, normalized size = 1.98

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^7(fx+e)}{3\cos(fx+e)^3}-\frac{4\left(\sin^7(fx+e)\right)}{3\cos(fx+e)}-\frac{4\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{3}+\frac{5fx}{2}+\frac{5e}{2}\right)+2ab\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\right.}{f}$
default	$\frac{b^2\left(\frac{\sin^7(fx+e)}{3\cos(fx+e)^3}-\frac{4\left(\sin^7(fx+e)\right)}{3\cos(fx+e)}-\frac{4\left(\sin^5(fx+e)+\frac{5\left(\sin^3(fx+e)\right)}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{3}+\frac{5fx}{2}+\frac{5e}{2}\right)+2ab\left(\frac{\sin^5(fx+e)}{\cos(fx+e)}+\right.}{f}$
risch	$\frac{x a^2}{2}-3xab+\frac{5x b^2}{2}+\frac{i e^{2i(fx+e)}a^2}{8f}-\frac{i e^{2i(fx+e)}ab}{4f}+\frac{i e^{2i(fx+e)}b^2}{8f}-\frac{i e^{-2i(fx+e)}a^2}{8f}+\frac{i e^{-2i(fx+e)}ab}{4f}-$ ie

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/3*sin(f*x+e)^7/cos(f*x+e)^3-4/3*sin(f*x+e)^7/cos(f*x+e)-4/3*(si
n(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/2*f*x+5/2*e)+2*a*
b*(sin(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x
-3/2*e)+a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A]

time = 0.50, size = 92, normalized size = 1.08

$$\frac{2\,b^2\tan\left(fx+e\right)^3+3\,(a^2-6\,ab+5\,b^2)(fx+e)+12\,(ab-b^2)\tan\left(fx+e\right)-\frac{3\,(a^2-2\,ab+b^2)\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 3*(a^2 - 6*a*b + 5*b^2)*(f*x + e) + 12*(a*b - b
^2)*tan(f*x + e) - 3*(a^2 - 2*a*b + b^2)*tan(f*x + e)/(tan(f*x + e)^2 + 1))
/f
```

Fricas [A]

time = 1.60, size = 99, normalized size = 1.16

$$\frac{3\,(a^2-6\,ab+5\,b^2)fx\cos\left(fx+e\right)^3-\left(3\,(a^2-2\,ab+b^2)\cos\left(fx+e\right)^4-2\,(6\,ab-7\,b^2)\cos\left(fx+e\right)^2-2\,b^2\right)\sin\left(fx+e\right)}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 - 6*a*b + 5*b^2)*f*x*cos(f*x + e)^3 - (3*(a^2 - 2*a*b + b^2)*co
s(f*x + e)^4 - 2*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f*c
os(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^2\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. 2(81) = 162.
time = 1.06, size = 1411, normalized size = 16.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*a^2*f*x*tan(f*x)^5*tan(e)^5 - 18*a*b*f*x*tan(f*x)^5*tan(e)^5 + 15*b^
2*f*x*tan(f*x)^5*tan(e)^5 + 3*a^2*f*x*tan(f*x)^5*tan(e)^3 - 18*a*b*f*x*tan(
f*x)^5*tan(e)^3 + 15*b^2*f*x*tan(f*x)^5*tan(e)^3 - 9*a^2*f*x*tan(f*x)^4*tan
(e)^4 + 54*a*b*f*x*tan(f*x)^4*tan(e)^4 - 45*b^2*f*x*tan(f*x)^4*tan(e)^4 + 3
*a^2*f*x*tan(f*x)^3*tan(e)^5 - 18*a*b*f*x*tan(f*x)^3*tan(e)^5 + 15*b^2*f*x*
tan(f*x)^3*tan(e)^5 + 3*a^2*tan(f*x)^5*tan(e)^4 - 18*a*b*tan(f*x)^5*tan(e)^
4 + 15*b^2*tan(f*x)^5*tan(e)^4 + 3*a^2*tan(f*x)^4*tan(e)^5 - 18*a*b*tan(f*x
)^4*tan(e)^5 + 15*b^2*tan(f*x)^4*tan(e)^5 - 9*a^2*f*x*tan(f*x)^4*tan(e)^2 +
 54*a*b*f*x*tan(f*x)^4*tan(e)^2 - 45*b^2*f*x*tan(f*x)^4*tan(e)^2 + 12*a^2*f
*x*tan(f*x)^3*tan(e)^3 - 72*a*b*f*x*tan(f*x)^3*tan(e)^3 + 60*b^2*f*x*tan(f*
x)^3*tan(e)^3 - 9*a^2*f*x*tan(f*x)^2*tan(e)^4 + 54*a*b*f*x*tan(f*x)^2*tan(e
)^4 - 45*b^2*f*x*tan(f*x)^2*tan(e)^4 - 12*a*b*tan(f*x)^5*tan(e)^2 + 10*b^2*
tan(f*x)^5*tan(e)^2 - 12*a^2*tan(f*x)^4*tan(e)^3 + 36*a*b*tan(f*x)^4*tan(e)
^3 - 30*b^2*tan(f*x)^4*tan(e)^3 - 12*a^2*tan(f*x)^3*tan(e)^4 + 36*a*b*tan(f
*x)^3*tan(e)^4 - 30*b^2*tan(f*x)^3*tan(e)^4 - 12*a*b*tan(f*x)^2*tan(e)^5 +
10*b^2*tan(f*x)^2*tan(e)^5 + 9*a^2*f*x*tan(f*x)^3*tan(e) - 54*a*b*f*x*tan(f
*x)^3*tan(e) + 45*b^2*f*x*tan(f*x)^3*tan(e) - 12*a^2*f*x*tan(f*x)^2*tan(e)^
2 + 72*a*b*f*x*tan(f*x)^2*tan(e)^2 - 60*b^2*f*x*tan(f*x)^2*tan(e)^2 + 9*a^2
*f*x*tan(f*x)*tan(e)^3 - 54*a*b*f*x*tan(f*x)*tan(e)^3 + 45*b^2*f*x*tan(f*x)
*tan(e)^3 - 2*b^2*tan(f*x)^5 + 24*a*b*tan(f*x)^4*tan(e) - 30*b^2*tan(f*x)^4
*tan(e) + 18*a^2*tan(f*x)^3*tan(e)^2 - 36*a*b*tan(f*x)^3*tan(e)^2 + 10*b^2*
tan(f*x)^3*tan(e)^2 + 18*a^2*tan(f*x)^2*tan(e)^3 - 36*a*b*tan(f*x)^2*tan(e)
```

```
^3 + 10*b^2*tan(f*x)^2*tan(e)^3 + 24*a*b*tan(f*x)*tan(e)^4 - 30*b^2*tan(f*x
)*tan(e)^4 - 2*b^2*tan(e)^5 - 3*a^2*f*x*tan(f*x)^2 + 18*a*b*f*x*tan(f*x)^2
- 15*b^2*f*x*tan(f*x)^2 + 9*a^2*f*x*tan(f*x)*tan(e) - 54*a*b*f*x*tan(f*x)*t
an(e) + 45*b^2*f*x*tan(f*x)*tan(e) - 3*a^2*f*x*tan(e)^2 + 18*a*b*f*x*tan(e)
^2 - 15*b^2*f*x*tan(e)^2 - 12*a*b*tan(f*x)^3 + 10*b^2*tan(f*x)^3 - 12*a^2*t
an(f*x)^2*tan(e) + 36*a*b*tan(f*x)^2*tan(e) - 30*b^2*tan(f*x)^2*tan(e) - 12
*a^2*tan(f*x)*tan(e)^2 + 36*a*b*tan(f*x)*tan(e)^2 - 30*b^2*tan(f*x)*tan(e)^
2 - 12*a*b*tan(e)^3 + 10*b^2*tan(e)^3 - 3*a^2*f*x + 18*a*b*f*x - 15*b^2*f*x
 + 3*a^2*tan(f*x) - 18*a*b*tan(f*x) + 15*b^2*tan(f*x) + 3*a^2*tan(e) - 18*a
*b*tan(e) + 15*b^2*tan(e))/(f*tan(f*x)^5*tan(e)^5 + f*tan(f*x)^5*tan(e)^3 -
 3*f*tan(f*x)^4*tan(e)^4 + f*tan(f*x)^3*tan(e)^5 - 3*f*tan(f*x)^4*tan(e)^2
+ 4*f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(e)^4 + 3*f*tan(f*x)^3*tan(e)
 - 4*f*tan(f*x)^2*tan(e)^2 + 3*f*tan(f*x)*tan(e)^3 - f*tan(f*x)^2 + 3*f*tan
(f*x)*tan(e) - f*tan(e)^2 - f)
```

Mupad [B]
time = 11.80, size = 114, normalized size = 1.34

$$\frac{\tan(e+f\,x)\,\left(2\,a\,b-2\,b^2\right)}{f}+\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\sin\left(2\,e+2\,f\,x\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{2\,f}+\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,(a-b)\,(a-5\,b)}{2\,\left(\frac{a^2}{2}-3\,a\,b+\frac{5\,b^2}{2}\right)}\right)\,(a-b)\,(a-5\,b)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*(2*a*b - 2*b^2))/f + (b^2*tan(e + f*x)^3)/(3*f) - (sin(2*e +
2*f*x)*(a^2/2 - a*b + b^2/2))/(2*f) + (atan((tan(e + f*x)*(a - b)*(a - 5*b)
)/(2*(a^2/2 - 3*a*b + (5*b^2)/2)))*(a - b)*(a - 5*b))/(2*f)
```

3.51 $\int \left(a+b\tan^2(e+fx)\right)^2 \, dx$

Optimal. Leaf size=46

$$(a-b)^2x+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] (a-b)^2*x+(2*a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3742, 398, 209}

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e\right.}{f}$$
$$= (a-b)^2x+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.39, size = 73, normalized size = 1.59

$$\frac{\tan(e+fx)\left(\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(e+fx)}\right)}{\sqrt{-\tan^2(e+fx)}}+b(6a-b(3-\tan^2(e+fx)))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (Tan[e + f*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[e + f*x]^2]])/Sqrt[-Tan[e + f
*x]^2] + b*(6*a - b*(3 - Tan[e + f*x]^2))))/(3*f)
```

Maple [A]
time = 0.03, size = 59, normalized size = 1.28

method	result	size
norman	$\left(a^2-2ab+b^2\right)x+\frac{(2a-b)b\tan(fx+e)}{f}+\frac{b^2\left(\tan^3(fx+e)\right)}{3f}$	49
derivativedivides	$\frac{\frac{b^2\left(\tan^3(fx+e)\right)}{3}+2ab\tan(fx+e)-b^2\tan(fx+e)+\left(a^2-2ab+b^2\right)\arctan(\tan(fx+e))}{f}$	59
default	$\frac{\frac{b^2\left(\tan^3(fx+e)\right)}{3}+2ab\tan(fx+e)-b^2\tan(fx+e)+\left(a^2-2ab+b^2\right)\arctan(\tan(fx+e))}{f}$	59
risch	$x\,a^2-2xab+x\,b^2-\frac{4ib\left(-3a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}+3b\,e^{2i(fx+e)}-3a+2b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*b^2*tan(f*x+e)^3+2*a*b*tan(f*x+e)-b^2*tan(f*x+e)+(a^2-2*a*b+b^2)*a
rctan(tan(f*x+e)))
```

Maxima [A]
time = 0.51, size = 63, normalized size = 1.37

$$a^2x - \frac{2(fx + e - \tan(fx + e))ab}{f} + \frac{(\tan(fx + e)^3 + 3fx + 3e - 3\tan(fx + e))b^2}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(f*x + e - tan(f*x + e))*a*b/f + 1/3*(tan(f*x + e)^3 + 3*f*x + 3*
e - 3*tan(f*x + e))*b^2/f
```

Fricas [A]
time = 2.17, size = 53, normalized size = 1.15

$$\frac{b^2\tan(fx + e)^3 + 3(a^2 - 2ab + b^2)fx + 3(2ab - b^2)\tan(fx + e)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*x + 3*(2*a*b - b^2)*tan(f
*x + e))/f
```

Sympy [A]
time = 0.12, size = 68, normalized size = 1.48

$$\begin{cases} a^2x - 2abx + \frac{2ab\tan(e+fx)}{f} + b^2x + \frac{b^2\tan^3(e+fx)}{3f} - \frac{b^2\tan(e+fx)}{f} & \text{for } f \neq 0 \\ x(a + b\tan^2(e))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(e + f*x)/f + b**2*x + b**2*tan(e +
f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)**2, Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(46) = 92.
time = 0.91, size = 382, normalized size = 8.30

$$\frac{3a^2fx\tan(fx)^3\tan(e)^3 - 6abfx\tan(fx)^3\tan(e)^3 + 3b^2fx\tan(fx)^3\tan(e)^3 - 9a^2fx\tan(fx)^2\tan(e)^2 + 18abfx\tan(fx)^2\tan(e)^2 - 9b^2fx\tan(fx)^2\tan(e)^2 - 6ab\tan(fx)^3\tan(e)^2 + 3b^2\tan(fx)^3\tan(e)^2 - 6ab\tan(fx)^2\tan(e)^3 + 3b^2\tan(fx)^2\tan(e)^3 + 9a^2fx\tan(fx)\tan(e) - 18abfx\tan(fx)\tan(e) + 9b^2fx\tan(fx)\tan(e) - b^2\tan(fx)^3 + 12ab\tan(fx)^2\tan(e) - 9b^2\tan(fx)^2\tan(e) + 12ab\tan(fx)\tan(e)^2 - 9b^2\tan(fx)\tan(e)^2 - b^2\tan(e)^3 - 3a^2fx + 6abfx - 3b^2fx - 6ab\tan(fx) + 3b^2\tan(fx) - 6ab\tan(e) + 3b^2\tan(e)}{3(f\tan(fx)^3\tan(e)^3 - 3f\tan(fx)^2\tan(e)^2 + 3f\tan(fx)\tan(e) - f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*f*x*tan(f*x)^3*tan(e)^3 - 6*a*b*f*x*tan(f*x)^3*tan(e)^3 + 3*b^2*
f*x*tan(f*x)^3*tan(e)^3 - 9*a^2*f*x*tan(f*x)^2*tan(e)^2 + 18*a*b*f*x*tan(f*
x)^2*tan(e)^2 - 9*b^2*f*x*tan(f*x)^2*tan(e)^2 - 6*a*b*tan(f*x)^3*tan(e)^2 +
 3*b^2*tan(f*x)^3*tan(e)^2 - 6*a*b*tan(f*x)^2*tan(e)^3 + 3*b^2*tan(f*x)^2*t
an(e)^3 + 9*a^2*f*x*tan(f*x)*tan(e) - 18*a*b*f*x*tan(f*x)*tan(e) + 9*b^2*f*
x*tan(f*x)*tan(e) - b^2*tan(f*x)^3 + 12*a*b*tan(f*x)^2*tan(e) - 9*b^2*tan(f
*x)^2*tan(e) + 12*a*b*tan(f*x)*tan(e)^2 - 9*b^2*tan(f*x)*tan(e)^2 - b^2*tan
(e)^3 - 3*a^2*f*x + 6*a*b*f*x - 3*b^2*f*x - 6*a*b*tan(f*x) + 3*b^2*tan(f*x)
 - 6*a*b*tan(e) + 3*b^2*tan(e))/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan
(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 11.90, size = 76, normalized size = 1.65

$$\frac{\tan(e+f\,x)\,\left(2\,a\,b-b^2\right)}{f}+\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,{(a-b)}^2}{a^2-2\,a\,b+b^2}\right)\,{(a-b)}^2}{f}+\frac{b^2\,{\tan(e+f\,x)}^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*(2*a*b - b^2))/f + (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*
b + b^2))*(a - b)^2)/f + (b^2*tan(e + f*x)^3)/(3*f)
```

3.52 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -a^2*cot(f*x+e)/f+2*a*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3744, 276}

$$-\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*Cot[e + f*x])/f) + (2*a*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*
f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 44, normalized size = 0.96

$$\frac{-3a^2\cot(e+fx)+b(6a-b+b\sec^2(e+fx))\tan(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
[Out] (-3*a^2*Cot[e + f*x] + b*(6*a - b + b*Sec[e + f*x]^2)*Tan[e + f*x])/(3*f)
```

Maple [A]
time = 0.11, size = 48, normalized size = 1.04

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}+2ab\tan(fx+e)-a^2\cot(fx+e)}{f}$
default	$\frac{\frac{b^2\left(\sin^3(fx+e)\right)}{3\cos(fx+e)^3}+2ab\tan(fx+e)-a^2\cot(fx+e)}{f}$
risch	$-\frac{2i(3a^2e^{6i(fx+e)}-6ab\,e^{6i(fx+e)}+3b^2e^{6i(fx+e)}+9a^2e^{4i(fx+e)}-6ab\,e^{4i(fx+e)}-3b^2e^{4i(fx+e)}+9a^2e^{2i(fx+e)}+6ab\,e^{2i(fx+}}{3f\left(e^{2i(fx+e)}-1\right)\left(e^{2i(fx+e)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3+2*a*b*tan(f*x+e)-a^2*cot(f*x+e))
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.96

$$\frac{b^2\tan(fx+e)^3+6\,ab\tan(fx+e)-\frac{3\,a^2}{\tan(fx+e)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) - 3*a^2/tan(f*x + e))/f
```

Fricas [A]
time = 1.16, size = 75, normalized size = 1.63

$$-\frac{(3\,a^2+6\,ab-b^2)\cos(fx+e)^4-2\,(3\,ab-b^2)\cos(fx+e)^2-b^2}{3\,f\cos(fx+e)^3\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a*b - b^2)*cos(f*x + e)^2
 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^2 \csc^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**2, x)
```

Giac [A]
time = 0.81, size = 44, normalized size = 0.96

$$\frac{b^2\tan\left(fx + e\right)^3 + 6\,ab\tan\left(fx + e\right) - \frac{3\,a^2}{\tan(fx+e)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) - 3*a^2/tan(f*x + e))/f
```

Mupad [B]
time = 11.86, size = 67, normalized size = 1.46

$$\frac{-3\,a^2\cos\left(e + f\,x\right)^4 + 6\,a\,b\cos\left(e + f\,x\right)^2\sin\left(e + f\,x\right)^2 + b^2\sin\left(e + f\,x\right)^4}{3\,f\cos\left(e + f\,x\right)^3\,\sin\left(e + f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^2,x)
```

```
[Out] (b^2*sin(e + f*x)^4 - 3*a^2*cos(e + f*x)^4 + 6*a*b*cos(e + f*x)^2*sin(e + f
*x)^2)/(3*f*cos(e + f*x)^3*sin(e + f*x))
```

3.53 $\int \csc^4(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx$

Optimal. Leaf size=70

$$-\frac{a(a+2b)\cot(e+fx)}{f}-\frac{a^2\cot^3(e+fx)}{3f}+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -a*(a+2*b)*cot(f*x+e)/f-1/3*a^2*cot(f*x+e)^3/f+b*(2*a+b)*tan(f*x+e)/f+1/3*b
^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3744, 459}

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{b(2a+b)\tan(e+fx)}{f}-\frac{a(a+2b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a*(a + 2*b)*Cot[e + f*x])/f) - (a^2*Cot[e + f*x]^3)/(3*f) + (b*(2*a + b)
*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+bx^2)^2}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b(2a+b)+\frac{a^2}{x^4}+\frac{a(a+2b)}{x^2}+b^2x^2\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a(a+2b)\cot(e+fx)}{f} - \frac{a^2\cot^3(e+fx)}{3f} + \frac{b(2a+b)\tan(e+}{f}$$

Mathematica [A]
time = 0.33, size = 59, normalized size = 0.84

$$\frac{-a\cot(e+fx)\left(2a+6b+a\csc^2(e+fx)\right)+b\left(6a+2b+b\sec^2(e+fx)\right)\tan(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-(a*Cot[e + f*x]*(2*a + 6*b + a*Csc[e + f*x]^2)) + b*(6*a + 2*b + b*Sec[e
+ f*x]^2)*Tan[e + f*x])/(3*f)
```

Maple [A]
time = 0.15, size = 81, normalized size = 1.16

method	result
derivativedivides	$\frac{-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+a^2\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)}{f}$
default	$\frac{-b^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+a^2\left(-\frac{2}{3}-\frac{\left(\csc^2(fx+e)\right)}{3}\right)\cot(fx+e)}{f}$
risch	$\frac{4i\left(3a^2e^{8i(fx+e)}-6ab\,e^{8i(fx+e)}+3b^2e^{8i(fx+e)}+8a^2e^{6i(fx+e)}-8b^2e^{6i(fx+e)}+6a^2e^{4i(fx+e)}+12ab\,e^{4i(fx+e)}+6b^2e^{4i(fx+}\right.}{3f\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e)
-2*cot(f*x+e))+a^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e))
```

Maxima [A]
time = 0.30, size = 70, normalized size = 1.00

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(2\,ab+b^2\right)\tan\left(fx+e\right)-\frac{3\left(a^2+2\,ab\right)\tan(fx+e)^2+a^2}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + b^2)*tan(f*x + e) - (3*(a^2 + 2*a*b)*t
an(f*x + e)^2 + a^2)/tan(f*x + e)^3)/f
```

Fricas [A]
time = 1.09, size = 98, normalized size = 1.40

$$-\frac{2\,(a^2+6\,ab+b^2)\cos{(fx+e)}^6-3\,(a^2+6\,ab+b^2)\cos{(fx+e)}^4+6\,ab\cos{(fx+e)}^2+b^2}{3\,\left(f\cos{(fx+e)}^5-f\cos{(fx+e)}^3\right)\sin{(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 6*a*b + b^2)*cos(f*x + e)^6 - 3*(a^2 + 6*a*b + b^2)*cos(f*x
+ e)^4 + 6*a*b*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*cos(f*x + e)^3)
*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2{(e+fx)}\right)^2\csc^4{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**4, x)
```

Giac [A]
time = 0.81, size = 84, normalized size = 1.20

$$\frac{b^2\tan{(fx+e)}^3+6\,ab\tan{(fx+e)}+3\,b^2\tan{(fx+e)}-\frac{3\,a^2\tan(fx+e)^2+6\,ab\tan(fx+e)^2+a^2}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 3*b^2*tan(f*x + e) - (3*a^2*
tan(f*x + e)^2 + 6*a*b*tan(f*x + e)^2 + a^2)/tan(f*x + e)^3)/f
```

Mupad [B]
time = 11.82, size = 69, normalized size = 0.99

$$\frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\tan(e+f\,x)^2\,(a^2+2\,b\,a)+\frac{a^2}{3}}{f\tan{(e+f\,x)}^3}+\frac{b\tan(e+f\,x)\,\left(2\,a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^4,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)^2*(2*a*b + a^2) + a^2/3)/(f*tan(
e + f*x)^3) + (b*tan(e + f*x)*(2*a + b))/f
```

3.54 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal. Leaf size=93

$$-\frac{(a^2+4ab+b^2)\cot(e+fx)}{f}-\frac{2a(a+b)\cot^3(e+fx)}{3f}-\frac{a^2\cot^5(e+fx)}{5f}+\frac{2b(a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(a^2+4*a*b+b^2)*cot(f*x+e)/f-2/3*a*(a+b)*cot(f*x+e)^3/f-1/5*a^2*cot(f*x+e)
^5/f+2*b*(a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3744, 459}

$$-\frac{(a^2+4ab+b^2)\cot(e+fx)}{f}-\frac{a^2\cot^5(e+fx)}{5f}+\frac{2b(a+b)\tan(e+fx)}{f}-\frac{2a(a+b)\cot^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a^2 + 4*a*b + b^2)*Cot[e + f*x])/f) - (2*a*(a + b)*Cot[e + f*x]^3)/(3*f
) - (a^2*Cot[e + f*x]^5)/(5*f) + (2*b*(a + b)*Tan[e + f*x])/f + (b^2*Tan[e
+ f*x]^3)/(3*f)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+bx^2)^2}{x^6} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(2b(a+b)+\frac{a^2}{x^6}+\frac{2a(a+b)}{x^4}+\frac{a^2+4ab+b^2}{x^2}+b^2x^2\right) dx, x, \text{ta}\right.}{f}$$
$$= -\frac{(a^2+4ab+b^2)\cot(e+fx)}{f} - \frac{2a(a+b)\cot^3(e+fx)}{3f} - \frac{a^2\,\text{co}}{}$$

Mathematica [A]
time = 0.57, size = 88, normalized size = 0.95

$$\frac{-\cot(e+fx)\left(8a^2+50ab+15b^2+2a(2a+5b)\csc^2(e+fx)+3a^2\csc^4(e+fx)\right)+5b(6a+5b+b\sec^2(e+fx))\tan(e+fx)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-(Cot[e + f*x]*(8*a^2 + 50*a*b + 15*b^2 + 2*a*(2*a + 5*b)*Csc[e + f*x]^2 +
 3*a^2*Csc[e + f*x]^4)) + 5*b*(6*a + 5*b + b*Sec[e + f*x]^2)*Tan[e + f*x])/
(15*f)
```

Maple [A]
time = 0.14, size = 136, normalized size = 1.46

method	result
derivativedivides	$\frac{b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\,\text{c}}{}\right.}{f}$
default	$\frac{b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\,\text{c}}{}\right.}{f}$
risch	$-\frac{16i\left(10a^2e^{10i(fx+e)}-20ab\,e^{10i(fx+e)}+10b^2e^{10i(fx+e)}+25a^2e^{8i(fx+e)}+10ab\,e^{8i(fx+e)}-35b^2e^{8i(fx+e)}+16a^2e^{6i(fx+e}\right.}{15f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/3/sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x
+e))+2*a*b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(
f*x+e))+a^2*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e))
```

Maxima [A]
time = 0.30, size = 93, normalized size = 1.00

$$\frac{5\,b^2\tan(fx+e)^3+30\,(ab+b^2)\tan(fx+e)-\frac{15\,(a^2+4\,ab+b^2)\tan(fx+e)^4+10\,(a^2+ab)\tan(fx+e)^2+3\,a^2}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + b^2)*tan(f*x + e) - (15*(a^2 + 4*a*b
 + b^2)*tan(f*x + e)^4 + 10*(a^2 + a*b)*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e
)^5)/f
```

Fricas [A]
time = 1.00, size = 145, normalized size = 1.56

$$-\frac{8\left(a^{2}+10\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{8}-20\left(a^{2}+10\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{6}+15\left(a^{2}+10\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{4}-10\left(3\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}-5\,b^{2}}{15\left(f\cos\left(fx+e\right)^{7}-2\,f\cos\left(fx+e\right)^{5}+f\cos\left(fx+e\right)^{3}\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 + 10*a*b + 5*b^2)*cos(f*x + e)^8 - 20*(a^2 + 10*a*b + 5*b^2)*
cos(f*x + e)^6 + 15*(a^2 + 10*a*b + 5*b^2)*cos(f*x + e)^4 - 10*(3*a*b + b^2
)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f
*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.84, size = 128, normalized size = 1.38

$$\frac{5\,b^{2}\tan\left(fx+e\right)^{3}+30\,ab\tan\left(fx+e\right)+30\,b^{2}\tan\left(fx+e\right)-\frac{15\,a^{2}\tan\left(fx+e\right)^{4}+60\,ab\tan\left(fx+e\right)^{4}+15\,b^{2}\tan\left(fx+e\right)^{4}+10\,a^{2}\tan\left(fx+e\right)^{2}+10\,ab\tan\left(fx+e\right)^{2}+3\,a^{2}}{\tan\left(fx+e\right)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 30*b^2*tan(f*x + e) - (1
5*a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a
^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e)^5)/f
```

Mupad [B]
time = 12.19, size = 90, normalized size = 0.97

$$\frac{b^2\,\mathrm{tan}\left(e+f\,x\right)^3}{3\,f}-\frac{\mathrm{tan}\left(e+f\,x\right)^4\,\left(a^2+4\,a\,b+b^2\right)+\frac{a^2}{5}+\mathrm{tan}\left(e+f\,x\right)^2\,\left(\frac{2\,a^2}{3}+\frac{2\,b\,a}{3}\right)}{f\,{\mathrm{tan}\left(e+f\,x\right)}^5}+\frac{2\,b\,\mathrm{tan}\left(e+f\,x\right)\,\left(a+b\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^6,x)
```

```
[Out] (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)^4*(4*a*b + a^2 + b^2) + a^2/5 +
tan(e + f*x)^2*((2*a*b)/3 + (2*a^2)/3))/(f*tan(e + f*x)^5) + (2*b*tan(e + f
*x)*(a + b))/f
```

3.55 $\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{7/2}f}-\frac{a^2\cos(e+fx)}{(a-b)^3f}+\frac{(2a-b)\cos^3(e+fx)}{3(a-b)^2f}-\frac{\cos^5(e+fx)}{5(a-b)f}$$

```
[Out] -a^2*cos(f*x+e)/(a-b)^3/f+1/3*(2*a-b)*cos(f*x+e)^3/(a-b)^2/f-1/5*cos(f*x+e)
^5/(a-b)/f-a^2*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(7/2)/f
```

Rubi [A]
time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3745, 472, 211}

$$-\frac{a^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{7/2}}-\frac{a^2\cos(e+fx)}{f(a-b)^3}-\frac{\cos^5(e+fx)}{5f(a-b)}+\frac{(2a-b)\cos^3(e+fx)}{3f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(7/2)*f
)) - (a^2*Cos[e + f*x])/((a - b)^3*f) + ((2*a - b)*Cos[e + f*x]^3)/(3*(a -
b)^2*f) - Cos[e + f*x]^5/(5*(a - b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
```

```
- 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(-1+x^2)^2}{x^6(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a-b)x^6}+\frac{-2a+b}{(a-b)^2x^4}+\frac{a^2}{(a-b)^3x^2}-\frac{a^2b}{(a-b)^3(a-b+bx^2)}\right)\,dx, x, \sec(e+fx)\right)}{f} \\
&= -\frac{a^2\cos(e+fx)}{(a-b)^3f}+\frac{(2a-b)\cos^3(e+fx)}{3(a-b)^2f}-\frac{\cos^5(e+fx)}{5(a-b)f}-\frac{(a^2b)\operatorname{Subst}\left(\int \frac{1}{a-b+}\right.}{(a} \\
&= -\frac{a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{7/2}f}-\frac{a^2\cos(e+fx)}{(a-b)^3f}+\frac{(2a-b)\cos^3(e+fx)}{3(a-b)^2f}-\frac{\cos}{5}
\end{aligned}
$$

Mathematica [A]
time = 2.04, size = 177, normalized size = 1.51

$$\frac{120a^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+120a^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+\sqrt{a-b}\,\cos(e+fx)\left(-89a^2-42ab+11b^2+4(7a^2-9ab+2b^2)\cos(2(e+fx))-3(a-b)^2\cos(4(e+fx))\right)}{120(a-b)^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (120*a^2*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]] +
 120*a^2*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]] +
 Sqrt[a - b]*Cos[e + f*x]*(-89*a^2 - 42*a*b + 11*b^2 + 4*(7*a^2 - 9*a*b + 2
*b^2)*Cos[2*(e + f*x)] - 3*(a - b)^2*Cos[4*(e + f*x)]))/(120*(a - b)^(7/2)*
f)
```

Maple [A]
time = 0.34, size = 144, normalized size = 1.23

method	result
derivativedivides	$\frac{-\frac{\frac{a^2\left(\cos^5(fx+e)\right)}{5}-\frac{2ab\left(\cos^5(fx+e)\right)}{5}+\frac{b^2\left(\cos^5(fx+e)\right)}{5}-\frac{2a^2\left(\cos^3(fx+e)\right)}{3}+ab\left(\cos^3(fx+e)\right)-\frac{b^2\left(\cos^3(fx+e)\right)}{3}+a^2\cos(fx+e)}{(a-b)^3}+}{f}$

default	$-\frac{-\frac{\frac{a^2\left(\cos^5(fx+e)\right)}{5}-\frac{2ab\left(\cos^5(fx+e)\right)}{5}+\frac{b^2\left(\cos^5(fx+e)\right)}{5}-\frac{2a^2\left(\cos^3(fx+e)\right)}{3}+ab\left(\cos^3(fx+e)\right)-\frac{b^2\left(\cos^3(fx+e)\right)}{3}+a^2\cos(fx+e)}{(a-b)^3}+\cdots}{f}$
risch	$-\frac{5\,e^{i(fx+e)}a^2}{16(a-b)^3 f}-\frac{e^{i(fx+e)}ab}{4(a-b)^3 f}+\frac{e^{i(fx+e)}b^2}{16(a-b)^3 f}-\frac{5\,e^{-i(fx+e)}a^2}{16(a-b)(a^2-2ab+b^2)f}-\frac{e^{-i(fx+e)}ab}{4(a-b)(a^2-2ab+b^2)f}+\frac{e^{-i(fx+e)}\cdots}{16(a-b)(a^2-2a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a-b)^3*(1/5*a^2*cos(f*x+e)^5-2/5*a*b*cos(f*x+e)^5+1/5*b^2*cos(f*x+
e)^5-2/3*a^2*cos(f*x+e)^3+a*b*cos(f*x+e)^3-1/3*b^2*cos(f*x+e)^3+a^2*cos(f*x
+e))+a^2*b/(a-b)^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))
)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]

time = 0.83, size = 304, normalized size = 2.60

$$\left[-\frac{6\,(a^2-2\,ab+b^2)\cos(fx+e)^5-10\,(2\,a^2-3\,ab+b^2)\cos(fx+e)^3+15\,a^2\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos(fx+e)^2-2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)+30\,a^2\cos(fx+e)}{30\,(a^3-3\,a^2b+3\,ab^2-b^3)f},\ -\frac{3\,(a^2-2\,ab+b^2)\cos(fx+e)^5-5\,(2\,a^2-3\,ab+b^2)\cos(fx+e)^3+15\,a^2\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)+15\,a^2\cos(fx+e)}{15\,(a^3-3\,a^2b+3\,ab^2-b^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 10*(2*a^2 - 3*a*b + b^2)*cos
(f*x + e)^3 + 15*a^2*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a -
 b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*a
^2*cos(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f), -1/15*(3*(a^2 - 2*a*b
 + b^2)*cos(f*x + e)^5 - 5*(2*a^2 - 3*a*b + b^2)*cos(f*x + e)^3 + 15*a^2*sq
rt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*a^2*cos(
f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(109) = 218.
time = 0.71, size = 377, normalized size = 3.22

$$-\frac{\frac{15\,a^2 b\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{(a^3-3a^2b+3ab^2-b^3)\sqrt{ab-b^2}}-\frac{2\left(8a^2+9ab-2b^2-\frac{40a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{30ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{10b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{10b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{90ab(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{30b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{15ab(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}\right)}{(a^3-3a^2b+3ab^2-b^3)\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*a^2*b*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b - b
^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(
a*b - b^2)) - 2*(8*a^2 + 9*a*b - 2*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 30*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 10*b^2*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + 10*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 90*a*b*(cos(f*x
+ e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3 + 15*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

Mupad [**B**]
time = 14.41, size = 643, normalized size = 5.50

$$-\frac{\frac{2(8a^2+9ab-2b^2)}{15(a-b)(a^2-2ab+b^2)}+\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(8a^2+b^2)}{3(a-b)(a^2-2ab+b^2)}+\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(4a^2+3ab-b^2)}{3(a-b)(a^2-2ab+b^2)}+\frac{4b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(3a-b)}{(a-b)(a^2-2ab+b^2)}+\frac{2ab\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8}{(a-b)(a^2-2ab+b^2)}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}-\frac{a^2\sqrt{b}\,\mathrm{atan}\left(\frac{\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a\sqrt{b}\,(16a^{10}b-96a^9b^2+240a^8b^3-320a^7b^4+240a^6b^5-96a^5b^6+16a^4b^7)}{2(a-b)^{13/2}}+\frac{a^3\sqrt{b}\,(a-2b)(16a^{12}-176a^{11}b+864a^{10}b^2-2496a^9b^3+4704a^8b^4-6048a^7b^5+5376a^6b^6-3264a^5b^7+1296a^4b^8-304a^3b^9+32a^2b^{10})}{8(a-b)^{21/2}}\right)+\frac{a^3\sqrt{b}\,(a-2b)(-16a^{12}+144a^{11}b-576a^{10}b^2+1344a^9b^3-2016a^8b^4+2016a^7b^5-1344a^6b^6+576a^5b^7-144a^4b^8+16a^3b^9)}{8(a-b)^{21/2}}\right)(a-b)^7}{4a^{12}b-24a^{11}b^2+60a^{10}b^3-80a^9b^4+60a^8b^5-24a^7b^6+4a^6b^7}\right)}{f(a-b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2),x)
```

```
[Out] - ((2*(9*a*b + 8*a^2 - 2*b^2))/(15*(a - b)*(a^2 - 2*a*b + b^2)) + (4*tan(e/
2 + (f*x)/2)^4*(8*a^2 + b^2))/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (4*tan(e/2
+ (f*x)/2)^2*(3*a*b + 4*a^2 - b^2))/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (4*b*
tan(e/2 + (f*x)/2)^6*(3*a - b))/((a - b)*(a^2 - 2*a*b + b^2)) + (2*a*b*tan(
e/2 + (f*x)/2)^8)/((a - b)*(a^2 - 2*a*b + b^2)))/(f*(5*tan(e/2 + (f*x)/2)^2
 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)
^8 + tan(e/2 + (f*x)/2)^10 + 1)) - (a^2*b^(1/2)*atan(((tan(e/2 + (f*x)/2)^2
```

```
*((a*b^(1/2)*(16*a^10*b + 16*a^4*b^7 - 96*a^5*b^6 + 240*a^6*b^5 - 320*a^7*b
^4 + 240*a^8*b^3 - 96*a^9*b^2))/(2*(a - b)^(13/2)) + (a^3*b^(1/2)*(a - 2*b)
*(16*a^12 - 176*a^11*b + 32*a^2*b^10 - 304*a^3*b^9 + 1296*a^4*b^8 - 3264*a^
5*b^7 + 5376*a^6*b^6 - 6048*a^7*b^5 + 4704*a^8*b^4 - 2496*a^9*b^3 + 864*a^1
0*b^2))/(8*(a - b)^(21/2))) + (a^3*b^(1/2)*(a - 2*b)*(144*a^11*b - 16*a^12
+ 16*a^3*b^9 - 144*a^4*b^8 + 576*a^5*b^7 - 1344*a^6*b^6 + 2016*a^7*b^5 - 20
16*a^8*b^4 + 1344*a^9*b^3 - 576*a^10*b^2))/(8*(a - b)^(21/2)))*(a - b)^7)/(
4*a^12*b + 4*a^6*b^7 - 24*a^7*b^6 + 60*a^8*b^5 - 80*a^9*b^4 + 60*a^10*b^3 -
 24*a^11*b^2)))/(f*(a - b)^(7/2))
```

3.56 $\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{a\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}-\frac{a\cos(e+fx)}{(a-b)^2f}+\frac{\cos^3(e+fx)}{3(a-b)f}$$

```
[Out] -a*cos(f*x+e)/(a-b)^2/f+1/3*cos(f*x+e)^3/(a-b)/f-a*arctan(sec(f*x+e)*b^(1/2
)/(a-b)^(1/2))*b^(1/2)/(a-b)^(5/2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3745, 464, 331, 211}

$$-\frac{a\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}+\frac{\cos^3(e+fx)}{3f(a-b)}-\frac{a\cos(e+fx)}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(5/2)*f))
 - (a*Cos[e + f*x])/((a - b)^2*f) + Cos[e + f*x]^3/(3*(a - b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\cos^3(e+fx)}{3(a-b)f} + \frac{a\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{(a-b)f}\\ &= -\frac{a\cos(e+fx)}{(a-b)^2 f} + \frac{\cos^3(e+fx)}{3(a-b)f} - \frac{(ab)\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \sec(e+fx)\right)}{(a-b)^2 f}\\ &= -\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{5/2} f} - \frac{a\cos(e+fx)}{(a-b)^2 f} + \frac{\cos^3(e+fx)}{3(a-b)f}\end{aligned}$$

Mathematica [A]
time = 0.45, size = 149, normalized size = 1.77

$$\frac{6a\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+6a\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+(a-b)\cos(e+fx)(-5a-b+(a-b)\cos(2(e+fx)))}{6(a-b)^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (6*a*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]] + 6*a*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x
)/2])/Sqrt[b]] + (a - b)*Cos[e + f*x]*(-5*a - b + (a - b)*Cos[2*(e + f*x)])
)/(6*(a - b)^3*f)
```

Maple [A]
time = 0.27, size = 87, normalized size = 1.04

method	result

derivativedivides	$\dfrac{\dfrac{\frac{a\left(\cos^{3}(fx+e)\right)}{3}-\frac{b\left(\cos^{3}(fx+e)\right)}{3}-\cos(fx+e)a}{(a-b)^{2}}+\dfrac{ab\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{(a-b)^{2}\sqrt{b(a-b)}}}{f}$
default	$\dfrac{\dfrac{\frac{a\left(\cos^{3}(fx+e)\right)}{3}-\frac{b\left(\cos^{3}(fx+e)\right)}{3}-\cos(fx+e)a}{(a-b)^{2}}+\dfrac{ab\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{(a-b)^{2}\sqrt{b(a-b)}}}{f}$
risch	$-\frac{3\,\mathrm{e}^{i(fx+e)}a}{8(-a+b)^{2}f}-\frac{\mathrm{e}^{i(fx+e)}b}{8(-a+b)^{2}f}-\frac{3\,\mathrm{e}^{-i(fx+e)}a}{8(-a+b)^{2}f}-\frac{\mathrm{e}^{-i(fx+e)}b}{8(-a+b)^{2}f}+\frac{i\sqrt{b(a-b)}\,a\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{b(a-b}}{a-b}\right.}{2(a-b)^{3}f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^2*(1/3*a*cos(f*x+e)^3-1/3*b*cos(f*x+e)^3-cos(f*x+e)*a)+a*b/(a-
b)^2/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.80, size = 214, normalized size = 2.55

$$\left[\frac{2\,(a-b)\cos\left(fx+e\right)^{3}+3\,a\sqrt{-\frac{b}{a-b}}\,\log\left(\frac{(a-b)\cos(fx+e)^{2}+2\,(a-b)\sqrt{-\frac{b}{a-b}}\,\cos(fx+e)-b}{(a-b)\cos(fx+e)^{2}+b}\right)-6\,a\cos\left(fx+e\right)}{6\,(a^{2}-2\,ab+b^{2})f},\ \frac{(a-b)\cos\left(fx+e\right)^{3}-3\,a\sqrt{\frac{b}{a-b}}\,\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\,\cos(fx+e)}{b}\right)-3\,a\cos\left(fx+e\right)}{3\,(a^{2}-2\,ab+b^{2})f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a - b)*cos(f*x + e)^3 + 3*a*sqrt(-b/(a - b))*log(((a - b)*cos(f*x
+ e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)
```

```
^2 + b)) - 6*a*cos(f*x + e))/((a^2 - 2*a*b + b^2)*f), 1/3*((a - b)*cos(f*x
+ e)^3 - 3*a*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b
) - 3*a*cos(f*x + e))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(77) = 154.
time = 0.69, size = 180, normalized size = 2.14

$$\frac{ab\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{(a^2-2\,ab+b^2)\sqrt{ab-b^2}\,f}+\frac{a^2f^5\cos\left(fx+e\right)^3-2\,abf^5\cos\left(fx+e\right)^3+b^2f^5\cos\left(fx+e\right)^3-3\,a^2f^5\cos\left(fx+e\right)+3\,abf^5\cos\left(fx+e\right)}{3\left(a^3f^6-3\,a^2bf^6+3\,ab^2f^6-b^3f^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] a*b*arctan((a*cos(f*x + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^2 - 2*a*b
 + b^2)*sqrt(a*b - b^2)*f) + 1/3*(a^2*f^5*cos(f*x + e)^3 - 2*a*b*f^5*cos(f*
x + e)^3 + b^2*f^5*cos(f*x + e)^3 - 3*a^2*f^5*cos(f*x + e) + 3*a*b*f^5*cos(
f*x + e))/(a^3*f^6 - 3*a^2*b*f^6 + 3*a*b^2*f^6 - b^3*f^6)
```

Mupad [**B**]
time = 13.37, size = 382, normalized size = 4.55

$$-\frac{\frac{2\,(2\,a+b)}{3\,(a-b)^2}+\frac{4\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{(a-b)^2}+\frac{2\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{(a-b)^2}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}-\frac{a\sqrt{b}\,\mathrm{atan}\left(\frac{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{\sqrt{b}\,(8\,a^7\,b-32\,a^6\,b^2+48\,a^5\,b^3-32\,a^4\,b^4+8\,a^3\,b^5)}{(a-b)^{9/2}}-\frac{a\sqrt{b}\,(a-2\,b)\,(-16\,a^9+128\,a^8\,b-432\,a^7\,b^2+800\,a^6\,b^3-880\,a^5\,b^4+576\,a^4\,b^5-208\,a^3\,b^6+32\,a^2\,b^7)}{8\,(a-b)^{15/2}}\right)-\frac{a\sqrt{b}\,(a-2\,b)\,(16\,a^9-96\,a^8\,b+240\,a^7\,b^2-320\,a^6\,b^3+240\,a^5\,b^4-96\,a^4\,b^5+16\,a^3\,b^6)}{8\,(a-b)^{15/2}}\right)(a-b)^5}{4\,a^8\,b-16\,a^7\,b^2+24\,a^6\,b^3-16\,a^5\,b^4+4\,a^4\,b^5}\right)}{f\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2),x)
```

```
[Out] - ((2*(2*a + b))/(3*(a - b)^2) + (4*a*tan(e/2 + (f*x)/2)^2)/(a - b)^2 + (2*
b*tan(e/2 + (f*x)/2)^4)/(a - b)^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 +
 (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 + 1)) - (a*b^(1/2)*atan(((tan(e/2 + (f*x
)/2)^2*((b^(1/2)*(8*a^7*b + 8*a^3*b^5 - 32*a^4*b^4 + 48*a^5*b^3 - 32*a^6*b^
2))/(a - b)^(9/2) - (a*b^(1/2)*(a - 2*b)*(128*a^8*b - 16*a^9 + 32*a^2*b^7 -
 208*a^3*b^6 + 576*a^4*b^5 - 880*a^5*b^4 + 800*a^6*b^3 - 432*a^7*b^2))/(8*(
a - b)^(15/2))) - (a*b^(1/2)*(a - 2*b)*(16*a^9 - 96*a^8*b + 16*a^3*b^6 - 96
*a^4*b^5 + 240*a^5*b^4 - 320*a^6*b^3 + 240*a^7*b^2))/(8*(a - b)^(15/2)))*(a
 - b)^5)/(4*a^8*b + 4*a^4*b^5 - 16*a^5*b^4 + 24*a^6*b^3 - 16*a^7*b^2)))/(f*
(a - b)^(5/2))
```

3.57 $\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}-\frac{\cos(e+fx)}{(a-b)f}$$

```
[Out] -cos(f*x+e)/(a-b)/f-arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(3
/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3745, 331, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{\cos(e+fx)}{f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(3/2)*f)) -
 Cos[e + f*x]/((a - b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
```

```
- 1)/2]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)}{(a-b)f} - \frac{b\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \sec(e+fx)\right)}{(a-b)f}$$
$$= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{\cos(e+fx)}{(a-b)f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 121 vs. 2(60) = 120.
time = 0.18, size = 121, normalized size = 2.02

$$\frac{\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+(-a+b)\cos(e+fx)}{(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2),x]
[Out] (Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b
]] + Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]] + (-a + b)*Cos[e + f*x])/((a - b)^2*f)
```

Maple [A]
time = 0.21, size = 61, normalized size = 1.02

method	result
derivativedivides	$\frac{-\frac{\cos(fx+e)}{a-b}+\frac{b\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{(a-b)\sqrt{b(a-b)}}}{f}$
default	$\frac{-\frac{\cos(fx+e)}{a-b}+\frac{b\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{(a-b)\sqrt{b(a-b)}}}{f}$

risch	$-\frac{e^{i(fx+e)}}{2(a-b)f}-\frac{e^{-i(fx+e)}}{2(a-b)f}-\frac{i\sqrt{b(a-b)}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{b(a-b)}\,e^{i(fx+e)}}{a-b}+1\right)}{2(a-b)^2 f}+\frac{i\sqrt{b(a-b)}\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a-b)*cos(f*x+e)+b/(a-b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b
*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.82, size = 164, normalized size = 2.73

$$\left[-\frac{\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos(fx+e)^2-2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)+2\cos(fx+e)}{2\,(a-b)f},\ -\frac{\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)+\cos(fx+e)}{(a-b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a
 - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*cos(f*x + e))/((
a - b)*f), -(sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b
) + cos(f*x + e))/((a - b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2), x)
```

Giac [A]
time = 0.71, size = 81, normalized size = 1.35

$$-\frac{f\cos\left(fx+e\right)}{af^2-bf^2}+\frac{b\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}\,(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -f*cos(f*x + e)/(a*f^2 - b*f^2) + b*arctan((a*cos(f*x + e) - b*cos(f*x + e)
)/sqrt(a*b - b^2))/(sqrt(a*b - b^2)*(a - b)*f)
```

Mupad [B]
time = 11.79, size = 112, normalized size = 1.87

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{-a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2+a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-3\,a\,b+2\,b^2}{2\,\sqrt{b}\,(a-b)^{3/2}}\right)}{f\,(a-b)^{3/2}}-\frac{2\,\sqrt{a-b}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2(a-b)^{3/2}+(a-b)^{3/2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (b^(1/2)*atan((a^2 - a^2*tan(e/2 + (f*x)/2)^2 - 3*a*b + 2*b^2 + a*b*tan(e/2
 + (f*x)/2)^2)/(2*b^(1/2)*(a - b)^(3/2))))/(f*(a - b)^(3/2)) - (2*(a - b)^(
1/2))/(f*(tan(e/2 + (f*x)/2)^2*(a - b)^(3/2) + (a - b)^(3/2)))
```

3.58 $\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}\,f}-\frac{\tanh^{-1}(\cos(e+fx))}{af}$$

```
[Out] -arctanh(cos(f*x+e))/a/f-arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a/f
/(a-b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3745, 400, 213, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{af\sqrt{a-b}}-\frac{\tanh^{-1}(\cos(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*f)) -
 ArcTanh[Cos[e + f*x]]/(a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
```

```
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{af} - \frac{b\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \sec(e+fx)\right)}{af}$$
$$= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}\,f} - \frac{\tanh^{-1}(\cos(e+fx))}{af}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(60) = 120.
time = 0.14, size = 144, normalized size = 2.40

$$\frac{\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-(a-b)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{a(a-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b
]] + Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]] - (a - b)*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))/(a*(a - b
)*f)
```

Maple [A]
time = 0.24, size = 70, normalized size = 1.17

method	result
derivativedivides	$\frac{-\frac{\ln(\cos(fx+e)+1)}{2a}+\frac{\ln(\cos(fx+e)-1)}{2a}+\frac{b\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{a\sqrt{b(a-b)}}}{f}$
default	$\frac{-\frac{\ln(\cos(fx+e)+1)}{2a}+\frac{\ln(\cos(fx+e)-1)}{2a}+\frac{b\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{a\sqrt{b(a-b)}}}{f}$

risch	$\frac{\ln(e^{i(fx+e)}-1)}{af} - \frac{\ln(e^{i(fx+e)}+1)}{af} + \frac{i\sqrt{b(a-b)}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{b(a-b)}\,e^{i(fx+e)}}{a-b}+1\right)}{2(a-b)fa} - \frac{i\sqrt{b(a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/a*ln(cos(f*x+e)+1)+1/2/a*ln(cos(f*x+e)-1)+b/a/(b*(a-b))^(1/2)*arc
tan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 1.16, size = 192, normalized size = 3.20

$$\left[\frac{\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos(fx+e)^2+2(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)-\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{2af}, -\frac{2\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)+\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{2af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a -
 b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x + e)
 + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/(a*f), -1/2*(2*sqrt(b/(a - b))*arct
an(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + log(1/2*cos(f*x + e) + 1/2) -
 log(-1/2*cos(f*x + e) + 1/2))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(54) = 108.
time = 0.92, size = 113, normalized size = 1.88

$$-\frac{\frac{2\,b\arctan\left(-\frac{a\cos\left(fx+e\right)-b\cos\left(fx+e\right)-b}{\sqrt{ab-b^2}\cos\left(fx+e\right)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}\,a}-\frac{\log\left(\frac{|-\cos\left(fx+e\right)+1|}{|\cos\left(fx+e\right)+1|}\right)}{a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b - b^2)*co
s(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*a) - log(abs(-cos(f*x + e)
+ 1)/abs(cos(f*x + e) + 1))/a)/f
```

Mupad [B]
time = 11.86, size = 91, normalized size = 1.52

$$\frac{\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{a\,f}-\frac{\sqrt{b}\,\,\mathrm{atan}\left(\frac{b-a\,\cos\left(e+f\,x\right)+b\,\cos\left(e+f\,x\right)}{2\,\sqrt{b}\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\sqrt{a-b}}\right)}{a\,f\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))/(a*f) - (b^(1/2)*atan((b - a*cos
(e + f*x) + b*cos(e + f*x))/(2*b^(1/2)*cos(e/2 + (f*x)/2)^2*(a - b)^(1/2)))
)/(a*f*(a - b)^(1/2))
```

3.59 $\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f}-\frac{(a-2b)\tanh^{-1}(\cos(e+fx))}{2a^2f}-\frac{\cot(e+fx)\csc(e+fx)}{2af}$$

```
[Out] -1/2*(a-2*b)*arctanh(cos(f*x+e))/a^2/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f-arctan
(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)*b^(1/2)/a^2/f
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 482, 536, 213, 211}

$$-\frac{\sqrt{b}\,\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f}-\frac{(a-2b)\tanh^{-1}(\cos(e+fx))}{2a^2f}-\frac{\cot(e+fx)\csc(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^2*f))
 - ((a - 2*b)*ArcTanh[Cos[e + f*x]])/(2*a^2*f) - (Cot[e + f*x]*Csc[e + f*x]
)/(2*a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2af} + \frac{\text{Subst}\left(\int \frac{a-b-bx^2}{(-1+x^2)(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{2af}\\
&= -\frac{\cot(e+fx)\csc(e+fx)}{2af} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{2a^2f} - \frac{((a-
\\
&= -\frac{\sqrt{a-b}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f} - \frac{(a-2b)\tanh^{-1}(\cos(e+fx))}{2a^2f} - \frac{\cot(e+
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(89) = 178.
time = 0.42, size = 195, normalized size = 2.19

$$\frac{8\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+8\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-a\csc^2\left(\frac{1}{2}(e+fx)\right)-4a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+8b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+4a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-8b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (8*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt
[b]] + 8*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2]
```

```
)/Sqrt[b]] - a*Csc[(e + f*x)/2]^2 - 4*a*Log[Cos[(e + f*x)/2]] + 8*b*Log[Cos
[(e + f*x)/2]] + 4*a*Log[Sin[(e + f*x)/2]] - 8*b*Log[Sin[(e + f*x)/2]] + a*
Sec[(e + f*x)/2]^2)/(8*a^2*f)
```

Maple [A]
time = 0.32, size = 117, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{1}{4a(\cos(fx+e)+1)}+\frac{(-a+2b)\ln(\cos(fx+e)+1)}{4a^2}+\frac{1}{4a(\cos(fx+e)-1)}+\frac{(a-2b)\ln(\cos(fx+e)-1)}{4a^2}+\frac{b(a-b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{a^2\sqrt{b(a-b)}}}{f}$
default	$\frac{\frac{1}{4a(\cos(fx+e)+1)}+\frac{(-a+2b)\ln(\cos(fx+e)+1)}{4a^2}+\frac{1}{4a(\cos(fx+e)-1)}+\frac{(a-2b)\ln(\cos(fx+e)-1)}{4a^2}+\frac{b(a-b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{a^2\sqrt{b(a-b)}}}{f}$
risch	$\frac{e^{3i(fx+e)}+e^{i(fx+e)}}{fa\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln(e^{i(fx+e)}+1)}{2af}+\frac{\ln(e^{i(fx+e)}+1)b}{a^2f}+\frac{\ln(e^{i(fx+e)}-1)}{2af}-\frac{\ln(e^{i(fx+e)}-1)b}{a^2f}-\frac{i\sqrt{ab-b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4/a/(cos(f*x+e)+1)+1/4/a^2*(-a+2*b)*ln(cos(f*x+e)+1)+1/4/a/(cos(f*x+
e)-1)+1/4*(a-2*b)/a^2*ln(cos(f*x+e)-1)+b*(a-b)/a^2/(b*(a-b))^(1/2)*arctan((
a-b)*cos(f*x+e)/(b*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 1.03, size = 345, normalized size = 3.88

$$\left[\frac{2\sqrt{-ab+b^2}\left(\cos(fx+e)^2-1\right)\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{-ab+b^2}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)+2a\cos(fx+e)-\left((a-2b)\cos(fx+e)^2-a+2b\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\left((a-2b)\cos(fx+e)^2-a+2b\right)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{4\left(a^2f\cos(fx+e)^2-a^2f\right)}, \frac{4\sqrt{ab-b^2}\left(\cos(fx+e)^2-1\right)\arctan\left(\frac{\sqrt{ab-b^2}\cos(fx+e)}{b}\right)+2a\cos(fx+e)-\left((a-2b)\cos(fx+e)^2-a+2b\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\left((a-2b)\cos(fx+e)^2-a+2b\right)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{4\left(a^2f\cos(fx+e)^2-a^2f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a*b + b^2)*(cos(f*x + e)^2 - 1)*log(-((a - b)*cos(f*x + e)^2
+ 2*sqrt(-a*b + b^2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*a*
cos(f*x + e) - ((a - 2*b)*cos(f*x + e)^2 - a + 2*b)*log(1/2*cos(f*x + e) +
1/2) + ((a - 2*b)*cos(f*x + e)^2 - a + 2*b)*log(-1/2*cos(f*x + e) + 1/2))/(
a^2*f*cos(f*x + e)^2 - a^2*f), 1/4*(4*sqrt(a*b - b^2)*(cos(f*x + e)^2 - 1)*
arctan(sqrt(a*b - b^2)*cos(f*x + e)/b) + 2*a*cos(f*x + e) - ((a - 2*b)*cos(
f*x + e)^2 - a + 2*b)*log(1/2*cos(f*x + e) + 1/2) + ((a - 2*b)*cos(f*x + e)
^2 - a + 2*b)*log(-1/2*cos(f*x + e) + 1/2))/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(81) = 162.
time = 0.77, size = 214, normalized size = 2.40

$$\frac{\frac{2\,(a-2\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2}-\frac{8\,\sqrt{ab-b^2}\,\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\,\cos(fx+e)+\sqrt{ab-b^2}}\right)}{a^2}+\frac{\left(a-\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{a^2(\cos(fx+e)-1)}-\frac{\cos(fx+e)-1}{a(\cos(fx+e)+1)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 2*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^2 - 8*
sqrt(a*b - b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b - b
^2)*cos(f*x + e) + sqrt(a*b - b^2)))/a^2 + (a - 2*a*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) +
 1)/(a^2*(cos(f*x + e) - 1)) - (cos(f*x + e) - 1)/(a*(cos(f*x + e) + 1)))/f
```

Mupad [B]
time = 13.39, size = 591, normalized size = 6.64

$$-\frac{a\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)+2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)-\frac{1}{2}\right)+4\,\mathrm{atan}\left(\frac{2a^4b\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-a^4b-10a^3b^2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2+4a^3b^2+20a^2b^3\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-6a^2b^3-18ab^4\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3ab^4+6b^5\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{6\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2(ab-b^2)^{5/2}-2a^2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2(ab-b^2)^{3/2}}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sqrt{ab-b^2}-4\,\mathrm{atan}\left(\frac{2a^4b\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-a^4b-10a^3b^2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2+4a^3b^2+20a^2b^3\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-6a^2b^3-18ab^4\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3ab^4+6b^5\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{6\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2(ab-b^2)^{5/2}-2a^2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2(ab-b^2)^{3/2}}\right)\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sqrt{ab-b^2}+4b\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)-4b\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{4a^2f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-4a^2f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] -(a*(cos(e/2 + (f*x)/2)^2 - 2*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/c
os(e/2 + (f*x)/2)) + 2*cos(e/2 + (f*x)/2)^4*log(sin(e/2 + (f*x)/2)/cos(e/2
+ (f*x)/2)) - 1/2) + 4*atan((6*b^5*cos(e/2 + (f*x)/2)^2 + 3*a*b^4 - a^4*b -
 6*a^2*b^3 + 4*a^3*b^2 + 20*a^2*b^3*cos(e/2 + (f*x)/2)^2 - 10*a^3*b^2*cos(e
/2 + (f*x)/2)^2 - 18*a*b^4*cos(e/2 + (f*x)/2)^2 + 2*a^4*b*cos(e/2 + (f*x)/2
)^2)/(6*cos(e/2 + (f*x)/2)^2*(a*b - b^2)^(5/2) - 2*a^2*cos(e/2 + (f*x)/2)^2
*(a*b - b^2)^(3/2)))*cos(e/2 + (f*x)/2)^2*(a*b - b^2)^(1/2) - 4*atan((6*b^5
*cos(e/2 + (f*x)/2)^2 + 3*a*b^4 - a^4*b - 6*a^2*b^3 + 4*a^3*b^2 + 20*a^2*b^
3*cos(e/2 + (f*x)/2)^2 - 10*a^3*b^2*cos(e/2 + (f*x)/2)^2 - 18*a*b^4*cos(e/2
 + (f*x)/2)^2 + 2*a^4*b*cos(e/2 + (f*x)/2)^2)/(6*cos(e/2 + (f*x)/2)^2*(a*b
- b^2)^(5/2) - 2*a^2*cos(e/2 + (f*x)/2)^2*(a*b - b^2)^(3/2)))*cos(e/2 + (f*
x)/2)^4*(a*b - b^2)^(1/2) + 4*b*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)
/cos(e/2 + (f*x)/2)) - 4*b*cos(e/2 + (f*x)/2)^4*log(sin(e/2 + (f*x)/2)/cos(
e/2 + (f*x)/2)))/(4*a^2*f*cos(e/2 + (f*x)/2)^2 - 4*a^2*f*cos(e/2 + (f*x)/2)
^4)
```

3.60 $\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=130

$$-\frac{(a-b)^{3/2}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f}-\frac{(3a^2-12ab+8b^2)\tanh^{-1}(\cos(e+fx))}{8a^3f}-\frac{(5a-4b)\cot(e+fx)\csc}{8a^2f}$$

```
[Out] -1/8*(3*a^2-12*a*b+8*b^2)*arctanh(cos(f*x+e))/a^3/f-1/8*(5*a-4*b)*cot(f*x+e
)*csc(f*x+e)/a^2/f-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f-(a-b)^(3/2)*arctan(sec(f
*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^3/f
```

Rubi [A]
time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3745, 481, 541, 536, 213, 211}

$$-\frac{\sqrt{b}\,(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f}-\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f}-\frac{(3a^2-12ab+8b^2)\tanh^{-1}(\cos(e+fx))}{8a^3f}-\frac{\cot^3(e+fx)\csc(e+fx)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^3*f
)) - ((3*a^2 - 12*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*a^3*f) - ((5*a - 4
*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f) - (Cot[e + f*x]^3*Csc[e + f*x])/(4
*a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-a+b+(-4a+3b)x^2}{(-1+x^2)^2(a-b+bx^2)}\,dx, x, \sec(e+fx)\right)}{4af}$$

$$= -\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-(3}{}\right.}{}$$

$$= -\frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af} - \frac{((a-b)^2b)\,\text{S}}{}$$

$$= -\frac{(a-b)^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f} - \frac{(3a^2-12ab+8b^2)\tanh^{-1}(\cos(e+fx}{8a^3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 326 vs. 2(130) = 260.

time = 6.18, size = 326, normalized size = 2.51

$$\frac{(a-b)^{3/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\,\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{a}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{a^3f}+\frac{(a-b)^{3/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\,\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{a^3f}+\frac{(-3a+4b)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32a^2f}-\frac{\csc^4\left(\frac{1}{2}(e+fx)\right)}{64af}+\frac{(-3a^2+12ab-8b^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8a^3f}+\frac{(3a^2-12ab+8b^2)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8a^3f}+\frac{(3a-4b)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32a^2f}+\frac{\sec^4\left(\frac{1}{2}(e+fx)\right)}{64af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - b)^(3/2)*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/
2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(a^3*f) + ((a - b)^(3/2)*Sqrt[b]*
ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2] + Sqrt[a]*Sin[(e + f
*x)/2]))/Sqrt[b]])/(a^3*f) + ((-3*a + 4*b)*Csc[(e + f*x)/2]^2)/(32*a^2*f) -
 Csc[(e + f*x)/2]^4/(64*a*f) + ((-3*a^2 + 12*a*b - 8*b^2)*Log[Cos[(e + f*x)
/2]])/(8*a^3*f) + ((3*a^2 - 12*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^3*f
) + ((3*a - 4*b)*Sec[(e + f*x)/2]^2)/(32*a^2*f) + Sec[(e + f*x)/2]^4/(64*a*
f)
```

Maple [A]
time = 0.39, size = 185, normalized size = 1.42

method	result
derivativedivides	$\frac{\frac{1}{16a(\cos(fx+e)+1)^2}-\frac{-3a+4b}{16a^2(\cos(fx+e)+1)}+\frac{(-3a^2+12ab-8b^2)\ln(\cos(fx+e)+1)}{16a^3}-\frac{1}{16a(\cos(fx+e)-1)^2}-\frac{-3a+4b}{16a^2(\cos(fx+e)-1)}+\frac{(3a^2}{}}{f}$
default	$\frac{\frac{1}{16a(\cos(fx+e)+1)^2}-\frac{-3a+4b}{16a^2(\cos(fx+e)+1)}+\frac{(-3a^2+12ab-8b^2)\ln(\cos(fx+e)+1)}{16a^3}-\frac{1}{16a(\cos(fx+e)-1)^2}-\frac{-3a+4b}{16a^2(\cos(fx+e)-1)}+\frac{(3a^2}{}}{f}$
risch	$\frac{3a\,e^{7i(fx+e)}-4b\,e^{7i(fx+e)}-11a\,e^{5i(fx+e)}+4b\,e^{5i(fx+e)}-11a\,e^{3i(fx+e)}+4b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)}-4b\,e^{i(fx+e)}}{4f\,a^2\left(e^{2i(fx+e)}-1\right)^4}-\frac{3\ln(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/16/a/(cos(f*x+e)+1)^2-1/16*(-3*a+4*b)/a^2/(cos(f*x+e)+1)+1/16/a^3*(-
3*a^2+12*a*b-8*b^2)*ln(cos(f*x+e)+1)-1/16/a/(cos(f*x+e)-1)^2-1/16*(-3*a+4*b
)/a^2/(cos(f*x+e)-1)+1/16*(3*a^2-12*a*b+8*b^2)/a^3*ln(cos(f*x+e)-1)+b*(a^2-
2*a*b+b^2)/a^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(122) = 244$.
time = 1.06, size = 658, normalized size = 5.06

$$\left[\frac{2\,(3\,a^2-4\,ab)\cos(fx+e)^3-8\left((a-b)\cos(fx+e)^4-2\,(a-b)\cos(fx+e)^2+a-b\right)\sqrt{-ab+b^2}\log\left(\frac{(a-b)\cos(fx+e)^2-2\sqrt{-ab+b^2}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)-2\,(5\,a^2-4\,ab)\cos(fx+e)-\left((3\,a^2-12\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,a^2-12\,ab+8\,b^2)\cos(fx+e)^2+3\,a^2-12\,ab+8\,b^2\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\left((3\,a^2-12\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,a^2-12\,ab+8\,b^2)\cos(fx+e)^2+3\,a^2-12\,ab+8\,b^2\right)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{16\left(a^3f\cos(fx+e)^4-2\,a^3f\cos(fx+e)^2+a^3f\right)},\ \frac{2\,(3\,a^2-4\,ab)\cos(fx+e)^3+16\left((a-b)\cos(fx+e)^4-2\,(a-b)\cos(fx+e)^2+a-b\right)\sqrt{ab-b^2}\arctan\left(\frac{\sqrt{ab-b^2}\cos(fx+e)}{b}\right)-2\,(5\,a^2-4\,ab)\cos(fx+e)-\left((3\,a^2-12\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,a^2-12\,ab+8\,b^2)\cos(fx+e)^2+3\,a^2-12\,ab+8\,b^2\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\left((3\,a^2-12\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,a^2-12\,ab+8\,b^2)\cos(fx+e)^2+3\,a^2-12\,ab+8\,b^2\right)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{16\left(a^3f\cos(fx+e)^4-2\,a^3f\cos(fx+e)^2+a^3f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 - 4*a*b)*cos(f*x + e)^3 - 8*((a - b)*cos(f*x + e)^4 - 2*(a
 - b)*cos(f*x + e)^2 + a - b)*sqrt(-a*b + b^2)*log(((a - b)*cos(f*x + e)^2 -
 2*sqrt(-a*b + b^2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 2*(5*
a^2 - 4*a*b)*cos(f*x + e) - ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3
*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(1/2*cos
(f*x + e) + 1/2) + ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 12
*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(-1/2*cos(f*x + e
) + 1/2))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/16*(2*
(3*a^2 - 4*a*b)*cos(f*x + e)^3 + 16*((a - b)*cos(f*x + e)^4 - 2*(a - b)*cos
(f*x + e)^2 + a - b)*sqrt(a*b - b^2)*arctan(sqrt(a*b - b^2)*cos(f*x + e)/b)
 - 2*(5*a^2 - 4*a*b)*cos(f*x + e) - ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^
4 - 2*(3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log
(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(-1/2*cos
(f*x + e) + 1/2))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(122) = 244$.
time = 0.73, size = 377, normalized size = 2.90

$$-\frac{\frac{\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2}-\frac{4\,(3\,a^2-12\,ab+8\,b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3}+\frac{64\,(a^2b-2\,ab^2+b^3)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}\,a^3}+\frac{\left(a^2-\frac{8\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{72\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{48\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{a^3(\cos(fx+e)-1)^2}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/64*((8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 - 4*(
3*a^2 - 12*a*b + 8*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a
^3 + 64*(a^2*b - 2*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) -
b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*a^3)
+ (a^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
- 72*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^2*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(a^3*(cos(f*x + e) - 1)^2)
)/f
```

Mupad [B]
time = 14.72, size = 740, normalized size = 5.69

$$\frac{a^2\left(\frac{3\cos(3e+3fx)}{4}-\frac{11\cos(e+fx)}{4}+\frac{9\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{8}-\frac{3\cos(2e+2fx)\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{2}+\frac{3\cos(4e+4fx)\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{8}\right)+3b^2\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)-a\left(b\cos(3e+3fx)-b\cos(e+fx)+\frac{9b\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{2}-6b\cos(2e+2fx)\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)+\frac{3b\cos(4e+4fx)\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{2}\right)-4b^2\cos(2e+2fx)\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)+b^2\cos(4e+4fx)\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)+3\sqrt{b}\,\mathrm{atan}\left(\frac{a^4\cos(e+fx)-a^3b-3ab^3+b^4\cos(e+fx)+b^4+3a^2b^2+6a^2b^2\cos(e+fx)-4ab^3\cos(e+fx)-4a^3b\cos(e+fx)}{2\sqrt{b}\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2(a-b)^{7/2}}\right)(a-b)^{3/2}-4\sqrt{b}\,\mathrm{atan}\left(\frac{a^4\cos(e+fx)-a^3b-3ab^3+b^4\cos(e+fx)+b^4+3a^2b^2+6a^2b^2\cos(e+fx)-4ab^3\cos(e+fx)-4a^3b\cos(e+fx)}{2\sqrt{b}\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2(a-b)^{7/2}}\right)\cos(2e+2fx)(a-b)^{3/2}+\sqrt{b}\,\mathrm{atan}\left(\frac{a^4\cos(e+fx)-a^3b-3ab^3+b^4\cos(e+fx)+b^4+3a^2b^2+6a^2b^2\cos(e+fx)-4ab^3\cos(e+fx)-4a^3b\cos(e+fx)}{2\sqrt{b}\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2(a-b)^{7/2}}\right)\cos(4e+4fx)(a-b)^{3/2}}{3a^3f-4a^3f\cos(2e+2fx)+a^3f\cos(4e+4fx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] (a^2*((3*cos(3*e + 3*f*x))/4 - (11*cos(e + f*x))/4 + (9*log(sin(e/2 + (f*x)
/2)/cos(e/2 + (f*x)/2)))/8 - (3*cos(2*e + 2*f*x)*log(sin(e/2 + (f*x)/2)/cos
(e/2 + (f*x)/2)))/2 + (3*cos(4*e + 4*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 +
(f*x)/2)))/8) + 3*b^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) - a*(b*cos
(3*e + 3*f*x) - b*cos(e + f*x) + (9*b*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x
)/2)))/2 - 6*b*cos(2*e + 2*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))
+ (3*b*cos(4*e + 4*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/2) - 4*
b^2*cos(2*e + 2*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) + b^2*cos(4
*e + 4*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) + 3*b^(1/2)*atan((a^
4*cos(e + f*x) - a^3*b - 3*a*b^3 + b^4*cos(e + f*x) + b^4 + 3*a^2*b^2 + 6*a
^2*b^2*cos(e + f*x) - 4*a*b^3*cos(e + f*x) - 4*a^3*b*cos(e + f*x))/(2*b^(1/
2)*cos(e/2 + (f*x)/2)^2*(a - b)^(7/2)))*(a - b)^(3/2) - 4*b^(1/2)*atan((a^4
*cos(e + f*x) - a^3*b - 3*a*b^3 + b^4*cos(e + f*x) + b^4 + 3*a^2*b^2 + 6*a^
2*b^2*cos(e + f*x) - 4*a*b^3*cos(e + f*x) - 4*a^3*b*cos(e + f*x))/(2*b^(1/2
)*cos(e/2 + (f*x)/2)^2*(a - b)^(7/2)))*cos(2*e + 2*f*x)*(a - b)^(3/2) + b^(
1/2)*atan((a^4*cos(e + f*x) - a^3*b - 3*a*b^3 + b^4*cos(e + f*x) + b^4 + 3*
a^2*b^2 + 6*a^2*b^2*cos(e + f*x) - 4*a*b^3*cos(e + f*x) - 4*a^3*b*cos(e + f
*x))/(2*b^(1/2)*cos(e/2 + (f*x)/2)^2*(a - b)^(7/2)))*cos(4*e + 4*f*x)*(a -
b)^(3/2))/(3*a^3*f - 4*a^3*f*cos(2*e + 2*f*x) + a^3*f*cos(4*e + 4*f*x))
```

3.61 $\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=178

$$\frac{(5a^3+15a^2b-5ab^2+b^3)\,x}{16(a-b)^4}-\frac{a^{5/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^4f}-\frac{(11a^2-4ab+b^2)\cos(e+fx)\sin(e+fx)}{16(a-b)^3f}+$$

```
[Out] 1/16*(5*a^3+15*a^2*b-5*a*b^2+b^3)*x/(a-b)^4-1/16*(11*a^2-4*a*b+b^2)*cos(f*x
+e)*sin(f*x+e)/(a-b)^3/f+1/8*(3*a-b)*cos(f*x+e)^3*sin(f*x+e)/(a-b)^2/f+1/6*
cos(f*x+e)^3*sin(f*x+e)^3/(a-b)/f-a^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2)
)*b^(1/2)/(a-b)^4/f
```

Rubi [A]
time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3744, 481, 592, 541, 536, 209, 211}

$$-\frac{a^{5/2}\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^4}-\frac{(11a^2-4ab+b^2)\sin(e+fx)\cos(e+fx)}{16f(a-b)^3}+\frac{x(5a^3+15a^2b-5ab^2+b^3)}{16(a-b)^4}+\frac{\sin^3(e+fx)\cos^3(e+fx)}{6f(a-b)}+\frac{(3a-b)\sin(e+fx)\cos^3(e+fx)}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*x)/(16*(a - b)^4) - (a^(5/2)*Sqrt[b]*Ar
cTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)^4*f) - ((11*a^2 - 4*a*b + b^
2)*Cos[e + f*x]*Sin[e + f*x])/(16*(a - b)^3*f) + ((3*a - b)*Cos[e + f*x]^3*
Sin[e + f*x])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*(a - b)*
f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6(a-b)f} - \frac{\text{Subst}\left(\int \frac{x^2(3a-3(2a-b)x^2)}{(1+x^2)^3(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{6(a-b)f}$$

$$= \frac{(3a-b)\cos^3(e+fx)\sin(e+fx)}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6(a-b)f} - \frac{\text{Subst}\left(\int \frac{3a(3a}{}\right.}{}$$

$$= -\frac{(11a^2-4ab+b^2)\cos(e+fx)\sin(e+fx)}{16(a-b)^3f} + \frac{(3a-b)\cos^3(e+fx)\sin(e+fx}{8(a-b)^2f}$$

$$= -\frac{(11a^2-4ab+b^2)\cos(e+fx)\sin(e+fx)}{16(a-b)^3f} + \frac{(3a-b)\cos^3(e+fx)\sin(e+fx}{8(a-b)^2f}$$

$$= \frac{(5a^3+15a^2b-5ab^2+b^3)\,x}{16(a-b)^4} - \frac{a^{5/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^4f} - \frac{(11a^2-4ab+}{}$$

Mathematica [A]

time = 0.39, size = 140, normalized size = 0.79

$$-\frac{-12(5a^3+15a^2b-5ab^2+b^3)(e+fx)+192a^{5/2}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)+3(a-b)(5a-b)(3a+b)\sin(2(e+fx))-3(a-b)^2(3a-b)\sin(4(e+fx))+(a-b)^3\sin(6(e+fx))}{192(a-b)^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/192*(-12*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(e + f*x) + 192*a^(5/2)*Sqrt
[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + 3*(a - b)*(5*a - b)*(3*a + b)*
Sin[2*(e + f*x)] - 3*(a - b)^2*(3*a - b)*Sin[4*(e + f*x)] + (a - b)^3*Sin[6
*(e + f*x)])/((a - b)^4*f)
```

Maple [A]

time = 0.38, size = 185, normalized size = 1.04

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{11}{16}a^3+\frac{15}{16}a^2b-\frac{5}{16}a\,b^2+\frac{1}{16}b^3\right)\left(\tan^5(fx+e)\right)+\left(-\frac{5}{6}a^3+\frac{1}{2}a^2b+\frac{1}{2}a\,b^2-\frac{1}{6}b^3\right)\left(\tan^3(fx+e)\right)+\left(-\frac{5}{16}a^3+\frac{1}{16}a^2b+\frac{5}{16}a\,b^2-\frac{1}{16}b^3\right)\text{ta}}{\left(1+\tan^2(fx+e)\right)^3}}{(a-b)^4}}{f}$

default	$\frac{\frac{\frac{\left(-\frac{11}{16}a^3+\frac{15}{16}a^2b-\frac{5}{16}a\,b^2+\frac{1}{16}b^3\right)\left(\tan^5(fx+e)\right)+\left(-\frac{5}{6}a^3+\frac{1}{2}a^2b+\frac{1}{2}a\,b^2-\frac{1}{6}b^3\right)\left(\tan^3(fx+e)\right)+\left(-\frac{5}{16}a^3+\frac{1}{16}a^2b+\frac{5}{16}a\,b^2-\frac{1}{16}b^3\right)\tan(}{\left(1+\tan^2(fx+e)\right)^3}}{(a-b)^4}}{f}$
risch	$\frac{5x\,a^3}{16(a-b)^4}+\frac{15x\,a^2b}{16(a-b)^4}-\frac{5xa\,b^2}{16(a-b)^4}+\frac{x\,b^3}{16(a-b)^4}+\frac{15i\mathrm{e}^{2i(fx+e)}a^2}{128(a-b)^3f}+\frac{i\mathrm{e}^{2i(fx+e)}ab}{64(a-b)^3f}-\frac{i\mathrm{e}^{2i(fx+e)}b^2}{128(a-b)^3f}-\frac{15i\mathrm{e}^{-2}}{128(a^2-2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^4*(((-11/16*a^3+15/16*a^2*b-5/16*a*b^2+1/16*b^3)*tan(f*x+e)^5+
(-5/6*a^3+1/2*a^2*b+1/2*a*b^2-1/6*b^3)*tan(f*x+e)^3+(-5/16*a^3+1/16*a^2*b+5
/16*a*b^2-1/16*b^3)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*(5*a^3+15*a^2*b-5*a
*b^2+b^3)*arctan(tan(f*x+e)))-a^3*b/(a-b)^4/(a*b)^(1/2)*arctan(b*tan(f*x+e)
/(a*b)^(1/2)))
```

Maxima [A]

time = 0.51, size = 313, normalized size = 1.76

$$-\frac{\frac{48\,a^3b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)\sqrt{ab}}-\frac{3\left(5\,a^3+15\,a^2b-5\,ab^2+b^3\right)(fx+e)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{3\left(11\,a^2-4\,ab+b^2\right)\tan(fx+e)^5+8\left(5\,a^2+2\,ab-b^2\right)\tan(fx+e)^3+3\left(5\,a^2+4\,ab-b^2\right)\tan(fx+e)}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\tan(fx+e)^6+3\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\tan(fx+e)^4+a^3-3\,a^2b+3\,ab^2-b^3+3\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\tan(fx+e)^2}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/48*(48*a^3*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 4*a^3*b + 6*a^2*b^
2 - 4*a*b^3 + b^4)*sqrt(a*b)) - 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(f*x +
 e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (3*(11*a^2 - 4*a*b + b^2)
*tan(f*x + e)^5 + 8*(5*a^2 + 2*a*b - b^2)*tan(f*x + e)^3 + 3*(5*a^2 + 4*a*b
 - b^2)*tan(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(f*x + e)^6 + 3*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(f*x + e)^4 + a^3 - 3*a^2*b + 3*a*b^2 - b
^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(f*x + e)^2))/f
```

Fricas [A]

time = 0.81, size = 539, normalized size = 3.03

$$\left[\frac{12\sqrt{-ab}\,a^2\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(fx+e)^4-2\left(3\,ab+b^2\right)\cos(fx+e)^2+4\left((a+b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab}\sin(fx+e)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(fx+e)^4+2\left(ab-b^2\right)\cos(fx+e)^2+b^2}\right)+3\left(5\,a^3+15\,a^2b-5\,ab^2+b^3\right)fx-\left(8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\cos(fx+e)^5-2\left(13\,a^3-33\,a^2b+27\,ab^2-7\,b^3\right)\cos(fx+e)^3+3\left(11\,a^3-15\,a^2b+5\,ab^2-b^3\right)\cos(fx+e)\right)\sin(fx+e)}{48\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)f},\ \frac{24\sqrt{ab}\,a^2\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{ab}}{2\,ab\cos(fx+e)\sin(fx+e)}\right)+3\left(5\,a^3+15\,a^2b-5\,ab^2+b^3\right)fx-\left(8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\cos(fx+e)^5-2\left(13\,a^3-33\,a^2b+27\,ab^2-7\,b^3\right)\cos(fx+e)^3+3\left(11\,a^3-15\,a^2b+5\,ab^2-b^3\right)\cos(fx+e)\right)\sin(fx+e)}{48\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(-a*b)*a^2*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + b^2)*cos(f*x + e)^2 + 4*((a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2
```

```
)*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*f*x - (8*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - 2*(13*a^3 - 33*a^2*b + 27*a*
b^2 - 7*b^3)*cos(f*x + e)^3 + 3*(11*a^3 - 15*a^2*b + 5*a*b^2 - b^3)*cos(f*x
 + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f), 1/48*
(24*sqrt(a*b)*a^2*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*co
s(f*x + e)*sin(f*x + e))) + 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*f*x - (8*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - 2*(13*a^3 - 33*a^2*b + 27*a
*b^2 - 7*b^3)*cos(f*x + e)^3 + 3*(11*a^3 - 15*a^2*b + 5*a*b^2 - b^3)*cos(f*
x + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.04, size = 292, normalized size = 1.64

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)a^3b}{(a^4-4a^3b+6a^2b^2-4ab^3+b^4)\sqrt{ab}}-\frac{3(5a^3+15a^2b-5ab^2+b^3)(fx+e)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{33a^2\tan(fx+e)^5-12ab\tan(fx+e)^5+3b^2\tan(fx+e)^5+40a^2\tan(fx+e)^3+16ab\tan(fx+e)^3-8b^2\tan(fx+e)^3+15a^2\tan(fx+e)+12ab\tan(fx+e)-3b^2\tan(fx+e)}{(a^3-3a^2b+3ab^2-b^3)\left(\tan(fx+e)^2+1\right)^3}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/48*(48*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b)))*a^3*b/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a*b)) - 3*(
5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(f*x + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4) + (33*a^2*tan(f*x + e)^5 - 12*a*b*tan(f*x + e)^5 + 3*b^2*tan(f
*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 16*a*b*tan(f*x + e)^3 - 8*b^2*tan(f*x +
 e)^3 + 15*a^2*tan(f*x + e) + 12*a*b*tan(f*x + e) - 3*b^2*tan(f*x + e))/((a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(f*x + e)^2 + 1)^3))/f
```

Mupad [**B**]
time = 17.02, size = 2500, normalized size = 14.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan(-((((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a
^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*
```

a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (tan(e + f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(1024*b^11 - 7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 14336*a^5*b^6 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^2))/(4096*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (tan(e + f*x)*(b^9 - 10*a*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3))/(128*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*1i)/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) + (tan(e + f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(1024*b^11 - 7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 14336*a^5*b^6 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^2))/(4096*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e + f*x)*(b^9 - 10*a*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3))/(128*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*1i)/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/(((a^3*b^8)/128 - (9*a^4*b^7)/128 + (23*a^5*b^6)/64 - (55*a^6*b^5)/64 + (145*a^7*b^4)/128 + (55*a^8*b^3)/128)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) + (((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (tan(e + f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(1024*b^11 - 7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 14336*a^5*b^6 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^2))/(4096*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (tan(e + f*x)*(b^9 - 10*a*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3))/(128*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) + (tan(e + f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(1024*b^11 -

```
7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 14336*a^5*b^6
 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^2))/(4096*(a^4
 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15
*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*
1i))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e + f*x)*(b^9
- 10*a*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6
*b^3))/(128*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a
^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(32*(a^4 - 4*a^3*b - 4*
a*b^3 + b^4 + 6*a^2*b^2))))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*1i)/(1
6*f*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - ((tan(e + f*x)*(4*a*b +
5*a^2 - b^2))/(16*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e + f*x)^5*(11*a^
2 - 4*a*b + b^2))/(16*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e + f*x)^3*(2
*a*b + 5*a^2 - b^2))/(6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(f*(3*tan(e + f*x
)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) - (atan((((-a^5*b)^(1/2)*((((
3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*a^5*b^8)/2 + 1
4*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 - 9*a^10*b^3 + (5
*a^11*b^2)/4)/(2*(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 -
 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2)) - (tan(e + f*x)*(-a^
5*b)^(1/2)*(1024*b^11 - 7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336
*a^4*b^7 + 14336*a^5*b^6 - 28672*a^6*b^5 + 2048...
```

3.62 $\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=129

$$\frac{(3a^2+6ab-b^2)\,x}{8(a-b)^3}-\frac{a^{3/2}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^3f}-\frac{(5a-b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f}+\frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f}$$

```
[Out] 1/8*(3*a^2+6*a*b-b^2)*x/(a-b)^3-1/8*(5*a-b)*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f
+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f-a^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1
/2))*b^(1/2)/(a-b)^3/f
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 481, 541, 536, 209, 211}

$$-\frac{a^{3/2}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^3}+\frac{x(3a^2+6ab-b^2)}{8(a-b)^3}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)}-\frac{(5a-b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((3*a^2 + 6*a*b - b^2)*x)/(8*(a - b)^3) - (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/((a - b)^3*f) - ((5*a - b)*Cos[e + f*x]*Sin[e + f*x
])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f} - \frac{\text{Subst}\left(\int \frac{a+(-4a+b)x^2}{(1+x^2)^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a-b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{a(3a+\cdots}{(1+\cdots}\right.}{\cdots}$$

$$= -\frac{(5a-b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f} - \frac{(a^2b)\,\text{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= \frac{(3a^2+6ab-b^2)\,x}{8(a-b)^3} - \frac{a^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^3f} - \frac{(5a-b)\cos(e+fx)\sin\cdots}{8(a-b)^2f}$$

Mathematica [A]
time = 0.19, size = 99, normalized size = 0.77

$$\frac{4(3a^2+6ab-b^2)(e+fx)-32a^{3/2}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)-8a(a-b)\sin(2(e+fx))+(a-b)^2\sin(4(e+fx))}{32(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (4*(3*a^2 + 6*a*b - b^2)*(e + f*x) - 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan
[e + f*x])/Sqrt[a]] - 8*a*(a - b)*Sin[2*(e + f*x)] + (a - b)^2*Sin[4*(e + f
*x)])/(32*(a - b)^3*f)
```

Maple [A]
time = 0.31, size = 131, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{5}{8}a^2+\frac{3}{4}ab-\frac{1}{8}b^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2+\frac{1}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(1+\tan^2(fx+e)\right)^2}+\frac{\left(3a^2+6ab-b^2\right)\arctan(\tan(fx+e))}{8}}{(a-b)^3}-\frac{a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^3\sqrt{ab}}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{5}{8}a^2+\frac{3}{4}ab-\frac{1}{8}b^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2+\frac{1}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(1+\tan^2(fx+e)\right)^2}+\frac{\left(3a^2+6ab-b^2\right)\arctan(\tan(fx+e))}{8}}{(a-b)^3}-\frac{a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^3\sqrt{ab}}}{f}$
risch	$\frac{3x\,a^2}{8(a-b)^3}+\frac{3xab}{4(a-b)^3}-\frac{x\,b^2}{8(a-b)^3}+\frac{ia\,e^{2i(fx+e)}}{8(a-b)^2f}-\frac{ia\,e^{-2i(fx+e)}}{8(a^2-2ab+b^2)f}+\frac{\sqrt{-ab}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2(a-b)^3f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^3*(((-5/8*a^2+3/4*a*b-1/8*b^2)*tan(f*x+e)^3+(-3/8*a^2+1/4*a*b+
1/8*b^2)*tan(f*x+e))/(1+tan(f*x+e)^2)^2+1/8*(3*a^2+6*a*b-b^2)*arctan(tan(f*
x+e)))-a^2*b/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 189, normalized size = 1.47

$$-\frac{\frac{8\,a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab}}-\frac{(3\,a^2+6\,ab-b^2)(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(5\,a-b)\tan(fx+e)^3+(3\,a+b)\tan(fx+e)}{(a^2-2\,ab+b^2)\tan(fx+e)^4+2\,(a^2-2\,ab+b^2)\tan(fx+e)^2+a^2-2\,ab+b^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^2*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 3*a^2*b + 3*a*b^2 -
b^3)*sqrt(a*b)) - (3*a^2 + 6*a*b - b^2)*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2
- b^3) + ((5*a - b)*tan(f*x + e)^3 + (3*a + b)*tan(f*x + e))/((a^2 - 2*a*b
+ b^2)*tan(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*tan(f*x + e)^2 + a^2 - 2*a*b
+ b^2))/f
```

Fricas [A]
time = 2.28, size = 399, normalized size = 3.09

$$\left[\frac{(3a^2+6ab-b^2)fx-2\sqrt{-ab}\,a\log\left(\frac{(a^2+6ab+b^2)\cos(fx+e)^4-2(3ab+b^2)\cos(fx+e)^2-4\left((a+b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab}\,\sin(fx+e)+b^2}{(a^2-2ab+b^2)\cos(fx+e)^4+2(ab-b^2)\cos(fx+e)^2+b^2}\right)+\left(2(a^2-2ab+b^2)\cos(fx+e)^3-(5a^2-6ab+b^2)\cos(fx+e)\right)\sin(fx+e)}{8(a^3-3a^2b+3ab^2-b^3)f},\ \frac{(3a^2+6ab-b^2)fx+4\sqrt{ab}\,a\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{ab}}{2ab\cos(fx+e)\sin(fx+e)}\right)+\left(2(a^2-2ab+b^2)\cos(fx+e)^3-(5a^2-6ab+b^2)\cos(fx+e)\right)\sin(fx+e)}{8(a^3-3a^2b+3ab^2-b^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^2 + 6*a*b - b^2)*f*x - 2*sqrt(-a*b)*a*log(((a^2 + 6*a*b + b^2)*c
os(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a + b)*cos(f*x + e)^3
- b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f
*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) + (2*(a^2 - 2*a*b + b^2)*c
os(f*x + e)^3 - (5*a^2 - 6*a*b + b^2)*cos(f*x + e))*sin(f*x + e))/((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f), 1/8*((3*a^2 + 6*a*b - b^2)*f*x + 4*sqrt(a*b)*a*
arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x + e)*sin(f*x
 + e))) + (2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 6*a*b + b^2)*cos
(f*x + e))*sin(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.74, size = 190, normalized size = 1.47

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)a^2b}{(a^3-3a^2b+3ab^2-b^3)\sqrt{ab}}-\frac{(3a^2+6ab-b^2)(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{5a\tan(fx+e)^3-b\tan(fx+e)^3+3a\tan(fx+e)+b\tan(fx+e)}{(a^2-2ab+b^2)\left(\tan(fx+e)^2+1\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*a^2*b/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)) - (3*a^2 + 6*a*b -
```

```
b^2)*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (5*a*tan(f*x + e)^3 - b*ta
n(f*x + e)^3 + 3*a*tan(f*x + e) + b*tan(f*x + e))/((a^2 - 2*a*b + b^2)*(tan
(f*x + e)^2 + 1)^2))/f
```

Mupad [B]
time = 15.74, size = 2500, normalized size = 19.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan(((((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b
^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((32*a*b^9 - 96*a^
2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^
3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^
3 + 15*a^4*b^2)) - (tan(e + f*x)*(-a^3*b)^(1/2)*(1280*a*b^8 - 256*b^9 - 230
4*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256
*a^7*b^2))/(64*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b
^4 + 6*a^2*b^2)))*(-a^3*b)^(1/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(-a^
3*b)^(1/2)*1i)/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (((tan(e + f*x)*(b^7 -
 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a
*b^3 + b^4 + 6*a^2*b^2)) - (((32*a*b^9 - 96*a^2*b^8 - 96*a^3*b^7 + 800*a^4*
b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*
a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(e + f
*x)*(-a^3*b)^(1/2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 12
80*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(64*(3*a*b^2 - 3*a
^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-a^3*b)^(1
/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(-a^3*b)^(1/2)*1i)/(2*(3*a*b^2 -
3*a^2*b + a^3 - b^3)))/((a^2*b^6 - 11*a^3*b^5 + 27*a^4*b^4 + 15*a^5*b^3)/(3
2*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) +
 (((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b^3))/(
32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((32*a*b^9 - 96*a^2*b^8
- 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^3 + 96
*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15
*a^4*b^2)) - (tan(e + f*x)*(-a^3*b)^(1/2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*
b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b
^2))/(64*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6
*a^2*b^2)))*(-a^3*b)^(1/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(-a^3*b)^(
1/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) - (((tan(e + f*x)*(b^7 - 12*a*b^6
 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^
4 + 6*a^2*b^2)) - (((32*a*b^9 - 96*a^2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 144
0*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6
*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(e + f*x)*(-a^3
*b)^(1/2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^
```

```
5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(64*(3*a*b^2 - 3*a^2*b + a^
3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-a^3*b)^(1/2))/(2*(
3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(-a^3*b)^(1/2))/(2*(3*a*b^2 - 3*a^2*b + a^
3 - b^3))))*(-a^3*b)^(1/2)*1i)/(f*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) - ((tan(
e + f*x)^3*(5*a - b))/(8*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)*(3*a + b))/(8
*(a^2 - 2*a*b + b^2)))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (atan(
((((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b^3))/(
32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((32*a*b^9 - 96*a^2*b^8
- 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^3 + 96
*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15
*a^4*b^2)) - (tan(e + f*x)*(6*a*b + 3*a^2 - b^2)*(1280*a*b^8 - 256*b^9 - 23
04*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 25
6*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4
*a*b^3 + b^4 + 6*a^2*b^2)))*(6*a*b + 3*a^2 - b^2))/(16*(a*b^2*3i - a^2*b*3i
 + a^3*1i - b^3*1i)))*(6*a*b + 3*a^2 - b^2)*1i)/(16*(a*b^2*3i - a^2*b*3i +
a^3*1i - b^3*1i)) + (((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b
^4 + 73*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((32*
a*b^9 - 96*a^2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4
 - 544*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^
4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(e + f*x)*(6*a*b + 3*a^2 - b^2)*(1280*a
*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4
+ 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)
*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(6*a*b + 3*a^2 - b^2))/(16*(
a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(6*a*b + 3*a^2 - b^2)*1i)/(16*(a*b
^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))/((a^2*b^6 - 11*a^3*b^5 + 27*a^4*b^4 +
 15*a^5*b^3)/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 +
 15*a^4*b^2)) + (((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 +
 73*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((32*a*b^
9 - 96*a^2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 5
44*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 -
20*a^3*b^3 + 15*a^4*b^2)) - (tan(e + f*x)*(6*a*b + 3*a^2 - b^2)*(1280*a*b^8
 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 12
80*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^
4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(6*a...
```

3.63 $\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{(a+b)x}{2(a-b)^2}-\frac{\sqrt{a}\,\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^2f}-\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f}$$

```
[Out] 1/2*(a+b)*x/(a-b)^2-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f-arctan(b^(1/2)*tan(f*
x+e)/a^(1/2))*a^(1/2)*b^(1/2)/(a-b)^2/f
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 482, 536, 209, 211}

$$-\frac{\sqrt{a}\,\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{f(a-b)^2}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)}+\frac{x(a+b)}{2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a + b)*x)/(2*(a - b)^2) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a]])/((a - b)^2*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f} + \frac{\text{Subst}\left(\int \frac{a-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2(a-b)f} \\
&= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f} - \frac{(ab)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2 f} + \frac{(a+b)\text{Su}}{} \\
&= \frac{(a+b)x}{2(a-b)^2} - \frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^2 f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 69, normalized size = 0.84

$$\frac{2(a+b)(e+fx) - 4\sqrt{a}\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) + (-a+b)\sin(2(e+fx))}{4(a-b)^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*(a + b)*(e + f*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a]] + (-a + b)*Sin[2*(e + f*x)])/(4*(a - b)^2*f)
```

Maple [A]
time = 0.31, size = 83, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan^2(fx+e)}+\frac{(a+b)\arctan(\tan(fx+e))}{2}}{(a-b)^2}-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^2\sqrt{ab}}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan^2(fx+e)}+\frac{(a+b)\arctan(\tan(fx+e))}{2}}{(a-b)^2}-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^2\sqrt{ab}}}{f}$
risch	$\frac{xa}{2(a-b)^2}+\frac{xb}{2(a-b)^2}+\frac{ie^{2i(fx+e)}}{8(a-b)f}-\frac{ie^{-2i(fx+e)}}{8(a-b)f}-\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2(a-b)^2f}+\frac{\sqrt{-ab}\,\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^2*((-1/2*a+1/2*b)*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+b)*arctan
(tan(f*x+e)))-a*b/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 97, normalized size = 1.18

$$-\frac{\frac{2\,ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{ab}}-\frac{(fx+e)(a+b)}{a^2-2\,ab+b^2}+\frac{\tan(fx+e)}{(a-b)\tan(fx+e)^2+a-b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2 - 2*a*b + b^2)*sqrt(a*b)
) - (f*x + e)*(a + b)/(a^2 - 2*a*b + b^2) + tan(f*x + e)/((a - b)*tan(f*x +
 e)^2 + a - b))/f
```

Fricas [A]
time = 3.99, size = 288, normalized size = 3.51

$$\left[\frac{2\,(a+b)fx-2\,(a-b)\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-ab}\,\log\left(\frac{(a^2+6\,ab+b^2)\cos(fx+e)^4-2\,(3\,ab+b^2)\cos(fx+e)^2+4\left((a+b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab}\,\sin(fx+e)+b^2}{(a^2-2\,ab+b^2)\cos(fx+e)^4+2\,(ab-b^2)\cos(fx+e)^2+b^2}\right)}{4\,(a^2-2\,ab+b^2)f},\ \frac{(a+b)fx-(a-b)\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{ab}\,\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{ab}}{2\,ab\cos(fx+e)\sin(fx+e)}\right)}{2\,(a^2-2\,ab+b^2)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*f*x - 2*(a - b)*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b)*log(
((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((
a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a^
2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)))/((a
^2 - 2*a*b + b^2)*f), 1/2*((a + b)*f*x - (a - b)*cos(f*x + e)*sin(f*x + e)
+ sqrt(a*b)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x
+ e)*sin(f*x + e))))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.72, size = 113, normalized size = 1.38

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)ab}{(a^2-2\,ab+b^2)\sqrt{ab}}-\frac{(fx+e)(a+b)}{a^2-2\,ab+b^2}+\frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)(a-b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*a*b/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - (f*x + e)*(a + b)/(a^2 - 2*a*b
+ b^2) + tan(f*x + e)/((tan(f*x + e)^2 + 1)*(a - b)))/f
```

Mupad [**B**]
time = 12.75, size = 190, normalized size = 2.32

$$\frac{b\,\sin\left(2\,e+2\,f\,x\right)-a\,\sin\left(2\,e+2\,f\,x\right)+2\,a\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)+2\,b\,\mathrm{atan}\left(\frac{\sin(e+f\,x)}{\cos(e+f\,x)}\right)-\mathrm{atan}\left(\frac{b^3\,\sin(e+f\,x)\,\sqrt{-a\,b}\,1\mathrm{i}-a\,b^2\,\sin(e+f\,x)\,\sqrt{-a\,b}\,2\mathrm{i}+a^2\,b\,\sin(e+f\,x)\,\sqrt{-a\,b}\,1\mathrm{i}}{\cos(e+f\,x)\,a^3\,b-2\,\cos(e+f\,x)\,a^2\,b^2+\cos(e+f\,x)\,a\,b^3}\right)\,\sqrt{-a\,b}\,4\mathrm{i}}{4\,f\,a^2-8\,f\,a\,b+4\,f\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (b*sin(2*e + 2*f*x) - a*sin(2*e + 2*f*x) + 2*a*atan(sin(e + f*x)/cos(e + f*
x)) + 2*b*atan(sin(e + f*x)/cos(e + f*x)) - atan((b^3*sin(e + f*x)*(-a*b)^(
1/2)*1i - a*b^2*sin(e + f*x)*(-a*b)^(1/2)*2i + a^2*b*sin(e + f*x)*(-a*b)^(1
/2)*1i)/(a*b^3*cos(e + f*x) - 2*a^2*b^2*cos(e + f*x) + a^3*b*cos(e + f*x)))
*(-a*b)^(1/2)*4i)/(4*a^2*f + 4*b^2*f - 8*a*b*f)
```

3.64 $\int \frac{1}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)f}$$

```
[Out] x/(a-b)-arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/(a-b)/f/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3741, 3756, 211}

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3741

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+b\tan^2(e+fx)}\,dx &= \frac{x}{a-b}-\frac{b\int \frac{\sec^2(e+fx)}{a+b\tan^2(e+fx)}\,dx}{a-b}\\
&= \frac{x}{a-b}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\tan(e+fx)\right)}{(a-b)f}\\
&= \frac{x}{a-b}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)f}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.98

$$
\frac{\text{ArcTan}(\tan(e+fx))-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}}{af-bf}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[e + f*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sq
rt[a])/(a*f - b*f)
```

Maple [A]
time = 0.13, size = 50, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
default	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)f}-\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)f}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

[Out] 1/f*(1/(a-b)*arctan(tan(f*x+e))-1/(a-b)*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))

Maxima [A]
time = 0.50, size = 50, normalized size = 1.00

$$-\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}-\frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="maxima")

[Out] -(b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b))/f

Fricas [A]
time = 3.09, size = 190, normalized size = 3.80

$$\left[\frac{4\,fx-\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{4\,(a-b)f},\ \frac{2\,fx-\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)}{2\,(a-b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="fricas")

[Out] [1/4*(4*f*x - sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), 1/2*(2*f*x - sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a - b)*f)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(37) = 74.
time = 1.28, size = 240, normalized size = 4.80

$$\begin{cases} \frac{\tilde{\infty}x}{\tan^2(e)} & \text{for } a=0 \wedge b=0 \wedge f=0 \\ \frac{-x-\frac{1}{f\tan(e+fx)}}{b} & \text{for } a=0 \\ \frac{fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}+\frac{fx}{2bf\tan^2(e+fx)+2bf}+\frac{\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} & \text{for } a=b \\ \frac{x}{a+b\tan^2(e)} & \text{for } f=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{2fx\sqrt{-\frac{a}{b}}}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}}-\frac{\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}}+\frac{\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x - 1/(f*ta
n(e + f*x)))/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*
b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan(e + f*x)/(2*b*f*tan(e + f*
x)**2 + 2*b*f), Eq(a, b)), (x/(a + b*tan(e)**2), Eq(f, 0)), (x/a, Eq(b, 0))
, (2*f*x*sqrt(-a/b)/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)) - log(-sqrt(-a/b)
 + tan(e + f*x))/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)) + log(sqrt(-a/b) + t
an(e + f*x))/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)), True))
```

Giac [A]
time = 0.64, size = 68, normalized size = 1.36

$$-\frac{\frac{\left(\pi \left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,(a-b)}-\frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*
b/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b))/f
```

Mupad [B]
time = 11.69, size = 948, normalized size = 18.96

$$-\frac{\mathrm{atan}\left(\frac{\frac{-4b^3\tan(e+fx)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}}{2a-2b}+\frac{-4b^3\tan(e+fx)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}}{2a-2b}}{\frac{\left(-4b^3\tan(e+fx)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}\right)1i}{2a-2b}-\frac{\left(-4b^3\tan(e+fx)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}\right)1i}{2a-2b}}\right)}{f\,(a-b)}+\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-ab}\left(2b^3\tan(e+fx)-\frac{\sqrt{-ab}\left(2b^4-4ab^3+2a^2b^2+\frac{\tan(e+fx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4\left(ab-a^2\right)}\right)}{2\left(ab-a^2\right)}\right)1i}{ab-a^2}+\frac{\sqrt{-ab}\left(2b^3\tan(e+fx)-\frac{\sqrt{-ab}\left(4ab^3-2b^4-2a^2b^2+\frac{\tan(e+fx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4\left(ab-a^2\right)}\right)}{2\left(ab-a^2\right)}\right)1i}{ab-a^2}}{\frac{\sqrt{-ab}\left(2b^3\tan(e+fx)-\frac{\sqrt{-ab}\left(2b^4-4ab^3+2a^2b^2+\frac{\tan(e+fx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4\left(ab-a^2\right)}\right)}{2\left(ab-a^2\right)}\right)}{ab-a^2}-\frac{\sqrt{-ab}\left(2b^3\tan(e+fx)-\frac{\sqrt{-ab}\left(4ab^3-2b^4-2a^2b^2+\frac{\tan(e+fx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4\left(ab-a^2\right)}\right)}{2\left(ab-a^2\right)}\right)}{ab-a^2}}\right)\sqrt{-ab}\,1i}{af\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 +
 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^
3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2) + ((-a*b)^(1/2)*
(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan(e +
 f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2
))))/(2*(a*b - a^2)))*1i)/(a*b - a^2))/(((-a*b)^(1/2)*(2*b^3*tan(e + f*x) -
 ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 + 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8
*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))
)/(a*b - a^2) - ((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3
- 2*b^4 - 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b
^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2))))/(a*b - a^2))*(-a*b)^(
```

```
1/2)*1i)/(a*f*(a - b)) - atan(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(e + f*
x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*
b) - 4*b^3*tan(e + f*x))/(2*a - 2*b) + (((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (ta
n(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(
2*a - 2*b) - 4*b^3*tan(e + f*x))/(2*a - 2*b))/(((((4*b^4 - 8*a*b^3 + 4*a^2*
b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*
b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b) - ((((8*a*b^3 - 4
*b^4 - 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*
1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b)))/(f
*(a - b))
```

3.65 $\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}$$

```
[Out] -cot(f*x+e)/a/f-arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(3/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3744, 331, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*f)) - Cot[e + f
*x]/(a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)}{af} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{af}$$
$$= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\cot(e+fx)}{af}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 1.00

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\cot(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*f)) - Cot[e + f
*x]/(a*f)
```

Maple [A]
time = 0.19, size = 44, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{a\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{a\tan(fx+e)}}{f}$
risch	$-\frac{2i}{fa\left(e^{2i(fx+e)}-1\right)}-\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a^2f}+\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/a/tan(f*x+e))
```

Maxima [A]
time = 0.51, size = 44, normalized size = 0.92

$$-\frac{\frac{b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{1}{a\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(42) = 84.
time = 3.08, size = 273, normalized size = 5.69

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(fx+e)^4-2\,(3\,ab+b^2)\cos(fx+e)^2+4\left((a^2+ab)\cos(fx+e)^3-ab\cos(fx+e)\right)\sqrt{-\frac{b}{a}}\sin(fx+e)+b^2}{(a^2-2\,ab+b^2)\cos(fx+e)^4+2\,(ab-b^2)\cos(fx+e)^2+b^2}\right)\sin\left(fx+e\right)-4\cos\left(fx+e\right)}{4\,af\sin\left(fx+e\right)}, \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)}{2\,af\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*
cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/
a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/(a*f*sin(f*x + e)), 1
/2*(sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x
+ e)*sin(f*x + e)))*sin(f*x + e) - 2*cos(f*x + e))/(a*f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{2}\left(e+fx\right)}{a+b\tan^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2), x)
```

Giac [A]
time = 0.69, size = 62, normalized size = 1.29

$$-\frac{\frac{\left(\pi \left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,a}+\frac{1}{a\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*
b/(sqrt(a*b)*a) + 1/(a*tan(f*x + e)))/f
```

Mupad [B]
time = 10.99, size = 40, normalized size = 0.83

$$-\frac{\cot(e+f\,x)}{a\,f}-\frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a}}\right)}{a^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] - cot(e + f*x)/(a*f) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2)))/(a^(3
/2)*f)
```

3.66 $\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$-\frac{(a-b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{(a-b)\cot(e+fx)}{a^2f}-\frac{\cot^3(e+fx)}{3af}$$

```
[Out] -(a-b)*cot(f*x+e)/a^2/f-1/3*cot(f*x+e)^3/a/f-(a-b)*arctan(b^(1/2)*tan(f*x+e
)/a^(1/2))*b^(1/2)/a^(5/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3744, 464, 331, 211}

$$-\frac{\sqrt{b}\,(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{(a-b)\cot(e+fx)}{a^2f}-\frac{\cot^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*f)) - (
(a - b)*Cot[e + f*x])/(a^2*f) - Cot[e + f*x]^3/(3*a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot^3(e+fx)}{3af}+\frac{(a-b)\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{af}\\ &= -\frac{(a-b)\cot(e+fx)}{a^2f}-\frac{\cot^3(e+fx)}{3af}-\frac{((a-b)b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+f\right.}{a^2f}\\ &= -\frac{(a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{(a-b)\cot(e+fx)}{a^2f}-\frac{\cot^3(e+fx)}{3af}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 73, normalized size = 0.96

$$\frac{3\sqrt{b}\,(-a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)-\sqrt{a}\,\cot(e+fx)\,(2a-3b+a\csc^2(e+fx))}{3a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (3*Sqrt[b]*(-a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*Cot[e
+ f*x]*(2*a - 3*b + a*Csc[e + f*x]^2))/(3*a^(5/2)*f)
```

Maple [A]
time = 0.25, size = 67, normalized size = 0.88

method	result

derivativedivides	$\frac{-\frac{1}{3a\tan(fx+e)^3}-\frac{a-b}{a^2\tan(fx+e)}-\frac{b(a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
default	$\frac{-\frac{1}{3a\tan(fx+e)^3}-\frac{a-b}{a^2\tan(fx+e)}-\frac{b(a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
risch	$\frac{2i(3b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-6b\,e^{2i(fx+e)}-2a+3b)}{3f\,a^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a^2f}-\frac{\sqrt{-ab}\,\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3/a/tan(f*x+e)^3-(a-b)/a^2/tan(f*x+e)-b*(a-b)/a^2/(a*b)^(1/2)*arcta
n(b*tan(f*x+e)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 71, normalized size = 0.93

$$-\frac{\frac{3\left(ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,(a-b)\tan(fx+e)^2+a}{a^2\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*(
a - b)*tan(f*x + e)^2 + a)/(a^2*tan(f*x + e)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(69) = 138.
time = 3.80, size = 395, normalized size = 5.20

$$\left[-\frac{4\,(2\,a-3\,b)\cos(fx+e)^3+3\left((a-b)\cos(fx+e)^2-a+b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(fx+e)^4-2\left(3\,ab+b^2\right)\cos(fx+e)^2-4\left(\left(a^2+ab\right)\cos(fx+e)^3-ab\cos(fx+e)\right)\sqrt{-\frac{b}{a}}\sin(fx+e)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(fx+e)^4+2\left(ab-b^2\right)\cos(fx+e)^2+b^2}\right)\sin(fx+e)-12\,(a-b)\cos(fx+e)}{12\left(a^2f\cos(fx+e)^2-a^2f\right)\sin(fx+e)},\ -\frac{2\,(2\,a-3\,b)\cos(fx+e)^3-3\left((a-b)\cos(fx+e)^2-a+b\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)-6\,(a-b)\cos(fx+e)}{6\left(a^2f\cos(fx+e)^2-a^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a - 3*b)*cos(f*x + e)^3 + 3*((a - b)*cos(f*x + e)^2 - a + b)*s
qrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x
 + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(
```

```
f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x
 + e)^2 + b^2))*sin(f*x + e) - 12*(a - b)*cos(f*x + e))/((a^2*f*cos(f*x + e
)^2 - a^2*f)*sin(f*x + e)), -1/6*(2*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b)
*cos(f*x + e)^2 - a + b)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*
sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a - b)*cos(f*x +
 e))/((a^2*f*cos(f*x + e)^2 - a^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2), x)
```

Giac [A]
time = 0.68, size = 97, normalized size = 1.28

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(ab-b^2)}{\sqrt{ab}\,a^2}+\frac{3\,a\tan(fx+e)^2-3\,b\tan(fx+e)^2+a}{a^2\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*(a*b - b^2)/(sqrt(a*b)*a^2) + (3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^
2 + a)/(a^2*tan(f*x + e)^3))/f
```

Mupad [B]
time = 11.05, size = 67, normalized size = 0.88

$$-\frac{\frac{1}{3\,a}+\frac{\tan(e+f\,x)^2\,(a-b)}{a^2}}{f\tan\left(e+f\,x\right)^3}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a}}\right)\,(a-b)}{a^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] - (1/(3*a) + (tan(e + f*x)^2*(a - b))/a^2)/(f*tan(e + f*x)^3) - (b^(1/2)*at
an((b^(1/2)*tan(e + f*x))/a^(1/2))*(a - b))/(a^(5/2)*f)
```

3.67 $\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=105

$$-\frac{(a-b)^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f}-\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{(2a-b)\cot^3(e+fx)}{3a^2f}-\frac{\cot^5(e+fx)}{5af}$$

```
[Out] -(a-b)^2*cot(f*x+e)/a^3/f-1/3*(2*a-b)*cot(f*x+e)^3/a^2/f-1/5*cot(f*x+e)^5/a
/f-(a-b)^2*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(7/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3744, 472, 211}

$$-\frac{\sqrt{b}\,(a-b)^2\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f}-\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{(2a-b)\cot^3(e+fx)}{3a^2f}-\frac{\cot^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*f)) -
 ((a - b)^2*Cot[e + f*x])/(a^3*f) - ((2*a - b)*Cot[e + f*x]^3)/(3*a^2*f) -
Cot[e + f*x]^5/(5*a*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6}+\frac{2a-b}{a^2x^4}+\frac{(a-b)^2}{a^3x^2}-\frac{(a-b)^2b}{a^3(a+bx^2)}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{(2a-b)\cot^3(e+fx)}{3a^2f}-\frac{\cot^5(e+fx)}{5af}-\frac{((a-b)^2b)\,\text{Sub}}{}$$
$$= -\frac{(a-b)^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f}-\frac{(a-b)^2\cot(e+fx)}{a^3f}-\frac{(2a-b)\cot^3(e+}{3a^2f}$$

Mathematica [A]
time = 0.55, size = 103, normalized size = 0.98

$$\frac{-15(a-b)^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)-\sqrt{a}\cot(e+fx)\left(8a^2-25ab+15b^2+a(4a-5b)\csc^2(e+fx)+3a^2\csc^4(e+fx)\right)}{15a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
[Out] (-15*(a - b)^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*Cot
[e + f*x]*(8*a^2 - 25*a*b + 15*b^2 + a*(4*a - 5*b)*Csc[e + f*x]^2 + 3*a^2*C
sc[e + f*x]^4))/(15*a^(7/2)*f)
```

Maple [A]
time = 0.30, size = 99, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{5a\tan(fx+e)^5}-\frac{2a-b}{3a^2\tan(fx+e)^3}-\frac{a^2-2ab+b^2}{a^3\tan(fx+e)}-\frac{b\left(a^2-2ab+b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}}{f}$
default	$\frac{-\frac{1}{5a\tan(fx+e)^5}-\frac{2a-b}{3a^2\tan(fx+e)^3}-\frac{a^2-2ab+b^2}{a^3\tan(fx+e)}-\frac{b\left(a^2-2ab+b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}}{f}$
risch	$\frac{2i\left(15ab\,e^{8i(fx+e)}-15b^2e^{8i(fx+e)}-90ab\,e^{6i(fx+e)}+60b^2e^{6i(fx+e)}-80a^2e^{4i(fx+e)}+160ab\,e^{4i(fx+e)}-90b^2e^{4i(fx+e)}+40a^2\right.}{15f\,a^3\left(e^{2i(fx+e)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5/a/tan(f*x+e)^5-1/3*(2*a-b)/a^2/tan(f*x+e)^3-(a^2-2*a*b+b^2)/a^3/t
an(f*x+e)-b*(a^2-2*a*b+b^2)/a^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)
))
```

Maxima [A]
time = 0.50, size = 108, normalized size = 1.03

$$-\frac{\frac{15\,(a^2b-2\,ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{15\,(a^2-2\,ab+b^2)\tan(fx+e)^4+5\,(2\,a^2-ab)\tan(fx+e)^2+3\,a^2}{a^3\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^2*b - 2*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*
b)*a^3) + (15*(a^2 - 2*a*b + b^2)*tan(f*x + e)^4 + 5*(2*a^2 - a*b)*tan(f*x
+ e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(97) = 194.
time = 4.45, size = 571, normalized size = 5.44

$$\left[-\frac{4\,(8\,a^2-25\,ab+15\,b^2)\cos(fx+e)^5-20\,(4\,a^2-11\,ab+6\,b^2)\cos(fx+e)^3-15\,\left((a^2-2\,ab+b^2)\cos(fx+e)^4-2\,(a^2-2\,ab+b^2)\cos(fx+e)^2+a^2-2\,ab+b^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(fx+e)^4-2\,(3\,ab+b^2)\cos(fx+e)^2+4\,\left((a^2+ab)\cos(fx+e)^3-ab\cos(fx+e)\right)\sqrt{-\frac{b}{a}}\sin(fx+e)+b^2}{(a^2-2\,ab+b^2)\cos(fx+e)^4+2\,(ab-b^2)\cos(fx+e)^2+b^2}\right)\sin(fx+e)+60\,(a^2-2\,ab+b^2)\cos(fx+e)}{60\,(a^3f\cos(fx+e)^4-2\,a^3f\cos(fx+e)^2+a^3f)\sin(fx+e)},\ -\frac{2\,(8\,a^2-25\,ab+15\,b^2)\cos(fx+e)^5-10\,(4\,a^2-11\,ab+6\,b^2)\cos(fx+e)^3-15\,\left((a^2-2\,ab+b^2)\cos(fx+e)^4-2\,(a^2-2\,ab+b^2)\cos(fx+e)^2+a^2-2\,ab+b^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)+30\,(a^2-2\,ab+b^2)\cos(fx+e)}{30\,(a^3f\cos(fx+e)^4-2\,a^3f\cos(fx+e)^2+a^3f)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 11*a*b + 6
*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*
a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*b + b^2)*sqrt(-b/a)*log(((a^2 + 6*a*b
 + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*co
s(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a
*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e
) + 60*(a^2 - 2*a*b + b^2)*cos(f*x + e))/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*c
os(f*x + e)^2 + a^3*f)*sin(f*x + e)), -1/30*(2*(8*a^2 - 25*a*b + 15*b^2)*co
s(f*x + e)^5 - 10*(4*a^2 - 11*a*b + 6*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*
b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*
b + b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos
(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^2 - 2*a*b + b^2)*cos(f*x + e)
)/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)*sin(f*x + e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.72, size = 151, normalized size = 1.44

$$-\frac{\frac{15\left(a^2b-2\,ab^2+b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,a^3}+\frac{15\,a^2\tan(fx+e)^4-30\,ab\tan(fx+e)^4+15\,b^2\tan(fx+e)^4+10\,a^2\tan(fx+e)^2-5\,ab\tan(fx+e)^2+3\,a^2}{a^3\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^2*b - 2*a*b^2 + b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + ar
ctan(b*tan(f*x + e)/sqrt(a*b)))/(sqrt(a*b)*a^3) + (15*a^2*tan(f*x + e)^4 -
30*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 - 5*a
*b*tan(f*x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

Mupad [B]
time = 11.39, size = 115, normalized size = 1.10

$$-\frac{\frac{1}{5\,a}+\frac{\tan(e+f\,x)^2\,(2\,a-b)}{3\,a^2}+\frac{\tan(e+f\,x)^4\,(a^2-2\,a\,b+b^2)}{a^3}}{f\tan\left(e+f\,x\right)^5}-\frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(a-b)^2}{\sqrt{a}\,(a^2-2\,a\,b+b^2)}\right)\,(a-b)^2}{a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)),x)
```

```
[Out] - (1/(5*a) + (tan(e + f*x)^2*(2*a - b))/(3*a^2) + (tan(e + f*x)^4*(a^2 - 2*
a*b + b^2))/a^3)/(f*tan(e + f*x)^5) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(
a - b)^2)/(a^(1/2)*(a^2 - 2*a*b + b^2)))*(a - b)^2)/(a^(7/2)*f)
```

3.68 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{a\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{9/2}f}-\frac{(5a^2+10ab-b^2)\cos(e+fx)}{5(a-b)^4f}+\frac{(10a-3b)\cos^3(e+fx)}{15(a-b)^3f}-\frac{}{5(a-b}$$

```
[Out] -1/5*(5*a^2+10*a*b-b^2)*cos(f*x+e)/(a-b)^4/f+1/15*(10*a-3*b)*cos(f*x+e)^3/(
a-b)^3/f-1/5*cos(f*x+e)^5/(a-b)/f/(a-b+b*sec(f*x+e)^2)-1/10*b*(5*a^2+2*b^2)
*sec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)-1/2*a*(3*a+4*b)*arctan(sec(f*x+e
)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(9/2)/f
```

Rubi [A]
time = 0.22, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 473, 467, 1275, 211}

$$-\frac{(5a^2+10ab-b^2)\cos(e+fx)}{5f(a-b)^4}-\frac{b(5a^2+2b^2)\sec(e+fx)}{10f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}-\frac{a\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{9/2}}+\frac{(10a-3b)\cos^3(e+fx)}{15f(a-b)^3}-\frac{\cos^5(e+fx)}{5f(a-b)\left(a+b\sec^2(e+fx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a
 - b)^(9/2)*f) - ((5*a^2 + 10*a*b - b^2)*Cos[e + f*x])/(5*(a - b)^4*f) + ((
10*a - 3*b)*Cos[e + f*x]^3)/(15*(a - b)^3*f) - Cos[e + f*x]^5/(5*(a - b)*f*
(a - b + b*Sec[e + f*x]^2)) - (b*(5*a^2 + 2*b^2)*Sec[e + f*x])/(10*(a - b)^
4*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{-10a+3b+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^2}\,dx, x, \sec(e+\right.}{5(a-b)f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10(a-b)^4f\left(a-b+b\sec^2(e+fx)\right)} -$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{10(a-b)^4f\left(a-b+b\sec^2(e+fx)\right)} -$$

$$= -\frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5(a-b)^4f} + \frac{(10a-3b)\cos^3(e+fx)}{15(a-b)^3f} - \frac{\cos^5(}{5(a-b)f\left(a-\right.}$$

$$= -\frac{a\sqrt{b}\,(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{9/2}f} - \frac{\left(5a^2+10ab-b^2\right)\cos(e+fx)}{5(a-b)^4f} + \frac{(10}{}$$

Mathematica [A]
time = 2.51, size = 215, normalized size = 1.05

$$\frac{\frac{120a\sqrt{b}(3a+4b)\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{9/2}}+\frac{120a\sqrt{b}(3a+4b)\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{9/2}}+\frac{-30\cos(e+fx)\left(18ab+b^2+a^2\left(5+\frac{8b}{a+b+(a-b)\cos(2(e+fx))}\right)\right)+(a-b)(5(5a+3b)\cos(3(e+fx))+3(-a+b)\cos(5(e+fx)))}{(a-b)^4}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((120*a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])
/Sqrt[b]])/(a - b)^(9/2) + (120*a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] +
 Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(9/2) + (-30*Cos[e + f*x]*(18*
a*b + b^2 + a^2*(5 + (8*b)/(a + b + (a - b)*Cos[2*(e + f*x)]))) + (a - b)*(
5*(5*a + 3*b)*Cos[3*(e + f*x)] + 3*(-a + b)*Cos[5*(e + f*x)]))/(a - b)^4)/(
240*f)
```

Maple [A]
time = 0.48, size = 197, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{\frac{a^2\left(\cos^5(fx+e)\right)}{5}-\frac{2ab\left(\cos^5(fx+e)\right)}{5}+\frac{b^2\left(\cos^5(fx+e)\right)}{5}-\frac{2a^2\left(\cos^3(fx+e)\right)}{3}+\frac{2ab\left(\cos^3(fx+e)\right)}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)}{\left(a^2-2ab+b^2\right)(a-b)^2}+}{f}$
default	$\frac{-\frac{\frac{a^2\left(\cos^5(fx+e)\right)}{5}-\frac{2ab\left(\cos^5(fx+e)\right)}{5}+\frac{b^2\left(\cos^5(fx+e)\right)}{5}-\frac{2a^2\left(\cos^3(fx+e)\right)}{3}+\frac{2ab\left(\cos^3(fx+e)\right)}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)}{\left(a^2-2ab+b^2\right)(a-b)^2}+}{f}$
risch	$-\frac{5\,e^{3i(fx+e)}a}{96(-a+b)^3f}-\frac{e^{3i(fx+e)}b}{32(-a+b)^3f}-\frac{5\,e^{i(fx+e)}a^2}{16f(a^2-2ab+b^2)(a-b)^2}-\frac{9\,e^{i(fx+e)}ab}{8f(a^2-2ab+b^2)(a-b)^2}-\frac{e^{i(fx+e)}b^2}{16f(a^2-2ab+b^2)(a-b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a^2-2*a*b+b^2)/(a-b)^2*(1/5*a^2*cos(f*x+e)^5-2/5*a*b*cos(f*x+e)^5+
1/5*b^2*cos(f*x+e)^5-2/3*a^2*cos(f*x+e)^3+2/3*a*b*cos(f*x+e)^3+a^2*cos(f*x+
e)+2*a*b*cos(f*x+e))+a*b/(a-b)^4*(-1/2*a*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)+1/2*(3*a+4*b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^
(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 5.25, size = 609, normalized size = 2.99

$$\left[-\frac{12\,(a^3-3\,a^2b+3\,ab^2-b^3)\cos(fx+e)^7-4\,(10\,a^3-23\,a^2b+16\,ab^2-3\,b^3)\cos(fx+e)^5+20\,(3\,a^3+a^2b-4\,ab^2)\cos(fx+e)^3-15\,(3\,a^2b+4\,ab^2+(3\,a^3+a^2b-4\,ab^2)\cos(fx+e)^2)\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos(fx+e)^2+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)+30\,(3\,a^2b+4\,ab^2)\cos(fx+e)}{60\,((a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)f\cos(fx+e)^2+(a^4b-4\,a^3b^2+6\,a^2b^3-4\,ab^4+b^5)f)},-\frac{6\,(a^3-3\,a^2b+3\,ab^2-b^3)\cos(fx+e)^7-2\,(10\,a^3-23\,a^2b+16\,ab^2-3\,b^3)\cos(fx+e)^5+10\,(3\,a^3+a^2b-4\,ab^2)\cos(fx+e)^3+15\,(3\,a^2b+4\,ab^2+(3\,a^3+a^2b-4\,ab^2)\cos(fx+e)^2)\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)+15\,(3\,a^2b+4\,ab^2)\cos(fx+e)}{30\,((a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)f\cos(fx+e)^2+(a^4b-4\,a^3b^2+6\,a^2b^3-4\,ab^4+b^5)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 4*(10*a^3 - 23*
a^2*b + 16*a*b^2 - 3*b^3)*cos(f*x + e)^5 + 20*(3*a^3 + a^2*b - 4*a*b^2)*cos
(f*x + e)^3 - 15*(3*a^2*b + 4*a*b^2 + (3*a^3 + a^2*b - 4*a*b^2)*cos(f*x + e
)^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a -
b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*(3*a^2*b + 4*a*b^2
)*cos(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f), -1/3
0*(6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 2*(10*a^3 - 23*a^2*b
+ 16*a*b^2 - 3*b^3)*cos(f*x + e)^5 + 10*(3*a^3 + a^2*b - 4*a*b^2)*cos(f*x +
 e)^3 + 15*(3*a^2*b + 4*a*b^2 + (3*a^3 + a^2*b - 4*a*b^2)*cos(f*x + e)^2)*s
qrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*(3*a^2*
b + 4*a*b^2)*cos(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*
b^4 - b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^
5)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(193) = 386.
time = 0.85, size = 561, normalized size = 2.75

$$-\frac{\frac{15\,(3\,a^{2}b+4\,ab^{2})\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^{2}}\cos(fx+e)+\sqrt{ab-b^{2}}}\right)}{(a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4})\sqrt{ab-b^{2}}}+\frac{30\left(a^{2}b+\frac{a^{2}b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,ab^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{(a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4})\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}\right)}-\frac{4\left(8\,a^{2}+34\,ab+3\,b^{2}-\frac{40\,a^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{140\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,a^{2}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{160\,ab(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{30\,b^{2}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{180\,ab(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{30\,ab(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{15\,b^{2}(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}\right)}{(a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4})\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^{5}}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*a^2*b + 4*a*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)
/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*sqrt(a*b - b^2)) + 30*(a^2*b + a^2*b*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 2*a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/((a^4
 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2)) - 4*(8*a^2 + 34*a*b + 3*b^2 - 40*a^2*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 140*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 160*a*b*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 + 30*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 - 180*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*a*b*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*b^2*(cos(f*x + e) - 1)^4/(cos(f*x +
e) + 1)^4)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*((cos(f*x + e) - 1)
/(cos(f*x + e) + 1) - 1)^5))/f
```

Mupad [B]
time = 15.49, size = 1049, normalized size = 5.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] - ((6*a*b^2 + 83*a^2*b + 16*a^3)/(15*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3
)) + (tan(e/2 + (f*x)/2)^8*(366*a*b^2 - 83*a^2*b + 32*a^3))/(3*(a - b)*(3*a
*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^4*(1336*a*b^2 + 223*a^2*
b + 16*a^3))/(15*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (2*tan(e/2 + (f
*x)/2)^10*(11*a*b^2 + 6*a^2*b + 4*b^3))/((a - b)*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3)) + (4*tan(e/2 + (f*x)/2)^6*(73*a*b^2 + 32*a^2*b - 12*a^3 + 12*b^3))/(
3*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (2*tan(e/2 + (f*x)/2)^2*(145*a
*b^2 + 134*a^2*b + 24*a^3 + 12*b^3))/(15*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3)) + (a*tan(e/2 + (f*x)/2)^12*(3*a*b + 4*b^2))/((a - b)*(3*a*b^2 - 3*a^
2*b + a^3 - b^3)))/(f*(a + tan(e/2 + (f*x)/2)^4*(a + 20*b) + tan(e/2 + (f*x
)/2)^10*(a + 20*b) + tan(e/2 + (f*x)/2)^2*(3*a + 4*b) + tan(e/2 + (f*x)/2)^
12*(3*a + 4*b) - tan(e/2 + (f*x)/2)^6*(5*a - 40*b) - tan(e/2 + (f*x)/2)^8*(
5*a - 40*b) + a*tan(e/2 + (f*x)/2)^14)) - (a*b^(1/2)*atan(((a - b)^9*(tan(e
```

```
/2 + (f*x)/2)^2*((b^(1/2)*(3*a + 4*b)*(24*a^12*b + 32*a^3*b^10 - 232*a^4*b^
9 + 704*a^5*b^8 - 1120*a^6*b^7 + 896*a^7*b^6 - 112*a^8*b^5 - 448*a^9*b^4 +
416*a^10*b^3 - 160*a^11*b^2))/(4*(a - b)^(17/2)) - (a*b^(1/2)*(a - 2*b)*(3*
a + 4*b)^2*(224*a^14*b - 16*a^15 + 32*a^2*b^13 - 400*a^3*b^12 + 2304*a^4*b^
11 - 8096*a^5*b^10 + 19360*a^6*b^9 - 33264*a^7*b^8 + 42240*a^8*b^7 - 40128*
a^9*b^6 + 28512*a^10*b^5 - 14960*a^11*b^4 + 5632*a^12*b^3 - 1440*a^13*b^2))
/(32*(a - b)^(27/2))) - (a*b^(1/2)*(a - 2*b)*(3*a + 4*b)^2*(16*a^15 - 192*a
^14*b + 16*a^3*b^12 - 192*a^4*b^11 + 1056*a^5*b^10 - 3520*a^6*b^9 + 7920*a^
7*b^8 - 12672*a^8*b^7 + 14784*a^9*b^6 - 12672*a^10*b^5 + 7920*a^11*b^4 - 35
20*a^12*b^3 + 1056*a^13*b^2))/(32*(a - b)^(27/2))))/(9*a^14*b + 16*a^4*b^11
 - 104*a^5*b^10 + 265*a^6*b^9 - 296*a^7*b^8 + 28*a^8*b^7 + 280*a^9*b^6 - 26
6*a^10*b^5 + 40*a^11*b^4 + 76*a^12*b^3 - 48*a^13*b^2))*(3*a + 4*b))/(2*f*(a
 - b)^(9/2))
```

3.69 $\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal. Leaf size=133

$$-\frac{\sqrt{b}\,(3a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f}-\frac{(a+b)\cos(e+fx)}{(a-b)^3f}+\frac{\cos^3(e+fx)}{3(a-b)^2f}-\frac{ab\sec(e+fx)}{2(a-b)^3f\,(a-b+b\sec^2(e+$$

```
[Out] -(a+b)*cos(f*x+e)/(a-b)^3/f+1/3*cos(f*x+e)^3/(a-b)^2/f-1/2*a*b*sec(f*x+e)/(
a-b)^3/f/(a-b+b*sec(f*x+e)^2)-1/2*(3*a+2*b)*arctan(sec(f*x+e)*b^(1/2)/(a-b)
^(1/2))*b^(1/2)/(a-b)^(7/2)/f
```

Rubi [A]
time = 0.13, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3745, 467, 1275, 211}

$$-\frac{\sqrt{b}\,(3a+2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}+\frac{\cos^3(e+fx)}{3f(a-b)^2}-\frac{(a+b)\cos(e+fx)}{f(a-b)^3}-\frac{ab\sec(e+fx)}{2f(a-b)^3\,(a+b\sec^2(e+fx)-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a -
 b)^(7/2)*f) - ((a + b)*Cos[e + f*x])/((a - b)^3*f) + Cos[e + f*x]^3/(3*(a
- b)^2*f) - (a*b*Sec[e + f*x])/(2*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a-b+bx^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{ab\sec(e+fx)}{2(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int \frac{\frac{2}{(a-b)b}-\frac{2ax^2}{(a-b)^2b}+\frac{ax^4}{(a-b)^3}}{x^4(a-b+bx^2)}\,dx, x, \text{se}\right.}{2f}$$

$$= -\frac{ab\sec(e+fx)}{2(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int\left(\frac{2}{(a-b)^2bx^4}+\frac{2(a+b)}{b(-a+b)^3x^2}+\frac{}{(a-}\right.\right.}{2f}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^3 f} + \frac{\cos^3(e+fx)}{3(a-b)^2 f} - \frac{ab\sec(e+fx)}{2(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)} -$$

$$= -\frac{\sqrt{b}\,(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f} - \frac{(a+b)\cos(e+fx)}{(a-b)^3 f} + \frac{\cos^3(e+fx)}{3(a-b)^2 f} -$$

Mathematica [A]
time = 2.18, size = 182, normalized size = 1.37

$$\frac{\frac{6\sqrt{b}\,(3a+2b)\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}+\frac{6\sqrt{b}\,(3a+2b)\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}-\frac{\cos(e+fx)\left(9a+15b+\frac{12ab}{a+b+(a-b)\cos(2(e+fx))}\right)+(-a+b)\cos(3(e+fx))}{(a-b)^3}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((6*Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqr
t[b]])/(a - b)^(7/2) + (6*Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[a - b] + Sqrt[a]
*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(7/2) - (Cos[e + f*x]*(9*a + 15*b + (1
```

```
2*a*b)/(a + b + (a - b)*Cos[2*(e + f*x)])) + (-a + b)*Cos[3*(e + f*x)])/(a
- b)^3)/(12*f)
```

Maple [A]
time = 0.40, size = 152, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\cos^3(fx+e)\right)}{3}-\frac{b\left(\cos^3(fx+e)\right)}{3}-\cos(fx+e)a-b\cos(fx+e)}{\left(a^2-2ab+b^2\right)(a-b)}+\frac{b\left(-\frac{a\cos(fx+e)}{2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}+\frac{(3a+2b)\arctan\left(\frac{(a-b}{\sqrt{b}}\right.}{2\sqrt{b(a-}}\right.}{(a-b)^3}}{f}$
default	$\frac{\frac{\frac{a\left(\cos^3(fx+e)\right)}{3}-\frac{b\left(\cos^3(fx+e)\right)}{3}-\cos(fx+e)a-b\cos(fx+e)}{\left(a^2-2ab+b^2\right)(a-b)}+\frac{b\left(-\frac{a\cos(fx+e)}{2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}+\frac{(3a+2b)\arctan\left(\frac{(a-b}{\sqrt{b}}\right.}{2\sqrt{b(a-}}\right.}{(a-b)^3}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24(a^2-2ab+b^2)f}-\frac{3\,e^{i(fx+e)}a}{8f(a^2-2ab+b^2)(a-b)}-\frac{5\,e^{i(fx+e)}b}{8f(a^2-2ab+b^2)(a-b)}-\frac{3\,e^{-i(fx+e)}a}{8(a^3-3a^2b+3a\,b^2-b^3)f}-\frac{5\,e^{-i(fx+}}{8(a^3-3a^2b+3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2-2*a*b+b^2)/(a-b)*(1/3*a*cos(f*x+e)^3-1/3*b*cos(f*x+e)^3-cos(f*x
+e)*a-b*cos(f*x+e))+b/(a-b)^3*(-1/2*a*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)+1/2*(3*a+2*b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/
2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 6.69, size = 470, normalized size = 3.53

$$\left[\frac{4\,(a^2-2\,ab+b^2)\cos\left(fx+e\right)^5-4\,(3\,a^2-ab-2\,b^2)\cos\left(fx+e\right)^3-3\,\left((3\,a^2-ab-2\,b^2)\cos\left(fx+e\right)^2+3\,ab+2\,b^2\right)\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos(fx+e)^2-2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)-6\,(3\,ab+2\,b^2)\cos\left(fx+e\right)}{12\,\left((a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)f\cos\left(fx+e\right)^2+(a^3b-3\,a^2b^2+3\,ab^3-b^4)f\right)},\ \frac{2\,(a^2-2\,ab+b^2)\cos\left(fx+e\right)^5-2\,(3\,a^2-ab-2\,b^2)\cos\left(fx+e\right)^3-3\,\left((3\,a^2-ab-2\,b^2)\cos\left(fx+e\right)^2+3\,ab+2\,b^2\right)\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)-3\,(3\,ab+2\,b^2)\cos\left(fx+e\right)}{6\,\left((a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)f\cos\left(fx+e\right)^2+(a^3b-3\,a^2b^2+3\,ab^3-b^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 4*(3*a^2 - a*b - 2*b^2)*cos(f
*x + e)^3 - 3*((3*a^2 - a*b - 2*b^2)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*
x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 6*(3*a*b + 2*b^2)*cos(f*x + e))
/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3
*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/6*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 2
*(3*a^2 - a*b - 2*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - a*b - 2*b^2)*cos(f*x +
e)^2 + 3*a*b + 2*b^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f
*x + e)/b) - 3*(3*a*b + 2*b^2)*cos(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(124) = 248.
time = 0.85, size = 368, normalized size = 2.77

$$\frac{a^4f^{11}\cos\left(fx+e\right)^3-4\,a^3bf^{11}\cos\left(fx+e\right)^3+6\,a^2b^2f^{11}\cos\left(fx+e\right)^3-4\,ab^3f^{11}\cos\left(fx+e\right)^3+b^4f^{11}\cos\left(fx+e\right)^3-3\,a^4f^{11}\cos\left(fx+e\right)+6\,a^3bf^{11}\cos\left(fx+e\right)-6\,ab^3f^{11}\cos\left(fx+e\right)+3\,b^4f^{11}\cos\left(fx+e\right)}{3\,(a^6f^{12}-6\,a^5bf^{12}+15\,a^4b^2f^{12}-20\,a^3b^3f^{12}+15\,a^2b^4f^{12}-6\,ab^5f^{12}+b^6f^{12})}-\frac{ab\cos\left(fx+e\right)}{2\,(a^3-3\,a^2b+3\,ab^2-b^3)\left(a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b\right)f}+\frac{(3\,ab+2\,b^2)\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{2\,(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab-b^2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^4*f^11*cos(f*x + e)^3 - 4*a^3*b*f^11*cos(f*x + e)^3 + 6*a^2*b^2*f^11
*cos(f*x + e)^3 - 4*a*b^3*f^11*cos(f*x + e)^3 + b^4*f^11*cos(f*x + e)^3 - 3
*a^4*f^11*cos(f*x + e) + 6*a^3*b*f^11*cos(f*x + e) - 6*a*b^3*f^11*cos(f*x +
 e) + 3*b^4*f^11*cos(f*x + e))/(a^6*f^12 - 6*a^5*b*f^12 + 15*a^4*b^2*f^12 -
 20*a^3*b^3*f^12 + 15*a^2*b^4*f^12 - 6*a*b^5*f^12 + b^6*f^12) - 1/2*a*b*cos
(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a*cos(f*x + e)^2 - b*cos(f*x +
```

```
e)^2 + b)*f) + 1/2*(3*a*b + 2*b^2)*arctan((a*cos(f*x + e) - b*cos(f*x + e))
/sqrt(a*b - b^2))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b - b^2)*f)
```

Mupad [B]
time = 14.72, size = 737, normalized size = 5.54

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{\sqrt{b}\,(3\,a+2\,b)\,\left(24\,a^9\,b-128\,a^8\,b^2+264\,a^7\,b^3-240\,a^6\,b^4+40\,a^5\,b^5+96\,a^4\,b^6-72\,a^3\,b^7+16\,a^2\,b^8\right)}{4\,a\,(a-b)^{13/2}}+\frac{\sqrt{b}\,(a-2\,b)\,(3\,a+2\,b)^2\,\left(16\,a^{12}-176\,a^{11}\,b+864\,a^{10}\,b^2-2496\,a^9\,b^3+4704\,a^8\,b^4-6048\,a^7\,b^5+5376\,a^6\,b^6-3264\,a^5\,b^7+1296\,a^4\,b^8-304\,a^3\,b^9+32\,a^2\,b^{10}\right)}{32\,a\,(a-b)^{21/2}}\right)+\frac{\sqrt{b}\,(a-2\,b)\,(3\,a+2\,b)^2\,\left(-16\,a^{12}+144\,a^{11}\,b-576\,a^{10}\,b^2+1344\,a^9\,b^3-2016\,a^8\,b^4+2016\,a^7\,b^5-1344\,a^6\,b^6+576\,a^5\,b^7-144\,a^4\,b^8+16\,a^3\,b^9\right)}{32\,a\,(a-b)^{21/2}}\right)\,(a-b)^7}{-9\,a^{10}\,b+42\,a^9\,b^2-67\,a^8\,b^3+24\,a^7\,b^4+45\,a^6\,b^5-46\,a^5\,b^6+3\,a^4\,b^7+12\,a^3\,b^8-4\,a^2\,b^9}\right)\,(3\,a+2\,b)}{2\,f\,(a-b)^{7/2}}-\frac{\frac{4\,a^2+11\,b\,a}{3\,(a-b)\,(a^2-2\,a\,b+b^2)}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,(2\,b^2+3\,a\,b)}{(a-b)\,(a^2-2\,a\,b+b^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(2\,a^2-3\,a\,b+11\,b^2)}{(a-b)\,(a^2-2\,a\,b+b^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(2\,a^2+9\,a\,b+19\,b^2)}{3\,(a-b)\,(a^2-2\,a\,b+b^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(-10\,a^2+22\,a\,b+33\,b^2)}{3\,(a-b)\,(a^2-2\,a\,b+b^2)}}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+(a+4\,b)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+(12\,b-2\,a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+(12\,b-2\,a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+(a+4\,b)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (b^(1/2)*atan(((tan(e/2 + (f*x)/2)^2*((b^(1/2)*(3*a + 2*b)*(24*a^9*b + 16*a
^2*b^8 - 72*a^3*b^7 + 96*a^4*b^6 + 40*a^5*b^5 - 240*a^6*b^4 + 264*a^7*b^3 -
 128*a^8*b^2))/(4*a*(a - b)^(13/2)) + (b^(1/2)*(a - 2*b)*(3*a + 2*b)^2*(16*
a^12 - 176*a^11*b + 32*a^2*b^10 - 304*a^3*b^9 + 1296*a^4*b^8 - 3264*a^5*b^7
 + 5376*a^6*b^6 - 6048*a^7*b^5 + 4704*a^8*b^4 - 2496*a^9*b^3 + 864*a^10*b^2
))/(32*a*(a - b)^(21/2))) + (b^(1/2)*(a - 2*b)*(3*a + 2*b)^2*(144*a^11*b -
16*a^12 + 16*a^3*b^9 - 144*a^4*b^8 + 576*a^5*b^7 - 1344*a^6*b^6 + 2016*a^7*
b^5 - 2016*a^8*b^4 + 1344*a^9*b^3 - 576*a^10*b^2))/(32*a*(a - b)^(21/2)))*(
a - b)^7)/(12*a^3*b^8 - 4*a^2*b^9 - 9*a^10*b + 3*a^4*b^7 - 46*a^5*b^6 + 45*
a^6*b^5 + 24*a^7*b^4 - 67*a^8*b^3 + 42*a^9*b^2))*(3*a + 2*b))/(2*f*(a - b)^
(7/2)) - ((11*a*b + 4*a^2)/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (tan(e/2 + (f*
x)/2)^8*(3*a*b + 2*b^2))/((a - b)*(a^2 - 2*a*b + b^2)) + (2*tan(e/2 + (f*x)
/2)^6*(2*a^2 - 3*a*b + 11*b^2))/((a - b)*(a^2 - 2*a*b + b^2)) + (2*tan(e/2
+ (f*x)/2)^2*(9*a*b + 2*a^2 + 19*b^2))/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (2
*tan(e/2 + (f*x)/2)^4*(22*a*b - 10*a^2 + 33*b^2))/(3*(a - b)*(a^2 - 2*a*b +
 b^2)))/(f*(a + tan(e/2 + (f*x)/2)^2*(a + 4*b) + tan(e/2 + (f*x)/2)^8*(a +
4*b) - tan(e/2 + (f*x)/2)^4*(2*a - 12*b) - tan(e/2 + (f*x)/2)^6*(2*a - 12*b
) + a*tan(e/2 + (f*x)/2)^10))
```

3.70 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f}-\frac{3\cos(e+fx)}{2(a-b)^2f}+\frac{\cos(e+fx)}{2(a-b)f\,(a-b+b\sec^2(e+fx))}$$

```
[Out] -3/2*cos(f*x+e)/(a-b)^2/f+1/2*cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)-3/2*a
rctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(5/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3745, 296, 331, 211}

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}-\frac{3\cos(e+fx)}{2f(a-b)^2}+\frac{\cos(e+fx)}{2f(a-b)\,(a+b\sec^2(e+fx)-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(2*(a - b)^(5/2)*f)
 - (3*Cos[e + f*x])/(2*(a - b)^2*f) + Cos[e + f*x]/(2*(a - b)*f*(a - b + b*
Sec[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)^2}\,dx,x,\sec(e+fx)\right)}{f}$$
$$= \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{3\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)}\,dx,x,\sec(e+fx)\right.}{2(a-b)f}$$
$$= -\frac{3\cos(e+fx)}{2(a-b)^2f} + \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e+fx)\right)} - \frac{(3b)\text{Subst}\left(\int \frac{1}{a-b+bx^2}\right.}{2(a-}$$
$$= -\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f} - \frac{3\cos(e+fx)}{2(a-b)^2f} + \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e\right.}$$

Mathematica [A]
time = 0.54, size = 146, normalized size = 1.45

$$\frac{\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{2\cos(e+fx)\left(-1-\frac{b}{a+b+(a-b)\cos(2(e+fx))}\right)}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((3*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a -
b)^(5/2) + (3*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[
b]])/(a - b)^(5/2) + (2*Cos[e + f*x]*(-1 - b/(a + b + (a - b)*Cos[2*(e + f*
x)])))/(a - b)^2)/(2*f)
```

Maple [A]
time = 0.34, size = 103, normalized size = 1.02

method	result
derivativedivides	$\frac{-\frac{\cos(fx+e)}{a^2-2ab+b^2}+\frac{b\left(-\frac{\cos(fx+e)}{2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}+\frac{3\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{(a-b)^2}}{f}$
default	$\frac{-\frac{\cos(fx+e)}{a^2-2ab+b^2}+\frac{b\left(-\frac{\cos(fx+e)}{2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}+\frac{3\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{(a-b)^2}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2(a^2-2ab+b^2)f}-\frac{e^{-i(fx+e)}}{2(a^2-2ab+b^2)f}+\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{f(-a+b)^2\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b\right)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a^2-2*a*b+b^2)*cos(f*x+e)+b/(a-b)^2*(-1/2*cos(f*x+e)/(a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)+3/2/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^
(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]

time = 7.36, size = 319, normalized size = 3.16

$$\left[-\frac{4\,(a-b)\cos(fx+e)^3-3\,\left((a-b)\cos(fx+e)^2+b\right)\sqrt{-\frac{b}{a-b}}\,\log\left(\frac{(a-b)\cos(fx+e)^2+2\,(a-b)\sqrt{-\frac{b}{a-b}}\,\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)+6\,b\cos(fx+e)}{4\,\left((a^3-3\,a^2b+3\,ab^2-b^3)f\cos(fx+e)^2+(a^2b-2\,ab^2+b^3)f\right)},\ -\frac{2\,(a-b)\cos(fx+e)^3+3\,\left((a-b)\cos(fx+e)^2+b\right)\sqrt{\frac{b}{a-b}}\,\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\,\cos(fx+e)}{b}\right)+3\,b\cos(fx+e)}{2\,\left((a^3-3\,a^2b+3\,ab^2-b^3)f\cos(fx+e)^2+(a^2b-2\,ab^2+b^3)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a - b)*cos(f*x + e)^3 - 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(-b/(a
 - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e
) - b)/((a - b)*cos(f*x + e)^2 + b)) + 6*b*cos(f*x + e))/((a^3 - 3*a^2*b +
3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f), -1/2*(2*(a -
b)*cos(f*x + e)^3 + 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(b/(a - b))*arctan(-
(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 3*b*cos(f*x + e))/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.84, size = 153, normalized size = 1.51

$$-\frac{f^3\cos(fx+e)}{a^2f^4-2\,abf^4+b^2f^4}+\frac{3\,b\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{2\,(a^2-2\,ab+b^2)\sqrt{ab-b^2}\,f}-\frac{b\cos(fx+e)}{2\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)(a^2-2\,ab+b^2)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -f^3*cos(f*x + e)/(a^2*f^4 - 2*a*b*f^4 + b^2*f^4) + 3/2*b*arctan((a*cos(f*x
 + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^2 - 2*a*b + b^2)*sqrt(a*b - b^
2)*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*(a^2
- 2*a*b + b^2)*f)
```

Mupad [B]
time = 13.86, size = 436, normalized size = 4.32

$$-\frac{\frac{2\,a+b}{(a-b)^2}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(2\,a^2-a\,b+2\,b^2)}{a\,(a^2-2\,a\,b+b^2)}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(-2\,a^2+4\,a\,b+b^2)}{a\,(a-b)^2}}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+(4\,b-a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+(4\,b-a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\right)}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{(a-b)^5\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{\sqrt{b}\,(18\,a^6\,b-72\,a^5\,b^2+108\,a^4\,b^3-72\,a^3\,b^4+18\,a^2\,b^5)}{a\,(a-b)^{9/2}}-\frac{9\,\sqrt{b}\,(a-2\,b)\,(-16\,a^9+128\,a^8\,b-432\,a^7\,b^2+800\,a^6\,b^3-880\,a^5\,b^4+576\,a^4\,b^5-208\,a^3\,b^6+32\,a^2\,b^7)}{32\,a\,(a-b)^{15/2}}\right)-\frac{9\,\sqrt{b}\,(a-2\,b)\,(16\,a^9-96\,a^8\,b+240\,a^7\,b^2-320\,a^6\,b^3+240\,a^5\,b^4-96\,a^4\,b^5+16\,a^3\,b^6)}{32\,a\,(a-b)^{15/2}}\right)}{9\,a^6\,b-36\,a^5\,b^2+54\,a^4\,b^3-36\,a^3\,b^4+9\,a^2\,b^5}\right)}{2\,f\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] - ((2*a + b)/(a - b)^2 + (tan(e/2 + (f*x)/2)^4*(2*a^2 - a*b + 2*b^2))/(a*(a
^2 - 2*a*b + b^2)) + (2*tan(e/2 + (f*x)/2)^2*(4*a*b - 2*a^2 + b^2))/(a*(a -
```

```
b)^2))/(f*(a - tan(e/2 + (f*x)/2)^2*(a - 4*b) - tan(e/2 + (f*x)/2)^4*(a -
4*b) + a*tan(e/2 + (f*x)/2)^6)) - (3*b^(1/2)*atan(((a - b)^5*(tan(e/2 + (f*
x)/2)^2*((b^(1/2)*(18*a^6*b + 18*a^2*b^5 - 72*a^3*b^4 + 108*a^4*b^3 - 72*a^
5*b^2))/(a*(a - b)^(9/2)) - (9*b^(1/2)*(a - 2*b)*(128*a^8*b - 16*a^9 + 32*a
^2*b^7 - 208*a^3*b^6 + 576*a^4*b^5 - 880*a^5*b^4 + 800*a^6*b^3 - 432*a^7*b^
2))/(32*a*(a - b)^(15/2))) - (9*b^(1/2)*(a - 2*b)*(16*a^9 - 96*a^8*b + 16*a
^3*b^6 - 96*a^4*b^5 + 240*a^5*b^4 - 320*a^6*b^3 + 240*a^7*b^2))/(32*a*(a -
b)^(15/2))))/(9*a^6*b + 9*a^2*b^5 - 36*a^3*b^4 + 54*a^4*b^3 - 36*a^5*b^2)))
/(2*f*(a - b)^(5/2))
```

3.71 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{(3a-2b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}f}-\frac{\tanh^{-1}(\cos(e+fx))}{a^2f}-\frac{b\sec(e+fx)}{2a(a-b)f\,(a-b+b\sec^2(e+fx))}$$

```
[Out] -arctanh(cos(f*x+e))/a^2/f-1/2*b*sec(f*x+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)-
1/2*(3*a-2*b)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^2/(a-b)^(3/2
)/f
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3745, 425, 536, 213, 211}

$$-\frac{\sqrt{b}\,(3a-2b)\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2f(a-b)^{3/2}}-\frac{\tanh^{-1}(\cos(e+fx))}{a^2f}-\frac{b\sec(e+fx)}{2af(a-b)\,(a+b\sec^2(e+fx)-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^2*
(a - b)^(3/2)*f) - ArcTanh[Cos[e + f*x]]/(a^2*f) - (b*Sec[e + f*x])/(2*a*(a
 - b)*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a-b+bx^2)^2}\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{b\sec(e+fx)}{2a(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(-1+x^2)(a-b+bx^2)}\,dx, x, \sec(e+\right.}{2a(a-b)f}\\
&= -\frac{b\sec(e+fx)}{2a(a-b)f\left(a-b+b\sec^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{a^2f} -\\
&= -\frac{(3a-2b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}f} - \frac{\tanh^{-1}(\cos(e+fx))}{a^2f} - \frac{b\,s}{2a(a-b)f\,(a}
\end{aligned}
$$

Mathematica [A]
time = 0.60, size = 184, normalized size = 1.67

$$
\frac{\frac{(3a-2b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}}+\frac{(3a-2b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}}-\frac{2ab\cos(e+fx)}{(a-b)(a+b+(a-b)\cos(2(e+fx)))}-2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2a^2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[
b]])/(a - b)^(3/2) + ((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan
```

```
[(e + f*x)/2])/Sqrt[b]])/(a - b)^(3/2) - (2*a*b*Cos[e + f*x])/((a - b)*(a +
 b + (a - b)*Cos[2*(e + f*x)])) - 2*Log[Cos[(e + f*x)/2]] + 2*Log[Sin[(e +
f*x)/2]])/(2*a^2*f)
```

Maple [A]
time = 0.41, size = 128, normalized size = 1.16

method	result
derivativedivides	$\frac{-\frac{\ln(\cos(fx+e)+1)}{2a^2}+\frac{\ln(\cos(fx+e)-1)}{2a^2}+\frac{b\left(-\frac{a\cos(fx+e)}{2(a-b)\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}+\frac{(3a-2b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2(a-b)\sqrt{b(a-b)}}\right)}{a^2}}{f}$
default	$\frac{-\frac{\ln(\cos(fx+e)+1)}{2a^2}+\frac{\ln(\cos(fx+e)-1)}{2a^2}+\frac{b\left(-\frac{a\cos(fx+e)}{2(a-b)\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}+\frac{(3a-2b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2(a-b)\sqrt{b(a-b)}}\right)}{a^2}}{f}$
risch	$-\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{af(-a+b)\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b\right)}-\frac{\ln\left(e^{i(fx+e)}+1\right)}{a^2f}+\frac{\ln\left(e^{i(fx+e)}-1\right)}{a^2f}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/a^2*ln(cos(f*x+e)+1)+1/2/a^2*ln(cos(f*x+e)-1)+b/a^2*(-1/2*a/(a-b)
*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+1/2*(3*a-2*b)/(a-b)/(b*(a-b))
^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(102) = 204$.

time = 8.65, size = 488, normalized size = 4.44

$$\left[-\frac{2\,ab\cos\left(fx+e\right)-\left(\left(3\,a^{2}-5\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{2}+3\,ab-2\,b^{2}\right)\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos(fx+e)^{2}+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^{2}+b}\right)+2\left(\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}+ab-b^{2}\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-2\left(\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}+ab-b^{2}\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{4\left(\left(a^{4}-2\,a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b-a^{2}b^{2}\right)f\right)},-\frac{ab\cos\left(fx+e\right)+\left(\left(3\,a^{2}-5\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{2}+3\,ab-2\,b^{2}\right)\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)+\left(\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}+ab-b^{2}\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-\left(\left(a^{2}-2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}+ab-b^{2}\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\left(\left(a^{4}-2\,a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b-a^{2}b^{2}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*cos(f*x + e) - ((3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 + 3*a*b
 - 2*b^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/
(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*((a^2 - 2*a*b
+ b^2)*cos(f*x + e)^2 + a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) - 2*((a^2 -
2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^
4 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f), -1/2*(a*b*c
os(f*x + e) + ((3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 + 3*a*b - 2*b^2)*sqrt
(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + ((a^2 - 2*a*b
 + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) - ((a^2 - 2
*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^4
 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(102) = 204.
time = 0.70, size = 270, normalized size = 2.45

$$-\frac{\frac{\left(3\,ab-2\,b^{2}\right)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^{2}}\cos(fx+e)+\sqrt{ab-b^{2}}}\right)}{\left(a^{3}-a^{2}b\right)\sqrt{ab-b^{2}}}+\frac{2\left(ab+\frac{ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{\left(a^{3}-a^{2}b\right)\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}\right)}-\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b - 2*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a
*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^3 - a^2*b)*sqrt(a*b - b^2))
+ 2*(a*b + a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^2*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1))/((a^3 - a^2*b)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*
```

```
x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2)) - log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e)
+ 1))/a^2)/f
```

Mupad [B]
time = 13.72, size = 1140, normalized size = 10.36

$$\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2 f}-\frac{\frac{b}{a\,(a-b)}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\,(a\,b-2\,b^2)}{a^2\,(a-b)}}{f\,\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+(4\,b-2\,a)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a\right)}+\frac{\sqrt{b}\,\mathrm{atan}\left(\text{[illegible]}\right)\,(3\,a-2\,b)}{2\,a^2\,f\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] log(tan(e/2 + (f*x)/2))/(a^2*f) - (b/(a*(a - b)) - (tan(e/2 + (f*x)/2)^2*(a
*b - 2*b^2))/(a^2*(a - b)))/(f*(a - tan(e/2 + (f*x)/2)^2*(2*a - 4*b) + a*ta
n(e/2 + (f*x)/2)^4)) + (b^(1/2)*atan(((tan(e/2 + (f*x)/2)^2*((((b^(3/2)*(3*
a - 2*b)^3*(2*a^10 - 58*a^9*b + 96*a^4*b^6 - 432*a^5*b^5 + 772*a^6*b^4 - 68
6*a^7*b^3 + 306*a^8*b^2))/(8*a^6*(a - b)^(9/2)*(3*a^4*b - a^5 + a^2*b^3 - 3
*a^3*b^2)) + (2*b^(1/2)*(3*a - 2*b)*(108*a*b^5 + 9*a^5*b - 24*b^6 - 188*a^2
*b^4 + 158*a^3*b^3 - 63*a^4*b^2))/(a^2*(a - b)^(3/2)*(3*a^4*b - a^5 + a^2*b
^3 - 3*a^3*b^2)))*(768*a*b^4 - 259*a^4*b + 27*a^5 - 192*b^5 - 1164*a^2*b^3
+ 820*a^3*b^2))/(2*a^5*(a - b)^(9/2)*(36*a*b^2 - 39*a^2*b + 16*a^3 - 12*b^3
)) - (((8*(4*b^4 - 12*a*b^3 + 9*a^2*b^2))/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*
b^2) - (b*(3*a - 2*b)^2*(2*a^8 - 35*a^7*b + 96*a^2*b^6 - 432*a^3*b^5 + 746*
a^4*b^4 - 611*a^5*b^3 + 234*a^6*b^2))/(2*a^4*(a - b)^3*(3*a^4*b - a^5 + a^2
*b^3 - 3*a^3*b^2)))*(2*a^4 - 47*a^3*b - 240*a*b^3 + 96*b^4 + 186*a^2*b^2))/
(a^5*b^(1/2)*(a - b)^3*(36*a*b^2 - 39*a^2*b + 16*a^3 - 12*b^3))) + (((b^(1/
2)*(3*a - 2*b)*(12*a^5*b - 20*a^2*b^4 + 60*a^3*b^3 - 53*a^4*b^2))/(a^2*(a -
 b)^(3/2)*(a^5 - 2*a^4*b + a^3*b^2)) + (b^(3/2)*(3*a - 2*b)^3*(4*a^10 - 24*
a^9*b + 16*a^6*b^4 - 48*a^7*b^3 + 52*a^8*b^2))/(16*a^6*(a - b)^(9/2)*(a^5 -
 2*a^4*b + a^3*b^2)))*(768*a*b^4 - 259*a^4*b + 27*a^5 - 192*b^5 - 1164*a^2*
b^3 + 820*a^3*b^2))/(2*a^5*(a - b)^(9/2)*(36*a*b^2 - 39*a^2*b + 16*a^3 - 12
*b^3)) - (((4*(4*b^4 - 12*a*b^3 + 9*a^2*b^2))/(a^5 - 2*a^4*b + a^3*b^2) - (
b*(3*a - 2*b)^2*(4*a^8 - 36*a^7*b + 32*a^4*b^4 - 96*a^5*b^3 + 96*a^6*b^2))/
(4*a^4*(a - b)^3*(a^5 - 2*a^4*b + a^3*b^2)))*(2*a^4 - 47*a^3*b - 240*a*b^3
+ 96*b^4 + 186*a^2*b^2))/(a^5*b^(1/2)*(a - b)^3*(36*a*b^2 - 39*a^2*b + 16*a
^3 - 12*b^3)))*(4*a^7*(a - b)^(9/2) - 12*a^6*b*(a - b)^(9/2) - 4*a^4*b^3*(a
 - b)^(9/2) + 12*a^5*b^2*(a - b)^(9/2)))/(9*a^2*b - 12*a*b^2 + 4*b^3))*(3*a
 - 2*b))/(2*a^2*f*(a - b)^(3/2))
```

3.72 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{(3a-4b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3\sqrt{a-b}\,f}-\frac{(a-4b)\tanh^{-1}(\cos(e+fx))}{2a^3f}-\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)}-\frac{b\sec(e+fx)}{a^2f\left(a-b+b\sec^2(e+fx)\right)}$$

```
[Out] -1/2*(a-4*b)*arctanh(cos(f*x+e))/a^3/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b
*sec(f*x+e)^2)-b*sec(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)-1/2*(3*a-4*b)*arctan
(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^3/f/(a-b)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3745, 482, 541, 536, 213, 211}

$$-\frac{\sqrt{b}\,(3a-4b)\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3f\sqrt{a-b}}-\frac{(a-4b)\tanh^{-1}(\cos(e+fx))}{2a^3f}-\frac{b\sec(e+fx)}{a^2f\left(a+b\sec^2(e+fx)-b\right)}-\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a+b\sec^2(e+fx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a^3*
Sqrt[a - b]*f) - ((a - 4*b)*ArcTanh[Cos[e + f*x]])/(2*a^3*f) - (Cot[e + f*x
]*Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]^2)) - (b*Sec[e + f*x])/(a^2*
f*(a - b + b*Sec[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a-b+bx^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\,(a-b+b\sec^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{a-b-3bx^2}{(-1+x^2)(a-b+bx^2)^2}\,dx, x, \sec(e+fx)\right)}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\,(a-b+b\sec^2(e+fx))} - \frac{b\sec(e+fx)}{a^2f\,(a-b+b\sec^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2(a}{}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\,(a-b+b\sec^2(e+fx))} - \frac{b\sec(e+fx)}{a^2f\,(a-b+b\sec^2(e+fx))} + \frac{(a-4b)\text{Sub}}{}$$

$$= -\frac{(3a-4b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3\sqrt{a-b}\,f} - \frac{(a-4b)\tanh^{-1}(\cos(e+fx))}{2a^3f} - \frac{c}{2a}$$

Mathematica [A]
time = 4.13, size = 218, normalized size = 1.48

$$\frac{\frac{4(3a-4b)\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{\sqrt{a-b}}+\frac{4(3a-4b)\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{\sqrt{a-b}}-\frac{8ab\cos(e+fx)}{a+b+(a-b)\cos(2(e+fx))}-a\csc^2\left(\frac{1}{2}(e+fx)\right)-4(a-4b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+4(a-4b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((4*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqr
t[b]])/Sqrt[a - b] + (4*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*T
an[(e + f*x)/2])/Sqrt[b]])/Sqrt[a - b] - (8*a*b*Cos[e + f*x])/(a + b + (a -
 b)*Cos[2*(e + f*x)]) - a*Csc[(e + f*x)/2]^2 - 4*(a - 4*b)*Log[Cos[(e + f*x
)/2]] + 4*(a - 4*b)*Log[Sin[(e + f*x)/2]] + a*Sec[(e + f*x)/2]^2)/(8*a^3*f)
```

Maple [A]
time = 0.38, size = 156, normalized size = 1.06

method	result
derivativedivides	$\frac{\frac{1}{4a^2(\cos(fx+e)+1)}+\frac{(-a+4b)\ln(\cos(fx+e)+1)}{4a^3}+\frac{1}{4a^2(\cos(fx+e)-1)}+\frac{(a-4b)\ln(\cos(fx+e)-1)}{4a^3}+ b\left(-\frac{a\cos(fx+e)}{2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e\cdots\right.\right.}\cdots\right.}{f}$
default	$\frac{\frac{1}{4a^2(\cos(fx+e)+1)}+\frac{(-a+4b)\ln(\cos(fx+e)+1)}{4a^3}+\frac{1}{4a^2(\cos(fx+e)-1)}+\frac{(a-4b)\ln(\cos(fx+e)-1)}{4a^3}+ b\left(-\frac{a\cos(fx+e)}{2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e\cdots\right.\right.}\cdots\right.}{f}$
risch	$\frac{a\,e^{7i(fx+e)}-2b\,e^{7i(fx+e)}+3a\,e^{5i(fx+e)}+2b\,e^{5i(fx+e)}+3a\,e^{3i(fx+e)}+2b\,e^{3i(fx+e)}+a\,e^{i(fx+e)}-2b\,e^{i(fx+e)}}{f\,a^2\left(e^{2i(fx+e)}-1\right)^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)}-\frac{\ln\left(e^{i(fx+e}\cdots\right.}{2a^2f}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4/a^2/(cos(f*x+e)+1)+1/4/a^3*(-a+4*b)*ln(cos(f*x+e)+1)+1/4/a^2/(cos(
f*x+e)-1)+1/4*(a-4*b)/a^3*ln(cos(f*x+e)-1)+b/a^3*(-1/2*a*cos(f*x+e)/(a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)+1/2*(3*a-4*b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f
*x+e)/(b*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(140) = 280$.
time = 4.52, size = 700, normalized size = 4.76

$$\left[\frac{2\,(a^2-2\,ab)\cos\left(fx+e\right)^3+4\,ab\cos\left(fx+e\right)-\left((3\,a^2-7\,ab+4\,b^2)\cos\left(fx+e\right)^4-(3\,a^2-10\,ab+8\,b^2)\cos\left(fx+e\right)^2-3\,ab+4\,b^2\right)\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos\left(fx+e\right)^2-2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos\left(fx+e\right)-b}{(a-b)\cos\left(fx+e\right)^2+b}\right)-\left((a^2-5\,ab+4\,b^2)\cos\left(fx+e\right)^4-(a^2-6\,ab+8\,b^2)\cos\left(fx+e\right)^2-ab+4\,b^2\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left((a^2-5\,ab+4\,b^2)\cos\left(fx+e\right)^4-(a^2-6\,ab+8\,b^2)\cos\left(fx+e\right)^2-ab+4\,b^2\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{4\left((a^4-a^3b)f\cos\left(fx+e\right)^4-a^3bf-(a^4-2\,a^3b)f\cos\left(fx+e\right)^2\right)},\ \frac{2\,(a^2-2\,ab)\cos\left(fx+e\right)^3+4\,ab\cos\left(fx+e\right)-2\left((3\,a^2-7\,ab+4\,b^2)\cos\left(fx+e\right)^4-(3\,a^2-10\,ab+8\,b^2)\cos\left(fx+e\right)^2-3\,ab+4\,b^2\right)\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos\left(fx+e\right)}{b}\right)-\left((a^2-5\,ab+4\,b^2)\cos\left(fx+e\right)^4-(a^2-6\,ab+8\,b^2)\cos\left(fx+e\right)^2-ab+4\,b^2\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left((a^2-5\,ab+4\,b^2)\cos\left(fx+e\right)^4-(a^2-6\,ab+8\,b^2)\cos\left(fx+e\right)^2-ab+4\,b^2\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{4\left((a^4-a^3b)f\cos\left(fx+e\right)^4-a^3bf-(a^4-2\,a^3b)f\cos\left(fx+e\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - 2*a*b)*cos(f*x + e)^3 + 4*a*b*cos(f*x + e) - ((3*a^2 - 7*a*b
 + 4*b^2)*cos(f*x + e)^4 - (3*a^2 - 10*a*b + 8*b^2)*cos(f*x + e)^2 - 3*a*b
+ 4*b^2)*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/
(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - ((a^2 - 5*a*b +
4*b^2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)
*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^4 - (a^2
 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)*log(-1/2*cos(f*x + e) + 1/2
))/((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x +
e)^2), 1/4*(2*(a^2 - 2*a*b)*cos(f*x + e)^3 + 4*a*b*cos(f*x + e) - 2*((3*a^2
 - 7*a*b + 4*b^2)*cos(f*x + e)^4 - (3*a^2 - 10*a*b + 8*b^2)*cos(f*x + e)^2
- 3*a*b + 4*b^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x +
e)/b) - ((a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f
*x + e)^2 - a*b + 4*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 5*a*b + 4*b^
2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)*log
(-1/2*cos(f*x + e) + 1/2))/((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4
 - 2*a^3*b)*f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. 2(140) = 280.

time = 0.72, size = 417, normalized size = 2.84

$$\frac{\frac{6(a-4b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3}-\frac{12(3ab-4b^2)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}\,a^3}-\frac{3(\cos(fx+e)-1)}{a^2(\cos(fx+e)+1)}+\frac{3a^2+\frac{4a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{28ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{16b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{2a^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{8ab(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}}{a^3\left(\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{4b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}\right)}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(6*(a - 4*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^3 - 1
2*(3*a*b - 4*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b -
 b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*a^3) - 3*(cos(f*x +
 e) - 1)/(a^2*(cos(f*x + e) + 1)) + (3*a^2 + 4*a^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 28*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 + 16*b^2*(cos(f*x + e) - 1)^2/(cos(f*x +
e) + 1)^2 - 2*a^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*a*b*(cos(f*
x + e) - 1)^3/(cos(f*x + e) + 1)^3)/(a^3*(a*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 2*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 4*b*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3))
)/f
```

Mupad [**B**]

time = 12.37, size = 917, normalized size = 6.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)^2/(8*a^2*f) - (a/2 - tan(e/2 + (f*x)/2)^2*(a - 6*b) + (t
an(e/2 + (f*x)/2)^4*(a^2 - 8*a*b + 16*b^2))/(2*a))/(f*(4*a^3*tan(e/2 + (f*x
)/2)^2 + 4*a^3*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^4*(16*a^2*b - 8*a^
3))) + (log(tan(e/2 + (f*x)/2))*(a - 4*b))/(2*a^3*f) + (b^(1/2)*atan((2*a^2
*((b^(1/2)*(3*a - 4*b)*(12*a^6*b - 160*a^3*b^4 + 240*a^4*b^3 - 106*a^5*b^2)
)/(2*a^9*(a - b)^(1/2)) + (b^(3/2)*(3*a - 4*b)^3*(8*a^11 - 32*a^10*b + 32*a
^9*b^2))/(32*a^15*(a - b)^(3/2)))*(a - b)*(15*a^4 - 182*a^3*b - 864*a*b^3 +
 384*b^4 + 648*a^2*b^2))/((9*a^2*b - 24*a*b^2 + 16*b^3)*(72*a*b^2 - 27*a^2*
b + 4*a^3 - 48*b^3)) - (4*a^7*tan(e/2 + (f*x)/2)^2*(a - b)^(3/2)*((((4*(16*
b^4 - 24*a*b^3 + 9*a^2*b^2))/a^5 - (b*(3*a - 4*b)^2*(2*a^8 - 46*a^7*b + 384
*a^4*b^4 - 672*a^5*b^3 + 344*a^6*b^2))/(4*a^11*(a - b)))*(a^4 - 31*a^3*b -
336*a*b^3 + 192*b^4 + 180*a^2*b^2))/(b^(1/2)*(b*(27*a^7 + b*(48*a^5*b - 72*
a^6)) - 4*a^8)) + (((b^(1/2)*(3*a - 4*b)*(192*a*b^5 + 9*a^5*b - 384*a^2*b^4
 + 268*a^3*b^3 - 78*a^4*b^2))/(a^8*(a - b)^(1/2)) - (b^(3/2)*(3*a - 4*b)^3*
```

```
(104*a^9*b - 4*a^10 + 192*a^7*b^3 - 288*a^8*b^2))/(16*a^14*(a - b)^(3/2)))*
(15*a^4 - 182*a^3*b - 864*a*b^3 + 384*b^4 + 648*a^2*b^2))/(2*a^5*(a - b)^(1
/2)*(72*a*b^2 - 27*a^2*b + 4*a^3 - 48*b^3))))/(9*a^2*b - 24*a*b^2 + 16*b^3)
 + (4*a^7*(a - b)^(3/2)*((2*(112*a*b^4 - 64*b^5 - 60*a^2*b^3 + 9*a^3*b^2))/
a^6 + (b*(3*a - 4*b)^2*(56*a^8*b - 4*a^9 + 128*a^6*b^3 - 160*a^7*b^2))/(8*a
^12*(a - b)))*(a^4 - 31*a^3*b - 336*a*b^3 + 192*b^4 + 180*a^2*b^2))/(b^(1/2
)*(b*(27*a^7 + b*(48*a^5*b - 72*a^6)) - 4*a^8)*(9*a^2*b - 24*a*b^2 + 16*b^3
)))*(3*a - 4*b))/(2*a^3*f*(a - b)^(1/2))
```

3.73 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=210

$$-\frac{3(a-2b)\sqrt{a-b}\,\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f}-\frac{3(a^2-8ab+8b^2)\tanh^{-1}(\cos(e+fx))}{8a^4f}-\frac{(5a-6b)\cot(e+\ldots}{8a^2f\,(a-b+b\ldots}$$

```
[Out] -3/8*(a^2-8*a*b+8*b^2)*arctanh(cos(f*x+e))/a^4/f-1/8*(5*a-6*b)*cot(f*x+e)*c
sc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a-b+b
*sec(f*x+e)^2)-3/8*(3*a-4*b)*b*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)-3/2*(a
-2*b)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)*b^(1/2)/a^4/f
```

Rubi [A]
time = 0.20, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3745, 481, 541, 536, 213, 211}

$$-\frac{3\sqrt{b}\,(a-2b)\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f}-\frac{3b(3a-4b)\sec(e+fx)}{8a^3f\,(a+b\sec^2(e+fx)-b)}-\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\,(a+b\sec^2(e+fx)-b)}-\frac{3(a^2-8ab+8b^2)\tanh^{-1}(\cos(e+fx))}{8a^4f}-\frac{\cot^3(e+fx)\csc(e+fx)}{4af\,(a+b\sec^2(e+fx)-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*(a - 2*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]
])/(2*a^4*f) - (3*(a^2 - 8*a*b + 8*b^2)*ArcTanh[Cos[e + f*x]])/(8*a^4*f) -
((5*a - 6*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*(a - b + b*Sec[e + f*x]^2)
) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*(a - b + b*Sec[e + f*x]^2)) - (3*(
3*a - 4*b)*b*Sec[e + f*x])/(8*a^3*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3(a-b+bx^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+b+(-4a+5b)x^2}{(-1+x^2)^2(a-b+bx^2)^2}\,dx, x, \sec(e+fx)\right.}{4af}$$

$$= -\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{3(3a}{8a^3f\,(a}$$

$$= -\frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)} - \frac{3(3a}{8a^3f\,(a}$$

$$= -\frac{3(a-2b)\sqrt{a-b}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f} - \frac{3(a^2-8ab+8b^2)\tanh^{-1}(\cos}{8a^4f}$$

Mathematica [A]
time = 6.25, size = 392, normalized size = 1.87

$$\frac{3(a-2b)\sqrt{a-b}\sqrt{b}\,\text{ArcTan}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{2a^4f} + \frac{3(a-2b)\sqrt{a-b}\sqrt{b}\,\text{ArcTan}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{2a^4f} + \frac{-ab\cos(e+fx)+b^2\cos(e+fx)}{a^3f(a+b+a\cos(2(e+fx))-b\cos(2(e+fx)))} + \frac{(-3a+8b)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32a^3f} - \frac{\csc^4\left(\frac{1}{2}(e+fx)\right)}{64a^2f} - \frac{3(a^2-8ab+8b^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8a^4f} + \frac{3(a^2-8ab+8b^2)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8a^4f} + \frac{(3a-8b)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32a^3f} + \frac{\sec^4\left(\frac{1}{2}(e+fx)\right)}{64a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*(a - 2*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[
(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(2*a^4*f) + (3*(a - 2*b
)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2
] + Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(2*a^4*f) + (-(a*b*Cos[e + f*x]) +
 b^2*Cos[e + f*x])/(a^3*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])
) + ((-3*a + 8*b)*Csc[(e + f*x)/2]^2)/(32*a^3*f) - Csc[(e + f*x)/2]^4/(64*a
^2*f) - (3*(a^2 - 8*a*b + 8*b^2)*Log[Cos[(e + f*x)/2]])/(8*a^4*f) + (3*(a^2
 - 8*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^4*f) + ((3*a - 8*b)*Sec[(e +
f*x)/2]^2)/(32*a^3*f) + Sec[(e + f*x)/2]^4/(64*a^2*f)
```

Maple [A]
time = 0.44, size = 232, normalized size = 1.10

method	result

derivativedivides	$\frac{\frac{1}{16a^2(\cos(fx+e)+1)^2}-\frac{-3a+8b}{16a^3(\cos(fx+e)+1)}+\frac{\left(-3a^2+24ab-24b^2\right)\ln(\cos(fx+e)+1)}{16a^4}-\frac{1}{16a^2(\cos(fx+e)-1)^2}-\frac{-3a+8b}{16a^3(\cos(fx+e)-1)}+}{f}$
default	$\frac{\frac{1}{16a^2(\cos(fx+e)+1)^2}-\frac{-3a+8b}{16a^3(\cos(fx+e)+1)}+\frac{\left(-3a^2+24ab-24b^2\right)\ln(\cos(fx+e)+1)}{16a^4}-\frac{1}{16a^2(\cos(fx+e)-1)^2}-\frac{-3a+8b}{16a^3(\cos(fx+e)-1)}+}{f}$
risch	$\frac{3a^2e^{11i(fx+e)}-15ab\,e^{11i(fx+e)}+12b^2e^{11i(fx+e)}-5a^2e^{9i(fx+e)}+21ab\,e^{9i(fx+e)}-36b^2e^{9i(fx+e)}-30a^2e^{7i(fx+e)}-6ab\,e^{7}}{4f\,a^3\left(e^{2i(fx+e)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/16/a^2/(cos(f*x+e)+1)^2-1/16*(-3*a+8*b)/a^3/(cos(f*x+e)+1)+1/16/a^4*
(-3*a^2+24*a*b-24*b^2)*ln(cos(f*x+e)+1)-1/16/a^2/(cos(f*x+e)-1)^2-1/16*(-3*
a+8*b)/a^3/(cos(f*x+e)-1)+1/16/a^4*(3*a^2-24*a*b+24*b^2)*ln(cos(f*x+e)-1)+b
/a^4*((-1/2*a^2+1/2*a*b)*cos(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+3/2*(
a^2-3*a*b+2*b^2)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(202) = 404.
time = 5.28, size = 1090, normalized size = 5.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^3 - 5*a^2*b + 4*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 24*a^2*b + 2
4*a*b^2)*cos(f*x + e)^3 - 12*((a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^6 - (2*a^2
 - 7*a*b + 6*b^2)*cos(f*x + e)^4 + (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 + a
*b - 2*b^2)*sqrt(-a*b + b^2)*log(((a - b)*cos(f*x + e)^2 - 2*sqrt(-a*b + b^
2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 6*(3*a^2*b - 4*a*b^2)*
cos(f*x + e) - 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^
3 - 19*a^2*b + 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3
+ (a^3 - 11*a^2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e)
 + 1/2) + 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^3 - 1
9*a^2*b + 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3 + (a^
3 - 11*a^2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1
/2))/((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*
x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2), 1/16*(6*(a^3 - 5*a^2*b + 4*a*
b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 24*a^2*b + 24*a*b^2)*cos(f*x + e)^3 + 24*(
(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^6 - (2*a^2 - 7*a*b + 6*b^2)*cos(f*x + e)
^4 + (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 + a*b - 2*b^2)*sqrt(a*b - b^2)*ar
ctan(sqrt(a*b - b^2)*cos(f*x + e)/b) - 6*(3*a^2*b - 4*a*b^2)*cos(f*x + e) -
 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^3 - 19*a^2*b +
 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3 + (a^3 - 11*a^
2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((
a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (2*a^3 - 19*a^2*b + 40*a
*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8*b^3 + (a^3 - 11*a^2*b +
 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^5 - a
^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^
5 - 3*a^4*b)*f*cos(f*x + e)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(202) = 404.
time = 0.77, size = 551, normalized size = 2.62

$$\frac{\frac{12\,(a^2-8\,ab+8\,b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4}-\frac{96\,(a^2b-3\,ab^2+2\,b^3)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}\,a^4}-\frac{\frac{8\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^4}-\frac{\left(a^2-\frac{8\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{16\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{144\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{144\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{a^4(\cos(fx+e)-1)^2}-\frac{64\left(a^2b-ab^2+\frac{a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{3\,ab^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2\,b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)a^4}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 8*a*b + 8*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e)
+ 1))/a^4 - 96*(a^2*b - 3*a*b^2 + 2*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*
x + e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b
^2)*a^4) - (8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
)/a^4 - (a^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*a*b*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 - 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 144*b^2*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(a^4*(cos(f*x + e
) - 1)^2) - 64*(a^2*b - a*b^2 + a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 - 3*a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*b^3*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1))/((a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e
) + 1)^2)*a^4))/f
```

Mupad [B]
time = 12.31, size = 1113, normalized size = 5.30

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{64\,a^2\,f}-\frac{\frac{a^2}{4}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{15\,a^2}{4}-32\,a\,b+32\,b^2\right)+\frac{3\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(a-2\,b)}{2}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(a^3-10\,a^2\,b+24\,a\,b^2-16\,b^3)}{a}}{f\,\left(16\,a^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+16\,a^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(64\,a^3\,b-32\,a^4)\right)}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(a-2\,b)}{8\,a^3\,f}+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(3\,a^2-24\,a\,b+24\,b^2)}{8\,a^4\,f}+\ldots$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)^4/(64*a^2*f) - (a^2/4 - tan(e/2 + (f*x)/2)^4*((15*a^2)/4
 - 32*a*b + 32*b^2) + (3*a*tan(e/2 + (f*x)/2)^2*(a - 2*b))/2 + (2*tan(e/2 +
 (f*x)/2)^6*(24*a*b^2 - 10*a^2*b + a^3 - 16*b^3))/a)/(f*(16*a^4*tan(e/2 + (
f*x)/2)^4 + 16*a^4*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^6*(64*a^3*b -
32*a^4))) + (tan(e/2 + (f*x)/2)^2*(a - 2*b))/(8*a^3*f) + (log(tan(e/2 + (f*
x)/2))*(3*a^2 - 24*a*b + 24*b^2))/(8*a^4*f) + (3*atan((8*a^10*tan(e/2 + (f*
x)/2)^2*((((756*a*b^6 - 216*b^7 - 1026*a^2*b^5 + 675*a^3*b^4 - 216*a^4*b^3
+ 27*a^5*b^2)/a^8 + (9*(a - 2*b)^2*(a*b - b^2)*(180*a^10*b - 6*a^11 + 2304*
a^6*b^5 - 5760*a^7*b^4 + 4944*a^8*b^3 - 1656*a^9*b^2))/(16*a^16))*(960*a*b^
4 - 38*a^4*b + a^5 - 384*b^5 - 840*a^2*b^3 + 300*a^3*b^2))/(2*a^5*(b*(a - b
))^(3/2)*(a^4 - 12*a^3*b - 96*a*b^3 + 48*b^4 + 60*a^2*b^2)) + (((27*(a - 2*
b)^3*(a*b - b^2)^(3/2)*(416*a^12*b - 16*a^13 + 768*a^10*b^3 - 1152*a^11*b^2
))/(64*a^20) - (3*(a - 2*b)*(a*b - b^2)^(1/2)*(27*a^8*b + 1728*a^2*b^7 - 60
48*a^3*b^6 + 8352*a^4*b^5 - 5760*a^5*b^4 + 2070*a^6*b^3 - 369*a^7*b^2))/(4*
a^12))*(4*a^4 - 60*a^3*b - 384*a*b^3 + 192*b^4 + 252*a^2*b^2))/(a^5*b*(144*
a*b^4 - 13*a^4*b + a^5 - 48*b^5 - 156*a^2*b^3 + 72*a^3*b^2))))/(27*a^2 - 10
8*a*b + 108*b^2) + (8*a^5*((27*(a - 2*b)^3*(a*b - b^2)^(3/2)*(32*a^14 - 128
*a^13*b + 128*a^12*b^2))/(128*a^21) + (3*(a - 2*b)*(a*b - b^2)^(1/2)*(36*a^
9*b - 1440*a^4*b^6 + 4320*a^5*b^5 - 4824*a^6*b^4 + 2448*a^7*b^3 - 540*a^8*b
^2))/(8*a^13))*(4*a^4 - 60*a^3*b - 384*a*b^3 + 192*b^4 + 252*a^2*b^2))/(b*(
27*a^2 - 108*a*b + 108*b^2)*(144*a*b^4 - 13*a^4*b + a^5 - 48*b^5 - 156*a^2*
b^3 + 72*a^3*b^2)) - (4*a^5*((864*b^8 - 3456*a*b^7 + 5508*a^2*b^6 - 4428*a^
```

```
3*b^5 + 1863*a^4*b^4 - 378*a^5*b^3 + 27*a^6*b^2)/(2*a^9) - (9*(a - 2*b)^2*(
a*b - b^2)*(12*a^12 - 240*a^11*b + 768*a^8*b^4 - 1536*a^9*b^3 + 1008*a^10*b
^2))/(32*a^17))*(960*a*b^4 - 38*a^4*b + a^5 - 384*b^5 - 840*a^2*b^3 + 300*a
^3*b^2))/((b*(a - b))^(3/2)*(27*a^2 - 108*a*b + 108*b^2)*(a^4 - 12*a^3*b -
96*a*b^3 + 48*b^4 + 60*a^2*b^2)))*(a - 2*b)*(a*b - b^2)^(1/2))/(2*a^4*f)
```

3.74 $\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal. Leaf size=196

$$\frac{3(a^2+6ab+b^2)\,x}{8(a-b)^4}-\frac{3\sqrt{a}\,\sqrt{b}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^4f}-\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))}+\frac{\cos^3(e}{4(a-b)f}$$

```
[Out] 3/8*(a^2+6*a*b+b^2)*x/(a-b)^4-3/2*(a+b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*
a^(1/2)*b^(1/2)/(a-b)^4/f-1/8*(5*a+b)*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f/(a+b*
tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)-3/8*b*
(3*a+b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.18, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 481, 541, 536, 209, 211}

$$\frac{3x(a^2+6ab+b^2)}{8(a-b)^4}-\frac{3\sqrt{a}\,\sqrt{b}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2f(a-b)^4}-\frac{3b(3a+b)\tan(e+fx)}{8f(a-b)^3\,(a+b\tan^2(e+fx))}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)\,(a+b\tan^2(e+fx))}-\frac{(5a+b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2\,(a+b\tan^2(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*(a^2 + 6*a*b + b^2)*x)/(8*(a - b)^4) - (3*Sqrt[a]*Sqrt[b]*(a + b)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*(a - b)^4*f) - ((5*a + b)*Cos[e + f*x]
*Sin[e + f*x])/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin
[e + f*x])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)) - (3*b*(3*a + b)*Tan[e + f*
x])/(8*(a - b)^3*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
```

```
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{a+(-4a-b)x^2}{(1+x^2)^2(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))} + \frac{\text{Subs}}{}$$

$$= -\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))} - \frac{3}{8(a-}$$

$$= -\frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))} - \frac{3}{8(a-}$$

$$= \frac{3(a^2+6ab+b^2)\,x}{8(a-b)^4} - \frac{3\sqrt{a}\,\sqrt{b}\,(a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^4f} - \frac{(5a+b)\cos(}{8(a-b)^2f\,(}$$

Mathematica [A]

time = 1.08, size = 136, normalized size = 0.69

$$\frac{12(a^2+6ab+b^2)(e+fx) - 48\sqrt{a}\,\sqrt{b}\,(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right) - 8(a-b)(a+b)\sin(2(e+fx)) - \frac{16a(a-b)b\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))} + (a-b)^2\sin(4(e+fx))}{32(a-b)^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (12*(a^2 + 6*a*b + b^2)*(e + f*x) - 48*Sqrt[a]*Sqrt[b]*(a + b)*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sin[2*(e + f*x)] - (16*a*(a
- b)*b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]) + (a - b)^2*Sin
[4*(e + f*x)])/(32*(a - b)^4*f)
```

Maple [A]

time = 0.41, size = 159, normalized size = 0.81

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{5}{8}a^2+\frac{1}{4}ab+\frac{3}{8}b^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2+\frac{5}{8}b^2-\frac{1}{4}ab\right)\tan(fx+e)}{\left(1+\tan^2(fx+e)\right)^2}+\frac{3\left(a^2+6ab+b^2\right)\arctan(\tan(fx+e))}{8}}{(a-b)^4} - \frac{ab\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}\right.}{}}{f}$

default	$\frac{\frac{\frac{\left(-\frac{5}{8}a^2+\frac{1}{4}ab+\frac{3}{8}b^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2+\frac{5}{8}b^2-\frac{1}{4}ab\right)\tan(fx+e)}{\left(1+\tan^2(fx+e)\right)^2}+\frac{3\left(a^2+6ab+b^2\right)\arctan(\tan(fx+e))}{8}}{(a-b)^4}-\frac{ab\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\cdots\right.}{\cdots}}{f}$
risch	$\frac{3x\,a^2}{8(a^2-2ab+b^2)(a-b)^2}+\frac{9xab}{4(a^2-2ab+b^2)(a-b)^2}+\frac{3x\,b^2}{8(a^2-2ab+b^2)(a-b)^2}-\frac{i\mathrm{e}^{4i(fx+e)}}{64(a-b)^2f}+\frac{i\mathrm{e}^{2i(fx+e)}a}{8(a-b)^3f}+\frac{i\mathrm{e}^{2i(fx+e)}}{8(a-b)^3f}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^4*(((-5/8*a^2+1/4*a*b+3/8*b^2)*tan(f*x+e)^3+(-3/8*a^2+5/8*b^2-
1/4*a*b)*tan(f*x+e))/(1+tan(f*x+e)^2)^2+3/8*(a^2+6*a*b+b^2)*arctan(tan(f*x+
e)))-a*b/(a-b)^4*((1/2*a-1/2*b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+3/2*(a+b)/(a*
b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.50, size = 320, normalized size = 1.63

$$\frac{\frac{3\left(a^2+6ab+b^2\right)(fx+e)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{12\left(a^2b+ab^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\sqrt{ab}}-\frac{3\left(3ab+b^2\right)\tan(fx+e)^5+\left(5a^2+14ab+5b^2\right)\tan(fx+e)^3+3\left(a^2+3ab\right)\tan(fx+e)}{\left(a^3b-3a^2b^2+3ab^3-b^4\right)\tan(fx+e)^6+\left(a^4-a^3b-3a^2b^2+5ab^3-2b^4\right)\tan(fx+e)^4+a^4-3a^3b+3a^2b^2-ab^3+\left(2a^4-5a^3b+3a^2b^2+ab^3-b^4\right)\tan(fx+e)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^2 + 6*a*b + b^2)*(f*x + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
 b^4) - 12*(a^2*b + a*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 4*a^3*b
 + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a*b)) - (3*(3*a*b + b^2)*tan(f*x + e)^5
+ (5*a^2 + 14*a*b + 5*b^2)*tan(f*x + e)^3 + 3*(a^2 + 3*a*b)*tan(f*x + e))/(
(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*tan(f*x + e)^6 + (a^4 - a^3*b - 3*a^2*b
^2 + 5*a*b^3 - 2*b^4)*tan(f*x + e)^4 + a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3 +
(2*a^4 - 5*a^3*b + 3*a^2*b^2 + a*b^3 - b^4)*tan(f*x + e)^2))/f
```

Fricas [A]
time = 4.49, size = 729, normalized size = 3.72

$$\left[\frac{3\left(a^3+5a^2b-5ab^2-b^3\right)fx\cos(fx+e)^2+3\left(a^2b+6ab^2+b^3\right)fx+3\left(\left(a^2-b^2\right)\cos(fx+e)^2+ab+b^2\right)\sqrt{-ab}\log\left(\frac{\left(a^2+6ab+b^2\right)\cos(fx+e)^4-2\left(3ab+b^2\right)\cos(fx+e)^2+4\left(\left(a+b\right)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab}\sin(fx+e)+b^2}{\left(a^2-2ab+b^2\right)\cos(fx+e)^4+2\left(ab-b^2\right)\cos(fx+e)^2+b^2}\right)+\left(2\left(a^3-3a^2b+3ab^2-b^3\right)\cos(fx+e)^5-\left(5a^3-9a^2b+3ab^2+b^3\right)\cos(fx+e)^3-3\left(3a^2b-2ab^2-b^3\right)\cos(fx+e)\right)\sin(fx+e)}{8\left(\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)f\cos(fx+e)^2+\left(a^4b-4a^3b^2+6a^2b^3-4ab^4+b^5\right)f\right)}, \frac{3\left(a^3+5a^2b-5ab^2-b^3\right)fx\cos(fx+e)^2+3\left(a^2b+6ab^2+b^3\right)fx+6\left(\left(a^2-b^2\right)\cos(fx+e)^2+ab+b^2\right)\sqrt{ab}\arctan\left(\frac{\left(\left(a+b\right)\cos(fx+e)^2-b\right)\sqrt{ab}}{2ab\cos(fx+e)\sin(fx+e)}\right)+\left(2\left(a^3-3a^2b+3ab^2-b^3\right)\cos(fx+e)^5-\left(5a^3-9a^2b+3ab^2+b^3\right)\cos(fx+e)^3-3\left(3a^2b-2ab^2-b^3\right)\cos(fx+e)\right)\sin(fx+e)}{8\left(\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)f\cos(fx+e)^2+\left(a^4b-4a^3b^2+6a^2b^3-4ab^4+b^5\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^3 + 5*a^2*b - 5*a*b^2 - b^3)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 6*a
*b^2 + b^3)*f*x + 3*((a^2 - b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a*b)*log
(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*(
```

```
(a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a
^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) + (
2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - (5*a^3 - 9*a^2*b + 3*a*b
^2 + b^3)*cos(f*x + e)^3 - 3*(3*a^2*b - 2*a*b^2 - b^3)*cos(f*x + e))*sin(f*
x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*
x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f), 1/8*(3*(a^3
+ 5*a^2*b - 5*a*b^2 - b^3)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 6*a*b^2 + b^3)*f
*x + 6*((a^2 - b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(a*b)*arctan(1/2*((a +
b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x + e)*sin(f*x + e))) + (2*(a^3
 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b
^3)*cos(f*x + e)^3 - 3*(3*a^2*b - 2*a*b^2 - b^3)*cos(f*x + e))*sin(f*x + e)
)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)
^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.80, size = 266, normalized size = 1.36

$$\frac{\frac{3\,(a^2+6\,ab+b^2)(fx+e)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{4\,ab\tan(fx+e)}{(a^3-3\,a^2b+3\,ab^2-b^3)\left(b\tan(fx+e)^2+a\right)}-\frac{12\,(a^2b+ab^2)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)\sqrt{ab}}-\frac{5\,a\tan(fx+e)^3+3\,b\tan(fx+e)^3+3\,a\tan(fx+e)+5\,b\tan(fx+e)}{(a^3-3\,a^2b+3\,ab^2-b^3)\left(\tan(fx+e)^2+1\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 6*a*b + b^2)*(f*x + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
 b^4) - 4*a*b*tan(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(b*tan(f*x + e)
^2 + a)) - 12*(a^2*b + a*b^2)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan
(b*tan(f*x + e)/sqrt(a*b)))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sq
rt(a*b)) - (5*a*tan(f*x + e)^3 + 3*b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 5*
b*tan(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(f*x + e)^2 + 1)^2))/f
```

Mupad [**B**]
time = 16.14, size = 2500, normalized size = 12.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^2,x)

[Out] (atan(((((tan(e + f*x)*(108*a*b^6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 153
*a^4*b^3))/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 1
5*a^4*b^2)) - (3*(((9*a*b^11)/2 - (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b
^8 + 231*a^5*b^7 - 147*a^6*b^6 + 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 +
(3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 -
126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (3*tan(e + f*x)*(a*b
*6i + a^2*1i + b^2*1i)*(256*b^11 - 1792*a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^
8 + 3584*a^4*b^7 + 3584*a^5*b^6 - 7168*a^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^
3 + 256*a^9*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6
*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a*b*6i +
a^2*1i + b^2*1i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*b*6i
 + a^2*1i + b^2*1i)*3i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) +
(((tan(e + f*x)*(108*a*b^6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^
3))/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b
^2)) + (3*(((9*a*b^11)/2 - (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 23
1*a^5*b^7 - 147*a^6*b^6 + 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10
*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4
*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) + (3*tan(e + f*x)*(a*b*6i + a
^2*1i + b^2*1i)*(256*b^11 - 1792*a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^8 + 358
4*a^4*b^7 + 3584*a^5*b^6 - 7168*a^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^3 + 256
*a^9*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b
- 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a*b*6i + a^2*1i
+ b^2*1i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*b*6i + a^2*
1i + b^2*1i)*3i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/(((27*a*
b^7)/64 + (135*a^2*b^6)/32 + (189*a^3*b^5)/16 + (297*a^4*b^4)/32 + (81*a^5*
b^3)/64)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4
*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (3*((tan(e + f*x)*(108*a*b^
6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^3))/(32*(a^6 - 6*a^5*b -
6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (3*(((9*a*b^11)/2
- (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 231*a^5*b^7 - 147*a^6*b^6 +
 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8
*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a
^6*b^3 + 36*a^7*b^2) - (3*tan(e + f*x)*(a*b*6i + a^2*1i + b^2*1i)*(256*b^11
 - 1792*a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^8 + 3584*a^4*b^7 + 3584*a^5*b^6
- 7168*a^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^3 + 256*a^9*b^2))/(512*(a^4 - 4*
a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*
b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(a*b*6i + a^2*1i + b^2*1i))/(16*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*b*6i + a^2*1i + b^2*1i))/(16*(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (3*((tan(e + f*x)*(108*a*b^6 + 9*b^
7 + 486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^3))/(32*(a^6 - 6*a^5*b - 6*a*b^5
+ b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (3*(((9*a*b^11)/2 - (69*a^
2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 231*a^5*b^7 - 147*a^6*b^6 + 42*a^7*
b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9
```

```
- b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 +
 36*a^7*b^2) + (3*tan(e + f*x)*(a*b*6i + a^2*1i + b^2*1i)*(256*b^11 - 1792*
a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^8 + 3584*a^4*b^7 + 3584*a^5*b^6 - 7168*a
^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^3 + 256*a^9*b^2))/(512*(a^4 - 4*a^3*b -
4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20
*a^3*b^3 + 15*a^4*b^2)))*(a*b*6i + a^2*1i + b^2*1i))/(16*(a^4 - 4*a^3*b - 4
*a*b^3 + b^4 + 6*a^2*b^2)))*(a*b*6i + a^2*1i + b^2*1i))/(16*(a^4 - 4*a^3*b
- 4*a*b^3 + b^4 + 6*a^2*b^2))))*(a*b*6i + a^2*1i + b^2*1i)*3i)/(8*f*(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - ((3*tan(e + f*x)^5*(3*a*b + b^2))/(
8*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e + f*x)^3*(14*a*b + 5*a^2 + 5*b^
2))/(8*(a - b)*(a^2 - 2*a*b + b^2)) + (3*tan(e + f*x)*(3*a*b + a^2))/(8*(a
- b)*(a^2 - 2*a*b + b^2)))/(f*(a + b*tan(e + f*x)^6 + tan(e + f*x)^2*(2*a +
 b) + tan(e + f*x)^4*(a + 2*b))) + (atan((((-a*b)^(1/2)*(a + b)*((tan(e + f
*x)*(108*a*b^6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 153*a^4*b^3))/(32*(a^6
 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (3*((
(9*a*b^11)/2 - (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^4*b^8 + 231*a^5*b^7 -
147*a^6*b^6 + 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/2 + (3*a^10*b^2)/2)/(9*
a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a
^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (3*tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(2
56*b^11 - 1792*a*b^10 + 5120*a^2*b^9 - 7168*a^3*b^8 + 3584*a^4*b^7 + 3584*a
^5*b^6 - 7168*a^6*b^5 + 5120*a^7*b^4 - 1792*a^8*b^3 + 256*a^9*b^2))/(128*(a
^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 +
15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*3i)/(4*(a^4...
```

3.75 $\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{(a+3b)x}{2(a-b)^3}-\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a-b)^3f}-\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))}-\frac{b\tan(e+fx)}{(a-b)^2f\,(a+b\tan^2(e+$$

```
[Out] 1/2*(a+3*b)*x/(a-b)^3-1/2*(3*a+b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2
)/(a-b)^3/f/a^(1/2)-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)-b*
tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 482, 541, 536, 209, 211}

$$-\frac{\sqrt{b}\,(3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\,f(a-b)^3}-\frac{b\tan(e+fx)}{f(a-b)^2\,(a+b\tan^2(e+fx))}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\,(a+b\tan^2(e+fx))}+\frac{x(a+3b)}{2(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a + 3*b)*x)/(2*(a - b)^3) - (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(2*Sqrt[a]*(a - b)^3*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a -
 b)*f*(a + b*Tan[e + f*x]^2)) - (b*Tan[e + f*x])/((a - b)^2*f*(a + b*Tan[e
+ f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{a-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right.}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))} - \frac{b\tan(e+fx)}{(a-b)^2f\,(a+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))} - \frac{b\tan(e+fx)}{(a-b)^2f\,(a+b\tan^2(e+fx))} - \frac{(b(3a+}{}$$

$$= \frac{(a+3b)x}{2(a-b)^3} - \frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a-b)^3f} - \frac{\cos(e+fx)\sin(e+fx}{2(a-b)f\,(a+b\tan^2(e+}$$

Mathematica [A]
time = 1.17, size = 111, normalized size = 0.80

$$-\frac{-2(a+3b)(e+fx)+\frac{2\sqrt{b}(3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}+(a-b)\sin(2(e+fx))+\frac{2(a-b)b\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))}}{4(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/4*(-2*(a + 3*b)*(e + f*x) + (2*Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a]])/Sqrt[a] + (a - b)*Sin[2*(e + f*x)] + (2*(a - b)*b*Sin[2*(e
 + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]))/((a - b)^3*f)
```

Maple [A]
time = 0.34, size = 120, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan^2(fx+e)}+\frac{(a+3b)\arctan(\tan(fx+e))}{2}}{(a-b)^3}-\frac{b\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(3a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^3}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan^2(fx+e)}+\frac{(a+3b)\arctan(\tan(fx+e))}{2}}{(a-b)^3}-\frac{b\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(3a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^3}}{f}$
risch	$\frac{xa}{2(a^2-2ab+b^2)(a-b)}+\frac{3xb}{2(a^2-2ab+b^2)(a-b)}+\frac{i e^{2i(fx+e)}}{8(a^2-2ab+b^2)f}-\frac{i e^{-2i(fx+e)}}{8(a^2-2ab+b^2)f}-\frac{ib(a\,e^{2i(f}}{f(-a+b)^3(-a\,e^{4i(fx+e)}+b\,e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^3*((-1/2*a+1/2*b)*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+3*b)*arct
an(tan(f*x+e)))-1/(a-b)^3*b*((1/2*a-1/2*b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/
2*(3*a+b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.50, size = 191, normalized size = 1.38

$$\frac{\frac{(fx+e)(a+3b)}{a^3-3a^2b+3ab^2-b^3}-\frac{(3ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-3a^2b+3ab^2-b^3)\sqrt{ab}}-\frac{2b\tan(fx+e)^3+(a+b)\tan(fx+e)}{(a^2b-2ab^2+b^3)\tan(fx+e)^4+a^3-2a^2b+ab^2+(a^3-a^2b-ab^2+b^3)\tan(fx+e)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a + 3*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a*b + b^2)*ar
ctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b))
- (2*b*tan(f*x + e)^3 + (a + b)*tan(f*x + e))/((a^2*b - 2*a*b^2 + b^3)*tan(
f*x + e)^4 + a^3 - 2*a^2*b + a*b^2 + (a^3 - a^2*b - a*b^2 + b^3)*tan(f*x +
e)^2))/f
```

Fricas [A]
time = 3.65, size = 590, normalized size = 4.28

$$\left[\frac{4\,(a^2+2\,ab-3\,b^2)fx\cos\left(fx+e\right)^2+4\,(ab+3\,b^2)fx-\left((3\,a^2-2\,ab-b^2)\cos\left(fx+e\right)^2+3\,ab+b^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(fx+e)^4-2\,(3\,ab+b^2)\cos(fx+e)^2-4\left((a^2+ab)\cos(fx+e)^3-ab\cos(fx+e)\right)\sqrt{-\frac{b}{a}}\sin(fx+e)+b^2}{(a^2-2\,ab+b^2)\cos(fx+e)^4+2\,(ab-b^2)\cos(fx+e)^2+b^2}\right)-4\left((a^2-2\,ab+b^2)\cos\left(fx+e\right)^3+2\,(ab-b^2)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\left((a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)f\cos\left(fx+e\right)^2+(a^3b-3\,a^2b^2+3\,ab^3-b^4)f\right)},\frac{2\,(a^2+2\,ab-3\,b^2)fx\cos\left(fx+e\right)^2+2\,(ab+3\,b^2)fx+\left((3\,a^2-2\,ab-b^2)\cos\left(fx+e\right)^2+3\,ab+b^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)-2\left((a^2-2\,ab+b^2)\cos\left(fx+e\right)^3+2\,(ab-b^2)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{4\left((a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)f\cos\left(fx+e\right)^2+(a^3b-3\,a^2b^2+3\,ab^3-b^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + 2*a*b - 3*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b + 3*b^2)*f*x - ((
3*a^2 - 2*a*b - b^2)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(((a^2 + 6
*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a^2 + a*b
)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 -
 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) - 4*((a
^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 2*(a*b - b^2)*cos(f*x + e))*sin(f*x + e)
)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b -
3*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/4*(2*(a^2 + 2*a*b - 3*b^2)*f*x*cos(f*x + e
)^2 + 2*(a*b + 3*b^2)*f*x + ((3*a^2 - 2*a*b - b^2)*cos(f*x + e)^2 + 3*a*b +
 b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*
x + e)*sin(f*x + e))) - 2*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 2*(a*b - b^
2)*cos(f*x + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)
*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.78, size = 195, normalized size = 1.41

$$\frac{\frac{(fx+e)(a+3\,b)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3\,ab+b^2)}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab}}-\frac{2\,b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+b\tan(fx+e)^2+a\right)(a^2-2\,ab+b^2)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 3*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (pi*floor((f*x +
 e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*(3*a*b + b^2)/((a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)) - (2*b*tan(f*x + e)^3 + a*tan(f*x +
 e) + b*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + b*tan(f*x + e
)^2 + a)*(a^2 - 2*a*b + b^2)))/f
```

Mupad [B]
time = 14.94, size = 2500, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (atan((((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))/(a^4 - 4
*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) + ((-a*b)^(1/2)*(3*a + b)*((10*a*b^8 -
2*b^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*
a^7*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*
b^2) - (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7
+ 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^3
 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))
)/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))*(3*a + b)*1i)/(4*(a*b^3 + 3*a^3*
b - a^4 - 3*a^2*b^2)) + ((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a
^2*b^3))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) - ((-a*b)^(1/2)*(3*a +
 b)*((10*a*b^8 - 2*b^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4
+ 10*a^6*b^3 - 2*a^7*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*
a^3*b^3 + 15*a^4*b^2) + (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(40*a*b^8 - 8*
b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^
7*b^2))/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b
^4 + 6*a^2*b^2))))/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))*(3*a + b)*1i)/(
4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))/((5*a*b^4 + (3*b^5)/2 + (3*a^2*b^3)
/2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)
- ((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))/(a^4 - 4*a^3*
b - 4*a*b^3 + b^4 + 6*a^2*b^2) + ((-a*b)^(1/2)*(3*a + b)*((10*a*b^8 - 2*b^9
 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7*b
^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)
- (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*
a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^3 + 3*
a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*
(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))*(3*a + b))/(4*(a*b^3 + 3*a^3*b - a^4
- 3*a^2*b^2)) + ((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))
/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) - ((-a*b)^(1/2)*(3*a + b)*((10
```

```
*a*b^8 - 2*b^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6
*b^3 - 2*a^7*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3
+ 15*a^4*b^2) + (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(40*a*b^8 - 8*b^9 - 72
*a^2*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/
(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a
^2*b^2))))/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))*(3*a + b))/(4*(a*b^3 +
3*a^3*b - a^4 - 3*a^2*b^2))))*(-a*b)^(1/2)*(3*a + b)*1i)/(2*f*(a*b^3 + 3*a^
3*b - a^4 - 3*a^2*b^2)) - (atan((((a + 3*b)*(((a + 3*b)*((10*a*b^8 - 2*b^9
- 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7*b^
2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2) -
 (tan(e + f*x)*(a + 3*b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 40*a
^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^2*3i - a^2*b*3i + a^
3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a*b^2*3i
- a^2*b*3i + a^3*1i - b^3*1i)) + (tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3
))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*1i)/(4*(a*b^2*3i - a^2*b*3i
 + a^3*1i - b^3*1i)) - ((a + 3*b)*(((a + 3*b)*((10*a*b^8 - 2*b^9 - 18*a^2*b
^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7*b^2)/(a^6 -
6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2) + (tan(e +
f*x)*(a + 3*b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 7
2*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3
*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a*b^2*3i - a^2*b*3i
 + a^3*1i - b^3*1i)) - (tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))/(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*1i)/(4*(a*b^2*3i - a^2*b*3i + a^3*1i
- b^3*1i)))/(((a + 3*b)*(((a + 3*b)*((10*a*b^8 - 2*b^9 - 18*a^2*b^7 + 10*a^
3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7*b^2)/(a^6 - 6*a^5*b -
6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2) - (tan(e + f*x)*(a +
3*b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4
+ 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^4
- 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i
- b^3*1i)) + (tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))/(a^4 - 4*a^3*b -
4*a*b^3 + b^4 + 6*a^2*b^2)))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) -
(5*a*b^4 + (3*b^5)/2 + (3*a^2*b^3)/2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a
^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2) + ((a + 3*b)*(((a + 3*b)*((10*a*b^8 - 2*b
^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 - 2*a^7
*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2
) + (tan(e + f*x)*(a + 3*b)*(40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 4
0*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*(a*b^2*3i - a^2*b*3i +
 a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a*b^2*
3i - a^2*b*3i + a^3*1i - b^3*1i)) - (tan(e + f*...
```

3.76 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{x}{(a-b)^2}-\frac{(3a-b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2f}-\frac{b\tan(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))}$$

```
[Out] x/(a-b)^2-1/2*(3*a-b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(3/2)/(a
-b)^2/f-1/2*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3742, 425, 536, 209, 211}

$$-\frac{\sqrt{b}\,(3a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\,(a+b\tan^2(e+fx))}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*f) - (b*Tan[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]
^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right.}{2a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2f} - \frac{((3}{}$$

$$= \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2f} - \frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+\right.}$$

Mathematica [A]
time = 0.74, size = 88, normalized size = 0.91

$$\frac{2\text{ArcTan}(\tan(e+fx)) + \frac{\sqrt{b}(-3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(-a+b)\tan(e+fx)}{a(a+b\tan^2(e+fx))}}{2(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] (2*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[e + f*x])/(a*(a + b*Tan[e + f*x]^2)))/
(2*(a - b)^2*f)
```

Maple [A]
time = 0.22, size = 93, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^2}-\frac{b\left(\frac{(a-b)\tan(fx+e)}{2a\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^2}-\frac{b\left(\frac{(a-b)\tan(fx+e)}{2a\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}}{f}$
risch	$\frac{x}{a^2-2ab+b^2}+\frac{ib\left(a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}+a-b\right)}{fa(-a+b)^2\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\right.}{4a(a-b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^2*arctan(tan(f*x+e))-1/(a-b)^2*b*(1/2/a*(a-b)*tan(f*x+e)/(a+b*
tan(f*x+e)^2)+1/2*(3*a-b)/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.52, size = 118, normalized size = 1.22

$$-\frac{\frac{b\tan(fx+e)}{a^3-a^2b+(a^2b-ab^2)\tan(fx+e)^2}+\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 - a^2*b + (a^2*b - a*b^2)*tan(f*x + e)^2) + (3*a*
b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b
)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2))/f
```

Fricas [A]
time = 2.58, size = 406, normalized size = 4.19

$$\left[\frac{8\,abfx\tan(fx+e)^2+8\,a^2fx-\left((3\,ab-b^2)\tan(fx+e)^2+3\,a^2-ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)-4\,(ab-b^2)\tan(fx+e)}{8\left((a^3b-2\,a^2b^2+ab^3)f\tan(fx+e)^2+(a^4-2\,a^3b+a^2b^2)f\right)},\ \frac{4\,abfx\tan(fx+e)^2+4\,a^2fx-\left((3\,ab-b^2)\tan(fx+e)^2+3\,a^2-ab\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)-2\,(ab-b^2)\tan(fx+e)}{4\left((a^3b-2\,a^2b^2+ab^3)f\tan(fx+e)^2+(a^4-2\,a^3b+a^2b^2)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")

[Out] [1/8*(8*a*b*f*x*tan(f*x + e)^2 + 8*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2 + 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), 1/4*(4*a*b*f*x*tan(f*x + e)^2 + 4*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2 + 3*a^2 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))) - 2*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f)]

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. $2(78) = 156$.
time = 15.70, size = 2125, normalized size = 21.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e)**2)**2,x)

[Out] Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**2, Eq(b, 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b**2, Eq(a, 0)), (3*f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 6*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 3*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 3*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 5*tan(e + f*x)/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x/(a + b*tan(e)**2)**2, Eq(f, 0)), (4*a**2*f*x*sqrt(-a/b)/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) - 3*a**2*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + 3*a**2*log(sqrt(-a/b) + tan(e + f*x))/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + 4*a*b*f*x*sqrt(-a/b)*tan(e + f*x)**2/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) - 2*a*b*sqrt(-a/b)*tan(e + f*x)/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sq

```
rt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) - 3*a*b*log(-sqrt(-a/b) +
 tan(e + f*x))*tan(e + f*x)**2/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)
*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e +
 f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2
) + a*b*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*s
qrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b
)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e
+ f*x)**2) + 3*a*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**4*f
*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b)
 - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*
a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) - a*b*log(sqrt(-a/b) + tan(e + f*x))/(
4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sq
rt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a
/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + 2*b**2*sqrt(-a/b)*tan(e + f*
x)/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*
f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqr
t(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + b**2*log(-sqrt(-a/b) + t
an(e + f*x))*tan(e + f*x)**2/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*t
an(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f
*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2)
- b**2*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**4*f*sqrt(-a/b)
+ 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b*
*2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqr
t(-a/b)*tan(e + f*x)**2), True))
```

Giac [A]

time = 0.58, size = 127, normalized size = 1.31

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)(a^2-ab)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(f*x + e)/(a^2 -
2*a*b + b^2) + b*tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a^2 - a*b)))/f
```

Mupad [B]

time = 13.53, size = 2489, normalized size = 25.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^2,x)

[Out] (2*atan((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 -
4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^
2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2
*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2
) + (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2
)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^
4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) +
 (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*
b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(
2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^
4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*a*b^7 - 12*a^2*b^
6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 +
a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5
+ 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^
2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(b^5 - 6*a
*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^
2) + (((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a
^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(e + f*x)*(16*a^2*b
^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a
^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^
2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2
*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((3*a*b^3)/2 - b^4/2)/(3*a^4*b - a^5 + a^
2*b^3 - 3*a^3*b^2))))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((-a^3*b)^(1/2)*
((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2))
- (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^
2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(3*
a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16
*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b
)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2
*a^4*b + a^3*b^2)) + ((-a^3*b)^(1/2)*((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2
*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5
 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^
2) + (tan(e + f*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a
^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2
)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b +
 a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2*a^4*b + a^3*b^2)))/((((3*a*b^3)/2 - b^
4/2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + ((-a^3*b)^(1/2)*((tan(e + f*x)
*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) - (((2*a*b^7 -
 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b -
a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2
*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*
(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a -
 b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)
```

```
) - ((-a^3*b)^(1/2)*((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 -
2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 1
8*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(e + f*x
)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b
^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b
+ a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a
- b))/(4*(a^5 - 2*a^4*b + a^3*b^2))))*(-a^3*b)^(1/2)*(3*a - b)*1i)/(2*f*(a^
5 - 2*a^4*b + a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b*tan(e + f*x)^2)*(a
 - b))
```

3.77 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{3\cot(e+fx)}{2a^2f}+\frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -3/2*cot(f*x+e)/a^2/f-3/2*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(5/2
)/f+1/2*cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3744, 296, 331, 211}

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{3\cot(e+fx)}{2a^2f}+\frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*a^(5/2)*f) - (3*Cot[
e + f*x])/(2*a^2*f) + Cot[e + f*x]/(2*a*f*(a + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{2af\,(a+b\tan^2(e+fx))} + \frac{3\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{3\cot(e+fx)}{2a^2f} + \frac{\cot(e+fx)}{2af\,(a+b\tan^2(e+fx))} - \frac{(3b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e-$$

$$= -\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{3\cot(e+fx)}{2a^2f} + \frac{\cot(e+fx)}{2af\,(a+b\tan^2(e+fx))}$$

Mathematica [A]
time = 0.46, size = 83, normalized size = 1.01

$$\frac{-3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right) + \sqrt{a}\left(-2\cot(e+fx) - \frac{b\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))}\right)}{2a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-2*Cot[e + f*
x] - (b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])))/(2*a^(5/2)*f
)
```

Maple [A]
time = 0.22, size = 69, normalized size = 0.84

method	result

derivativedivides	$\frac{-\frac{1}{a^2\tan(fx+e)}-\frac{b\left(\frac{\tan(fx+e)}{2a+2b\left(\tan^2(fx+e)\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}}{f}$
default	$\frac{-\frac{1}{a^2\tan(fx+e)}-\frac{b\left(\frac{\tan(fx+e)}{2a+2b\left(\tan^2(fx+e)\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}}{f}$
risch	$-\frac{i(2a^2e^{4i(fx+e)}-3ab\,e^{4i(fx+e)}+3b^2e^{4i(fx+e)}+4a^2e^{2i(fx+e)}-6b^2e^{2i(fx+e)}+2a^2-5ab+3b^2)}{(a-b)f\,a^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)\left(e^{2i(fx+e)}-1\right)}-\frac{3\sqrt{-ab}\,\ln\left(e^{2i(f}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2/tan(f*x+e)-1/a^2*b*(1/2*tan(f*x+e)/(a+b*tan(f*x+e)^2)+3/2/(a*b)
^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.52, size = 77, normalized size = 0.94

$$-\frac{\frac{3\,b\tan(fx+e)^2+2\,a}{a^2b\tan(fx+e)^3+a^3\tan(fx+e)}+\frac{3\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b*tan(f*x + e)^2 + 2*a)/(a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))
+ 3*b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(72) = 144.
time = 2.28, size = 395, normalized size = 4.82

$$\left[-\frac{4\,(2\,a-3\,b)\cos(fx+e)^3-3\left((a-b)\cos(fx+e)^2+b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(fx+e)^4-2\,(3\,ab+b^2)\cos(fx+e)^2+4\left((a^2+ab)\cos(fx+e)^3-ab\cos(fx+e)\right)\sqrt{-\frac{b}{a}}\sin(fx+e)+b^2}{(a^2-2\,ab+b^2)\cos(fx+e)^4+2\,(ab-b^2)\cos(fx+e)^2+b^2}\right)\sin(fx+e)+12\,b\cos(fx+e)}{8\left(a^2bf+(a^3-a^2b)f\cos(fx+e)^2\right)\sin(fx+e)},-\frac{2\,(2\,a-3\,b)\cos(fx+e)^3-3\left((a-b)\cos(fx+e)^2+b\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)+6\,b\cos(fx+e)}{4\left(a^2bf+(a^3-a^2b)f\cos(fx+e)^2\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(-
b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)
^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x +
 e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)
^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/((a^2*b*f + (a^3 - a^2*b)*f*co
s(f*x + e)^2)*sin(f*x + e)), -1/4*(2*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b
)*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqr
t(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 6*b*cos(f*x + e))/((a^
2*b*f + (a^3 - a^2*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.69, size = 93, normalized size = 1.13

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,a^2}+\frac{3\,b\tan(fx+e)^2+2\,a}{\left(b\tan(fx+e)^3+a\tan(fx+e)\right)a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*b/(sqrt(a*b)*a^2) + (3*b*tan(f*x + e)^2 + 2*a)/((b*tan(f*x + e)^3 + a
*tan(f*x + e))*a^2))/f
```

Mupad [B]
time = 11.50, size = 70, normalized size = 0.85

$$-\frac{\frac{1}{a}+\frac{3\,b\,\tan(e+f\,x)^2}{2\,a^2}}{f\,\left(b\tan\left(e+f\,x\right)^3+a\tan\left(e+f\,x\right)\right)}-\frac{3\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a}}\right)}{2\,a^{5/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] - (1/a + (3*b*tan(e + f*x)^2)/(2*a^2))/(f*(a*tan(e + f*x) + b*tan(e + f*x)^
3)) - (3*b^(1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2)))/(2*a^(5/2)*f)
```

3.78 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} dx$

Optimal. Leaf size=116

$$-\frac{(3a-5b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{(a-2b)\cot(e+fx)}{a^3f}-\frac{\cot^3(e+fx)}{3a^2f}-\frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -(a-2*b)*cot(f*x+e)/a^3/f-1/3*cot(f*x+e)^3/a^2/f-1/2*(3*a-5*b)*arctan(b^(1/
2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(7/2)/f-1/2*(a-b)*b*tan(f*x+e)/a^3/f/(a+b*
tan(f*x+e)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3744, 467, 1275, 211}

$$-\frac{\sqrt{b}\,(3a-5b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{b(a-b)\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)}-\frac{(a-2b)\cot(e+fx)}{a^3f}-\frac{\cot^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - 5*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*
f) - ((a - 2*b)*Cot[e + f*x])/(a^3*f) - Cot[e + f*x]^3/(3*a^2*f) - ((a - b)
*b*Tan[e + f*x])/(2*a^3*f*(a + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int \frac{-\frac{2}{ab}-\frac{2(a-b)x^2}{a^2b}+\frac{(a-b)x^4}{a^3}}{x^4(a+bx^2)}\,dx, x, \tan(e+fx\right.}{2f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int\left(-\frac{2}{a^2bx^4}-\frac{2(a-2b)}{a^3bx^2}+\frac{3a-5b}{a^3(a+bx^2)}\right)dx, x,\right.}{2f}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^3f} - \frac{\cot^3(e+fx)}{3a^2f} - \frac{(a-b)b\tan(e+fx)}{2a^3f\left(a+b\tan^2(e+fx)\right)} - \frac{((3a-}{}$$

$$= -\frac{(3a-5b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{(a-2b)\cot(e+fx)}{a^3f} - \frac{\cot^3(e+fx)}{3a^2f}$$

Mathematica [A]
time = 0.66, size = 112, normalized size = 0.97

$$\frac{3\sqrt{b}\,(-3a+5b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}\,\left(-2\cot(e+fx)\left(2a-6b+a\csc^2(e+fx)\right)+\frac{3b(-a+b)\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))}\right)}{6a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*(-3*a + 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-
2*Cot[e + f*x]*(2*a - 6*b + a*Csc[e + f*x]^2) + (3*b*(-a + b)*Sin[2*(e + f*
x)])/(a + b + (a - b)*Cos[2*(e + f*x)])))/(6*a^(7/2)*f)
```

Maple [A]
time = 0.31, size = 100, normalized size = 0.86

method	result
derivativedivides	$\frac{-\frac{1}{3a^{2}\tan(fx+e)^{3}}-\frac{a-2b}{a^{3}\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^{2}(fx+e)\right)}+\frac{(3a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}}}{f}$
default	$\frac{-\frac{1}{3a^{2}\tan(fx+e)^{3}}-\frac{a-2b}{a^{3}\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^{2}(fx+e)\right)}+\frac{(3a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}}}{f}$
risch	$\frac{i\left(9ab\,e^{8i(fx+e)}-15b^{2}e^{8i(fx+e)}+12a^{2}e^{6i(fx+e)}-6ab\,e^{6i(fx+e)}+60b^{2}e^{6i(fx+e)}+20a^{2}e^{4i(fx+e)}+4ab\,e^{4i(fx+e)}-90b^{2}e^{4i(\right.}{3f\,a^{3}\left(e^{2i(fx+e)}-1\right)^{3}\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3/a^2/tan(f*x+e)^3-(a-2*b)/a^3/tan(f*x+e)-1/a^3*b*((1/2*a-1/2*b)*ta
n(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(3*a-5*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(
a*b)^(1/2))))
```

Maxima [A]
time = 0.52, size = 120, normalized size = 1.03

$$-\frac{\frac{3\,(3\,ab-5\,b^{2})\tan(fx+e)^{4}+2\,(3\,a^{2}-5\,ab)\tan(fx+e)^{2}+2\,a^{2}}{a^{3}b\tan(fx+e)^{5}+a^{4}\tan(fx+e)^{3}}+\frac{3\,(3\,ab-5\,b^{2})\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*((3*(3*a*b - 5*b^2)*tan(f*x + e)^4 + 2*(3*a^2 - 5*a*b)*tan(f*x + e)^2
+ 2*a^2)/(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3) + 3*(3*a*b - 5*b^2)*ar
ctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(107) = 214.
time = 3.07, size = 615, normalized size = 5.30

$$\left[-\frac{4\,(4\,a^{2}-19\,ab+15\,b^{2})\cos(fx+e)^{5}-8\,(3\,a^{2}-14\,ab+15\,b^{2})\cos(fx+e)^{3}+3\,\left((3\,a^{2}-8\,ab+5\,b^{2})\cos(fx+e)^{4}-(3\,a^{2}-11\,ab+10\,b^{2})\cos(fx+e)^{2}-3\,ab+5\,b^{2}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{(a^{2}+6\,ab+b^{2})\cos(fx+e)^{4}-2\,(3\,ab+b^{2})\cos(fx+e)^{2}-4\left((a^{2}+ab)\cos(fx+e)^{3}-ab\cos(fx+e)\right)\sqrt{-\frac{b}{a}}\sin(fx+e)+b^{2}}{(a^{2}-2\,ab+b^{2})\cos(fx+e)^{4}+2\,(ab-b^{2})\cos(fx+e)^{2}+b^{2}}\right)\sin(fx+e)-12\,(3\,ab-5\,b^{2})\cos(fx+e)}{24\left((a^{4}-a^{3}b)f\cos(fx+e)^{4}-a^{3}bf-(a^{4}-2\,a^{3}b)f\cos(fx+e)^{2}\right)\sin(fx+e)},\ -\frac{2\,(4\,a^{2}-19\,ab+15\,b^{2})\cos(fx+e)^{5}-4\,(3\,a^{2}-14\,ab+15\,b^{2})\cos(fx+e)^{3}-3\,\left((3\,a^{2}-8\,ab+5\,b^{2})\cos(fx+e)^{4}-(3\,a^{2}-11\,ab+10\,b^{2})\cos(fx+e)^{2}-3\,ab+5\,b^{2}\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(fx+e)^{2}-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)-6\,(3\,ab-5\,b^{2})\cos(fx+e)}{12\left((a^{4}-a^{3}b)f\cos(fx+e)^{4}-a^{3}bf-(a^{4}-2\,a^{3}b)f\cos(fx+e)^{2}\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(4*a^2 - 19*a*b + 15*b^2)*cos(f*x + e)^5 - 8*(3*a^2 - 14*a*b + 15
*b^2)*cos(f*x + e)^3 + 3*((3*a^2 - 8*a*b + 5*b^2)*cos(f*x + e)^4 - (3*a^2 -
 11*a*b + 10*b^2)*cos(f*x + e)^2 - 3*a*b + 5*b^2)*sqrt(-b/a)*log(((a^2 + 6*
a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a^2 + a*b)
*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 -
2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x
+ e) - 12*(3*a*b - 5*b^2)*cos(f*x + e))/(((a^4 - a^3*b)*f*cos(f*x + e)^4 -
a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/12*(2*(4*a^2
- 19*a*b + 15*b^2)*cos(f*x + e)^5 - 4*(3*a^2 - 14*a*b + 15*b^2)*cos(f*x + e
)^3 - 3*((3*a^2 - 8*a*b + 5*b^2)*cos(f*x + e)^4 - (3*a^2 - 11*a*b + 10*b^2)
*cos(f*x + e)^2 - 3*a*b + 5*b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)
^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(3*a*b -
5*b^2)*cos(f*x + e))/(((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2*
a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2)**2, x)
```

Giac [A]
time = 0.73, size = 142, normalized size = 1.22

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3ab-5b^2)}{\sqrt{ab}\,a^3}+\frac{3\left(ab\tan(fx+e)-b^2\tan(fx+e)\right)}{\left(b\tan(fx+e)^2+a\right)a^3}+\frac{2\left(3a\tan(fx+e)^2-6b\tan(fx+e)^2+a\right)}{a^3\tan(fx+e)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*(3*a*b - 5*b^2)/(sqrt(a*b)*a^3) + 3*(a*b*tan(f*x + e) - b^2*tan(f*x +
 e))/((b*tan(f*x + e)^2 + a)*a^3) + 2*(3*a*tan(f*x + e)^2 - 6*b*tan(f*x + e
)^2 + a)/(a^3*tan(f*x + e)^3))/f
```

Mupad [B]
time = 11.37, size = 108, normalized size = 0.93

$$-\frac{\frac{1}{3\,a}+\frac{\tan(e+f\,x)^2\,(3\,a-5\,b)}{3\,a^2}+\frac{b\tan(e+f\,x)^4\,(3\,a-5\,b)}{2\,a^3}}{f\,\left(b\tan\left(e+f\,x\right)^5+a\tan\left(e+f\,x\right)^3\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a}}\right)\,(3\,a-5\,b)}{2\,a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] - (1/(3*a) + (tan(e + f*x)^2*(3*a - 5*b))/(3*a^2) + (b*tan(e + f*x)^4*(3*a
- 5*b))/(2*a^3))/(f*(a*tan(e + f*x)^3 + b*tan(e + f*x)^5)) - (b^(1/2)*atan(
(b^(1/2)*tan(e + f*x))/a^(1/2))*(3*a - 5*b))/(2*a^(7/2)*f)
```

3.79 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=182

$$-\frac{(3a-7b)(a-b)\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f}-\frac{(5a^2-20ab+14b^2)\cot(e+fx)}{5a^4f}-\frac{(10a-7b)\cot^3(e+fx)}{15a^3f}-$$

```
[Out] -1/5*(5*a^2-20*a*b+14*b^2)*cot(f*x+e)/a^4/f-1/15*(10*a-7*b)*cot(f*x+e)^3/a^
3/f-1/2*(3*a-7*b)*(a-b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(9/2)/
f-1/5*cot(f*x+e)^5/a/f/(a+b*tan(f*x+e)^2)-1/10*b*(5*a^2-10*a*b+7*b^2)*tan(f
*x+e)/a^4/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 473, 467, 1275, 211}

$$-\frac{\sqrt{b}\,(3a-7b)(a-b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f}-\frac{(10a-7b)\cot^3(e+fx)}{15a^3f}-\frac{b(5a^2-10ab+7b^2)\tan(e+fx)}{10a^4f\,(a+b\tan^2(e+fx))}-\frac{(5a^2-20ab+14b^2)\cot(e+fx)}{5a^4f}-\frac{\cot^5(e+fx)}{5af\,(a+b\tan^2(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*((3*a - 7*b)*(a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(
a^(9/2)*f) - ((5*a^2 - 20*a*b + 14*b^2)*Cot[e + f*x])/(5*a^4*f) - ((10*a -
7*b)*Cot[e + f*x]^3)/(15*a^3*f) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e + f*x]
^2)) - (b*(5*a^2 - 10*a*b + 7*b^2)*Tan[e + f*x])/(10*a^4*f*(a + b*Tan[e + f
*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{10a-7b+5ax^2}{x^4(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)} - \frac{b(5a^2-10ab+7b^2)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)} - \frac{b(5a^2-10ab+7b^2)\tan(e+fx)}{10a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{b\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(5a^2-20ab+14b^2)\cot(e+fx)}{5a^4f} - \frac{(10a-7b)\cot^3(e+fx)}{15a^3f} - \frac{\cot^5(e}{5af\left(a+b\,\text{ta}\right.}$$

$$= -\frac{(3a-7b)(a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f} - \frac{(5a^2-20ab+14b^2)\cot(e+}{5a^4f}$$

Mathematica [A]

time = 1.27, size = 151, normalized size = 0.83

$$\frac{-15\sqrt{b}\left(3a^2-10ab+7b^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}\left(-2\cot(e+fx)\left(8a^2-50ab+45b^2+2a(2a-5b)\csc^2(e+fx)+3a^2\csc^4(e+fx)\right)-\frac{15(a-b)^2b\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))}\right)}{30a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-15*Sqrt[b]*(3*a^2 - 10*a*b + 7*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
] + Sqrt[a]*(-2*Cot[e + f*x]*(8*a^2 - 50*a*b + 45*b^2 + 2*a*(2*a - 5*b)*Csc
[e + f*x]^2 + 3*a^2*Csc[e + f*x]^4) - (15*(a - b)^2*b*Sin[2*(e + f*x)])/(a
+ b + (a - b)*Cos[2*(e + f*x)])))/(30*a^(9/2)*f)
```

Maple [A]

time = 0.36, size = 144, normalized size = 0.79

method	result
derivativedivides	$\frac{-\frac{1}{5a^2\tan(fx+e)^5}-\frac{2a-2b}{3a^3\tan(fx+e)^3}-\frac{a^2-4ab+3b^2}{a^4\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{1}{2}a^2-ab+\frac{1}{2}b^2\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{\left(3a^2-10ab+7b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}}{f}$
default	$\frac{-\frac{1}{5a^2\tan(fx+e)^5}-\frac{2a-2b}{3a^3\tan(fx+e)^3}-\frac{a^2-4ab+3b^2}{a^4\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{1}{2}a^2-ab+\frac{1}{2}b^2\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{\left(3a^2-10ab+7b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}}{f}$
risch	$i\left(105b^3-240a^3e^{6i(fx+e)}-2100b^3e^{6i(fx+e)}-16a^3e^{4i(fx+e)}+105b^3e^{12i(fx+e)}+1575b^3e^{4i(fx+e)}+48a^3e^{2i(fx+e)}-630b^3e^{2i}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5/a^2/tan(f*x+e)^5-1/3*(2*a-2*b)/a^3/tan(f*x+e)^3-(a^2-4*a*b+3*b^2)
/a^4/tan(f*x+e)-b/a^4*((1/2*a^2-a*b+1/2*b^2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+
1/2*(3*a^2-10*a*b+7*b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]

time = 0.50, size = 167, normalized size = 0.92

$$-\frac{\frac{15\left(3a^2b-10ab^2+7b^3\right)\tan(fx+e)^6+10\left(3a^3-10a^2b+7ab^2\right)\tan(fx+e)^4+6a^3+2\left(10a^3-7a^2b\right)\tan(fx+e)^2}{a^4b\tan(fx+e)^7+a^5\tan(fx+e)^5}+\frac{15\left(3a^2b-10ab^2+7b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*((15*(3*a^2*b - 10*a*b^2 + 7*b^3)*tan(f*x + e)^6 + 10*(3*a^3 - 10*a^2
*b + 7*a*b^2)*tan(f*x + e)^4 + 6*a^3 + 2*(10*a^3 - 7*a^2*b)*tan(f*x + e)^2)
/(a^4*b*tan(f*x + e)^7 + a^5*tan(f*x + e)^5) + 15*(3*a^2*b - 10*a*b^2 + 7*b
^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(171) = 342$.
time = 3.30, size = 889, normalized size = 4.88

$$\left[-\frac{4(16a^3-131a^2b+220ab^2-105b^3)\cos(fx+e)^7-4(40a^3-321a^2b+590ab^2-315b^3)\cos(fx+e)^5+20(6a^3-47a^2b+104ab^2-63b^3)\cos(fx+e)^3-15\left((3a^3-13a^2b+17ab^2-7b^3)\cos(fx+e)^6-(6a^3-29a^2b+44ab^2-21b^3)\cos(fx+e)^4+3a^2b-10ab^2+7b^3+(3a^3-19a^2b+37ab^2-21b^3)\cos(fx+e)^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6ab+b^2)\cos(fx+e)^4-2(3ab+b^2)\cos(fx+e)^2+4\left((a^2+ab)\cos(fx+e)^3-ab\cos(fx+e)\right)\sqrt{-\frac{b}{a}}\sin(fx+e)+b^2}{(a^2-2ab+b^2)\cos(fx+e)^4+2(ab-b^2)\cos(fx+e)^2+b^2}\right)\sin(fx+e)+60(3a^2b-10ab^2+7b^3)\cos(fx+e)}{120\left((a^5-a^4b)f\cos(fx+e)^6+a^4bf-(2a^5-3a^4b)f\cos(fx+e)^4+(a^5-3a^4b)f\cos(fx+e)^2\right)\sin(fx+e)}, -\frac{2(16a^3-131a^2b+220ab^2-105b^3)\cos(fx+e)^7-2(40a^3-321a^2b+590ab^2-315b^3)\cos(fx+e)^5+10(6a^3-47a^2b+104ab^2-63b^3)\cos(fx+e)^3-15\left((3a^3-13a^2b+17ab^2-7b^3)\cos(fx+e)^6-(6a^3-29a^2b+44ab^2-21b^3)\cos(fx+e)^4+3a^2b-10ab^2+7b^3+(3a^3-19a^2b+37ab^2-21b^3)\cos(fx+e)^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{((a+b)\cos(fx+e)^2-b)\sqrt{\frac{b}{a}}}{2b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)+30(3a^2b-10ab^2+7b^3)\cos(fx+e)}{60\left((a^5-a^4b)f\cos(fx+e)^6+a^4bf-(2a^5-3a^4b)f\cos(fx+e)^4+(a^5-3a^4b)f\cos(fx+e)^2\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 4*(4
0*a^3 - 321*a^2*b + 590*a*b^2 - 315*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 47*a^
2*b + 104*a*b^2 - 63*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2
 - 7*b^3)*cos(f*x + e)^6 - (6*a^3 - 29*a^2*b + 44*a*b^2 - 21*b^3)*cos(f*x +
 e)^4 + 3*a^2*b - 10*a*b^2 + 7*b^3 + (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)
*cos(f*x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*
a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e
))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(
a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^2*b - 10*a*b^2 + 7
*b^3)*cos(f*x + e))/(((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3
*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2)*sin(f*x + e)),
 -1/60*(2*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 2*(40
*a^3 - 321*a^2*b + 590*a*b^2 - 315*b^3)*cos(f*x + e)^5 + 10*(6*a^3 - 47*a^2
*b + 104*a*b^2 - 63*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2
- 7*b^3)*cos(f*x + e)^6 - (6*a^3 - 29*a^2*b + 44*a*b^2 - 21*b^3)*cos(f*x +
e)^4 + 3*a^2*b - 10*a*b^2 + 7*b^3 + (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*
cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)
/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(3*a^2*b - 10*a*b^2 + 7*b
^3)*cos(f*x + e))/(((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a
^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.79, size = 212, normalized size = 1.16

$$-\frac{\frac{15\,(3\,a^2b-10\,ab^2+7\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,a^4}+\frac{15\,(a^2b\tan(fx+e)-2\,ab^2\tan(fx+e)+b^3\tan(fx+e))}{\left(b\tan(fx+e)^2+a\right)a^4}+\frac{2\left(15\,a^2\tan(fx+e)^4-60\,ab\tan(fx+e)^4+45\,b^2\tan(fx+e)^4+10\,a^2\tan(fx+e)^2-10\,ab\tan(fx+e)^2+3\,a^2\right)}{a^4\tan(fx+e)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*a^2*b - 10*a*b^2 + 7*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
 + arctan(b*tan(f*x + e)/sqrt(a*b)))/(sqrt(a*b)*a^4) + 15*(a^2*b*tan(f*x +
e) - 2*a*b^2*tan(f*x + e) + b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a)*a^4)
+ 2*(15*a^2*tan(f*x + e)^4 - 60*a*b*tan(f*x + e)^4 + 45*b^2*tan(f*x + e)^4
+ 10*a^2*tan(f*x + e)^2 - 10*a*b*tan(f*x + e)^2 + 3*a^2)/(a^4*tan(f*x + e)^
5))/f
```

Mupad [B]
time = 12.37, size = 178, normalized size = 0.98

$$-\frac{\frac{1}{5\,a}+\frac{\tan(e+f\,x)^4\,(3\,a^2-10\,a\,b+7\,b^2)}{3\,a^3}+\frac{\tan(e+f\,x)^2\,(10\,a-7\,b)}{15\,a^2}+\frac{b\tan(e+f\,x)^6\,(3\,a^2-10\,a\,b+7\,b^2)}{2\,a^4}}{f\,\left(b\tan\left(e+f\,x\right)^7+a\tan\left(e+f\,x\right)^5\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)\,(a-b)\,(3\,a-7\,b)}{\sqrt{a}\,(3\,a^2-10\,a\,b+7\,b^2)}\right)\,(a-b)\,(3\,a-7\,b)}{2\,a^{9/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^2),x)
```

```
[Out] - (1/(5*a) + (tan(e + f*x)^4*(3*a^2 - 10*a*b + 7*b^2))/(3*a^3) + (tan(e + f
*x)^2*(10*a - 7*b))/(15*a^2) + (b*tan(e + f*x)^6*(3*a^2 - 10*a*b + 7*b^2))/
(2*a^4))/(f*(a*tan(e + f*x)^5 + b*tan(e + f*x)^7)) - (b^(1/2)*atan((b^(1/2)
*tan(e + f*x)*(a - b)*(3*a - 7*b))/(a^(1/2)*(3*a^2 - 10*a*b + 7*b^2)))*(a -
 b)*(3*a - 7*b))/(2*a^(9/2)*f)
```

3.80 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt{b}\,(15a^2+40ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{11/2}f}-\frac{(5a^2+20ab+2b^2)\cos(e+fx)}{5(a-b)^5f}+\frac{(10a-b)\cos^3(e+fx)}{15(a-b)^4f}$$

```
[Out] -1/5*(5*a^2+20*a*b+2*b^2)*cos(f*x+e)/(a-b)^5/f+1/15*(10*a-b)*cos(f*x+e)^3/(
a-b)^4/f-1/5*cos(f*x+e)^5/(a-b)/f/(a-b+b*sec(f*x+e)^2)^2-1/20*b*(5*a^2+4*b^
2)*sec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)^2-1/40*b*(35*a^2+40*a*b+24*b^2
)*sec(f*x+e)/(a-b)^5/f/(a-b+b*sec(f*x+e)^2)-1/8*(15*a^2+40*a*b+8*b^2)*arcta
n(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/(a-b)^(11/2)/f
```

Rubi [A]
time = 0.30, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3745, 473, 467, 1273, 1275, 211}

$$-\frac{\sqrt{b}\,(15a^2+40ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8f(a-b)^{11/2}}-\frac{(5a^2+20ab+2b^2)\cos(e+fx)}{5f(a-b)^5}-\frac{b(35a^2+40ab+24b^2)\sec(e+fx)}{40f(a-b)^5\left(a+b\sec^2(e+fx)-b\right)}-\frac{b(5a^2+4b^2)\sec(e+fx)}{20f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^2}+\frac{(10a-b)\cos^3(e+fx)}{15f(a-b)^4}-\frac{\cos^5(e+fx)}{5f(a-b)\left(a+b\sec^2(e+fx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[
a - b]])/((a - b)^(11/2)*f) - ((5*a^2 + 20*a*b + 2*b^2)*Cos[e + f*x])/(5*(a
 - b)^5*f) + ((10*a - b)*Cos[e + f*x]^3)/(15*(a - b)^4*f) - Cos[e + f*x]^5/
(5*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^2) - (b*(5*a^2 + 4*b^2)*Sec[e + f*x
])/(20*(a - b)^4*f*(a - b + b*Sec[e + f*x]^2)^2) - (b*(35*a^2 + 40*a*b + 24
*b^2)*Sec[e + f*x])/(40*(a - b)^5*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x^6(a-b+bx^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\,(a-b+b\sec^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{-10a+b+5(a-b)x^2}{x^4(a-b+bx^2)^3}\,dx, x, \sec(e\right.}{5(a-b)f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\,(a-b+b\sec^2(e+fx))^2} - \frac{b(5a^2+4b^2)\sec(e+fx)}{20(a-b)^4f\,(a-b+b\sec^2(e+fx))}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\,(a-b+b\sec^2(e+fx))^2} - \frac{b(5a^2+4b^2)\sec(e+fx)}{20(a-b)^4f\,(a-b+b\sec^2(e+fx))}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\,(a-b+b\sec^2(e+fx))^2} - \frac{b(5a^2+4b^2)\sec(e+fx)}{20(a-b)^4f\,(a-b+b\sec^2(e+fx))}$$

$$= -\frac{(5a^2+20ab+2b^2)\cos(e+fx)}{5(a-b)^5f} + \frac{(10a-b)\cos^3(e+fx)}{15(a-b)^4f} - \frac{\cos}{5(a-b)f\,(a-}$$

$$= -\frac{\sqrt{b}\,(15a^2+40ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{11/2}f} - \frac{(5a^2+20ab+2b^2)\cos(e}{5(a-b)^5f}$$

Mathematica [A]
time = 3.69, size = 278, normalized size = 1.05

$$\frac{\frac{30\sqrt{b}\,(15a^2+40ab+8b^2)\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{11/2}} + \frac{30\sqrt{b}\,(15a^2+40ab+8b^2)\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{11/2}} + \frac{-30\cos(e+fx)\left(11b^2+16ab\left(2+\frac{b}{a+b+(a-b)\cos(2(e+fx))}\right)+a^2\left(5-\frac{8b^2}{(a+b+(a-b)\cos(2(e+fx)))^2}+\frac{18b}{a+b+(a-b)\cos(2(e+fx))}\right)\right)+(a-b)(5(5a+7b)\cos(3(e+fx))+3(-a+b)\cos(5(e+fx)))}{(a-b)^5}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((30*Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e
 + f*x)/2])/Sqrt[b]])/(a - b)^(11/2) + (30*Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2
)*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(11/2)
+ (-30*Cos[e + f*x]*(11*b^2 + 16*a*b*(2 + b/(a + b + (a - b)*Cos[2*(e + f*x
)])) + a^2*(5 - (8*b^2)/(a + b + (a - b)*Cos[2*(e + f*x)])^2 + (18*b)/(a +
b + (a - b)*Cos[2*(e + f*x)]))) + (a - b)*(5*(5*a + 7*b)*Cos[3*(e + f*x)] +
 3*(-a + b)*Cos[5*(e + f*x)]))/(a - b)^5)/(240*f)
```

Maple [A]
time = 0.66, size = 282, normalized size = 1.07

method	result
derivativedivides	$-\frac{\frac{a^2\left(\cos^5(fx+e)\right)}{5}-\frac{2ab\left(\cos^5(fx+e)\right)}{5}+\frac{b^2\left(\cos^5(fx+e)\right)}{5}-\frac{2a^2\left(\cos^3(fx+e)\right)}{3}+\frac{ab\left(\cos^3(fx+e)\right)}{3}+\frac{b^2\left(\cos^3(fx+e)\right)}{3}+a^2\cos(fx+e)+4}{\left(a^3-3a^2b+3a\,b^2-b^3\right)\left(a^2-2ab+b^2\right)}$
default	$-\frac{\frac{a^2\left(\cos^5(fx+e)\right)}{5}-\frac{2ab\left(\cos^5(fx+e)\right)}{5}+\frac{b^2\left(\cos^5(fx+e)\right)}{5}-\frac{2a^2\left(\cos^3(fx+e)\right)}{3}+\frac{ab\left(\cos^3(fx+e)\right)}{3}+\frac{b^2\left(\cos^3(fx+e)\right)}{3}+a^2\cos(fx+e)+4}{\left(a^3-3a^2b+3a\,b^2-b^3\right)\left(a^2-2ab+b^2\right)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*(1/5*a^2*cos(f*x+e)^5-2/5
*a*b*cos(f*x+e)^5+1/5*b^2*cos(f*x+e)^5-2/3*a^2*cos(f*x+e)^3+1/3*a*b*cos(f*x
+e)^3+1/3*b^2*cos(f*x+e)^3+a^2*cos(f*x+e)+4*a*b*cos(f*x+e)+b^2*cos(f*x+e))+
b/(a-b)^5*((-1/8*a*(9*a^2-a*b-8*b^2)*cos(f*x+e)^3+(-7/8*a^2*b-a*b^2)*cos(f*
x+e))/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2+1/8*(15*a^2+40*a*b+8*b^2)/(b*(a-b
))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 4.29, size = 1040, normalized size = 3.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(48*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 16
*(10*a^4 - 31*a^3*b + 33*a^2*b^2 - 13*a*b^3 + b^4)*cos(f*x + e)^7 + 16*(15*
a^4 + 10*a^3*b - 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^5 + 50*(15*a^3
*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^3 + 15*((15*a^4 + 10*a^3*b
 - 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 +
8*b^4 + 2*(15*a^3*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-
b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*
x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 40*a*b^3 + 8*b
^4)*cos(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 -
21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^
4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^5*b^
2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*f), -1/120*(24*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 8*(10*a^4 - 31*a^
3*b + 33*a^2*b^2 - 13*a*b^3 + b^4)*cos(f*x + e)^7 + 8*(15*a^4 + 10*a^3*b -
57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^5 + 25*(15*a^3*b + 25*a^2*b^2 -
 32*a*b^3 - 8*b^4)*cos(f*x + e)^3 + 15*((15*a^4 + 10*a^3*b - 57*a^2*b^2 + 2
4*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 + 8*b^4 + 2*(15*a^3
*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(b/(a - b))*arctan(
-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*(15*a^2*b^2 + 40*a*b^3 + 8*b^
4)*cos(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 - 2
1*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^4
*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^5*b^2
 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 885 vs. 2(253) = 506.
time = 1.19, size = 885, normalized size = 3.35

$$-\frac{\frac{15\left(15 a^{2} b+40 a b^{2}+8 b^{3}\right) \arctan\left(-\frac{a \cos(fx+e)-b \cos(fx+e)-b}{\sqrt{ab-b^{2}} \cos(fx+e)+\sqrt{ab-b^{2}}}\right)}{\left(a^{5}-5 a^{4} b+10 a^{3} b^{2}-10 a^{2} b^{3}+5 a b^{4}-b^{5}\right) \sqrt{ab-b^{2}}}+\frac{30\left(9 a^{3} b+6 a^{2} b^{2}+\frac{27 a^{3} b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{32 a^{2} b^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{40 a b^{3}(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{27 a^{3} b(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{54 a^{2} b^{2}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{24 a b^{3}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{48 b^{4}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{9 a^{3} b(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{16 a^{2} b^{2}(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{8 a b^{3}(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{\left(a^{5}-5 a^{4} b+10 a^{3} b^{2}-10 a^{2} b^{3}+5 a b^{4}-b^{5}\right)\left(a+\frac{2 a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4 b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}\right)^{2}}-\frac{16\left(8 a^{2}+59 a b+23 b^{2}-\frac{40 a^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{250 a b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{70 b^{2}(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80 a^{2}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{320 a b(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{140 b^{2}(\cos(fx+e)-1)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{270 a b(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{90 b^{2}(\cos(fx+e)-1)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{45 a b(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{45 b^{2}(\cos(fx+e)-1)^{4}}{(\cos(fx+e)+1)^{4}}\right)}{\left(a^{5}-5 a^{4} b+10 a^{3} b^{2}-10 a^{2} b^{3}+5 a b^{4}-b^{5}\right)\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^{5}}}{120 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b + 40*a*b^2 + 8*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*
x + e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - 5*a^4
```

*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sqrt(a*b - b^2)) + 30*(9*a^3*
b + 6*a^2*b^2 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 32*a^2*b^2
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 40*a*b^3*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 54*a^2*
b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 24*a*b^3*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2 + 48*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 16*a^2*b^2*(cos(f*x +
e) - 1)^3/(cos(f*x + e) + 1)^3 - 8*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(a + 2
a(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 16*(8*a^2 + 5
9*a*b + 23*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 250*a*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 70*b^2*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 320*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 140*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 270*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 90*b^2*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 45*a*b*(cos(f*x + e) - 1)^4/(cos(
f*x + e) + 1)^4 + 45*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^5 -
5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*((cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 1)^5))/f

Mupad [B]
time = 16.33, size = 1536, normalized size = 5.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^3,x)

[Out] (b^(1/2)*atan(((a - b)^11*(2*tan(e/2 + (f*x)/2)^2*((b^(1/2)*(40*a*b + 15*a^
2 + 8*b^2)*(640*a^3*b^12 - 128*a^2*b^13 - 240*a^14*b + 400*a^4*b^11 - 11040
*a^5*b^10 + 39120*a^6*b^9 - 73344*a^7*b^8 + 84000*a^8*b^7 - 58560*a^9*b^6 +
20640*a^10*b^5 + 1280*a^11*b^4 - 4528*a^12*b^3 + 1760*a^13*b^2))/(16*a*(a
- b)^(21/2)) - (b^(1/2)*(a - 2*b)*(40*a*b + 15*a^2 + 8*b^2)^2*(128*a^18 - 2
176*a^17*b + 256*a^2*b^16 - 3968*a^3*b^15 + 28800*a^4*b^14 - 129920*a^5*b^1
3 + 407680*a^6*b^12 - 943488*a^7*b^11 + 1665664*a^8*b^10 - 2288000*a^9*b^9
+ 2471040*a^10*b^8 - 2104960*a^11*b^7 + 1409408*a^12*b^6 - 733824*a^13*b^5
+ 291200*a^14*b^4 - 85120*a^15*b^3 + 17280*a^16*b^2))/(512*a*(a - b)^(33/2)
)) - (b^(1/2)*(a - 2*b)*(40*a*b + 15*a^2 + 8*b^2)^2*(1920*a^17*b - 128*a^18
+ 128*a^3*b^15 - 1920*a^4*b^14 + 13440*a^5*b^13 - 58240*a^6*b^12 + 174720*
a^7*b^11 - 384384*a^8*b^10 + 640640*a^9*b^9 - 823680*a^10*b^8 + 823680*a^11
*b^7 - 640640*a^12*b^6 + 384384*a^13*b^5 - 174720*a^14*b^4 + 58240*a^15*b^3
- 13440*a^16*b^2))/(256*a*(a - b)^(33/2))))/(225*a^16*b + 64*a^2*b^15 - 16
80*a^4*b^13 + 3920*a^5*b^12 + 7665*a^6*b^11 - 50778*a^7*b^10 + 104685*a^8*b
^9 - 111960*a^9*b^8 + 57330*a^10*b^7 + 2660*a^11*b^6 - 20286*a^12*b^5 + 924

```
0*a^13*b^4 - 35*a^14*b^3 - 1050*a^15*b^2))*(40*a*b + 15*a^2 + 8*b^2))/(8*f*
(a - b)^(11/2)) - ((607*a^3*b + 64*a^4 + 274*a^2*b^2)/(60*(a - b)*(a^4 - 4*
a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e/2 + (f*x)/2)^14*(128*a*b^3 + 1
5*a^3*b + 24*b^4 + 85*a^2*b^2))/(2*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
 6*a^2*b^2)) + (tan(e/2 + (f*x)/2)^12*(936*a*b^3 - 365*a^3*b + 64*a^4 + 936
*b^4 + 1075*a^2*b^2))/(6*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2
)) + (tan(e/2 + (f*x)/2)^10*(4268*a*b^3 + 921*a^3*b - 224*a^4 + 1872*b^4 -
1545*a^2*b^2))/(6*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (t
an(e/2 + (f*x)/2)^4*(6224*a*b^3 - 671*a^3*b - 128*a^4 + 1832*b^4 + 5973*a^2
*b^2))/(30*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e/2
+ (f*x)/2)^6*(20696*a*b^3 + 867*a^3*b - 448*a^4 + 6280*b^4 - 935*a^2*b^2))/
(30*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e/2 + (f*x)
/2)^8*(21740*a*b^3 - 4064*a^3*b + 1312*a^4 + 12560*b^4 + 1527*a^2*b^2))/(30
*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e/2 + (f*x)/2)
^2*(1036*a*b^3 + 447*a^3*b + 32*a^4 + 2265*a^2*b^2))/(30*(a - b)*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (b*tan(e/2 + (f*x)/2)^16*(8*a*b^2 + 40
*a^2*b + 15*a^3))/(4*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/
(f*(a^2*tan(e/2 + (f*x)/2)^18 + tan(e/2 + (f*x)/2)^4*(24*a*b - 4*a^2 + 16*b
^2) + tan(e/2 + (f*x)/2)^14*(24*a*b - 4*a^2 + 16*b^2) + tan(e/2 + (f*x)/2)^
6*(8*a*b - 4*a^2 + 80*b^2) + tan(e/2 + (f*x)/2)^12*(8*a*b - 4*a^2 + 80*b^2)
 + tan(e/2 + (f*x)/2)^8*(6*a^2 - 40*a*b + 160*b^2) + tan(e/2 + (f*x)/2)^10*
(6*a^2 - 40*a*b + 160*b^2) + a^2 + tan(e/2 + (f*x)/2)^2*(8*a*b + a^2) + tan
(e/2 + (f*x)/2)^16*(8*a*b + a^2)))
```

3.81 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=180

$$-\frac{5\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{9/2}f}-\frac{(a+2b)\cos(e+fx)}{(a-b)^4f}+\frac{\cos^3(e+fx)}{3(a-b)^3f}-\frac{ab\sec(e+fx)}{4(a-b)^3f\,(a-b+b\sec^2(e-}$$

```
[Out] -(a+2*b)*cos(f*x+e)/(a-b)^4/f+1/3*cos(f*x+e)^3/(a-b)^3/f-1/4*a*b*sec(f*x+e)
/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^2-1/8*b*(7*a+4*b)*sec(f*x+e)/(a-b)^4/f/(a-b
+b*sec(f*x+e)^2)-5/8*(3*a+4*b)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/
2)/(a-b)^(9/2)/f
```

Rubi [A]
time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 467, 1273, 1275, 211}

$$-\frac{5\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8f(a-b)^{9/2}}+\frac{\cos^3(e+fx)}{3f(a-b)^3}-\frac{(a+2b)\cos(e+fx)}{f(a-b)^4}-\frac{b(7a+4b)\sec(e+fx)}{8f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}-\frac{ab\sec(e+fx)}{4f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-5*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(8*(a -
 b)^(9/2)*f) - ((a + 2*b)*Cos[e + f*x])/((a - b)^4*f) + Cos[e + f*x]^3/(3*(
a - b)^3*f) - (a*b*Sec[e + f*x])/(4*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2)^
2) - (b*(7*a + 4*b)*Sec[e + f*x])/(8*(a - b)^4*f*(a - b + b*Sec[e + f*x]^2)
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a-b+bx^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b\text{Subst}\left(\int \frac{\frac{4}{(a-b)b}-\frac{4ax^2}{(a-b)^2b}+\frac{3ax^4}{(a-b)^3}}{x^4(a-b+bx^2)^2}\,dx, x, \text{s}\right.}{4f}$$

$$= -\frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b(7a+4b)\sec(e+fx)}{8(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)} -$$

$$= -\frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{b(7a+4b)\sec(e+fx)}{8(a-b)^4 f\left(a-b+b\sec^2(e+fx)\right)} -$$

$$= -\frac{(a+2b)\cos(e+fx)}{(a-b)^4 f} + \frac{\cos^3(e+fx)}{3(a-b)^3 f} - \frac{ab\sec(e+fx)}{4(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^2}$$

$$= -\frac{5\sqrt{b}\,(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{9/2} f} - \frac{(a+2b)\cos(e+fx)}{(a-b)^4 f} + \frac{\cos^3(e+fx)}{3(a-b)^3 f}$$

Mathematica [A]
time = 3.71, size = 230, normalized size = 1.28

$$\frac{\frac{15\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{9/2}} + \frac{15\sqrt{b}\,(3a+4b)\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{9/2}} + \frac{2\left(3\cos(e+fx)\left(a\left(-3+\frac{4b^2}{(a+b+(a-b)\cos(2(e+fx)))^2}-\frac{9b}{a+b+(a-b)\cos(2(e+fx))}\right)+b\left(-9-\frac{4b}{a+b+(a-b)\cos(2(e+fx))}\right)\right)+(a-b)\cos(3(e+fx))\right)}{(a-b)^4}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((15*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]])/(a - b)^(9/2) + (15*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] + Sqrt[
a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(9/2) + (2*(3*Cos[e + f*x]*(a*(-3 +
(4*b^2)/(a + b + (a - b)*Cos[2*(e + f*x)])^2 - (9*b)/(a + b + (a - b)*Cos[2
*(e + f*x)])) + b*(-9 - (4*b)/(a + b + (a - b)*Cos[2*(e + f*x)]))) + (a - b
)*Cos[3*(e + f*x)]))/(a - b)^4)/(24*f)
```

Maple [A]
time = 0.52, size = 196, normalized size = 1.09

method	result

derivativedivides	$\frac{\frac{\frac{a\left(\cos^3(fx+e)\right)}{3}-\frac{b\left(\cos^3(fx+e)\right)}{3}-\cos(fx+e)a-2b\cos(fx+e)}{\left(a^3-3a^2b+3a\,b^2-b^3\right)(a-b)}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2+\frac{5}{8}ab+\frac{1}{2}b^2\right)\left(\cos^3(fx+e)\right)+\left(-\frac{7}{8}ab-\frac{1}{2}b^2\right)\cos(fx+e)}{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)^2}+\cdots\right.}{(a-b)^4}}{f}$
default	$\frac{\frac{\frac{a\left(\cos^3(fx+e)\right)}{3}-\frac{b\left(\cos^3(fx+e)\right)}{3}-\cos(fx+e)a-2b\cos(fx+e)}{\left(a^3-3a^2b+3a\,b^2-b^3\right)(a-b)}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2+\frac{5}{8}ab+\frac{1}{2}b^2\right)\left(\cos^3(fx+e)\right)+\left(-\frac{7}{8}ab-\frac{1}{2}b^2\right)\cos(fx+e)}{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)^2}+\cdots\right.}{(a-b)^4}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24(a^3-3a^2b+3a\,b^2-b^3)f}-\frac{3\,e^{i(fx+e)}a}{8(a^3-3a^2b+3a\,b^2-b^3)(a-b)f}-\frac{9\,e^{i(fx+e)}b}{8(a^3-3a^2b+3a\,b^2-b^3)(a-b)f}-\frac{3\,e^{-i(fx+e\cdots}}{8(a^4-4a^3b+6a^2b^2-\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*(1/3*a*cos(f*x+e)^3-1/3*b*cos(f*x+e)
^3-cos(f*x+e)*a-2*b*cos(f*x+e))+b/(a-b)^4*(((-9/8*a^2+5/8*a*b+1/2*b^2)*cos(
f*x+e)^3+(-7/8*a*b-1/2*b^2)*cos(f*x+e))/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2
+5/8*(3*a+4*b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(171) = 342$.
time = 3.75, size = 795, normalized size = 4.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 16*(3*a^3 - 2*a^
2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^5 - 50*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*
x + e)^3 + 15*((3*a^3 - 2*a^2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^4 + 3*a*b^2
 + 4*b^3 + 2*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^2)*sqrt(-b/(a - b))*log
(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a
 - b)*cos(f*x + e)^2 + b)) - 30*(3*a*b^2 + 4*b^3)*cos(f*x + e))/((a^6 - 6*a
^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*x + e)
^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*f*cos(
f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f), 1/24*(8*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 8*(3*a^3 - 2*a^2*b - 5*a*b
^2 + 4*b^3)*cos(f*x + e)^5 - 25*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^3 -
15*((3*a^3 - 2*a^2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^4 + 3*a*b^2 + 4*b^3 +
2*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)
*sqrt(b/(a - b))*cos(f*x + e)/b) - 15*(3*a*b^2 + 4*b^3)*cos(f*x + e))/((a^6
 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*
x + e)^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*
f*cos(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(171) = 342.
time = 1.15, size = 563, normalized size = 3.13

$$\frac{a^6 f^{17}\cos(fx+e)^3 - 6a^5 b f^{17}\cos(fx+e)^3 + 15a^4b^2 f^{17}\cos(fx+e)^3 - 20a^3b^3 f^{17}\cos(fx+e)^3 + 15a^2b^4 f^{17}\cos(fx+e)^3 - 6ab^5 f^{17}\cos(fx+e)^3 + b^6 f^{17}\cos(fx+e)^3 - 3a^6 f^{17}\cos(fx+e) + 9a^5 b f^{17}\cos(fx+e) - 30a^3b^3 f^{17}\cos(fx+e) + 45a^2b^4 f^{17}\cos(fx+e) - 27ab^5 f^{17}\cos(fx+e) + 6b^6 f^{17}\cos(fx+e)}{3\left(a^9 f^{18} - 9a^8 b f^{18} + 36a^7b^2 f^{18} - 84a^6b^3 f^{18} + 126a^5b^4 f^{18} - 126a^4b^5 f^{18} + 84a^3b^6 f^{18} - 36a^2b^7 f^{18} + 9ab^8 f^{18} - b^9 f^{18}\right)} + \frac{5\left(3ab + 4b^2\right)\arctan\left(\frac{a\cos(fx+e) - b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{8\left(a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4\right)\sqrt{ab-b^2}\,f} - \frac{\frac{9a^2b\cos(fx+e)^3}{f} - \frac{5ab^2\cos(fx+e)^3}{f} - \frac{4b^3\cos(fx+e)^3}{f} + \frac{7ab^2\cos(fx+e)}{f} + \frac{4b^3\cos(fx+e)}{f}}{8\left(a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4\right)\left(a\cos(fx+e)^2 - b\cos(fx+e)^2 + b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/3*(a^6*f^17*cos(f*x + e)^3 - 6*a^5*b*f^17*cos(f*x + e)^3 + 15*a^4*b^2*f^1
7*cos(f*x + e)^3 - 20*a^3*b^3*f^17*cos(f*x + e)^3 + 15*a^2*b^4*f^17*cos(f*x
 + e)^3 - 6*a*b^5*f^17*cos(f*x + e)^3 + b^6*f^17*cos(f*x + e)^3 - 3*a^6*f^1
7*cos(f*x + e) + 9*a^5*b*f^17*cos(f*x + e) - 30*a^3*b^3*f^17*cos(f*x + e) +
 45*a^2*b^4*f^17*cos(f*x + e) - 27*a*b^5*f^17*cos(f*x + e) + 6*b^6*f^17*cos
(f*x + e))/(a^9*f^18 - 9*a^8*b*f^18 + 36*a^7*b^2*f^18 - 84*a^6*b^3*f^18 + 1
26*a^5*b^4*f^18 - 126*a^4*b^5*f^18 + 84*a^3*b^6*f^18 - 36*a^2*b^7*f^18 + 9*
```

```
a*b^8*f^18 - b^9*f^18) + 5/8*(3*a*b + 4*b^2)*arctan((a*cos(f*x + e) - b*cos
(f*x + e))/sqrt(a*b - b^2))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sq
rt(a*b - b^2)*f) - 1/8*(9*a^2*b*cos(f*x + e)^3/f - 5*a*b^2*cos(f*x + e)^3/f
 - 4*b^3*cos(f*x + e)^3/f + 7*a*b^2*cos(f*x + e)/f + 4*b^3*cos(f*x + e)/f)/
((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a*cos(f*x + e)^2 - b*cos(f*x
+ e)^2 + b)^2)
```

Mupad [B]
time = 15.45, size = 1154, normalized size = 6.41

$$-\frac{\frac{16a^3+83a^2b+6ab^2}{12(a-b)(a^3-3a^2b+3ab^2-b^3)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(-8a^3+299ab^2+24b^3)}{6(a-b)(a^3-3a^2b+3ab^2-b^3)}+\frac{5a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}(4b^2+3ab)}{4(a-b)(a^3-3a^2b+3ab^2-b^3)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}(8a^4-32a^3b+93a^2b^2+28ab^3+8b^4)}{2a(a-b)(a^3-3a^2b+3ab^2-b^3)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(56a^4-144a^3b+31a^2b^2+546ab^3+36b^4)}{3a(a-b)(a^3-3a^2b+3ab^2-b^3)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(-96a^4+71a^3b+344a^2b^2+1208ab^3+48b^4)}{12a(a-b)(a^3-3a^2b+3ab^2-b^3)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(-176a^4+569a^3b-666a^2b^2+1704ab^3+144b^4)}{12a(a-b)(a^3-3a^2b+3ab^2-b^3)}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(8ab-a^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}(8ab-a^2)+a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(-3a^2+8ab+16b^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}(-3a^2+8ab+16b^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(3a^2-16ab+48b^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(3a^2-16ab+48b^2)+a^2\right)}-\frac{5\sqrt{b}\,\mathrm{atan}\left(\frac{2\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{5\sqrt{b}(3a+4b)(240a^{11}b-1600a^{10}b^2+4160a^9b^3-4480a^8b^4-1120a^7b^5+8960a^6b^6-11200a^5b^7+7040a^4b^8-2320a^3b^9+320a^2b^{10})}{16a(a-b)^{17/2}}-\frac{25\sqrt{b}(a-2b)(3a+4b)^2(-128a^{15}+1792a^{14}b-11520a^{13}b^2+45056a^{12}b^3-119680a^{11}b^4+228096a^{10}b^5-321024a^9b^6+337920a^8b^7-266112a^7b^8+154880a^6b^9-64768a^5b^{10}+18432a^4b^{11}-3200a^3b^{12}+256a^2b^{13})}{512a(a-b)^{27/2}}\right)-\frac{25\sqrt{b}(a-2b)(3a+4b)^2(128a^{15}-1536a^{14}b+8448a^{13}b^2-28160a^{12}b^3+63360a^{11}b^4-101376a^{10}b^5+118272a^9b^6-101376a^8b^7+63360a^7b^8-28160a^6b^9+8448a^5b^{10}-1536a^4b^{11}+128a^3b^{12})}{512a(a-b)^{27/2}}\right)(a-b)^9}{225a^{12}b-1200a^{11}b^2+1900a^{10}b^3+1000a^9b^4-6650a^8b^5+7000a^7b^6+700a^6b^7-7400a^5b^8+6625a^4b^9-2600a^3b^{10}+400a^2b^{11}}\right)(3a+4b)}{8f(a-b)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] - ((6*a*b^2 + 83*a^2*b + 16*a^3)/(12*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3
)) + (tan(e/2 + (f*x)/2)^2*(299*a*b^2 - 8*a^3 + 24*b^3))/(6*(a - b)*(3*a*b^
2 - 3*a^2*b + a^3 - b^3)) + (5*a*tan(e/2 + (f*x)/2)^12*(3*a*b + 4*b^2))/(4*
(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^10*(28*a*b^3
 - 32*a^3*b + 8*a^4 + 8*b^4 + 93*a^2*b^2))/(2*a*(a - b)*(3*a*b^2 - 3*a^2*b
+ a^3 - b^3)) + (tan(e/2 + (f*x)/2)^6*(546*a*b^3 - 144*a^3*b + 56*a^4 + 36*
b^4 + 31*a^2*b^2))/(3*a*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2
 + (f*x)/2)^4*(1208*a*b^3 + 71*a^3*b - 96*a^4 + 48*b^4 + 344*a^2*b^2))/(12*
a*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^8*(1704*a*
b^3 + 569*a^3*b - 176*a^4 + 144*b^4 - 666*a^2*b^2))/(12*a*(a - b)*(3*a*b^2
- 3*a^2*b + a^3 - b^3)))/(f*(tan(e/2 + (f*x)/2)^2*(8*a*b - a^2) + tan(e/2 +
 (f*x)/2)^12*(8*a*b - a^2) + a^2*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*x)/2)
^4*(8*a*b - 3*a^2 + 16*b^2) + tan(e/2 + (f*x)/2)^10*(8*a*b - 3*a^2 + 16*b^2
) + tan(e/2 + (f*x)/2)^6*(3*a^2 - 16*a*b + 48*b^2) + tan(e/2 + (f*x)/2)^8*(
3*a^2 - 16*a*b + 48*b^2) + a^2)) - (5*b^(1/2)*atan((2*(tan(e/2 + (f*x)/2)^2
*((5*b^(1/2)*(3*a + 4*b)*(240*a^11*b + 320*a^2*b^10 - 2320*a^3*b^9 + 7040*a
^4*b^8 - 11200*a^5*b^7 + 8960*a^6*b^6 - 1120*a^7*b^5 - 4480*a^8*b^4 + 4160*
a^9*b^3 - 1600*a^10*b^2))/(16*a*(a - b)^(17/2)) - (25*b^(1/2)*(a - 2*b)*(3*
a + 4*b)^2*(1792*a^14*b - 128*a^15 + 256*a^2*b^13 - 3200*a^3*b^12 + 18432*a
^4*b^11 - 64768*a^5*b^10 + 154880*a^6*b^9 - 266112*a^7*b^8 + 337920*a^8*b^7
 - 321024*a^9*b^6 + 228096*a^10*b^5 - 119680*a^11*b^4 + 45056*a^12*b^3 - 11
520*a^13*b^2))/(512*a*(a - b)^(27/2))) - (25*b^(1/2)*(a - 2*b)*(3*a + 4*b)^
2*(128*a^15 - 1536*a^14*b + 128*a^3*b^12 - 1536*a^4*b^11 + 8448*a^5*b^10 -
28160*a^6*b^9 + 63360*a^7*b^8 - 101376*a^8*b^7 + 118272*a^9*b^6 - 101376*a^
10*b^5 + 63360*a^11*b^4 - 28160*a^12*b^3 + 8448*a^13*b^2))/(512*a*(a - b)^(
27/2)))*(a - b)^9)/(225*a^12*b + 400*a^2*b^11 - 2600*a^3*b^10 + 6625*a^4*b^
9 - 7400*a^5*b^8 + 700*a^6*b^7 + 7000*a^7*b^6 - 6650*a^8*b^5 + 1000*a^9*b^4
 + 1900*a^10*b^3 - 1200*a^11*b^2))*(3*a + 4*b))/(8*f*(a - b)^(9/2))
```

3.82 $\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal. Leaf size=138

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f}-\frac{15\cos(e+fx)}{8(a-b)^3f}+\frac{\cos(e+fx)}{4(a-b)f\,(a-b+b\sec^2(e+fx))^2}+\frac{5\cos(e+}{8(a-b)^2f\,(a-b+b}$$

```
[Out] -15/8*cos(f*x+e)/(a-b)^3/f+1/4*cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)^2+5/
8*cos(f*x+e)/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)-15/8*arctan(sec(f*x+e)*b^(1/2)/
(a-b)^(1/2))*b^(1/2)/(a-b)^(7/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3745, 296, 331, 211}

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}-\frac{15\cos(e+fx)}{8f(a-b)^3}+\frac{5\cos(e+fx)}{8f(a-b)^2\,(a+b\sec^2(e+fx)-b)}+\frac{\cos(e+fx)}{4f(a-b)\,(a+b\sec^2(e+fx)-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(8*(a - b)^(7/2)*f
) - (15*Cos[e + f*x])/(8*(a - b)^3*f) + Cos[e + f*x]/(4*(a - b)*f*(a - b +
b*Sec[e + f*x]^2)^2) + (5*Cos[e + f*x])/(8*(a - b)^2*f*(a - b + b*Sec[e + f
*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)^3}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2}+\frac{5\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)^2}\,dx, x, \sec(e+fx\right.}{4(a-b)f}\\ &= \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2}+\frac{5\cos(e+fx)}{8(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)}+\\ &= -\frac{15\cos(e+fx)}{8(a-b)^3f}+\frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2}+\frac{5\cos(e}{8(a-b)^2f\,(a-b}\\ &= -\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f}-\frac{15\cos(e+fx)}{8(a-b)^3f}+\frac{\cos(e+fx)}{4(a-b)f\,(a-b+b\sec^2}\end{aligned}$$

Mathematica [A]
time = 1.12, size = 170, normalized size = 1.23

$$\frac{\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}+\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}+\frac{2\cos(e+fx)\left(-4+\frac{4b^2}{(a+b+(a-b)\cos(2(e+fx)))^2}-\frac{9b}{a+b+(a-b)\cos(2(e+fx))}\right)}{(a-b)^3}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((15*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a -
 b)^(7/2) + (15*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqr
t[b]])/(a - b)^(7/2) + (2*Cos[e + f*x]*(-4 + (4*b^2)/(a + b + (a - b)*Cos[2
```

```
*(e + f*x)])^2 - (9*b)/(a + b + (a - b)*Cos[2*(e + f*x)])))/(a - b)^3)/(8*f
)
```

Maple [A]
time = 0.39, size = 132, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{\cos(fx+e)}{a^3-3a^2b+3a\,b^2-b^3}+\frac{b\left(\frac{\left(-\frac{9a}{8}+\frac{9b}{8}\right)\left(\cos^3(fx+e)\right)-\frac{7b\cos(fx+e)}{8}}{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)^2}+\frac{15\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{8\sqrt{b(a-b)}}\right)}{(a-b)^3}}{f}$
default	$\frac{-\frac{\cos(fx+e)}{a^3-3a^2b+3a\,b^2-b^3}+\frac{b\left(\frac{\left(-\frac{9a}{8}+\frac{9b}{8}\right)\left(\cos^3(fx+e)\right)-\frac{7b\cos(fx+e)}{8}}{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)^2}+\frac{15\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{8\sqrt{b(a-b)}}\right)}{(a-b)^3}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2(a^3-3a^2b+3a\,b^2-b^3)f}-\frac{e^{-i(fx+e)}}{2(a^3-3a^2b+3a\,b^2-b^3)f}-\frac{b(-9a\,e^{7i(fx+e)}+9b\,e^{7i(fx+e)}-27a\,e^{5i(fx+e)}-b\,e^{5i(fx+e)}-2}{4(-a+b)^3(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a^3-3*a^2*b+3*a*b^2-b^3)*cos(f*x+e)+b/(a-b)^3*(((-9/8*a+9/8*b)*cos
(f*x+e)^3-7/8*b*cos(f*x+e))/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2+15/8/(b*(a-
b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(128) = 256$.

time = 2.84, size = 574, normalized size = 4.16

$$\left[-\frac{16\,(a^2-2\,ab+b^2)\cos(fx+e)^5+50\,(ab-b^2)\cos(fx+e)^3+30\,b^2\cos(fx+e)+15\,((a^2-2\,ab+b^2)\cos(fx+e)^4+2\,(ab-b^2)\cos(fx+e)^2+b^2)\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos(fx+e)^2-2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)}{16\,((a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)f\cos(fx+e)^4+2\,(a^4b-4\,a^3b^2+6\,a^2b^3-4\,ab^4+b^5)f\cos(fx+e)^2+(a^3b^2-3\,a^2b^3+3\,ab^4-b^5)f)},\ -\frac{8\,(a^2-2\,ab+b^2)\cos(fx+e)^5+25\,(ab-b^2)\cos(fx+e)^3+15\,b^2\cos(fx+e)+15\,((a^2-2\,ab+b^2)\cos(fx+e)^4+2\,(ab-b^2)\cos(fx+e)^2+b^2)\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)}{8\,((a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)f\cos(fx+e)^4+2\,(a^4b-4\,a^3b^2+6\,a^2b^3-4\,ab^4+b^5)f\cos(fx+e)^2+(a^3b^2-3\,a^2b^3+3\,ab^4-b^5)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 + 50*(a*b - b^2)*cos(f*x + e)
^3 + 30*b^2*cos(f*x + e) + 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b
- b^2)*cos(f*x + e)^2 + b^2)*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2
- 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)
))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e
)^4 + 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 +
(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f), -1/8*(8*(a^2 - 2*a*b + b^2)*cos(f
*x + e)^5 + 25*(a*b - b^2)*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) + 15*((a^2
- 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)*sqrt(b/
(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b))/((a^5 - 5*a^4*b +
 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^4 + 2*(a^4*b - 4*a
^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^3*b^2 - 3*a^2*b^3
 + 3*a*b^4 - b^5)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.09, size = 223, normalized size = 1.62

$$-\frac{f^5\cos(fx+e)}{a^3f^6-3\,a^2bf^6+3\,ab^2f^6-b^3f^6}+\frac{15\,b\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab-b^2}\,f}-\frac{\frac{9\,ab\cos(fx+e)^3}{f}-\frac{9\,b^2\cos(fx+e)^3}{f}+\frac{7\,b^2\cos(fx+e)}{f}}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -f^5*cos(f*x + e)/(a^3*f^6 - 3*a^2*b*f^6 + 3*a*b^2*f^6 - b^3*f^6) + 15/8*b*
arctan((a*cos(f*x + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*sqrt(a*b - b^2)*f) - 1/8*(9*a*b*cos(f*x + e)^3/f - 9*b^2*co
s(f*x + e)^3/f + 7*b^2*cos(f*x + e)/f)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a*
cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^2)
```

Mupad [B]

time = 14.87, size = 780, normalized size = 5.65

$$-\frac{\frac{8\,a^2+9\,a\,b-2\,b^2}{4\,\left(a-b\right)\,\left(a^2-2\,a\,b+b^2\right)}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(16\,a^4-41\,a^3\,b+27\,a^2\,b^2-40\,a\,b^3+8\,b^4\right)}{2\,a^2\,\left(a-b\right)\,\left(a^2-2\,a\,b+b^2\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(24\,a^4-64\,a^3\,b+53\,a^2\,b^2+40\,a\,b^3-8\,b^4\right)}{2\,a^2\,\left(a-b\right)\,\left(a^2-2\,a\,b+b^2\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(8\,a^3-9\,a^2\,b+24\,a\,b^2-8\,b^3\right)}{4\,a\,\left(a-b\right)\,\left(a^2-2\,a\,b+b^2\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(-16\,a^3+23\,a^2\,b+27\,a\,b^2-4\,b^3\right)}{2\,a\,\left(a-b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(8\,a\,b-3\,a^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(8\,a\,b-3\,a^2\right)+a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(2\,a^2-8\,a\,b+16\,b^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(2\,a^2-8\,a\,b+16\,b^2\right)+a^2\right)}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{{\left(a-b\right)}^7\,\left(2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(\frac{\sqrt{b}\,\left(225\,a^8\,b-1350\,a^7\,b^2+3375\,a^6\,b^3-4500\,a^5\,b^4+3375\,a^4\,b^5-1350\,a^3\,b^6+225\,a^2\,b^7\right)}{a\,{\left(a-b\right)}^{13/2}}+\frac{225\,\sqrt{b}\,\left(a-2\,b\right)\,\left(128\,a^{12}-1408\,a^{11}\,b+6912\,a^{10}\,b^2-19968\,a^9\,b^3+37632\,a^8\,b^4-48384\,a^7\,b^5+43008\,a^6\,b^6-26112\,a^5\,b^7+10368\,a^4\,b^8-2432\,a^3\,b^9+256\,a^2\,b^{10}\right)}{512\,a\,{\left(a-b\right)}^{21/2}}\right)+\frac{225\,\sqrt{b}\,\left(a-2\,b\right)\,\left(-128\,a^{12}+1152\,a^{11}\,b-4608\,a^{10}\,b^2+10752\,a^9\,b^3-16128\,a^8\,b^4+16128\,a^7\,b^5-10752\,a^6\,b^6+4608\,a^5\,b^7-1152\,a^4\,b^8+128\,a^3\,b^9\right)}{256\,a\,{\left(a-b\right)}^{21/2}}\right)}{225\,a^8\,b-1350\,a^7\,b^2+3375\,a^6\,b^3-4500\,a^5\,b^4+3375\,a^4\,b^5-1350\,a^3\,b^6+225\,a^2\,b^7}\right)}{8\,f\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] - ((9*a*b + 8*a^2 - 2*b^2)/(4*(a - b)*(a^2 - 2*a*b + b^2)) - (tan(e/2 + (f*
x)/2)^6*(16*a^4 - 41*a^3*b - 40*a*b^3 + 8*b^4 + 27*a^2*b^2))/(2*a^2*(a - b)
*(a^2 - 2*a*b + b^2)) + (tan(e/2 + (f*x)/2)^4*(40*a*b^3 - 64*a^3*b + 24*a^4
 - 8*b^4 + 53*a^2*b^2))/(2*a^2*(a - b)*(a^2 - 2*a*b + b^2)) + (tan(e/2 + (f
*x)/2)^8*(24*a*b^2 - 9*a^2*b + 8*a^3 - 8*b^3))/(4*a*(a - b)*(a^2 - 2*a*b +
b^2)) + (tan(e/2 + (f*x)/2)^2*(27*a*b^2 + 23*a^2*b - 16*a^3 - 4*b^3))/(2*a*
(a - b)*(a^2 - 2*a*b + b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(8*a*b - 3*a^2) + ta
n(e/2 + (f*x)/2)^8*(8*a*b - 3*a^2) + a^2*tan(e/2 + (f*x)/2)^10 + tan(e/2 +
(f*x)/2)^4*(2*a^2 - 8*a*b + 16*b^2) + tan(e/2 + (f*x)/2)^6*(2*a^2 - 8*a*b +
 16*b^2) + a^2)) - (15*b^(1/2)*atan(((a - b)^7*(2*tan(e/2 + (f*x)/2)^2*((b^
(1/2)*(225*a^8*b + 225*a^2*b^7 - 1350*a^3*b^6 + 3375*a^4*b^5 - 4500*a^5*b^4
 + 3375*a^6*b^3 - 1350*a^7*b^2))/(a*(a - b)^(13/2)) + (225*b^(1/2)*(a - 2*b
)*(128*a^12 - 1408*a^11*b + 256*a^2*b^10 - 2432*a^3*b^9 + 10368*a^4*b^8 - 2
6112*a^5*b^7 + 43008*a^6*b^6 - 48384*a^7*b^5 + 37632*a^8*b^4 - 19968*a^9*b^
3 + 6912*a^10*b^2))/(512*a*(a - b)^(21/2))) + (225*b^(1/2)*(a - 2*b)*(1152*
a^11*b - 128*a^12 + 128*a^3*b^9 - 1152*a^4*b^8 + 4608*a^5*b^7 - 10752*a^6*b
^6 + 16128*a^7*b^5 - 16128*a^8*b^4 + 10752*a^9*b^3 - 4608*a^10*b^2))/(256*a
*(a - b)^(21/2))))/(225*a^8*b + 225*a^2*b^7 - 1350*a^3*b^6 + 3375*a^4*b^5 -
 4500*a^5*b^4 + 3375*a^6*b^3 - 1350*a^7*b^2)))/(8*f*(a - b)^(7/2))
```

3.83 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}f}-\frac{\tanh^{-1}(\cos(e+fx))}{a^3f}-\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+f\right.}$$

```
[Out] -arctanh(cos(f*x+e))/a^3/f-1/4*b*sec(f*x+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)^
2-1/8*(7*a-4*b)*b*sec(f*x+e)/a^2/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)-1/8*(15*a^2
-20*a*b+8*b^2)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^3/(a-b)^(5/
2)/f
```

Rubi [A]
time = 0.16, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3745, 425, 541, 536, 213, 211}

$$-\frac{\tanh^{-1}(\cos(e+fx))}{a^3f}-\frac{b(7a-4b)\sec(e+fx)}{8a^2f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)}-\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3f(a-b)^{5/2}}-\frac{b\sec(e+fx)}{4af(a-b)\left(a+b\sec^2(e+fx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[
a - b]])/(a^3*(a - b)^(5/2)*f) - ArcTanh[Cos[e + f*x]]/(a^3*f) - (b*Sec[e +
 f*x])/(4*a*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^2) - ((7*a - 4*b)*b*Sec[e
+ f*x])/(8*a^2*(a - b)^2*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a-b+bx^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-b-3bx^2}{(-1+x^2)(a-b+bx^2)^2}\,dx, x, \text{sec}\right.}{4a(a-b)f}$$

$$= -\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-4b)b\sec(e+fx)}{8a^2(a-b)^2f\left(a-b+b\sec^2(e+fx\right.}$$

$$= -\frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-4b)b\sec(e+fx)}{8a^2(a-b)^2f\left(a-b+b\sec^2(e+fx\right.}$$

$$= -\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}f} - \frac{\tanh^{-1}(\cos(e+fx))}{a^3f} - \frac{\ }{4}$$

Mathematica [A]

time = 2.11, size = 247, normalized size = 1.49

$$\frac{\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{b}\,(15a^2-20ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{8a^2b^2\cos(e+fx)}{(a-b)^2(a+b+(a-b)\cos(2(e+fx)))^2} - \frac{2a(9a-4b)b\cos(e+fx)}{(a-b)^2(a+b+(a-b)\cos(2(e+fx)))} - 8\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right) + 8\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e +
f*x)/2])/Sqrt[b]])/(a - b)^(5/2) + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTa
n[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(5/2) + (8*a^2
*b^2*Cos[e + f*x])/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2) - (2*a*
(9*a - 4*b)*b*Cos[e + f*x])/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)]))
- 8*Log[Cos[(e + f*x)/2]] + 8*Log[Sin[(e + f*x)/2]])/(8*a^3*f)
```

Maple [A]

time = 0.53, size = 183, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{\ln(\cos(fx+e)+1)}{2a^3}+\frac{\ln(\cos(fx+e)-1)}{2a^3}+\frac{b\left(\frac{-\frac{(9a-4b)a\left(\cos^3(fx+e)\right)}{8(a-b)}-\frac{ab(7a-4b)\cos(fx+e)}{8\left(a^2-2ab+b^2\right)}}{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)^2}+\frac{\left(15a^2-20ab+8b^2\right)\arctan\left(\frac{(a-b)\,c}{\sqrt{b(}}\right.}{8\left(a^2-2ab+b^2\right)\sqrt{b(a-}}\right.}{a^3}}{f}$

default	$\frac{-\frac{\ln(\cos(fx+e)+1)}{2a^3}+\frac{\ln(\cos(fx+e)-1)}{2a^3}+\frac{b\left(\frac{-\frac{(9a-4b)a\left(\cos^3(fx+e)\right)}{8(a-b)}-\frac{ab(7a-4b)\cos(fx+e)}{8\left(a^2-2ab+b^2\right)}}{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)^2}+\frac{\left(15a^2-20ab+8b^2\right)\arctan\left(\frac{(a-b)\cos}{\sqrt{b(a}}\right.}{8\left(a^2-2ab+b^2\right)\sqrt{b(a-b}}\right.}{a^3}}{f}$
risch	$-\frac{b\left(9a^2e^{7i(fx+e)}-13ab\,e^{7i(fx+e)}+4b^2e^{7i(fx+e)}+27a^2e^{5i(fx+e)}-11ab\,e^{5i(fx+e)}-4b^2e^{5i(fx+e)}+27a^2e^{3i(fx+e)}-11ab\,e^{3i}\right.}{4(a^2-2ab+b^2)\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/a^3*ln(cos(f*x+e)+1)+1/2/a^3*ln(cos(f*x+e)-1)+b/a^3*((-1/8*(9*a-4
*b)*a/(a-b)*cos(f*x+e)^3-1/8*a*b*(7*a-4*b)/(a^2-2*a*b+b^2)*cos(f*x+e))/(a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)^2+1/8*(15*a^2-20*a*b+8*b^2)/(a^2-2*a*b+b^2)/(
b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(158) = 316$.
time = 1.38, size = 1078, normalized size = 6.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(9*a^3*b - 13*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^3 - ((15*a^4 - 50*a
^3*b + 63*a^2*b^2 - 36*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 20*a*b^
3 + 8*b^4 + 2*(15*a^3*b - 35*a^2*b^2 + 28*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sq
rt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos
(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*(7*a^2*b^2 - 4*a*b^3)*cos(
```

```
f*x + e) + 8*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 +
a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e
)^2)*log(1/2*cos(f*x + e) + 1/2) - 8*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
+ b^4)*cos(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 4*a^6*b
+ 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2
+ 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f
), -1/8*((9*a^3*b - 13*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^3 + ((15*a^4 - 50*a^
3*b + 63*a^2*b^2 - 36*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 20*a*b^3
 + 8*b^4 + 2*(15*a^3*b - 35*a^2*b^2 + 28*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqr
t(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + (7*a^2*b^2 -
 4*a*b^3)*cos(f*x + e) + 4*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos
(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^
4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) - 4*((a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b -
3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((
a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*
b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^
3 + a^3*b^4)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(158) = 316.
time = 0.98, size = 534, normalized size = 3.22

$$-\frac{\frac{(15a^2b-20ab^2+8b^3)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{(a^5-2a^4b+a^3b^2)\sqrt{ab-b^2}}+\frac{2\left(9a^3b-6a^2b^2+\frac{27a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{68a^2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{32ab^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{27a^3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{90a^2b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{120ab^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{48b^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{9a^3b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{28a^2b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{16ab^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}\right)}{(a^5-2a^4b+a^3b^2)\left(a+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}-\frac{4\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 20*a*b^2 + 8*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e
) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - 2*a^4*b +
a^3*b^2)*sqrt(a*b - b^2)) + 2*(9*a^3*b - 6*a^2*b^2 + 27*a^3*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - 68*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + 32*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2 - 90*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
```

```
 + 1)^2 + 120*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 48*b^4*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f
*x + e) + 1)^3 - 28*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 16*
a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 - 2*a^4*b + a^3*b^2)
*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 4*log(
abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^3)/f
```

Mupad [B]
time = 16.27, size = 1844, normalized size = 11.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] log(tan(e/2 + (f*x)/2))/(a^3*f) - ((3*(3*a*b - 2*b^2))/(4*a*(a^2 - 2*a*b +
b^2)) + (3*tan(e/2 + (f*x)/2)^4*(40*a*b^3 + 9*a^3*b - 16*b^4 - 30*a^2*b^2))
/(4*a^3*(a^2 - 2*a*b + b^2)) - (tan(e/2 + (f*x)/2)^6*(9*a^2*b - 28*a*b^2 +
16*b^3))/(4*a^2*(a^2 - 2*a*b + b^2)) - (tan(e/2 + (f*x)/2)^2*(27*a^2*b - 68
*a*b^2 + 32*b^3))/(4*a^2*(a^2 - 2*a*b + b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(8*
a*b - 4*a^2) + tan(e/2 + (f*x)/2)^6*(8*a*b - 4*a^2) + a^2*tan(e/2 + (f*x)/2
)^8 + tan(e/2 + (f*x)/2)^4*(6*a^2 - 16*a*b + 16*b^2) + a^2)) + (b^(1/2)*ata
n(((tan(e/2 + (f*x)/2)^2*((((b^(3/2)*(15*a^2 - 20*a*b + 8*b^2)^3*(4096*a^15
*b - 128*a^16 + 6144*a^7*b^9 - 46080*a^8*b^8 + 150784*a^9*b^7 - 281216*a^10
*b^6 + 327168*a^11*b^5 - 243584*a^12*b^4 + 113920*a^13*b^3 - 31104*a^14*b^2
))/(32768*a^9*(a - b)^(15/2)*(a^11 - 6*a^10*b + a^5*b^6 - 6*a^6*b^5 + 15*a^
7*b^4 - 20*a^8*b^3 + 15*a^9*b^2)) - (b^(1/2)*(15*a^2 - 20*a*b + 8*b^2)*(153
6*a*b^9 + 720*a^9*b - 11520*a^2*b^8 + 37760*a^3*b^7 - 70400*a^4*b^6 + 81384
*a^5*b^5 - 59564*a^6*b^4 + 26864*a^7*b^3 - 6780*a^8*b^2))/(128*a^3*(a - b)^
(5/2)*(a^11 - 6*a^10*b + a^5*b^6 - 6*a^6*b^5 + 15*a^7*b^4 - 20*a^8*b^3 + 15
*a^9*b^2)))*(3072*a*b^4 - 1090*a^4*b + 111*a^5 - 768*b^5 - 4752*a^2*b^3 + 3
424*a^3*b^2))/(2*a^5*(a - b)^(13/2)*(960*a*b^4 - 1055*a^4*b + 256*a^5 - 192
*b^5 - 1920*a^2*b^3 + 1960*a^3*b^2)) + (((576*a*b^6 - 64*b^7 - 1920*a^2*b^5
 + 3160*a^3*b^4 - 2625*a^4*b^3 + 900*a^5*b^2)/(8*(a^11 - 6*a^10*b + a^5*b^6
 - 6*a^6*b^5 + 15*a^7*b^4 - 20*a^8*b^3 + 15*a^9*b^2)) + (b*(15*a^2 - 20*a*b
 + 8*b^2)^2*(2768*a^12*b - 128*a^13 + 6144*a^4*b^9 - 46080*a^5*b^8 + 150656
*a^6*b^7 - 279104*a^7*b^6 + 318672*a^8*b^5 - 228160*a^9*b^4 + 99424*a^10*b^
3 - 24192*a^11*b^2))/(2048*a^6*(a - b)^5*(a^11 - 6*a^10*b + a^5*b^6 - 6*a^6
*b^5 + 15*a^7*b^4 - 20*a^8*b^3 + 15*a^9*b^2)))*(1344*a*b^4 - 205*a^4*b + 8*
a^5 - 384*b^5 - 1752*a^2*b^3 + 980*a^3*b^2))/(a^5*b^(1/2)*(a - b)^6*(960*a*
b^4 - 1055*a^4*b + 256*a^5 - 192*b^5 - 1920*a^2*b^3 + 1960*a^3*b^2))) + (((
b^(1/2)*(15*a^2 - 20*a*b + 8*b^2)*(240*a^8*b - 320*a^3*b^6 + 1600*a^4*b^5 -
 3232*a^5*b^4 + 3208*a^6*b^3 - 1505*a^7*b^2))/(64*a^3*(a - b)^(5/2)*(a^10 -
 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)) + (b^(3/2)*(15*a^2 - 20*a*b +
```

```
8*b^2)^3*(64*a^15 - 512*a^14*b + 256*a^9*b^6 - 1280*a^10*b^5 + 2624*a^11*b^
4 - 2816*a^12*b^3 + 1664*a^13*b^2))/(16384*a^9*(a - b)^(15/2)*(a^10 - 4*a^9
*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)))*(3072*a*b^4 - 1090*a^4*b + 111*a^5
- 768*b^5 - 4752*a^2*b^3 + 3424*a^3*b^2))/(2*a^5*(a - b)^(13/2)*(960*a*b^4
- 1055*a^4*b + 256*a^5 - 192*b^5 - 1920*a^2*b^3 + 1960*a^3*b^2)) - (((64*b^
6 - 320*a*b^5 + 640*a^2*b^4 - 600*a^3*b^3 + 225*a^4*b^2)/(4*(a^10 - 4*a^9*b
 + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)) - (b*(15*a^2 - 20*a*b + 8*b^2)^2*(64*a
^12 - 752*a^11*b + 512*a^6*b^6 - 2560*a^7*b^5 + 5216*a^8*b^4 - 5424*a^9*b^3
 + 2944*a^10*b^2))/(1024*a^6*(a - b)^5*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^
3 + 6*a^8*b^2)))*(1344*a*b^4 - 205*a^4*b + 8*a^5 - 384*b^5 - 1752*a^2*b^3 +
 980*a^3*b^2))/(a^5*b^(1/2)*(a - b)^6*(960*a*b^4 - 1055*a^4*b + 256*a^5 - 1
92*b^5 - 1920*a^2*b^3 + 1960*a^3*b^2)))*(256*a^13*(a - b)^(15/2) - 1536*a^1
2*b*(a - b)^(15/2) + 256*a^7*b^6*(a - b)^(15/2) - 1536*a^8*b^5*(a - b)^(15/
2) + 3840*a^9*b^4*(a - b)^(15/2) - 5120*a^10*b^3*(a - b)^(15/2) + 3840*a^11
*b^2*(a - b)^(15/2)))/(225*a^4*b - 320*a*b^4 + 64*b^5 + 640*a^2*b^3 - 600*a
^3*b^2))*(15*a^2 - 20*a*b + 8*b^2))/(8*a^3*f*(a - b)^(5/2))
```

3.84 $\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal. Leaf size=205

$$-\frac{\sqrt{b}\,(15a^2-40ab+24b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{3/2}f}-\frac{(a-6b)\tanh^{-1}(\cos(e+fx))}{2a^4f}-\frac{\cot(e+fx)\csc(e+f}{2af\,(a-b+b\sec^2(e+}$$

```
[Out] -1/2*(a-6*b)*arctanh(cos(f*x+e))/a^4/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b
*sec(f*x+e)^2)^2-3/4*b*sec(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^2-1/8*(11*a-12
*b)*b*sec(f*x+e)/a^3/(a-b)/f/(a-b+b*sec(f*x+e)^2)-1/8*(15*a^2-40*a*b+24*b^2
)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^4/(a-b)^(3/2)/f
```

Rubi [A]
time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3745, 482, 541, 536, 213, 211}

$$-\frac{(a-6b)\tanh^{-1}(\cos(e+fx))}{2a^4f}-\frac{b(11a-12b)\sec(e+fx)}{8a^3f(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3b\sec(e+fx)}{4a^2f\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\sqrt{b}\,(15a^2-40ab+24b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^4f(a-b)^{3/2}}-\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a+b\sec^2(e+fx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt
[a - b]])/(a^4*(a - b)^(3/2)*f) - ((a - 6*b)*ArcTanh[Cos[e + f*x]])/(2*a^4*
f) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*x]^2)^2) - (3*
b*Sec[e + f*x])/(4*a^2*f*(a - b + b*Sec[e + f*x]^2)^2) - ((11*a - 12*b)*b*S
ec[e + f*x])/(8*a^3*(a - b)*f*(a - b + b*Sec[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
```

```
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a-b+bx^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a-b-5bx^2}{(-1+x^2)(a-b+bx^2)^3}\,dx, x, \sec(e+fx)\right.}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(11a}{8a^3(a-b)}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(11a}{8a^3(a-b)}$$

$$= -\frac{\sqrt{b}\,(15a^2-40ab+24b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{3/2}f} - \frac{(a-6b)\tanh^{-1}(\cos(e+}{2a^4f}$$

Mathematica [A]
time = 5.62, size = 286, normalized size = 1.40

$$\frac{\frac{\sqrt{b}\,(15a^2-40ab+24b^2)\text{ArcTan}\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{b}\,(15a^2-40ab+24b^2)\text{ArcTan}\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} + \frac{8a^2b^2\cos(e+fx)}{(a-b)(a+b+(a-b)\cos(2(e+fx)))^2} - \frac{2a(9a-8b)b\cos(e+fx)}{(a-b)(a+b+(a-b)\cos(2(e+fx)))} - a\csc^2\left(\frac{1}{2}(e+fx)\right) - 4(a-6b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right) + 4(a-6b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right) + a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e +
 f*x)/2])/Sqrt[b]])/(a - b)^(3/2) + (Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*Arc
Tan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(3/2) + (8*a
^2*b^2*Cos[e + f*x])/((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2) - (2*a*
(9*a - 8*b)*b*Cos[e + f*x])/((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])) -
a*Csc[(e + f*x)/2]^2 - 4*(a - 6*b)*Log[Cos[(e + f*x)/2]] + 4*(a - 6*b)*Log[
Sin[(e + f*x)/2]] + a*Sec[(e + f*x)/2]^2)/(8*a^4*f)
```

Maple [A]
time = 0.53, size = 206, normalized size = 1.00

method	result

derivativedivides	$\frac{\frac{1}{4a^3(\cos(fx+e)+1)}+\frac{(-a+6b)\ln(\cos(fx+e)+1)}{4a^4}+\frac{1}{4a^3(\cos(fx+e)-1)}+\frac{(a-6b)\ln(\cos(fx+e)-1)}{4a^4}+\frac{b\left(\frac{-\frac{(9a-8b)a\left(\cos^3(fx+e)\right)}{8}-\frac{ab}{}}{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(\right.\right.}\right.}{}}{f}$
default	$\frac{\frac{1}{4a^3(\cos(fx+e)+1)}+\frac{(-a+6b)\ln(\cos(fx+e)+1)}{4a^4}+\frac{1}{4a^3(\cos(fx+e)-1)}+\frac{(a-6b)\ln(\cos(fx+e)-1)}{4a^4}+\frac{b\left(\frac{-\frac{(9a-8b)a\left(\cos^3(fx+e)\right)}{8}-\frac{ab}{}}{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(\right.\right.}\right.}{}}{f}$
risch	$\frac{4a^3e^{11i(fx+e)}-21a^2b\,e^{11i(fx+e)}+29a\,b^2e^{11i(fx+e)}-12b^3e^{11i(fx+e)}+20a^3e^{9i(fx+e)}-37a^2b\,e^{9i(fx+e)}-15a\,b^2e^{9i(fx+e)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4/a^3/(cos(f*x+e)+1)+1/4/a^4*(-a+6*b)*ln(cos(f*x+e)+1)+1/4/a^3/(cos(
f*x+e)-1)+1/4*(a-6*b)/a^4*ln(cos(f*x+e)-1)+b/a^4*((-1/8*(9*a-8*b)*a*cos(f*x
+e)^3-1/8*a*b*(7*a-8*b)/(a-b)*cos(f*x+e))/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
^2+1/8*(15*a^2-40*a*b+24*b^2)/(a-b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)
/(b*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(196) = 392$.
time = 1.76, size = 1457, normalized size = 7.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*a^4 - 21*a^3*b + 29*a^2*b^2 - 12*a*b^3)*cos(f*x + e)^5 + 2*(17*
a^3*b - 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^3 - ((15*a^4 - 70*a^3*b + 119*a
^2*b^2 - 88*a*b^3 + 24*b^4)*cos(f*x + e)^6 - (15*a^4 - 100*a^3*b + 229*a^2*
b^2 - 216*a*b^3 + 72*b^4)*cos(f*x + e)^4 - 15*a^2*b^2 + 40*a*b^3 - 24*b^4 -
 (30*a^3*b - 125*a^2*b^2 + 168*a*b^3 - 72*b^4)*cos(f*x + e)^2)*sqrt(-b/(a -
 b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e)
 - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 - 12*a*b^3)*cos(f*x + e
) - 4*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^
4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7
*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)
*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 +
6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*c
os(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^
3 - 18*b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 3*a^6*b +
 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 - (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4
*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2
 - (a^5*b^2 - a^4*b^3)*f), 1/8*((4*a^4 - 21*a^3*b + 29*a^2*b^2 - 12*a*b^3)*
cos(f*x + e)^5 + (17*a^3*b - 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^3 - ((15*a
^4 - 70*a^3*b + 119*a^2*b^2 - 88*a*b^3 + 24*b^4)*cos(f*x + e)^6 - (15*a^4 -
 100*a^3*b + 229*a^2*b^2 - 216*a*b^3 + 72*b^4)*cos(f*x + e)^4 - 15*a^2*b^2
+ 40*a*b^3 - 24*b^4 - (30*a^3*b - 125*a^2*b^2 + 168*a*b^3 - 72*b^4)*cos(f*x
 + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) +
(11*a^2*b^2 - 12*a*b^3)*cos(f*x + e) - 2*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*
a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 1
8*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 +
 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 2*((a^4 -
 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b
+ 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^
4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)*log(-1/2*cos
(f*x + e) + 1/2))/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 -
 (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^
5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2 - (a^5*b^2 - a^4*b^3)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(196) = 392.

time = 1.03, size = 622, normalized size = 3.03

$$-\frac{-\frac{(15\,a^2b-40\,ab^2+24\,b^3)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{(a^5-a^4b)\sqrt{ab-b^2}}+\frac{2\left(9\,a^3b-10\,a^2b^2+\frac{27\,a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{80\,a^2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{56\,ab^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{27\,a^3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{102\,a^2b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{152\,ab^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{80\,b^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{9\,a^3b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{32\,a^2b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{24\,ab^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}\right)}{(a^5-a^4b)\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}-\frac{2\,(a-6\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4}-\frac{\left(a-\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{12\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{a^4(\cos(fx+e)-1)}+\frac{\cos(fx+e)-1}{a^3(\cos(fx+e)+1)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 40*a*b^2 + 24*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x +
e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - a^4*b)*sq
rt(a*b - b^2)) + 2*(9*a^3*b - 10*a^2*b^2 + 27*a^3*b*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) - 80*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 56*a*b^
3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2 - 102*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 152*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 80*b^4*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
 1)^3 - 32*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 24*a*b^3*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 - a^4*b)*(a + 2*a*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 2*(a - 6*b)*log(abs(-cos(f*
x + e) + 1)/abs(cos(f*x + e) + 1))/a^4 - (a - 2*a*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + 12*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) +
1)/(a^4*(cos(f*x + e) - 1)) + (cos(f*x + e) - 1)/(a^3*(cos(f*x + e) + 1)))/
f
```

Mupad [B]
time = 13.22, size = 1652, normalized size = 8.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)^2/(8*a^3*f) - (a^2/2 + (tan(e/2 + (f*x)/2)^4*(96*a*b^2 -
 38*a^2*b + 3*a^3 - 64*b^3))/(a - b) + (tan(e/2 + (f*x)/2)^8*(64*a*b^2 - 19
*a^2*b + a^3 - 48*b^3))/(2*(a - b)) - (tan(e/2 + (f*x)/2)^2*(14*a*b^2 - 15*
a^2*b + 2*a^3))/(a - b) - (tan(e/2 + (f*x)/2)^6*(2*a^4 - 33*a^3*b - 152*a*b
^3 + 80*b^4 + 106*a^2*b^2))/(a*(a - b)))/(f*(4*a^5*tan(e/2 + (f*x)/2)^2 + 4
*a^5*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^6*(24*a^5 - 64*a^4*b + 64*a
^3*b^2) + tan(e/2 + (f*x)/2)^4*(32*a^4*b - 16*a^5) + tan(e/2 + (f*x)/2)^8*(
32*a^4*b - 16*a^5))) + (log(tan(e/2 + (f*x)/2))*(a - 6*b))/(2*a^4*f) + (b^(
1/2)*atan(((tan(e/2 + (f*x)/2)^2*((((b^(3/2)*(15*a^2 - 40*a*b + 24*b^2)^3*(
128*a^16 - 3712*a^15*b + 6144*a^10*b^6 - 27648*a^11*b^5 + 49408*a^12*b^4 -
43904*a^13*b^3 + 19584*a^14*b^2))/(32768*a^12*(a - b)^(9/2)*(3*a^10*b - a^1
1 + a^8*b^3 - 3*a^9*b^2)) + (b^(1/2)*(15*a^2 - 40*a*b + 24*b^2)*(360*a^9*b
- 13824*a^2*b^8 + 66816*a^3*b^7 - 132864*a^4*b^6 + 139776*a^5*b^5 - 83240*a
```

^6*b^4 + 27836*a^7*b^3 - 4860*a^8*b^2))/(128*a^4*(a - b)^(3/2)*(3*a^10*b -
a^11 + a^8*b^3 - 3*a^9*b^2)))*(63*a^6 - 1013*a^5*b - 9600*a*b^5 + 2304*b^6
+ 15792*a^2*b^4 - 12888*a^3*b^3 + 5342*a^4*b^2))/(2*a^5*(a - b)^(9/2)*(5760
*a*b^4 - 735*a^4*b + 64*a^5 - 1728*b^5 - 6960*a^2*b^3 + 3600*a^3*b^2)) - ((
(6912*a*b^6 - 1728*b^7 - 10800*a^2*b^5 + 8240*a^3*b^4 - 3075*a^4*b^3 + 450*
a^5*b^2)/(8*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)) + (b*(15*a^2 - 40*a*b
+ 24*b^2)^2*(1936*a^12*b - 64*a^13 + 18432*a^6*b^7 - 86016*a^7*b^6 + 161664
*a^8*b^5 - 155008*a^9*b^4 + 78736*a^10*b^3 - 19680*a^11*b^2))/(2048*a^8*(a
- b)^3*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))*(3072*a*b^4 - 145*a^4*b +
4*a^5 - 1152*b^5 - 2856*a^2*b^3 + 1080*a^3*b^2))/(a^5*b^(1/2)*(a - b)^3*(57
60*a*b^4 - 735*a^4*b + 64*a^5 - 1728*b^5 - 6960*a^2*b^3 + 3600*a^3*b^2))) +
(((b^(3/2)*(15*a^2 - 40*a*b + 24*b^2)^3*(128*a^16 - 768*a^15*b + 512*a^12*
b^4 - 1536*a^13*b^3 + 1664*a^14*b^2))/(32768*a^12*(a - b)^(9/2)*(a^11 - 2*a
^10*b + a^9*b^2)) + (b^(1/2)*(15*a^2 - 40*a*b + 24*b^2)*(240*a^9*b - 5760*a
^4*b^6 + 19200*a^5*b^5 - 23776*a^6*b^4 + 13344*a^7*b^3 - 3250*a^8*b^2))/(12
8*a^4*(a - b)^(3/2)*(a^11 - 2*a^10*b + a^9*b^2)))*(63*a^6 - 1013*a^5*b - 96
00*a*b^5 + 2304*b^6 + 15792*a^2*b^4 - 12888*a^3*b^3 + 5342*a^4*b^2))/(2*a^5
(a - b)^(9/2)(5760*a*b^4 - 735*a^4*b + 64*a^5 - 1728*b^5 - 6960*a^2*b^3 +
3600*a^3*b^2)) - (((12096*a*b^6 - 3456*b^7 - 15840*a^2*b^5 + 9520*a^3*b^4
- 2550*a^4*b^3 + 225*a^5*b^2)/(8*(a^11 - 2*a^10*b + a^9*b^2)) + (b*(15*a^2
- 40*a*b + 24*b^2)^2*(1248*a^12*b - 64*a^13 + 3072*a^8*b^5 - 9728*a^9*b^4 +
11328*a^10*b^3 - 5856*a^11*b^2))/(2048*a^8*(a - b)^3*(a^11 - 2*a^10*b + a^
9*b^2)))*(3072*a*b^4 - 145*a^4*b + 4*a^5 - 1152*b^5 - 2856*a^2*b^3 + 1080*a
^3*b^2))/(a^5*b^(1/2)*(a - b)^3*(5760*a*b^4 - 735*a^4*b + 64*a^5 - 1728*b^5
- 6960*a^2*b^3 + 3600*a^3*b^2)))*(256*a^13*(a - b)^(9/2) - 768*a^12*b*(a -
b)^(9/2) - 256*a^10*b^3*(a - b)^(9/2) + 768*a^11*b^2*(a - b)^(9/2)))/(225*
a^4*b - 1920*a*b^4 + 576*b^5 + 2320*a^2*b^3 - 1200*a^3*b^2))*(15*a^2 - 40*a
*b + 24*b^2))/(8*a^4*f*(a - b)^(3/2))

3.85 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=259

$$-\frac{3\sqrt{b}\,(5a^2-20ab+16b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^5\sqrt{a-b}\,f}-\frac{3(a^2-12ab+16b^2)\tanh^{-1}(\cos(e+fx))}{8a^5f}-\frac{(5a-8b)\,c}{8a^2f\,(a-}$$

```
[Out] -3/8*(a^2-12*a*b+16*b^2)*arctanh(cos(f*x+e))/a^5/f-1/8*(5*a-8*b)*cot(f*x+e)
*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^2-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a
-b+b*sec(f*x+e)^2)^2-1/8*(7*a-12*b)*b*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)
^2-3/2*(a-2*b)*b*sec(f*x+e)/a^4/f/(a-b+b*sec(f*x+e)^2)-3/8*(5*a^2-20*a*b+16
*b^2)*arctan(sec(f*x+e)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^5/f/(a-b)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3745, 481, 541, 536, 213, 211}

$$-\frac{3b(a-2b)\sec(e+fx)}{2a^4f\,(a+b\sec^2(e+fx)-b)}-\frac{b(7a-12b)\sec(e+fx)}{8a^3f\,(a+b\sec^2(e+fx)-b)^2}-\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\,(a+b\sec^2(e+fx)-b)^2}-\frac{3\sqrt{b}\,(5a^2-20ab+16b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^5f\sqrt{a-b}}-\frac{3(a^2-12ab+16b^2)\tanh^{-1}(\cos(e+fx))}{8a^5f}-\frac{\cot^3(e+fx)\csc(e+fx)}{4af\,(a+b\sec^2(e+fx)-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-3*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a
- b]])/(8*a^5*Sqrt[a - b]*f) - (3*(a^2 - 12*a*b + 16*b^2)*ArcTanh[Cos[e + f
*x]])/(8*a^5*f) - ((5*a - 8*b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*(a - b +
 b*Sec[e + f*x]^2)^2) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*(a - b + b*Sec
[e + f*x]^2)^2) - ((7*a - 12*b)*b*Sec[e + f*x])/(8*a^3*f*(a - b + b*Sec[e +
 f*x]^2)^2) - (3*(a - 2*b)*b*Sec[e + f*x])/(2*a^4*f*(a - b + b*Sec[e + f*x]
^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3(a-b+bx^2)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-a+b+(-4a+7b)x^2}{(-1+x^2)^2(a-b+bx^2)^3}\,dx, x, \sec(e+f\right.}{4af}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\text{Subs}}{}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7}{8a^3f}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7}{8a^3f}$$

$$= -\frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7}{8a^3f}$$

$$= -\frac{3\sqrt{b}\,(5a^2-20ab+16b^2)\tan^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^5\sqrt{a-b}\,f} - \frac{3(a^2-12ab+16b^2)\tan}{8a^5f}$$

Mathematica [A]
time = 6.38, size = 468, normalized size = 1.81

$$-\frac{3\sqrt{a-b}\sqrt{b}\,(5a^2-20ab+16b^2)\,\text{ArcTan}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{8a^5(-a+b)f} - \frac{3\sqrt{a-b}\sqrt{b}\,(5a^2-20ab+16b^2)\,\text{ArcTan}\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{8a^5(-a+b)f} + \frac{b^2\cos(e+fx)}{a^3f(a+b+a\cos(2(e+fx))-b\cos(2(e+fx)))^2} - \frac{3(3ab\cos(e+fx)-4b^2\cos(e+fx))}{4a^4f(a+b+a\cos(2(e+fx))-b\cos(2(e+fx)))} - \frac{3(a-4b)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32a^4f} - \frac{\csc^4\left(\frac{1}{2}(e+fx)\right)}{64a^3f} - \frac{3(a^2-12ab+16b^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8a^5f} + \frac{3(a^2-12ab+16b^2)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8a^5f} + \frac{3(a-4b)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32a^4f} + \frac{\sec^4\left(\frac{1}{2}(e+fx)\right)}{64a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-3*Sqrt[a - b]*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sec[(e + f*x)/2]*
(Sqrt[a - b]*Cos[(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(8*a^5
*(-a + b)*f) - (3*Sqrt[a - b]*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sec
[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2] + Sqrt[a]*Sin[(e + f*x)/2]))/Sq
rt[b]])/(8*a^5*(-a + b)*f) + (b^2*Cos[e + f*x])/(a^3*f*(a + b + a*Cos[2*(e
+ f*x)] - b*Cos[2*(e + f*x)])^2) - (3*(3*a*b*Cos[e + f*x] - 4*b^2*Cos[e + f
*x]))/(4*a^4*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])) - (3*(a -
 4*b)*Csc[(e + f*x)/2]^2)/(32*a^4*f) - Csc[(e + f*x)/2]^4/(64*a^3*f) - (3*(
a^2 - 12*a*b + 16*b^2)*Log[Cos[(e + f*x)/2]])/(8*a^5*f) + (3*(a^2 - 12*a*b
+ 16*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^5*f) + (3*(a - 4*b)*Sec[(e + f*x)/2]^
2)/(32*a^4*f) + Sec[(e + f*x)/2]^4/(64*a^3*f)
```

Maple [A]
time = 0.49, size = 265, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{1}{16a^3(\cos(fx+e)+1)^2}-\frac{-3a+12b}{16a^4(\cos(fx+e)+1)}+\frac{(-3a^2+36ab-48b^2)\ln(\cos(fx+e)+1)}{16a^5}-\frac{1}{16a^3(\cos(fx+e)-1)^2}-\frac{-3a+12b}{16a^4(\cos(fx+e)-1)}+\frac{(}{}}{}$
default	$\frac{\frac{1}{16a^3(\cos(fx+e)+1)^2}-\frac{-3a+12b}{16a^4(\cos(fx+e)+1)}+\frac{(-3a^2+36ab-48b^2)\ln(\cos(fx+e)+1)}{16a^5}-\frac{1}{16a^3(\cos(fx+e)-1)^2}-\frac{-3a+12b}{16a^4(\cos(fx+e)-1)}+\frac{(}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/16/a^3/(cos(f*x+e)+1)^2-1/16*(-3*a+12*b)/a^4/(cos(f*x+e)+1)+1/16/a^5
*(-3*a^2+36*a*b-48*b^2)*ln(cos(f*x+e)+1)-1/16/a^3/(cos(f*x+e)-1)^2-1/16*(-3
*a+12*b)/a^4/(cos(f*x+e)-1)+1/16/a^5*(3*a^2-36*a*b+48*b^2)*ln(cos(f*x+e)-1)
+b/a^5*((-3/8*a*(3*a^2-7*a*b+4*b^2)*cos(f*x+e)^3+(-7/8*a^2*b+3/2*a*b^2)*cos
(f*x+e))/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2+3/8*(5*a^2-20*a*b+16*b^2)/(b*(
a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(251) = 502$.
time = 1.67, size = 1741, normalized size = 6.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^4 - 9*a^3*b + 16*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 -
 46*a^3*b + 108*a^2*b^2 - 72*a*b^3)*cos(f*x + e)^5 - 2*(19*a^3*b - 72*a^2*b
^2 + 72*a*b^3)*cos(f*x + e)^3 + 3*((5*a^4 - 30*a^3*b + 61*a^2*b^2 - 52*a*b^
3 + 16*b^4)*cos(f*x + e)^8 - 2*(5*a^4 - 35*a^3*b + 86*a^2*b^2 - 88*a*b^3 +
32*b^4)*cos(f*x + e)^6 + (5*a^4 - 50*a^3*b + 166*a^2*b^2 - 216*a*b^3 + 96*b
^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 20*a*b^3 + 16*b^4 + 2*(5*a^3*b - 30*a^2*b^
2 + 56*a*b^3 - 32*b^4)*cos(f*x + e)^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*
x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x +
e)^2 + b)) - 24*(a^2*b^2 - 2*a*b^3)*cos(f*x + e) - 3*((a^4 - 14*a^3*b + 41*
a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(a^4 - 15*a^3*b + 54*a^2*b^
2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18*a^3*b + 94*a^2*b^2 - 168*
a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b^3 + 16*b^4 + 2*(a^3*b - 1
4*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2)
+ 3*((a^4 - 14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(
a^4 - 15*a^3*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18
*a^3*b + 94*a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b
^3 + 16*b^4 + 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*lo
g(-1/2*cos(f*x + e) + 1/2))/((a^7 - 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a
^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b
+ 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*x + e)^2), 1/
16*(6*(a^4 - 9*a^3*b + 16*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 46
*a^3*b + 108*a^2*b^2 - 72*a*b^3)*cos(f*x + e)^5 - 2*(19*a^3*b - 72*a^2*b^2
+ 72*a*b^3)*cos(f*x + e)^3 - 6*((5*a^4 - 30*a^3*b + 61*a^2*b^2 - 52*a*b^3 +
 16*b^4)*cos(f*x + e)^8 - 2*(5*a^4 - 35*a^3*b + 86*a^2*b^2 - 88*a*b^3 + 32*
b^4)*cos(f*x + e)^6 + (5*a^4 - 50*a^3*b + 166*a^2*b^2 - 216*a*b^3 + 96*b^4)
*cos(f*x + e)^4 + 5*a^2*b^2 - 20*a*b^3 + 16*b^4 + 2*(5*a^3*b - 30*a^2*b^2 +
 56*a*b^3 - 32*b^4)*cos(f*x + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/
(a - b))*cos(f*x + e)/b) - 24*(a^2*b^2 - 2*a*b^3)*cos(f*x + e) - 3*((a^4 -
14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(a^4 - 15*a^3
*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18*a^3*b + 94*
a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b^3 + 16*b^4
+ 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*
x + e) + 1/2) + 3*((a^4 - 14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*
x + e)^8 - 2*(a^4 - 15*a^3*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)
^6 + (a^4 - 18*a^3*b + 94*a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^
2*b^2 - 12*a*b^3 + 16*b^4 + 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(
f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 2*a^6*b + a^5*b^2)*f*cos(
f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (
a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f
*x + e)^2)]
```

<u>**Sympy**</u> [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 922 vs. 2(251) = 502.
time = 1.08, size = 922, normalized size = 3.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 12*a*b + 16*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e
) + 1))/a^5 - 24*(5*a^2*b - 20*a*b^2 + 16*b^3)*arctan(-(a*cos(f*x + e) - b*
cos(f*x + e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a
*b - b^2)*a^5) - (8*a^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a^2*b*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) - a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e
) + 1)^2)/a^6 - (a^4 - 4*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*a^3
*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^4*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2 + 216*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 30
4*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 20*a^4*(cos(f*x + e)
- 1)^3/(cos(f*x + e) + 1)^3 + 360*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e)
+ 1)^3 - 1024*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 896*a*b^3
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 5*a^4*(cos(f*x + e) - 1)^4/(co
s(f*x + e) + 1)^4 + 64*a^3*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 19
2*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 256*a*b^3*(cos(f*x +
e) - 1)^4/(cos(f*x + e) + 1)^4 - 256*b^4*(cos(f*x + e) - 1)^4/(cos(f*x + e)
 + 1)^4 + 16*a^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 168*a^3*b*(cos
(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 384*a^2*b^2*(cos(f*x + e) - 1)^5/(c
os(f*x + e) + 1)^5 - 256*a*b^3*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 +
6*a^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 - 72*a^3*b*(cos(f*x + e) -
1)^6/(cos(f*x + e) + 1)^6 + 96*a^2*b^2*(cos(f*x + e) - 1)^6/(cos(f*x + e) +
 1)^6)/(a^5*(a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 4*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)^2))/f
```

Mupad [B]
time = 12.82, size = 1357, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)^4/(64*a^3*f) + (tan(e/2 + (f*x)/2)^2*((3*(a - 2*b))/(16*
a^4) - 1/(16*a^3)))/f + (tan(e/2 + (f*x)/2)^4*(100*a*b^2 - 72*a^2*b + (13*a
^3)/2) - tan(e/2 + (f*x)/2)^10*(144*a*b^2 - 42*a^2*b + 2*a^3 - 128*b^3) - t
an(e/2 + (f*x)/2)^6*(496*a*b^2 - 174*a^2*b + 11*a^3 - 416*b^3) + tan(e/2 +
(f*x)/2)^2*(4*a^2*b - a^3) - a^3/4 + (tan(e/2 + (f*x)/2)^8*(31*a^4 - 592*a^
3*b - 2944*a*b^3 + 1792*b^4 + 2016*a^2*b^2))/(4*a))/(f*(16*a^6*tan(e/2 + (f
*x)/2)^4 + 16*a^6*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^8*(96*a^6 - 25
6*a^5*b + 256*a^4*b^2) + tan(e/2 + (f*x)/2)^6*(128*a^5*b - 64*a^6) + tan(e/
2 + (f*x)/2)^10*(128*a^5*b - 64*a^6))) + (log(tan(e/2 + (f*x)/2))*(3*a^2 -
36*a*b + 48*b^2))/(8*a^5*f) + (3*b^(1/2)*atan((a^13*(a - b)^(3/2)*((256*((3
456*b^8 - 11232*a*b^7 + 14256*a^2*b^6 - 8910*a^3*b^5 + 2835*a^4*b^4 - (3375
*a^5*b^3)/8 + (675*a^6*b^2)/32)/a^12 - (9*b*(5*a^2 - 20*a*b + 16*b^2)^2*(19
2*a^14 - 4992*a^13*b + 24576*a^10*b^4 - 43008*a^11*b^3 + 24576*a^12*b^2))/(
8192*a^22*(a - b)))*(1728*a*b^4 - 45*a^4*b + a^5 - 768*b^5 - 1344*a^2*b^3 +
 420*a^3*b^2))/(b^(1/2)*(b*(b*(b*(1680*a^7 + b*(768*a^5*b - 1920*a^6)) - 60
0*a^8) + 75*a^9) - 4*a^10)) - 256*tan(e/2 + (f*x)/2)^2*((((4752*a*b^6 - 172
8*b^7 - 4860*a^2*b^5 + 2295*a^3*b^4 - (2025*a^4*b^3)/4 + (675*a^5*b^2)/16)/
a^11 + (9*b*(5*a^2 - 20*a*b + 16*b^2)^2*(3552*a^12*b - 96*a^13 + 73728*a^8*
b^5 - 165888*a^9*b^4 + 125952*a^10*b^3 - 36480*a^11*b^2))/(4096*a^21*(a - b
)))*(1728*a*b^4 - 45*a^4*b + a^5 - 768*b^5 - 1344*a^2*b^3 + 420*a^3*b^2))/(
b^(1/2)*(b*(b*(b*(1680*a^7 + b*(768*a^5*b - 1920*a^6)) - 600*a^8) + 75*a^9)
 - 4*a^10)) - (((b^(3/2)*(5*a^2 - 20*a*b + 16*b^2)^3*((351*a^15*b)/128 - (2
7*a^16)/256 + (81*a^13*b^3)/16 - (243*a^14*b^2)/32))/(a^26*(a - b)^(3/2)) -
 (3*b^(1/2)*(5*a^2 - 20*a*b + 16*b^2)*(540*a^9*b + 110592*a^3*b^7 - 331776*
a^4*b^6 + 389376*a^5*b^5 - 225792*a^6*b^4 + 67248*a^7*b^3 - 9720*a^8*b^2))/
(256*a^16*(a - b)^(1/2)))*(4224*a*b^4 - 330*a^4*b + 17*a^5 - 1536*b^5 - 422
4*a^2*b^3 + 1848*a^3*b^2))/(2*a^5*(a - b)^(1/2)*(1920*a*b^4 - 75*a^4*b + 4*
a^5 - 768*b^5 - 1680*a^2*b^3 + 600*a^3*b^2))) + (128*((b^(3/2)*(5*a^2 - 20*
a*b + 16*b^2)^3*((27*a^17)/256 - (27*a^16*b)/64 + (27*a^15*b^2)/64))/(a^27*
(a - b)^(3/2)) + (3*b^(1/2)*(5*a^2 - 20*a*b + 16*b^2)*(720*a^10*b - 92160*a
^5*b^6 + 230400*a^6*b^5 - 210816*a^7*b^4 + 85824*a^8*b^3 - 14760*a^9*b^2))/
(512*a^17*(a - b)^(1/2)))*(4224*a*b^4 - 330*a^4*b + 17*a^5 - 1536*b^5 - 422
4*a^2*b^3 + 1848*a^3*b^2))/(a^5*(a - b)^(1/2)*(1920*a*b^4 - 75*a^4*b + 4*a^
5 - 768*b^5 - 1680*a^2*b^3 + 600*a^3*b^2))))/(675*a^4*b - 17280*a*b^4 + 691
2*b^5 + 15120*a^2*b^3 - 5400*a^3*b^2))*(5*a^2 - 20*a*b + 16*b^2))/(8*a^5*f*
(a - b)^(1/2))
```

3.86 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=250

$$\frac{3(a^2+10ab+5b^2)\,x}{8(a-b)^5}-\frac{3\sqrt{b}\,(5a^2+10ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a-b)^5f}-\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,\left(a+b\tan^2(e+fx)\right)^2}+$$

```
[Out] 3/8*(a^2+10*a*b+5*b^2)*x/(a-b)^5-3/8*(5*a^2+10*a*b+b^2)*arctan(b^(1/2)*tan(
f*x+e)/a^(1/2))*b^(1/2)/(a-b)^5/f/a^(1/2)-1/8*(5*a+3*b)*cos(f*x+e)*sin(f*x+
e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*
tan(f*x+e)^2)^2-1/8*b*(7*a+5*b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)^2-3
/2*b*(a+b)*tan(f*x+e)/(a-b)^4/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.23, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 481, 541, 536, 209, 211}

$$-\frac{3\sqrt{b}\,(5a^2+10ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,f(a-b)^5}+\frac{3x(a^2+10ab+5b^2)}{8(a-b)^5}-\frac{3b(a+b)\tan(e+fx)}{2f(a-b)^4\,(a+b\tan^2(e+fx))}-\frac{b(7a+5b)\tan(e+fx)}{8f(a-b)^3\,(a+b\tan^2(e+fx))^2}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)\,(a+b\tan^2(e+fx))^2}-\frac{(5a+3b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2\,(a+b\tan^2(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 + 10*a*b + 5*b^2)*x)/(8*(a - b)^5) - (3*Sqrt[b]*(5*a^2 + 10*a*b + b
^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^5*f) - ((5*a
 + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)^2)
 + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - (
b*(7*a + 5*b)*Tan[e + f*x])/(8*(a - b)^3*f*(a + b*Tan[e + f*x]^2)^2) - (3*b
*(a + b)*Tan[e + f*x])/(2*(a - b)^4*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
```

```
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{\text{Subst}\left(\int \frac{a+(-4a-3b)x^2}{(1+x^2)^2(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subs}}{}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{}{8(a-}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{}{8(a-}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{}{8(a-}$$

$$= \frac{3(a^2+10ab+5b^2)\,x}{8(a-b)^5} - \frac{3\sqrt{b}\,(5a^2+10ab+b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a-b)^5f} - \frac{(5a+}{8(a-}$$

Mathematica [A]
time = 0.61, size = 194, normalized size = 0.78

$$\frac{12(a^2+10ab+5b^2)(e+fx) - \frac{12\sqrt{b}\,(5a^2+10ab+b^2)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}} - 8(a-b)(a+2b)\sin(2(e+fx)) + \frac{16a(a-b)b^2\sin(2(e+fx))}{(a+b+(a-b)\cos(2(e+fx)))^2} - \frac{4(a-b)b(9a+5b)\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))} + (a-b)^2\sin(4(e+fx))}{32(a-b)^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (12*(a^2 + 10*a*b + 5*b^2)*(e + f*x) - (12*Sqrt[b]*(5*a^2 + 10*a*b + b^2)*A
rcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sqrt[a] - 8*(a - b)*(a + 2*b)*Sin[2*
(e + f*x)] + (16*a*(a - b)*b^2*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e
+ f*x)])^2 - (4*(a - b)*b*(9*a + 5*b)*Sin[2*(e + f*x)])/(a + b + (a - b)*Co
s[2*(e + f*x)]) + (a - b)^2*Sin[4*(e + f*x)])/(32*(a - b)^5*f)
```

Maple [A]
time = 0.54, size = 209, normalized size = 0.84

method	result

derivativedivides	$\frac{\frac{\frac{\left(-\frac{1}{4}ab+\frac{7}{8}b^2-\frac{5}{8}a^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2-\frac{3}{4}ab+\frac{9}{8}b^2\right)\tan(fx+e)}{\left(1+\tan^2(fx+e)\right)^2}+\frac{3\left(a^2+10ab+5b^2\right)\arctan(\tan(fx+e))}{8}}{(a-b)^5}-b\left(\frac{\left(\frac{7}{8}a^2b-\frac{1}{4}ab^2-\frac{5}{8}\right.}{}\right.}{f}$
default	$\frac{\frac{\frac{\left(-\frac{1}{4}ab+\frac{7}{8}b^2-\frac{5}{8}a^2\right)\left(\tan^3(fx+e)\right)+\left(-\frac{3}{8}a^2-\frac{3}{4}ab+\frac{9}{8}b^2\right)\tan(fx+e)}{\left(1+\tan^2(fx+e)\right)^2}+\frac{3\left(a^2+10ab+5b^2\right)\arctan(\tan(fx+e))}{8}}{(a-b)^5}-b\left(\frac{\left(\frac{7}{8}a^2b-\frac{1}{4}ab^2-\frac{5}{8}\right.}{}\right.}{f}$
risch	$\frac{3x\,a^2}{8(a^3-3a^2b+3a\,b^2-b^3)(a-b)^2}+\frac{15xab}{4(a^3-3a^2b+3a\,b^2-b^3)(a-b)^2}+\frac{15x\,b^2}{8(a^3-3a^2b+3a\,b^2-b^3)(a-b)^2}-\frac{i\mathrm{e}^{-2i(f}}{4(a-b)(a^3-3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^5*(((-1/4*a*b+7/8*b^2-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-3/4*a*b+
9/8*b^2)*tan(f*x+e))/(1+tan(f*x+e)^2)^2+3/8*(a^2+10*a*b+5*b^2)*arctan(tan(f
*x+e)))-b/(a-b)^5*(((7/8*a^2*b-1/4*a*b^2-5/8*b^3)*tan(f*x+e)^3+3/8*a*(3*a^2
-2*a*b-b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+3/8*(5*a^2+10*a*b+b^2)/(a*b)^(
1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.52, size = 470, normalized size = 1.88

$$\frac{\frac{3\left(a^2+10\,ab+5\,b^2\right)(fx+e)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5}-\frac{3\left(5\,a^2b+10\,ab^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\left(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5\right)\sqrt{ab}}-\frac{12\left(ab^2+b^3\right)\tan(fx+e)^7+\left(19\,a^2b+34\,ab^2+19\,b^3\right)\tan(fx+e)^5+\left(5\,a^3+31\,a^2b+31\,ab^2+5\,b^3\right)\tan(fx+e)^3+3\left(a^3+6\,a^2b+ab^2\right)\tan(fx+e)}{\left(a^4b^2-4\,a^3b^3+6\,a^2b^4-4\,ab^5+b^6\right)\tan(fx+e)^8+2\left(a^5b-3\,a^4b^2+2\,a^3b^3+2\,a^2b^4-3\,ab^5+b^6\right)\tan(fx+e)^6+a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3+a^2b^4+\left(a^6-9\,a^4b^2+16\,a^3b^3-9\,a^2b^4+b^6\right)\tan(fx+e)^4+2\left(a^6-3\,a^5b+2\,a^4b^2+2\,a^3b^3-3\,a^2b^4+ab^5\right)\tan(fx+e)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^2 + 10*a*b + 5*b^2)*(f*x + e)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^
2*b^3 + 5*a*b^4 - b^5) - 3*(5*a^2*b + 10*a*b^2 + b^3)*arctan(b*tan(f*x + e)
/sqrt(a*b))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sqrt
(a*b)) - (12*(a*b^2 + b^3)*tan(f*x + e)^7 + (19*a^2*b + 34*a*b^2 + 19*b^3)*
tan(f*x + e)^5 + (5*a^3 + 31*a^2*b + 31*a*b^2 + 5*b^3)*tan(f*x + e)^3 + 3*(
a^3 + 6*a^2*b + a*b^2)*tan(f*x + e))/((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*
a*b^5 + b^6)*tan(f*x + e)^8 + 2*(a^5*b - 3*a^4*b^2 + 2*a^3*b^3 + 2*a^2*b^4
- 3*a*b^5 + b^6)*tan(f*x + e)^6 + a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a
^2*b^4 + (a^6 - 9*a^4*b^2 + 16*a^3*b^3 - 9*a^2*b^4 + b^6)*tan(f*x + e)^4 +
2*(a^6 - 3*a^5*b + 2*a^4*b^2 + 2*a^3*b^3 - 3*a^2*b^4 + a*b^5)*tan(f*x + e)^
2))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(241) = 482$.
time = 2.97, size = 1223, normalized size = 4.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^4 + 8*a^3*b - 14*a^2*b^2 + 5*b^4)*f*x*cos(f*x + e)^4 + 24*(a^3
*b + 9*a^2*b^2 - 5*a*b^3 - 5*b^4)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 10*a*b
^3 + 5*b^4)*f*x - 3*((5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)^4 +
5*a^2*b^2 + 10*a*b^3 + b^4 + 2*(5*a^3*b + 5*a^2*b^2 - 9*a*b^3 - b^4)*cos(f*
x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b
^2)*cos(f*x + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt
(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b
^2)*cos(f*x + e)^2 + b^2)) + 4*(2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^
4)*cos(f*x + e)^7 - (5*a^4 - 12*a^3*b + 6*a^2*b^2 + 4*a*b^3 - 3*b^4)*cos(f*
x + e)^5 - (19*a^3*b - 21*a^2*b^2 - 15*a*b^3 + 17*b^4)*cos(f*x + e)^3 - 12*
(a^2*b^2 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 -
35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*
(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*
f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6
 - b^7)*f), 1/16*(6*(a^4 + 8*a^3*b - 14*a^2*b^2 + 5*b^4)*f*x*cos(f*x + e)^4
 + 12*(a^3*b + 9*a^2*b^2 - 5*a*b^3 - 5*b^4)*f*x*cos(f*x + e)^2 + 6*(a^2*b^2
 + 10*a*b^3 + 5*b^4)*f*x + 3*((5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x
+ e)^4 + 5*a^2*b^2 + 10*a*b^3 + b^4 + 2*(5*a^3*b + 5*a^2*b^2 - 9*a*b^3 - b^
4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b
/a)/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*
b^3 + b^4)*cos(f*x + e)^7 - (5*a^4 - 12*a^3*b + 6*a^2*b^2 + 4*a*b^3 - 3*b^4
)*cos(f*x + e)^5 - (19*a^3*b - 21*a^2*b^2 - 15*a*b^3 + 17*b^4)*cos(f*x + e)
^3 - 12*(a^2*b^2 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 7*a^6*b + 21*a^
5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e
)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6
 + b^7)*f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 +
 5*a*b^6 - b^7)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.18, size = 399, normalized size = 1.60

$$\frac{\frac{3\left(a^2+10ab+5b^2\right)(fx+e)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5}-\frac{3\left(5a^2b+10ab^2+b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5)\sqrt{ab}}-\frac{12ab^2\tan(fx+e)^7+12b^3\tan(fx+e)^7+19a^2b\tan(fx+e)^5+34ab^2\tan(fx+e)^5+19b^3\tan(fx+e)^5+5a^3\tan(fx+e)^3+31a^2b\tan(fx+e)^3+31ab^2\tan(fx+e)^3+5b^3\tan(fx+e)^3+3a^3\tan(fx+e)+18a^2b\tan(fx+e)+3ab^2\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+b\tan(fx+e)^2+a\right)^2(a^4-4a^3b+6a^2b^2-4ab^3+b^4)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 10*a*b + 5*b^2)*(f*x + e)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^
2*b^3 + 5*a*b^4 - b^5) - 3*(5*a^2*b + 10*a*b^2 + b^3)*(pi*floor((f*x + e)/p
i + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^5 - 5*a^4*b + 10*a^
3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sqrt(a*b)) - (12*a*b^2*tan(f*x + e)^7 +
 12*b^3*tan(f*x + e)^7 + 19*a^2*b*tan(f*x + e)^5 + 34*a*b^2*tan(f*x + e)^5
+ 19*b^3*tan(f*x + e)^5 + 5*a^3*tan(f*x + e)^3 + 31*a^2*b*tan(f*x + e)^3 +
31*a*b^2*tan(f*x + e)^3 + 5*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) + 18*a^
2*b*tan(f*x + e) + 3*a*b^2*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e
)^2 + b*tan(f*x + e)^2 + a)^2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)))
/f
```

Mupad [B]
time = 16.39, size = 2500, normalized size = 10.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (atan(((((tan(e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^2*b^5 + 540*a^3*b^4 + 1
17*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 +
 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) + (3*((6*a*b^13 - (3*b^14)/2 + 21*a
^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9 + 1638*a^6*b^8 -
1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b^4 + 6*a^11*b^3 -
(3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*a^2*b^10 - 220*a^
3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*b^5 + 495*a^8*b^4
 - 220*a^9*b^3 + 66*a^10*b^2) - (3*tan(e + f*x)*(10*a*b + a^2 + 5*b^2)*(115
2*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^3*b^10 - 11520*a^4*b^9 + 5376*
a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600*a^8*b^5 - 4480*a^9*b^4 + 1152
*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^
2*b^3*10i + a^3*b^2*10i)*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a
^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(10*a*b + a^2 + 5*b^2))/(1
6*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i)))*(10
*a*b + a^2 + 5*b^2)*3i)/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^
3*10i + a^3*b^2*10i)) + (((tan(e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^2*b^5
```

+ 540*a^3*b^4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b
^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (3*((6*a*b^13 -
(3*b^14)/2 + 21*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9
+ 1638*a^6*b^8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b^
4 + 6*a^11*b^3 - (3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*
a^2*b^10 - 220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*
b^5 + 495*a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) + (3*tan(e + f*x)*(10*a*b +
a^2 + 5*b^2)*(1152*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^3*b^10 - 1152
0*a^4*b^9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600*a^8*b^5 - 44
80*a^9*b^4 + 1152*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^4*5i - a^4*b*5i + a^5
*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i)*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 +
28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(10*a*b +
a^2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a
^3*b^2*10i)))*(10*a*b + a^2 + 5*b^2)*3i)/(16*(a*b^4*5i - a^4*b*5i + a^5*1i
- b^5*1i - a^2*b^3*10i + a^3*b^2*10i)))/(((1755*a*b^7)/64 + (135*b^8)/64 +
(2511*a^2*b^6)/32 + (2511*a^3*b^5)/32 + (1755*a^4*b^4)/64 + (135*a^5*b^3)/6
4)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*a^2*b^10 - 220*a^3*b^9 + 495*a
^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*b^5 + 495*a^8*b^4 - 220*a^9*b^
3 + 66*a^10*b^2) - (3*((tan(e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^2*b^5 + 5
40*a^3*b^4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6
- 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) + (3*((6*a*b^13 - (3*
b^14)/2 + 21*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9 + 1
638*a^6*b^8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b^4 +
6*a^11*b^3 - (3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*a^2
*b^10 - 220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*b^5
+ 495*a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) - (3*tan(e + f*x)*(10*a*b + a^2
+ 5*b^2)*(1152*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^3*b^10 - 11520*a
^4*b^9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600*a^8*b^5 - 4480*
a^9*b^4 + 1152*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^4*5i - a^4*b*5i + a^5*1i
- b^5*1i - a^2*b^3*10i + a^3*b^2*10i)*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*
a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(10*a*b + a^
2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*
b^2*10i)))*(10*a*b + a^2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*
1i - a^2*b^3*10i + a^3*b^2*10i)) + (3*((tan(e + f*x)*(540*a*b^6 + 117*b^7 +
990*a^2*b^5 + 540*a^3*b^4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b
^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (3*
((6*a*b^13 - (3*b^14)/2 + 21*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 -
1332*a^5*b^9 + 1638*a^6*b^8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5
+ 21*a^10*b^4 + 6*a^11*b^3 - (3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11
+ b^12 + 66*a^2*b^10 - 220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b
^6 - 792*a^7*b^5 + 495*a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) + (3*tan(e + f*
x)*(10*a*b + a^2 + 5*b^2)*(1152*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^
3*b^10 - 11520*a^4*b^9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600
*a^8*b^5 - 4480*a^9*b^4 + 1152*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^4*5i - a
^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i)*(a^8 - 8*a^7*b - 8*a

```
*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2
)))*(10*a*b + a^2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^
2*b^3*10i + a^3*b^2*10i)))*(10*a*b + a^2 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i
 + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i))))*(10*a*b + a^2 + 5*b^2)*3
i)/(8*f*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*b^2*10i)
) - ((3*tan(e + f*x)*(a*b^2 + 6*a^2*b + a^3))/(...
```

3.87 $\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal. Leaf size=193

$$\frac{(a+5b)x}{2(a-b)^4}-\frac{\sqrt{b}\,(15a^2+10ab-b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^4f}-\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))^2}-\frac{3b\tan}{4(a-b)^2f\,(a}$$

```
[Out] 1/2*(a+5*b)*x/(a-b)^4-1/8*(15*a^2+10*a*b-b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(
1/2))*b^(1/2)/a^(3/2)/(a-b)^4/f-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(
f*x+e)^2)^2-3/4*b*tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^2-1/8*b*(11*a+b)*
tan(f*x+e)/a/(a-b)^3/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.17, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 482, 541, 536, 209, 211}

$$-\frac{\sqrt{b}\,(15a^2+10ab-b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}f(a-b)^4}-\frac{b(11a+b)\tan(e+fx)}{8af(a-b)^3\,(a+b\tan^2(e+fx))}-\frac{3b\tan(e+fx)}{4f(a-b)^2\,(a+b\tan^2(e+fx))^2}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\,(a+b\tan^2(e+fx))^2}+\frac{x(a+5b)}{2(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((a + 5*b)*x)/(2*(a - b)^4) - (Sqrt[b]*(15*a^2 + 10*a*b - b^2)*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(3/2)*(a - b)^4*f) - (Cos[e + f*x]*Sin[e +
 f*x])/(2*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - (3*b*Tan[e + f*x])/(4*(a -
b)^2*f*(a + b*Tan[e + f*x]^2)^2) - (b*(11*a + b)*Tan[e + f*x])/(8*a*(a - b)
^3*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
```

```
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{a-5bx^2}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subst}}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\,(a+b\tan^2(e+fx))^2} - \frac{b}{8a(a}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\,(a+b\tan^2(e+fx))^2} - \frac{b}{8a(a}$$

$$= \frac{(a+5b)x}{2(a-b)^4} - \frac{\sqrt{b}\,(15a^2+10ab-b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^4f} - \frac{\cos(e+fx)}{2(a-b)f\,(a+b}$$

Mathematica [A]
time = 1.67, size = 164, normalized size = 0.85

$$\frac{4(a+5b)(e+fx) + \frac{\sqrt{b}\,(-15a^2-10ab+b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} - 2(a-b)\sin(2(e+fx)) + \frac{4(a-b)b^2\sin(2(e+fx))}{(a+b+(a-b)\cos(2(e+fx)))^2} - \frac{(a-b)b(9a+b)\sin(2(e+fx))}{a(a+b+(a-b)\cos(2(e+fx)))}}{8(a-b)^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (4*(a + 5*b)*(e + f*x) + (Sqrt[b]*(-15*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/a^(3/2) - 2*(a - b)*Sin[2*(e + f*x)] + (4*(a - b)*b
^2*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])^2 - ((a - b)*b*(9*a
 + b)*Sin[2*(e + f*x)])/(a*(a + b + (a - b)*Cos[2*(e + f*x)])))/(8*(a - b)^
4*f)
```

Maple [A]
time = 0.46, size = 172, normalized size = 0.89

method	result

derivativedivides	$\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan^2(fx+e)}+\frac{(a+5b)\arctan(\tan(fx+e))}{2}}{(a-b)^4}-\frac{b\left(\frac{\frac{b\left(7a^2-6ab-b^2\right)\left(\tan^3(fx+e)\right)}{8a}+\left(\frac{9}{8}a^2-\frac{5}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{(15a^2+10a}{}\right.}{(a-b)^4}}{f}$
default	$\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan^2(fx+e)}+\frac{(a+5b)\arctan(\tan(fx+e))}{2}}{(a-b)^4}-\frac{b\left(\frac{\frac{b\left(7a^2-6ab-b^2\right)\left(\tan^3(fx+e)\right)}{8a}+\left(\frac{9}{8}a^2-\frac{5}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{(15a^2+10a}{}\right.}{(a-b)^4}}{f}$
risch	$\frac{xa}{2(a^3-3a^2b+3a\,b^2-b^3)(a-b)}+\frac{5xb}{2(a^3-3a^2b+3a\,b^2-b^3)(a-b)}+\frac{i\mathrm{e}^{2i(fx+e)}}{8f(a^3-3a^2b+3a\,b^2-b^3)}-\frac{i\mathrm{e}^{-2i(fx+e)}}{8f(a^3-3a^2b+3a\,b^2-b^3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^4*((-1/2*a+1/2*b)*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+5*b)*arct
an(tan(f*x+e)))-1/(a-b)^4*b*((1/8*b*(7*a^2-6*a*b-b^2)/a*tan(f*x+e)^3+(9/8*a
^2-5/4*a*b+1/8*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(15*a^2+10*a*b-b^2
)/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.51, size = 354, normalized size = 1.83

$$\frac{\frac{4\,(fx+e)(a+5\,b)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(15\,a^2b+10\,ab^2-b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)\sqrt{ab}}-\frac{(11\,ab^2+b^3)\tan(fx+e)^5+(17\,a^2b+6\,ab^2+b^3)\tan(fx+e)^3+(4\,a^3+9\,a^2b-ab^2)\tan(fx+e)}{(a^4b^2-3\,a^3b^3+3\,a^2b^4-ab^5)\tan(fx+e)^6+a^6-3\,a^5b+3\,a^4b^2-a^3b^3+(2\,a^5b-5\,a^4b^2+3\,a^3b^3+a^2b^4-ab^5)\tan(fx+e)^4+(a^6-a^5b-3\,a^4b^2+5\,a^3b^3-2\,a^2b^4)\tan(fx+e)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*(f*x + e)*(a + 5*b)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (1
5*a^2*b + 10*a*b^2 - b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5 - 4*a^4*b
+ 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - ((11*a*b^2 + b^3)*tan(f*x + e
)^5 + (17*a^2*b + 6*a*b^2 + b^3)*tan(f*x + e)^3 + (4*a^3 + 9*a^2*b - a*b^2)
*tan(f*x + e))/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*tan(f*x + e)^6 +
a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3 + (2*a^5*b - 5*a^4*b^2 + 3*a^3*b^3 + a^
2*b^4 - a*b^5)*tan(f*x + e)^4 + (a^6 - a^5*b - 3*a^4*b^2 + 5*a^3*b^3 - 2*a^
2*b^4)*tan(f*x + e)^2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(183) = 366.
time = 2.75, size = 1106, normalized size = 5.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^4 + 3*a^3*b - 9*a^2*b^2 + 5*a*b^3)*f*x*cos(f*x + e)^4 + 32*(a^
3*b + 4*a^2*b^2 - 5*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 5*a*b^3)*f*x
- ((15*a^4 - 20*a^3*b - 6*a^2*b^2 + 12*a*b^3 - b^4)*cos(f*x + e)^4 + 15*a^2
*b^2 + 10*a*b^3 - b^4 + 2*(15*a^3*b - 5*a^2*b^2 - 11*a*b^3 + b^4)*cos(f*x +
 e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)
*cos(f*x + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b
/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)
*cos(f*x + e)^2 + b^2)) - 4*(4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(f*x
+ e)^5 + (17*a^3*b - 33*a^2*b^2 + 15*a*b^3 + b^4)*cos(f*x + e)^3 + (11*a^2*
b^2 - 10*a*b^3 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 6*a^6*b + 15*a^5*
b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*f*cos(f*x + e)^4 + 2*(a^
6*b - 5*a^5*b^2 + 10*a^4*b^3 - 10*a^3*b^4 + 5*a^2*b^5 - a*b^6)*f*cos(f*x +
e)^2 + (a^5*b^2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6)*f), 1/16*(8*(a
^4 + 3*a^3*b - 9*a^2*b^2 + 5*a*b^3)*f*x*cos(f*x + e)^4 + 16*(a^3*b + 4*a^2*
b^2 - 5*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2 + 5*a*b^3)*f*x + ((15*a^4 -
20*a^3*b - 6*a^2*b^2 + 12*a*b^3 - b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b
^3 - b^4 + 2*(15*a^3*b - 5*a^2*b^2 - 11*a*b^3 + b^4)*cos(f*x + e)^2)*sqrt(b
/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f
*x + e))) - 2*(4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + (17*a
^3*b - 33*a^2*b^2 + 15*a*b^3 + b^4)*cos(f*x + e)^3 + (11*a^2*b^2 - 10*a*b^3
 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b
^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*f*cos(f*x + e)^4 + 2*(a^6*b - 5*a^5*b^
2 + 10*a^4*b^3 - 10*a^3*b^4 + 5*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 + (a^5*b^
2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.12, size = 282, normalized size = 1.46

$$\frac{\frac{4\,(fx+e)(a+5\,b)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(15\,a^2b+10\,ab^2-b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)\sqrt{ab}}-\frac{4\,\tan(fx+e)}{(a^3-3\,a^2b+3\,ab^2-b^3)\left(\tan(fx+e)^2+1\right)}-\frac{7\,ab^2\tan(fx+e)^3+b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)-ab^2\tan(fx+e)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\left(b\tan(fx+e)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*(f*x + e)*(a + 5*b)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (1
5*a^2*b + 10*a*b^2 - b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*t
an(f*x + e)/sqrt(a*b)))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sq
rt(a*b)) - 4*tan(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(f*x + e)^2
+ 1)) - (7*a*b^2*tan(f*x + e)^3 + b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e)
 - a*b^2*tan(f*x + e))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(b*tan(f*x + e)
^2 + a)^2))/f
```

Mupad [B]
time = 16.49, size = 2500, normalized size = 12.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] - ((tan(e + f*x)^5*(11*a*b^2 + b^3))/(8*a*(3*a*b^2 - 3*a^2*b + a^3 - b^3))
+ (tan(e + f*x)*(9*a*b + 4*a^2 - b^2))/(8*(a - b)*(a^2 - 2*a*b + b^2)) + (b
*tan(e + f*x)^3*(6*a*b + 17*a^2 + b^2))/(8*a*(a - b)*(a^2 - 2*a*b + b^2)))/
(f*(tan(e + f*x)^2*(2*a*b + a^2) + tan(e + f*x)^4*(2*a*b + b^2) + a^2 + b^2
*tan(e + f*x)^6)) - (atan(((((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 +
460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a
^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((((17*a^2*b^11)/2 - (a*b^12)/2 - 48*a
^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 + 18*a^8*b^
5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b - a^11 + a^2*b
^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a
^8*b^3 - 36*a^9*b^2) - (tan(e + f*x)*(a*1i + b*5i)*(256*a^2*b^11 - 1792*a^3
*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^
8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^4 - 4*a^3*b -
 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b
^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3*b - 4*a*b^3
+ b^4 + 6*a^2*b^2)))*(a*1i + b*5i)*1i)/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
6*a^2*b^2)) + (((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 + 460*a^3*b^4 +
 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a
^5*b^3 + 15*a^6*b^2)) + ((((17*a^2*b^11)/2 - (a*b^12)/2 - 48*a^3*b^10 + 138
*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 + 18*a^8*b^5 + (39*a^9*b
^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^
8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a
^9*b^2) + (tan(e + f*x)*(a*1i + b*5i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*
a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*
a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^4 - 4*a^3*b - 4*a*b^3 + b^
4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b
^3 + 15*a^6*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2
*b^2)))*(a*1i + b*5i)*1i)/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/
```

(((39*a^2*b^5)/4 - (5*b^7)/64 - (3*a*b^6)/32 + (475*a^3*b^4)/32 + (165*a^4*b^3)/64)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 + 460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((((17*a^2*b^11)/2 - (a*b^12)/2 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 + 18*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(e + f*x)*(a*1i + b*5i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 + 460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + ((((17*a^2*b^11)/2 - (a*b^12)/2 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 + 18*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) + (tan(e + f*x)*(a*1i + b*5i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))*(a*1i + b*5i)*1i)/(2*f*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (atan(((((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^5 + 460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((-a^3*b)^(1/2)*(((17*a^2*b^11)/2 - (a*b^12)/2 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 + 18*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(10*a*b + 15*a^2 - b^2)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(512*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(10*a*b + 15*a^2 - b^2))/(16*(a^7 - 4*a^6*b +...

3.88 $\int \frac{1}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal. Leaf size=150

$$\frac{x}{(a-b)^3}-\frac{\sqrt{b}\,(15a^2-10ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3 f}-\frac{b\tan(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2}-\frac{(7a-}{8a^2(a-b)^2}$$

```
[Out] x/(a-b)^3-1/8*(15*a^2-10*a*b+3*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1
/2)/a^(5/2)/(a-b)^3/f-1/4*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(
7*a-3*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3742, 425, 541, 536, 209, 211}

$$-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\,(a+b\tan^2(e+fx))}-\frac{\sqrt{b}\,(15a^2-10ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b\tan(e+fx)}{4af(a-b)\,(a+b\tan^2(e+fx))^2}+\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*f) - (b*Tan[e + f*x])/(4*a*(a - b)*f*(a
+ b*Tan[e + f*x]^2)^2) - ((7*a - 3*b)*b*Tan[e + f*x])/(8*a^2*(a - b)^2*f*(a
 + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\,(a+b\tan^2(e+fx))} + \frac{\text{Sul}}{}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\,(a+b\tan^2(e+fx))} + \frac{\text{Sul}}{}$$

$$= \frac{x}{(a-b)^3} - \frac{\sqrt{b}\,(15a^2-10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3f} - \frac{b\tan(e}{4a(a-b)f\,(a+}$$

Mathematica [A]
time = 1.34, size = 138, normalized size = 0.92

$$-\frac{-8\text{ArcTan}(\tan(e+fx))+\frac{\sqrt{b}\,(15a^2-10ab+3b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2(a-b)^2b\tan(e+fx)}{a(a+b\tan^2(e+fx))^2}+\frac{(7a-3b)(a-b)b\tan(e+fx)}{a^2(a+b\tan^2(e+fx))}}{8(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] -1/8*(-8*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(
Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[e + f*x])/(a*(
a + b*Tan[e + f*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[e + f*x])/(a^2*(a + b
*Tan[e + f*x]^2)))/((a - b)^3*f)
```

Maple [A]
time = 0.32, size = 142, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^3}-\frac{b\left(\frac{\frac{b\left(7a^2-10ab+3b^2\right)\left(\tan^3(fx+e)\right)}{8a^2}+\frac{\left(9a^2-14ab+5b^2\right)\tan(fx+e)}{8a}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2-10ab+3b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right.}{8a^2\sqrt{ab}}\right.}{(a-b)^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^3}-\frac{b\left(\frac{\frac{b\left(7a^2-10ab+3b^2\right)\left(\tan^3(fx+e)\right)}{8a^2}+\frac{\left(9a^2-14ab+5b^2\right)\tan(fx+e)}{8a}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2-10ab+3b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right.}{8a^2\sqrt{ab}}\right.}{(a-b)^3}}{f}$
risch	$\frac{x}{a^3-3a^2b+3a\,b^2-b^3}+\frac{i\left(9a^3\mathrm{e}^{6i(fx+e)}+a^2b\,\mathrm{e}^{6i(fx+e)}-13a\,b^2\mathrm{e}^{6i(fx+e)}+3b^3\mathrm{e}^{6i(fx+e)}+27a^3\mathrm{e}^{4i(fx+e)}+9a^2b\,\mathrm{e}^{4i(fx+e)}\right.}{4\left(-a\,\mathrm{e}^{4i(fx+e)}+b\,\mathrm{e}^{4i(f\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^3*arctan(tan(f*x+e))-b/(a-b)^3*((1/8*b*(7*a^2-10*a*b+3*b^2)/a^
2*tan(f*x+e)^3+1/8*(9*a^2-14*a*b+5*b^2)/a*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+
1/8*(15*a^2-10*a*b+3*b^2)/a^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))
)
```

Maxima [A]
time = 0.51, size = 233, normalized size = 1.55

$$-\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}+\frac{(7\,ab^2-3\,b^3)\tan(fx+e)^3+(9\,a^2b-5\,ab^2)\tan(fx+e)}{a^6-2\,a^5b+a^4b^2+(a^4b^2-2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b-2\,a^4b^2+a^3b^3)\tan(fx+e)^2}-\frac{8\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5
- 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) + ((7*a*b^2 - 3*b^3)*tan(f*x +
e)^3 + (9*a^2*b - 5*a*b^2)*tan(f*x + e))/(a^6 - 2*a^5*b + a^4*b^2 + (a^4*b^
2 - 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^3)*t
an(f*x + e)^2) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(141) = 282$.
time = 3.40, size = 766, normalized size = 5.11

$$\left[\frac{32 a^2b^2fx\tan(fx+e)^4 + 64a^3bfx\tan(fx+e)^2 + 32a^4fx - 4(7a^2b^2 - 10ab^3 + 3b^4)\tan(fx+e)^3 - \left((15a^2b^2 - 10ab^3 + 3b^4)\tan(fx+e)^4 + 15a^4 - 10a^3b + 3a^2b^2 + 2(15a^3b - 10a^2b^2 + 3ab^3)\tan(fx+e)^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4 - 6ab\tan(fx+e)^2 + a^2 + 4\left(ab\tan(fx+e)^3 - a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4 + 2ab\tan(fx+e)^2 + a^2}\right) - 4(9a^3b - 14a^2b^2 + 5ab^3)\tan(fx+e)}{32\left((a^5b^2 - 3a^4b^3 + 3a^3b^4 - a^2b^5)f\tan(fx+e)^4 + 2(a^6b - 3a^5b^2 + 3a^4b^3 - a^3b^4)f\tan(fx+e)^2 + (a^7 - 3a^6b + 3a^5b^2 - a^4b^3)f\right)}, \frac{16a^2b^2fx\tan(fx+e)^4 + 32a^3bfx\tan(fx+e)^2 + 16a^4fx - 2(7a^2b^2 - 10ab^3 + 3b^4)\tan(fx+e)^3 - \left((15a^2b^2 - 10ab^3 + 3b^4)\tan(fx+e)^4 + 15a^4 - 10a^3b + 3a^2b^2 + 2(15a^3b - 10a^2b^2 + 3ab^3)\tan(fx+e)^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2 - a\right)\sqrt{\frac{b}{a}}}{2b\tan(fx+e)}\right) - 2(9a^3b - 14a^2b^2 + 5ab^3)\tan(fx+e)}{16\left((a^5b^2 - 3a^4b^3 + 3a^3b^4 - a^2b^5)f\tan(fx+e)^4 + 2(a^6b - 3a^5b^2 + 3a^4b^3 - a^3b^4)f\tan(fx+e)^2 + (a^7 - 3a^6b + 3a^5b^2 - a^4b^3)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a^2*b^2*f*x*tan(f*x + e)^4 + 64*a^3*b*f*x*tan(f*x + e)^2 + 32*a^4
*f*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b
 - 10*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4
 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*s
qrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(9*a^3*b
- 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*ta
n(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/16*(16*a^2*b^2*f
*x*tan(f*x + e)^4 + 32*a^3*b*f*x*tan(f*x + e)^2 + 16*a^4*f*x - 2*(7*a^2*b^2
 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(
f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a^2*b^2 + 3*a
*b^3)*tan(f*x + e)^2)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)
/(b*tan(f*x + e))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8964 vs. $2(133) = 266$.
time = 78.80, size = 8964, normalized size = 59.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**6, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**3, Eq(b,
 0)), ((-x - 1/(f*tan(e + f*x)) + 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e +
f*x)**5))/b**3, Eq(a, 0)), (15*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)*
*6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) +
 45*f*x*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x
)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*x)**2/(48
*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f
*x)**2 + 48*b**3*f) + 15*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e
+ f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 15*tan(e + f*x)**5/(4
8*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e +
f*x)**2 + 48*b**3*f) + 40*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*
b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 33*tan(e
 + f*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*
f*tan(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x/(a + b*tan(e)**2)**3, Eq(f, 0
)), (16*a**4*f*x*sqrt(-a/b)/(16*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*
tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e
+ f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt
(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a
/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)
*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5
*f*sqrt(-a/b)*tan(e + f*x)**4) - 15*a**4*log(-sqrt(-a/b) + tan(e + f*x))/(1
6*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*
sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqr
t(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-
a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*
b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**
4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4)
 + 15*a**4*log(sqrt(-a/b) + tan(e + f*x))/(16*a**7*f*sqrt(-a/b) + 32*a**6*b
*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqr
t(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a*
*5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*
b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**
4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2
- 16*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4) + 32*a**3*b*f*x*sqrt(-a/b)*tan
(e + f*x)**2/(16*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2
 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*
a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**
4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x
)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**
4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b)*t
an(e + f*x)**4) - 18*a**3*b*sqrt(-a/b)*tan(e + f*x)/(16*a**7*f*sqrt(-a/b) +
 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*
b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)*
*2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4
+ 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 4
```

```
8*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e
+ f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4) - 30*a**3*b*log(-sqr
t(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(16*a**7*f*sqrt(-a/b) + 32*a**6*b*f
*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(
-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**5
*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*b*
*3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**4*
f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 -
16*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4) + 10*a**3*b*log(-sqrt(-a/b) + ta
n(e + f*x))/(16*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2
- 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a
**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**4
*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)
**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4
 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b)*ta
n(e + f*x)**4) + 30*a**3*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(
16*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f
*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sq
rt(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(
-a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4
*b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b*
*4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*...
```

Giac [A]
time = 0.69, size = 213, normalized size = 1.42

$$-\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}-\frac{8\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{7\,ab^2\tan(fx+e)^3-3\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)-5\,ab^2\tan(fx+e)}{(a^4-2\,a^3b+a^2b^2)\left(b\tan(fx+e)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (7*a*b^2*tan(f*x +
 e)^3 - 3*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e))
/((a^4 - 2*a^3*b + a^2*b^2)*(b*tan(f*x + e)^2 + a)^2))/f
```

Mupad [B]
time = 14.93, size = 2500, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (atan((((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300
*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b
^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^
6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^1
0 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2))
- (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 128
0*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 128
0*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8
 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10
*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b
 + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b)^(1/2)*
((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3)
)/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^2*b^10 -
 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 +
5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 -
 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(-a^5*b
)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^
7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b
^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 -
4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a
^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^
7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a
^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3
 + 15*a^8*b^2)) - ((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^
2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^
3 + 6*a^6*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^
7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^
2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15
*a^8*b^2)) - (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^
4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^
9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^
6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(
15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a
^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b
)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*
a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^
2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^
7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a
^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)
*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 230
4*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 25
6*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^
4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/
(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2))/(16*
(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2))))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b +
```

```
3*b^2)*1i)/(8*f*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) - ((tan(e + f*x)^3*(
7*a*b^2 - 3*b^3))/(8*a^2*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)*(9*a*b - 5*b^
2))/(8*a*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan(e +
 f*x)^2)) - (2*atan((((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^
5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^1
0*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3
+ 15*a^8*b^2)) - (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 -
 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)
*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a
^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*
x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 -
 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 -
2*b^3) - ((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760
*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(
a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2
)) + (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^
6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a
^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(9*b^7 -
60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a
^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((51
*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a^3*b^3)/(32...
```

3.89 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f}-\frac{15\cot(e+fx)}{8a^3f}+\frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2}+\frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -15/8*cot(f*x+e)/a^3/f-15/8*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(7
/2)/f+1/4*cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)^2+5/8*cot(f*x+e)/a^2/f/(a+b*tan
(f*x+e)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3744, 296, 331, 211}

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f}-\frac{15\cot(e+fx)}{8a^3f}+\frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)}+\frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(7/2)*f) - (15*Co
t[e + f*x])/(8*a^3*f) + Cot[e + f*x]/(4*a*f*(a + b*Tan[e + f*x]^2)^2) + (5*
Cot[e + f*x])/(8*a^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)} + \frac{15\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)}\right.}{8a}$$

$$= -\frac{15\cot(e+fx)}{8a^3f} + \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)} -$$

$$= -\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f} - \frac{15\cot(e+fx)}{8a^3f} + \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2}$$

Mathematica [A]
time = 0.68, size = 144, normalized size = 1.29

$$\frac{-15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right) - 8\sqrt{a}\,\cot(e+fx) + \frac{4a^{3/2}b^2\sin(2(e+fx))}{(a-b)(a+b+(a-b)\cos(2(e+fx)))^2} - \frac{\sqrt{a}\,(9a-7b)b\sin(2(e+fx))}{(a-b)(a+b+(a-b)\cos(2(e+fx)))}}{8a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - 8*Sqrt[a]*Cot[e + f*x
] + (4*a^(3/2)*b^2*Sin[2*(e + f*x)])/((a - b)*(a + b + (a - b)*Cos[2*(e + f
*x)])^2) - (Sqrt[a]*(9*a - 7*b)*b*Sin[2*(e + f*x)])/((a - b)*(a + b + (a -
b)*Cos[2*(e + f*x)])))/(8*a^(7/2)*f)
```

Maple [A]
time = 0.27, size = 83, normalized size = 0.74

method	result
derivativedivides	$\frac{-\frac{1}{a^{3}\tan\left(fx+e\right)}-\frac{b\left(\frac{\frac{7b\left(\tan^{3}\left(fx+e\right)\right)}{8}+\frac{9a\tan\left(fx+e\right)}{8}}{\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{2}}+\frac{15\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}}{f}$
default	$\frac{-\frac{1}{a^{3}\tan\left(fx+e\right)}-\frac{b\left(\frac{\frac{7b\left(\tan^{3}\left(fx+e\right)\right)}{8}+\frac{9a\tan\left(fx+e\right)}{8}}{\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{2}}+\frac{15\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}}{f}$
risch	$-\frac{i\left(8a^{4}e^{8i(fx+e)}-23a^{3}b\,e^{8i(fx+e)}+45a^{2}b^{2}e^{8i(fx+e)}-45a\,b^{3}e^{8i(fx+e)}+15b^{4}e^{8i(fx+e)}+32a^{4}e^{6i(fx+e)}-46a^{3}b\,e^{6i(fx+e)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^3/tan(f*x+e)-1/a^3*b*((7/8*b*tan(f*x+e)^3+9/8*a*tan(f*x+e))/(a+b*
tan(f*x+e)^2)^2+15/8/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.50, size = 111, normalized size = 0.99

$$-\frac{\frac{15\,b^{2}\tan\left(fx+e\right)^{4}+25\,ab\tan\left(fx+e\right)^{2}+8\,a^{2}}{a^{3}b^{2}\tan\left(fx+e\right)^{5}+2\,a^{4}b\tan\left(fx+e\right)^{3}+a^{5}\tan\left(fx+e\right)}+\frac{15\,b\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*b^2*tan(f*x + e)^4 + 25*a*b*tan(f*x + e)^2 + 8*a^2)/(a^3*b^2*tan(
f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*tan(f*x + e)) + 15*b*arctan(b*tan
(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(102) = 204$.
time = 4.36, size = 583, normalized size = 5.21

$$\left[-\frac{4\left(8a^{2}-25ab+15b^{2}\right)\cos\left(fx+e\right)^{5}+20\left(5ab-6b^{2}\right)\cos\left(fx+e\right)^{3}-15\left(\left(a^{2}-2ab+b^{2}\right)\cos\left(fx+e\right)^{4}+2\left(ab-b^{2}\right)\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^{2}+6ab+b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3ab+b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a^{2}+ab\right)\cos\left(fx+e\right)^{3}-ab\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}\sin\left(fx+e\right)+b^{2}}{\left(a^{2}-2ab+b^{2}\right)\cos\left(fx+e\right)^{4}+2\left(ab-b^{2}\right)\cos\left(fx+e\right)^{2}+b^{2}}\right)\sin\left(fx+e\right)+60b^{2}\cos\left(fx+e\right)}{32\left(a^{3}b^{2}f+\left(a^{5}-2a^{4}b+a^{3}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{4}b-a^{3}b^{2}\right)f\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)},\ -\frac{2\left(8a^{2}-25ab+15b^{2}\right)\cos\left(fx+e\right)^{5}+10\left(5ab-6b^{2}\right)\cos\left(fx+e\right)^{3}-15\left(\left(a^{2}-2ab+b^{2}\right)\cos\left(fx+e\right)^{4}+2\left(ab-b^{2}\right)\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(\left(a+b\right)\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{b}{a}}}{2b\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+30b^{2}\cos\left(fx+e\right)}{16\left(a^{3}b^{2}f+\left(a^{5}-2a^{4}b+a^{3}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{4}b-a^{3}b^{2}\right)f\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x + e)^5 + 20*(5*a*b - 6*b^2)*cos
(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*
x + e)^2 + b^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a
*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e)
)*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a
*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/((a^3*
b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^4 + 2*(a^4*b - a^3*b^2)*f*
cos(f*x + e)^2)*sin(f*x + e)), -1/16*(2*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x +
 e)^5 + 10*(5*a*b - 6*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x
 + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(1/2*((a + b)
*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e)
+ 30*b^2*cos(f*x + e))/((a^3*b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x +
e)^4 + 2*(a^4*b - a^3*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.06, size = 109, normalized size = 0.97

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,a^3}+\frac{7\,b^2\tan(fx+e)^3+9\,ab\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^2a^3}+\frac{8}{a^3\tan(fx+e)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b)))*b/(sqrt(a*b)*a^3) + (7*b^2*tan(f*x + e)^3 + 9*a*b*tan(f*x + e))/((b*
tan(f*x + e)^2 + a)^2*a^3) + 8/(a^3*tan(f*x + e)))/f
```

Mupad [**B**]
time = 11.22, size = 102, normalized size = 0.91

$$-\frac{\frac{1}{a}+\frac{25\,b\tan(e+f\,x)^2}{8\,a^2}+\frac{15\,b^2\tan(e+f\,x)^4}{8\,a^3}}{f\,\left(a^2\tan\left(e+f\,x\right)+2\,a\,b\tan\left(e+f\,x\right)^3+b^2\tan\left(e+f\,x\right)^5\right)}-\frac{15\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a}}\right)}{8\,a^{7/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] - (1/a + (25*b*tan(e + f*x)^2)/(8*a^2) + (15*b^2*tan(e + f*x)^4)/(8*a^3))/(
f*(a^2*tan(e + f*x) + b^2*tan(e + f*x)^5 + 2*a*b*tan(e + f*x)^3)) - (15*b^(
1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2)))/(8*a^(7/2)*f)
```

3.90 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{5(3a-7b)\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f}-\frac{(a-3b)\cot(e+fx)}{a^4f}-\frac{\cot^3(e+fx)}{3a^3f}-\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2}-$$

```
[Out] -(a-3*b)*cot(f*x+e)/a^4/f-1/3*cot(f*x+e)^3/a^3/f-5/8*(3*a-7*b)*arctan(b^(1/
2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(9/2)/f-1/4*(a-b)*b*tan(f*x+e)/a^3/f/(a+b*
tan(f*x+e)^2)^2-1/8*(7*a-11*b)*b*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 467, 1273, 1275, 211}

$$-\frac{5\sqrt{b}\,(3a-7b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f}-\frac{b(7a-11b)\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)}-\frac{(a-3b)\cot(e+fx)}{a^4f}-\frac{b(a-b)\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2}-\frac{\cot^3(e+fx)}{3a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-5*(3*a - 7*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(9/2)*
f) - ((a - 3*b)*Cot[e + f*x])/(a^4*f) - Cot[e + f*x]^3/(3*a^3*f) - ((a - b)
*b*Tan[e + f*x])/(4*a^3*f*(a + b*Tan[e + f*x]^2)^2) - ((7*a - 11*b)*b*Tan[e
 + f*x])/(8*a^4*f*(a + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b\text{Subst}\left(\int \frac{-\frac{4}{ab}-\frac{4(a-b)x^2}{a^2b}+\frac{3(a-b)x^4}{a^3}}{x^4(a+bx^2)^2}\,dx, x, \tan(e+f\right.}{4f}$$

$$= -\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-11b)b\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-8ab-8(a\right.}{}$$

$$= -\frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-11b)b\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8b}{x^4}-\right.\right.}{}$$

$$= -\frac{(a-3b)\cot(e+fx)}{a^4f} - \frac{\cot^3(e+fx)}{3a^3f} - \frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-}{8a^4f(}$$

$$= -\frac{5(3a-7b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f} - \frac{(a-3b)\cot(e+fx)}{a^4f} - \frac{\cot^3(e+fx}{3a^3f}$$

Mathematica [A]
time = 1.21, size = 146, normalized size = 0.95

$$\frac{15\sqrt{b}\,(-3a+7b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}\left(-8\cot(e+fx)\left(2a-9b+a\csc^2(e+fx)\right)-\frac{3b\left(9a^2-6ab-11b^2+(9a^2-20ab+11b^2)\cos(2(e+fx))\right)\sin(2(e+fx))}{(a+b+(a-b)\cos(2(e+fx)))^2}\right)}{24a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*(-3*a + 7*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(
-8*Cot[e + f*x]*(2*a - 9*b + a*Csc[e + f*x]^2) - (3*b*(9*a^2 - 6*a*b - 11*b
^2 + (9*a^2 - 20*a*b + 11*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b +
 (a - b)*Cos[2*(e + f*x)])^2))/(24*a^(9/2)*f)
```

Maple [A]
time = 0.38, size = 123, normalized size = 0.80

method	result

derivativedivides	$\frac{-\frac{1}{3a^3\tan(fx+e)^3}-\frac{a-3b}{a^4\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{11}{8}b^2\right)\left(\tan^3(fx+e)\right)+\frac{a(9a-13b)\tan(fx+e)}{8}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{5(3a-7b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}}{f}$
default	$\frac{-\frac{1}{3a^3\tan(fx+e)^3}-\frac{a-3b}{a^4\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{11}{8}b^2\right)\left(\tan^3(fx+e)\right)+\frac{a(9a-13b)\tan(fx+e)}{8}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{5(3a-7b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}}{f}$
risch	$\frac{i\left(147a^3b-16a^4+176a^4\mathrm{e}^{8i(fx+e)}-1575b^4\mathrm{e}^{8i(fx+e)}+224a^4\mathrm{e}^{6i(fx+e)}+2100b^4\mathrm{e}^{6i(fx+e)}+96a^4\mathrm{e}^{4i(fx+e)}-1575b^4\mathrm{e}^{4i(fx+e)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3/a^3/tan(f*x+e)^3-(a-3*b)/a^4/tan(f*x+e)-1/a^4*b*(((7/8*a*b-11/8*b
^2)*tan(f*x+e)^3+1/8*a*(9*a-13*b)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+5/8*(3*a
-7*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.51, size = 165, normalized size = 1.07

$$-\frac{\frac{15\,(3\,ab^2-7\,b^3)\tan(fx+e)^6+25\,(3\,a^2b-7\,ab^2)\tan(fx+e)^4+8\,a^3+8\,(3\,a^3-7\,a^2b)\tan(fx+e)^2}{a^4b^2\tan(fx+e)^7+2\,a^5b\tan(fx+e)^5+a^6\tan(fx+e)^3}+\frac{15\,(3\,ab-7\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*((15*(3*a*b^2 - 7*b^3)*tan(f*x + e)^6 + 25*(3*a^2*b - 7*a*b^2)*tan(f*
x + e)^4 + 8*a^3 + 8*(3*a^3 - 7*a^2*b)*tan(f*x + e)^2)/(a^4*b^2*tan(f*x + e
)^7 + 2*a^5*b*tan(f*x + e)^5 + a^6*tan(f*x + e)^3) + 15*(3*a*b - 7*b^2)*arc
tan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(145) = 290.
time = 5.95, size = 891, normalized size = 5.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(4*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 4*(24
*a^3 - 206*a^2*b + 485*a*b^2 - 315*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 62*
a*b^2 + 63*b^3)*cos(f*x + e)^3 + 15*((3*a^3 - 13*a^2*b + 17*a*b^2 - 7*b^3)*
cos(f*x + e)^6 - (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*cos(f*x + e)^4 - 3*
a*b^2 + 7*b^3 - (6*a^2*b - 23*a*b^2 + 21*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*lo
g(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*
((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b
^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^
2))*sin(f*x + e) - 60*(3*a*b^2 - 7*b^3)*cos(f*x + e))/(((a^6 - 2*a^5*b + a^
4*b^2)*f*cos(f*x + e)^6 - a^4*b^2*f - (a^6 - 4*a^5*b + 3*a^4*b^2)*f*cos(f*x
 + e)^4 - (2*a^5*b - 3*a^4*b^2)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/48*(2*(
16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 2*(24*a^3 - 206*
a^2*b + 485*a*b^2 - 315*b^3)*cos(f*x + e)^5 - 10*(15*a^2*b - 62*a*b^2 + 63*
b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2 - 7*b^3)*cos(f*x + e
)^6 - (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*cos(f*x + e)^4 - 3*a*b^2 + 7*b
^3 - (6*a^2*b - 23*a*b^2 + 21*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a
 + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x
+ e) - 30*(3*a*b^2 - 7*b^3)*cos(f*x + e))/(((a^6 - 2*a^5*b + a^4*b^2)*f*cos
(f*x + e)^6 - a^4*b^2*f - (a^6 - 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^4 - (2
*a^5*b - 3*a^4*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.03, size = 175, normalized size = 1.14

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3ab-7b^2)}{\sqrt{ab}\,a^4}+\frac{3\left(7ab^2\tan(fx+e)^3-11b^3\tan(fx+e)^3+9a^2b\tan(fx+e)-13ab^2\tan(fx+e)\right)}{\left(b\tan(fx+e)^2+a\right)^2a^4}+\frac{8\left(3a\tan(fx+e)^2-9b\tan(fx+e)^2+a\right)}{a^4\tan(fx+e)^3}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b)))*(3*a*b - 7*b^2)/(sqrt(a*b)*a^4) + 3*(7*a*b^2*tan(f*x + e)^3 - 11*b^
3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) - 13*a*b^2*tan(f*x + e))/((b*tan(f*
```

```
x + e)^2 + a)^2*a^4) + 8*(3*a*tan(f*x + e)^2 - 9*b*tan(f*x + e)^2 + a)/(a^4
*tan(f*x + e)^3))/f
```

Mupad [B]
time = 12.28, size = 147, normalized size = 0.95

$$-\frac{\frac{1}{3\,a}+\frac{\tan(e+f\,x)^2\,(3\,a-7\,b)}{3\,a^2}+\frac{25\,b\tan(e+f\,x)^4\,(3\,a-7\,b)}{24\,a^3}+\frac{5\,b^2\tan(e+f\,x)^6\,(3\,a-7\,b)}{8\,a^4}}{f\,\left(a^2\tan\left(e+f\,x\right)^3+2\,a\,b\tan\left(e+f\,x\right)^5+b^2\tan\left(e+f\,x\right)^7\right)}-\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a}}\right)\,(3\,a-7\,b)}{8\,a^{9/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^3),x)
```

```
[Out] - (1/(3*a) + (tan(e + f*x)^2*(3*a - 7*b))/(3*a^2) + (25*b*tan(e + f*x)^4*(3
*a - 7*b))/(24*a^3) + (5*b^2*tan(e + f*x)^6*(3*a - 7*b))/(8*a^4))/(f*(a^2*t
an(e + f*x)^3 + b^2*tan(e + f*x)^7 + 2*a*b*tan(e + f*x)^5)) - (5*b^(1/2)*at
an((b^(1/2)*tan(e + f*x))/a^(1/2))*(3*a - 7*b))/(8*a^(9/2)*f)
```

3.91 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{b}\,(15a^2-70ab+63b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f}-\frac{(5a^2-30ab+27b^2)\cot(e+fx)}{5a^5f}-\frac{(10a-9b)\cot^3(e+}{15a^4f}$$

```
[Out] -1/5*(5*a^2-30*a*b+27*b^2)*cot(f*x+e)/a^5/f-1/15*(10*a-9*b)*cot(f*x+e)^3/a^
4/f-1/8*(15*a^2-70*a*b+63*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a
^(11/2)/f-1/5*cot(f*x+e)^5/a/f/(a+b*tan(f*x+e)^2)^2-1/20*b*(5*a^2-10*a*b+9*
b^2)*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)^2-1/40*b*(35*a^2-110*a*b+99*b^2)*t
an(f*x+e)/a^5/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.21, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3744, 473, 467, 1273, 1275, 211}

$$-\frac{(10a-9b)\cot^3(e+fx)}{15a^4f}-\frac{\sqrt{b}\,(15a^2-70ab+63b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f}-\frac{b(35a^2-110ab+99b^2)\tan(e+fx)}{40a^5f\,(a+b\tan^2(e+fx))}-\frac{(5a^2-30ab+27b^2)\cot(e+fx)}{5a^5f}-\frac{b(5a^2-10ab+9b^2)\tan(e+fx)}{20a^4f\,(a+b\tan^2(e+fx))^2}-\frac{\cot^5(e+fx)}{5af\,(a+b\tan^2(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -1/8*(Sqrt[b]*(15*a^2 - 70*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a]])/(a^(11/2)*f) - ((5*a^2 - 30*a*b + 27*b^2)*Cot[e + f*x])/(5*a^5*f) - (
(10*a - 9*b)*Cot[e + f*x]^3)/(15*a^4*f) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[
e + f*x]^2)^2) - (b*(5*a^2 - 10*a*b + 9*b^2)*Tan[e + f*x])/(20*a^4*f*(a + b
*Tan[e + f*x]^2)^2) - (b*(35*a^2 - 110*a*b + 99*b^2)*Tan[e + f*x])/(40*a^5*
f*(a + b*Tan[e + f*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
```

```
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{10a-9b+5ax^2}{x^4(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{\cot^5(e+fx)}{5af\,(a+b\tan^2(e+fx))^2} - \frac{b(5a^2-10ab+9b^2)\tan(e+fx)}{20a^4f\,(a+b\tan^2(e+fx))^2} - \frac{b\text{Subst}\left(\int \frac{4}{\cdots}\right.}{\cdots}$$

$$= -\frac{\cot^5(e+fx)}{5af\,(a+b\tan^2(e+fx))^2} - \frac{b(5a^2-10ab+9b^2)\tan(e+fx)}{20a^4f\,(a+b\tan^2(e+fx))^2} - \frac{b(35a^2-11\cdots}{40a^5f\,(\cdots}$$

$$= -\frac{\cot^5(e+fx)}{5af\,(a+b\tan^2(e+fx))^2} - \frac{b(5a^2-10ab+9b^2)\tan(e+fx)}{20a^4f\,(a+b\tan^2(e+fx))^2} - \frac{b(35a^2-11\cdots}{40a^5f\,(\cdots}$$

$$= -\frac{(5a^2-30ab+27b^2)\cot(e+fx)}{5a^5f} - \frac{(10a-9b)\cot^3(e+fx)}{15a^4f} - \frac{\cot^5(e+\cdots}{5af\,(a+b\tan\cdots}$$

$$= -\frac{\sqrt{b}\,(15a^2-70ab+63b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f} - \frac{(5a^2-30ab+27b^2)\cot(\cdots}{5a^5f}$$

Mathematica [A]
time = 1.12, size = 346, normalized size = 1.50

$$\frac{-960\sqrt{b}\,(15a^2-70ab+63b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)-\frac{2\sqrt{a}\,(1600a^4-165a^3b+637a^2b^2-28875ab^3+33075b^4+4(416a^4-447a^3b-1400a^2b^2+13125ab^3-13230b^4)\cos(2(e+fx))-4(32a^4-257a^3b-2821a^2b^2+8925ab^3-6615b^4)\cos(4(e+fx))-128a^4\cos(6(e+fx))+1788a^3b\cos(6(e+fx))-8800a^2b^2\cos(6(e+fx))+14700ab^3\cos(6(e+fx))-7560b^4\cos(6(e+fx))+64a^4\cos(8(e+fx))-863a^3b\cos(8(e+fx))+2479a^2b^2\cos(8(e+fx))-2625ab^3\cos(8(e+fx))+945b^4\cos(8(e+fx)))\cot(e+fx)\csc^4(e+fx)}{(a+b+(a-b)\cos(2(e+fx)))^2}}{7680a^{11/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-960*Sqrt[b]*(15*a^2 - 70*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a]] - (2*Sqrt[a]*(1600*a^4 - 165*a^3*b + 637*a^2*b^2 - 28875*a*b^3 + 33075
*b^4 + 4*(416*a^4 - 447*a^3*b - 1400*a^2*b^2 + 13125*a*b^3 - 13230*b^4)*Cos
[2*(e + f*x)] - 4*(32*a^4 - 257*a^3*b - 2821*a^2*b^2 + 8925*a*b^3 - 6615*b^
4)*Cos[4*(e + f*x)] - 128*a^4*Cos[6*(e + f*x)] + 1788*a^3*b*Cos[6*(e + f*x)
] - 8800*a^2*b^2*Cos[6*(e + f*x)] + 14700*a*b^3*Cos[6*(e + f*x)] - 7560*b^4
*Cos[6*(e + f*x)] + 64*a^4*Cos[8*(e + f*x)] - 863*a^3*b*Cos[8*(e + f*x)] +
2479*a^2*b^2*Cos[8*(e + f*x)] - 2625*a*b^3*Cos[8*(e + f*x)] + 945*b^4*Cos[8
*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^4)/(a + b + (a - b)*Cos[2*(e + f*x)]
)^2)/(7680*a^(11/2)*f)
```

Maple [A]
time = 0.40, size = 175, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{1}{5a^3\tan(fx+e)^5}-\frac{2a-3b}{3a^4\tan(fx+e)^3}-\frac{a^2-6ab+6b^2}{a^5\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b-\frac{11}{4}ab^2+\frac{15}{8}b^3\right)\left(\tan^3(fx+e)\right)+\frac{a\left(9a^2-26ab+17b^2\right)\tan(fx+e)}{8}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\right.}{a^5}}{f}$
default	$\frac{-\frac{1}{5a^3\tan(fx+e)^5}-\frac{2a-3b}{3a^4\tan(fx+e)^3}-\frac{a^2-6ab+6b^2}{a^5\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b-\frac{11}{4}ab^2+\frac{15}{8}b^3\right)\left(\tan^3(fx+e)\right)+\frac{a\left(9a^2-26ab+17b^2\right)\tan(fx+e)}{8}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\right.}{a^5}}{f}$
risch	$i\left(863a^3b-64a^4-2624a^4e^{8i(fx+e)}-66150b^4e^{8i(fx+e)}-896a^4e^{6i(fx+e)}+52920b^4e^{6i(fx+e)}+256a^4e^{4i(fx+e)}-26460b^4e^{4\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/5/a^3/tan(f*x+e)^5-1/3*(2*a-3*b)/a^4/tan(f*x+e)^3-(a^2-6*a*b+6*b^2)
/a^5/tan(f*x+e)-b/a^5*(((7/8*a^2*b-11/4*a*b^2+15/8*b^3)*tan(f*x+e)^3+1/8*a*
(9*a^2-26*a*b+17*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(15*a^2-70*a*b+6
3*b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.51, size = 220, normalized size = 0.95

$$-\frac{\frac{15\left(15a^2b^2-70ab^3+63b^4\right)\tan(fx+e)^8+25\left(15a^3b-70a^2b^2+63ab^3\right)\tan(fx+e)^6+8\left(15a^4-70a^3b+63a^2b^2\right)\tan(fx+e)^4+24a^4+8\left(10a^4-9a^3b\right)\tan(fx+e)^2}{a^5b^2\tan(fx+e)^9+2a^6b\tan(fx+e)^7+a^7\tan(fx+e)^5}+\frac{15\left(15a^2b-70ab^2+63b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^5}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/120*((15*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*tan(f*x + e)^8 + 25*(15*a^3*b
- 70*a^2*b^2 + 63*a*b^3)*tan(f*x + e)^6 + 8*(15*a^4 - 70*a^3*b + 63*a^2*b^2
)*tan(f*x + e)^4 + 24*a^4 + 8*(10*a^4 - 9*a^3*b)*tan(f*x + e)^2)/(a^5*b^2*t
an(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(f*x + e)^5) + 15*(15*a^2*b
 - 70*a*b^2 + 63*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(220) = 440.
time = 7.49, size = 1239, normalized size = 5.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(64*a^4 - 863*a^3*b + 2479*a^2*b^2 - 2625*a*b^3 + 945*b^4)*cos(f
*x + e)^9 - 4*(160*a^4 - 2173*a^3*b + 7158*a^2*b^2 - 8925*a*b^3 + 3780*b^4)
*cos(f*x + e)^7 + 4*(120*a^4 - 1685*a^3*b + 7104*a^2*b^2 - 11025*a*b^3 + 56
70*b^4)*cos(f*x + e)^5 + 20*(75*a^3*b - 530*a^2*b^2 + 1155*a*b^3 - 756*b^4)
*cos(f*x + e)^3 - 15*((15*a^4 - 100*a^3*b + 218*a^2*b^2 - 196*a*b^3 + 63*b^
4)*cos(f*x + e)^8 - 2*(15*a^4 - 115*a^3*b + 303*a^2*b^2 - 329*a*b^3 + 126*b
^4)*cos(f*x + e)^6 + (15*a^4 - 160*a^3*b + 573*a^2*b^2 - 798*a*b^3 + 378*b^
4)*cos(f*x + e)^4 + 15*a^2*b^2 - 70*a*b^3 + 63*b^4 + 2*(15*a^3*b - 100*a^2*
b^2 + 203*a*b^3 - 126*b^4)*cos(f*x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b
^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*
x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b +
 b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) +
60*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*cos(f*x + e))/(((a^7 - 2*a^6*b + a^5*b^
2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x +
 e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2
)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/240*(2*(64*a^4 - 863*a^3*b + 2479*a^2
*b^2 - 2625*a*b^3 + 945*b^4)*cos(f*x + e)^9 - 2*(160*a^4 - 2173*a^3*b + 715
8*a^2*b^2 - 8925*a*b^3 + 3780*b^4)*cos(f*x + e)^7 + 2*(120*a^4 - 1685*a^3*b
 + 7104*a^2*b^2 - 11025*a*b^3 + 5670*b^4)*cos(f*x + e)^5 + 10*(75*a^3*b - 5
30*a^2*b^2 + 1155*a*b^3 - 756*b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 100*a^3*b
 + 218*a^2*b^2 - 196*a*b^3 + 63*b^4)*cos(f*x + e)^8 - 2*(15*a^4 - 115*a^3*b
 + 303*a^2*b^2 - 329*a*b^3 + 126*b^4)*cos(f*x + e)^6 + (15*a^4 - 160*a^3*b
+ 573*a^2*b^2 - 798*a*b^3 + 378*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 70*a*b^3
 + 63*b^4 + 2*(15*a^3*b - 100*a^2*b^2 + 203*a*b^3 - 126*b^4)*cos(f*x + e)^2
)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x +
e)*sin(f*x + e)))*sin(f*x + e) + 30*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*cos(f*
x + e))/(((a^7 - 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 -
 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(
f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.95, size = 263, normalized size = 1.14

$$-\frac{\frac{15\,(15\,a^2b-70\,ab^2+63\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,a^5}+\frac{15\left(7\,a^2b^2\tan(fx+e)^3-22\,ab^3\tan(fx+e)^3+15\,b^4\tan(fx+e)^3+9\,a^3b\tan(fx+e)-26\,a^2b^2\tan(fx+e)+17\,ab^3\tan(fx+e)\right)}{\left(b\tan(fx+e)^2+a\right)^2a^5}+\frac{8\left(15\,a^2\tan(fx+e)^4-90\,ab\tan(fx+e)^4+90\,b^2\tan(fx+e)^4+10\,a^2\tan(fx+e)^2-15\,ab\tan(fx+e)^2+3\,a^2\right)}{a^5\tan(fx+e)^5}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")

[Out] -1/120*(15*(15*a^2*b - 70*a*b^2 + 63*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/(sqrt(a*b)*a^5) + 15*(7*a^2*b^2*tan(f*x + e)^3 - 22*a*b^3*tan(f*x + e)^3 + 15*b^4*tan(f*x + e)^3 + 9*a^3*b*tan(f*x + e) - 26*a^2*b^2*tan(f*x + e) + 17*a*b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a)^2*a^5) + 8*(15*a^2*tan(f*x + e)^4 - 90*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 + 3*a^2)/(a^5*tan(f*x + e)^5))/f

Mupad [B]
time = 13.31, size = 199, normalized size = 0.86

$$-\frac{\frac{1}{5\,a}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^4\,\left(15\,a^2-70\,a\,b+63\,b^2\right)}{15\,a^3}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(10\,a-9\,b\right)}{15\,a^2}+\frac{5\,b\,{\mathrm{tan}\left(e+f\,x\right)}^6\,\left(15\,a^2-70\,a\,b+63\,b^2\right)}{24\,a^4}+\frac{b^2\,{\mathrm{tan}\left(e+f\,x\right)}^8\,\left(15\,a^2-70\,a\,b+63\,b^2\right)}{8\,a^5}}{f\,\left(a^2\,{\mathrm{tan}\left(e+f\,x\right)}^5+2\,a\,b\,{\mathrm{tan}\left(e+f\,x\right)}^7+b^2\,{\mathrm{tan}\left(e+f\,x\right)}^9\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\mathrm{tan}\left(e+f\,x\right)}{\sqrt{a}}\right)\,\left(15\,a^2-70\,a\,b+63\,b^2\right)}{8\,a^{11/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^3),x)

[Out] - (1/(5*a) + (tan(e + f*x)^4*(15*a^2 - 70*a*b + 63*b^2))/(15*a^3) + (tan(e + f*x)^2*(10*a - 9*b))/(15*a^2) + (5*b*tan(e + f*x)^6*(15*a^2 - 70*a*b + 63*b^2))/(24*a^4) + (b^2*tan(e + f*x)^8*(15*a^2 - 70*a*b + 63*b^2))/(8*a^5))/(f*(a^2*tan(e + f*x)^5 + b^2*tan(e + f*x)^9 + 2*a*b*tan(e + f*x)^7)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2))*(15*a^2 - 70*a*b + 63*b^2))/(8*a^(11/2)*f)

3.92 $\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}+\frac{2(5a-4b)\cos^3(e+fx)\,(a}{15(a-}$$

```
[Out] 2/15*(5*a-4*b)*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)^2/f-1/5*cos(f*
x+e)^5*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+arctanh(sec(f*x+e)*b^(1/2)/(a-b+b
*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3745, 473, 462, 283, 223, 212}

$$-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{5f(a-b)}+\frac{2(5a-4b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{15f(a-b)^2}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f + (2*(5*a - 4*b)*Cos[e +
f*x]^3*(a - b + b*Sec[e + f*x]^2)^(3/2))/(15*(a - b)^2*f) - (Cos[e + f*x]^5
*(a - b + b*Sec[e + f*x]^2)^(3/2))/(5*(a - b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2\sqrt{a-b+bx^2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\,(a-b+b\sec^2(e+fx))^{3/2}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{(-2(5a-4b)+}{}\right.}{}$$

$$= \frac{2(5a-4b)\cos^3(e+fx)\,(a-b+b\sec^2(e+fx))^{3/2}}{15(a-b)^2 f} - \frac{\cos^5(e+fx}{}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{2(5a-4b)\cos^3(e+fx}{15(}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{2(5a-4b)\cos^3(e+fx}{15(}$$

$$= \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b}}{f}$$

Mathematica [A]
time = 2.23, size = 208, normalized size = 1.29

$$\frac{\cos(e+fx)\left(120\sqrt{2}\,(a-b)^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\,\sqrt{b}}\right)+\sqrt{a+b+(a-b)\cos(2(e+fx))}\,(-89a^2+254ab-149b^2+4(7a^2-15ab+8b^2)\cos(2(e+fx))-3(a-b)^2\cos(4(e+fx)))\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{120\sqrt{2}\,(a-b)^2 f\sqrt{a+b+(a-b)\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(120*Sqrt[2]*(a - b)^2*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*C
os[2*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]
]*(-89*a^2 + 254*a*b - 149*b^2 + 4*(7*a^2 - 15*a*b + 8*b^2)*Cos[2*(e + f*x)
] - 3*(a - b)^2*Cos[4*(e + f*x)]))*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*
Sec[e + f*x]^2])/(120*Sqrt[2]*(a - b)^2*f*Sqrt[a + b + (a - b)*Cos[2*(e + f
*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7043 vs. 2(145) = 290.
time = 1.39, size = 7044, normalized size = 43.75

method	result	size

default	Expression too large to display	7044

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.50, size = 215, normalized size = 1.34

$$\frac{\frac{20\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a-b} - 30\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e) - 15\sqrt{b}\,\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)+\sqrt{b}}\right) - \frac{2\left(3\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-5\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3\right)}{a^2-2ab+b^2}}{30f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30*(20*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/(a - b) - 30*sqrt(
a - b + b/cos(f*x + e)^2)*cos(f*x + e) - 15*sqrt(b)*log((sqrt(a - b + b/cos
(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*
x + e) + sqrt(b))) - 2*(3*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 -
 5*(a - b + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3)/(a^2 - 2*a*b + b^2))/
f
```

Fricas [A]
time = 3.38, size = 396, normalized size = 2.46

$$\left[\frac{15\left(a^2-2ab+b^2\right)\sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)-2\left(3\left(a^2-2ab+b^2\right)\cos(fx+e)^5-\left(10a^2-21ab+11b^2\right)\cos(fx+e)^3+\left(15a^2-40ab+23b^2\right)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{30\left(a^2-2ab+b^2\right)f},\ -\frac{15\left(a^2-2ab+b^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)+\left(3\left(a^2-2ab+b^2\right)\cos(fx+e)^5-\left(10a^2-21ab+11b^2\right)\cos(fx+e)^3+\left(15a^2-40ab+23b^2\right)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{15\left(a^2-2ab+b^2\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(a^2 - 2*a*b + b^2)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt
(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/c
os(f*x + e)^2) - 2*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - (10*a^2 - 21*a*b
 + 11*b^2)*cos(f*x + e)^3 + (15*a^2 - 40*a*b + 23*b^2)*cos(f*x + e))*sqrt((
(a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 - 2*a*b + b^2)*f), -1/15
*(15*(a^2 - 2*a*b + b^2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + (3*(a^2 - 2*a*b + b^2)*cos(f*x +
 e)^5 - (10*a^2 - 21*a*b + 11*b^2)*cos(f*x + e)^3 + (15*a^2 - 40*a*b + 23*b
^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2
- 2*a*b + b^2)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2695 vs. 2(153) = 306.
time = 1.40, size = 2695, normalized size = 16.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*b*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
- sqrt(a))/sqrt(-b))/sqrt(-b) - 2*(15*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^9*b + 165*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))^8*sqrt(a)*b - 320*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
7*a^2 + 540*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a*b + 32
0*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan
(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*b^2 + 640*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(5/2) - 2940*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(3/2)*b + 2960*(sqrt(a)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*b^2 + 832*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 - 1246*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b - 2464*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a))^5*a*b^2 + 2848*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))^5*b^3 - 2560*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
```

```
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))^4*a^(7/2) + 11590*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4
*a^(5/2)*b - 16400*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a
^(3/2)*b^2 + 6560*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*sq
rt(a)*b^3 + 320*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^4
- 4500*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*
a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3*b + 14720
*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b^2 - 16320*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^3 + 5120*(sqrt(a)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^4 + 3200*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(9/2) - 15980*(sqrt(a)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2)*b + 27760*(sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*t
an(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b^2 - 18880*(sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*t
an(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^3 + 3840*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^4 - 2880*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))*a^5 + 17735*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))*a^4*b - 41760*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*a^3*b^2 + 46240*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
)*a^2*b^3 - 23040*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^
4 + 3840*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^5 + 768*a^(
11/2) - 5379*a^(9/2)*b + 14864*a^(7/2)*b^2 - 20448*a^(5/2)*b^3 + 14080*a^(3
/2)*b^4 - 3840*sqrt(a)*b^5)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 ...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+fx\right)^{5}\sqrt{b\tan\left(e+fx\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.93 $\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}+\frac{\cos^3(e+fx)\,(a-b+b\,\mathrm{s}}{3(a-b)f}$$

```
[Out] 1/3*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+arctanh(sec(f*x+e)*b^(1
/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(
1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3745, 462, 283, 223, 212}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3f(a-b)}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f + (Cos[e + f*x]^3*(a - b
+ b*Sec[e + f*x]^2)^(3/2))/(3*(a - b)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a-b+bx^2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx}}{x^2}\right.}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a-b+b\right.}{3(a-b)}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a-b+b\right.}{3(a-b)}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b}}{f}$$

Mathematica [A]
time = 0.74, size = 170, normalized size = 1.50

$$\frac{\cos(e+fx)\left(6\sqrt{2}(a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{a+b+(a-b)\cos(2(e+fx))}\,(-5a+7b+(a-b)\cos(2(e+fx)))\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{6\sqrt{2}(a-b)f\sqrt{a+b+(a-b)\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(6*Sqrt[2]*(a - b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2
*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*(-
5*a + 7*b + (a - b)*Cos[2*(e + f*x)]))*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x
)])*Sec[e + f*x]^2])/(6*Sqrt[2]*(a - b)*f*Sqrt[a + b + (a - b)*Cos[2*(e + f
*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4295 vs. $2(101) = 202$.
time = 0.35, size = 4296, normalized size = 38.02

method	result	size
default	Expression too large to display	4296

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(cos(f*x+e)-1)^2*(-6*a^(5/2)*b^(1/2)*cos(f*x+e)^2*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)+12*a^(3/2)*b^(3/2)*cos(f*x+e)^2*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-6*a^(1/2)*b^(5/2)*
cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-3*a
^(7/2)*4^(1/2)*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos
(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(1/2)*b^(1/2)*4^(1/2))*b+4*a^(5/2)*b^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-8*a^(3/2)*b^(3/2)*cos(f*x+e)*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-6*a^(1/2)*4^(1/2)*b^(7
/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*a^(1/2)*b^
(5/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)
+9*4^(1/2)*b^(7/2)*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)
^2/a^(1/2))*a-9*4^(1/2)*b^(7/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos
(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+
b)/sin(f*x+e)^2/a^(1/2))*a+9*a^(5/2)*4^(1/2)*arctanh(1/8*(cos(f*x+e)-1)*(4^
(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*b^2-9*4^(1/2)*b^(5/2
)*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^2+9
*4^(1/2)*b^(5/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2
/a^(1/2))*a^2-9*a^(3/2)*4^(1/2)*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x
+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
```

```
/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*b^3+3*4^(1/2)*b^(3/2)*ln(-4*(cos(
f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^3-3*4^(1/2)*b^(3
/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^3
+3*a^(1/2)*4^(1/2)*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2
*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*b^(1/2)*4^(1/2))*b^4-2*a^(5/2)*b^(1/2)*cos(f*x+e)^4*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)+4*a^(3/2)*b^(3/2)*cos(f*x+e
)^4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-2*a^(1/2)*b^
(5/2)*cos(f*x+e)^4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/
2)-8*a^(5/2)*b^(1/2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(3/2)+3*a^(7/2)*4^(1/2)*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)-12*a^(5/2)*4^(1/2)*b^(3/2)*((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+15*a^(3/2)*4^(1/2)*b^(5/2)*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+16*a^(3/2)*b^(3/2)*cos(f*
x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-8*a^(1/2)
*b^(5/2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(3/2)-3*4^(1/2)*b^(9/2)*cos(f*x+e)*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*
cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/
2)+b)/sin(f*x+e)^2/a^(1/2))+3*4^(1/2)*b^(9/2)*cos(f*x+e)*ln(-2*(cos(f*x+e)-
1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))-3*4^(1/2)*b^(9/2)*ln(-4*(
cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))+3*4^(1/2)*b^(9
/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))+4*a
^(5/2)*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)-8
*a^(3/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)
+4*a^(1/2)*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/
2)-3*a^(7/2)*4^(1/2)*cos(f*x+e)*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x
+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*b-6*a...
```

Maxima [A]

time = 0.52, size = 140, normalized size = 1.24

$$\frac{\frac{2\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a-b}-6\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)-3\sqrt{b}\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)+\sqrt{b}}\right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/(a - b) - 6*sqrt(a -
 b + b/cos(f*x + e)^2)*cos(f*x + e) - 3*sqrt(b)*log((sqrt(a - b + b/cos(f*x
 + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x +
e) + sqrt(b))))/f
```

Fricas [**A**]
time = 6.20, size = 293, normalized size = 2.59

$$\left[\frac{3(a-b)\sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)+2\left((a-b)\cos(fx+e)^3-(3a-4b)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{6(a-b)f},-\frac{3(a-b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)-\left((a-b)\cos(fx+e)^3-(3a-4b)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3(a-b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a - b)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a -
 b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2)
 + 2*((a - b)*cos(f*x + e)^3 - (3*a - 4*b)*cos(f*x + e))*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/((a - b)*f), -1/3*(3*(a - b)*sqrt(-b)*arct
an(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - ((a - b)*cos(f*x + e)^3 - (3*a - 4*b)*cos(f*x + e))*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/((a - b)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(107) = 214.
time = 0.92, size = 498, normalized size = 4.41

$$\frac{1}{3}\left(\frac{\left(3ab\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)-3b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3a\sqrt{-b}\sqrt{b}-4\sqrt{-b}\,b^{\frac{3}{2}}\right)\mathrm{sgn}(f)\,\mathrm{sgn}(\cos(fx+e))}{a\sqrt{-b}f^2-\sqrt{-b}bf^2}-\frac{3b\arctan\left(\frac{\sqrt{a\cos(fx+e)^2-b\cos(fx+e)^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(f)\,\mathrm{sgn}(\cos(fx+e))}{\sqrt{-b}f^2}+\frac{(a\cos(fx+e)^2-b\cos(fx+e)^2+b)^{\frac{3}{2}}a^2f^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos(fx+e))-3\sqrt{a\cos(fx+e)^2-b\cos(fx+e)^2+b}\,a^3f^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos(fx+e))-2(a\cos(fx+e)^2-b\cos(fx+e)^2+b)^{\frac{3}{2}}abf^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos(fx+e))+9\sqrt{a\cos(fx+e)^2-b\cos(fx+e)^2+b}\,a^2bf^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos(fx+e))+(a\cos(fx+e)^2-b\cos(fx+e)^2+b)^{\frac{3}{2}}b^2f^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos(fx+e))-9\sqrt{a\cos(fx+e)^2-b\cos(fx+e)^2+b}\,ab^2f^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos(fx+e))+3\sqrt{a\cos(fx+e)^2-b\cos(fx+e)^2+b}\,b^3f^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos(fx+e))}{a^3f^6-3a^2bf^6+3ab^2f^6-b^3f^6}\right)|f|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((3*a*b*arctan(sqrt(b)/sqrt(-b)) - 3*b^2*arctan(sqrt(b)/sqrt(-b)) + 3*a
*sqrt(-b)*sqrt(b) - 4*sqrt(-b)*b^(3/2))*sgn(f)*sgn(cos(f*x + e))/(a*sqrt(-b
)*f^2 - sqrt(-b)*b*f^2) - 3*b*arctan(sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)
^2 + b)/sqrt(-b))*sgn(f)*sgn(cos(f*x + e))/(sqrt(-b)*f^2) + ((a*cos(f*x + e
)^2 - b*cos(f*x + e)^2 + b)^(3/2)*a^2*f^4*sgn(f)*sgn(cos(f*x + e)) - 3*sqrt
(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a^3*f^4*sgn(f)*sgn(cos(f*x + e))
- 2*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*a*b*f^4*sgn(f)*sgn(cos(
f*x + e)) + 9*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a^2*b*f^4*sgn(f
)*sgn(cos(f*x + e)) + (a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*b^2*f
^4*sgn(f)*sgn(cos(f*x + e)) - 9*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 +
b)*a*b^2*f^4*sgn(f)*sgn(cos(f*x + e)) + 3*sqrt(a*cos(f*x + e)^2 - b*cos(f*x
 + e)^2 + b)*b^3*f^4*sgn(f)*sgn(cos(f*x + e)))/(a^3*f^6 - 3*a^2*b*f^6 + 3*a
*b^2*f^6 - b^3*f^6))*abs(f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin{(e+f\,x)}^3\,\sqrt{b\tan{(e+f\,x)}^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.94 $\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}$$

```
[Out] arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)
*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3745, 283, 223, 212}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/f
- (Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\right.}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, -\right.}{}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b+}}{f}$$

Mathematica [A]
time = 0.43, size = 140, normalized size = 1.94

$$-\frac{\left(-2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2}\sqrt{a+b+(a-b)\cos(2(e+fx))}\right)\csc(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\sin(2(e+fx))}{4f\sqrt{a+b+(a-b)\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*((-2*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]/(Sqrt[2]*S
qrt[b])] + Sqrt[2]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])*Csc[e + f*x]*Sqr
t[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(f*S
qrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(64) = 128.
time = 0.14, size = 144, normalized size = 2.00

method	result

default	$\frac{\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{\cos(fx+e)^2}}\cos(fx+e)\left(\sqrt{b}\,\ln\left(\frac{2\sqrt{b}\,\sqrt{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}+2}{\cos(fx+e)}\right.\right.}{f\sqrt{a\left(\cos^2\left(fx+e\right)\right)-\left(\cos^2\left(fx+e\right)\right)b+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(b^(1
/2)*ln(2*(b^(1/2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2)+b)/cos(f*x+e))-(a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2))/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1
/2)
```

Maxima [A]
time = 0.53, size = 103, normalized size = 1.43

$$-\frac{2\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+\sqrt{b}\log\left(\frac{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+\sqrt{b}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)*log((sqrt(a -
 b + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)
^2)*cos(f*x + e) + sqrt(b))))/f
```

Fricas [A]
time = 2.72, size = 217, normalized size = 3.01

$$\left[-\frac{2\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2f}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{b}\right)+\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - s
qrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -(sqrt(-b)*a
```

```
rctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/b) + sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2{(e + fx)}}\sin{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(68) = 136.
time = 0.87, size = 142, normalized size = 1.97

$$-\left(\frac{b\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}}{\sqrt{-b}}\right)\operatorname{sgn}\left(f\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{\sqrt{-b}f^{2}}+\frac{\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}\operatorname{sgn}\left(f\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{f^{2}}-\frac{\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\sqrt{b}\right)\operatorname{sgn}\left(f\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{\sqrt{-b}f^{2}}\right)|f|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)/sqrt(-b))*sgn(f)*s
gn(cos(f*x + e))/(sqrt(-b)*f^2) + sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2
+ b)*sgn(f)*sgn(cos(f*x + e))/f^2 - (b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*
sqrt(b))*sgn(f)*sgn(cos(f*x + e))/(sqrt(-b)*f^2))*abs(f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.95 $\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}$$

```
[Out] -arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(s
ec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3745, 399, 223, 212, 385, 213}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/
f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} + \frac{b\text{Subst}\left(\int \right.}{}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{b\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{}{\sqrt{a-}}\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 300 vs. 2(84) = 168.
time = 4.68, size = 300, normalized size = 3.57

$$\frac{\cos(e+fx)\left(4\sqrt{b}\tanh^{-1}\left(\frac{-\sqrt{a}\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{2\sqrt{b}}\right)+\sqrt{a}\left(2\tanh^{-1}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\frac{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)+\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{2f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(4*Sqrt[b]*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sq
rt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])] +
Sqrt[a]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1
 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + S
qrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]))*Sec[
(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(2*
f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(72) = 144$.
time = 0.38, size = 721, normalized size = 8.58

method	result
default	$-\frac{\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{\cos(fx+e)^2}}\ \sqrt{4}\ \cos(fx+e)(\cos(fx+e)-1)\left(2\,\mathrm{arctanh}\left(\frac{(\cos(fx+e)-1)\left(\sqrt{4}\ \cos(fx+e)-\sqrt{4}\ -2\cos\right.}{8\sin(fx+e)^2\sqrt{\frac{a(\cos^2(fx+e))-(\cos}{(\cos(fx+e)}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*4^(1/2)*cos(f
*x+e)*(cos(f*x+e)-1)*(2*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1
/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*a^(1/2)*b+2*ln(-2*(cos(f*x+e)-1)*(cos(f*x
+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)-2*b^(3/2)*ln(-4*(cos(f*x+e)-
1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))-ln(-2*(cos(f*x+e)-1)*(cos
(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a*b^(1/2)-a*ln(-4*(cos(f*x+e)*a^
(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos
(f*x+e)+b)/(cos(f*x+e)-1))*b^(1/2))/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)/a^(1/2)/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e), x)
```

Fricas [A]
time = 2.57, size = 546, normalized size = 6.50

$$\left[\frac{\sqrt{a}\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)+\sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2f}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{b}\right)-\sqrt{a}\log\left(-\frac{2\left((a-b)\cos(fx+e)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)}{2f}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{a}\right)+\sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2f}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{a}\right)-\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{b}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))
 + sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -1/2*(2*
sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e)/b) - sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt((
(a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x
+ e)^2 - 1)))/f, 1/2*(2*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + sqrt(b)*log(-((a - b)*cos(f*x + e
)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e) + 2*b)/cos(f*x + e)^2))/f, (sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) - sqrt(-b)*arctan(sqrt(-b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\,\csc\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

3.96 $\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=127

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2\sqrt{a}\,f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\csc(e}{}$$

```
[Out] -1/2*(a+b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/f/a^(1/2)
+arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*cot(f
*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3745, 478, 537, 223, 212, 385, 213}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2\sqrt{a}\,f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/2*((a + b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/(Sqrt[a]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec
[e + f*x]^2]])/f - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a-b+bx^2}}{(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\Big(\int - \ldots}{\ldots}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{b\text{Subst}\Big(\int \ldots}{\ldots}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{b\text{Subst}\Big(\int \ldots}{\ldots}$$

$$= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\sqrt{b}\tanh^{-1}\Big(-\ldots}{\ldots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 586 vs. 2(127) = 254.
time = 2.53, size = 586, normalized size = 4.61

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/2*(Cot[e + f*x]*Csc[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec
[e + f*x]^2]*(-(a*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan
[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]) - b*Log[a - 2*b - a*Tan[
(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x
)/2]^2)^2]] + a*Cos[e + f*x]*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*S
qrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + b*Cos[e + f*
x]*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2
 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + (Sqrt[a]*Sqrt[(a + b + (a - b)*Cos[2*(
e + f*x)])*Sec[(e + f*x)/2]^4])/Sqrt[2] - 16*Sqrt[a]*Sqrt[b]*ArcTanh[(-(Sqr
t[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan
[(e + f*x)/2]^2)^2])/(2*Sqrt[b])]*Sin[(e + f*x)/2]^2 - 4*(a + b)*ArcTanh[Ta
n[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2
)^2]/Sqrt[a]]*Sin[(e + f*x)/2]^2))/(Sqrt[a]*f*Sqrt[(a + b + (a - b)*Cos[2*(
e + f*x)])*Sec[(e + f*x)/2]^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. $2(109) = 218$.
time = 0.56, size = 2075, normalized size = 16.34

method	result	size
default	Expression too large to display	2075

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(cos(f*x+e)-1)*(4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(1/2)*cos(f*x+e)^2*b^(3/2)*a^(1/2)*4^(1/2)-4*((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2*b^(1/2)*a^(3/2)*4^(1/2)+3*ln(-2
*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^2
*b^(3/2)*4^(1/2)*a-ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e
)^2*b^(3/2)*4^(1/2)*a-4*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f
*x+e)^2/a^(1/2))*cos(f*x+e)^2*b^(3/2)*4^(1/2)*a+4*arctanh(1/8*(cos(f*x+e)-1
)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*cos(f*x+e)^2*a^
(3/2)*4^(1/2)*b+8*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2
)*cos(f*x+e)^2*b^(1/2)*a^(1/2)+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*cos(f*x+e)*b^(1/2)*a^(3/2)*4^(1/2)-ln(-2*(cos(f*x+e)-1)*(c
os(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^2*b^(1/2)*4^(1/2)*a
^2-ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(
1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)^2*b^(1/2)*4^(1
/2)*a^2+16*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*cos(f
*x+e)*b^(1/2)*a^(1/2)-4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(1/2)*b^(3/2)*a^(1/2)*4^(1/2)+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*b^(1/2)*a^(3/2)*4^(1/2)-3*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e
)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)*4^(1/2)*a+ln(-4*(cos(f*x+e)*a^
(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos
(f*x+e)+b)/(cos(f*x+e)-1))*b^(3/2)*4^(1/2)*a+4*ln(-4*(cos(f*x+e)-1)*(cos(f*
```

```
x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*b^(3/2)*4^(1/2)*a-4*arctanh(1/8*(co
s(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*a^(3/
2)*4^(1/2)*b+8*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*b
^(1/2)*a^(1/2)+ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a
^(1/2))*b^(1/2)*4^(1/2)*a^2+ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(c
os(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*
b^(1/2)*4^(1/2)*a^2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+
e)^2)^(1/2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)/sin(
f*x+e)^4/a^(3/2)/b^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

Fricas [**A**]
time = 5.38, size = 905, normalized size = 7.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2
*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
 + b)/(cos(f*x + e)^2 - 1)) + 2*(a*cos(f*x + e)^2 - a)*sqrt(b)*log(-((a - b
)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^2 - a*f), 1/2*((
(a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + a*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + (a*cos(f*x + e)^2 - a)*sqrt(b)*lo
g(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^2 - a*
```

```
f), -1/4*(4*(a*cos(f*x + e)^2 - a)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - 2*a*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - ((a + b)*cos(f*x + e)^2 - a
- b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)))
/(a*f*cos(f*x + e)^2 - a*f), 1/2*(((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)
*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e)/a) - 2*(a*cos(f*x + e)^2 - a)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + a*sqrt(((a - b)*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(a*f*cos(f*x + e)^2 - a*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2{(e + fx)}}\csc^3{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,{\tan\left(e+f\,x\right)}^2+a}}{{\sin\left(e+f\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

3.97 $\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=187

$$-\frac{(3a^2+6ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}-\frac{(3a+b}{}$$

```
[Out] -1/8*(3*a^2+6*a*b-b^2)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2
))/a^(3/2)/f+arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)
/f-1/8*(3*a+b)*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a/f-1/4*cot
(f*x+e)*csc(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3745, 478, 592, 537, 223, 212, 385, 213}

$$-\frac{(3a^2+6ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4f}-\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/8*((3*a^2 + 6*a*b - b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*S
ec[e + f*x]^2]])/(a^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt
[a - b + b*Sec[e + f*x]^2]])/f - ((3*a + b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[
a - b + b*Sec[e + f*x]^2])/(8*a*f) - (Cot[e + f*x]*Csc[e + f*x]^3*Sqrt[a -
b + b*Sec[e + f*x]^2])/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\frac{\text{Subst}\left(\int \frac{x^4\sqrt{a-b+bx^2}}{(-1+x^2)^3}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{4f}+\frac{\text{Subst}\Big(\int}{}$$

$$=-\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af}-\text{co}$$

$$=-\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af}-\text{co}$$

$$=-\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af}-\text{co}$$

$$=-\frac{(3a^2+6ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\ \text{ta}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1049 vs. $2(187) = 374$.

time = 6.46, size = 1049, normalized size = 5.61

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*(((-3*a*Cos[e + f*x] - b*Cos[e + f*x])*Csc[e + f*x]^2)/(8*a) - (Cot[e
+ f*x]*Csc[e + f*x]^3)/4))/f + (((3*a^2 - 2*a*b - b^2)*(1 + Cos[e + f*x])*S
qrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[
2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]*ArcTanh[(-(Sqrt[a]*(-1 + T
an[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2
]^2)^2])/(2*Sqrt[b])] - Sqrt[b]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Ta
n[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b -
a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e
 + f*x)/2]^2)^2]]))*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt
[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x
)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[
```

```
(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
 f*x)/2]^2)^2]) - ((3*a^2 + 14*a*b - b^2)*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[
2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])
/(1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/
2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*S
qrt[b])] + Sqrt[b]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2
]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*
x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)
^2]]))*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e
+ f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(
4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e
+ f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2
]))/(8*a*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5377 vs. $2(165) = 330$.
time = 0.31, size = 5378, normalized size = 28.76

method	result	size
default	Expression too large to display	5378

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

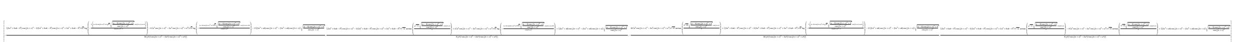
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

Fricas [A]
time = 7.02, size = 1345, normalized size = 7.19



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 6*a*b - b^2)*cos
(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 +
```

```
 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 a + b)/(cos(f*x + e)^2 - 1)) - 8*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^
2 + a^2)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*((
3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x +
 e)^2 + a^2*f), 1/8*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 6*a
*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(-a)*arctan(sqrt(-a)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + 4*(a^2*co
s(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b)*log(-((a - b)*cos(f*x +
e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + 2*b)/cos(f*x + e)^2) + ((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)
*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*co
s(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f), -1/16*(16*(a^2*cos(f*x + e)
^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a^2 + 6*a*b - b^2)*c
os(f*x + e)^4 - 2*(3*a^2 + 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^
2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) -
2*((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f
*x + e)^2 + a^2*f), 1/8*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 +
 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(-a)*arctan(sqrt(-a
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) - 8*(a^
2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a^2 +
a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 +
a^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2{(e + fx)}}\,\csc^5{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

3.98 $\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=189

$$\frac{(3a^2-12ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{3/2}f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(3a-4b)\,\text{co}}{}$$

```
[Out] 1/8*(3*a^2-12*a*b+8*b^2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(
1/2))/(a-b)^(3/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^
(1/2)/f-1/8*(3*a-4*b)*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)/
f-1/4*cos(f*x+e)*sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3744, 478, 592, 537, 223, 212, 385, 209}

$$\frac{(3a^2-12ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}-\frac{(3a-4b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 12*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/(8*(a - b)^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b*Tan[e + f*x]^2]])/f - ((3*a - 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sq
rt[a + b*Tan[e + f*x]^2])/(8*(a - b)*f) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt
[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{3}{(1+x}\right.}{}$$

$$= -\frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e}{}$$

$$= -\frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e}{}$$

$$= -\frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e}{}$$

$$= \frac{(3a^2-12ab+8b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{3/2}f} + \frac{\sqrt{b}\tanh^{-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.56, size = 330, normalized size = 1.75

$$\frac{\left(-((a-b)(7a^2-11b^2+6(a^2-3ab+2b^2)\cos(2(e+fx))-(a-b)^2\cos(4(e+fx))))+2\sqrt{2}a(3a^2-7ab+4b^2)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)-2\sqrt{2}a(3a^2-12ab+8b^2)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sec^2(e+fx)\sin(2(e+fx))}{32\sqrt{2}(a-b)^2f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-((a - b)*(7*a^2 - 11*b^2 + 6*(a^2 - 3*a*b + 2*b^2)*Cos[2*(e + f*x)] - (a
 - b)^2*Cos[4*(e + f*x)])) + 2*Sqrt[2]*a*(3*a^2 - 7*a*b + 4*b^2)*Sqrt[((a +
 b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2
]*a*(3*a^2 - 12*a*b + 8*b^2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)
])/(32*Sqrt[2]*(a - b)^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e +
f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.65, size = 2498, normalized size = 13.22

method	result	size
default	Expression too large to display	2498

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*a^2-4*((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*a*b+2*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*b^2-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)*cos(f*x+e)^4*a^2+4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f
*x+e)^4*a*b-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*b^2+6*
2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)
*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^
(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/s
in(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a
+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)-24
*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e
)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)
^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/
sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-
a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)+1
6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+
e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b
)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a
)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)
-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)-
3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+
e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b
)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a
)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/
sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*
b^2)/a^2)^(1/2))*a^2*sin(f*x+e)+4*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2
)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+
b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^
(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b*sin(f*x+e)+16*2^(1/2)*
((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos
```

```
(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*El
lipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e
),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^
(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)-16*2^(1/2)*
((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos
(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*El
lipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e
),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^
(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)-5*((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^3*a^2+13*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^3*a*b-8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)*cos(f*x+e)^3*b^2+5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+
e)^2*a^2-13*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b+8*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^2-5*((2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)*a*b+6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)*cos(f*x+e)*b^2+5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-6*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)*sin(f*x+e)*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)-1)/(a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(177) = 354$.
time = 16.92, size = 2154, normalized size = 11.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((3*a^2 - 12*a*b + 8*b^2)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2
*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a
*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3
 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*
b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 -
```

```
8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b +
 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)
*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-
a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 16
*(a^2 - 2*a*b + b^2)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*
(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
+ 8*b^2)/cos(f*x + e)^4) + 8*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2
 - 11*a*b + 6*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/((a^2 - 2*a*b + b^2)*f), -1/64*(32*(a^2 - 2*a*b + b^2
)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x +
 e)^2 + b^2)*sin(f*x + e))) - (3*a^2 - 12*a*b + 8*b^2)*sqrt(-a + b)*log(128
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*
a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b
+ 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2
*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 1
6*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^
7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2
*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^
3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e)) - 8*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 11*
a*b + 6*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e))/((a^2 - 2*a*b + b^2)*f), 1/32*((3*a^2 - 12*a*b + 8*b^2)*sqr
t(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b
 + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*
b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*
b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 8*(a^2 - 2*a*b + b^2)*sqrt(b)
*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2
 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4
*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 11*a*b + 6*b^2)*cos(f*x +
 e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2
- 2*a*b + b^2)*f), 1/32*((3*a^2 - 12*a*b + 8*b^2)*sqrt(a - b)*arctan(-1/4*(
8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)
^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e
)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x +
 e)^2)*sin(f*x + e))) - 16*(a^2 - 2*a*b + b^2)*sqrt(-b)*arctan(1/2*((a - 2*
b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e))) +
4*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 11*a*b + 6*b^2)*cos(f*x
+ e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2
 - 2*a*b + b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2(e+fx)}\,\sin^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+f\,x)^4\,\sqrt{b\tan(e+f\,x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.99 $\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=128

$$\frac{(a-2b)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{a-b}\,f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sin(e+f}{}$$

```
[Out] 1/2*(a-2*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)
^(1/2)+arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*c
os(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3744, 478, 537, 223, 212, 385, 209}

$$\frac{(a-2b)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*Sqrt[a - b]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/f - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*
f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{}{(1+}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\text{Subs}}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\text{Subs}}{}$$

$$= \frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{a-b}\,f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{}{\sqrt{a-}}\right.}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 3.99, size = 273, normalized size = 2.13

$$-\frac{\left((a-b)(a+b+(a-b)\cos(2(e+fx)))+\sqrt{2}a(-a+b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+\sqrt{2}a(a-2b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sec^2(e+fx)\sin(2(e+fx))}{4\sqrt{2}(a-b)f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*(((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*a*(-a + b)*Sqrt
[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + Sq
rt[2]*a*(a - 2*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/
b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])
*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(Sqrt[2]
*(a - b)*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.29, size = 1342, normalized size = 10.48

method	result	size

default	Expression too large to display	1342

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(-4*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*
x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-
b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b*sin(f
*x+e)+2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(
f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*
(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+
1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*
b+8*b^2)/a^2)^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1
/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1
/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*
a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2))*a*sin(f*x+e)+4*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2
)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+
b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2))*b*sin(f*x+e)+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos
(f*x+e)^3*a-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^3*b-((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a+((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*
cos(f*x+e)*b-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b)*cos(f*x+e)*sin(f*
x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)-1)/
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(117) = 234$.
time = 2.89, size = 1929, normalized size = 15.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)*sin(f*x + e) - (a - 2*b)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*
b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3
+ 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b
^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 -
8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b +
5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*
cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a
 + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 4*(
a - b)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*
cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f
*x + e)^4))/((a - b)*f), -1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + 8*(a - b)*sqrt(-b)*arctan(1/2*
((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x +
 e))) - (a - 2*b)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*
cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos
(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4
- 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b
^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^
3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((
a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a - b)*f), -1/8
*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*
sin(f*x + e) - sqrt(a - b)*(a - 2*b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos
(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2
)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2
*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 2
*(a - b)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2
)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*
```

```
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos
(f*x + e)^4))/((a - b)*f), -1/8*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - sqrt(a - b)*(a - 2*b)*arctan(
-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*
x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f
*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos
(f*x + e)^2)*sin(f*x + e))) + 4*(a - b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/((a - b)
*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2{(e + fx)}}\sin^2{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.100 $\int \sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+arcta
nh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3742, 399, 223, 212, 385, 209}

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\right.}{f}$$

$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\right.}{}$$

$$= \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+f}\right.}{f}$$

Mathematica [A]
time = 0.49, size = 108, normalized size = 1.27

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{b}\,\tan^2(e+fx)-\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{b}\,\log\left(-\sqrt{b}\,\tan(e+fx)+\sqrt{a+b\tan^2(e+fx)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[e + f*x]^2 - Tan[e + f*x]*Sqrt
[a + b*Tan[e + f*x]^2])/Sqrt[a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Tan[e + f*x])
+ Sqrt[a + b*Tan[e + f*x]^2]])/f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(73) = 146$.
time = 0.08, size = 167, normalized size = 1.96

method	result
derivativedivides	$\frac{b\left(\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+}{\sqrt{b^4(a-b)}\,\sqrt{a+b(\mathrm{ta}}}\right.}{b^2(a-b)}\right.}{f}$
default	$\frac{b\left(\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+}{\sqrt{b^4(a-b)}\,\sqrt{a+b(\mathrm{ta}}}\right.}{b^2(a-b)}\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-(b^4*(a-b))
^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e)))+a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(
1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 1.25, size = 432, normalized size = 5.08

$$\left[\frac{\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{b}\tan\left(fx+e\right)+a\right)+\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(fx+e)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)}{2\,f}, \frac{2\,\sqrt{a-b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{a-b}\tan(fx+e)}\right)+\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{b}\tan\left(fx+e\right)+a\right)}{2\,f}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-b}}{b\tan(fx+e)}\right)-\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(fx+e)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)}{2\,f}, \frac{\sqrt{a-b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{a-b}\tan(fx+e)}\right)-\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-b}}{b\tan(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)
*tan(f*x + e) + a) + sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)))/
f, 1/2*(2*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f
*x + e))) + sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(b)*tan(f*x + e) + a))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
 e)^2 + 1)))/f, (sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b
)*tan(f*x + e))) - sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*t
an(f*x + e))))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2), x)
```

3.101 $\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)*(
a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3744, 283, 223, 212}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a}}\right.}{f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.18, size = 156, normalized size = 2.36

$$-\frac{\left((a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)-\sqrt{2}\,b\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\tan(e+fx)}{\sqrt{2}\,f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - Sqrt[2]*b*Sqrt[((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((
a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan[e +
f*x])/(Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.59, size = 1213, normalized size = 18.38

method	result	size
default	Expression too large to display	1213

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(Elli
pticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),(
(8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1
/2))*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f
*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(
a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1
)/a)^(1/2)*cos(f*x+e)*sin(f*x+e)*b-2*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2
)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*
a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2))*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/
2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b
^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(
f*x+e)+1)/a)^(1/2)*cos(f*x+e)*sin(f*x+e)*b+2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(
a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1
)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*(
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/
2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*b*sin(f*x+e)-2*2^
(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a
-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1
/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1
/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin
(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*
b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b*sin(f*x+e)-((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a+((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b
)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/sin(f*x+e)/((2*I*b^(1/2)*(a-b)^(1/2)+a-
2*b)/a)^(1/2)
```

Maxima [A]
time = 0.29, size = 50, normalized size = 0.76

$$\frac{\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) - sqrt(b*tan(f*x + e)^2 + a)/tan
(f*x + e))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(62) = 124$.
time = 1.35, size = 355, normalized size = 5.38

$$\left[\frac{\sqrt{b}\log\left(\frac{(a^2-8ab+8b^2)\cos(fx+e)^4+8(ab-2b^2)\cos(fx+e)^2+4\left((a-2b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)\sin(fx+e)-4\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{4f\sin(fx+e)}, -\frac{\sqrt{-b}\arctan\left(\frac{\left((a-2b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left((ab-b^2)\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin(fx+e)+2\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{2f\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*c
os(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4)*sin(f*x + e) - 4*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e))/(f*sin(f*x + e)), -1/2*(sqrt(-b)*arctan(1/2*((a - 2*b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x +
 e) + 2*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*
sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2(e + fx)}\,\csc^2(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

3.102 $\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx\right)}{3af}$$

```
[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)*(
a+b*tan(f*x+e)^2)^(1/2)/f-1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/a/f
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3744, 462, 283, 223, 212}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f - (Cot[e + f*x]^3*(a + b*Tan[e +
f*x]^2)^(3/2))/(3*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)\sqrt{a+bx^2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,d\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b\tan^2(\right.}{3af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b\tan^2(\right.}{3af}$$

$$= \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\,\text{ta}}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 4.55, size = 204, normalized size = 2.04

$$-\frac{\left((6a^2+11ab+3b^2+4(a^2-3ab-b^2)\cos(2(e+fx))+(-2a^2+ab+b^2)\cos(4(e+fx)))\csc^4(e+fx)-12\sqrt{2}\,ab\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\tan(e+fx)}{12\sqrt{2}\,af\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/12*(((6*a^2 + 11*a*b + 3*b^2 + 4*(a^2 - 3*a*b - b^2)*Cos[2*(e + f*x)] +
(-2*a^2 + a*b + b^2)*Cos[4*(e + f*x)])*Csc[e + f*x]^4 - 12*Sqrt[2]*a*b*Sqrt
[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan
[e + f*x])/(Sqrt[2]*a*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x
]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.47, size = 2441, normalized size = 24.41

method	result	size
default	Expression too large to display	2441

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2
)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+
b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^
(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b-6*sin(f*x+e)*cos(f*x+e
)^3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*
x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a
-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)
/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/
2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2
)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b+3*sin(f*x+
e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^
(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e
)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(c
os(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-
2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*
a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b-6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*co
s(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e
)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2
)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*Elliptic
Pi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b-3*sin(f*x+e)*cos(f*x+e)*2^(1
/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
```

```
)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2
)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*
x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a
^2)^(1/2))*a*b+6*sin(f*x+e)*cos(f*x+e)*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)
^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)
^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+
e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2))*a*b-3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(
1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(
I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f
*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(
a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b*sin(f*x+e)+6*2^(1/2)*((I*cos(
f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+
b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*
(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi
((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)-2*((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a^2+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)*cos(f*x+e)^4*a*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x
+e)^4*b^2+3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^2-4*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b-2*((2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^2+3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)*a*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/sin(f*x+e)^3/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/a
```

Maxima [A]
time = 0.29, size = 81, normalized size = 0.81

$$\frac{3\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)-\frac{3\sqrt{b\tan\left(fx+e\right)^2+a}}{\tan(fx+e)}-\frac{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}{a\tan(fx+e)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) - 3*sqrt(b*tan(f*x + e)^2
+ a)/tan(f*x + e) - (b*tan(f*x + e)^2 + a)^(3/2)/(a*tan(f*x + e)^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(94) = 188$.

time = 3.19, size = 465, normalized size = 4.65

$$\left[\frac{3\,\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{b}\log\left(\frac{\left(a^2-8\,ab+8\,b^2\right)\cos(fx+e)^4+8\,\left(ab-2\,b^2\right)\cos(fx+e)^2+4\,\left(\left(a-2\,b\right)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin(fx+e)+8\,b^2}{\cos(fx+e)^4}\right)\sin\left(fx+e\right)-4\,\left(\left(2\,a+b\right)\cos\left(fx+e\right)^3-\left(3\,a+b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{12\,\left(af\cos\left(fx+e\right)^2-af\right)\sin\left(fx+e\right)},-\frac{3\,\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{-b}\arctan\left(\frac{\left(\left(a-2\,b\right)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{2\,\left(\left(ab-b^2\right)\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin\left(fx+e\right)+2\,\left(\left(2\,a+b\right)\cos\left(fx+e\right)^3-\left(3\,a+b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{6\,\left(af\cos\left(fx+e\right)^2-af\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x
+ e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b
*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*si
n(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + b)*cos(f*x + e
)^3 - (3*a + b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2))/((a*f*cos(f*x + e)^2 - a*f)*sin(f*x + e)), -1/6*(3*(a*cos(f*x + e)^2
- a)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*
x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((2*a + b)*cos(f*x + e)^3 -
 (3*a + b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/((a*f*cos(f*x + e)^2 - a*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\tan^2\left(e+fx\right)}\,\csc^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}}{{\mathrm{sin}\left(e+f\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

3.103 $\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{2(5a-b)\cot^3(e+fx)\,(a+b\tan$$

[Out] arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f-2/15*(5*a-b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/a^2/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2)/a/f

Rubi [A]
time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3744, 473, 462, 283, 223, 212}

$$-\frac{2(5a-b)\cot^3(e+fx)\,(a+b\tan^2(e+fx))^{3/2}}{15a^2f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\,(a+b\tan^2(e+fx))^{3/2}}{5af}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]

[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f - (2*(5*a - b)*Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2))/(15*a^2*f) - (Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2))/(5*a*f)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2\sqrt{a+bx^2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{5af} + \frac{\text{Subst}\left(\int \frac{(2(5a-b)+5ax^2)\sqrt{}}{x^4}\right.}{}$$

$$= -\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cot^5(e+fx)\,(}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{2(5a-b)\cot^3(e+fx)\,(a-}{15a^2f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{2(5a-b)\cot^3(e+fx)\,(a-}{15a^2f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 3.57, size = 287, normalized size = 2.04

$$-\frac{\left((80a^3+198a^2b+98ab^2-20b^3+(40a^3-241a^2b-149ab^2+30b^3)\cos(2(e+fx))+(-32a^3+42a^2b+62ab^2-12b^3)\cos(4(e+fx))+8a^3\cos(6(e+fx))+a^2b\cos(6(e+fx))-11ab^2\cos(6(e+fx))+2b^3\cos(6(e+fx)))\csc^6(e+fx)-240\sqrt{2}a^2b\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\tan(e+fx)}{240\sqrt{2}a^2f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/240*(((80*a^3 + 198*a^2*b + 98*a*b^2 - 20*b^3 + (40*a^3 - 241*a^2*b - 14
9*a*b^2 + 30*b^3)*Cos[2*(e + f*x)] + (-32*a^3 + 42*a^2*b + 62*a*b^2 - 12*b^
3)*Cos[4*(e + f*x)] + 8*a^3*Cos[6*(e + f*x)] + a^2*b*Cos[6*(e + f*x)] - 11*
a*b^2*Cos[6*(e + f*x)] + 2*b^3*Cos[6*(e + f*x)])*Csc[e + f*x]^6 - 240*Sqrt[
2]*a^2*b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Ellipt
icF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt
[2]], 1])*Tan[e + f*x])/(Sqrt[2]*a^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x
)])*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.37, size = 3769, normalized size = 26.73

method	result	size
default	Expression too large to display	3769

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(32*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^2-25
*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b-31*cos(f*x+e)
^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^2+cos(f*x+e)^6*((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b-11*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)*a*b^2+9*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)*a^2*b+30*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)
^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(
cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/
2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a
^2*b*sin(f*x+e)-15*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/
2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2*b*sin(f*x+e)-2*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)*b^3+8*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)*a^3-20*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3+15*
cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3+2*cos(f*x+e)^6*(
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3-6*cos(f*x+e)^4*((2*I*b^(1/2)*(
a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3+6*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)/a)^(1/2)*b^3+15*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b+10*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^2+30*2^(1/2)*((I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+
e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-
cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)
-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^4*a^2*b-15*sin(f*x+e)*co
s(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*
x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2
-8*a*b+8*b^2)/a^2)^(1/2))*a^2*b-60*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/
2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/
```

```
2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a
+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^3*a^2*b+30*2^(1/2)*((I*cos(f*x+e)*b^
(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f
*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/
2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a
-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)
*cos(f*x+e)^3*a^2*b-60*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)
*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*co
s(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e
)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I
*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2))*sin(f*x+e)*cos(f*x+e)^2*a^2*b+30*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^
(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e
)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*
I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^
2*a^2*b+30*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*
x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-
b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x
+e)*cos(f*x+e)*a^2*b-15*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2
)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos...
```

Maxima [A]
time = 0.29, size = 140, normalized size = 0.99

$$\frac{15\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)-\frac{15\sqrt{b\tan\left(fx+e\right)^2+a}}{\tan(fx+e)}-\frac{10\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}{a\tan(fx+e)^3}+\frac{2\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}b}{a^2\tan(fx+e)^3}-\frac{3\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}{a\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) - 15*sqrt(b*tan(f*x + e)
^2 + a)/tan(f*x + e) - 10*(b*tan(f*x + e)^2 + a)^(3/2)/(a*tan(f*x + e)^3) +
 2*(b*tan(f*x + e)^2 + a)^(3/2)*b/(a^2*tan(f*x + e)^3) - 3*(b*tan(f*x + e)^
2 + a)^(3/2)/(a*tan(f*x + e)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(133) = 266$.
time = 2.97, size = 623, normalized size = 4.42

$$\left[\frac{15\left(a^2\cos\left(fx+e\right)^4-2a^2\cos\left(fx+e\right)^2+a^2\right)\sqrt{b}\log\left(\frac{\left(a^2-8ab+8b^2\right)\cos(fx+e)^4+8\left(ab-2b^2\right)\cos(fx+e)^2+4\left(\left(a-2b\right)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)\sin\left(fx+e\right)-4\left(\left(8a^2+9ab-2b^2\right)\cos\left(fx+e\right)^5-\left(20a^2+19ab-4b^2\right)\cos\left(fx+e\right)^3+\left(15a^2+10ab-2b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{60\left(a^2f\cos\left(fx+e\right)^4-2a^2f\cos\left(fx+e\right)^2+a^2f\right)\sin\left(fx+e\right)}, -\frac{15\left(a^2\cos\left(fx+e\right)^4-2a^2\cos\left(fx+e\right)^2+a^2\right)\sqrt{-b}\arctan\left(\frac{\left(\left(a-2b\right)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{2\left(\left(ab-b^2\right)\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin\left(fx+e\right)+2\left(\left(8a^2+9ab-2b^2\right)\cos\left(fx+e\right)^5-\left(20a^2+19ab-4b^2\right)\cos\left(fx+e\right)^3+\left(15a^2+10ab-2b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{30\left(a^2f\cos\left(fx+e\right)^4-2a^2f\cos\left(fx+e\right)^2+a^2f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b)*log(((a
^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a
 - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e
) - 4*((8*a^2 + 9*a*b - 2*b^2)*cos(f*x + e)^5 - (20*a^2 + 19*a*b - 4*b^2)*c
os(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x +
e)^2 + a^2*f)*sin(f*x + e)), -1/30*(15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x
+ e)^2 + a^2)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^
2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((8*a^2 + 9*a*b -
2*b^2)*cos(f*x + e)^5 - (20*a^2 + 19*a*b - 4*b^2)*cos(f*x + e)^3 + (15*a^2
+ 10*a*b - 2*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/((a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f)*sin(f*x +
e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\,\csc^6\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^6,x)
[Out] int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^6, x)
```

3.104 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=227

$$\frac{(3a-7b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{(3a-7b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f}-\frac{(3a$$

```
[Out] -1/3*(3*a-7*b)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+2/3*cos(f*x+e)
^3*(a-b+b*sec(f*x+e)^2)^(5/2)/(a-b)/f-1/5*cos(f*x+e)^5*(a-b+b*sec(f*x+e)^2)
^(5/2)/(a-b)/f+1/2*(3*a-7*b)*arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2
)^(1/2))*b^(1/2)/f+1/2*(3*a-7*b)*b*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a
-b)/f
```

Rubi [A]
time = 0.15, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3745, 473, 464, 283, 201, 223, 212}

$$\frac{b(3a-7b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{5f(a-b)}+\frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{3f(a-b)}-\frac{(3a-7b)\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3f(a-b)}+\frac{\sqrt{b}\,(3a-7b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a - 7*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
f*x]^2]])/(2*f) + ((3*a - 7*b)*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/(2*(a - b)*f) - ((3*a - 7*b)*Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/
2))/(3*(a - b)*f) + (2*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*
(a - b)*f) - (Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(5/2))/(5*(a - b)*f
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx=\frac{\text{Subst}\left(\int\frac{(-1+x^2)^2(a-b+bx^2)^{3/2}}{x^6}dx,x,\sec(e+fx)\right)}{f}$$

$$=-\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{5(a-b)f}+\frac{\text{Subst}\left(\int\frac{(-10(a-}{}\right.}{}$$

$$=\frac{2\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{3(a-b)f}-\frac{\cos^5(e+fx)\,(a-}{5(}$$

$$=-\frac{(3a-7b)\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f}+\frac{2\cos^3(e}{}$$

$$=\frac{(3a-7b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f}-\frac{(3a-7b)\,c}{}$$

$$=\frac{(3a-7b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f}-\frac{(3a-7b)\,c}{}$$

$$=\frac{(3a-7b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{(3a-7}{}$$

Mathematica [A]
time = 5.80, size = 233, normalized size = 1.03

$$\frac{\cos(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(120\sqrt{2}\sqrt{b}\,(3a^2-10ab+7b^2)\tanh^{-1}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\sqrt{b}}\right)+2\sqrt{a+b+(a-b)\cos(2(e+fx))}\left(-89a^2+474ab-409b^2+4(7a^2-20ab+13b^2)\cos(2(e+fx))-3(a-b)^2\cos(4(e+fx))+60(a-b)b\sec^2(e+fx)\right)\right)}{240\sqrt{2}\,(a-b)f\sqrt{a+b+(a-b)\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(120*
Sqrt[2]*Sqrt[b]*(3*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b + (a - b)*Cos[2
*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + 2*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*
(-89*a^2 + 474*a*b - 409*b^2 + 4*(7*a^2 - 20*a*b + 13*b^2)*Cos[2*(e + f*x)]
 - 3*(a - b)^2*Cos[4*(e + f*x)] + 60*(a - b)*b*Sec[e + f*x]^2)))/(240*Sqrt[
2]*(a - b)*f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2398 vs. $2(203) = 406$.
time = 0.61, size = 2399, normalized size = 10.57

method	result	size
default	Expression too large to display	2399

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/f*(cos(f*x+e)-1)^3*(32*a^(5/2)*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^5+20*a^(9/2)*b^(1/2)*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^4-52*a^(7/2)*b^(3/2
)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^4+3
2*a^(5/2)*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)*cos(f*x+e)^4-30*a^(9/2)*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*cos(f*x+e)^3+140*a^(7/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^3-116*a^(5/2)*b^(5/2)*((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^3-30*a^(9/2)
*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x
+e)^2+140*a^(7/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)*cos(f*x+e)^2-116*a^(5/2)*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2+15*a^(7/2)*b^(3/2)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-15*a^(5/2)*b^(5/2)*((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-15*b^(9/2)*ln(-4*(cos(f*x+e)-
1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^2*a+15*b^(9/2)
*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*
x+e)^2*a-30*b^(7/2)*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e
)^2/a^(1/2))*cos(f*x+e)^2*a^2+30*b^(7/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a
^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^2*a^2+45*b^(5/2)*ln(-4*(cos(f*
x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(c
os(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^2*a^3-45*
b^(5/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))
*cos(f*x+e)^2*a^3-150*a^(7/2)*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x+e
)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*cos(f*x+e)^2*b^2+105*a^(5/2)*arctan
```

```
h(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)
^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/
2))*cos(f*x+e)^2*b^3+45*a^(9/2)*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x
+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*cos(f*x+e)^2*b+15*a^(7/2)*b^(3/2)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-15*a
^(5/2)*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*c
os(f*x+e)-6*a^(9/2)*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*cos(f*x+e)^7+12*a^(7/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^7-6*a^(5/2)*b^(5/2)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^7-6*a^(9/2)*b^(1/2)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^6+12*a
^(7/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*c
os(f*x+e)^6-6*a^(5/2)*b^(5/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)*cos(f*x+e)^6+20*a^(9/2)*b^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^5-52*a^(7/2)*b^(3/2)*((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^5)*cos(f*x+e)*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)/((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)^6/(a-b)/a^(5/2)/b^(1/
2)
```

Maxima [**A**]
time = 0.52, size = 350, normalized size = 1.54

$$-\frac{\frac{12\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5}{a-b}-40\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3+60\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,(a-b)\cos(fx+e)-120\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)-60\,b^{\frac{3}{2}}\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)-\frac{30\,(ab-b^2)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2-b}+\frac{45\,(ab-b^2)\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{\sqrt{b}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/60*(12*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5/(a - b) - 40*(a -
 b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 + 60*sqrt(a - b + b/cos(f*x + e
)^2)*(a - b)*cos(f*x + e) - 120*sqrt(a - b + b/cos(f*x + e)^2)*b*cos(f*x +
e) - 60*b^(3/2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))
/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 30*(a*b - b^2)*
sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b + b/cos(f*x + e)^2)*cos
(f*x + e)^2 - b) + 45*(a*b - b^2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f
*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)))
/sqrt(b))/f
```

Fricas [**A**]
time = 2.65, size = 448, normalized size = 1.97

$$\left[-\frac{15\,(3a^2-10ab+7b^2)\sqrt{b}\cos(fx+e)\log\left(-\frac{(a-b)\cos(fx+e)^2-2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)+2\left(6(a^2-2ab+b^2)\cos(fx+e)^6-4(5a^2-13ab+8b^2)\cos(fx+e)^4+2(15a^2-70ab+58b^2)\cos(fx+e)^2-15ab+15b^2\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{60\,(a-b)f\cos(fx+e)},\ -\frac{15\,(3a^2-10ab+7b^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos(fx+e)+\left(6(a^2-2ab+b^2)\cos(fx+e)^6-4(5a^2-13ab+8b^2)\cos(fx+e)^4+2(15a^2-70ab+58b^2)\cos(fx+e)^2-15ab+15b^2\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{30\,(a-b)f\cos(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(3*a^2 - 10*a*b + 7*b^2)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(
f*x + e)^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^6
 - 4*(5*a^2 - 13*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(15*a^2 - 70*a*b + 58*b^2)
*cos(f*x + e)^2 - 15*a*b + 15*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2))/((a - b)*f*cos(f*x + e)), -1/30*(15*(3*a^2 - 10*a*b + 7*b^2)*sqr
t(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e)/b)*cos(f*x + e) + (6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^6 - 4*(5*a^
2 - 13*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(15*a^2 - 70*a*b + 58*b^2)*cos(f*x +
 e)^2 - 15*a*b + 15*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/((a - b)*f*cos(f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(e+f\,x)^5\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.105 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=186

$$\frac{(3a-5b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{(3a-5b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f}-\frac{(3a$$

```
[Out] -1/3*(3*a-5*b)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+1/3*cos(f*x+e)
^3*(a-b+b*sec(f*x+e)^2)^(5/2)/(a-b)/f+1/2*(3*a-5*b)*arctanh(sec(f*x+e)*b^(1
/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*(3*a-5*b)*b*sec(f*x+e)*(a-b+b
*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]
time = 0.10, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3745, 464, 283, 201, 223, 212}

$$\frac{b(3a-5b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f(a-b)}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{3f(a-b)}-\frac{(3a-5b)\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3f(a-b)}+\frac{\sqrt{b}(3a-5b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
f*x]^2]])/(2*f) + ((3*a - 5*b)*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/(2*(a - b)*f) - ((3*a - 5*b)*Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/
2))/(3*(a - b)*f) + (Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*(a
 - b)*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)^{3/2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{3(a-b)f} + \frac{(3a-5b)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(3a-5b)\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f} + \frac{\cos^3(e+}{}$$

$$= \frac{(3a-5b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-5b)\,c}{}$$

$$= \frac{(3a-5b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-5b)\,c}{}$$

$$= \frac{(3a-5b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-5}{}$$

Mathematica [A]
time = 2.01, size = 188, normalized size = 1.01

$$\frac{\left(12\sqrt{2}(3a-5b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\sqrt{b}}\right)\cos^2(e+fx)+\sqrt{a+b+(a-b)\cos(2(e+fx))}\left(-9a+37b-8(a-3b)\cos(2(e+fx))+(a-b)\cos(4(e+fx))\right)\right)\sec(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{24\sqrt{2}f\sqrt{a+b+(a-b)\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((12*Sqrt[2]*(3*a - 5*b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*
x)]]/(Sqrt[2]*Sqrt[b])]*Cos[e + f*x]^2 + Sqrt[a + b + (a - b)*Cos[2*(e + f*
x)]]*(-9*a + 37*b - 8*(a - 3*b)*Cos[2*(e + f*x)] + (a - b)*Cos[4*(e + f*x)]
))*Sec[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(2
4*Sqrt[2]*f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(166) = 332$.
time = 0.28, size = 1104, normalized size = 5.94

method	result	size
default	Expression too large to display	1104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/f*(cos(f*x+e)-1)^3*(2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*a^(7/2)*b^(1/2)*cos(f*x+e)^5-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)*cos(f*x+e)^5+2*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)*b^(1/2)*cos(f*x+e)^4-2*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)*co
s(f*x+e)^4-6*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(
7/2)*b^(1/2)*cos(f*x+e)^3+14*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)*a^(5/2)*b^(3/2)*cos(f*x+e)^3-6*((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)*b^(1/2)*cos(f*x+e)^2+9*arctanh(1/8*(cos
(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*a^(7/2
)*cos(f*x+e)^2*b+14*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*a^(5/2)*b^(3/2)*cos(f*x+e)^2-6*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*
cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/
2)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*cos(f*x+e)^2*a+6*ln(-4*(cos(f*x+e)-1)*(
cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*cos(f*x+e)^2*a+3*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)*cos(
f*x+e)-15*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+
e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*b^(1/2)*4^(1/2))*a^(5/2)*cos(f*x+e)^2*b^2+3*a^(5/2)*b^(3/2)*((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)/((a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)^6/a^(5/2)/b^(1/2)
```

<u>Maxima</u> [A]
time = 0.52, size = 314, normalized size = 1.69

$$\frac{4\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-12\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,(a-b)\cos(fx+e)+12\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)+6\,b^{\frac{3}{2}}\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)+\frac{6\,(ab-b^2)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2-b}-\frac{9\,(ab-b^2)\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{\sqrt{b}}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 12*sqrt(a - b + b
/cos(f*x + e)^2)*(a - b)*cos(f*x + e) + 12*sqrt(a - b + b/cos(f*x + e)^2)*b
*cos(f*x + e) + 6*b^(3/2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)
```

```
- sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) + 6*(a*
b - b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b + b/cos(f*x +
e)^2)*cos(f*x + e)^2 - b) - 9*(a*b - b^2)*log((sqrt(a - b + b/cos(f*x + e)^
2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + s
qrt(b)))/sqrt(b))/f
```

Fricas [A]
time = 3.19, size = 327, normalized size = 1.76

$$\left[-\frac{3\,(3\,a-5\,b)\sqrt{b}\,\cos\left(fx+e\right)\log\left(-\frac{(a-b)\cos(fx+e)^2-2\,\sqrt{b}\,\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\,\left(2\,(a-b)\cos\left(fx+e\right)^4-2\,(3\,a-7\,b)\cos\left(fx+e\right)^2+3\,b\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{12\,f\cos\left(fx+e\right)},\ -\frac{3\,(3\,a-5\,b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\cos(fx+e)}{b}\right)\cos\left(fx+e\right)-\left(2\,(a-b)\cos\left(fx+e\right)^4-2\,(3\,a-7\,b)\cos\left(fx+e\right)^2+3\,b\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{6\,f\cos\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a - 5*b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)^2 - 2
*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2
*b)/cos(f*x + e)^2) - 2*(2*(a - b)*cos(f*x + e)^4 - 2*(3*a - 7*b)*cos(f*x +
 e)^2 + 3*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x
+ e)), -1/6*(3*(3*a - 5*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) - (2*(a - b)*cos(f*
x + e)^4 - 2*(3*a - 7*b)*cos(f*x + e)^2 + 3*b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.84, size = 326, normalized size = 1.75

$$-\frac{1}{6}\left(\frac{3\,(3\,ab\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right))-5\,b^2\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right)))\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,f^2}-\frac{3\left(\sqrt{a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b}\,ab\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right))-\sqrt{a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b}\,b^2\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right))\right)}{\left(a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2\right)f^2}-\frac{2\left(\left(a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b\right)^{\frac{3}{2}}f^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right))-3\,\sqrt{a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b}\,af^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right))+6\,\sqrt{a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b}\,bf^4\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right))\right)}{f^6}\right)|f|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*(3*a*b*sgn(f)*sgn(cos(f*x + e)) - 5*b^2*sgn(f)*sgn(cos(f*x + e)))*a
rctan(sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)/sqrt(-b))/(sqrt(-b)*f^2
) - 3*(sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a*b*sgn(f)*sgn(cos(f*x
```

```
 + e)) - sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*b^2*sgn(f)*sgn(cos(f
*x + e)))/((a*cos(f*x + e)^2 - b*cos(f*x + e)^2)*f^2) - 2*((a*cos(f*x + e)^
2 - b*cos(f*x + e)^2 + b)^(3/2)*f^4*sgn(f)*sgn(cos(f*x + e)) - 3*sqrt(a*cos
(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a*f^4*sgn(f)*sgn(cos(f*x + e)) + 6*sqrt
(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*b*f^4*sgn(f)*sgn(cos(f*x + e)))/f
^6)*abs(f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.106 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=113

$$\frac{3(a-b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{3b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\,(}{}$$

```
[Out] -cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/f+3/2*(a-b)*arctanh(sec(f*x+e)*b^(1/
2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+3/2*b*sec(f*x+e)*(a-b+b*sec(f*x+e)
^2)^(1/2)/f
```

Rubi [A]

time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 283, 201, 223, 212}

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{f}+\frac{3\sqrt{b}\,(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*
x]^2]])/(2*f) + (3*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*f) - (
Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2))/f
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{f} + \frac{(3b)\text{Subst}\left(\int \sqrt{a-b}\right.}{}$$

$$= \frac{3b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a-b+\right.}{f}$$

$$= \frac{3b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\left(a-b+\right.}{f}$$

$$= \frac{3(a-b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{3b\sec(e+}{}$$

Mathematica [A]
time = 1.43, size = 170, normalized size = 1.50

$$\frac{\left(6\sqrt{2}\,(a-b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\,\sqrt{b}}\right)\cos^2(e+fx)-2(a-2b+(a-b)\cos(2(e+fx)))\sqrt{a+b+(a-b)\cos(2(e+fx))}\right)\sec(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{4\sqrt{2}\,f\sqrt{a+b+(a-b)\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((6*Sqrt[2]*(a - b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]/
(Sqrt[2]*Sqrt[b])]*Cos[e + f*x]^2 - 2*(a - 2*b + (a - b)*Cos[2*(e + f*x)])*
Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])*Sec[e + f*x]*Sqrt[(a + b + (a - b)*
Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(4*Sqrt[2]*f*Sqrt[a + b + (a - b)*Cos[2*
(e + f*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(99) = 198$.
time = 0.11, size = 359, normalized size = 3.18

method	result
default	$-\frac{\left(\frac{a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)\left(3b^{\frac{5}{2}}\ln\left(\frac{2\sqrt{b}\,\sqrt{a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b}+2b}{\cos(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)*(3
*b^(5/2)*ln(2*(b^(1/2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2)+b)/cos(f*x+e
))*cos(f*x+e)^2-3*b^(3/2)*ln(2*(b^(1/2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(
1/2)+b)/cos(f*x+e))*a*cos(f*x+e)^2+(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(3/2)*
a*cos(f*x+e)^2-(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(3/2)*b*cos(f*x+e)^2-(a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)^(5/2)+3*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2
)*a*b*cos(f*x+e)^2-3*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(1/2)*b^2*cos(f*x+e)
^2)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^(3/2)/b
```

Maxima [A]
time = 0.52, size = 186, normalized size = 1.65

$$-\frac{4\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,(a-b)\cos(fx+e)-\frac{2\,(ab-b^2)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)}{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2-b}+\frac{3\,(ab-b^2)\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)+\sqrt{b}}\right)}{\sqrt{b}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(a - b + b/cos(f*x + e)^2)*(a - b)*cos(f*x + e) - 2*(a*b - b^2)
*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b + b/cos(f*x + e)^2)*co
s(f*x + e)^2 - b) + 3*(a*b - b^2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f
*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)))
/sqrt(b))/f
```

Fricas [A]
time = 4.69, size = 286, normalized size = 2.53

$$\left[-\frac{3\,(a-b)\sqrt{b}\cos\left(fx+e\right)\log\left(-\frac{\frac{(a-b)\cos\left(fx+e\right)^{2}-2\sqrt{b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^{2}}}{}\right)+2\left(2\,(a-b)\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4\,f\cos\left(fx+e\right)},\ -\frac{3\,(a-b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{b}\right)\cos\left(fx+e\right)+\left(2\,(a-b)\cos\left(fx+e\right)^{2}-b\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{2\,f\cos\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a - b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)^2 - 2*sqrt
(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/c
os(f*x + e)^2) + 2*(2*(a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/2*(3*(a - b)*sqrt(-b)*arctan
(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
*cos(f*x + e) + (2*(a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\sin\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*sin(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(105) = 210.
time = 1.42, size = 277, normalized size = 2.45

$$-\frac{1}{2}\left(\frac{3\left(ab\,\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-b^{2}\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,f^{2}}+\frac{2\left(\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}\,a\,\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}\,b\,\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)\right)}{f^{2}}-\frac{\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}\,ab\,\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}\,b^{2}\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{\left(a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}\right)f^{2}}\right)|f|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*(a*b*sgn(f)*sgn(cos(f*x + e)) - b^2*sgn(f)*sgn(cos(f*x + e)))*arcta
n(sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)/sqrt(-b))/(sqrt(-b)*f^2) +
2*(sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a*sgn(f)*sgn(cos(f*x + e))
 - sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*b*sgn(f)*sgn(cos(f*x + e))
)/f^2 - (sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a*b*sgn(f)*sgn(cos(f
*x + e)) - sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*b^2*sgn(f)*sgn(cos
(f*x + e)))/((a*cos(f*x + e)^2 - b*cos(f*x + e)^2)*f^2))*abs(f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+fx)\,\left(b\tan(e+fx)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.107 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}+\frac{(3a-b)\sqrt{b}\ \tanh^{-1}\left(\frac{\sqrt{b}\ \sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{b\sec(e+fx}{}$$

```
[Out] -a^(3/2)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/f+1/2*(3*a-
b)*arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*s
ec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3745, 427, 537, 223, 212, 385, 213}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}+\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2f}+\frac{\sqrt{b}\,(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\ \sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/
f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-b+bx^2)^{3/2}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{2a^2-3ab+b^2+}{(-1+x^2)\sqrt{a-}}\right.}{}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{}{(-1+x^2)\sqrt{a}}\right.}{}$$

$$= \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f} + \frac{a^2\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx,\right.}{}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{(3a-b)\sqrt{b}\,\tanh}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 492 vs. 2(127) = 254.
time = 5.23, size = 492, normalized size = 3.87

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sec[(e + f*x)/2]^2*(-4*Sqrt[b]*(-3*a + b)*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e
+ f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2
])/(2*Sqrt[b])]*Cos[e + f*x]^2 + 4*a^(3/2)*ArcTanh[Tan[(e + f*x)/2]^2 - Sqr
t[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]]*Cos[e +
f*x]^2 + a^(3/2)*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[
(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + a^(3/2)*Cos[2*(e + f*x)]
*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 +
 a*(-1 + Tan[(e + f*x)/2]^2)^2]] + (b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)
])*Sec[(e + f*x)/2]^4])/Sqrt[2] + (b*Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos
[2*(e + f*x)])*Sec[(e + f*x)/2]^4])/Sqrt[2])*Sec[e + f*x]*Sqrt[(a + b + (a
- b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(4*f*Sqrt[(a + b + (a - b)*Cos[2*(e
 + f*x)])*Sec[(e + f*x)/2]^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. 2(109) = 218.
time = 0.26, size = 1248, normalized size = 9.83

method	result	size
default	Expression too large to display	1248

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)*cos(f*
x+e)*(cos(f*x+e)-1)^3*(4*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)
+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(
f*x+e)^2/a^(1/2))*b^(7/2)*cos(f*x+e)^2*a-4*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)
*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x
+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*cos(f*x+e)^2*a-3*arctanh(1/8*(c
os(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*a^(7
/2)*cos(f*x+e)^2*b-6*cos(f*x+e)^2*b^(5/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*
a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^2+6*cos(f*x+e)^2*b^(5/2)*ln(-4*(cos(f*
x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(c
os(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^2+arctanh(1/8*(cos
(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*a^(5/2
)*cos(f*x+e)^2*b^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(5/2)*b^(3/2)*cos(f*x+e)-a^(5/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)+cos(f*x+e)^2*b^(1/2)*ln(-2*(cos(f*x+e)-1)*(co
s(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^4+cos(f*x+e)^2*b^(1/2)*ln(-4*
(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(
f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*a^4)/sin(f*x+e)^6/((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/a^(5/2)/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

Fricas [A]
time = 4.48, size = 799, normalized size = 6.29

$$\left[\frac{2 a^{\frac{3}{2}} \cos(fx+e) \log\left(-\frac{2\left((a-b)\cos(fx+e)^2 - 2\sqrt{a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) + a + b\right)}{\cos(fx+e)^2-1}\right) - (3a-b)\sqrt{b}\cos(fx+e)\log\left(-\frac{(a-b)\cos(fx+e)^2 - 2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) + 2b}{\cos(fx+e)^2}\right) + 2b\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4f\cos(fx+e)}, -\frac{(3a-b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos(fx+e) - a^{\frac{3}{2}}\cos(fx+e)\log\left(-\frac{2\left((a-b)\cos(fx+e)^2 - 2\sqrt{a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) + a + b\right)}{\cos(fx+e)^2-1}\right) - b\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2f\cos(fx+e)}, \frac{4\sqrt{-a}\,a\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a}\right)\cos(fx+e) - (3a-b)\sqrt{b}\cos(fx+e)\log\left(-\frac{(a-b)\cos(fx+e)^2 - 2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) + 2b}{\cos(fx+e)^2}\right) + 2b\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4f\cos(fx+e)}, \frac{2\sqrt{-a}\,a\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a}\right)\cos(fx+e) - (3a-b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos(fx+e) + b\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2f\cos(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^(3/2)*cos(f*x + e)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f
*x + e)^2 - 1)) - (3*a - b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)
^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(f*cos(f*x + e)), -1/2*((3*a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a
 - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) - a^
(3/2)*cos(f*x + e)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2
- 1)) - b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e
)), 1/4*(4*sqrt(-a)*a*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)/a)*cos(f*x + e) - (3*a - b)*sqrt(b)*cos(f*x + e)*
log(-((a - b)*cos(f*x + e)^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), 1/2*(2*sqrt(-a)*a*arcta
n(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a
)*cos(f*x + e) - (3*a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) + b*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}} \csc\left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*csc(e + f*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

3.108 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{b\,\mathrm{se}}{}$$

```
[Out] -1/2*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/f-1/2*(a+3*b)*arctanh
(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*(3*a+b)*arcta
nh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+b*sec(f*x+e)*(a
-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3745, 478, 542, 537, 223, 212, 385, 213}

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}+\frac{\sqrt{b}\,(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}-\frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/f + (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
 b + b*Sec[e + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x
]^2])/f - (Cot[e + f*x]*Csc[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2))/(2*f
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{x^2(a-b+bx^2)^{3/2}}{(-1+x^2)^2}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=-\frac{\cot(e+fx)\csc(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{2f}+\frac{\text{Subst}\Big(}{}$$

$$=\frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}-\frac{\cot(e+fx)\csc(e+f}{}$$

$$=\frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}-\frac{\cot(e+fx)\csc(e+f}{}$$

$$=\frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}-\frac{\cot(e+fx)\csc(e+f}{}$$

$$=-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{\sqrt{b}\,(3a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1012 vs. $2(167) = 334$.

time = 6.64, size = 1012, normalized size = 6.06

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*(-1/2*(a*Cot[e + f*x]*Csc[e + f*x]) + (b*Sec[e + f*x])/2))/f + (((a^2
- b^2)*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]
*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]
*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^
2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])] - Sqrt[b]*(2*ArcTanh[Tan[(
e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2
]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e +
 f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]))*(-1 + Tan[(e + f*x)/2]^2)*(1
 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)
/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a -
 b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f
```

```
*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]) - ((a^2 + 6*a*b + b^2)*(1 + Cos[
e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (
a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]*ArcTanh[(-(Sqrt
[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[
(e + f*x)/2]^2)^2])/(2*Sqrt[b])] + Sqrt[b]*(2*ArcTanh[Tan[(e + f*x)/2]^2 -
Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log
[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(
-1 + Tan[(e + f*x)/2]^2)^2]]))*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)
/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + T
an[(e + f*x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f
*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1
 + Tan[(e + f*x)/2]^2)^2]))/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2903 vs. $2(145) = 290$.

time = 0.29, size = 2904, normalized size = 17.39

method	result	size
default	Expression too large to display	2904

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(cos(f*x+e)-1)^2*(10*cos(f*x+e)^3*b^(7/2)*ln(-2*(cos(f*x+e)-1)*(cos(f
*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a-10*cos(f*x+e)^3*b^(7/2)*ln(-4*(c
os(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a-6*cos(f*x+e)^
3*a^(7/2)*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+
e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*b^(1/2)*4^(1/2))*b-18*cos(f*x+e)^3*b^(5/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x
+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^2+18*cos(f*x+e)^3*b^(5/2)*ln(-4*(c
os(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^2-2*cos(f*x+e
)^3*a^(5/2)*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)-2*cos(f*
x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)*b^(1/2)*4^(1/2))*b^2-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*a^(7/2)*b^(1/2)*cos(f*x+e)^2-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*b^(3/2)*cos(f*x+e)^2-10*ln(-2*(cos(f*x+e
)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
```

```
2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*b^(7/2)*cos(f*x+e)^2*a+
10*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*b^(7
/2)*cos(f*x+e)^2*a+6*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/2)*cos(f*x+e)-4^(1/2)
-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*a^(7/2)*cos(f*x+e)^2*b+3*cos(f*x+e)^3*b^(3/2
)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^3+3
*cos(f*x+e)^3*b^(3/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*a^3+18
*cos(f*x+e)^2*b^(5/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x
+e)^2/a^(1/2))*a^2-18*cos(f*x+e)^2*b^(5/2)*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)
*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x
+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^2+2*arctanh(1/8*(cos(f*x+e)-1)*(4^(1/
2)*cos(f*x+e)-4^(1/2)-2*cos(f*x+e)-2)/sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*b^(1/2)*4^(1/2))*a^(5/2)*cos(f*x+e)^2*b^
2+cos(f*x+e)^3*b^(1/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*
x+e)^2/a^(1/2))*a^4+cos(f*x+e)^3*b^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(co
s(f*x+e)-1))*a^4-3*cos(f*x+e)^2*b^(3/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^
(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*
a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^3-3*cos(f*x+e)^2*b^(3/2)*ln(-4*(cos(f*x+
e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-
b*cos(f*x+e)+b)/(cos(f*x+e)-1))*a^3+2*a^(5/2)*b^(3/2)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2*b^(1/2)*ln(-2*(cos(f*x+e
)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(
f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^4-cos(f*x+e)^2*b^(1/2
)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1
/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*a^4)*cos(f*x+e)*((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)/((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

Fricas [A]
time = 4.62, size = 1062, normalized size = 6.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/4*(((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(a)*log(-2*((
a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + ((3*a + b)*cos(f*x +
 e)^3 - (3*a + b)*cos(f*x + e))*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sq
rt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)
/cos(f*x + e)^2) + 2*((a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/4*(2*((3*
a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - ((a + 3*b
)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(f*x
+ e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e) + a + b)/(cos(f*x + e)^2 - 1)) - 2*((a + b)*cos(f*x + e)^2 - b)*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*
x + e)), 1/4*(2*((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(-a
)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)/a) + ((3*a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(b)*log(
-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((a + b)*cos(f*x + e)^2
 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3
- f*cos(f*x + e)), 1/2*(((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))
*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)/a) - ((3*a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt
(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)/b) + ((a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

3.109 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=223

$$-\frac{3(a^2+6ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8\sqrt{a}\,f}+\frac{3\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}$$

```
[Out] -1/4*cot(f*x+e)*csc(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(3/2)/f-3/8*(a^2+6*a*b+b^
2)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/f/a^(1/2)+3/2*(a+
b)*arctanh(sec(f*x+e)*b^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+3/8*(a+
3*b)*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f-3/8*(a+b)*csc(f*x+e)^2*sec(f*x
+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.23, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3745, 478, 591, 596, 537, 223, 212, 385, 213}

$$-\frac{3(a^2+6ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8\sqrt{a}\,f}+\frac{3(a+3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8f}-\frac{3(a+b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8f}+\frac{3\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a^2 + 6*a*b + b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]])/(8*Sqrt[a]*f) + (3*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*
x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(2*f) + (3*(a + 3*b)*Sec[e + f*x]*Sqrt
[a - b + b*Sec[e + f*x]^2])/(8*f) - (3*(a + b)*Csc[e + f*x]^2*Sec[e + f*x]*
Sqrt[a - b + b*Sec[e + f*x]^2])/(8*f) - (Cot[e + f*x]*Csc[e + f*x]^3*(a - b
 + b*Sec[e + f*x]^2)^(3/2))/(4*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{x^4\left(a-b+bx^2\right)^{3/2}}{\left(-1+x^2\right)^3}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{4f}+\frac{\text{Subs}}{}$$

$$=-\frac{3(a+b)\csc^2(e+fx)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f}$$

$$=\frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f}-\frac{3(a+b)\csc}{}$$

$$=\frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f}-\frac{3(a+b)\csc}{}$$

$$=\frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f}-\frac{3(a+b)\csc}{}$$

$$=-\frac{3(a^2+6ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8\sqrt{a}\,f}+\frac{3\sqrt{}}{}$$

Mathematica [A]
time = 4.74, size = 415, normalized size = 1.86

$$\frac{\cos(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-2\csc^2(e+fx)\left(3a+5b+2a\csc^2(e+fx)\right)+8b\sec^2(e+fx)+\frac{3\left(16\sqrt{a}\sqrt{b}(a+b)\tanh^{-1}\left(\frac{-\sqrt{a}\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{2\sqrt{b}}\right)+(a^2+6ab+b^2)\left(2\tanh^{-1}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\frac{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)+\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\cos^2(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}}{\sqrt{a}\sqrt{\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}\right)}{16\sqrt{2}\,f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-2*C
sc[e + f*x]^2*(3*a + 5*b + 2*a*Csc[e + f*x]^2) + 8*b*Sec[e + f*x]^2 + (3*(1
6*Sqrt[a]*Sqrt[b]*(a + b)*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) + S
qrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])] +
 (a^2 + 6*a*b + b^2)*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)
/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e +
f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^
2)^2]]))*Sec[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]^2*Sec[(e + f*x)/2]^4])/(Sqrt[
a]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 +
Tan[(e + f*x)/2]^2)^2])))/(16*Sqrt[2]*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6193 vs. $2(195) = 390$.
time = 0.33, size = 6194, normalized size = 27.78

method	result	size
default	Expression too large to display	6194

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

Fricas [**A**]
time = 4.12, size = 1449, normalized size = 6.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos(f*
x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(f*
x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
```

```
f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + a*b)*cos(f*x + e)^5 -
2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x + e))*sqrt(b)*log(-((a -
 b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3*(a^2 + 3*a*b)*cos(f*x + e)
^4 - (5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2
+ b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(
f*x + e)), 1/8*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^
2)*cos(f*x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(-a)*arctan(sqrt(
-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + 6*(
(a^2 + a*b)*cos(f*x + e)^5 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos
(f*x + e))*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + (3
*(a^2 + 3*a*b)*cos(f*x + e)^4 - (5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*
f*cos(f*x + e)^3 + a*f*cos(f*x + e)), -1/16*(24*((a^2 + a*b)*cos(f*x + e)^5
 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x + e))*sqrt(-b)*arctan
(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
 - 3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos(f*x +
e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(f*x +
e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + a + b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a^2 + 3*a*b)*cos(f*x + e)^4 - (
5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x +
e)), 1/8*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos
(f*x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(-a)*arctan(sqrt(-a)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) - 12*((a^2
+ a*b)*cos(f*x + e)^5 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x
+ e))*sqrt(-b)*arctan(sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e)/b) + (3*(a^2 + 3*a*b)*cos(f*x + e)^4 - (5*a^2 + 13*a*b)*
cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

3.110 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=222

$$\frac{3(a^2-8ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{a-b}\,f}+\frac{3(a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{3(\cdots}{\cdots}$$

```
[Out] 3/8*(a^2-8*a*b+8*b^2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2
))/f/(a-b)^(1/2)+3/2*(a-2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^
(1/2))*b^(1/2)/f-3/8*(a-4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/8*(a-2
*b)*sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/4*cos(f*x+e)*sin(f
*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A]
time = 0.22, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3744, 478, 591, 596, 537, 223, 212, 385, 209}

$$\frac{3(a^2-8ab+8b^2)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f\sqrt{a-b}}-\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{3(a-2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{3\sqrt{b}\,(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\sin^3(e+fx)\cos(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a^2 - 8*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(8*Sqrt[a - b]*f) + (3*(a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[
e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*f) - (3*(a - 4*b)*Tan[e + f*x]*Sq
rt[a + b*Tan[e + f*x]^2])/(8*f) + (3*(a - 2*b)*Sin[e + f*x]^2*Tan[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/(8*f) - (Cos[e + f*x]*Sin[e + f*x]^3*(a + b*Tan
[e + f*x]^2)^(3/2))/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^{3/2}}{(1+x^2)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cos(e+fx)\sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{4f}+\frac{\text{Subst}\Big(}{}$$

$$=\frac{3(a-2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}-\frac{\text{co}}{}$$

$$=-\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{3(a-2b)\sin}{}$$

$$=-\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{3(a-2b)\sin}{}$$

$$=-\frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{3(a-2b)\sin}{}$$

$$=\frac{3(a^2-8ab+8b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{a-b}\,f}+\frac{3(a-2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 5.21, size = 278, normalized size = 1.25

$$\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(\frac{3a\csc^2(e+fx)\left((a^2-5ab+4b^2)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)-(a^2-8ab+8b^2)\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sin(2(e+fx))}{\sqrt{2}(a-b)b\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}+\frac{1}{4}((-8a+18b)\sin(2(e+fx))+(a-b)\sin(4(e+fx))+16b\tan(e+fx))\right)}{8\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((3*a*Csc[e + f*x]
^2*((a^2 - 5*a*b + 4*b^2)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - (a^2 - 8*a*b + 8*b^2)*EllipticPi
[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1])*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)*b*Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]) + ((-8*a + 18*b)*Sin[2*(e + f*x
)] + (a - b)*Sin[4*(e + f*x)] + 16*b*Tan[e + f*x])/4))/(8*Sqrt[2]*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.64, size = 2630, normalized size = 11.85

method	result	size
default	Expression too large to display	2630

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(17*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*a*b-17*((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a*b-((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^3*a*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2)*cos(f*x+e)^2*a*b-5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x
+e)^5*a^2+2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^7*b^2-3*2^
(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a
-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1
/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1
/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(
f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)
/a^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2+6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(
a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1
)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2+48*2^(1/2)*((I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)
/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a
```

```
-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((
cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*b^2-48*2^(1
/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2
)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f
*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)
/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^
2*b^2-48*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+c
os(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+
e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b
)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+
e)*cos(f*x+e)^2*a*b-4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2+12*sin(
f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a
-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f
*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b
)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1
/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b+24*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(
(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(
f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b
^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*Ell
ipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e)
,1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(
1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b-4*((2*I*b^(1/2)*(a-b)^(
1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^7*a*b+4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2)*cos(f*x+e)^6*a*b-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x
+e)^6*b^2+2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^7*a^2-2*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a^2-12*((2*I*b^(1/2)*(
a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*b^2+5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)/a)^(1/2)*cos(f*x+e)^4*a^2+12*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*
cos(f*x+e)^4*b^2+6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^3*b
^2-6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^2+4*((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)*b^2)*cos(f*x+e)*((a*cos(f*x+e)^
2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(cos(f*x+e)-1)/(a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)^2/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(206) = 412$.
time = 160.74, size = 2261, normalized size = 10.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 - 8*a*b + 8*b^2)*sqrt(-a + b)*cos(f*x + e)*log(128*(a^4 - 4*
a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*
a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b
^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256
*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos
(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^
3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*
b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x
 + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e)) + 24*(a^2 - 3*a*b + 2*b^2)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b
+ 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos
(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 8*(2*(a^2 - 2*a*b +
b^2)*cos(f*x + e)^4 - 5*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*a*b - 4*b^
2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a - b)
*f*cos(f*x + e)), -1/64*(48*(a^2 - 3*a*b + 2*b^2)*sqrt(-b)*arctan(1/2*((a -
 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))
*cos(f*x + e) + 3*(a^2 - 8*a*b + 8*b^2)*sqrt(-a + b)*cos(f*x + e)*log(128*(
a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^
3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b +
77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b
^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*
b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7
- 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b
 + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)
*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e)) - 8*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 5*(a^2 - 3*a*b
 + 2*b^2)*cos(f*x + e)^2 + 4*a*b - 4*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/((a - b)*f*cos(f*x + e)), 1/32*(3*(a^2 - 8*a
*b + 8*b^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 -
```

```
 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e
))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 -
 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - 1
2*(a^2 - 3*a*b + 2*b^2)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos
(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos(f*x + e)^3
+ 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(2*(a^2 - 2*a*b + b^2)*cos(f*x
 + e)^4 - 5*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*a*b - 4*b^2)*sqrt(((a
- b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a - b)*f*cos(f*x +
 e)), 1/32*(3*(a^2 - 8*a*b + 8*b^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b
 + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*
a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b +
3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x
 + e)))*cos(f*x + e) - 24*(a^2 - 3*a*b + 2*b^2)*sqrt(-b)*arctan(1/2*((a - 2
*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*c
os(f*x + e) + 4*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 5*(a^2 - 3*a*b + 2*
b^2)*cos(f*x + e)^2 + 4*a*b - 4*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/((a - b)*f*cos(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(e+f\,x)^4\,\left(b\,\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.111 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$\frac{(a-4b)\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{(3a-4b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan}{}$$

```
[Out] 1/2*(a-4*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(
1/2)/f+1/2*(3*a-4*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b
^(1/2)/f+b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/2*cos(f*x+e)*sin(f*x+e)*
(a+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A]
time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3744, 478, 542, 537, 223, 212, 385, 209}

$$\frac{(a-4b)\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\sqrt{b}\,(3a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 4*b)*Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e +
 f*x]^2]])/(2*f) + ((3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b*Tan[e + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]
)/f - (Cos[e + f*x]*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^{3/2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx}{2}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx}{2}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx}{2}$$

$$= \frac{(a-4b)\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3a-4b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 5.74, size = 324, normalized size = 1.96

$$-\frac{\sec^2(e+fx)\left(-4\sqrt{2}a(a-2b)\cot(e+fx)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+4\sqrt{2}a(a-4b)\cot(e+fx)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+(3a^2-6ab-5b^2+4(a-b)^2\cos(2(e+fx))+(a-b)^2\cos(4(e+fx)))\csc(e+fx)\sec(e+fx)\right)\sin(2(e+fx))\tan(e+fx)}{16\sqrt{2}f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/16*(Sec[e + f*x]^2*(-4*Sqrt[2]*a*(a - 2*b)*Cot[e + f*x]*Sqrt[((a + b + (
a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 4*Sqrt[2]*a*(a
 - 4*b)*Cot[e + f*x]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^
2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x
)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + (3*a^2 - 6*a*b - 5*b^2 + 4*(a - b)^2*
Cos[2*(e + f*x)] + (a - b)^2*Cos[4*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])*S
in[2*(e + f*x)]*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f
*x)])*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 2261, normalized size = 13.70

method	result	size
default	Expression too large to display	2261

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(-2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*
x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a
-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*
x+e)*cos(f*x+e)^2*a^2+10*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*
cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x
+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b
)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(
2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a*b-8*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^
(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e
)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*b^2+2^(1/2)*((I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*
x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2
)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e
)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-
b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*
cos(f*x+e)^2*a^2-2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(
a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1
)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(
f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*(
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/
2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b-6*sin(f*x+e)*
cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/
2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b
^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(
f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(
```

```
a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b+8
*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e
)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)
^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/
sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a
+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*
x+e)^2*b^2+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*a^2-2*((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*a*b+((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*b^2-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)
^(1/2)*cos(f*x+e)^4*a^2+2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x
+e)^4*a*b-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*b^2-((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)*b^2+((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)*sin(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/cos(f*x+e)^2)^(3/2)/(cos(f*x+e)-1)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2
/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(152) = 304$.
time = 27.08, size = 2025, normalized size = 12.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((a - 4*b)*sqrt(-a + b)*cos(f*x + e)*log(128*(a^4 - 4*a^3*b + 6*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*
b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3
+ 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b
^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 +
8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b +
5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*
cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a
 + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 2*(
3*a - 4*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 +
```

```
8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x
 + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e) + 8*b^2)/cos(f*x + e)^4) + 8*((a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), -1/16*(
4*(3*a - 4*b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^
2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + (a - 4*b)*sqrt(-a +
b)*cos(f*x + e)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x
 + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6
+ 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a
^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34
*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e
)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a
^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*((a - b)*cos(f*x + e)^2 - b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x
+ e)), 1/8*(sqrt(a - b)*(a - 4*b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*
x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*c
os(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^
3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*
x + e) - (3*a - 4*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*
x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((a - b)*cos(f*x + e)^2 - b)*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)
), 1/8*(sqrt(a - b)*(a - 4*b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x +
e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f
*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 -
(a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
e) - 2*(3*a - 4*b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(
f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b
 - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*((a - b)*cos(
f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(f*cos(f*x + e))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}} \sin^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*sin(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.112 $\int (a+b\tan^2(e+fx))^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)}{}$$

```
[Out] (a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/2*(
3*a-2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2
*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$,
Rules used = {3742, 427, 537, 223, 212, 385, 209}

$$\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{(1+x^2)\sqrt{a+bx^2}} dx, x, \tan(e\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}} dx,\right.}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2} dx, x, \frac{}{\sqrt{}}\right.}{f}$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a+}}\right.}{2f}$$

Mathematica [A]
time = 0.88, size = 142, normalized size = 1.14

$$\frac{-2(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{b}\tan^2(e+fx)-\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{b}\,(-3a+2b)\log\left(-\sqrt{b}\,\tan(e+fx)+\sqrt{a+b\tan^2(e+fx)}\right)+b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - b)^(3/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[e + f*x]^2 - Tan[e + f*x]*S
qrt[a + b*Tan[e + f*x]^2])/Sqrt[a - b]] + Sqrt[b]*(-3*a + 2*b)*Log[-(Sqrt[b
]*Tan[e + f*x]) + Sqrt[a + b*Tan[e + f*x]^2]] + b*Tan[e + f*x]*Sqrt[a + b*T
an[e + f*x]^2])/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(107) = 214$.
time = 0.06, size = 327, normalized size = 2.62

method	result

derivativedivides	$\frac{b^{2}\left(\frac{\tan(fx+e)\sqrt{a+b\left(\tan^{2}(fx+e)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^{2}(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\text{ta}\right.}{}\right.}{}$
default	$\frac{b^{2}\left(\frac{\tan(fx+e)\sqrt{a+b\left(\tan^{2}(fx+e)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^{2}(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\text{ta}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/2*tan(f*x+e)/b*(a+b*tan(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2
)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e
)^2)^(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b)
)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))+2*a*b*(ln(b^(1/2)*tan(f*x+e)+
(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a
-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))+a^2*(b^4*(a-b))
^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A]
time = 4.52, size = 567, normalized size = 4.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(a - b)*sqrt(-a + b)*log(-((a - 2*b)*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/
```

```
(tan(f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, -1/
2*((3*a - 2*b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f
*x + e))) - (-a + b)^(3/2)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - sqrt(b
*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/4*(4*(a - b)^(3/2)*arctan(-sqrt(b
*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*sqrt(b)*log(
2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)
 + 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/2*(2*(a - b)^(3/2)*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*s
qrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt
(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2), x)
```

3.113 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=100

$$\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e\right.}{f}$$

```
[Out] 3/2*a*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+3/2*b*
(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3744, 283, 201, 223, 212}

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2
*f) + (3*b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f) - (Cot[e + f*x]*(
a + b*Tan[e + f*x]^2)^(3/2))/f
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f} + \frac{(3b)\text{Subst}\left(\int \sqrt{a+bx^2}\right.}{f}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b\tan^2\right.}{f}$$

$$= \frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b\tan^2\right.}{f}$$

$$= \frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{3b\tan(e+fx)\sqrt{a}}{2f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.82, size = 220, normalized size = 2.20

$$\frac{\csc(e+fx)\sec^3(e+fx)\left(-6a^2-ab+3b^2-4(2a^2+b^2)\cos(2(e+fx))-2a^2\cos(4(e+fx))+ab\cos(4(e+fx))+b^2\cos(4(e+fx))+3\sqrt{2}\,ab\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\sin^2(2(e+fx))\right)}{8\sqrt{2}\,f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Csc[e + f*x]*Sec[e + f*x]^3*(-6*a^2 - a*b + 3*b^2 - 4*(2*a^2 + b^2)*Cos[2*
(e + f*x)] - 2*a^2*Cos[4*(e + f*x)] + a*b*Cos[4*(e + f*x)] + b^2*Cos[4*(e +
 f*x)] + 3*Sqrt[2]*a*b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]/Sqrt[2]], 1]*Sin[2*(e + f*x)]^2))/(8*Sqrt[2]*f*Sqrt[(a + b + (a -
 b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 1355, normalized size = 13.55

method	result	size
default	Expression too large to display	1355

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)*(-
3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1
/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I
*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*
x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b
)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a
-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b+6*sin(f*x+e)*cos(f*x+e)^3*2^(1
/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2
)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f
*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)
/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b-3*sin(f*x+e)*cos(f
*x+e)^2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+co
s(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2
)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e
)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*
a*b+8*b^2)/a^2)^(1/2))*a*b+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(co
s(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^
(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b+2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)*cos(f*x+e)^4*a^2-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+
```

```
e)^4*a*b-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*b^2+((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b+2*((2*I*b^(1/2)*(a-b)^(
1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^2-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(
1/2)*b^2)/sin(f*x+e)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2/((2*I*b^(1/2)*(a-b
)^(1/2)+a-2*b)/a)^(1/2)
```

Maxima [A]
time = 0.29, size = 78, normalized size = 0.78

$$\frac{3\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)+3\,\sqrt{b\tan\left(fx+e\right)^2+a}\,b\tan\left(fx+e\right)-\frac{2\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}{\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) + 3*sqrt(b*tan(f*x + e)^
2 + a)*b*tan(f*x + e) - 2*(b*tan(f*x + e)^2 + a)^(3/2)/tan(f*x + e))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(92) = 184.
time = 1.88, size = 415, normalized size = 4.15

$$\left[\frac{3\,a\sqrt{b}\,\cos\left(fx+e\right)\log\left(\frac{\left(a^2-8\,ab+8\,b^2\right)\cos(fx+e)^4+8\,\left(ab-2\,b^2\right)\cos(fx+e)^2+4\left(\left(a-2\,b\right)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin(fx+e)+8\,b^2}{\cos(fx+e)^4}\right)\sin\left(fx+e\right)-4\left(\left(2\,a+b\right)\cos\left(fx+e\right)^2-b\right)\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{8\,f\cos\left(fx+e\right)\sin\left(fx+e\right)},-\frac{3\,a\sqrt{-b}\,\arctan\left(\frac{\left(\left(a-2\,b\right)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{2\left(\left(ab-b^2\right)\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+2\left(\left(2\,a+b\right)\cos\left(fx+e\right)^2-b\right)\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{4\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 +
8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + b)*cos(f*x + e)^2 - b)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x
+ e)), -1/4*(3*a*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b -
 b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) + 2*((
2*a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/(f*cos(f*x + e)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*csc(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

3.114 $\int \csc^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=162

$$\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}-\frac{(3a+2b)\,\mathrm{co}}{}$$

```
[Out] 1/2*(3*a+2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/
f+1/2*b*(3*a+2*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/a/f-1/3*(3*a+2*b)*cot
(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/a/f-1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(5
/2)/a/f
```

Rubi [A]
time = 0.09, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3744, 464, 283, 201, 223, 212}

$$\frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af}-\frac{(3a+2b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/(2*f) + (b*(3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*
f) - ((3*a + 2*b)*Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(3*a*f) - (Cot
[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/(3*a*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+bx^2)^{3/2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af} + \frac{(3a+2b)\text{Subst}\left(\int \frac{(a+ \cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{(3a+2b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} - \frac{\cot^3(e+fx)\cdots}{\cdots}$$

$$= \frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+2b)\cot(e \cdots}{\cdots}$$

$$= \frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+2b)\cot(e \cdots}{\cdots}$$

$$= \frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+2b)\,t\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 1.96, size = 177, normalized size = 1.09

$$\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-4(a+2b)\cot(e+fx)-2a\cot(e+fx)\csc^2(e+fx)+\frac{3\sqrt{2}(3a+2b)\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)}{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}+3b\tan(e+fx)\right)}{6\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-4*(a + 2*b)*Cot[
e + f*x] - 2*a*Cot[e + f*x]*Csc[e + f*x]^2 + (3*Sqrt[2]*(3*a + 2*b)*Cot[e +
 f*x]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]/Sqrt[2]], 1])/Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b] + 3*b*Tan[e + f*x]))/(6*Sqrt[2]*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 4594, normalized size = 28.36

method	result	size
default	Expression too large to display	4594

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a*b+3*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b+12*2^(1/2)*((I*cos(f
*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b
)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(
a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi(
(cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*b^2+9*sin(f
*x+e)*cos(f*x+e)^5*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/
2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b-18*sin(f*x+e)*cos(f*x+e)^5*2^(1/2)*((
I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f
*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*Elli
pticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),
1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1
/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b+9*sin(f*x+e)*cos(f*x+e)^
4*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+
e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b
)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a
)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/
sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*
b^2)/a^2)^(1/2))*a*b-18*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*((I*cos(f*x+e)*b^(1
/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x
+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e
)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b-3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/
2)*b^2-9*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)
-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)
*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b
*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(
```

```
1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b+18*sin(f*x+e)*cos(f*x+e
)^3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*
x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a
-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)
/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/
2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2
)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b-9*sin(f*x+
e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^
(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e
)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(c
os(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-
2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*
a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b+18*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*c
os(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+
e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*Ellipti
cPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)
/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b-7*((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)*cos(f*x+e)^6*a*b+6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1
/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1
/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*
a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*
b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^5*
b^2-12*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)
*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)
+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2...
```

Maxima [A]
time = 0.28, size = 187, normalized size = 1.15

$$\frac{9\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)+6\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)+9\,\sqrt{b\tan\left(fx+e\right)^2+a}\,b\tan\left(fx+e\right)+\frac{6\,\sqrt{b\tan\left(fx+e\right)^2+a}\,b^2\tan(fx+e)}{a}-\frac{6\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}{\tan(fx+e)}-\frac{4\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}b}{a\tan(fx+e)}-\frac{2\left(b\tan(fx+e)^2+a\right)^{\frac{5}{2}}}{a\tan(fx+e)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(9*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) + 6*b^(3/2)*arcsinh(b*ta
n(f*x + e)/sqrt(a*b)) + 9*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e) + 6*sqr
t(b*tan(f*x + e)^2 + a)*b^2*tan(f*x + e)/a - 6*(b*tan(f*x + e)^2 + a)^(3/2)
/tan(f*x + e) - 4*(b*tan(f*x + e)^2 + a)^(3/2)*b/(a*tan(f*x + e)) - 2*(b*ta
n(f*x + e)^2 + a)^(5/2)/(a*tan(f*x + e)^3))/f
```

Fricas [A]
time = 2.74, size = 531, normalized size = 3.28

$$\left[\frac{3\left((3a+2b)\cos(fx+e)^3-(3a+2b)\cos(fx+e)\right)\sqrt{b}\log\left(\frac{(a^2-8ab+8b^2)\cos(fx+e)^4+8(ab-2b^2)\cos(fx+e)^2+4\left((a-2b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)\sin(fx+e)-4\left((4a+11b)\cos(fx+e)^4-2(3a+7b)\cos(fx+e)^2+3b\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24\left(f\cos(fx+e)^3-f\cos(fx+e)\right)\sin(fx+e)}, -\frac{3\left((3a+2b)\cos(fx+e)^3-(3a+2b)\cos(fx+e)\right)\sqrt{-b}\arctan\left(\frac{\left((a-2b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left((ab-b^2)\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin(fx+e)+2\left((4a+11b)\cos(fx+e)^4-2(3a+7b)\cos(fx+e)^2+3b\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12\left(f\cos(fx+e)^3-f\cos(fx+e)\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((3*a + 2*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt(b)*lo
g(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 +
4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*
x + e) - 4*((4*a + 11*b)*cos(f*x + e)^4 - 2*(3*a + 7*b)*cos(f*x + e)^2 + 3*
b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 -
f*cos(f*x + e))*sin(f*x + e)), -1/12*(3*((3*a + 2*b)*cos(f*x + e)^3 - (3*a
+ 2*b)*cos(f*x + e))*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a
*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((4*a + 11*
b)*cos(f*x + e)^4 - 2*(3*a + 7*b)*cos(f*x + e)^2 + 3*b)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x + e))*sin(f*
x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2}}{{\sin\left(e+f\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

3.115 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=196

$$\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}-\frac{(3a+4b)\,c}{}$$

```
[Out] 1/2*(3*a+4*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/
f+1/2*b*(3*a+4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/a/f-1/3*(3*a+4*b)*cot
(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/a/f-2/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(5
/2)/a/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(5/2)/a/f
```

Rubi [A]
time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3744, 473, 464, 283, 201, 223, 212}

$$\frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}+\frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5af}-\frac{2\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af}-\frac{(3a+4b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/(2*f) + (b*(3*a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*
f) - ((3*a + 4*b)*Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(3*a*f) - (2*C
ot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/(3*a*f) - (Cot[e + f*x]^5*(a +
b*Tan[e + f*x]^2)^(5/2))/(5*a*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+bx^2)^{3/2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5af} + \frac{\text{Subst}\left(\int \frac{(10a+5ax^2)(a}{x^4}\right.}{}$$

$$= -\frac{2\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af} - \frac{\cot^5(e+fx)(a+}{5a}$$

$$= -\frac{(3a+4b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} - \frac{2\cot^3(e+}{}$$

$$= \frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+4b)\cot(}{}$$

$$= \frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+4b)\cot(}{}$$

$$= \frac{\sqrt{b}\,(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+4b)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.23, size = 213, normalized size = 1.09

$$\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-\frac{2(8a^2+34ab+3b^2)\cot(e+fx)}{a} - 4(2a+3b)\cot(e+fx)\csc^2(e+fx) - 6a\cot(e+fx)\csc^4(e+fx) + \frac{15\sqrt{2}(3a+4b)\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)}{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}} + 15b\tan(e+fx)\right)}{30\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((-2*(8*a^2 + 34*a
*b + 3*b^2)*Cot[e + f*x])/a - 4*(2*a + 3*b)*Cot[e + f*x]*Csc[e + f*x]^2 - 6
*a*Cot[e + f*x]*Csc[e + f*x]^4 + (15*Sqrt[2]*(3*a + 4*b)*Cot[e + f*x]*Ellip
ticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqr
```

```
t[2]], 1])/Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b] + 15
*b*Tan[e + f*x]))/(30*Sqrt[2]*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.48, size = 6988, normalized size = 35.65

method	result	size
default	Expression too large to display	6988

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] result too large to display
```

Maxima [A]
time = 0.30, size = 216, normalized size = 1.10

$$\frac{45\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)+60\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)+45\sqrt{b\tan\left(fx+e\right)^2+a}\,b\tan\left(fx+e\right)+\frac{60\sqrt{b\tan\left(fx+e\right)^2+a}\,b^2\tan(fx+e)}{a}-\frac{30\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}{\tan(fx+e)}-\frac{40\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}b}{a\tan(fx+e)}-\frac{20\left(b\tan(fx+e)^2+a\right)^{\frac{5}{2}}}{a\tan(fx+e)^3}-\frac{6\left(b\tan(fx+e)^2+a\right)^{\frac{5}{2}}}{a\tan(fx+e)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] 1/30*(45*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) + 60*b^(3/2)*arcsinh(b
*tan(f*x + e)/sqrt(a*b)) + 45*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e) + 6
0*sqrt(b*tan(f*x + e)^2 + a)*b^2*tan(f*x + e)/a - 30*(b*tan(f*x + e)^2 + a)
^(3/2)/tan(f*x + e) - 40*(b*tan(f*x + e)^2 + a)^(3/2)*b/(a*tan(f*x + e)) -
20*(b*tan(f*x + e)^2 + a)^(5/2)/(a*tan(f*x + e)^3) - 6*(b*tan(f*x + e)^2 +
a)^(5/2)/(a*tan(f*x + e)^5))/f
```

Fricas [A]
time = 5.95, size = 695, normalized size = 3.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/120*(15*((3*a^2 + 4*a*b)*cos(f*x + e)^5 - 2*(3*a^2 + 4*a*b)*cos(f*x + e)
^3 + (3*a^2 + 4*a*b)*cos(f*x + e))*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f
*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 +
2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((16*a^2 + 83*a*b +
 6*b^2)*cos(f*x + e)^6 - (40*a^2 + 193*a*b + 12*b^2)*cos(f*x + e)^4 + (30*a
^2 + 125*a*b + 6*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/((a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*co
```

```
s(f*x + e))*sin(f*x + e)), -1/60*(15*((3*a^2 + 4*a*b)*cos(f*x + e)^5 - 2*(3
*a^2 + 4*a*b)*cos(f*x + e)^3 + (3*a^2 + 4*a*b)*cos(f*x + e))*sqrt(-b)*arcta
n(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin
(f*x + e)))*sin(f*x + e) + 2*((16*a^2 + 83*a*b + 6*b^2)*cos(f*x + e)^6 - (4
0*a^2 + 193*a*b + 12*b^2)*cos(f*x + e)^4 + (30*a^2 + 125*a*b + 6*b^2)*cos(f
*x + e)^2 - 15*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a*
f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^6,x)
[Out] int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^6, x)
```

3.116 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=144

$$-\frac{(15a^2-10ab+3b^2)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^3f}+\frac{2(5a-3b)\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx}}{15(a-b)^2f}$$

```
[Out] -1/15*(15*a^2-10*a*b+3*b^2)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^3/f
+2/15*(5*a-3*b)*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^2/f-1/5*cos(f
*x+e)^5*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]
time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3745, 473, 464, 270}

$$-\frac{(15a^2-10ab+3b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{15f(a-b)^3}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{5f(a-b)}+\frac{2(5a-3b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{15f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/15*((15*a^2 - 10*a*b + 3*b^2)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2
])/((a - b)^3*f) + (2*(5*a - 3*b)*Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x
]^2])/(15*(a - b)^2*f) - (Cos[e + f*x]^5*Sqrt[a - b + b*Sec[e + f*x]^2])/(5
*(a - b)*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{5(a-b)f} + \frac{\text{Subst}\left(\int \frac{-2(5a-3b)+5(a-b)x^2}{x^4\sqrt{a-b+bx^2}}\,dx, x,\right.}{5(a-b)f}$$

$$= \frac{2(5a-3b)\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^2f} - \frac{\cos^5(e+fx)\sqrt{a-b+}}{5(a-b)}$$

$$= -\frac{\left(15a^2-10ab+3b^2\right)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^3f} + \frac{2(5a-3b)\,\text{co}}{}$$

Mathematica [A]
time = 42.12, size = 112, normalized size = 0.78

$$\frac{\cos(e+fx)\left(-89a^2+34ab-9b^2+4\left(7a^2-10ab+3b^2\right)\cos(2(e+fx))-3(a-b)^2\cos(4(e+fx))\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{120\sqrt{2}\,(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-89*a^2 + 34*a*b - 9*b^2 + 4*(7*a^2 - 10*a*b + 3*b^2)*Cos[2*
(e + f*x)] - 3*(a - b)^2*Cos[4*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e +
 f*x)])*Sec[e + f*x]^2])/(120*Sqrt[2]*(a - b)^3*f)
```

Maple [A]
time = 0.42, size = 169, normalized size = 1.17

method	result
default	$-\frac{(a(\cos^2(fx+e))-(\cos^2(fx+e))b+b)(3(\cos^4(fx+e))a^2-6(\cos^4(fx+e))ab+3(\cos^4(fx+e))b^2-10(\cos^2(fx+e))a^2+16(\cos^2(fx+e)}{15f\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{\cos(fx+e)^2}}\cos(fx+e)(a-b)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(3*cos(f*x+e)^4*a^2-6*cos(f*x+e)^
4*a*b+3*cos(f*x+e)^4*b^2-10*cos(f*x+e)^2*a^2+16*cos(f*x+e)^2*a*b-6*cos(f*x+
e)^2*b^2+15*a^2-10*a*b+3*b^2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)
^2)^(1/2)/cos(f*x+e)/(a-b)^3
```

Maxima [A]
time = 0.31, size = 226, normalized size = 1.57

$$-\frac{\frac{15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a-b}+\frac{3\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-10\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3+15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b^2\cos(fx+e)}{a^3-3a^2b+3ab^2-b^3}-\frac{10\left(\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^2-2ab+b^2}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a - b) + (3*(a - b +
 b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a - b + b/cos(f*x + e)^2)^(3/
2)*b*cos(f*x + e)^3 + 15*sqrt(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e))/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f
*x + e)^3 - 3*sqrt(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^2 - 2*a*b +
 b^2))/f
```

Fricas [A]
time = 2.25, size = 129, normalized size = 0.90

$$-\frac{\left(3\left(a^2-2ab+b^2\right)\cos(fx+e)^5-2\left(5a^2-8ab+3b^2\right)\cos(fx+e)^3+\left(15a^2-10ab+3b^2\right)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{15\left(a^3-3a^2b+3ab^2-b^3\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 2*(5*a^2 - 8*a*b + 3*b^2)*cos
(f*x + e)^3 + (15*a^2 - 10*a*b + 3*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. $2(138) = 276$.
time = 1.34, size = 1376, normalized size = 9.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 256/15*(5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a - 10*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(3/2) + 15*(sqrt(a)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*b - 13*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2 - 14*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))^5*a*b + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^5*b^2 + 40*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a))^4*a^(5/2) - 55*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*
a^(3/2)*b - 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3 +
60*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b - 40*(sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^2 - 50*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2) + 65*(sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b + 45*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))*a^4 - 110*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))*a^3*b + 60*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))*a^2*b^2 - 12*a^(9/2) + 39*a^(7/2)*b - 32*a^(5/2)*b^2)/(((sqrt(a)*tan(1/2
```

```
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*
f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a + 4*b)^5*f*sgn(tan(1/2*f*x + 1/2*e)^2 -
1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(e+fx)^5}{\sqrt{b\tan(e+fx)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.117 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=88

$$-\frac{(3a-b)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)^2f}+\frac{\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)f}$$

```
[Out] -1/3*(3*a-b)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^2/f+1/3*cos(f*x+e)
^3*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]

time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {3745, 464, 270}

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)}-\frac{(3a-b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/3*((3*a - b)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/((a - b)^2*f)
+ (Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a - b)*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
```

```
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)f} + \frac{(3a-b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-b+bx^2}}\,d\right.}{3(a-b)f}$$

$$= -\frac{(3a-b)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)^2f} + \frac{\cos^3(e+fx)\sqrt{a-b+b\,\text{se}}}{3(a-b)f}$$

Mathematica [A]
time = 1.54, size = 74, normalized size = 0.84

$$\frac{\cos(e+fx)(-5a+b+(a-b)\cos(2(e+fx)))\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{6\sqrt{2}(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-5*a + b + (a - b)*Cos[2*(e + f*x)])*Sqrt[(a + b + (a - b)*C
os[2*(e + f*x)])*Sec[e + f*x]^2])/(6*Sqrt[2]*(a - b)^2*f)
```

Maple [A]
time = 0.29, size = 104, normalized size = 1.18

method	result	size
default	$\frac{(a(\cos^2(fx+e))-(\cos^2(fx+e))b+b)(a(\cos^2(fx+e))-(\cos^2(fx+e))b-3a+b)}{3f\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{\cos(fx+e)^2}}\cos(fx+e)(a-b)^2}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b-3*a+
b)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(a-b)^
2
```

Maxima [A]
time = 0.36, size = 112, normalized size = 1.27

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{a-b}-\frac{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos(fx+e)}{a^2-2\,ab+b^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a - b) - ((a - b + b/c
os(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a - b + b/cos(f*x + e)^2)*b*co
s(f*x + e))/(a^2 - 2*a*b + b^2))/f
```

Fricas [A]
time = 2.93, size = 79, normalized size = 0.90

$$\frac{\left((a-b)\cos\left(fx+e\right)^3-(3\,a-b)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\,(a^2-2\,ab+b^2)\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - b)*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^2 - 2*a*b + b^2)*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(84) = 168.
time = 1.20, size = 205, normalized size = 2.33

$$\frac{\left(3\,a\sqrt{b}-b^{\frac{3}{2}}\right)\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right))}{3\,(a^2|f|-2\,ab|f|+b^2|f|)}+\frac{\left(a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b\right)^{\frac{3}{2}}f^2}{3\,(a|f|\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-b|f|\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right))(af^2-bf^2)}-\frac{\sqrt{a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b}\;a}{a^2|f|\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)-2\,ab|f|\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)+b^2|f|\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a*sqrt(b) - b^(3/2))*sgn(f)*sgn(cos(f*x + e))/(a^2*abs(f) - 2*a*b*ab
s(f) + b^2*abs(f)) + 1/3*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*f^
2/((a*abs(f)*sgn(f)*sgn(cos(f*x + e)) - b*abs(f)*sgn(f)*sgn(cos(f*x + e)))*
(a*f^2 - b*f^2)) - sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a/(a^2*abs
(f)*sgn(f)*sgn(cos(f*x + e)) - 2*a*b*abs(f)*sgn(f)*sgn(cos(f*x + e)) + b^2*
abs(f)*sgn(f)*sgn(cos(f*x + e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.118 $\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=37

$$-\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{(a-b)f}$$

```
[Out] -cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]

time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3745, 270}

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/((a - b)*f))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{(a-b)f}\end{aligned}$$

Mathematica [A]
time = 0.62, size = 52, normalized size = 1.41

$$\frac{\cos(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{\sqrt{2}\,(-a+b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqr
t[2]*(-a + b)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(35) = 70$.
time = 0.22, size = 78, normalized size = 2.11

method	result	size
default	$-\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{f\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{\cos(fx+e)^2}}\,\cos(fx+e)(a-b)}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)*(a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(a-b)
```

Maxima [A]
time = 0.30, size = 37, normalized size = 1.00

$$-\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,\cos(fx+e)}{(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*f)
```

Fricas [A]
time = 2.53, size = 48, normalized size = 1.30

$$-\frac{\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\cos(fx+e)}{(a-b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{(e+fx)}}{\sqrt{a+b\tan^2{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(37) = 74.
time = 1.12, size = 88, normalized size = 2.38

$$\frac{\sqrt{b}\,\mathrm{sgn}(f)\,\mathrm{sgn}(\cos{(fx+e)})}{a|f|-b|f|}-\frac{\sqrt{a\cos{(fx+e)}^2-b\cos{(fx+e)}^2+b}}{a|f|\mathrm{sgn}\,(f)\,\mathrm{sgn}\,(\cos{(fx+e)})-b|f|\mathrm{sgn}\,(f)\,\mathrm{sgn}\,(\cos{(fx+e)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b)*sgn(f)*sgn(cos(f*x + e))/(a*abs(f) - b*abs(f)) - sqrt(a*cos(f*x + e
)^2 - b*cos(f*x + e)^2 + b)/(a*abs(f)*sgn(f)*sgn(cos(f*x + e)) - b*abs(f)*s
gn(f)*sgn(cos(f*x + e)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin{(e+f\,x)}}{\sqrt{b\tan{(e+f\,x)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.119 $\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/f/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3745, 385, 213}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(Sqrt[a]*f
))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 221 vs. 2(42) = 84.
time = 2.40, size = 221, normalized size = 5.26

$$\frac{\cos(e+fx)\left(2\tanh^{-1}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\frac{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)+\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{2\sqrt{a}\,f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 +
 a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]
^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]])
*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]
)/(2*Sqrt[a]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(36) = 72.
time = 0.32, size = 351, normalized size = 8.36

method	result
default	$\frac{\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{(\cos(fx+e)+1)^2}}\left(\ln\left(-\frac{2(\cos(fx+e)-1)\left(\cos(fx+e)\sqrt{a}\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{(\cos(fx+e)+1)^2}}-\cos(fx+e)a+\right.}{\sin(fx+e)^2\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*(ln(-2*(co
s(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))+ln(-4*(cos(f*x+e
)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)-1)))*sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 5.02, size = 142, normalized size = 3.38

$$\left[\frac{\log\left(-\frac{2\left((a-b)\cos(fx+e)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)}{2\sqrt{a}\,f}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))/(sqrt(a)
*f), sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)/a)/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{b\,{\tan\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2)), x)
```

3.120 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af}$$

```
[Out] -1/2*(a-b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(3/2)/f
-1/2*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3745, 482, 12, 385, 213}

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/2*((a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/(a^(3/2)*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])
/(2*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{a-\cdots}{(-1+x^2)\sqrt{a-\cdots}}\cdots\right.}{\cdots}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af} + \frac{(a-b)\text{Subst}\left(\int \frac{\cdots}{(-1+\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{-1+a\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{\cdots}}{2af\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 367 vs. 2(91) = 182.
time = 3.19, size = 367, normalized size = 4.03

$$\frac{\cot^2(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(2(-a+b)\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)-\left(-2(a-b)\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)+\sqrt{2}\sqrt{a}\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)}\right)\sec(e+fx)+8(a-b)\tanh^{-1}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\frac{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)\sec(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)\right)}{4a^{3/2}f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Cot[e + f*x]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(2*
(-a + b)*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x
)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] - (-2*(a - b)*Log[a - 2*b - a*Tan[
(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x
)/2]^2)^2]] + Sqrt[2]*Sqrt[a]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(
e + f*x)/2]^4])*Sec[e + f*x] + 8*(a - b)*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[
4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]]*Sec[e + f*
x]*Sin[(e + f*x)/2]^2))/(4*a^(3/2)*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)]
)*Sec[(e + f*x)/2]^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2800 vs. $2(79) = 158$.
time = 0.38, size = 2801, normalized size = 30.78

method	result	size
default	Expression too large to display	2801

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*(((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(co
s(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^3*a^2
-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+
e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos
(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)^3*a*b+((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(
1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(
f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)^3*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(co
s(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*c
os(f*x+e)^3*a*b-2*cos(f*x+e)^2*a^(5/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*
x+e)^2/a^(1/2))*cos(f*x+e)^2*a^2-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2
/a^(1/2))*cos(f*x+e)^2*a*b+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
```

```
)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(co
s(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)^2*a^2-
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e
)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)^2*a*b+2*cos(f*x+e)^2*a^(3/2)*b-((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)-
1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)*a^2+((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(cos(
f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*cos(f*x+e)*a*b-((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(co
s(f*x+e)-1))*cos(f*x+e)*a^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(c
os(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)*a*b-(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)
-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^2+((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(
1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*
a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a
^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a*b-((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos
(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*a^2+((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^
(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos
(f*x+e)+b)/(cos(f*x+e)-1))*a*b-2*a^(3/2)*b)*sin(f*x+e)^2/(cos(f*x+e)-1)^2/c
os(f*x+e)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e
)+1)^2/a^(5/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [**A**]
time = 2.39, size = 302, normalized size = 3.32

$$\left[\frac{2 a \sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)-((a-b)\cos(fx+e)^2-a+b)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos(fx+e)^2+2\sqrt{a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)}{4\left(a^2 f\cos(fx+e)^2-a^2 f\right)}, \frac{((a-b)\cos(fx+e)^2-a+b)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a}\right)+a\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{2\left(a^2 f\cos(fx+e)^2-a^2 f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) -
((a - b)*cos(f*x + e)^2 - a + b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2
*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
 + b)/(cos(f*x + e)^2 - 1)))/(a^2*f*cos(f*x + e)^2 - a^2*f), 1/2*(((a - b)*
cos(f*x + e)^2 - a + b)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + a*sqrt(((a - b)*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*cos(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2)), x)
```

3.121 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=143

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{5/2}f}-\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f}$$

```
[Out] -3/8*(a-b)^2*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(5/2)
/f-1/8*(5*a-3*b)*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a^2/f-1/4
*cot(f*x+e)^3*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3745, 481, 541, 12, 385, 213}

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{5/2}f}-\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{8a^2f}-\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-3*(a - b)^2*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/(8*a^(5/2)*f) - ((5*a - 3*b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a - b + b*Se
c[e + f*x]^2])/(8*a^2*f) - (Cot[e + f*x]^3*Csc[e + f*x]*Sqrt[a - b + b*Sec[
e + f*x]^2])/(4*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3\sqrt{a-b+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{-a+b-2(2}{(-1+x^2)^2\sqrt{a-}}\right.}{}$$

$$= -\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)}{}$$

$$= -\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)}{}$$

$$= -\frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)}{}$$

$$= -\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a-3b)\cot(e+fx)\csc(}{}$$

Mathematica [A]
time = 4.27, size = 273, normalized size = 1.91

$$\frac{\left(-\sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\left(3a-3b+2a\csc^2(e+fx)\right)+\frac{3(a-b)^2\cos(e+fx)\left(2\tanh^{-1}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\frac{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)+\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)}}\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{16a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-(Sqrt[2]*Sqrt[a]*Cot[e + f*x]*Csc[e + f*x]*(3*a - 3*b + 2*a*Csc[e + f*x]
^2)) + (3*(a - b)^2*Cos[e + f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*T
an[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b -
 a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(
e + f*x)/2]^2)^2]])*Sec[(e + f*x)/2]^2)/Sqrt[(a + b + (a - b)*Cos[2*(e + f*
x)])*Sec[(e + f*x)/2]^4])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f
*x]^2])/(16*a^(5/2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6333 vs. 2(127) = 254.
time = 0.34, size = 6334, normalized size = 44.29

method	result	size
default	Expression too large to display	6334

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^5/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 4.56, size = 461, normalized size = 3.22

$$\left[\frac{3\left(\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^4-2\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^2+a^2-2ab+b^2\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos(fx+e)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)+2\left(3\left(a^2-ab\right)\cos\left(fx+e\right)^3-\left(5a^2-3ab\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{16\left(a^3f\cos\left(fx+e\right)^4-2a^3f\cos\left(fx+e\right)^2+a^3f\right)}, \frac{3\left(\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^4-2\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^2+a^2-2ab+b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{a}\right)+\left(3\left(a^2-ab\right)\cos\left(fx+e\right)^3-\left(5a^2-3ab\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{8\left(a^3f\cos\left(fx+e\right)^4-2a^3f\cos\left(fx+e\right)^2+a^3f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 - 2*a*b + b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sq
rt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a +
b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 - a*b)*cos(f*x + e)^3 - (5*a^2 - 3*a*b
)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*f*c
os(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/8*(3*((a^2 - 2*a*b + b^2
)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*b + b^2
)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)/a) + (3*(a^2 - a*b)*cos(f*x + e)^3 - (5*a^2 - 3*a*b)*cos(f*x
 + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*f*cos(f*x +
e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(csc(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(135) = 270$.
time = 1.18, size = 906, normalized size = 6.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] 1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a)*(tan(1/2*f*x + 1/2*e)^2/a + 3*(3*a - 2*b)/a^2) - 24
*(a^2 - 2*a*b + b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 12*(a^(5/2) - 2*a^(3/2)*b + sqrt(a)*b^2)*
log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a + a^(3/2) -
 2*sqrt(a)*b))/a^3 + 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))^3*a^2 - 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b +
 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 - 3*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) - 6*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 + 16*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))*a^2*b - 10*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))*a*b^2 + 5*a^(7/2) - 4*a^(5/2)*b)/(((sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^2 - a)^2*a^2))/(f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^5\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2)),x)
[Out] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2)), x)
```

3.122 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=146

$$\frac{3a^2\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{5/2}f}-\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2f}+\frac{\cos^3(e+}{}$$

```
[Out] 3/8*a^2*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)
/f-1/8*(5*a-2*b)*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)^2/f+1
/4*cos(f*x+e)^3*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]
time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3744, 481, 541, 12, 385, 209}

$$\frac{3a^2\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{5/2}}+\frac{\sin(e+fx)\cos^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f(a-b)}-\frac{(5a-2b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(8*(a
 - b)^(5/2)*f) - ((5*a - 2*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e +
f*x]^2])/(8*(a - b)^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e +
f*x]^2])/(4*(a - b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)f} - \frac{\text{Subst}\left(\int \frac{a-2(2a-b)x^2}{(1+x^2)^2\sqrt{a+bx^2}}\right.}{4(a-b}$$

$$= -\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin}{}$$

$$= -\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin}{}$$

$$= -\frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin}{}$$

$$= \frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{5/2}f} - \frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{}}{8(a-b)^2f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 4.37, size = 314, normalized size = 2.15

$$-\frac{\left((a-b)\left(7a^2+8ab-3b^2+2(3a^2-5ab+2b^2)\cos(2(e+fx))-(a-b)^2\cos(4(e+fx))\right)+6\sqrt{2}a^2(-a+b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+6\sqrt{2}a^3\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sec^2(e+fx)\sin(2(e+fx))}{32\sqrt{2}(a-b)^3f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/32*(((a - b)*(7*a^2 + 8*a*b - 3*b^2 + 2*(3*a^2 - 5*a*b + 2*b^2)*Cos[2*(e
 + f*x)] - (a - b)^2*Cos[4*(e + f*x)]) + 6*Sqrt[2]*a^2*(-a + b)*Sqrt[((a +
b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 6*Sqrt[2]
*a^3*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticPi
[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)^3*f*
Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 1169, normalized size = 8.01

method	result
default	$\frac{\sin(fx+e)\left(2\sqrt{\frac{2i\sqrt{b}\,\sqrt{a-b}+a-2b}{a}}\,(\cos^5(fx+e))a^2-4\sqrt{\frac{2i\sqrt{b}\,\sqrt{a-b}+a-2b}{a}}\,(\cos^5(fx+e))ab+2\sqrt{\frac{2i\sqrt{b}\,\sqrt{a-}}{a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*sin(f*x+e)*(2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*
a^2-4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*a*b+2*((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^5*b^2-2*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a^2+4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)*cos(f*x+e)^4*a*b-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+
e)^4*b^2+6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*
x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a
-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2*si
n(f*x+e)-3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*
x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2
-8*a*b+8*b^2)/a^2)^(1/2))*a^2*sin(f*x+e)-5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)*cos(f*x+e)^3*a^2+9*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(
f*x+e)^3*a*b-4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^3*b^2+5
*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^2-9*((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b+4*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)*cos(f*x+e)^2*b^2-5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
*cos(f*x+e)*a*b+2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)*b^2+
5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-2*((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)*b^2)/(cos(f*x+e)-1)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos
(f*x+e)^2)^(1/2)/cos(f*x+e)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)
^2
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(138) = 276$.
time = 5.70, size = 817, normalized size = 5.60

$$\left[-\frac{3 a^{2} \sqrt{-a+b} \log\left(128 \left(a^{4}-4 a^{3} b+6 a^{2} b^{2}-4 a b^{3}+b^{4}\right) \cos\left(fx+e\right)^{8}-256 \left(a^{4}-5 a^{3} b+9 a^{2} b^{2}-7 a b^{3}+2 b^{4}\right) \cos\left(fx+e\right)^{6}+32 \left(5 a^{4}-34 a^{3} b+77 a^{2} b^{2}-72 a b^{3}+24 b^{4}\right) \cos\left(fx+e\right)^{4}+a^{4}-32 a^{3} b+160 a^{2} b^{2}-256 a b^{3}+128 b^{4}-32 \left(a^{4}-11 a^{3} b+34 a^{2} b^{2}-40 a b^{3}+16 b^{4}\right) \cos\left(fx+e\right)^{2}+8 \left(16 \left(a^{3}-3 a^{2} b+3 a b^{2}-b^{3}\right) \cos\left(fx+e\right)^{7}-24 \left(a^{3}-4 a^{2} b+5 a b^{2}-2 b^{3}\right) \cos\left(fx+e\right)^{5}+2 \left(5 a^{3}-29 a^{2} b+48 a b^{2}-24 b^{3}\right) \cos\left(fx+e\right)^{3}-\left(a^{3}-10 a^{2} b+24 a b^{2}-16 b^{3}\right) \cos\left(fx+e\right)\right) \sqrt{-a+b} \sqrt{\frac{(a-b) \cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}} \sin\left(fx+e\right)\right)-8 \left(2 \left(a^{2}-2 a b+b^{2}\right) \cos\left(fx+e\right)^{3}-\left(5 a^{2}-7 a b+2 b^{2}\right) \cos\left(fx+e\right)\right) \sqrt{\frac{(a-b) \cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}} \sin\left(fx+e\right)}{64 \left(a^{3}-3 a^{2} b+3 a b^{2}-b^{3}\right) f}, \frac{3 \sqrt{a-b}\, a^{2} \arctan\left(-\frac{\left(8 \left(a^{2}-2 a b+b^{2}\right) \cos\left(fx+e\right)^{5}-8 \left(a^{2}-3 a b+2 b^{2}\right) \cos\left(fx+e\right)^{3}+\left(a^{2}-8 a b+8 b^{2}\right) \cos\left(fx+e\right)\right) \sqrt{a-b} \sqrt{\frac{(a-b) \cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{4 \left(2 \left(a^{3}-3 a^{2} b+3 a b^{2}-b^{3}\right) \cos\left(fx+e\right)^{4}-a^{2} b+3 a b^{2}-2 b^{3}-\left(a^{3}-6 a^{2} b+9 a b^{2}-4 b^{3}\right) \cos\left(fx+e\right)^{2}\right) \sin\left(fx+e\right)}\right)+4 \left(2 \left(a^{2}-2 a b+b^{2}\right) \cos\left(fx+e\right)^{3}-\left(5 a^{2}-7 a b+2 b^{2}\right) \cos\left(fx+e\right)\right) \sqrt{\frac{(a-b) \cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}} \sin\left(fx+e\right)}{32 \left(a^{3}-3 a^{2} b+3 a b^{2}-b^{3}\right) f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*a^2*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(
f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x
 + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11
*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*
cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 -
(a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a -
b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*(a^2 - 2*a*b +
b^2)*cos(f*x + e)^3 - (5*a^2 - 7*a*b + 2*b^2)*cos(f*x + e))*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2
- b^3)*f), 1/32*(3*sqrt(a - b)*a^2*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f
*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*
cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b
^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(
2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 7*a*b + 2*b^2)*cos(f*x + e)
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{4}\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.123 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=93

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{3/2}f}-\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f}$$

```
[Out] 1/2*a*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f
-1/2*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3744, 482, 12, 385, 209}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{3/2}}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (a*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*(a - b
)^(3/2)*f) - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a -
 b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f} + \frac{\text{Subst}\left(\int \frac{a}{(1+x^2)\sqrt{a+bx^2}}\, \cdots\right)}{2(a-b) \cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f} + \frac{a\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}} \cdots\right)}{2(a-b \cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f} + \frac{a\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x \cdots\right)}{2(a- \cdots}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+ \cdots}}{2(a-b)f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 3.34, size = 270, normalized size = 2.90

$$-\frac{\left((a-b)(a+b+(a-b)\cos(2(e+fx)))+\sqrt{2}\,a(-a+b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+\sqrt{2}\,a^2\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sec^2(e+fx)\sin(2(e+fx))}{4\sqrt{2}\,(a-b)^2 f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*(((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*a*(-a + b)*Sqrt
[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + Sq
rt[2]*a^2*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Ellip
ticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)
^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 795, normalized size = 8.55

method	result
default	$-\frac{\sin(fx+e)\left(\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)\sqrt{b}\,\sqrt{a-b}-i\sqrt{b}\,\sqrt{a-b}+\cos(fx+e)a-b\cos(fx+e)+b}{(\cos(fx+e)+1)a}}\,\sqrt{-\frac{2\left(i\cos(fx+e)\sqrt{b}\right.}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*sin(f*x+e)*(2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/
2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*
x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2
)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)*cos(f*x+e)^3*a-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*
x+e)^3*b-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a+((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b+((2*I*b^(1/2)*(a-b)^(1/2)+a
```

```
-2*b)/a)^(1/2)*cos(f*x+e)*b-((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b)/(c
os(f*x+e)-1)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x
+e)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(86) = 172$.
time = 2.74, size = 723, normalized size = 7.77

$$\left[-\frac{8(a-b)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - a\sqrt{-a+b}\log\left(128(a^4-4a^3b+6a^2b^2-4ab^3+b^4)\cos(fx+e)^8 - 256(a^4-5a^3b+9a^2b^2-7ab^3+2b^4)\cos(fx+e)^6 + 32(5a^4-34a^3b+77a^2b^2-72ab^3+24b^4)\cos(fx+e)^4 + a^4 - 32a^3b + 160a^2b^2 - 256ab^3 + 128b^4 - 32(a^4-11a^3b+34a^2b^2-40ab^3+16b^4)\cos(fx+e)^2 - 8\left(16(a^3-3a^2b+3ab^2-b^3)\cos(fx+e)^7 - 24(a^3-4a^2b+5ab^2-2b^3)\cos(fx+e)^5 + 2(5a^3-29a^2b+48ab^2-24b^3)\cos(fx+e)^3 - (a^3-10a^2b+24ab^2-16b^3)\cos(fx+e)\right)\sqrt{-a+b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)\right)}{16(a^2-2ab+b^2)f}, -\frac{4(a-b)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \sqrt{a-b}\,a\arctan\left(-\frac{\left(8(a^2-2ab+b^2)\cos(fx+e)^5 - 8(a^2-3ab+2b^2)\cos(fx+e)^3 + (a^2-8ab+8b^2)\cos(fx+e)\right)\sqrt{a-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2(a^3-3a^2b+3ab^2-b^3)\cos(fx+e)^4 - a^2b + 3ab^2 - 2b^3 - (a^3-6a^2b+9ab^2-4b^3)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{8(a^2-2ab+b^2)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)*sin(f*x + e) - a*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*
b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4
)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*
(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2
- 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x
+ e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^2 - 2*a*
b + b^2)*f), -1/8*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)*sin(f*x + e) - sqrt(a - b)*a*arctan(-1/4*(8*(a^2 - 2*a*b +
 b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*
b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*
a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x +
 e))))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.124 $\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3742, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[e + f*x]^2],x]
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Maple [A]
time = 0.08, size = 67, normalized size = 1.46

method	result	size
derivativedivides	$\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b(\tan^2(fx+e))}}\right)}{f\,b^2(a-b)}$	67
default	$\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b(\tan^2(fx+e))}}\right)}{f\,b^2(a-b)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 6.73, size = 131, normalized size = 2.85

$$\left[-\frac{\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)}{2\,(a-b)f},\ \frac{\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{a-b}\,\tan(fx+e)}\right)}{\sqrt{a-b}\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))/((a - b)*f), arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e)))/(sqrt(a - b)*f)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [B]
time = 12.36, size = 40, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\tan(e+f\,x)\,\sqrt{a-b}}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\right)}{f\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] atan((tan(e + f*x)*(a - b)^(1/2))/(a + b*tan(e + f*x)^2)^(1/2))/(f*(a - b)^
(1/2))
```

3.125 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

`[Out] -cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f`

Rubi [A]
time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3744, 270}

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

`[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]`

`[Out] -((Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a*f))`

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 49, normalized size = 1.63

$$-\frac{\cot(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{\sqrt{2}\,af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(S
qrt[2]*a*f))
```

Maple [A]
time = 0.28, size = 57, normalized size = 1.90

method	result	size
default	$-\frac{\sqrt{\frac{a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b}{\cos(fx+e)^2}}\,\cos(fx+e)}{f\sin(fx+e)a}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(
f*x+e)/a
```

Maxima [A]
time = 0.28, size = 32, normalized size = 1.07

$$-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{af\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*tan(f*x + e)^2 + a)/(a*f*tan(f*x + e))
```

Fricas [A]
time = 7.86, size = 53, normalized size = 1.77

$$-\frac{\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f*sin(f*
x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [B]
time = 12.86, size = 36, normalized size = 1.20

$$-\frac{\cot(e+f\,x)\,\sqrt{a+\frac{b\sin\left(e+f\,x\right)^2}{\cos\left(e+f\,x\right)^2}}}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

```
[Out] -(cot(e + f*x)*(a + (b*sin(e + f*x)^2)/cos(e + f*x)^2)^(1/2))/(a*f)
```

3.126 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

```
[Out] -1/3*(3*a-2*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)^3*(
a+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3744, 464, 270}

$$-\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/3*((3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a^2*f) - (Cot[e
 + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
```

```
+ 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{(3a-2b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \right.}{3af}$$

$$= -\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e}}{3af}$$

Mathematica [**A**]
time = 0.45, size = 68, normalized size = 0.92

$$-\frac{\cot(e+fx)\left(2a-2b+a\csc^2(e+fx)\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{3\sqrt{2}\,a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/3*(Cot[e + f*x]*(2*a - 2*b + a*Csc[e + f*x]^2)*Sqrt[(a + b + (a - b)*Cos
[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*a^2*f)
```

Maple [**A**]
time = 0.30, size = 86, normalized size = 1.16

method	result	size
default	$\frac{\left(2a\left(\cos^2(fx+e)\right)-2\left(\cos^2(fx+e)\right)b-3a+2b\right)\sqrt{\frac{a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b}{\cos(fx+e)^2}}\ \cos(fx+e)}{3f\sin(fx+e)^3a^2}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*(2*a*cos(f*x+e)^2-2*cos(f*x+e)^2*b-3*a+2*b)*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)^3/a^2
```

Maxima [A]
time = 0.27, size = 93, normalized size = 1.26

$$-\frac{\frac{3\sqrt{b\tan\left(fx+e\right)^2+a}}{a\tan(fx+e)}-\frac{2\sqrt{b\tan\left(fx+e\right)^2+a}\,b}{a^2\tan(fx+e)}+\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{a\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)) - 2*sqrt(b*tan(f*x + e)
^2 + a)*b/(a^2*tan(f*x + e)) + sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)^3
))/f
```

Fricas [A]
time = 5.27, size = 96, normalized size = 1.30

$$-\frac{\left(2\,(a-b)\cos\left(fx+e\right)^3-(3\,a-2\,b)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a - b)*cos(f*x + e)^3 - (3*a - 2*b)*cos(f*x + e))*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2*f*cos(f*x + e)^2 - a^2*f)*sin(f*x +
 e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^4\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [B]
time = 18.82, size = 145, normalized size = 1.96

$$-\frac{2\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{a+\frac{b\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}^2}{{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^2}}\,\left(a\,1{}\mathrm{i}-b\,1{}\mathrm{i}-a\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,4{}\mathrm{i}+a\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}+b\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,2{}\mathrm{i}-b\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}\right)}{3\,a^2\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

```
[Out] -(2*(exp(e*2i + f*x*2i) + 1)*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e
*2i + f*x*2i) + 1)^2)^(1/2)*(a*1i - b*1i - a*exp(e*2i + f*x*2i)*4i + a*exp(
e*4i + f*x*4i)*1i + b*exp(e*2i + f*x*2i)*2i - b*exp(e*4i + f*x*4i)*1i))/(3*
a^2*f*(exp(e*2i + f*x*2i) - 1)^3)
```

3.127 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{(15a^2-20ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}-\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\text{c}$$

```
[Out] -1/15*(15*a^2-20*a*b+8*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/f-2/15*
(5*a-2*b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/5*cot(f*x+e)^5*(a+b
*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3744, 473, 464, 270}

$$-\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{(15a^2-20ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/15*((15*a^2 - 20*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(
a^3*f) - (2*(5*a - 2*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^2*
f) - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*a*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{2(5a-2b)+5ax^2}{x^4\sqrt{a+bx^2}}\,dx, x, \tan(e+\ldots\right.}{5af}$$

$$= -\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(\ldots}}{5af}$$

$$= -\frac{(15a^2-20ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} - \frac{2(5a-2b)\cot^3(e\ldots}{}$$

Mathematica [A]
time = 1.97, size = 90, normalized size = 0.73

$$-\frac{\cot(e+fx)\left(8(a-b)^2+4a(a-b)\csc^2(e+fx)+3a^2\csc^4(e+fx)\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{15\sqrt{2}\,a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/15*(Cot[e + f*x]*(8*(a - b)^2 + 4*a*(a - b)*Csc[e + f*x]^2 + 3*a^2*Csc[e
 + f*x]^4)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2
]*a^3*f)
```

Maple [A]
time = 0.35, size = 148, normalized size = 1.20

method	result
default	$-\frac{\left(8\left(\cos^{4}(fx+e)\right)a^{2}-16\left(\cos^{4}(fx+e)\right)ab+8\left(\cos^{4}(fx+e)\right)b^{2}-20\left(\cos^{2}(fx+e)\right)a^{2}+36\left(\cos^{2}(fx+e)\right)ab-16\left(\cos^{2}(fx+e)\right)b^{2}+15a^{2}-20\right.}{15f\sin(fx+e)^{5}a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(8*cos(f*x+e)^4*a^2-16*cos(f*x+e)^4*a*b+8*cos(f*x+e)^4*b^2-20*cos(f
*x+e)^2*a^2+36*cos(f*x+e)^2*a*b-16*cos(f*x+e)^2*b^2+15*a^2-20*a*b+8*b^2)*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)^
5/a^3
```

Maxima [A]
time = 0.29, size = 185, normalized size = 1.50

$$-\frac{\frac{15\sqrt{b\tan(fx+e)^{2}+a}}{a\tan(fx+e)}-\frac{20\sqrt{b\tan(fx+e)^{2}+a}\,b}{a^{2}\tan(fx+e)}+\frac{8\sqrt{b\tan(fx+e)^{2}+a}\,b^{2}}{a^{3}\tan(fx+e)}+\frac{10\sqrt{b\tan(fx+e)^{2}+a}}{a\tan(fx+e)^{3}}-\frac{4\sqrt{b\tan(fx+e)^{2}+a}\,b}{a^{2}\tan(fx+e)^{3}}+\frac{3\sqrt{b\tan(fx+e)^{2}+a}}{a\tan(fx+e)^{5}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)) - 20*sqrt(b*tan(f*x +
 e)^2 + a)*b/(a^2*tan(f*x + e)) + 8*sqrt(b*tan(f*x + e)^2 + a)*b^2/(a^3*tan
(f*x + e)) + 10*sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)^3) - 4*sqrt(b*ta
n(f*x + e)^2 + a)*b/(a^2*tan(f*x + e)^3) + 3*sqrt(b*tan(f*x + e)^2 + a)/(a*
tan(f*x + e)^5))/f
```

Fricas [A]
time = 10.25, size = 149, normalized size = 1.21

$$-\frac{\left(8\left(a^{2}-2ab+b^{2}\right)\cos(fx+e)^{5}-4\left(5a^{2}-9ab+4b^{2}\right)\cos(fx+e)^{3}+\left(15a^{2}-20ab+8b^{2}\right)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^{2}+b}{\cos(fx+e)^{2}}}}{15\left(a^{3}f\cos(fx+e)^{4}-2a^{3}f\cos(fx+e)^{2}+a^{3}f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 4*(5*a^2 - 9*a*b + 4*b^2)*cos
(f*x + e)^3 + (15*a^2 - 20*a*b + 8*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^
2 + a^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{6}(e+fx)}{\sqrt{a+b\tan^{2}(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [B]
time = 22.11, size = 761, normalized size = 6.19

$$\frac{\left(\frac{(a-b)\,\left(-64\,a^2+32\,a\,b+32\,b^2\right)}{120\,a^3\,f\,(a\,1\mathrm{i}-b\,1\mathrm{i})}-\frac{(a-b)\,\left(64\,a^2-96\,a\,b+32\,b^2\right)}{120\,a^3\,f\,(a\,1\mathrm{i}-b\,1\mathrm{i})}\right)\,\sqrt{a+\frac{b\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}}\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^2\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}+\frac{\sqrt{a+\frac{b\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}}\,\left(\frac{2\,(3\,a-3\,b)}{3\,a\,f\,(a\,1\mathrm{i}-b\,1\mathrm{i})}+\frac{(3\,a-3\,b)\,(96\,a-64\,b)}{240\,a^2\,f\,(a\,1\mathrm{i}-b\,1\mathrm{i})}+\frac{(3\,a-3\,b)\,(256\,a+64\,b)}{240\,a^2\,f\,(a\,1\mathrm{i}-b\,1\mathrm{i})}\right)\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^4\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}+\frac{\left(\frac{(a-b)\,(32\,a-16\,b)}{30\,a^2\,f\,(a\,1\mathrm{i}-b\,1\mathrm{i})}+\frac{(a-b)\,(32\,a+48\,b)}{30\,a^2\,f\,(a\,1\mathrm{i}-b\,1\mathrm{i})}\right)\,\sqrt{a+\frac{b\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}}\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}-\frac{{(a-b)}^2\,\sqrt{a+\frac{b\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}}\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)\,8\mathrm{i}}{15\,a^3\,f\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}+\frac{8\,(2\,a-2\,b)\,\sqrt{a+\frac{b\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}}\,\left(2\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+1\right)}{5\,a\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^5\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,(a\,1\mathrm{i}-b\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

```
[Out] ((((a - b)*(32*a*b - 64*a^2 + 32*b^2))/(120*a^3*f*(a*1i - b*1i)) - ((a - b)
*(64*a^2 - 96*a*b + 32*b^2))/(120*a^3*f*(a*1i - b*1i)))*(a + (b*(exp(e*2i +
 f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i
) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^2*(exp(e*2i + f*x*2i
) + 1)) + ((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)
^2)^(1/2)*((2*(3*a - 3*b))/(3*a*f*(a*1i - b*1i)) + ((3*a - 3*b)*(96*a - 64*
b))/(240*a^2*f*(a*1i - b*1i)) + ((3*a - 3*b)*(256*a + 64*b))/(240*a^2*f*(a*
1i - b*1i)))*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i +
f*x*2i) - 1)^4*(exp(e*2i + f*x*2i) + 1)) + ((((a - b)*(32*a - 16*b))/(30*a^
2*f*(a*1i - b*1i)) + ((a - b)*(32*a + 48*b))/(30*a^2*f*(a*1i - b*1i)))*(a +
 (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*ex
p(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^3*(ex
p(e*2i + f*x*2i) + 1)) - ((a - b)^2*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)
/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4
i) + 1)*8i)/(15*a^3*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)) +
(8*(2*a - 2*b)*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i)
+ 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/(5*a*f*(exp(
e*2i + f*x*2i) - 1)^5*(exp(e*2i + f*x*2i) + 1)*(a*1i - b*1i))
```

3.128 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{(15a^2+10ab-b^2)\cos(e+fx)}{15(a-b)^3f\sqrt{a-b+b\sec^2(e+fx)}}+\frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}-\frac{\cos^5(e+fx)}{5(a-b)f\sqrt{a-b+b\sec^2(}$$

```
[Out] -1/15*(15*a^2+10*a*b-b^2)*cos(f*x+e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^(1/2)+2
/15*(5*a-2*b)*cos(f*x+e)^3/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1/5*cos(f*x
+e)^5/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)-2/15*b*(15*a^2+10*a*b-b^2)*sec(f*x
+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3745, 473, 464, 277, 197}

$$-\frac{2b(15a^2+10ab-b^2)\sec(e+fx)}{15f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(15a^2+10ab-b^2)\cos(e+fx)}{15f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos^5(e+fx)}{5f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}+\frac{2(5a-2b)\cos^3(e+fx)}{15f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*((15*a^2 + 10*a*b - b^2)*Cos[e + f*x])/((a - b)^3*f*Sqrt[a - b + b*Se
c[e + f*x]^2]) + (2*(5*a - 2*b)*Cos[e + f*x]^3)/(15*(a - b)^2*f*Sqrt[a - b
+ b*Sec[e + f*x]^2]) - Cos[e + f*x]^5/(5*(a - b)*f*Sqrt[a - b + b*Sec[e + f
*x]^2]) - (2*b*(15*a^2 + 10*a*b - b^2)*Sec[e + f*x])/(15*(a - b)^4*f*Sqrt[a
 - b + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= -\frac{\cos^5(e+fx)}{5(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-2(5a-2b)+5(a-b)x^2}{x^4\left(a-b+bx^2\right)^{3/2}}\,dx, x, \sec\right.}{5(a-b)f} \\
&= \frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} \\
&= -\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15(a-b)^3f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+f}} \\
&= -\frac{\left(15a^2+10ab-b^2\right)\cos(e+fx)}{15(a-b)^3f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{2(5a-2b)\cos^3(e+fx)}{15(a-b)^2f\sqrt{a-b+b\sec^2(e+f}}
\end{aligned}$$

Mathematica [A]
time = 2.00, size = 186, normalized size = 0.93

$$-\frac{\left(150a^3+1078a^2b+338ab^2-30b^3+\left(125a^3+169a^2b-329ab^2+35b^3\right)\cos(2(e+fx))-2(a-b)^2(11a+b)\cos(4(e+fx))+3a^3\cos(6(e+fx))-9a^2b\cos(6(e+fx))+9ab^2\cos(6(e+fx))-3b^3\cos(6(e+fx))\right)\sec(e+fx)}{240\sqrt{2}\,(a-b)^4f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/240*((150*a^3 + 1078*a^2*b + 338*a*b^2 - 30*b^3 + (125*a^3 + 169*a^2*b -
 329*a*b^2 + 35*b^3)*Cos[2*(e + f*x)] - 2*(a - b)^2*(11*a + b)*Cos[4*(e + f
*x)] + 3*a^3*Cos[6*(e + f*x)] - 9*a^2*b*Cos[6*(e + f*x)] + 9*a*b^2*Cos[6*(e
 + f*x)] - 3*b^3*Cos[6*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^4*f*Sqrt[
(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 67747 vs. $2(183) = 366$.
time = 5.38, size = 67748, normalized size = 340.44

method	result	size
default	Expression too large to display	67748

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(191) = 382$.
time = 0.38, size = 407, normalized size = 2.05

$$-\frac{\frac{15\,b^3}{(a^4-4a^3b+6a^2b^2-4ab^3+b^4)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}+\frac{15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2-2ab+b^2}+\frac{3\left(\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-5\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3+15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b^2\cos(fx+e)\right)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{10\left(\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-6\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^3-3a^2b+3ab^2-b^3}+\frac{30\,b^2}{(a^3-3a^2b+3ab^2-b^3)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}+\frac{15\,b}{(a^2-2ab+b^2)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^3/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a - b + b/c
os(f*x + e)^2)*cos(f*x + e)) + 15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x +
e)/(a^2 - 2*a*b + b^2) + 3*((a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5
 - 5*(a - b + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a - b + b/
cos(f*x + e)^2)*b^2*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^
4) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a - b + b
/cos(f*x + e)^2)*b*cos(f*x + e))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 30*b^2/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)
) + 15*b/((a^2 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))
/f
```

Fricas [A]
time = 3.23, size = 240, normalized size = 1.21

$$-\frac{\left(3\left(a^3-3a^2b+3ab^2-b^3\right)\cos(fx+e)^7-2\left(5a^3-12a^2b+9ab^2-2b^3\right)\cos(fx+e)^5+\left(15a^3-5a^2b-11ab^2+b^3\right)\cos(fx+e)^3+2\left(15a^2b+10ab^2-b^3\right)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{15\left(\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)f\cos(fx+e)^2+\left(a^4b-4a^3b^2+6a^2b^3-4ab^4+b^5\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 2*(5*a^3 - 12*a^2
*b + 9*a*b^2 - 2*b^3)*cos(f*x + e)^5 + (15*a^3 - 5*a^2*b - 11*a*b^2 + b^3)*
cos(f*x + e)^3 + 2*(15*a^2*b + 10*a*b^2 - b^3)*cos(f*x + e))*sqrt(((a - b)*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b
^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a
*b^4 + b^5)*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3432 vs. $2(191) = 382$.
time = 2.32, size = 3432, normalized size = 17.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*((a^5*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 3*a^4*b^3*sgn(tan(1/2*
f*x + 1/2*e)^2 - 1) + 3*a^3*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^2*b^5*s
gn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1/2*f*x + 1/2*e)^2/(a^7*b - 7*a^6*b^2 +
 21*a^5*b^3 - 35*a^4*b^4 + 35*a^3*b^5 - 21*a^2*b^6 + 7*a*b^7 - b^8) + (a^5*
b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 3*a^4*b^3*sgn(tan(1/2*f*x + 1/2*e)^2
- 1) + 3*a^3*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^2*b^5*sgn(tan(1/2*f*x
+ 1/2*e)^2 - 1))/(a^7*b - 7*a^6*b^2 + 21*a^5*b^3 - 35*a^4*b^4 + 35*a^3*b^5
- 21*a^2*b^6 + 7*a*b^7 - b^8))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + 4*(15*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^9*a*b + 165*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^8*a^(3/2)*b - 60*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))^8*sqrt(a)*b^2 + 320*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
```

+ 1/2*e)^2 + a))^7*a^3 - 420*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))^7*a^2*b + 480*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
7*a*b^2 - 80*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*b^3 - 6
40*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(7/2) + 900*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(5/2)*b + 160*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*b^3 - 832*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^4 - 542*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3*b + 1728*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b^2 - 272*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*
f*x + 1/2*e)^2 + a))^5*a*b^3 + 128*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a))^5*b^4 + 2560*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))^4*a^(9/2) - 3130*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4
*a^(7/2)*b - 5880*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^
(5/2)*b^2 + 8480*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(
3/2)*b^3 - 2240*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt
(a)*b^4 - 320*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^5 +
4140*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*
tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^4*b - 2080*(s
qrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2
*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3*b^2 - 8560*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b^3 + 7680*(sqrt(a)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^4 - 1280*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^5 - 3200*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^2*a^(11/2) + 1940*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2

```
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*
f*x + 1/2*e)^2 + a))^2*a^(9/2)*b + 15840*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a))^2*a^(7/2)*b^2 - 23200*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a))^2*a^(5/2)*b^3 + 8320*(sqrt(a...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (e+f\,x)^5}{\left(b\tan (e+f\,x)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.129 $\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{(3a+b)\cos(e+fx)}{3(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}+\frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}-\frac{2b(3a+b)\sec(e+fx)}{3(a-b)^3f\sqrt{a-b+b\sec^2(e+fx)}}$$

```
[Out] -1/3*(3*a+b)*cos(f*x+e)/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)+1/3*cos(f*x+e)
^3/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)-2/3*b*(3*a+b)*sec(f*x+e)/(a-b)^3/f/(a
-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3745, 464, 277, 197}

$$-\frac{2b(3a+b)\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}+\frac{\cos^3(e+fx)}{3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(3a+b)\cos(e+fx)}{3f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*((3*a + b)*Cos[e + f*x])/((a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2])
+ Cos[e + f*x]^3/(3*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2]) - (2*b*(3*a +
 b)*Sec[e + f*x])/(3*(a - b)^3*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a-b+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(3a+b)\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)^{3/2}}\,dx, x, \text{s}\right.}{3(a-b)f}$$

$$= -\frac{(3a+b)\cos(e+fx)}{3(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}$$

$$= -\frac{(3a+b)\cos(e+fx)}{3(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}$$

Mathematica [**A**]
time = 1.21, size = 106, normalized size = 0.81

$$-\frac{(9a^2+46ab+9b^2+8(a^2-b^2)\cos(2(e+fx))-(a-b)^2\cos(4(e+fx)))\sec(e+fx)}{12\sqrt{2}\,(a-b)^3f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/12*((9*a^2 + 46*a*b + 9*b^2 + 8*(a^2 - b^2)*Cos[2*(e + f*x)] - (a - b)^2
*Cos[4*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^3*f*Sqrt[(a + b + (a - b)
*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 14990 vs. 2(119) = 238.
time = 1.44, size = 14991, normalized size = 114.44

method	result	size

default	Expression too large to display	14991

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.29, size = 226, normalized size = 1.73

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^2-2ab+b^2}-\frac{\left(a-b+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-6\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)}{a^3-3a^2b+3ab^2-b^3}+\frac{3b^2}{\left(a^3-3a^2b+3ab^2-b^3\right)\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}+\frac{3b}{\left(a^2-2ab+b^2\right)\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^2 - 2*a*b + b^2) - (
(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a - b + b/cos(f*x
+ e)^2)*b*cos(f*x + e))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 3*b^2/((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)) + 3*b/((
a^2 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))/f
```

Fricas [A]
time = 4.04, size = 164, normalized size = 1.25

$$\frac{\left(\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-\left(3\,a^2-2\,ab-b^2\right)\cos\left(fx+e\right)^3-2\left(3\,ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)f\cos\left(fx+e\right)^2+\left(a^3b-3\,a^2b^2+3\,ab^3-b^4\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - (3*a^2 - 2*a*b - b^2)*cos(f*x + e
)^3 - 2*(3*a*b + b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 +
(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. $2(125) = 250$.
time = 1.77, size = 1181, normalized size = 9.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*a*sqrt(b) + b^(3/2))*sgn(f)*sgn(cos(f*x + e))/(a^3*abs(f) - 3*a^2*b*
abs(f) + 3*a*b^2*abs(f) - b^3*abs(f)) - a*b/((a^3*abs(f)*sgn(f)*sgn(cos(f*x
 + e)) - 3*a^2*b*abs(f)*sgn(f)*sgn(cos(f*x + e)) + 3*a*b^2*abs(f)*sgn(f)*sg
n(cos(f*x + e)) - b^3*abs(f)*sgn(f)*sgn(cos(f*x + e)))*sqrt(a*cos(f*x + e)^
2 - b*cos(f*x + e)^2 + b)) + 1/3*((a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)
^(3/2)*a^6*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 - 3*sqrt(a*cos(f*x + e)^2 - b*c
os(f*x + e)^2 + b)*a^7*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 - 6*(a*cos(f*x + e)
^2 - b*cos(f*x + e)^2 + b)^(3/2)*a^5*b*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 + 1
5*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a^6*b*f^2*sgn(f)^2*sgn(cos(
f*x + e))^2 + 15*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*a^4*b^2*f^
2*sgn(f)^2*sgn(cos(f*x + e))^2 - 27*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^
2 + b)*a^5*b^2*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 - 20*(a*cos(f*x + e)^2 - b*
cos(f*x + e)^2 + b)^(3/2)*a^3*b^3*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 + 15*sqr
t(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a^4*b^3*f^2*sgn(f)^2*sgn(cos(f*x
 + e))^2 + 15*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*a^2*b^4*f^2*s
gn(f)^2*sgn(cos(f*x + e))^2 + 15*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 +
 b)*a^3*b^4*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 - 6*(a*cos(f*x + e)^2 - b*cos(
f*x + e)^2 + b)^(3/2)*a*b^5*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 - 27*sqrt(a*co
s(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a^2*b^5*f^2*sgn(f)^2*sgn(cos(f*x + e))
^2 + (a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*b^6*f^2*sgn(f)^2*sgn(c
os(f*x + e))^2 + 15*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a*b^6*f^2
*sgn(f)^2*sgn(cos(f*x + e))^2 - 3*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2
+ b)*b^7*f^2*sgn(f)^2*sgn(cos(f*x + e))^2)/(a^9*f^2*abs(f)*sgn(f)^3*sgn(cos
(f*x + e))^3 - 9*a^8*b*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 + 36*a^7*b^2
*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 - 84*a^6*b^3*f^2*abs(f)*sgn(f)^3*s
gn(cos(f*x + e))^3 + 126*a^5*b^4*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 -
126*a^4*b^5*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 + 84*a^3*b^6*f^2*abs(f)
*sgn(f)^3*sgn(cos(f*x + e))^3 - 36*a^2*b^7*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x
+ e))^3 + 9*a*b^8*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 - b^9*f^2*abs(f)*
sgn(f)^3*sgn(cos(f*x + e))^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.130 $\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{\cos(e+fx)}{(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}-\frac{2b\sec(e+fx)}{(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}$$

```
[Out] -cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)-2*b*sec(f*x+e)/(a-b)^2/f/(a-
b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3745, 277, 197}

$$-\frac{2b\sec(e+fx)}{f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos(e+fx)}{f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]/((a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f
*x])/((a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{3/2}}\,dx, x, \sec(e\right.}{(a-b)f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{2b\sec(e+fx)}{(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}$$

Mathematica [A]
time = 1.69, size = 72, normalized size = 0.95

$$-\frac{(a+3b+(a-b)\cos(2(e+fx)))\sec(e+fx)}{\sqrt{2}\,(a-b)^2f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
[Out] -(((a + 3*b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^2*f*
Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Maple [A]
time = 0.12, size = 103, normalized size = 1.36

method	result	size
default	$-\frac{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+2b\right)}{f\left(\frac{a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3(a-b)^2}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+2*b)/
((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/cos(f*x+e)^3/(a-b)^2
```

Maxima [A]
time = 0.29, size = 87, normalized size = 1.14

$$-\frac{\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2-2ab+b^2} + \frac{b}{(a^2-2ab+b^2)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^2 - 2*a*b + b^2) + b/((a^2
 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))/f
```

Fricas [A]
time = 5.51, size = 109, normalized size = 1.43

$$-\frac{\left((a-b)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3})f\cos\left(fx+e\right)^{2}+(a^{2}b-2\,ab^{2}+b^{3})f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^
2*b - 2*a*b^2 + b^3)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(76) = 152.
time = 1.60, size = 178, normalized size = 2.34

$$-\frac{f^{2}\left(\frac{\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}}{a|f|\mathrm{sgn}(f)\mathrm{sgn}(\cos(fx+e))-b|f|\mathrm{sgn}(f)\mathrm{sgn}(\cos(fx+e))}+\frac{b}{(a|f|\mathrm{sgn}(f)\mathrm{sgn}(\cos(fx+e))-b|f|\mathrm{sgn}(f)\mathrm{sgn}(\cos(fx+e)))\sqrt{a\cos\left(fx+e\right)^{2}-b\cos\left(fx+e\right)^{2}+b}}\right)}{af^{2}-bf^{2}}+\frac{2\sqrt{b}\,\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right))}{a^{2}|f|-2\,ab|f|+b^{2}|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -f^2*(sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)/(a*abs(f)*sgn(f)*sgn(co
s(f*x + e)) - b*abs(f)*sgn(f)*sgn(cos(f*x + e))) + b/((a*abs(f)*sgn(f)*sgn(
cos(f*x + e)) - b*abs(f)*sgn(f)*sgn(cos(f*x + e)))*sqrt(a*cos(f*x + e)^2 -
```

```
b*cos(f*x + e)^2 + b)))/(a*f^2 - b*f^2) + 2*sqrt(b)*sgn(f)*sgn(cos(f*x + e)
)/(a^2*abs(f) - 2*a*b*abs(f) + b^2*abs(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (e+f x)}{\left(b \tan (e+f x)^2+a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.131 $\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}$$

```
[Out] -arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(3/2)/f-b*sec(f*x
+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3745, 390, 385, 213}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\sec(e+fx)}{af(a-b)\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(a^(3/2)*f
)) - (b*Sec[e + f*x])/(a*(a - b)*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a-b+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a-b+bx^2}}\,dx,\right.}{af}$$

$$= -\frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1+ax^2}\,dx, x, \frac{\sec(}{\sqrt{a-b+b}}\right.}{af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+f}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1007 vs. 2(84) = 168.
time = 6.74, size = 1007, normalized size = 11.99

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*b*Cos[e + f*x]*Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(
1 + Cos[2*(e + f*x)])])/(a*(a - b)*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*
(e + f*x)])) + (((1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e
+ f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*
(4*Sqrt[a]*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e
+ f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])] - Sqrt[b]*(2*Arc
Tanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*
```

```
x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*
b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]))*(-1 + Tan[(e + f*x
)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan
[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a
 + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*
Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]) - ((1 + Cos[e + f*x])*
Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos
[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]*ArcTanh[(-(Sqrt[a]*(-1 +
Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/
2]^2)^2])/(2*Sqrt[b])] + Sqrt[b]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*T
an[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b -
 a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(
e + f*x)/2]^2)^2]]))*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqr
t[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*
x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt
[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e
+ f*x)/2]^2)^2]))/(a*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3490 vs. $2(76) = 152$.
time = 0.36, size = 3491, normalized size = 41.56

method	result	size
default	Expression too large to display	3491

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e
)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))
*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^3*a^
3-2*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^3*a^2*b+
ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^3*a*b^2+((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^
(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos
(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)^3*a^3-2*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-co
```

s(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1)
)*cos(f*x+e)^3*a^2*b+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+
e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a
^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)^3*a*b^2+2*co
s(f*x+e)^2*a^(5/2)*b+ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e
)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+
e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*co
s(f*x+e)^2*a^3-2*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos
(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2
/a^(1/2))*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*
x+e)^2*a^2*b+ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(
1/2))*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)
^2*a*b^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(
cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f
*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)^2*a^3-2*((a*cos(f*x+e)^2
-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(
cos(f*x+e)-1))*cos(f*x+e)^2*a^2*b+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b
+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+
1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)
^2*a*b^2-2*cos(f*x+e)^2*a^(3/2)*b^2+ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2
)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b
*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1
/2)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)
+1)^2)^(1/2)*cos(f*x+e)*a^2*b-ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f
*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)
/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)
^(1/2)*cos(f*x+e)*a*b^2+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2
)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)*a^2*b-((a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a
^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*co
s(f*x+e)+b)/(cos(f*x+e)-1))*cos(f*x+e)*a*b^2+ln(-2*(cos(f*x+e)-1)*(cos(f*x+

```
e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(c
os(f*x+e)+1)^2)^(1/2)*a^2*b-ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(80) = 160.
time = 3.66, size = 373, normalized size = 4.44

$$\left[-\frac{2\,ab\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\left(\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^2+ab-b^2\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos(fx+e)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)}{2\left(\left(a^4-2\,a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^3b-a^2b^2\right)f\right)}, -\frac{ab\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)-\left(\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^2+ab-b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos(fx+e)}{a}\right)}{\left(a^4-2\,a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^3b-a^2b^2\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
 - ((a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*sqrt(a)*log(-2*((a - b)
*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)))/((a^4 - 2*a^3*b + a^2*b^2)*
f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f), -(a*b*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e) - ((a^2 - 2*a*b + b^2)*cos(f*x + e)^2 +
a*b - b^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/a))/((a^4 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (
a^3*b - a^2*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2)), x)
```

3.132 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{5/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}}-\frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+f}}$$

```
[Out] -1/2*(a-3*b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(5/2)
/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(1/2)-3/2*b*sec(f*x+e
)/a^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3745, 482, 541, 12, 385, 213}

$$-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{3b\sec(e+fx)}{2a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*((a - 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^
2]])/(a^(5/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*Sqrt[a - b + b*Sec[e
+ f*x]^2]) - (3*b*Sec[e + f*x])/(2*a^2*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a-b+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{(-1+x^2)(a-b+bx^2)^{3/2}}\,dx, x, \sec(e+\right.}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{\text{Subst}\Big(}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(a-3b)}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}} + \frac{(a-3b)}{}$$

$$= -\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 304 vs. 2(127) = 254.
time = 3.65, size = 304, normalized size = 2.39

$$\frac{-\frac{(a+3b+(a-3b)\cos(2(e+fx)))\csc^2(e+fx)\sec(e+fx)}{\sqrt{2}a^2\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}} + \frac{(a-3b)\cos(e+fx)\left(2\tanh^{-1}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\frac{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)+\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{2a^{5/2}\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(((a + 3*b + (a - 3*b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2*Sec[e + f*x])/(Sq
rt[2]*a^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])) + ((a -
 3*b)*Cos[e + f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2
]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*
x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)
^2]])*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*
x]^2])/(2*a^(5/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^
4]))/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5632 vs. 2(111) = 222.
time = 0.38, size = 5633, normalized size = 44.35

method	result	size
default	Expression too large to display	5633

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.80, size = 479, normalized size = 3.77

$$\left[-\frac{\left((a^2-4ab+3b^2)\cos(fx+e)^4-(a^2-5ab+6b^2)\cos(fx+e)^2-ab+3b^2\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos(fx+e)^2+2\sqrt{a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)-2\left((a^2-3ab)\cos(fx+e)^3+3ab\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left((a^4-a^3b)f\cos(fx+e)^4-a^3bf-(a^4-2a^3b)f\cos(fx+e)^2\right)}, \frac{\left((a^2-4ab+3b^2)\cos(fx+e)^4-(a^2-5ab+6b^2)\cos(fx+e)^2-ab+3b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a}\right)+\left((a^2-3ab)\cos(fx+e)^3+3ab\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left((a^4-a^3b)f\cos(fx+e)^4-a^3bf-(a^4-2a^3b)f\cos(fx+e)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^4 - (a^2 - 5*a*b + 6*b^2)*cos(f*
x + e)^2 - a*b + 3*b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(co
s(f*x + e)^2 - 1)) - 2*((a^2 - 3*a*b)*cos(f*x + e)^3 + 3*a*b*cos(f*x + e))*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 - a^3*b)*f*cos(f*x
 + e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2), 1/2*(((a^2 - 4*a*b +
 3*b^2)*cos(f*x + e)^4 - (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 - a*b + 3*b^2
)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)/a) + ((a^2 - 3*a*b)*cos(f*x + e)^3 + 3*a*b*cos(f*x + e))*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 - a^3*b)*f*cos(f*x +
e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2)), x)
```

3.133 $\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{3(a-5b)(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{7/2}f}-\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}}-\frac{\cot^3(e+fx)}{4af\sqrt{a-b+}}$$

```
[Out] -3/8*(a-5*b)*(a-b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a
^(7/2)/f-5/8*(a-b)*cot(f*x+e)*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1
/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1/8*(13*a-15*b)*b
*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3745, 481, 541, 12, 385, 213}

$$-\frac{3(a-5b)(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{7/2}f}-\frac{b(13a-15b)\sec(e+fx)}{8a^3f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a+b\sec^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a - 5*b)*(a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e +
 f*x]^2]])/(8*a^(7/2)*f) - (5*(a - b)*Cot[e + f*x]*Csc[e + f*x])/(8*a^2*f*S
qrt[a - b + b*Sec[e + f*x]^2]) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*a*f*Sqrt[
a - b + b*Sec[e + f*x]^2]) - ((13*a - 15*b)*b*Sec[e + f*x])/(8*a^3*f*Sqrt[a
 - b + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3(a-b+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a+b-4(a-b)x^2}{(-1+x^2)^2(a-b+bx^2)^{3/2}}\,dx, x, \sec(e+\right.}{4af}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(1}{8a^3f}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(1}{8a^3f}$$

$$= -\frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(1}{8a^3f}$$

$$= -\frac{3(a-5b)(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a-b)\cot(e+}{8a^2f\sqrt{a-b+}}$$

Mathematica [A]
time = 4.86, size = 345, normalized size = 1.84

$$\frac{\frac{\left((-8a^2+52ab-60b^2)\cos(2(e+fx))+(a-b)(-11a-45b+3(a-5b)\cos(4(e+fx)))\right)\csc^4(e+fx)\sec(e+fx)}{4\sqrt{2}a^3\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}} + \frac{3(a-5b)(a-b)\cos(e+fx)\left(2\tanh^{-1}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\frac{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)+\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{2a^{7/2}\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((((-8*a^2 + 52*a*b - 60*b^2)*Cos[2*(e + f*x)] + (a - b)*(-11*a - 45*b + 3*
(a - 5*b)*Cos[4*(e + f*x)]))*Csc[e + f*x]^4*Sec[e + f*x])/(4*Sqrt[2]*a^3*Sq
rt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]) + (3*(a - 5*b)*(a -
b)*Cos[e + f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2
 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/
2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]
])*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^
2])/(2*a^(7/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
)/(8*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10581 vs. $2(167) = 334$.
time = 0.34, size = 10582, normalized size = 56.59

method	result	size
default	Expression too large to display	10582

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 1.72, size = 735, normalized size = 3.93

$$\left[\frac{3\left((a^3-7a^2b+11ab^2-5b^3)\cos(fx+e)^6-(2a^3-15a^2b+28ab^2-15b^3)\cos(fx+e)^4+a^2b-6ab^2+5b^3+(a^3-9a^2b+23ab^2-15b^3)\cos(fx+e)^2\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos(fx+e)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)+2\left(3(a^3-6a^2b+5ab^2)\cos(fx+e)^5-(5a^3-31a^2b+30ab^2)\cos(fx+e)^3-(13a^2b-15ab^2)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{16\left((a^5-a^4b)f\cos(fx+e)^6+a^4bf-(2a^5-3a^4b)f\cos(fx+e)^4+(a^5-3a^4b)f\cos(fx+e)^2\right)},\ \frac{3\left((a^3-7a^2b+11ab^2-5b^3)\cos(fx+e)^6-(2a^3-15a^2b+28ab^2-15b^3)\cos(fx+e)^4+a^2b-6ab^2+5b^3+(a^3-9a^2b+23ab^2-15b^3)\cos(fx+e)^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a}\right)+\left(3(a^3-6a^2b+5ab^2)\cos(fx+e)^5-(5a^3-31a^2b+30ab^2)\cos(fx+e)^3-(13a^2b-15ab^2)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{8\left((a^5-a^4b)f\cos(fx+e)^6+a^4bf-(2a^5-3a^4b)f\cos(fx+e)^4+(a^5-3a^4b)f\cos(fx+e)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^3 - 7*a^2*b + 11*a*b^2 - 5*b^3)*cos(f*x + e)^6 - (2*a^3 - 15*a
^2*b + 28*a*b^2 - 15*b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + 5*b^3 + (a^3 -
 9*a^2*b + 23*a*b^2 - 15*b^3)*cos(f*x + e)^2)*sqrt(a)*log(-2*((a - b)*cos(f
*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^3 - 6*a^2*b + 5*a*b^2)*c
os(f*x + e)^5 - (5*a^3 - 31*a^2*b + 30*a*b^2)*cos(f*x + e)^3 - (13*a^2*b -
15*a*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e
)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2), 1/8*(3*((a^3 - 7*a^2*b + 11*a*b^2
- 5*b^3)*cos(f*x + e)^6 - (2*a^3 - 15*a^2*b + 28*a*b^2 - 15*b^3)*cos(f*x +
e)^4 + a^2*b - 6*a*b^2 + 5*b^3 + (a^3 - 9*a^2*b + 23*a*b^2 - 15*b^3)*cos(f*
x + e)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)/a) + (3*(a^3 - 6*a^2*b + 5*a*b^2)*cos(f*x + e)^5 - (
5*a^3 - 31*a^2*b + 30*a*b^2)*cos(f*x + e)^3 - (13*a^2*b - 15*a*b^2)*cos(f*x
 + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^5 - a^4*b)*f*
```

```
cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^
4*b)*f*cos(f*x + e)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1318 vs. $2(177) = 354$.
time = 1.58, size = 1318, normalized size = 7.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(((((a^8*b - a^7*b^2)*tan(1/2*f*x + 1/2*e)^2/(a^9*b*sgn(tan(1/2*f*x +
1/2*e)^2 - 1) - a^8*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)) + (7*a^8*b - 17*a^
7*b^2 + 10*a^6*b^3)/(a^9*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^8*b^2*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2 - (17*a^8*b - 145*a^7*b^
2 + 248*a^6*b^3 - 120*a^5*b^4)/(a^9*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^8
*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2 + (9*a^8*b +
41*a^7*b^2 - 114*a^6*b^3 + 64*a^5*b^4)/(a^9*b*sgn(tan(1/2*f*x + 1/2*e)^2 -
1) - a^8*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)))/sqrt(a*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - 24*(a^2
- 6*a*b + 5*b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))/sqrt(-a))/(sqrt(-a)*a^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)) - 12*(a^(5/2) -
 6*a^(3/2)*b + 5*sqrt(a)*b^2)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))*a + a^(3/2) - 2*sqrt(a)*b))/(a^4*sgn(tan(1/2*f*x + 1/2*e)^
2 - 1)) + 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2 -
 16*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 14*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 - 3*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
```

```
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 4*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b - 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))*a^3 + 20*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))*a^2*b - 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^
2 + 5*a^(7/2) - 8*a^(5/2)*b)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a))^2 - a)^2*a^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)))/f
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2)),x)
[Out] \text{Hanged}
```

3.134 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{3a(a+4b)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{7/2}f}-\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}+\frac{\cos^3(e+fx)\sin(e+\cdots}{4(a-b)f\sqrt{a+b\tan^2(e\cdots}}$$

```
[Out] 3/8*a*(a+4*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)
^(7/2)/f-5/8*a*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/4
*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/8*b*(13*a+2*b)*
tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3744, 481, 541, 12, 385, 209}

$$\frac{3a(a+4b)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{7/2}}-\frac{b(13a+2b)\tan(e+fx)}{8f(a-b)^3\sqrt{a+b\tan^2(e+fx)}}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{5a\sin(e+fx)\cos(e+fx)}{8f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a*(a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/(8*(a - b)^(7/2)*f) - (5*a*Cos[e + f*x]*Sin[e + f*x])/(8*(a - b)^2*f*Sqr
t[a + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*(a - b)*f*Sqrt[
a + b*Tan[e + f*x]^2]) - (b*(13*a + 2*b)*Tan[e + f*x])/(8*(a - b)^3*f*Sqrt[
a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a-4ax^2}{(1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \tan(e+j\right.}{4(a-b)f}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Su}}{}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{}{8(}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{}{8(}$$

$$= -\frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{}{8(}$$

$$= \frac{3a(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{7/2}f} - \frac{5a\cos(e+fx)\sin(e+f}{8(a-b)^2f\sqrt{a+b\tan^2(e+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 3.59, size = 325, normalized size = 1.74

$$\frac{\left(-\left((a-b)\left(7a^2+48ab+5b^2+(6a^2-2ab-4b^2)\cos(2(e+fx))-(a-b)^2\cos(4(e+fx))\right)\right)+6\sqrt{2}\,a(a^2+3ab-4b^2)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)-6\sqrt{2}\,a^2(a+4b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sec^2(e+fx)\sin(2(e+fx))}{32\sqrt{2}\,(a-b)^4f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-((a - b)*(7*a^2 + 48*a*b + 5*b^2 + (6*a^2 - 2*a*b - 4*b^2)*Cos[2*(e + f*
x)] - (a - b)^2*Cos[4*(e + f*x)])) + 6*Sqrt[2]*a*(a^2 + 3*a*b - 4*b^2)*Sqrt
[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sq
```

```
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 6*
Sqrt[2]*a^2*(a + 4*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(32*
Sqrt[2]*(a - b)^4*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]
)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.85, size = 6028, normalized size = 32.24

method	result	size
default	Expression too large to display	6028

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(177) = 354$.
time = 101.42, size = 1081, normalized size = 5.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*(a^2*b + 4*a*b^2 + (a^3 + 3*a^2*b - 4*a*b^2)*cos(f*x + e)^2)*sqrt(
-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8
- 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*
a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*
a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2
 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*
```

```
(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 2
4*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*co
s(f*x + e)^5 - 5*(a^3 - 2*a^2*b + a*b^2)*cos(f*x + e)^3 - (13*a^2*b - 11*a*
b^2 - 2*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f), 1/32
*(3*(a^2*b + 4*a*b^2 + (a^3 + 3*a^2*b - 4*a*b^2)*cos(f*x + e)^2)*sqrt(a - b
)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^
2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((
a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4
*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*cos(f*x + e)^5 - 5*(a^3 - 2*a^2*b + a*b^2)*cos(f*x + e)^3 - (13*a^2*b - 11
*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^
5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral(sin(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(e+f\,x)^4}{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.135 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(a+2b)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{5/2}f}-\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}}-\frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] 1/2*(a+2*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(
5/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-3/2*b*tan
(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3744, 482, 541, 12, 385, 209}

$$\frac{(a+2b)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{5/2}}-\frac{3b\tan(e+fx)}{2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*(a - b)^(5/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f*Sqrt[a + b*Ta
n[e + f*x]^2]) - (3*b*Tan[e + f*x])/(2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-2bx^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+f\right.}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a-}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{(a-}{}$$

$$= \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.89, size = 282, normalized size = 2.10

$$-\frac{\left((a-b)(a+5b+(a-b)\cos(2(e+fx))) - \sqrt{2}(a^2+ab-2b^2)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right) + \sqrt{2}a(a+2b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sec^2(e+fx)\sin(2(e+fx))}{4\sqrt{2}(a-b)^3f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(((a - b)*(a + 5*b + (a - b)*Cos[2*(e + f*x)]) - Sqrt[2]*(a^2 + a*b -
2*b^2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Elliptic
F[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2
]], 1] + Sqrt[2]*a*(a + 2*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)
```

```
])/(Sqrt[2]*(a - b)^3*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x
]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 1615, normalized size = 12.05

method	result	size
default	Expression too large to display	1615

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b
)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a
)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x
+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-
4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*sin(f*x+e)-2*2^(1/
2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*
cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)
-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)
*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*
x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)
/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)+((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)*b-((2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)/a)^(1/2)*b)*sin(f*x+e)/(cos(f*x+e)-1)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/cos(f*x+e)^2)^(3/2)/cos(f*x+e)^3/a/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2)/(a-b)-1/4/f*(-cos(2*f*x+2*e)^2*(a-b)^(3/2)*a^3*b+2*cos(2*f*x+2*e)^2*(a-
b)^(3/2)*a^2*b^2-cos(2*f*x+2*e)^2*(a-b)^(3/2)*a*b^3+4*(b^4*(a-b))^(1/2)*arc
tan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+
2*e)+1))^(1/2)/sin(2*f*x+2*e)*b^2*(a-b)/(b^4*(a-b))^(1/2))*sin(2*f*x+2*e)*(
(a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)*a*(a-b)^(
3/2)-2*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2
*e)+1))^(1/2)*arctan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)
+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^4*b+8*sin(2*f
*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)*
arctan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f
*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^3*b^2-10*sin(2*f*x+2*e)*((a
*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)*arctan((cos
(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))
^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^2*b^3+4*sin(2*f*x+2*e)*((a*cos(2*f*x+2
*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)*arctan((cos(2*f*x+2*e)-
1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2
*f*x+2*e)*(a-b)^(1/2))*a*b^4-2*cos(2*f*x+2*e)*(a-b)^(3/2)*a^2*b^2-2*cos(2*f
*x+2*e)*(a-b)^(3/2)*a*b^3+4*cos(2*f*x+2*e)*(a-b)^(3/2)*b^4+(a-b)^(3/2)*a^3*
```

```
b+3*(a-b)^(3/2)*a*b^3-4*(a-b)^(3/2)*b^4)/sin(2*f*x+2*e)/((a*cos(2*f*x+2*e)-
b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/(a-b)^(9/2)/a/b
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(125) = 250$.
time = 6.00, size = 941, normalized size = 7.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((a^2 + a*b - 2*b^2)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a + b)*log
(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4
- 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^
3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160
*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3
 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x +
 e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29
*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 1
6*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) + 8*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 3*(a*b - b
^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3
*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/8*(((a^2 + a*b - 2*b^2)*cos(f*x + e)^
2 + a*b + 2*b^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e
)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*
x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (
a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*((a^2 -
 2*a*b + b^2)*cos(f*x + e)^3 + 3*(a*b - b^2)*cos(f*x + e))*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2
 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(e + fx)}{(a + b\tan^2(e + fx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral(sin(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(e + f\,x)^2}{\left(b\tan(e + f\,x)^2 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.136 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*tan
(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3742, 390, 385, 209}

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Tan[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e\cdots\right.}{(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e\cdots}{\sqrt{a+b\tan\cdots}}\right.}{(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.33, size = 214, normalized size = 2.52

$$\frac{4\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(a(a-b)\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\tan^2(e+fx)+\frac{15\left(3a+2b\tan^2(e+fx)\right)\left(-2\text{ArcSin}\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\right)\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)+a\sqrt{\frac{(a-b)\sin^2(2(e+fx))\left(a+b\tan^2(e+fx)\right)}{a^2}}\right)}{\left(\frac{(a-b)\sin^2(2(e+fx))\left(a+b\tan^2(e+fx)\right)}{a^2}\right)^{3/2}}\right)}{15a^4f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] (4*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]*(a*(a - b)*Hyperg
eometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2 + (15*(3*
a + 2*b*Tan[e + f*x]^2)*(-2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a*Cos
[e + f*x]^2 + b*Sin[e + f*x]^2) + a*Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b
*Tan[e + f*x]^2))/a^2]))/(((a - b)*Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2
))/a^2)^(3/2)))/(15*a^4*f)
```

Maple [A]
time = 0.07, size = 102, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{(a-b)^2b^2}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\left(\tan^2(fx+e)\right)}}}{f}$
default	$\frac{\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{(a-b)^2b^2}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\left(\tan^2(fx+e)\right)}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+
b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/(a-b)*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)
^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.86, size = 324, normalized size = 3.81

$$\left[\frac{\left(ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2b)\tan(fx+e)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\left(ab-b^2\right)\tan\left(fx+e\right)}{2\left(\left(a^3b-2a^2b^2+ab^3\right)f\tan\left(fx+e\right)^2+\left(a^4-2a^3b+a^2b^2\right)f\right)}, \frac{\left(ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{a-b}\,\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{a-b}\,\tan(fx+e)}\right)-\sqrt{b\tan\left(fx+e\right)^2+a}\,\left(ab-b^2\right)\tan\left(fx+e\right)}{\left(a^3b-2a^2b^2+ab^3\right)f\tan\left(fx+e\right)^2+\left(a^4-2a^3b+a^2b^2\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/2*((a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), ((a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan^{2}\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e)**2)**(3/2),x)

[Out] Integral((a + b*tan(e + f*x)**2)**(-3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*tan(f*x + e)^2 + a)^(-3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*tan(e + f*x)^2)^(3/2),x)

[Out] int(1/(a + b*tan(e + f*x)^2)^(3/2), x)

3.137 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}}-\frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)^(1/2)-2*b*tan(f*x+e)/a^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3744, 277, 197}

$$-\frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]/(a*f*Sqrt[a + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f*x])/(a^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{af}$$
$$= -\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [A]
time = 0.87, size = 74, normalized size = 1.19

$$-\frac{(a+2b+(a-2b)\cos(2(e+fx)))\csc(e+fx)\sec(e+fx)}{\sqrt{2}\,a^2f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
[Out] -(((a + 2*b + (a - 2*b)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/(Sqrt[
2]*a^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Maple [A]
time = 0.40, size = 109, normalized size = 1.76

method	result	size
default	$-\frac{\left(a\left(\cos^2(fx+e)\right)-2\left(\cos^2(fx+e)\right)b+2b\right)\left(\cos^3(fx+e)\right)\left(\frac{a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}}{f\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)^2\sin(fx+e)a^2}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(a*cos(f*x+e)^2-2*cos(f*x+e)^2*b+2
*b)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/sin
(f*x+e)/a^2
```

Maxima [A]
time = 0.28, size = 62, normalized size = 1.00

$$-\frac{\frac{2\,b\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^2} + \frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^2) + 1/(sqrt(b*tan(f*x + e
)^2 + a)*a*tan(f*x + e)))/f
```

Fricas [A]
time = 1.12, size = 96, normalized size = 1.55

$$-\frac{\left((a-2\,b)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{\left(a^{2}bf+(a^{3}-a^{2}b)f\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a^2*b*f + (a^3 - a^2*b)*f*cos(f*x + e)^2)*sin(f*x +
 e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{2}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [B]
time = 18.30, size = 2978, normalized size = 48.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2)),x)

[Out] ((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(exp(e*2i + f*x*2i)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (3*((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (3*((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))) - exp(e*4i + f*x*4i)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) - (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (3*((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) - ((a - b)^4*(a + 2*b))/(4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (3*((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)

```
^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b
)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
 b*1i)) - (3*((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a
+ 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(8*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i))) + ((a + 3*b)*((((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(
a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(
a + 2*b)*(a*1i - b*1i)) - (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*
b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f*(a*b
^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (3*((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((
a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(4*f
*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))
/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/((exp(e*2i + f*x*2i) + 1
)*(b - a - exp(e*2i + f*x*2i)*(a + 3*b) + exp(e*4i + f*x*4i)*(a + 3*b) + ex
p(e*6i + f*x*6i)*(a - b)))
```

3.138 $\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}}-\frac{2(3a-4b)b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -1/3*(3*a-4*b)*cot(f*x+e)/a^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*cot(f*x+e)^3/a
/f/(a+b*tan(f*x+e)^2)^(1/2)-2/3*(3*a-4*b)*b*tan(f*x+e)/a^3/f/(a+b*tan(f*x+e
)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3744, 464, 277, 197}

$$-\frac{2b(3a-4b)\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*((3*a - 4*b)*Cot[e + f*x])/(a^2*f*Sqrt[a + b*Tan[e + f*x]^2]) - Cot[e
+ f*x]^3/(3*a*f*Sqrt[a + b*Tan[e + f*x]^2]) - (2*(3*a - 4*b)*b*Tan[e + f*x]
)/(3*a^3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{3/2}}\,dx, x, \tan(e+f\right.}{3af}$$

$$= -\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}} - \frac{(2(3a-4b)b)\text{S}}{}$$

$$= -\frac{(3a-4b)\cot(e+fx)}{3a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3af\sqrt{a+b\tan^2(e+fx)}} - \frac{2(3a-4b)b\,\text{ta}}{3a^3f\sqrt{a+b\,\text{ta}}}$$

Mathematica [A]
time = 1.00, size = 119, normalized size = 1.04

$$\frac{(-3a^2-7ab+12b^2-2(a^2-6ab+8b^2)\cos(2(e+fx))+(a^2-5ab+4b^2)\cos(4(e+fx)))\csc^3(e+fx)\sec(e+fx)}{6\sqrt{2}\,a^3f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-3*a^2 - 7*a*b + 12*b^2 - 2*(a^2 - 6*a*b + 8*b^2)*Cos[2*(e + f*x)] + (a^2
 - 5*a*b + 4*b^2)*Cos[4*(e + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*Sqrt[2]
*a^3*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [A]
time = 0.33, size = 170, normalized size = 1.49

method	result
default	$\frac{\left(2\left(\cos^4(fx+e)\right)a^2-10\left(\cos^4(fx+e)\right)ab+8\left(\cos^4(fx+e)\right)b^2-3\left(\cos^2(fx+e)\right)a^2+16\left(\cos^2(fx+e)\right)ab-16\left(\cos^2(fx+e)\right)b^2-6ab+8b^2\right)(c}{3f\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)^2\sin(fx+e)^3a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(2*cos(f*x+e)^4*a^2-10*cos(f*x+e)
^4*a*b+8*cos(f*x+e)^4*b^2-3*cos(f*x+e)^2*a^2+16*cos(f*x+e)^2*a*b-16*cos(f*x
+e)^2*b^2-6*a*b+8*b^2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(
f*x+e)^2)^(3/2)/sin(f*x+e)^3/a^3
```

Maxima [A]
time = 0.30, size = 151, normalized size = 1.32

$$-\frac{\frac{6\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}\,a^2}-\frac{8\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}\,a^3}+\frac{3}{\sqrt{b\tan(fx+e)^2+a}\,a\tan(fx+e)}-\frac{4\,b}{\sqrt{b\tan(fx+e)^2+a}\,a^2\tan(fx+e)}+\frac{1}{\sqrt{b\tan(fx+e)^2+a}\,a\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(6*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^2) - 8*b^2*tan(f*x + e
)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 3/(sqrt(b*tan(f*x + e)^2 + a)*a*tan(f*
x + e)) - 4*b/(sqrt(b*tan(f*x + e)^2 + a)*a^2*tan(f*x + e)) + 1/(sqrt(b*tan
(f*x + e)^2 + a)*a*tan(f*x + e)^3))/f
```

Fricas [A]
time = 4.04, size = 163, normalized size = 1.43

$$-\frac{\left(2\left(a^2-5\,ab+4\,b^2\right)\cos(fx+e)^5-\left(3\,a^2-16\,ab+16\,b^2\right)\cos(fx+e)^3-2\left(3\,ab-4\,b^2\right)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left((a^4-a^3b)f\cos(fx+e)^4-a^3bf-(a^4-2\,a^3b)f\cos(fx+e)^2\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^5 - (3*a^2 - 16*a*b + 16*b^2)*co
s(f*x + e)^3 - 2*(3*a*b - 4*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/(((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2
*a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [B]
time = 35.94, size = 2500, normalized size = 21.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

```
[Out] ((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*
(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(((a + 3*b)*(((a + 3*b)*(((
a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a -
 b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a
^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a
 + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a
^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b -
a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a
 - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i
- b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b
 - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a
 + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*
(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^
6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1
i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2)
 + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2
))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)
*(a*1i - b*1i))))/(a - b) - ((a - b)^8*(a + 2*b))/(3072*a^4*f*(a*b^2 - a^2*
```

b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^4*(a + 2*b)*(56*a^3*b - 84*a^4 - 8
*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
- (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (
a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*
(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + (((((a - b)*(a - 2*b) - (a + 2*b
)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a +
3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^3*(9*a + 4*b))/(768*a^3*f*(
a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((a + 3*b)*
((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + ((
(a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a
- b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*
(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
- ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*
(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^
2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b)^
5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^6*(a + 2
b)(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((
(((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a
- b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a -
b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)) + ((a
- b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
b*1i))))/(a - b) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b
))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)
^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
1i)) + (((((a - b)(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^
2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a
^2))*(a - b)*(56*a^3*b - 84*a^4 - 8*b^4 + 36*a^2*b^2))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a + 2*b)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^
4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(3*a
+ b))/(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^4*(
a + 2*b)*(64*a^3*b - 72*a*b^3 - 182*a^4 + 30*b^4 + 96*a^2*b^2))/(3072*a^4*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) + (((a + 2*b)^3 + (((a - b)*(a
- 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a
b - a^2))(a - b)^3*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1
i - b*1i))))/(a - b) - ((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a
- b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a
- b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a +
2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*
a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3*b)*(((
a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2
*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i -
b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b
- a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3...

3.139 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}} dx$

Optimal. Leaf size=171

$$-\frac{(15a^2-40ab+24b^2)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+fx)}}-\frac{2b(15a^2-40}{15a^4f\sqrt{}}$$

```
[Out] -1/15*(15*a^2-40*a*b+24*b^2)*cot(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(1/2)-2/15
*(5*a-3*b)*cot(f*x+e)^3/a^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/5*cot(f*x+e)^5/a/f
/(a+b*tan(f*x+e)^2)^(1/2)-2/15*b*(15*a^2-40*a*b+24*b^2)*tan(f*x+e)/a^4/f/(a
+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3744, 473, 464, 277, 197}

$$-\frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{2b(15a^2-40ab+24b^2)\tan(e+fx)}{15a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{(15a^2-40ab+24b^2)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*((15*a^2 - 40*a*b + 24*b^2)*Cot[e + f*x])/(a^3*f*Sqrt[a + b*Tan[e + f
*x]^2]) - (2*(5*a - 3*b)*Cot[e + f*x]^3)/(15*a^2*f*Sqrt[a + b*Tan[e + f*x]^
2]) - Cot[e + f*x]^5/(5*a*f*Sqrt[a + b*Tan[e + f*x]^2]) - (2*b*(15*a^2 - 40
*a*b + 24*b^2)*Tan[e + f*x])/(15*a^4*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2(5a-3b)+5ax^2}{x^4(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5af\sqrt{a+b\tan^2(e+fx)}} - \frac{(-15a^2+8(}{}$$

$$= -\frac{(15a^2-8(5a-3b)b)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}} - \frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{}{5af\sqrt{}}$$

$$= -\frac{(15a^2-8(5a-3b)b)\cot(e+fx)}{15a^3f\sqrt{a+b\tan^2(e+fx)}} - \frac{2(5a-3b)\cot^3(e+fx)}{15a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{}{5af\sqrt{}}$$

Mathematica [A]

time = 1.50, size = 135, normalized size = 0.79

$$-\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(\cot(e+fx)\left(8a^2-41ab+33b^2+a(4a-9b)\csc^2(e+fx)+3a^2\csc^4(e+fx)\right)+\frac{15(a-b)^2b\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))}\right)}{15\sqrt{2}\,a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(Cot[e + f*x
]*(8*a^2 - 41*a*b + 33*b^2 + a*(4*a - 9*b)*Csc[e + f*x]^2 + 3*a^2*Csc[e + f
*x]^4) + (15*(a - b)^2*b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)
])))/(Sqrt[2]*a^4*f)
```

Maple [A]
time = 0.42, size = 264, normalized size = 1.54

method	result
default	$-\frac{(8(\cos^6(fx+e))a^3-64(\cos^6(fx+e))a^2b+104(\cos^6(fx+e))a\,b^2-48(\cos^6(fx+e))b^3-20(\cos^4(fx+e))a^3+164(\cos^4(fx+e))a^2b-28}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(8*cos(f*x+e)^6*a^3-64*cos(f*x+
e)^6*a^2*b+104*cos(f*x+e)^6*a*b^2-48*cos(f*x+e)^6*b^3-20*cos(f*x+e)^4*a^3+1
64*cos(f*x+e)^4*a^2*b-288*cos(f*x+e)^4*a*b^2+144*cos(f*x+e)^4*b^3+15*cos(f*
x+e)^2*a^3-130*cos(f*x+e)^2*a^2*b+264*cos(f*x+e)^2*a*b^2-144*cos(f*x+e)^2*b
^3+30*a^2*b-80*a*b^2+48*b^3)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)^5/a^4
```

Maxima [A]
time = 0.29, size = 273, normalized size = 1.60

$$-\frac{\frac{30\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}\,a^2}-\frac{80\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}\,a^3}+\frac{48\,b^3\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}\,a^4}+\frac{15}{\sqrt{b\tan(fx+e)^2+a}\,a\tan(fx+e)}-\frac{40\,b}{\sqrt{b\tan(fx+e)^2+a}\,a^2\tan(fx+e)}+\frac{24\,b^2}{\sqrt{b\tan(fx+e)^2+a}\,a^3\tan(fx+e)}+\frac{10}{\sqrt{b\tan(fx+e)^2+a}\,a\tan(fx+e)^3}-\frac{6\,b}{\sqrt{b\tan(fx+e)^2+a}\,a^2\tan(fx+e)^3}+\frac{3}{\sqrt{b\tan(fx+e)^2+a}\,a\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(30*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^2) - 80*b^2*tan(f*x
+ e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 48*b^3*tan(f*x + e)/(sqrt(b*tan(f*x
 + e)^2 + a)*a^4) + 15/(sqrt(b*tan(f*x + e)^2 + a)*a*tan(f*x + e)) - 40*b/(
sqrt(b*tan(f*x + e)^2 + a)*a^2*tan(f*x + e)) + 24*b^2/(sqrt(b*tan(f*x + e)^
2 + a)*a^3*tan(f*x + e)) + 10/(sqrt(b*tan(f*x + e)^2 + a)*a*tan(f*x + e)^3)
 - 6*b/(sqrt(b*tan(f*x + e)^2 + a)*a^2*tan(f*x + e)^3) + 3/(sqrt(b*tan(f*x
+ e)^2 + a)*a*tan(f*x + e)^5))/f
```

Fricas [A]
time = 86.68, size = 242, normalized size = 1.42

$$-\frac{\left(8\left(a^{3}-8a^{2}b+13ab^{2}-6b^{3}\right)\cos\left(fx+e\right)^{7}-4\left(5a^{3}-41a^{2}b+72ab^{2}-36b^{3}\right)\cos\left(fx+e\right)^{5}+\left(15a^{3}-130a^{2}b+264ab^{2}-144b^{3}\right)\cos\left(fx+e\right)^{3}+2\left(15a^{2}b-40ab^{2}+24b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{15\left((a^{5}-a^{4}b)f\cos\left(fx+e\right)^{6}+a^{4}bf-(2a^{5}-3a^{4}b)f\cos\left(fx+e\right)^{4}+(a^{5}-3a^{4}b)f\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^3 - 8*a^2*b + 13*a*b^2 - 6*b^3)*cos(f*x + e)^7 - 4*(5*a^3 - 41*
a^2*b + 72*a*b^2 - 36*b^3)*cos(f*x + e)^5 + (15*a^3 - 130*a^2*b + 264*a*b^2
 - 144*b^3)*cos(f*x + e)^3 + 2*(15*a^2*b - 40*a*b^2 + 24*b^3)*cos(f*x + e))
*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^5 - a^4*b)*f*cos(f*
x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f
*cos(f*x + e)^2)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{6}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

```
[Out] \text{Hanged}
```

3.140 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{(5a^2+10ab+b^2)\cos(e+fx)}{5(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}+\frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2 f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cos^5(e+fx)}{5(a-b)f\left(a-b+b\sec^2(e\right.}$$

```
[Out] -1/5*(5*a^2+10*a*b+b^2)*cos(f*x+e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^(3/2)+2/1
5*(5*a-b)*cos(f*x+e)^3/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/5*cos(f*x+e)^
5/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-4/15*b*(5*a^2+10*a*b+b^2)*sec(f*x+e)/(
a-b)^4/f/(a-b+b*sec(f*x+e)^2)^(3/2)-8/15*b*(5*a^2+10*a*b+b^2)*sec(f*x+e)/(a
-b)^5/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3745, 473, 464, 277, 198, 197}

$$-\frac{8b(5a^2+10ab+b^2)\sec(e+fx)}{15f(a-b)^5\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b(5a^2+10ab+b^2)\sec(e+fx)}{15f(a-b)^4\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{(5a^2+10ab+b^2)\cos(e+fx)}{5f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\cos^5(e+fx)}{5f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}+\frac{2(5a-b)\cos^3(e+fx)}{15f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/5*((5*a^2 + 10*a*b + b^2)*Cos[e + f*x])/((a - b)^3*f*(a - b + b*Sec[e +
f*x]^2)^(3/2)) + (2*(5*a - b)*Cos[e + f*x]^3)/(15*(a - b)^2*f*(a - b + b*Se
c[e + f*x]^2)^(3/2)) - Cos[e + f*x]^5/(5*(a - b)*f*(a - b + b*Sec[e + f*x]^
2)^(3/2)) - (4*b*(5*a^2 + 10*a*b + b^2)*Sec[e + f*x])/(15*(a - b)^4*f*(a -
b + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(5*a^2 + 10*a*b + b^2)*Sec[e + f*x])/(1
5*(a - b)^5*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
```

```
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2}{x^6(a-b+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5(a-b)f\,(a-b+b\sec^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{-2(5a-b)+5(a-b)x^2}{x^4(a-b+bx^2)^{5/2}}\,dx, x, \text{sec}\right.}{5(a-b)f}$$

$$= \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2 f\,(a-b+b\sec^2(e+fx))^{3/2}} - \frac{\cos^5(e+fx)}{5(a-b)f\,(a-b+b\sec^2(e+fx))}$$

$$= -\frac{(5a^2+10ab+b^2)\cos(e+fx)}{5(a-b)^3 f\,(a-b+b\sec^2(e+fx))^{3/2}} + \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2 f\,(a-b+b\sec^2(e+f}$$

$$= -\frac{(5a^2+10ab+b^2)\cos(e+fx)}{5(a-b)^3 f\,(a-b+b\sec^2(e+fx))^{3/2}} + \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2 f\,(a-b+b\sec^2(e+f}$$

$$= -\frac{(5a^2+10ab+b^2)\cos(e+fx)}{5(a-b)^3 f\,(a-b+b\sec^2(e+fx))^{3/2}} + \frac{2(5a-b)\cos^3(e+fx)}{15(a-b)^2 f\,(a-b+b\sec^2(e+f}$$

Mathematica [A]
time = 2.42, size = 294, normalized size = 1.19

$$-\frac{\cos(e+fx)\left(425a^4+4700a^3b+6134a^2b^2+4700ab^3+425b^4+48(11a^4+106a^3b-106ab^3-11b^4)\cos(2(e+fx))+12(a-b)^2(7a^2+50ab+7b^2)\cos(4(e+fx))-16a^4\cos(6(e+fx))+32a^3b\cos(6(e+fx))-32ab^3\cos(6(e+fx))+16b^4\cos(6(e+fx))+3a^4\cos(8(e+fx))-12a^3b\cos(8(e+fx))+18a^2b^2\cos(8(e+fx))-12ab^3\cos(8(e+fx))+3b^4\cos(8(e+fx))\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{480\sqrt{2}(a-b)^5 f(a+b+(a-b)\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/480*(Cos[e + f*x]*(425*a^4 + 4700*a^3*b + 6134*a^2*b^2 + 4700*a*b^3 + 42
5*b^4 + 48*(11*a^4 + 106*a^3*b - 106*a*b^3 - 11*b^4)*Cos[2*(e + f*x)] + 12*
(a - b)^2*(7*a^2 + 50*a*b + 7*b^2)*Cos[4*(e + f*x)] - 16*a^4*Cos[6*(e + f*x
)] + 32*a^3*b*Cos[6*(e + f*x)] - 32*a*b^3*Cos[6*(e + f*x)] + 16*b^4*Cos[6*(
e + f*x)] + 3*a^4*Cos[8*(e + f*x)] - 12*a^3*b*Cos[8*(e + f*x)] + 18*a^2*b^2
*Cos[8*(e + f*x)] - 12*a*b^3*Cos[8*(e + f*x)] + 3*b^4*Cos[8*(e + f*x)])*Sqr
t[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*(a - b)^5*f*
(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 1.95, size = 391, normalized size = 1.58

method	result

default	$\frac{(a-b)^2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)\left(3\left(\cos^8(fx+e)\right)a^4-12\left(\cos^8(fx+e)\right)a^3b+18\left(\cos^8(fx+e)\right)a^2b^2-12\left(\cos^8(fx+e)\right)a\,b^3+3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/f*(a-b)^2*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(3*cos(f*x+e)^8*a^4-12*cos
(f*x+e)^8*a^3*b+18*cos(f*x+e)^8*a^2*b^2-12*cos(f*x+e)^8*a*b^3+3*cos(f*x+e)^
8*b^4-10*cos(f*x+e)^6*a^4+32*cos(f*x+e)^6*a^3*b-36*cos(f*x+e)^6*a^2*b^2+16*
cos(f*x+e)^6*a*b^3-2*cos(f*x+e)^6*b^4+15*cos(f*x+e)^4*a^4-42*cos(f*x+e)^4*a
^2*b^2+24*cos(f*x+e)^4*a*b^3+3*cos(f*x+e)^4*b^4+60*cos(f*x+e)^2*a^3*b+60*co
s(f*x+e)^2*a^2*b^2-108*cos(f*x+e)^2*a*b^3-12*cos(f*x+e)^2*b^4+40*a^2*b^2+80
*a*b^3+8*b^4)*4^(1/2)*a^7/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^
(5/2)/cos(f*x+e)^5/((-b*(a-b))^(1/2)+a-b)^7/((-b*(a-b))^(1/2)-a+b)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(238) = 476$.
time = 0.30, size = 558, normalized size = 2.25

$$-\frac{\frac{15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{3\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-20\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3+90\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b^2\cos(fx+e)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5}-\frac{10\left(\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-9\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{5\left(12\left(a-b+\frac{b}{\cos(fx+e)^2}\right)b^3\cos(fx+e)^2-b^4\right)}{\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}+\frac{10\left(9\left(a-b+\frac{b}{\cos(fx+e)^2}\right)b^2\cos(fx+e)^2-b^3\right)}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}+\frac{5\left(6\left(a-b+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2\right)}{\left(a^3-3a^2b+3ab^2-b^3\right)\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) + (3*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a - b
 + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a - b + b/cos(f*x + e
)^2)*b^2*cos(f*x + e))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
 b^5) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a - b
+ b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4) + 5*(12*(a - b + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(a - b + b/cos(f*x + e)^2
)^(3/2)*cos(f*x + e)^3) + 10*(9*(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e)
^2 - b^3)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a - b + b/cos(f*x +
 e)^2)^(3/2)*cos(f*x + e)^3) + 5*(6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x +
e)^2 - b^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/
2)*cos(f*x + e)^3))/f
```

Fricas [A]
time = 5.59, size = 379, normalized size = 1.53

$$-\frac{\left(3\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\cos(fx+e)^9-2\left(5a^4-16a^3b+18a^2b^2-8ab^3+b^4\right)\cos(fx+e)^7+3\left(5a^4-14a^2b^2+8ab^3+b^4\right)\cos(fx+e)^5+12\left(5a^3b+5a^2b^2-9ab^3-b^4\right)\cos(fx+e)^3+8\left(5a^2b^2+10ab^3+b^4\right)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{15\left(\left(a^7-7a^6b+21a^5b^2-35a^4b^3+35a^3b^4-21a^2b^5+7ab^6-b^7\right)f\cos(fx+e)^4+2\left(a^6b-6a^5b^2+15a^4b^3-20a^3b^4+15a^2b^5-6ab^6+b^7\right)f\cos(fx+e)^2+\left(a^5b^2-5a^4b^3+10a^3b^4-10a^2b^5+5ab^6-b^7\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 2*(5*
a^4 - 16*a^3*b + 18*a^2*b^2 - 8*a*b^3 + b^4)*cos(f*x + e)^7 + 3*(5*a^4 - 14
*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)^5 + 12*(5*a^3*b + 5*a^2*b^2 - 9*a*b^
3 - b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 10*a*b^3 + b^4)*cos(f*x + e))*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^7 - 7*a^6*b + 21*a^5*b^2
- 35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 +
2*(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7
)*f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b
^6 - b^7)*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5506 vs. 2(238) = 476.
time = 4.38, size = 5506, normalized size = 22.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*(5*((((6*a^16*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 73*a^15*b^4*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1) + 403*a^14*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) -
1326*a^13*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2860*a^12*b^7*sgn(tan(1/2*f
*x + 1/2*e)^2 - 1) - 4147*a^11*b^8*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 3861*a
^10*b^9*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1716*a^9*b^10*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1) - 858*a^8*b^11*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2145*a^7*b^12
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1859*a^6*b^13*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1) + 962*a^5*b^14*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 312*a^4*b^15*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1) + 59*a^3*b^16*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 5*
a^2*b^17*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1/2*f*x + 1/2*e)^2/(a^18*b^2
- 18*a^17*b^3 + 153*a^16*b^4 - 816*a^15*b^5 + 3060*a^14*b^6 - 8568*a^13*b^7
 + 18564*a^12*b^8 - 31824*a^11*b^9 + 43758*a^10*b^10 - 48620*a^9*b^11 + 437
58*a^8*b^12 - 31824*a^7*b^13 + 18564*a^6*b^14 - 8568*a^5*b^15 + 3060*a^4*b^
```

16 - 816*a^3*b^17 + 153*a^2*b^18 - 18*a*b^19 + b^20) - 3*(2*a^16*b^3*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1) - 31*a^15*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2
13*a^14*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 858*a^13*b^6*sgn(tan(1/2*f*x
+ 1/2*e)^2 - 1) + 2236*a^12*b^7*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 3861*a^11
*b^8*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 4147*a^10*b^9*sgn(tan(1/2*f*x + 1/2*
e)^2 - 1) - 1716*a^9*b^10*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2574*a^8*b^11*s
gn(tan(1/2*f*x + 1/2*e)^2 - 1) + 5863*a^7*b^12*sgn(tan(1/2*f*x + 1/2*e)^2 -
 1) - 6149*a^6*b^13*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 4134*a^5*b^14*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1) - 1872*a^4*b^15*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) +
557*a^3*b^16*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 99*a^2*b^17*sgn(tan(1/2*f*x
+ 1/2*e)^2 - 1) + 8*a*b^18*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^18*b^2 - 18*
a^17*b^3 + 153*a^16*b^4 - 816*a^15*b^5 + 3060*a^14*b^6 - 8568*a^13*b^7 + 18
564*a^12*b^8 - 31824*a^11*b^9 + 43758*a^10*b^10 - 48620*a^9*b^11 + 43758*a^
8*b^12 - 31824*a^7*b^13 + 18564*a^6*b^14 - 8568*a^5*b^15 + 3060*a^4*b^16 -
816*a^3*b^17 + 153*a^2*b^18 - 18*a*b^19 + b^20))*tan(1/2*f*x + 1/2*e)^2 - 3
*(2*a^16*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 31*a^15*b^4*sgn(tan(1/2*f*x
+ 1/2*e)^2 - 1) + 213*a^14*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 858*a^13*b
^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2236*a^12*b^7*sgn(tan(1/2*f*x + 1/2*e)
^2 - 1) - 3861*a^11*b^8*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 4147*a^10*b^9*sgn
(tan(1/2*f*x + 1/2*e)^2 - 1) - 1716*a^9*b^10*sgn(tan(1/2*f*x + 1/2*e)^2 - 1
) - 2574*a^8*b^11*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 5863*a^7*b^12*sgn(tan(1
/2*f*x + 1/2*e)^2 - 1) - 6149*a^6*b^13*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 41
34*a^5*b^14*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1872*a^4*b^15*sgn(tan(1/2*f*x
 + 1/2*e)^2 - 1) + 557*a^3*b^16*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 99*a^2*b^
17*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 8*a*b^18*sgn(tan(1/2*f*x + 1/2*e)^2 -
1))/(a^18*b^2 - 18*a^17*b^3 + 153*a^16*b^4 - 816*a^15*b^5 + 3060*a^14*b^6 -
 8568*a^13*b^7 + 18564*a^12*b^8 - 31824*a^11*b^9 + 43758*a^10*b^10 - 48620*
a^9*b^11 + 43758*a^8*b^12 - 31824*a^7*b^13 + 18564*a^6*b^14 - 8568*a^5*b^15
 + 3060*a^4*b^16 - 816*a^3*b^17 + 153*a^2*b^18 - 18*a*b^19 + b^20))*tan(1/2
*f*x + 1/2*e)^2 + (6*a^16*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 73*a^15*b^4
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 403*a^14*b^5*sgn(tan(1/2*f*x + 1/2*e)^2
- 1) - 1326*a^13*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2860*a^12*b^7*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1) - 4147*a^11*b^8*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) +
 3861*a^10*b^9*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1716*a^9*b^10*sgn(tan(1/2*
f*x + 1/2*e)^2 - 1) - 858*a^8*b^11*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2145*a
^7*b^12*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1859*a^6*b^13*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1) + 962*a^5*b^14*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 312*a^4*b^15*
sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 59*a^3*b^16*sgn(tan(1/2*f*x + 1/2*e)^2 -
1) - 5*a^2*b^17*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^18*b^2 - 18*a^17*b^3 +
153*a^16*b^4 - 816*a^15*b^5 + 3060*a^14*b^6 - 8568*a^13*b^7 + 18564*a^12*b^
8 - 31824*a^11*b^9 + 43758*a^10*b^10 - 48620*a^9*b^11 + 43758*a^8*b^12 - 31
824*a^7*b^13 + 18564*a^6*b^14 - 8568*a^5*b^15 + 3060*a^4*b^16 - 816*a^3*b^1
7 + 153*a^2*b^18 - 18*a*b^19 + b^20))/(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) + 4*(30*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +

```
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^9*a*b + 15*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^9*b^2 + 330*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))^8*a^(3/2)*b - 15*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))^8*sqrt(a)*b^2 + 320*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^7*a^3 - 200*(sqrt(a)*tan(1/2*f*...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+f\,x)^5}{\left(b\tan(e+f\,x)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.141 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=168

$$-\frac{(a+b)\cos(e+fx)}{(a-b)^2 f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}-\frac{4b(a+b)\sec(e+f}{3(a-b)^3 f\left(a-b+b\sec^2(e\right.}$$

```
[Out] -(a+b)*cos(f*x+e)/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(3/2)+1/3*cos(f*x+e)^3/(a-
b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-4/3*b*(a+b)*sec(f*x+e)/(a-b)^3/f/(a-b+b*sec
(f*x+e)^2)^(3/2)-8/3*b*(a+b)*sec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)^(1/2
)
```

Rubi [A]
time = 0.11, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3745, 464, 277, 198, 197}

$$-\frac{8b(a+b)\sec(e+fx)}{3f(a-b)^4\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b(a+b)\sec(e+fx)}{3f(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^{3/2}}+\frac{\cos^3(e+fx)}{3f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{(a+b)\cos(e+fx)}{f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a + b)*Cos[e + f*x])/((a - b)^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2))) +
Cos[e + f*x]^3/(3*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(a + b
)*Sec[e + f*x])/(3*(a - b)^3*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(a
+ b)*Sec[e + f*x])/(3*(a - b)^4*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a-b+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{x^2\left(a-b+bx^2\right)^{5/2}}\,dx, x, \text{s}\right.}{(a-b)f}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3}}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3}}$$

$$= -\frac{(a+b)\cos(e+fx)}{(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3}}$$

Mathematica [A]
time = 1.56, size = 205, normalized size = 1.22

$$-\frac{\cos(e+fx)\left(26a^3+186a^2b+190ab^2+110b^3+3(11a^3+63a^2b-31ab^2-43b^3)\cos(2(e+fx))+6(a-b)^2(a+3b)\cos(4(e+fx))-a^3\cos(6(e+fx))+3a^2b\cos(6(e+fx))-3ab^2\cos(6(e+fx))+b^3\cos(6(e+fx))\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{24\sqrt{2}\,(a-b)^4f(a+b+(a-b)\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/24*(Cos[e + f*x]*(26*a^3 + 186*a^2*b + 190*a*b^2 + 110*b^3 + 3*(11*a^3 +
 63*a^2*b - 31*a*b^2 - 43*b^3)*Cos[2*(e + f*x)] + 6*(a - b)^2*(a + 3*b)*Cos
[4*(e + f*x)] - a^3*Cos[6*(e + f*x)] + 3*a^2*b*Cos[6*(e + f*x)] - 3*a*b^2*C
os[6*(e + f*x)] + b^3*Cos[6*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*
x)])*Sec[e + f*x]^2])/(Sqrt[2]*(a - b)^4*f*(a + b + (a - b)*Cos[2*(e + f*x)
])^2)
```

Maple [A]
time = 0.81, size = 262, normalized size = 1.56

method	result
default	$-\frac{(a-b)\left(a\left(\cos^{2}(fx+e)\right)-\left(\cos^{2}(fx+e)\right)b+b\right)\left(\left(\cos^{6}(fx+e)\right)a^{3}-3\left(\cos^{6}(fx+e)\right)a^{2}b+3\left(\cos^{6}(fx+e)\right)a\,b^{2}-\left(\cos^{6}(fx+e)\right)b^{3}-3\left(\cos^{4}(\right.\right.}{6f\left(\frac{a\left(\cos^{2}(fx+e)\right)-\left(\cos^{2}(fx+e)\right)b+b}{\cos(fx+e)^{2}}\right)^{\frac{5}{2}}\cos(fx+e)^{5}\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(a-b)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(cos(f*x+e)^6*a^3-3*cos(f*x+
e)^6*a^2*b+3*cos(f*x+e)^6*a*b^2-cos(f*x+e)^6*b^3-3*cos(f*x+e)^4*a^3+3*cos(f
*x+e)^4*a^2*b+3*cos(f*x+e)^4*a*b^2-3*cos(f*x+e)^4*b^3-12*cos(f*x+e)^2*a^2*b
+12*cos(f*x+e)^2*b^3-8*a*b^2-8*b^3)*4^(1/2)*a^5/((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/((-b*(a-b))^(1/2)+a-b)^5/((-b*(a-b
))^(1/2)-a+b)^5
```

Maxima [A]
time = 0.29, size = 322, normalized size = 1.92

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos(fx+e)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{\left(a-b+\frac{b}{\cos(fx+e)^{2}}\right)^{\frac{3}{2}}\cos(fx+e)^{3}-9\sqrt{a-b+\frac{b}{\cos\left(fx+e\right)^{2}}}\,b\cos(fx+e)}{a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}}+\frac{9\left(a-b+\frac{b}{\cos(fx+e)^{2}}\right)b^{2}\cos(fx+e)^{2}-b^{3}}{(a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4})\left(a-b+\frac{b}{\cos(fx+e)^{2}}\right)^{\frac{3}{2}}\cos(fx+e)^{3}}+\frac{6\left(a-b+\frac{b}{\cos(fx+e)^{2}}\right)b\cos(fx+e)^{2}-b^{2}}{(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3})\left(a-b+\frac{b}{\cos(fx+e)^{2}}\right)^{\frac{3}{2}}\cos(fx+e)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) - ((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a - b
+ b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4) + (9*(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e)^2 - b^3)/((a^4 - 4*a^
3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x +
 e)^3) + (6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3))/f
```

Fricas [A]
time = 6.22, size = 278, normalized size = 1.65

$$\frac{\left((a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3})\cos\left(fx+e\right)^{7}-3\,(a^{3}-a^{2}b-ab^{2}+b^{3})\cos\left(fx+e\right)^{5}-12\,(a^{2}b-b^{3})\cos\left(fx+e\right)^{3}-8\,(ab^{2}+b^{3})\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\left((a^{6}-6\,a^{5}b+15\,a^{4}b^{2}-20\,a^{3}b^{3}+15\,a^{2}b^{4}-6\,ab^{5}+b^{6})f\cos\left(fx+e\right)^{4}+2\,(a^{5}b-5\,a^{4}b^{2}+10\,a^{3}b^{3}-10\,a^{2}b^{4}+5\,ab^{5}-b^{6})f\cos\left(fx+e\right)^{2}+(a^{4}b^{2}-4\,a^{3}b^{3}+6\,a^{2}b^{4}-4\,ab^{5}+b^{6})f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 3*(a^3 - a^2*b - a*b^
2 + b^3)*cos(f*x + e)^5 - 12*(a^2*b - b^3)*cos(f*x + e)^3 - 8*(a*b^2 + b^3)
*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^6 - 6*
a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*x + e
)^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*f*cos
(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1511 vs. 2(162) = 324.
time = 2.81, size = 1511, normalized size = 8.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/3*(a*b + b^2)*sgn(f)*sgn(cos(f*x + e))/(a^4*sqrt(b)*abs(f) - 4*a^3*b^(3/2
)*abs(f) + 6*a^2*b^(5/2)*abs(f) - 4*a*b^(7/2)*abs(f) + b^(9/2)*abs(f)) + 1/
3*((a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*a^8*f^2*sgn(f)^2*sgn(cos
(f*x + e))^2 - 3*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a^9*f^2*sgn(
f)^2*sgn(cos(f*x + e))^2 - 8*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2
)*a^7*b*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 + 18*sqrt(a*cos(f*x + e)^2 - b*cos
(f*x + e)^2 + b)*a^8*b*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 + 28*(a*cos(f*x + e
)^2 - b*cos(f*x + e)^2 + b)^(3/2)*a^6*b^2*f^2*sgn(f)^2*sgn(cos(f*x + e))^2
- 36*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a^7*b^2*f^2*sgn(f)^2*sgn
(cos(f*x + e))^2 - 56*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*a^5*b
^3*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 + 70*(a*cos(f*x + e)^2 - b*cos(f*x + e)
^2 + b)^(3/2)*a^4*b^4*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 + 126*sqrt(a*cos(f*x
 + e)^2 - b*cos(f*x + e)^2 + b)*a^5*b^4*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 -
56*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*a^3*b^5*f^2*sgn(f)^2*sgn
(cos(f*x + e))^2 - 252*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a^4*b^
5*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 + 28*(a*cos(f*x + e)^2 - b*cos(f*x + e)^
```

```
2 + b)^(3/2)*a^2*b^6*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 + 252*sqrt(a*cos(f*x
+ e)^2 - b*cos(f*x + e)^2 + b)*a^3*b^6*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 - 8
*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2)*a*b^7*f^2*sgn(f)^2*sgn(cos
(f*x + e))^2 - 144*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a^2*b^7*f^
2*sgn(f)^2*sgn(cos(f*x + e))^2 + (a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^
(3/2)*b^8*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 + 45*sqrt(a*cos(f*x + e)^2 - b*c
os(f*x + e)^2 + b)*a*b^8*f^2*sgn(f)^2*sgn(cos(f*x + e))^2 - 6*sqrt(a*cos(f*
x + e)^2 - b*cos(f*x + e)^2 + b)*b^9*f^2*sgn(f)^2*sgn(cos(f*x + e))^2)/(a^1
2*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 - 12*a^11*b*f^2*abs(f)*sgn(f)^3*s
gn(cos(f*x + e))^3 + 66*a^10*b^2*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 -
220*a^9*b^3*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 + 495*a^8*b^4*f^2*abs(f
)*sgn(f)^3*sgn(cos(f*x + e))^3 - 792*a^7*b^5*f^2*abs(f)*sgn(f)^3*sgn(cos(f*
x + e))^3 + 924*a^6*b^6*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 - 792*a^5*b
^7*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 + 495*a^4*b^8*f^2*abs(f)*sgn(f)^
3*sgn(cos(f*x + e))^3 - 220*a^3*b^9*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3
 + 66*a^2*b^10*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e))^3 - 12*a*b^11*f^2*abs(
f)*sgn(f)^3*sgn(cos(f*x + e))^3 + b^12*f^2*abs(f)*sgn(f)^3*sgn(cos(f*x + e)
)^3) - 1/3*(6*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*a*b + 3*(a*cos(f*x
+ e)^2 - b*cos(f*x + e)^2 + b)*b^2 - a*b^2)/((a^4*abs(f)*sgn(f)*sgn(cos(f*x
 + e)) - 4*a^3*b*abs(f)*sgn(f)*sgn(cos(f*x + e)) + 6*a^2*b^2*abs(f)*sgn(f)*
sgn(cos(f*x + e)) - 4*a*b^3*abs(f)*sgn(f)*sgn(cos(f*x + e)) + b^4*abs(f)*sg
n(f)*sgn(cos(f*x + e)))*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^(3/2))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (e+f\,x)^3}{{\left(b\tan (e+f\,x)^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.142 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}-\frac{4b\sec(e+fx)}{3(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}-\frac{8b\sec(e+fx)}{3(a-b)^3f\sqrt{a-b+b\sec^2(e}$$

```
[Out] -cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-4/3*b*sec(f*x+e)/(a-b)^2/f/(
a-b+b*sec(f*x+e)^2)^(3/2)-8/3*b*sec(f*x+e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^(
1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3745, 277, 198, 197}

$$-\frac{8b\sec(e+fx)}{3f(a-b)^3\sqrt{a+b\sec^2(e+fx)-b}}-\frac{4b\sec(e+fx)}{3f(a-b)^2\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\cos(e+fx)}{f(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]/((a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2))) - (4*b*Sec[e +
 f*x])/(3*(a - b)^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (8*b*Sec[e + f*x]
)/(3*(a - b)^3*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a-b+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(4b)\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{5/2}}\,dx, x, \text{sec}\right.}{(a-b)f}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3(a-b)^2f\left(a-b+b\sec^2(e+fx)\right.}$$

$$= -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3(a-b)^2f\left(a-b+b\sec^2(e+fx)\right.}$$

Mathematica [A]
time = 1.31, size = 124, normalized size = 1.05

$$-\frac{\cos(e+fx)\left((3a+5b)^2+12(a^2+2ab-3b^2)\cos(2(e+fx))+3(a-b)^2\cos(4(e+fx))\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{6\sqrt{2}\,(a-b)^3f(a+b+(a-b)\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*(Cos[e + f*x]*((3*a + 5*b)^2 + 12*(a^2 + 2*a*b - 3*b^2)*Cos[2*(e + f*x
)] + 3*(a - b)^2*Cos[4*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*
Sec[e + f*x]^2])/(Sqrt[2]*(a - b)^3*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Maple [A]
time = 0.12, size = 147, normalized size = 1.25

method	result
default	$-\frac{\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)\left(3\left(\cos^4(fx+e)\right)a^2-6\left(\cos^4(fx+e)\right)ab+3\left(\cos^4(fx+e)\right)b^2+12\left(\cos^2(fx+e)\right)ab-12\left(\cos^2(fx+\right.\right.}{3f\left(\frac{a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5(a-b)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)*(3*cos(f*x+e)^4*a^2-6*cos(f*x+e)^4
*a*b+3*cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-12*cos(f*x+e)^2*b^2+8*b^2)/((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5/(a-b)^3
```

Maxima [A]
time = 0.28, size = 141, normalized size = 1.19

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{6\left(a-b+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2}{(a^3-3a^2b+3ab^2-b^3)\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) + (6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3))/f
```

Fricas [A]
time = 2.67, size = 209, normalized size = 1.77

$$-\frac{(3(a^2-2ab+b^2)\cos(fx+e)^5+12(ab-b^2)\cos(fx+e)^3+8b^2\cos(fx+e))\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3((a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5)f\cos(fx+e)^4+2(a^4b-4a^3b^2+6a^2b^3-4ab^4+b^5)f\cos(fx+e)^2+(a^3b^2-3a^2b^3+3ab^4-b^5)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 + 12*(a*b - b^2)*cos(f*x + e)^3
+ 8*b^2*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a
^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^4 +
2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^3*b
^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [A]
time = 2.56, size = 223, normalized size = 1.89

$$-\frac{f^4\left(\frac{3\sqrt{a\cos\left(fx+e\right)^2-b\cos\left(fx+e\right)^2+b}}{a|f|\mathrm{sgn}(f)\mathrm{sgn}(\cos(fx+e))-b|f|\mathrm{sgn}(f)\mathrm{sgn}(\cos(fx+e))}+\frac{6\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)b-b^2}{(a|f|\mathrm{sgn}(f)\mathrm{sgn}(\cos(fx+e))-b|f|\mathrm{sgn}(f)\mathrm{sgn}(\cos(fx+e)))\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^{\frac{3}{2}}}\right)}{3\left(af^2-bf^2\right)^2}+\frac{8\sqrt{b}\,\mathrm{sgn}(f)\,\mathrm{sgn}(\cos\left(fx+e\right))}{3\left(a^3|f|-3\,a^2b|f|+3\,ab^2|f|-b^3|f|\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*f^4*(3*sqrt(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)/(a*abs(f)*sgn(f)*
sgn(cos(f*x + e)) - b*abs(f)*sgn(f)*sgn(cos(f*x + e))) + (6*(a*cos(f*x + e)
^2 - b*cos(f*x + e)^2 + b)*b - b^2)/((a*abs(f)*sgn(f)*sgn(cos(f*x + e)) - b
*abs(f)*sgn(f)*sgn(cos(f*x + e)))*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)
^(3/2)))/(a*f^2 - b*f^2)^2 + 8/3*sqrt(b)*sgn(f)*sgn(cos(f*x + e))/(a^3*abs(
f) - 3*a^2*b*abs(f) + 3*a*b^2*abs(f) - b^3*abs(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.143 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{5/2}f}-\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}-\frac{(5a-3b)b\sec(e+}{3a^2(a-b)^2f\sqrt{a-b+b\sec}}$$

```
[Out] -arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*sec
(f*x+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/3*(5*a-3*b)*b*sec(f*x+e)/a^2
/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3745, 425, 541, 12, 385, 213}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b(5a-3b)\sec(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{b\sec(e+fx)}{3af(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(a^(5/2)*f
)) - (b*Sec[e + f*x])/(3*a*(a - b)*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((
5*a - 3*b)*b*Sec[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Sec[e + f*x]^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a-b+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-b-2bx^2}{(-1+x^2)(a-b+bx^2)^{3/2}}\,dx, x,\right.}{3a(a-b)f}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+}}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+}}$$

$$= -\frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 300 vs. 2(136) = 272.

time = 4.60, size = 300, normalized size = 2.21

$$\frac{\cos(e+fx)\left(-\frac{2\sqrt{2}\sqrt{a}\,b\left(6a^2+ab-3b^2+3(2a^2-3ab+b^2)\cos(2(e+fx))\right)}{(a-b)^2\left(a+b+(a-b)\cos(2(e+fx))\right)^2}+\frac{3\left(2\tanh^{-1}\left(\frac{\tan^2\left(\frac{1}{2}(e+fx)\right)-\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)+\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\left(a+b+(a-b)\cos(2(e+fx))\right)\sec^4\left(\frac{1}{2}(e+fx)\right)}}\right)\sqrt{\left(a+b+(a-b)\cos(2(e+fx))\right)\sec^2(e+fx)}}{6a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]*((-2*Sqrt[2]*Sqrt[a]*b*(6*a^2 + a*b - 3*b^2 + 3*(2*a^2 - 3*a*
b + b^2)*Cos[2*(e + f*x)]))/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2
) + (3*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1
+ Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sq
rt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]])*Sec[(e
 + f*x)/2]^2)/Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])*
Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(6*a^(5/2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 27447 vs. 2(122) = 244.

time = 3.79, size = 27448, normalized size = 201.82

method	result	size
default	Expression too large to display	27448

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(128) = 256.
time = 3.04, size = 720, normalized size = 5.29

$$\left[\frac{3\left(\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\cos(fx+e)^4+a^2b^2-2ab^3+b^4+2\left(a^3b-3a^2b^2+3ab^3-b^4\right)\cos(fx+e)^2\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos(fx+e)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)-2\left(3\left(2a^3b-3a^2b^2+ab^3\right)\cos(fx+e)^3+\left(5a^2b^2-3ab^3\right)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{6\left(\left(a^7-4a^6b+6a^5b^2-4a^4b^3+a^3b^4\right)f\cos(fx+e)^4+2\left(a^6b-3a^5b^2+3a^4b^3-a^3b^4\right)f\cos(fx+e)^2+\left(a^5b^2-2a^4b^3+a^3b^4\right)f\right)}, \frac{3\left(\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\cos(fx+e)^4+a^2b^2-2ab^3+b^4+2\left(a^3b-3a^2b^2+3ab^3-b^4\right)\cos(fx+e)^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a}\right)-\left(3\left(2a^3b-3a^2b^2+ab^3\right)\cos(fx+e)^3+\left(5a^2b^2-3ab^3\right)\cos(fx+e)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(\left(a^7-4a^6b+6a^5b^2-4a^4b^3+a^3b^4\right)f\cos(fx+e)^4+2\left(a^6b-3a^5b^2+3a^4b^3-a^3b^4\right)f\cos(fx+e)^2+\left(a^5b^2-2a^4b^3+a^3b^4\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*b
^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)^2)*
sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) - 2*(
3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 - 3*a*b^3)*cos(
f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 4*a^6*
b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^
2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)
*f), 1/3*(3*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a
^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)
^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/a) - (3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*
a^2*b^2 - 3*a*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^
4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2
 - 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+f\,x)\,\left(b\tan(e+f\,x)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(5/2)),x)
[Out] int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(5/2)), x)
```

3.144 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{(a-5b)\tanh^{-1}\left(\frac{\sqrt{a}\ \sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{7/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}}-\frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+\right.}$$

```
[Out] -1/2*(a-5*b)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(7/2)
/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(3/2)-5/6*b*sec(f*x+e
)/a^2/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/6*(13*a-15*b)*b*sec(f*x+e)/a^3/(a-b)/f
/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3745, 482, 541, 12, 385, 213}

$$-\frac{(a-5b)\tanh^{-1}\left(\frac{\sqrt{a}\ \sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{7/2}f}-\frac{b(13a-15b)\sec(e+fx)}{6a^3f(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{5b\sec(e+fx)}{6a^2f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\cot(e+fx)\csc(e+fx)}{2af\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/2*((a - 5*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^
2]])/(a^(7/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*(a - b + b*Sec[e + f*
x]^2)^(3/2)) - (5*b*Sec[e + f*x])/(6*a^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2)
) - ((13*a - 15*b)*b*Sec[e + f*x])/(6*a^3*(a - b)*f*Sqrt[a - b + b*Sec[e +
f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a-b+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\,(a-b+b\sec^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-b-4bx^2}{(-1+x^2)(a-b+bx^2)^{5/2}}\,dx, x, \sec(e\right.}{2af}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\,(a-b+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\,(a-b+b\sec^2(e+fx))^{3/2}} + \text{Sub}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\,(a-b+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\,(a-b+b\sec^2(e+fx))^{3/2}} - \frac{}{6a^3}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\,(a-b+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\,(a-b+b\sec^2(e+fx))^{3/2}} - \frac{}{6a^3}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\,(a-b+b\sec^2(e+fx))^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\,(a-b+b\sec^2(e+fx))^{3/2}} - \frac{}{6a^3}$$

$$= -\frac{(a-5b)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cot(e+fx)\csc(e+}{2af\,(a-b+b\sec^2(e+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 380 vs. $2(177) = 354$.
time = 2.69, size = 380, normalized size = 2.15

$$\frac{\frac{\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{1+\cos(2(e+fx))}}\left(8ab^2\cos(e+fx)-24(a-b)b\cos(e+fx)(a+b+(a-b)\cos(2(e+fx)))-3(a-b)(a+b+(a-b)\cos(2(e+fx)))^2\cot(e+fx)\csc(e+fx)\right)}{3a^3(a-b)(a+b+(a-b)\cos(2(e+fx)))^2} + \frac{(a-5b)\cos(e+fx)\left(2\tanh^{-1}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\frac{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)+\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{2a^{7/2}\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^4\left(\frac{1}{2}(e+fx)\right)}}}{2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(8*a*b^2*
Cos[e + f*x] - 24*(a - b)*b*Cos[e + f*x]*(a + b + (a - b)*Cos[2*(e + f*x)])
 - 3*(a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]
))/(3*a^3*(a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2) + ((a - 5*b)*Cos[e
+ f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1
+ Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sq
rt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]])*Sec[(e
```

```
 + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(2*a^
(7/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4]))/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 38485 vs. $2(157) = 314$.
time = 5.31, size = 38486, normalized size = 217.44

method	result	size
default	Expression too large to display	38486

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(166) = 332$.
time = 4.20, size = 919, normalized size = 5.19

$$\left[-\frac{3\left(\left(a^4-8a^3b+18a^2b^2-16ab^3+5b^4\right)\cos\left(fx+e\right)^6-\left(a^4-10a^3b+32a^2b^2-38ab^3+15b^4\right)\cos\left(fx+e\right)^4-a^2b^2+6ab^3-5b^4-\left(2a^3b-15a^2b^2+28ab^3-15b^4\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos\left(fx+e\right)^2+2\sqrt{a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+a+b\right)}{\cos\left(fx+e\right)^2-1}\right)-2\left(3\left(a^4-7a^3b+11a^2b^2-5ab^3\right)\cos\left(fx+e\right)^5+2\left(9a^3b-23a^2b^2+15ab^3\right)\cos\left(fx+e\right)^3+\left(13a^2b^2-15ab^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{12\left(\left(a^7-3a^6b+3a^5b^2-a^4b^3\right)f\cos\left(fx+e\right)^6-\left(a^7-5a^6b+7a^5b^2-3a^4b^3\right)f\cos\left(fx+e\right)^4-\left(2a^6b-5a^5b^2+3a^4b^3\right)f\cos\left(fx+e\right)^2-\left(a^5b^2-a^4b^3\right)f\right)}, \frac{3\left(\left(a^4-8a^3b+18a^2b^2-16ab^3+5b^4\right)\cos\left(fx+e\right)^6-\left(a^4-10a^3b+32a^2b^2-38ab^3+15b^4\right)\cos\left(fx+e\right)^4-a^2b^2+6ab^3-5b^4-\left(2a^3b-15a^2b^2+28ab^3-15b^4\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a}\right)+\left(3\left(a^4-7a^3b+11a^2b^2-5ab^3\right)\cos\left(fx+e\right)^5+2\left(9a^3b-23a^2b^2+15ab^3\right)\cos\left(fx+e\right)^3+\left(13a^2b^2-15ab^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{6\left(\left(a^7-3a^6b+3a^5b^2-a^4b^3\right)f\cos\left(fx+e\right)^6-\left(a^7-5a^6b+7a^5b^2-3a^4b^3\right)f\cos\left(fx+e\right)^4-\left(2a^6b-5a^5b^2+3a^4b^3\right)f\cos\left(fx+e\right)^2-\left(a^5b^2-a^4b^3\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((a^4 - 8*a^3*b + 18*a^2*b^2 - 16*a*b^3 + 5*b^4)*cos(f*x + e)^6 -
 (a^4 - 10*a^3*b + 32*a^2*b^2 - 38*a*b^3 + 15*b^4)*cos(f*x + e)^4 - a^2*b^2
 + 6*a*b^3 - 5*b^4 - (2*a^3*b - 15*a^2*b^2 + 28*a*b^3 - 15*b^4)*cos(f*x + e
)^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))
- 2*(3*(a^4 - 7*a^3*b + 11*a^2*b^2 - 5*a*b^3)*cos(f*x + e)^5 + 2*(9*a^3*b -
 23*a^2*b^2 + 15*a*b^3)*cos(f*x + e)^3 + (13*a^2*b^2 - 15*a*b^3)*cos(f*x +
e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 3*a^6*b + 3*
a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 - (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4*b^
3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2 -
(a^5*b^2 - a^4*b^3)*f), 1/6*(3*((a^4 - 8*a^3*b + 18*a^2*b^2 - 16*a*b^3 + 5*
```

```
b^4)*cos(f*x + e)^6 - (a^4 - 10*a^3*b + 32*a^2*b^2 - 38*a*b^3 + 15*b^4)*cos
(f*x + e)^4 - a^2*b^2 + 6*a*b^3 - 5*b^4 - (2*a^3*b - 15*a^2*b^2 + 28*a*b^3
- 15*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + (3*(a^4 - 7*a^3*b + 11*a^2*b^2
- 5*a*b^3)*cos(f*x + e)^5 + 2*(9*a^3*b - 23*a^2*b^2 + 15*a*b^3)*cos(f*x + e
)^3 + (13*a^2*b^2 - 15*a*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6
 - (a^7 - 5*a^6*b + 7*a^5*b^2 - 3*a^4*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*
a^5*b^2 + 3*a^4*b^3)*f*cos(f*x + e)^2 - (a^5*b^2 - a^4*b^3)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(e+f\,x)^3\,\left(b\,\tan(e+f\,x)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(5/2)), x)
```

3.145 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{(3a^2-30ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{9/2}f}-\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+}{4af\,(a-b}$$

```
[Out] -1/8*(3*a^2-30*a*b+35*b^2)*arctanh(sec(f*x+e)*a^(1/2)/(a-b+b*sec(f*x+e)^2)^
(1/2))/a^(9/2)/f-1/8*(5*a-7*b)*cot(f*x+e)*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e
)^2)^(3/2)-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/24*
(23*a-35*b)*b*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)^(3/2)-5/24*(11*a-21*b)*
b*sec(f*x+e)/a^4/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3745, 481, 541, 12, 385, 213}

$$-\frac{5b(11a-21b)\sec(e+fx)}{24a^4f\sqrt{a+b\sec^2(e+fx)-b}}-\frac{b(23a-35b)\sec(e+fx)}{24a^3f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{(3a^2-30ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a}\,\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{8a^{9/2}f}-\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a+b\sec^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/8*((3*a^2 - 30*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b +
 b*Sec[e + f*x]^2]])/(a^(9/2)*f) - ((5*a - 7*b)*Cot[e + f*x]*Csc[e + f*x])/
(8*a^2*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (Cot[e + f*x]^3*Csc[e + f*x])/
(4*a*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((23*a - 35*b)*b*Sec[e + f*x])/(
24*a^3*f*(a - b + b*Sec[e + f*x]^2)^(3/2)) - (5*(11*a - 21*b)*b*Sec[e + f*x
])/(24*a^4*f*Sqrt[a - b + b*Sec[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^3(a-b+bx^2)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a+b-2(2a-3b)x^2}{(-1+x^2)^2(a-b+bx^2)^{5/2}}\,dx, x, \sec(e\right.}{4af}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \text{Sul}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{24a}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{24a}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{24a}$$

$$= -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{}{24a}$$

$$= -\frac{(3a^2-30ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a-7b)\cot(}{8a^2f\,(a-b+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1132 vs. $2(237) = 474$.

time = 6.60, size = 1132, normalized size = 4.78

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*((4*b^2*Cos[e + f*x])/(3*a^3*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e
+ f*x)])^2) - (2*(2*a*b*Cos[e + f*x] - 3*b^2*Cos[e + f*x]))/(a^4*(a + b + a
*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])) + ((-3*a*Cos[e + f*x] + 11*b*Cos[e
 + f*x])*Csc[e + f*x]^2)/(8*a^4) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4*a^3))/
```

```
f + ((3*a^2 - 30*a*b + 35*b^2)*(((1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*
x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos
[2*(e + f*x)])]*(4*Sqrt[a]*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) +
Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])]
- Sqrt[b]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(
-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^2 +
 Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]))*(-
1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]
^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt[a]
*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]
^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]) - ((1
+ Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a +
 b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]*ArcTanh[(
-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1
+ Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])] + Sqrt[b]*(2*ArcTanh[Tan[(e + f*x)/2
]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]]
 + Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2
 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]))*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e
+ f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/
(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*
(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 +
 a*(-1 + Tan[(e + f*x)/2]^2)^2])))/(8*a^4*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 49916 vs. $2(213) = 426$.
time = 6.61, size = 49917, normalized size = 210.62

method	result	size
default	Expression too large to display	49917

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(225) = 450$.
time = 4.10, size = 1073, normalized size = 4.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((3*a^4 - 36*a^3*b + 98*a^2*b^2 - 100*a*b^3 + 35*b^4)*cos(f*x + e)
^8 - 2*(3*a^4 - 39*a^3*b + 131*a^2*b^2 - 165*a*b^3 + 70*b^4)*cos(f*x + e)^6
 + (3*a^4 - 48*a^3*b + 233*a^2*b^2 - 390*a*b^3 + 210*b^4)*cos(f*x + e)^4 +
3*a^2*b^2 - 30*a*b^3 + 35*b^4 + 2*(3*a^3*b - 36*a^2*b^2 + 95*a*b^3 - 70*b^4
)*cos(f*x + e)^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt((
(a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x
+ e)^2 - 1)) + 2*(3*(3*a^4 - 33*a^3*b + 65*a^2*b^2 - 35*a*b^3)*cos(f*x + e)
^7 - (15*a^4 - 177*a^3*b + 445*a^2*b^2 - 315*a*b^3)*cos(f*x + e)^5 - (78*a^
3*b - 305*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^3 - 5*(11*a^2*b^2 - 21*a*b^3)*c
os(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 2*a
^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2
)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*
b - 2*a^5*b^2)*f*cos(f*x + e)^2), 1/24*(3*((3*a^4 - 36*a^3*b + 98*a^2*b^2 -
 100*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(3*a^4 - 39*a^3*b + 131*a^2*b^2 - 1
65*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (3*a^4 - 48*a^3*b + 233*a^2*b^2 - 390*a
*b^3 + 210*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 30*a*b^3 + 35*b^4 + 2*(3*a^3*b
 - 36*a^2*b^2 + 95*a*b^3 - 70*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-a)
*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/a) + (3*(3*
a^4 - 33*a^3*b + 65*a^2*b^2 - 35*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 177*a^3*
b + 445*a^2*b^2 - 315*a*b^3)*cos(f*x + e)^5 - (78*a^3*b - 305*a^2*b^2 + 315
*a*b^3)*cos(f*x + e)^3 - 5*(11*a^2*b^2 - 21*a*b^3)*cos(f*x + e))*sqrt(((a -
 b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 2*a^6*b + a^5*b^2)*f*cos(f
*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^6 + (a
^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*
x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. $2(225) = 450$.

time = 2.20, size = 1728, normalized size = 7.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(((((3*((a^17*b^2 - 2*a^16*b^3 + a^15*b^4)*tan(1/2*f*x + 1/2*e)^2/(a^
18*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*a^17*b^3*sgn(tan(1/2*f*x + 1/2*e
)^2 - 1) + a^16*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)) + (5*a^17*b^2 - 24*a^1
6*b^3 + 33*a^15*b^4 - 14*a^14*b^5)/(a^18*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1
) - 2*a^17*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^16*b^4*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2 - 2*(45*a^17*b^2 - 498*a^16*b^3 + 1
421*a^15*b^4 - 1528*a^14*b^5 + 560*a^13*b^6)/(a^18*b^2*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1) - 2*a^17*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^16*b^4*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2 + 6*(25*a^17*b^2 - 248*a^
16*b^3 + 989*a^15*b^4 - 1894*a^14*b^5 + 1688*a^13*b^6 - 560*a^12*b^7)/(a^18
*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*a^17*b^3*sgn(tan(1/2*f*x + 1/2*e)^
2 - 1) + a^16*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2
 - 3*(35*a^17*b^2 - 102*a^16*b^3 - 365*a^15*b^4 + 1664*a^14*b^5 - 2000*a^13*
b^6 + 768*a^12*b^7)/(a^18*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*a^17*b^3*
sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^16*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))
)*tan(1/2*f*x + 1/2*e)^2 + (27*a^17*b^2 + 264*a^16*b^3 - 1249*a^15*b^4 + 15
98*a^14*b^5 - 640*a^13*b^6)/(a^18*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*a
^17*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^16*b^4*sgn(tan(1/2*f*x + 1/2*e)
^2 - 1)))/(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a)^(3/2) - 24*(3*a^2 - 30*a*b + 35*b^2)*arctan(-(sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a^4*sgn(
tan(1/2*f*x + 1/2*e)^2 - 1)) - 12*(3*a^(5/2) - 30*a^(3/2)*b + 35*sqrt(a)*b^
2)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a + a^(3/2
) - 2*sqrt(a)*b))/(a^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)) + 12*(4*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2 - 20*(sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 22*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))^3*b^2 - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))^2*a^(5/2) + 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
2*a^(3/2)*b - 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
```

```
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 +
24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b - 26*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^2 + 5*a^(7/2) - 12*a^(5/2)
*b)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a
*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^2*a^4*sgn
(tan(1/2*f*x + 1/2*e)^2 - 1)))/f
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(5/2)),x)
[Out] \text{Hanged}
```

3.146 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=246

$$\frac{(3a^2+24ab+8b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{9/2}f}-\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+f}{4(a-b)f\,(a+}$$

```
[Out] 1/8*(3*a^2+24*a*b+8*b^2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(
1/2))/(a-b)^(9/2)/f-5/24*b*(11*a+10*b)*tan(f*x+e)/(a-b)^4/f/(a+b*tan(f*x+e)
^2)^(1/2)-1/8*(5*a+2*b)*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^
(3/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-1/24*b*(
23*a+12*b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = $\{3744, 481, 541, 12, 385, 209\}$

$$\frac{(3a^2+24ab+8b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8f(a-b)^{9/2}}-\frac{5b(11a+10b)\tan(e+fx)}{24f(a-b)^4\sqrt{a+b\tan^2(e+fx)}}-\frac{b(23a+12b)\tan(e+fx)}{24f(a-b)^3\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\sin(e+fx)\cos^3(e+fx)}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(5a+2b)\sin(e+fx)\cos(e+fx)}{8f(a-b)^2\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a^2 + 24*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/(8*(a - b)^(9/2)*f) - ((5*a + 2*b)*Cos[e + f*x]*Sin[e + f*x])
/(8*(a - b)^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x
])/(4*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (b*(23*a + 12*b)*Tan[e + f*
x])/(24*(a - b)^3*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (5*b*(11*a + 10*b)*Tan[
e + f*x])/(24*(a - b)^4*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{a-2(2a+b)x^2}{(1+x^2)^2(a+bx^2)^{5/2}}\,dx, x, \tan(e+\right.}{4(a-b)f}$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} +$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} -$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} -$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} -$$

$$= -\frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} -$$

$$= \frac{(3a^2+24ab+8b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{9/2}f} - \frac{(5a+2b)\cos(e+f}{8(a-b)^2f\,(a+b\,\text{ta}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 3.75, size = 378, normalized size = 1.54

$$-\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-3\sqrt{2}\,ab(3a^2+24ab+8b^2)\left(\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}\right)^{3/2}\left(2(a-b)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)-2a\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sin^2(e+fx)\sin(2(e+fx))-a(a-b)\left(64ab^2\sin(2(e+fx))-64b(3a+2b)(a+b+(a-b)\cos(2(e+fx)))\sin(2(e+fx))-6(4a+7b)(a+b+(a-b)\cos(2(e+fx)))^2\sin(2(e+fx))+3(a-b)(a+b+(a-b)\cos(2(e+fx)))^2\sin(4(e+fx))\right)\right)}{96\sqrt{2}\,a(a-b)^5f(a+b+(a-b)\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/96*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-3*Sqrt[2]*
a*b*(3*a^2 + 24*a*b + 8*b^2)*(((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
```

```
*x]^2)/b)^(3/2)*(2*(a - b)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 2*a*EllipticPi[-(b/(a - b)), Ar
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
1])*Sin[e + f*x]^2*Sin[2*(e + f*x)] - a*(a - b)*(64*a*b^2*Sin[2*(e + f*x)]
- 64*b*(3*a + 2*b)*(a + b + (a - b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)] - 6*
(4*a + 7*b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Sin[2*(e + f*x)] + 3*(a -
b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Sin[4*(e + f*x)])))/(Sqrt[2]*a*(a -
 b)^5*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7942 vs. $2(222) = 444$.
time = 2.51, size = 7943, normalized size = 32.29

method	result	size
default	Expression too large to display	7943

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.147 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=181

$$\frac{(a+4b)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{7/2}f}-\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{5b\tan(e+fx)}{6(a-b)^2f\,(a+b\tan^2(e+f}$$

```
[Out] 1/2*(a+4*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(
7/2)/f-1/6*b*(13*a+2*b)*tan(f*x+e)/a/(a-b)^3/f/(a+b*tan(f*x+e)^2)^(1/2)-1/2
*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-5/6*b*tan(f*x+e)/(a
-b)^2/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3744, 482, 541, 12, 385, 209}

$$\frac{(a+4b)\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f(a-b)^{7/2}}-\frac{b(13a+2b)\tan(e+fx)}{6af(a-b)^3\sqrt{a+b\tan^2(e+fx)}}-\frac{5b\tan(e+fx)}{6f(a-b)^2\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\sin(e+fx)\cos(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(
2*(a - b)^(7/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*(a - b)*f*(a + b*Tan[e
+ f*x]^2)^(3/2)) - (5*b*Tan[e + f*x])/(6*(a - b)^2*f*(a + b*Tan[e + f*x]^2)
^(3/2)) - (b*(13*a + 2*b)*Tan[e + f*x])/(6*a*(a - b)^3*f*Sqrt[a + b*Tan[e +
 f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-4bx^2}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+\right.}{2(a-b)f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \cdots$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \cdots$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \cdots$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \cdots$$

$$= \frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/}}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.92, size = 309, normalized size = 1.71

$$-\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-((a-b)(8ab^2-4b(6a+b)(a+b+(a-b)\cos(2(e+fx)))-3a(a+b+(a-b)\cos(2(e+fx)))^2)\sin(2(e+fx)))-\frac{3ab(a+4b)\left(\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}\right)^{3/2}\left(2(a-b)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)-2a\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sin^2(e+fx)\sin(2(e+fx))}{\sqrt{2}}\right)}{12\sqrt{2}\,a(a-b)^4f(a+b+(a-b)\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/12*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-((a - b)*(
8*a*b^2 - 4*b*(6*a + b)*(a + b + (a - b)*Cos[2*(e + f*x)]) - 3*a*(a + b + (
a - b)*Cos[2*(e + f*x)])^2)*Sin[2*(e + f*x)]) - (3*a*b*(a + 4*b)*(((a + b +
```

```
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*(2*(a - b)*EllipticF[Ar
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
1] - 2*a*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e + f*x]^2*Sin[2*(e + f*x)])/Sq
rt[2]))/(Sqrt[2]*a*(a - b)^4*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2510 vs. $2(161) = 322$.
time = 0.38, size = 2511, normalized size = 13.87

method	result	size
default	Expression too large to display	2511

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(cos(2*f*x+2*e)-1)*(-3*cos(2*f*x+2*e)*sin(2*f*x+2*e)*(b^4*(a-b))^(1/
2)*arctan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(
2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*b^2*(a-b)/(b^4*(a-b))^(1/2))*((a*cos(2*
f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*a^2+6*cos(2*f*x+2*
e)^2*a^3*b^3-14*cos(2*f*x+2*e)^2*a^2*b^4+10*cos(2*f*x+2*e)^2*a*b^5-2*cos(2*
f*x+2*e)^2*b^6-3*(b^4*(a-b))^(1/2)*arctan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+
2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*b^2*(a-
b)/(b^4*(a-b))^(1/2))*sin(2*f*x+2*e)*a^2*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e
)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)+10*cos(2*f*x+2*e)*a^2*b^4-14*cos(2*f*x+2*e
)*a*b^5+4*cos(2*f*x+2*e)*b^6-6*a^3*b^3+4*a^2*b^4+4*a*b^5-2*b^6)/sin(2*f*x+2
*e)^3/(a-b)^3/a^2/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+
1))^(3/2)/b^2-1/12/f*(cos(2*f*x+2*e)-1)*(-8*(a-b)^(3/2)*b^6-8*cos(2*f*x+2*e
)^2*(a-b)^(3/2)*b^6-80*cos(2*f*x+2*e)*(a-b)^(3/2)*a*b^5+3*cos(2*f*x+2*e)^3*
(a-b)^(3/2)*a^5*b-9*cos(2*f*x+2*e)^3*(a-b)^(3/2)*a^4*b^2+3*cos(2*f*x+2*e)^2
*(a-b)^(3/2)*a^5*b+3*cos(2*f*x+2*e)^2*(a-b)^(3/2)*a^4*b^2-3*(a-b)^(3/2)*a^5
*b*cos(2*f*x+2*e)+9*(a-b)^(3/2)*a^4*b^2*cos(2*f*x+2*e)-24*cos(2*f*x+2*e)*si
n(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(
3/2)*arctan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(co
s(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^4*b^2+30*cos(2*f*x+2*e
)*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1
))^(3/2)*arctan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)
/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^3*b^3-12*cos(2*f*x
+2*e)*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*
e)+1))^(3/2)*arctan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+
a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^2*b^4-24*(b^4*
(a-b))^(1/2)*arctan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+
a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*b^2*(a-b)/(b^4*(a-b))^(1/2))*
sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))
^(3/2)*a^2*(a-b)^(3/2)+6*cos(2*f*x+2*e)*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b
```

```
*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((cos(2*f*x+2*e)-1)/((
a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+
2*e)*(a-b)^(1/2))*a^5*b+9*cos(2*f*x+2*e)^3*(a-b)^(3/2)*a^3*b^3-3*cos(2*f*x+
2*e)^3*(a-b)^(3/2)*a^2*b^4+21*cos(2*f*x+2*e)^2*(a-b)^(3/2)*a^3*b^3-71*cos(2
*f*x+2*e)^2*(a-b)^(3/2)*a^2*b^4+52*cos(2*f*x+2*e)^2*(a-b)^(3/2)*a*b^5+3*cos
(2*f*x+2*e)*(a-b)^(3/2)*a^3*b^3+55*cos(2*f*x+2*e)*(a-b)^(3/2)*a^2*b^4+6*sin
(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3
/2)*arctan((cos(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos
(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^5*b-24*sin(2*f*x+2*e)*(
(a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((c
os(2*f*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1
))^(1/2)/sin(2*f*x+2*e)*(a-b)^(1/2))*a^4*b^2+30*sin(2*f*x+2*e)*((a*cos(2*f*
x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((cos(2*f*x+2*
e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/si
n(2*f*x+2*e)*(a-b)^(1/2))*a^3*b^3-12*sin(2*f*x+2*e)*((a*cos(2*f*x+2*e)-b*co
s(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((cos(2*f*x+2*e)-1)/((a*c
os(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/2)/sin(2*f*x+2*e
)*(a-b)^(1/2))*a^2*b^4-24*cos(2*f*x+2*e)*sin(2*f*x+2*e)*(a-b)^(3/2)*((a*cos
(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(3/2)*arctan((cos(2*f
*x+2*e)-1)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+1))^(1/
2)/sin(2*f*x+2*e)*b^2*(a-b)/(b^4*(a-b))^(1/2))*(b^4*(a-b))^(1/2)*a^2-33*(a-
b)^(3/2)*a^3*b^3+19*(a-b)^(3/2)*a^2*b^4+28*(a-b)^(3/2)*a*b^5+16*cos(2*f*x+2
*e)*(a-b)^(3/2)*b^6-3*(a-b)^(3/2)*a^5*b-3*(a-b)^(3/2)*a^4*b^2)/sin(2*f*x+2*
e)^3/(a-b)^(11/2)/((a*cos(2*f*x+2*e)-b*cos(2*f*x+2*e)+a+b)/(cos(2*f*x+2*e)+
1))^(3/2)/a^2/b
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
[Out] Integral(sin(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.148 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f}-\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*(
5*a-2*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+
e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3742, 425, 541, 12, 385, 209}

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Tan[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((5*
a - 2*b)*b*Tan[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e\right.}{3a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{3a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.17, size = 1331, normalized size = 9.93

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]] -
 (3150*(a - b)*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2)/a +
 (1575*(a - b)^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4)/a
^2 + (2100*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/a - (
4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[
e + f*x]^2)/a^2 + (2100*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]
]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (840*b^2*ArcSin[Sqrt[((a - b)*Sin[e
+ f*x]^2)/a]]*Tan[e + f*x]^4)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*
```

```
Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 + (840*(a - b)^2*b^2
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4*Tan[e + f*x]^4)/a^
4 + 2100*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + 96*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)
/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*
x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + (2800*b*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e + f*x]^
2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (168*b*Hypergeometri
c2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7
/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (48
*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*(((a
- b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Ta
n[e + f*x]^2))/a])/a + (1120*b^2*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e +
 f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 + (72*b^2*Hype
rgeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]
^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]
)/a^2 + (24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x
]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x
]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 - 1575*Sqrt[((a - b)*Cos[e + f*x]^2*Sin
[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (2100*b*Tan[e + f*x]^2*Sqrt[((a
- b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])/a - (840*b
^2*Tan[e + f*x]^4*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e
+ f*x]^2))/a^2])/a^2))/(315*a^2*f*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[a
 + b*Tan[e + f*x]^2]*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]*(1 + (
b*Tan[e + f*x]^2)/a))
```

Maple [A]

time = 0.07, size = 163, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{a-b}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}\,\sqrt{a}}\right.}{(a-b)^3b^2}}{f}$
default	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{a-b}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}\,\sqrt{a}}\right.}{(a-b)^3b^2}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a-b)*b*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+
e)/(a+b*tan(f*x+e)^2)^(1/2))+1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-
```

```
b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/(a-b)^2*b*tan(f
*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(126) = 252$.
time = 5.09, size = 581, normalized size = 4.34

$$\left[-\frac{3\left(a^{2}b^{2}\tan\left(fx+e\right)^{4}+2\,a^{3}b\tan\left(fx+e\right)^{2}+a^{4}\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2b)\tan(fx+e)^{2}-2\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)+2\left(\left(5\,a^{2}b^{2}-7\,ab^{3}+2\,b^{4}\right)\tan\left(fx+e\right)^{3}+3\left(2\,a^{3}b-3\,a^{2}b^{2}+ab^{3}\right)\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}}{6\left(\left(a^{5}b^{2}-3\,a^{4}b^{3}+3\,a^{3}b^{4}-a^{2}b^{5}\right)f\tan\left(fx+e\right)^{4}+2\left(a^{6}b-3\,a^{5}b^{2}+3\,a^{4}b^{3}-a^{3}b^{4}\right)f\tan\left(fx+e\right)^{2}+\left(a^{7}-3\,a^{6}b+3\,a^{5}b^{2}-a^{4}b^{3}\right)f\right)},\ \frac{3\left(a^{2}b^{2}\tan\left(fx+e\right)^{4}+2\,a^{3}b\tan\left(fx+e\right)^{2}+a^{4}\right)\sqrt{a-b}\,\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{a-b}\,\tan(fx+e)}\right)-\left(\left(5\,a^{2}b^{2}-7\,ab^{3}+2\,b^{4}\right)\tan\left(fx+e\right)^{3}+3\left(2\,a^{3}b-3\,a^{2}b^{2}+ab^{3}\right)\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}}{3\left(\left(a^{5}b^{2}-3\,a^{4}b^{3}+3\,a^{3}b^{4}-a^{2}b^{5}\right)f\tan\left(fx+e\right)^{4}+2\left(a^{6}b-3\,a^{5}b^{2}+3\,a^{4}b^{3}-a^{3}b^{4}\right)f\tan\left(fx+e\right)^{2}+\left(a^{7}-3\,a^{6}b+3\,a^{5}b^{2}-a^{4}b^{3}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(-a +
b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((5*a^2*b^2 - 7*a*b^3 + 2*b^
4)*tan(f*x + e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*ta
n(f*x + e)^2 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x +
 e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7
 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/3*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^
3*b*tan(f*x + e)^2 + a^4)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(s
qrt(a - b)*tan(f*x + e))) - ((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*tan(f*x + e)^3 +
 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/
((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b -
3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*
b^2 - a^4*b^3)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan^{2}\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.149 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{8b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -8/3*b*tan(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(1/2)-cot(f*x+e)/a/f/(a+b*tan(f*
x+e)^2)^(3/2)-4/3*b*tan(f*x+e)/a^2/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3744, 277, 198, 197}

$$-\frac{8b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cot[e + f*x]/(a*f*(a + b*Tan[e + f*x]^2)^(3/2))) - (4*b*Tan[e + f*x])/(3*
a^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*b*Tan[e + f*x])/(3*a^3*f*Sqrt[a +
b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4b)\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{af}$$
$$= -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(8b)\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b\tan}{3a^3f\sqrt{a+}}$$

Mathematica [A]
time = 0.70, size = 133, normalized size = 1.37

$$-\frac{(3(3a^2+4ab+8b^2)+4(3a^2-8b^2)\cos(2(e+fx))+(3a^2-12ab+8b^2)\cos(4(e+fx)))\cot(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{6\sqrt{2}\,a^3f(a+b+(a-b)\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*((3*(3*a^2 + 4*a*b + 8*b^2) + 4*(3*a^2 - 8*b^2)*Cos[2*(e + f*x)] + (3*
a^2 - 12*a*b + 8*b^2)*Cos[4*(e + f*x)])*Cot[e + f*x]*Sqrt[(a + b + (a - b)*
Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*a^3*f*(a + b + (a - b)*Cos[2*(e
 + f*x)])^2)
```

Maple [A]
time = 0.43, size = 153, normalized size = 1.58

method	result

default	$-\frac{(3(\cos^4(fx+e))a^2-12(\cos^4(fx+e))ab+8(\cos^4(fx+e))b^2+12(\cos^2(fx+e))ab-16(\cos^2(fx+e))b^2+8b^2)(\cos^5(fx+e))\left(\frac{a(\cos^2(fx+}{}\right.}{3f(a(\cos^2(fx+e))-(\cos^2(fx+e))b+b)^4\sin(fx+e)a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^4*(3*cos(f*x+e)^4*a^2-12*cos(f*x+e
)^4*a*b+8*cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-16*cos(f*x+e)^2*b^2+8*b^2)*c
os(f*x+e)^5*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/sin(f*x+
e)/a^3
```

Maxima [A]

time = 0.29, size = 91, normalized size = 0.94

$$-\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^3}+\frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}a^2}+\frac{3}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}a\tan(fx+e)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 4*b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a)^(3/2)*a^2) + 3/((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(
f*x + e)))/f
```

Fricas [A]

time = 13.55, size = 164, normalized size = 1.69

$$-\frac{\left((3\,a^2-12\,ab+8\,b^2)\cos\left(fx+e\right)^5+4\,(3\,ab-4\,b^2)\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{3\left(a^3b^2f+(a^5-2\,a^4b+a^3b^2)f\cos\left(fx+e\right)^4+2\,(a^4b-a^3b^2)f\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^5 + 4*(3*a*b - 4*b^2)*cos(f*x +
 e)^3 + 8*b^2*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)/((a^3*b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^4 + 2*(a^4*b - a^
3*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [B]
time = 27.44, size = 324, normalized size = 3.34

$$-\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{a+\frac{b\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}{{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}}\,\left(-a\,b\,12\mathrm{i}+a^2\,3\mathrm{i}+b^2\,8\mathrm{i}+a^2\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,12\mathrm{i}+a^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,18\mathrm{i}+a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,12\mathrm{i}+a^2\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,3\mathrm{i}-b^2\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,32\mathrm{i}+b^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,48\mathrm{i}-b^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,32\mathrm{i}+b^2\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,8\mathrm{i}+a\,b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,24\mathrm{i}-a\,b\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,12\mathrm{i}\right)}{3\,a^3\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,{\left(a-b+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+2\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

```
[Out] -((exp(e*2i + f*x*2i) + 1)*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2
i + f*x*2i) + 1)^2)^(1/2)*(a^2*3i - a*b*12i + b^2*8i + a^2*exp(e*2i + f*x*2
i)*12i + a^2*exp(e*4i + f*x*4i)*18i + a^2*exp(e*6i + f*x*6i)*12i + a^2*exp(
e*8i + f*x*8i)*3i - b^2*exp(e*2i + f*x*2i)*32i + b^2*exp(e*4i + f*x*4i)*48i
 - b^2*exp(e*6i + f*x*6i)*32i + b^2*exp(e*8i + f*x*8i)*8i + a*b*exp(e*4i +
f*x*4i)*24i - a*b*exp(e*8i + f*x*8i)*12i))/(3*a^3*f*(exp(e*2i + f*x*2i) - 1
)*(a - b + 2*a*exp(e*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 2*b*exp(e*2i + f
*x*2i) - b*exp(e*4i + f*x*4i))^2)
```

3.150 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{4(a-2b)b\tan(e+fx)}{3a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{8(a-2b)b\tan(}{3a^4f\sqrt{a+b\tan^2}}$$

```
[Out] -8/3*(a-2*b)*b*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)^(1/2)-(a-2*b)*cot(f*x+e)
/a^2/f/(a+b*tan(f*x+e)^2)^(3/2)-1/3*cot(f*x+e)^3/a/f/(a+b*tan(f*x+e)^2)^(3/
2)-4/3*(a-2*b)*b*tan(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3744, 464, 277, 198, 197}

$$-\frac{8b(a-2b)\tan(e+fx)}{3a^4f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b(a-2b)\tan(e+fx)}{3a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a - 2*b)*Cot[e + f*x])/(a^2*f*(a + b*Tan[e + f*x]^2)^(3/2))) - Cot[e +
f*x]^3/(3*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (4*(a - 2*b)*b*Tan[e + f*x])/
(3*a^3*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (8*(a - 2*b)*b*Tan[e + f*x])/(3*a^
4*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \tan(e+\right.}{af}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4(a-2b)b}{}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4(a-2b}{3a^3f\,(a+b}$$

$$= -\frac{(a-2b)\cot(e+fx)}{a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4(a-2b}{3a^3f\,(a+b}$$

Mathematica [A]

time = 0.78, size = 140, normalized size = 0.96

$$\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-\cot(e+fx)\left(2a-8b+a\csc^2(e+fx)\right)+\frac{2b\left(-3a^2+2ab+4b^2+\left(-3a^2+7ab-4b^2\right)\cos(2(e+fx))\right)\sin(2(e+fx))}{(a+b+(a-b)\cos(2(e+fx)))^2}\right)}{3\sqrt{2}\,a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-(Cot[e + f*x]*(2
*a - 8*b + a*Csc[e + f*x]^2)) + (2*b*(-3*a^2 + 2*a*b + 4*b^2 + (-3*a^2 + 7*
a*b - 4*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e
+ f*x)])^2))/(3*Sqrt[2]*a^4*f)
```

Maple [A]
time = 0.55, size = 245, normalized size = 1.68

method	result
default	$\frac{(2(\cos^6(fx+e))a^3-18(\cos^6(fx+e))a^2b+32(\cos^6(fx+e))a\,b^2-16(\cos^6(fx+e))b^3-3(\cos^4(fx+e))a^3+30(\cos^4(fx+e))a^2b-72(\cos^4(}{3f(a(\cos^2(fx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^4*(2*cos(f*x+e)^6*a^3-18*cos(f*x+e)
^6*a^2*b+32*cos(f*x+e)^6*a*b^2-16*cos(f*x+e)^6*b^3-3*cos(f*x+e)^4*a^3+30*co
s(f*x+e)^4*a^2*b-72*cos(f*x+e)^4*a*b^2+48*cos(f*x+e)^4*b^3-12*cos(f*x+e)^2*
a^2*b+48*cos(f*x+e)^2*a*b^2-48*cos(f*x+e)^2*b^3-8*a*b^2+16*b^3)*cos(f*x+e)^
5*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)/sin(f*x+e)^3/a^4
```

Maxima [A]
time = 0.29, size = 209, normalized size = 1.43

$$-\frac{\frac{8\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{3}}+\frac{4\,b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{16\,b^{2}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a^{4}}-\frac{8\,b^{2}\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{3}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a\tan\left(fx+e\right)}-\frac{6\,b}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{2}\tan\left(fx+e\right)}+\frac{1}{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 4*b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a)^(3/2)*a^2) - 16*b^2*tan(f*x + e)/(sqrt(b*tan(f*x +
e)^2 + a)*a^4) - 8*b^2*tan(f*x + e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^3) + 3/
((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(f*x + e)) - 6*b/((b*tan(f*x + e)^2 + a)
^(3/2)*a^2*tan(f*x + e)) + 1/((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(f*x + e)^3
))/f
```

Fricas [A]
time = 95.74, size = 250, normalized size = 1.71

$$-\frac{\left(2\left(a^{3}-9\,a^{2}b+16\,ab^{2}-8\,b^{3}\right)\cos\left(fx+e\right)^{7}-3\left(a^{3}-10\,a^{2}b+24\,ab^{2}-16\,b^{3}\right)\cos\left(fx+e\right)^{5}-12\left(a^{2}b-4\,ab^{2}+4\,b^{3}\right)\cos\left(fx+e\right)^{3}-8\left(ab^{2}-2\,b^{3}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{\left(a-b\right)\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\left(\left(a^{6}-2\,a^{5}b+a^{4}b^{2}\right)f\cos\left(fx+e\right)^{6}-a^{4}b^{2}f-\left(a^{6}-4\,a^{5}b+3\,a^{4}b^{2}\right)f\cos\left(fx+e\right)^{4}-\left(2\,a^{5}b-3\,a^{4}b^{2}\right)f\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^7 - 3*(a^3 - 10*a^2
*b + 24*a*b^2 - 16*b^3)*cos(f*x + e)^5 - 12*(a^2*b - 4*a*b^2 + 4*b^3)*cos(f
*x + e)^3 - 8*(a*b^2 - 2*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/(((a^6 - 2*a^5*b + a^4*b^2)*f*cos(f*x + e)^6 - a^4*b^2*f
 - (a^6 - 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^4 - (2*a^5*b - 3*a^4*b^2)*f*c
os(f*x + e)^2)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.151 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{(5a^2-20ab+16b^2)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{4b(5a^2-2}{15a^4f\,(a}$$

```
[Out] -8/15*b*(5*a^2-20*a*b+16*b^2)*tan(f*x+e)/a^5/f/(a+b*tan(f*x+e)^2)^(1/2)-1/5
*(5*a^2-20*a*b+16*b^2)*cot(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(3/2)-2/15*(5*a-
4*b)*cot(f*x+e)^3/a^2/f/(a+b*tan(f*x+e)^2)^(3/2)-1/5*cot(f*x+e)^5/a/f/(a+b*
tan(f*x+e)^2)^(3/2)-4/15*b*(5*a^2-20*a*b+16*b^2)*tan(f*x+e)/a^4/f/(a+b*tan(
f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3744, 473, 464, 277, 198, 197}

$$-\frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{8b(5a^2-20ab+16b^2)\tan(e+fx)}{15a^5f\sqrt{a+b\tan^2(e+fx)}}-\frac{4b(5a^2-20ab+16b^2)\tan(e+fx)}{15a^4f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(5a^2-20ab+16b^2)\cot(e+fx)}{5a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\cot^5(e+fx)}{5af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/5*((5*a^2 - 20*a*b + 16*b^2)*Cot[e + f*x])/(a^3*f*(a + b*Tan[e + f*x]^2)
^(3/2)) - (2*(5*a - 4*b)*Cot[e + f*x]^3)/(15*a^2*f*(a + b*Tan[e + f*x]^2)^(
3/2)) - Cot[e + f*x]^5/(5*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (4*b*(5*a^2 -
 20*a*b + 16*b^2)*Tan[e + f*x])/(15*a^4*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (
8*b*(5*a^2 - 20*a*b + 16*b^2)*Tan[e + f*x])/(15*a^5*f*Sqrt[a + b*Tan[e + f*
x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{2(5a-4b)+5ax^2}{x^4(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= -\frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\cot^5(e+fx)}{5af\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(-15a^2+1}{}$$

$$= -\frac{(5a^2-4(5a-4b)b)\cot(e+fx)}{5a^3f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{}{5af\,(a}$$

$$= -\frac{(5a^2-4(5a-4b)b)\cot(e+fx)}{5a^3f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{}{5af\,(a}$$

$$= -\frac{(5a^2-4(5a-4b)b)\cot(e+fx)}{5a^3f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{2(5a-4b)\cot^3(e+fx)}{15a^2f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{}{5af\,(a}$$

Mathematica [A]
time = 1.54, size = 174, normalized size = 0.79

$$\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-\cot(e+fx)\left(8a^2-66ab+73b^2+2a(2a-7b)\csc^2(e+fx)+3a^2\csc^4(e+fx)\right)+\frac{5b(-a+b)(6a^2-7ab-11b^2+(6a^2-17ab+11b^2)\cos(2(e+fx)))\sin(2(e+fx))}{(a+b+(a-b)\cos(2(e+fx)))^2}\right)}{15\sqrt{2}\,a^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-(Cot[e + f*x]*(8
*a^2 - 66*a*b + 73*b^2 + 2*a*(2*a - 7*b)*Csc[e + f*x]^2 + 3*a^2*Csc[e + f*x
]^4)) + (5*b*(-a + b)*(6*a^2 - 7*a*b - 11*b^2 + (6*a^2 - 17*a*b + 11*b^2)*C
os[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])^2))/(
15*Sqrt[2]*a^5*f)
```

Maple [A]
time = 0.37, size = 371, normalized size = 1.69

method	result
default	$-\frac{(8(\cos^8(fx+e))a^4-112(\cos^8(fx+e))a^3b+328(\cos^8(fx+e))a^2b^2-352(\cos^8(fx+e))a\,b^3+128(\cos^8(fx+e))b^4-20(\cos^6(fx+e))a^4+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^4*(8*cos(f*x+e)^8*a^4-112*cos(f*x
+e)^8*a^3*b+328*cos(f*x+e)^8*a^2*b^2-352*cos(f*x+e)^8*a*b^3+128*cos(f*x+e)^
8*b^4-20*cos(f*x+e)^6*a^4+292*cos(f*x+e)^6*a^3*b-976*cos(f*x+e)^6*a^2*b^2+1
216*cos(f*x+e)^6*a*b^3-512*cos(f*x+e)^6*b^4+15*cos(f*x+e)^4*a^4-240*cos(f*x
+e)^4*a^3*b+1008*cos(f*x+e)^4*a^2*b^2-1536*cos(f*x+e)^4*a*b^3+768*cos(f*x+e
)^4*b^4+60*cos(f*x+e)^2*a^3*b-400*cos(f*x+e)^2*a^2*b^2+832*cos(f*x+e)^2*a*b
^3-512*cos(f*x+e)^2*b^4+40*a^2*b^2-160*a*b^3+128*b^4)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/cos(f*x+e)^2)^(5/2)*cos(f*x+e)^5/sin(f*x+e)^5/a^5
```

Maxima [A]
time = 0.30, size = 361, normalized size = 1.65

$$-\frac{\frac{40\,b\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^3}+\frac{20\,b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^2}-\frac{160\,b^2\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^4}-\frac{80\,b^2\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^3}+\frac{128\,b^3\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}\,a^5}+\frac{64\,b^3\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^4}+\frac{15}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a\tan\left(fx+e\right)}-\frac{60\,b}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^2\tan\left(fx+e\right)}+\frac{48\,b^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^3\tan\left(fx+e\right)}+\frac{10}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a\tan\left(fx+e\right)^3}-\frac{8\,b}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^2\tan\left(fx+e\right)^3}+\frac{3}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(40*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 20*b*tan(f*x +
e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^2) - 160*b^2*tan(f*x + e)/(sqrt(b*tan(f*
x + e)^2 + a)*a^4) - 80*b^2*tan(f*x + e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^3)
 + 128*b^3*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^5) + 64*b^3*tan(f*x +
 e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^4) + 15/((b*tan(f*x + e)^2 + a)^(3/2)*a
*tan(f*x + e)) - 60*b/((b*tan(f*x + e)^2 + a)^(3/2)*a^2*tan(f*x + e)) + 48*
b^2/((b*tan(f*x + e)^2 + a)^(3/2)*a^3*tan(f*x + e)) + 10/((b*tan(f*x + e)^2
 + a)^(3/2)*a*tan(f*x + e)^3) - 8*b/((b*tan(f*x + e)^2 + a)^(3/2)*a^2*tan(f
*x + e)^3) + 3/((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(f*x + e)^5))/f
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{6}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.152 $\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx$

Optimal. Leaf size=92

$$\frac{\cos^2(e+fx)^{\frac{1}{2}+p}\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1+m+2p);\frac{1}{2}(3+m+2p);\sin^2(e+fx)\right)(d\sin(e+fx))^m\tan(e+fx)\left(\right.}{f(1+m+2p)}$$

[Out] (cos(f*x+e)^2)^(1/2+p)*hypergeom([1/2+p, 1/2+1/2*m+p],[3/2+1/2*m+p],sin(f*x+e)^2)*(d*sin(f*x+e))^m*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1+m+2*p)

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3739, 2682, 2657}

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\sin(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);\sin^2(e+fx)\right)}{f(m+2p+1)}$$

Antiderivative was successfully verified.

[In] Int[(d*Sin[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]

[Out] ((Cos[e + f*x]^2)^(1/2 + p)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m + 2*p)/2, (3 + m + 2*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*FracPart[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\sin(e+fx))^m \tan^{2p}(e+$$

$$= \left(d\cos^{2p}(e+fx)\sin(e+fx)(d\sin(e+fx))^{-1-2p}\left(b\tan^2(e+fx)\right)\right.$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}+p}\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1+m+2p);\frac{1}{2}(3+m+2p);\sin\right.}{f(1+m+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.37, size = 292, normalized size = 3.17

$$\frac{(3+m+2p)F_1\left(\frac{1}{2}+\frac{m}{2}+p;2p,1+m;\frac{3}{2}+\frac{m}{2}+p;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)(d\sin(e+fx))^m\left(b\tan^2(e+fx)\right)^p}{f(1+m+2p)\left((3+m+2p)F_1\left(\frac{1}{2}+\frac{m}{2}+p;2p,1+m;\frac{3}{2}+\frac{m}{2}+p;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((1+m)F_1\left(\frac{3}{2}+\frac{m}{2}+p;2p,2+m;\frac{5}{2}+\frac{m}{2}+p;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2pF_1\left(\frac{3}{2}+\frac{m}{2}+p;1+2p,1+m;\frac{5}{2}+\frac{m}{2}+p;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((3 + m + 2*p)*AppellF1[1/2 + m/2 + p, 2*p, 1 + m, 3/2 + m/2 + p, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(b*Tan[e +
f*x]^2)^p)/(f*(1 + m + 2*p)*((3 + m + 2*p)*AppellF1[1/2 + m/2 + p, 2*p, 1 +
 m, 3/2 + m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((1 + m)*Ap
pellF1[3/2 + m/2 + p, 2*p, 2 + m, 5/2 + m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] - 2*p*AppellF1[3/2 + m/2 + p, 1 + 2*p, 1 + m, 5/2 + m/2 + p,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^m \left(b\left(\tan^2(fx+e)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^p\left(d\sin\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
[Out] Integral((b*tan(e + f*x)**2)**p*(d*sin(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
[Out] int((d*sin(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

3.153 $\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=121

$$\frac{F_1\left(\frac{1+m}{2};\frac{2+m}{2},-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p \ldots}{f(1+m)}$$

```
[Out] AppellF1(1/2+1/2*m,1+1/2*m,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(s
ec(f*x+e)^2)^(1/2*m)*(d*sin(f*x+e))^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(1+
m)/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3748, 525, 524}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+2}{2},-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/a)]*(Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*Tan[e + f*x]*(a + b
*Tan[e + f*x]^2)^p)/(f*(1 + m)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3748

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff
```

```
*(d*Sin[e + f*x])^m*((Sec[e + f*x]^2)^(m/2)/(f*Tan[e + f*x]^m)), Subst[Int[
(ff*x)^m*((a + b*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]
/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \frac{\left(\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \tan^{-m}(e+fx)\right) \mathrm{Subst}\left(\int\right.}{f}$$

$$= \frac{\left(\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \tan^{-m}(e+fx)\left(a+b\tan\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1+m}{2};\frac{2+m}{2},-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\sec^2(e}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 275 vs. 2(121) = 242.
time = 1.61, size = 275, normalized size = 2.27

$$\frac{a(3+m)F_1\left(\frac{1+m}{2};\frac{2+m}{2},-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p}{f(1+m)\left(a(3+m)F_1\left(\frac{1+m}{2};\frac{2+m}{2},-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)+\left(2bpF_1\left(\frac{3+m}{2};\frac{2+m}{2},1-p;\frac{5+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)-a(2+m)F_1\left(\frac{3+m}{2};\frac{4+m}{2},-p;\frac{5+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\right)\tan^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (a*(3 + m)*AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)]*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(a + b
*Tan[e + f*x]^2)^p)/(f*(1 + m)*(a*(3 + m)*AppellF1[(1 + m)/2, (2 + m)/2, -p
, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[(3
 + m)/2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/a)] - a*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^m \left(a+b\left(\tan^2(fx+e)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d \sin (e + f x))^m \left(b \tan(e + f x)^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
[Out] int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

3.154 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=208

$$\frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f}-\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{5(a-b)f}-\frac{(15a^2 \cdots}{}$$

```
[Out] 1/15*(-2*b*p+10*a-7*b)*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1+p)/(a-b)^2/f-1/
5*cos(f*x+e)^5*(a-b+b*sec(f*x+e)^2)^(1+p)/(a-b)/f-1/15*(15*a^2-20*a*b*(1+p)
+4*b^2*(p^2+3*p+2))*cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/(
a-b))*(a-b+b*sec(f*x+e)^2)^p/(a-b)^2/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3745, 473, 464, 372, 371}

$$-\frac{\left(15a^2-20ab(p+1)+4b^2(p^2+3p+2)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)}{15f(a-b)^2}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{5f(a-b)}+\frac{(10a-2bp-7b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{15f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((10*a - 7*b - 2*b*p)*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(1
5*(a - b)^2*f) - (Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(5*(a
- b)*f) - ((15*a^2 - 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cos[e + f*x]*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))]*(a - b + b*S
ec[e + f*x]^2)^p)/(15*(a - b)^2*f*(1 + (b*Sec[e + f*x]^2)/(a - b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a-b+bx^2)^p}{x^6}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=-\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{5(a-b)f}+\frac{\text{Subst}\left(\int \frac{(-10a+b(7+\right.}{}$$

$$=\frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f}-\frac{\cos^5}{}$$

$$=\frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f}-\frac{\cos^5}{}$$

$$=\frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f}-\frac{\cos^5}{}$$

Mathematica [A]
time = 5.02, size = 283, normalized size = 1.36

$$-\frac{2^{3+p}\cos(e+fx)\sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\left(15a^2-20ab(1+p)+4b^2(2+3p+p^2)\right)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)+\frac{1}{4}(a+b+(a-b)\cos(2(e+fx)))(-17a+b(11+4p)+3(a-b)\cos(2(e+fx)))\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^p\right)}{15(a-b)^2f\left(3\left(\frac{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}{a-b}\right)^p-2^{2+p}\cos(2(e+fx))\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^p+2^p\cos(4(e+fx))\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^p\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/15*(2^(3 + p)*Cos[e + f*x]*Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p*((15*
a^2 - 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Hypergeometric2F1[-1/2, -p, 1
/2, -((b*Sec[e + f*x]^2)/(a - b))] + ((a + b + (a - b)*Cos[2*(e + f*x)])*(-
17*a + b*(11 + 4*p) + 3*(a - b)*Cos[2*(e + f*x)])*((a + b*Tan[e + f*x]^2)/(
a - b))^p)/4))/((a - b)^2*f*(3*(((a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e +
 f*x]^2)/(a - b))^p - 2^(2 + p)*Cos[2*(e + f*x)]*((a + b*Tan[e + f*x]^2)/(a
 - b))^p + 2^p*Cos[4*(e + f*x)]*((a + b*Tan[e + f*x]^2)/(a - b))^p))
```

Maple [**F**]
time = 0.80, size = 0, normalized size = 0.00

$$\int \left(\sin^{5}\left(f x +e \right)\right) \left(a +b \left(\tan^{2}\left(f x +e \right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*tan(f*x + e)^2 + a)^p*s
in(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^5\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^p, x)
```

3.155 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=140

$$\frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f}-\frac{(3a-2b(1+p))\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)(a-b}{3(a-b)f}$$

```
[Out] 1/3*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1+p)/(a-b)/f-1/3*(3*a-2*b*(1+p))*cos
(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/(a-b))*(a-b+b*sec(f*x+e)
^2)^p/(a-b)/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3745, 464, 372, 371}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{3f(a-b)}-\frac{(3a-2b(p+1))\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(3*(a - b)*f) - ((3*a -
 2*b*(1 + p))*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*
x]^2)/(a - b))]*(a - b + b*Sec[e + f*x]^2)^p)/(3*(a - b)*f*(1 + (b*Sec[e +
f*x]^2)/(a - b))^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a-b+bx^2)^p}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f} + \frac{(3a-2b(1+p))\text{Subs}}{}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f} + \frac{\left((3a-2b(1+p))(a\right.}{}$$

$$= \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f} - \frac{(3a-2b(1+p))\cos(}{}$$

Mathematica [A]
time = 2.70, size = 184, normalized size = 1.31

$$\frac{\sin(e+fx)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left((-3a+2b(1+p))\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)+\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^p\right)}{f\left(3a\sec^2(e+fx)\left(\frac{a-b+b\sec^2(e+fx)}{a-b}\right)^p-3(a-b)\left(\frac{a+b\tan^2(e+fx)}{a-b}\right)^{1+p}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Sin[e + f*x]*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((-3*a + 2*b*(1 + p))*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))] + (a*Cos[e +
 f*x]^2 + b*Sin[e + f*x]^2)*((a + b*Tan[e + f*x]^2)/(a - b))^p))/(f*(3*a*Se
c[e + f*x]^2*((a - b + b*Sec[e + f*x]^2)/(a - b))^p - 3*(a - b)*((a + b*Tan
[e + f*x]^2)/(a - b))^(1 + p)))
```

Maple [F]
time = 0.70, size = 0, normalized size = 0.00

$$\int \left(\sin^3\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+f\,x)^3\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^p, x)
```

3.156 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=79

$$-\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}}{f}$$

```
[Out] -cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/(a-b))*(a-b+b*sec(f*
x+e)^2)^p/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3745, 372, 371}

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a -
b))]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a - b))^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a-b+bx^2)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)}{x^2}\right.}{f}$$
$$= -\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)\left(a-b+b\sec^2(e+fx\right.}{f}$$

Mathematica [A]
time = 0.64, size = 80, normalized size = 1.01

$$-\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a-b}\right)\left(\frac{a-b+b\sec^2(e+fx)}{a-b}\right)^{-p}\left(a+b\tan^2(e+fx)\right)^p}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a -
b))]*(a + b*Tan[e + f*x]^2)^p)/(f*((a - b + b*Sec[e + f*x]^2)/(a - b))^p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sin\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^p, x)
```

3.157 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec(e+fx)\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}}{f}$$

```
[Out] -AppellF1(1/2,1,-p,3/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/(a-b))*sec(f*x+e)*(a-b+
b*sec(f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3745, 441, 440}

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))]
*Sec[e + f*x]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a -
 b))^p))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
```

```
 - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a-b+bx^2)^p}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left((a-b+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right.}{-1+x}\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec(e+fx)(a-b+}{f}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1215 vs. $2(88) = 176$.
time = 14.07, size = 1215, normalized size = 13.81

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(2*p)*((2*AppellF1[-1/2 - p, -1/2, -p,
 1/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/(
(1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1,
 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 +
 (b*Tan[e + f*x]^2)/a)^p))/(2*f*(b*p*Sec[e + f*x]^2*Tan[e + f*x]*(a + b*Tan
[e + f*x]^2)^(-1 + p)*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*
x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + (a*Cot
[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/a
)^p) + ((a + b*Tan[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p,
-Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Sqrt[Sec[e + f*x]^2]
)/((1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (4*a*p*Ap
pellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b
)]*Cot[e + f*x]*(1 + (a*Cot[e + f*x]^2)/b)^(-1 - p)*Sqrt[Csc[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2])/(b*(1 + 2*p)) + (2*((-2*a*(-1/2 - p)*p*AppellF1[1/2 - p
, -1/2, 1 - p, 3/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f
*x]*Csc[e + f*x]^2)/(b*(1/2 - p)) - ((-1/2 - p)*AppellF1[1/2 - p, 1/2, -p,
3/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x
]^2)/(1/2 - p))*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)
```

```
^p*Sqrt[Csc[e + f*x]^2]) + (2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e
+ f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2]*Tan[e + f*x])/((1 +
 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (2*b*p*AppellF1[
1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan
[e + f*x]^3*(1 + (b*Tan[e + f*x]^2)/a)^(-1 - p))/a - (2*AppellF1[1, 1/2, -p
, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])
/(1 + (b*Tan[e + f*x]^2)/a)^p - (Tan[e + f*x]^2*((b*p*AppellF1[2, 1/2, 1 -
p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]
)/a - (AppellF1[2, 3/2, -p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Se
c[e + f*x]^2*Tan[e + f*x])/2))/(1 + (b*Tan[e + f*x]^2)/a)^p))/2))
```

Maple [**F**]
time = 0.21, size = 0, normalized size = 0.00

$$\int \csc\left(fx+e\right)\left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p}{\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x), x)
```

3.158 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/(a-b))*sec(f*x+e)^3*
(a-b+b*sec(f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/(a-b))^p)
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3745, 525, 524}

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))]*S
ec[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/(a
 - b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3745

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
```

```
m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1)
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-b+bx^2\right)^p}{\left(-1+x^2\right)^2}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a-b+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\right.}{\left(-1+\right.}\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec^3(e+fx)\left(a-b+\right.}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 252 vs. 2(92) = 184.
time = 15.05, size = 252, normalized size = 2.74

$$\frac{b(-3+2p)F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\cot^2(e+fx),-\frac{a\cot^2(e+fx)}{b}\right)\cot(e+fx)\csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p}{f(-1+2p)\left(b(-3+2p)F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\cot^2(e+fx),-\frac{a\cot^2(e+fx)}{b}\right)-\left(2apF_1\left(\frac{3}{2}-p;-\frac{1}{2},1-p;\frac{5}{2}-p;-\cot^2(e+fx),-\frac{a\cot^2(e+fx)}{b}\right)+bF_1\left(\frac{3}{2}-p;\frac{1}{2},-p;\frac{5}{2}-p;-\cot^2(e+fx),-\frac{a\cot^2(e+fx)}{b}\right)\right)\cot^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (b*(-3 + 2*p)*AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -((a*Co
t[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(-
1 + 2*p)*(b*(-3 + 2*p)*AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2
, -((a*Cot[e + f*x]^2)/b)] - (2*a*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2 - p,
 -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2, -p, 5
/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)])*Cot[e + f*x]^2))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(\csc^3(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan(e+f\,x)^2+a\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^3,x)
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

3.159 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^3*(a+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3744, 525, 524}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
```

```
+ 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^p}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{(1+x^2)^2}\right.}{f}$$
$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\,(a+b}{3f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.67, size = 3698, normalized size = 44.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(AppellF1[1/2, 2,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2
, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2,
 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e +
 f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
 f*x]^2])*Tan[e + f*x]^2))*(-1/4*(Cos[2*(e + f*x)]^3*(a + b*Tan[e + f*x]^2)
^p) + (I/4)*Sin[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p + (Sin[2*(e + f*x)]^2
*(a + b*Tan[e + f*x]^2)^p)/2 - (I/4)*Sin[2*(e + f*x)]^3*(a + b*Tan[e + f*x]
^2)^p + Cos[2*(e + f*x)]^2*((a + b*Tan[e + f*x]^2)^p/2 - (I/4)*Sin[2*(e + f
*x)]*(a + b*Tan[e + f*x]^2)^p) + Cos[2*(e + f*x)]*(-1/4*(a + b*Tan[e + f*x]
^2)^p - (Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2)^p)/4)))/(f*(6*a*b*p*Sin[
e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p)*(AppellF1[1/2, 2, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -T
```

an[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 3*a*Cos[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 9*a*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 3*a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (4*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/3)/(-3*a*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (2*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Sec[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(-4*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Sec[e + f*x]^2*Tan[

```
e + f*x] - 3*a*((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (4*AppellF1[3/2, 3,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)...
```

Maple [**F**]
time = 0.73, size = 0, normalized size = 0.00

$$\int \left(\sin^2 \left(f x + e\right)\right) \left(a + b \left(\tan^2 \left(f x + e\right)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e)^2 + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^p, x)
```

3.160 $\int (a+b\tan^2(e+fx))^p\,dx$

Optimal. Leaf size=78

$$\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan(e+fx)\,(a+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)*(a+b*tan(
f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 441, 440}

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx))^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx, x, \tan(e\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2}; 1, -p; \frac{3}{2}; -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a}\right) \tan(e+fx)\left(a+b\tan^2(e+fx\right.}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 192 vs. 2(78) = 156.
time = 0.35, size = 192, normalized size = 2.46

$$\frac{3aF_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{b\tan^2(e+fx)}{a}, -\tan^2(e+fx)\right)\sin(2(e+fx))\left(a+b\tan^2(e+fx)\right)^p}{6afF_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{b\tan^2(e+fx)}{a}, -\tan^2(e+fx)\right) + 4f\left(bpF_1\left(\frac{3}{2}; 1-p, 1; \frac{5}{2}; -\frac{b\tan^2(e+fx)}{a}, -\tan^2(e+fx)\right) - aF_1\left(\frac{3}{2}; -p, 2; \frac{5}{2}; -\frac{b\tan^2(e+fx)}{a}, -\tan^2(e+fx)\right)\right)\tan^2(e+fx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Si
n[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p)/(6*a*f*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 4*f*(b*p*AppellF1[3/2, 1 - p, 1,
 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^{2}\left(e + fx\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**p,x)
[Out] Integral((a + b*tan(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p,x)
[Out] int((a + b*tan(e + f*x)^2)^p, x)
```

3.161 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=68

$$-\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] -cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^2
)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3744, 372, 371}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,d\right.}{f}$$

$$= -\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^p}{f}$$

Mathematica [**A**]
time = 0.43, size = 68, normalized size = 1.00

$$-\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Maple [**F**]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(\csc^2(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan(e+f\,x)^2+a\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^2,x)
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

3.162 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=120

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af}-\frac{(3a-b(1-2p))\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)(a+b\tan$$

```
[Out] -1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1+p)/a/f-1/3*(3*a-b*(1-2*p))*cot(f*x+
e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^2)^p/a/f/(
(1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3744, 464, 372, 371}

$$-\frac{(3a-b(1-2p))\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{3af}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/(a*f) - ((3*a - b*(1 -
 2*p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a
)]*(a + b*Tan[e + f*x]^2)^p)/(3*a*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+bx^2)^p}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af} - \frac{(-3a-b(-3+2(1+$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af} - \frac{\left((-3a-b(-3+2(1\right.}{}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af} - \frac{(3a-b(1-2p))\cot(}{}$$

Mathematica [A]
time = 0.94, size = 111, normalized size = 0.92

$$\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(-a-b\tan^2(e+fx)-(3a+b(-1+2p))\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)\tan^2(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)}{3af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p*(-a - b*Tan[e + f*x]^2 - ((3*a + b
*(-1 + 2*p))*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Tan[
e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/a)^p))/(3*a*f)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(\csc^4(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan(e+f\,x)^2+a\right)^p}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^4,x)
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

3.163 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=180

$$-\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{5af}-\frac{(15a^2-b(\cdots$$

```
[Out] -1/15*(10*a-b*(3-2*p))*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1+p)/a^2/f-1/5*cot(
f*x+e)^5*(a+b*tan(f*x+e)^2)^(1+p)/a/f-1/15*(15*a^2-b*(10*a-b*(3-2*p))*(1-2*
p))*cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e
)^2)^p/a^2/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.13, antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3744, 473, 464, 372, 371}

$$-\frac{\left(15-\frac{b(1-2p)(10a-b(3-2p))}{a^2}\right)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)}{15f}-\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{15a^2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/15*((10*a - b*(3 - 2*p))*Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/
(a^2*f) - (Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(1 + p))/(5*a*f) - ((15 -
(b*(10*a - b*(3 - 2*p))*(1 - 2*p))/a^2)*Cot[e + f*x]*Hypergeometric2F1[-1/2
, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(a + b*Tan[e + f*x]^2)^p)/(15*f*(1 + (b
*Tan[e + f*x]^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+bx^2)^p}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{5af} + \frac{\text{Subst}\left(\int \frac{(10a-b(3-2p)+}{x}\right.}{}$$

$$= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cot}{}$$

$$= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cot}{}$$

$$= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cot}{}$$

Mathematica [A]

time = 0.71, size = 141, normalized size = 0.78

$$-\frac{\cot(e+fx)\left(3\cot^4(e+fx)\,{}_2F_1\left(-\frac{5}{2},-p;-\frac{3}{2};-\frac{b\tan^2(e+fx)}{a}\right)+10\cot^2(e+fx)\,{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)+15\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a}\right)\right)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/15*(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -((
b*Tan[e + f*x]^2)/a)] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -p, -1/2,
 -((b*Tan[e + f*x]^2)/a)] + 15*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e
+ f*x]^2)/a)])*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Maple [**F**]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\csc^{6}\left(fx+e\right)\right) \left(a+b\left(\tan^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\tan(e+f\,x)^2+a\right)^p}{\sin\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^6,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^6, x)
```

3.164 $\int (d\sin(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=98

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\, {}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1+m+np);\frac{1}{2}(3+m+np);\sin^2(e+fx)\right)(d\sin(e+fx))^m \tan(e+fx}{f(1+m+np)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p+1/2, 1/2*n*p+1/2*m+1/2],[1/
2*n*p+1/2*m+3/2],sin(f*x+e)^2)*(d*sin(f*x+e))^m*tan(f*x+e)*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p+m+1)
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3740, 2682, 2657}

$$\frac{\tan(e+fx)(d\sin(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(np+1)}\left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(m+np+1);\frac{1}{2}(m+np+3);\sin^2(e+fx)\right)}{f(m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m + n*p
)/2, (3 + m + n*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
```

```
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\sin(e+fx))^m($$

$$= \left(d\cos^{np}(e+fx)\sin(e+fx)(d\sin(e+fx))^{-1-np}\left(b(c\tan(e+\right.\right.$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1+m+np);\frac{1}{2}(3+m+\right.}{f(1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.32, size = 295, normalized size = 3.01

$$\frac{(3+m+np)F_1\left(\frac{1}{2}(1+m+np);np,1+m;\frac{1}{2}(3+m+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)(d\sin(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p}{f(1+m+np)\left((3+m+np)F_1\left(\frac{1}{2}(1+m+np);np,1+m;\frac{1}{2}(3+m+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((1+m)F_1\left(\frac{1}{2}(3+m+np);np,2+m;\frac{1}{2}(5+m+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-npF_1\left(\frac{1}{2}(3+m+np);1+np,1+m;\frac{1}{2}(5+m+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((3 + m + n*p)*AppellF1[(1 + m + n*p)/2, n*p, 1 + m, (3 + m + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(b*(c*T
an[e + f*x])^n)^p)/(f*(1 + m + n*p)*((3 + m + n*p)*AppellF1[(1 + m + n*p)/2
, n*p, 1 + m, (3 + m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2
*((1 + m)*AppellF1[(3 + m + n*p)/2, n*p, 2 + m, (5 + m + n*p)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*p*AppellF1[(3 + m + n*p)/2, 1 + n*p, 1 +
 m, (5 + m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x
)/2]^2))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^m \left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p (d\sin(e+fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*sin(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\sin(e+f\,x))^m\,(b\,(c\tan(e+f\,x))^n)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

3.165 $\int \sin^2(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=63

$$\frac{{}_2F_1\left(2,\frac{1}{2}(3+np);\frac{1}{2}(5+np);-\tan^2(e+fx)\right)\tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3+np)}$$

```
[Out] hypergeom([2, 1/2*n*p+3/2],[1/2*n*p+5/2],-tan(f*x+e)^2)*tan(f*x+e)^3*(b*(c*
tan(f*x+e))^n)^p/f/(n*p+3)
```

Rubi [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2671, 371}

$$\frac{\tan^3(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+3);\frac{1}{2}(np+5);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[2, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sin^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sin^2(e+fx)(c\tan(e+ \ldots$$

$$= \frac{(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p)\,\mathrm{Subst}\left(\int \frac{x^{2+np}}{(c^2+x^2)^2}\,dx, x, c \ldots\right.}{f}$$

$$= \frac{{}_2F_1\left(2, \frac{1}{2}(3+np); \frac{1}{2}(5+np); -\tan^2(e+fx)\right)\tan^3(e+fx)\left(b(c\tan \ldots\right.}{f(3+np)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.67, size = 517, normalized size = 8.21

$$\frac{8(6+2np)\left(F_1\left(\frac{1}{2}(1+np); np, 2; \frac{1}{2}(3+np); \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right) - F_1\left(\frac{1}{2}(1+np); np, 3; \frac{1}{2}(3+np); \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\cos^5\left(\frac{1}{2}(e+fx)\right)\sin^3\left(\frac{1}{2}(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)\left(2(3+np)F_1\left(\frac{1}{2}(1+np); np, 2; \frac{1}{2}(3+np); \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right) - 2(3+np)F_1\left(\frac{1}{2}(1+np); np, 3; \frac{1}{2}(3+np); \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right) + 2\left(2F_1\left(\frac{1}{2}(3+np); np, 3; \frac{1}{2}(5+np); \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right) - 3F_1\left(\frac{1}{2}(3+np); np, 4; \frac{1}{2}(5+np); \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right) + np\left(-F_1\left(\frac{1}{2}(3+np); 1+np, 2; \frac{1}{2}(5+np); \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right) + F_1\left(\frac{1}{2}(3+np); 1+np, 3; \frac{1}{2}(5+np); \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)(-1+\cos(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (8*(6 + 2*n*p)*(AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] - AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^5*Sin[(e + f*x)/2]^3*
(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)*(2*(3 + n*p)*AppellF1[(1 + n*p)/2, n
*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/
2]^2 - 2*(3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(2*AppellF1[(3 + n*p)/2,
 n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 3*AppellF1
[(3 + n*p)/2, n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + n*p*(-AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*(-1 + Cos[e + f*x])))
```

Maple [F]
time = 0.64, size = 0, normalized size = 0.00

$$\int \left(\sin^2(fx+e)\right)\left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \sin^2(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sin(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin(e+f\,x)^2\,(b\,(c\tan(e+f\,x))^n)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int(sin(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

3.166 $\int (b(c\tan(e+fx))^n)^p \, dx$

Optimal. Leaf size=61

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*ta
n(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int (b(c\tan(e+fx))^n)^p\, dx &= \left((c\tan(e+fx))^{-np}(b(c\tan(e+fx))^n)^p\right)\int (c\tan(e+fx))^{np}\, dx \\
&= \frac{(c(c\tan(e+fx))^{-np}(b(c\tan(e+fx))^n)^p)\operatorname{Subst}\left(\int \frac{x^{np}}{c^2+x^2}\, dx, x, c\tan(e+fx)\right)}{f} \\
&= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)}{f(1+np)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.97

$$
\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f+fnp}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f + f*n*p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int (b(c\tan(fx+e))^n)^p\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\,(c\tan(e+f\,x))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p, x)
```

3.167 $\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=33

$$-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

```
[Out] -cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+1)
```

Rubi [A]

time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2671, 30}

$$-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^2(e+fx)(c\tan(e-$$
$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int x^{-2+np}\,dx,x,c\right.}{f}$$
$$= -\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 0.94

$$\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(-1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
[Out] (Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-1 + n*p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.62, size = 10285, normalized size = 311.67

method	result	size
risch	Expression too large to display	10285

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [A]
time = 0.31, size = 39, normalized size = 1.18

$$\frac{b^p c^{np}\left(\tan\left(fx+e\right)^n\right)^p}{(np-1)f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 1)*f*tan(f*x + e))
```

Fricas [A]
time = 1.44, size = 55, normalized size = 1.67

$$\frac{\cos\left(fx+e\right)e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}}{(fnp-f)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] cos(f*x + e)*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))/((f*n*p -
f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \csc^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(b\,(c\tan(e+f\,x))^n)^p}{\sin(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^2,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^2, x)
```

3.168 $\int \csc^4(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=69

$$-\frac{\cot(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1-np)}-\frac{\cot^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3-np)}$$

```
[Out] -cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+1)-cot(f*x+e)^3*(b*(c*tan(f*x+e)
)^n)^p/f/(-n*p+3)
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2671, 14}

$$-\frac{\cot^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3-np)}-\frac{\cot(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p))) - (Cot[e + f*x]^3*
(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \csc^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^4(e+fx)(c\tan(e$$

$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int x^{-4+np}(c^2+x\right.}{f}$$

$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int (c^2x^{-4+np}+x\right.}{f}$$

$$= -\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)} - \frac{\cot^3(e+fx)\left(b(c\tan(e+f\right.}{f(3-np)}$$

Mathematica [A]
time = 0.11, size = 59, normalized size = 0.86

$$\frac{(-2+np+\cos(2(e+fx)))\cot(e+fx)\csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(-3+np)(-1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((-2 + n*p + Cos[2*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2*(b*(c*Tan[e + f*
x])^n)^p)/(f*(-3 + n*p)*(-1 + n*p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.80, size = 34276, normalized size = 496.75

method	result	size
risch	Expression too large to display	34276

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.31, size = 77, normalized size = 1.12

$$\frac{\frac{b^p c^{np}\left(\tan(fx+e)^n\right)^p}{(np-1)\tan(fx+e)} + \frac{b^p c^{np}\left(\tan(fx+e)^n\right)^p}{(np-3)\tan(fx+e)^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

[Out] (b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 1)*tan(f*x + e)) + b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 3)*tan(f*x + e)^3))/f

Fricas [A]
time = 1.06, size = 110, normalized size = 1.59

$$\frac{\left(2\,\cos\left(fx+e\right)^{3}+\left(np-3\right)\cos\left(fx+e\right)\right)e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log\left(b\right)\right)}}{\left(fn^{2}p^{2}-4\,fnp-\left(fn^{2}p^{2}-4\,fnp+3\,f\right)\cos\left(fx+e\right)^{2}+3\,f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (2*cos(f*x + e)^3 + (n*p - 3)*cos(f*x + e))*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))/((f*n^2*p^2 - 4*f*n*p - (f*n^2*p^2 - 4*f*n*p + 3*f)*cos(f*x + e)^2 + 3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b(c\tan\left(e+fx\right))^{n}\right)^{p}\csc^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^{n}\right)}^{p}}{{\sin\left(e+f\,x\right)}^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^4,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^4, x)
```

3.169 $\int \csc^6(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=104

$$-\frac{\cot(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1-np)}-\frac{2\cot^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3-np)}-\frac{\cot^5(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(5-np)}$$

```
[Out] -cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+1)-2*cot(f*x+e)^3*(b*(c*tan(f*x+
e))^n)^p/f/(-n*p+3)-cot(f*x+e)^5*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+5)
```

Rubi [A]

time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2671, 276}

$$-\frac{\cot^5(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(5-np)}-\frac{2\cot^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3-np)}-\frac{\cot(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p))) - (2*Cot[e + f*x]^
3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p)) - (Cot[e + f*x]^5*(b*(c*Tan[e + f
*x])^n)^p)/(f*(5 - n*p))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^6(e+fx)(c\tan(e$$

$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int x^{-6+np}(c^2+x^2}{f}$$

$$= \frac{\left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \left(c^4x^{-6+np}+2c\right.\right.}{f}$$

$$= -\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)} - \frac{2\cot^3(e+fx)\left(b(c\tan(e+f\right.}{f(3-np)}$$

Mathematica [A]
time = 0.20, size = 89, normalized size = 0.86

$$\frac{(8-6np+n^2p^2+2(-3+np)\cos(2(e+fx))+\cos(4(e+fx)))\cot(e+fx)\csc^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(-5+np)(-3+np)(-1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((8 - 6*n*p + n^2*p^2 + 2*(-3 + n*p)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*C
ot[e + f*x]*Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(-5 + n*p)*(-3 + n*
p)*(-1 + n*p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.08, size = 70269, normalized size = 675.66

method	result	size
risch	Expression too large to display	70269

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.32, size = 114, normalized size = 1.10

$$\frac{\frac{b^pc^{np}(\tan(fx+e)^n)^p}{(np-1)\tan(fx+e)} + \frac{2\,b^pc^{np}(\tan(fx+e)^n)^p}{(np-3)\tan(fx+e)^3} + \frac{b^pc^{np}(\tan(fx+e)^n)^p}{(np-5)\tan(fx+e)^5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 1)*tan(f*x + e)) + 2*b^p*c^(n*p)*(t
an(f*x + e)^n)^p/((n*p - 3)*tan(f*x + e)^3) + b^p*c^(n*p)*(tan(f*x + e)^n)^
p/((n*p - 5)*tan(f*x + e)^5))/f
```

Fricas [A]
time = 0.76, size = 188, normalized size = 1.81

$$\frac{\left(8 \cos\left(fx+e\right)^{5}+4\left(np-5\right)\cos\left(fx+e\right)^{3}+\left(n^{2}p^{2}-8\,np+15\right)\cos\left(fx+e\right)\right)e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}}{\left(fn^{3}p^{3}-9\,fn^{2}p^{2}+\left(fn^{3}p^{3}-9\,fn^{2}p^{2}+23\,fnp-15\,f\right)\cos\left(fx+e\right)^{4}+23\,fnp-2\left(fn^{3}p^{3}-9\,fn^{2}p^{2}+23\,fnp-15\,f\right)\cos\left(fx+e\right)^{2}-15\,f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (8*cos(f*x + e)^5 + 4*(n*p - 5)*cos(f*x + e)^3 + (n^2*p^2 - 8*n*p + 15)*cos
(f*x + e))*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))/((f*n^3*p^3
- 9*f*n^2*p^2 + (f*n^3*p^3 - 9*f*n^2*p^2 + 23*f*n*p - 15*f)*cos(f*x + e)^4
+ 23*f*n*p - 2*(f*n^3*p^3 - 9*f*n^2*p^2 + 23*f*n*p - 15*f)*cos(f*x + e)^2 -
 15*f)*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\left(c\tan(e+f\,x)\right)^{n}\right)^{p}}{\sin\left(e+f\,x\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^6,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^6, x)
```

3.170 $\int \sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=93

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(4+np);\frac{1}{2}(6+np);\sin^2(e+fx)\right)\sin^3(e+fx)\tan(e+fx)\left(b(c\tan(e+\right.}{f(4+np)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p+2, 1/2*n*p+1/2],[1/2*n*p+3]
,sin(f*x+e)^2)*sin(f*x+e)^3*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+4)
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2682, 2657}

$$\frac{\sin^3(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4);\frac{1}{2}(np+6);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2,
 (6 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(4 + n*p))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
```

```
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sin^3(e+fx)(c\tan(e$$
$$= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(4+np);\frac{1}{2}(6+np);\sin^2(e+\right.}{f(4+np)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.82, size = 506, normalized size = 5.44

$$\frac{4(4+np)\left(F_1\left(1+\frac{np}{2};np,3;2+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-F_1\left(1+\frac{np}{2};np,4;2+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\sin\left(\frac{1}{2}(e+fx)\right)\sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(2+np)\left(2(4+np)F_1\left(1+\frac{np}{2};np,3;2+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)-2(4+np)F_1\left(1+\frac{np}{2};np,4;2+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+2\left(3F_1\left(2+\frac{np}{2};np,4;3+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4F_1\left(2+\frac{np}{2};np,5;3+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+np\left(-F_1\left(2+\frac{np}{2};1+np,3;3+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+F_1\left(2+\frac{np}{2};1+np,4;3+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)(-1+\cos(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (4*(4 + n*p)*(AppellF1[1 + (n*p)/2, n*p, 3, 2 + (n*p)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - AppellF1[1 + (n*p)/2, n*p, 4, 2 + (n*p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*Sin[
e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p)*(2*(4 + n*p)*AppellF1[1 +
 (n*p)/2, n*p, 3, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[(e + f*x)/2]^2 - 2*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 4, 2 + (n*p)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(3*AppellF1[2
 + (n*p)/2, n*p, 4, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 4*AppellF1[2 + (n*p)/2, n*p, 5, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + n*p*(-AppellF1[2 + (n*p)/2, 1 + n*p, 3, 3 + (n*p)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[2 + (n*p)/2, 1 + n*p, 4, 3 + (n
*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*(-1 + Cos[e + f*x])))
```

Maple [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int \left(\sin^3\left(fx+e\right)\right)\left(b(c\tan\left(fx+e\right))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,{\left(b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int(sin(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

3.171 $\int \sin(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=91

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np);\frac{1}{2}(4+np);\sin^2(e+fx)\right)\sin(e+fx)\tan(e+fx)\,(b(c\tan(e+\cdots}{f(2+np)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p+1, 1/2*n*p+1/2],[1/2*n*p+2]
,sin(f*x+e)^2)*sin(f*x+e)*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+2)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3740, 2682, 2657}

$$\frac{\sin(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2);\frac{1}{2}(np+4);\sin^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2,
 (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x])
^n)^p)/(f*(2 + n*p))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
```

```
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx &= \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sin(e+fx)(c\tan(e+ \\ &= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+f \\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np);\frac{1}{2}(4+np);\sin^2(e+f}{f(2+np)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.90, size = 284, normalized size = 3.12

$$\frac{8(4+np)F_1\left(1+\frac{np}{2};np,2;2+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2+np)\left(2(4+np)F_1\left(1+\frac{np}{2};np,2;2+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+2\left(2F_1\left(2+\frac{np}{2};np,3;3+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-npF_1\left(2+\frac{np}{2};1+np,2;3+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(-1+\cos(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
[Out] (8*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 2, 2 + (n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^4*Sin[(e + f*x)/2]^2*(b*(c*Tan[e + f
*x])^n)^p)/(f*(2 + n*p)*(2*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 2, 2 + (n*p
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(2*App
ellF1[2 + (n*p)/2, n*p, 3, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] - n*p*AppellF1[2 + (n*p)/2, 1 + n*p, 2, 3 + (n*p)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sin\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b(c\tan\left(e+fx\right))^{n}\right)^{p}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sin(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,{\left(b\,{\left(c\,\mathrm{tan}(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(sin(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

3.172 $\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=81

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(1+np);\frac{1}{2}(2+np);\sin^2(e+fx)\right)\sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p, 1/2*n*p+1/2],[1/2*n*p+1],s
in(f*x+e)^2)*sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/n/p
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3740, 2681, 2657}

$$\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+2);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (2
+ n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
```

```
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc(e+fx)(c\tan(e+$$

$$= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(1+np);\frac{1}{2}(2+np);\sin^2(e+fx)\right)\text{se}}{fnp}$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 0.95

$$\frac{{}_2F_1\left(\frac{np}{2},np;1+\frac{np}{2};\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^{np}\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e + f*x)/2]^2]*(Cos[e +
f*x]*Sec[(e + f*x)/2]^2)^(n*p)*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \csc\left(fx+e\right)\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \csc(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,(c\tan(e+f\,x))^n)^p}{\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x),x)
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x), x)
```

3.173 $\int \csc^3(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=92

$$-\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\csc^2(e+fx)\,{}_2F_1\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np);\frac{np}{2};\sin^2(e+fx)\right)\sec(e+fx)\,(b(c\tan(e+fx
}{f(2-np)}$$

```
[Out] -(cos(f*x+e)^2)^(1/2*n*p+1/2)*csc(f*x+e)^2*hypergeom([1/2*n*p-1, 1/2*n*p+1/
2],[1/2*n*p],sin(f*x+e)^2)*sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+2)
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2681, 2657}

$$-\frac{\csc^2(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,{}_2F_1\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1);\frac{np}{2};\sin^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(2-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -(((Cos[e + f*x]^2)^((1 + n*p)/2)*Csc[e + f*x]^2*Hypergeometric2F1[(-2 + n*
p)/2, (1 + n*p)/2, (n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*x]
)^n)^p)/(f*(2 - n*p)))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
```

```
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \csc^3(e+fx)(c\tan(e-$$

$$= \left(\cos^{np}(e+fx)\sin^{-np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^{-np}(e+$$

$$= -\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\csc^2(e+fx)\,{}_2F_1\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np);\frac{np}{2};\right.}{f(2-np)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 14.76, size = 1399, normalized size = 15.21

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[(e + f*x)/2]^2*Hypergeometric2F1[n*p, -1 + (n*p)/2, (n*p)/2, Tan[(e +
f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n*p)*(b*(c*Tan[e + f*x])^n)^p
)/(f*(-8 + 4*n*p)) + ((4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]^2*(b*(c*Tan[e + f*x])
^n)^p)/(8*f*(2 + n*p)*((4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + AppellF1[2 +
 (n*p)/2, n*p, 2, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(-1
 + Cos[e + f*x]) + 2*n*p*AppellF1[2 + (n*p)/2, 1 + n*p, 1, 3 + (n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2)) + (Hypergeometri
c2F1[n*p, 1 + (n*p)/2, 2 + (n*p)/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(
e + f*x)/2]^2)^(n*p)*Tan[(e + f*x)/2]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(8 + 4
*n*p)) + (Cot[(e + f*x)/2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n*p)*((2 + n*
p)*Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e + f*x)/2]^2] - n*p*A
ppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^(n*p)*(b*(c*Tan[e + f*x])^n)^p)/(8
*f*n*p*(2 + n*p)*(((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + n*p)*(-(Sec[(e +
 f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
)*((2 + n*p)*Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e + f*x)/2]^
2] - n*p*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^(n*p))/(2*(2 + n*p)) + (
```

```
(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n*p)*(-(n*p*AppellF1[1 + (n*p)/2, n*p, 1
, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2]) - n*p*Tan[(e + f*x)/2]^2*(-(((1 + (n*p)/2)*AppellF1[2 + (
n*p)/2, n*p, 2, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + (n*p)/2)) + (n*p*(1 + (n*p)/2)*AppellF
1[2 + (n*p)/2, 1 + n*p, 1, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + (n*p)/2)) + (n*p*(2 + n*p)*
Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(-Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*
p)/2, Tan[(e + f*x)/2]^2] + (1 - Tan[(e + f*x)/2]^2)^(-(n*p))))/2)*Tan[e +
f*x]^(n*p))/(2*n*p*(2 + n*p)) + ((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n*p)*Se
c[e + f*x]^2*((2 + n*p)*Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e
 + f*x)/2]^2] - n*p*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^(-1 + n*p))/(
2*(2 + n*p))))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\csc^3 \left(fx+e\right)\right) \left(b(c\tan \left(fx+e\right))^n\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \csc^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,(c\tan(e+f\,x))^n)^p}{\sin(e+f\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^3,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^3, x)
```

3.174 $\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx$

Optimal. Leaf size=28

$$\text{Int}((d\sin(e+fx))^m (a+b\tan^n(e+fx))^p, x)$$

```
[Out] Unintegrable((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p, x]
```

Rubi steps

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx = \int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx$$

Mathematica [A]
time = 1.73, size = 0, normalized size = 0.00

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p,x]
```

```
[Out] Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p, x]
```

Maple [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int (d\sin(fx+e))^m (a+b(\tan^n(fx+e)))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
[Out] int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*tan(f*x+e)**n)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (d\,\sin\left(e+f\,x\right))^m\,\left(a+b\,\tan(e+f\,x)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^n)^p,x)
[Out] int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^n)^p, x)
```

3.175 $\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=99

$$\frac{(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+2p)} \, {}_2F_1\left(\frac{1}{2}(1+2p), \frac{1}{2}(1-m+2p); \frac{1}{2}(3+2p); \sin^2(e+fx)\right) \tan(e+fx)}{f(1+2p)}$$

```
[Out] (d*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2-1/2*m+p)*hypergeom([1/2+p, 1/2-1/2*m+p
],[3/2+p],sin(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1+2*p)
```

Rubi [A]
time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3739, 2683, 2697}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+2p+1)} \, {}_2F_1\left(\frac{1}{2}(2p+1), \frac{1}{2}(-m+2p+1); \frac{1}{2}(2p+3); \sin^2(e+fx)\right)}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + 2*p)/2)*Hypergeometric2F1[(1
 + 2*p)/2, (1 - m + 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\cos(e+fx))^m \tan^{2p}(e+$$
$$= \left((d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m \tan^{-2p}(e+fx)\left(b\tan^2(e+f\right.\right.$$
$$= \frac{(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+2p)}\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1-m+\right.}{f(1+2}$$

Mathematica [A]
time = 0.36, size = 81, normalized size = 0.82

$$\frac{(d\cos(e+fx))^m\,{}_2F_1\left(1+\frac{m}{2},\frac{1}{2}+p;\frac{3}{2}+p;-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}\tan(e+fx)\left(b\tan^2(e+fx)\right)^p}{f(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*Hypergeometric2F1[1 + m/2, 1/2 + p, 3/2 + p, -Tan[e + f
*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 + 2*
p))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (d\cos(fx+e))^m \left(b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(e+fx\right)\right)^p \left(d\cos\left(e+fx\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
[Out] Integral((b*tan(e + f*x)**2)**p*(d*cos(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\cos\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
[Out] int((d*cos(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

3.176 $\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=108

$$\frac{F_1\left(\frac{1}{2};\frac{2+m}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+fx)\left(a+b\tan^2(e+\right.}{f}$$

```
[Out] AppellF1(1/2,1+1/2*m,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*cos(f*x+e))
^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e
)^2/a)^p)
```

Rubi [A]

time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3750, 3760, 441, 440}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{m+2}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(a + b*Tan[e + f*x
]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(d*Cos[e + f*x])^FracPart[m]*(Sec
[e + f*x]/d)^FracPart[m], Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sec[e + f*x]/d)
^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Rule 3760

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff
*((d*Sec[e + f*x])^m/(f*(Sec[e + f*x]^2)^(m/2))), Subst[Int[(1 + ff^2*x^2)^
(m/2 - 1)*(a + b*ff^2*x^2)^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx = \left((d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m\right)\int \left(\frac{\sec(e+fx)}{d}\right)^{-m}$$

$$= \frac{\left((d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\right)\operatorname{Subst}\left(\int \left(1+x^2\right)^{-1-\frac{m}{2}}\right.}{f}$$

$$= \frac{\left((d\cos(e+fx))^m \sec^2(e+fx)^{m/2} \left(a+b\tan^2(e+fx)\right)^p \left(1\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{2+m}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\cos(e+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2033 vs. $2(108) = 216$.
time = 15.12, size = 2033, normalized size = 18.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*(a + b*Ta
n[e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m
)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)*((
6*a*b*p*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/a)]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p))/((Sec[e + f*x]^2)^
(m/2)*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -p, 5/2, -Tan
```

```
[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)) + (3*a*AppellF1[1/2
, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(a + b*Tan[
e + f*x]^2)^p)/((Sec[e + f*x]^2)^(m/2)*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)
/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*Appel
lF1[3/2, (4 + m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan
[e + f*x]^2)) + (6*a*(-1 - m/2)*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]^2
*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4
+ m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)
 + (3*a*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, (2 +
 m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2
*Tan[e + f*x])/(3*a) - ((2 + m)*AppellF1[3/2, 1 + (2 + m)/2, -p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/3)*(a + b
*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) - (3*a
*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(2*(2*b
*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/a)])*Sec[e + f*x]^2*Tan[e + f*x] + 3*a*((2*b*p*AppellF1[3/2
, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e +
f*x]^2*Tan[e + f*x])/(3*a) - ((2 + m)*AppellF1[3/2, 1 + (2 + m)/2, -p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/3)
+ Tan[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/2, (2 + m)/2, 2 - p, 7/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*
a) - (3*(2 + m)*AppellF1[5/2, 1 + (2 + m)/2, 1 - p, 7/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/5) - a*(2 + m)*((6*b*p
*AppellF1[5/2, (4 + m)/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a) - (3*(4 + m)*AppellF1[5/2, 1 + (4 +
 m)/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Ta
n[e + f*x])/5))))/(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -
p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)^2))
```

Maple [F]

time = 0.27, size = 0, normalized size = 0.00

$$\int \left(d \cos \left(f x +e \right)\right)^{m} \left(a +b \left(\tan^{2}\left(f x +e \right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\cos(e+f\,x))^m\,\left(b\,\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

3.177 $\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx$

Optimal. Leaf size=101

$$\frac{(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+np)} {}_2F_1\left(\frac{1}{2}(1+np), \frac{1}{2}(1-m+np); \frac{1}{2}(3+np); \sin^2(e+fx)\right) \tan(e+fx)}{f(1+np)}$$

```
[Out] (d*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2*n*p-1/2*m+1/2)*hypergeom([1/2*n*p+1/2,
 1/2*n*p-1/2*m+1/2],[1/2*n*p+3/2],sin(f*x+e)^2)*tan(f*x+e)*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3740, 2683, 2697}

$$\frac{\tan(e+fx)(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(-m+np+1)} (b(c\tan(e+fx))^n)^p {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(-m+np+1); \frac{1}{2}(np+3); \sin^2(e+fx)\right)}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n*p)/2)*Hypergeometric2F1[(1
 + n*p)/2, (1 - m + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
```

```
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\cos(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\cos(e+fx))^m$$

$$= \left((d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m (c\tan(e+fx))^{-np}\,(b(c\,t\right.$$

$$= \frac{(d\cos(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(1-m+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1-\right.}{f}$$

Mathematica [A]
time = 0.34, size = 91, normalized size = 0.90

$$\frac{(d\cos(e+fx))^m\,{}_2F_1\left(\frac{2+m}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((d*Cos[e + f*x])^m*Hypergeometric2F1[(2 + m)/2, (1 + n*p)/2, (3 + n*p)/2,
-Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)
^p)/(f*(1 + n*p))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (d\cos(fx+e))^m\left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\left(d\cos\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*cos(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\cos\left(e+f\,x\right)\right)}^m\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

3.178 $\int (d\cos(e+fx))^m (a+b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=57

$$(d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m \operatorname{Int}\left(\left(\frac{\sec(e+fx)}{d}\right)^{-m} (a+b(c\tan(e+fx))^n)^p, x\right)$$

```
[Out] (d*cos(f*x+e))^m*(sec(f*x+e)/d)^m*Unintegrable((a+b*(c*tan(f*x+e))^n)^p/((s
ec(f*x+e)/d)^m),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\cos(e+fx))^m (a+b(c\tan(e+fx))^n)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*Cos[e + f*x])^m*(Sec[e + f*x]/d)^m*Defer[Int][(a + b*(c*Tan[e + f*x])^n)
^p/(Sec[e + f*x]/d)^m, x]
```

Rubi steps

$$\int (d\cos(e+fx))^m (a+b(c\tan(e+fx))^n)^p\,dx = \left((d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m\right) \int \left(\frac{\sec(e+fx)}{d}\right)$$

Mathematica [A]
time = 1.75, size = 0, normalized size = 0.00

$$\int (d\cos(e+fx))^m (a+b(c\tan(e+fx))^n)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int (d\cos(fx+e))^m (a+b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\cos(e+fx))^m \left(a+b\left(c\tan(e+fx)\right)^n\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*cos(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p, x)
```

3.179 $\int (a+a\tan^2(c+dx))^4\,dx$

Optimal. Leaf size=65

$$\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}$$

```
[Out] a^4*tan(d*x+c)/d+a^4*tan(d*x+c)^3/d+3/5*a^4*tan(d*x+c)^5/d+1/7*a^4*tan(d*x+
c)^7/d
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3738, 12, 3852}

$$\frac{a^4\tan^7(c+dx)}{7d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^4,x]
```

```
[Out] (a^4*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/d + (3*a^4*Tan[c + d*x]^5)/(5*d
) + (a^4*Tan[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+a\tan^2(c+dx)\right)^4\,dx &= \int a^4\sec^8(c+dx)\,dx\\ &= a^4\int \sec^8(c+dx)\,dx\\ &= -\frac{a^4\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 46, normalized size = 0.71

$$\frac{a^4\left(\tan(c+dx)+\tan^3(c+dx)+\frac{3}{5}\tan^5(c+dx)+\frac{1}{7}\tan^7(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^4,x]
```

```
[Out] (a^4*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c + d*x]^5)/5 + Tan[c + d*x]^7
/7))/d
```

Maple [A]
time = 0.04, size = 43, normalized size = 0.66

method	result	size
derivativedivides	$\frac{a^4\left(\frac{\left(\tan^7(dx+c)\right)}{7}+\frac{3\left(\tan^5(dx+c)\right)}{5}+\tan^3(dx+c)+\tan(dx+c)\right)}{d}$	43
default	$\frac{a^4\left(\frac{\left(\tan^7(dx+c)\right)}{7}+\frac{3\left(\tan^5(dx+c)\right)}{5}+\tan^3(dx+c)+\tan(dx+c)\right)}{d}$	43
risch	$\frac{32ia^4\left(35\,e^{6i(dx+c)}+21\,e^{4i(dx+c)}+7\,e^{2i(dx+c)}+1\right)}{35d\left(e^{2i(dx+c)}+1\right)^7}$	58
norman	$\frac{a^4\tan(dx+c)}{d}+\frac{a^4\left(\tan^3(dx+c)\right)}{d}+\frac{3a^4\left(\tan^5(dx+c)\right)}{5d}+\frac{a^4\left(\tan^7(dx+c)\right)}{7d}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^4*(1/7*tan(d*x+c)^7+3/5*tan(d*x+c)^5+tan(d*x+c)^3+tan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(61) = 122.
time = 0.49, size = 157, normalized size = 2.42

$$a^4x+\frac{\left(15\,\tan\left(dx+c\right)^7-21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+105\,dx+105\,c-105\,\tan\left(dx+c\right)\right)a^4}{105\,d}+\frac{4\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a^4}{15\,d}+\frac{2\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^4}{d}-\frac{4\left(dx+c-\tan\left(dx+c\right)\right)a^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 +
105*d*x + 105*c - 105*tan(d*x + c))*a^4/d + 4/15*(3*tan(d*x + c)^5 - 5*tan(
d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^4/d + 2*(tan(d*x + c)^3 + 3
*d*x + 3*c - 3*tan(d*x + c))*a^4/d - 4*(d*x + c - tan(d*x + c))*a^4/d
```

Fricas [A]
time = 4.69, size = 56, normalized size = 0.86

$$\frac{5\,a^4\tan\left(dx+c\right)^7+21\,a^4\tan\left(dx+c\right)^5+35\,a^4\tan\left(dx+c\right)^3+35\,a^4\tan\left(dx+c\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a^4*tan(d*x + c)^7 + 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)^3
+ 35*a^4*tan(d*x + c))/d
```

Sympy [A]
time = 0.20, size = 68, normalized size = 1.05

$$\begin{cases} \frac{a^4\tan^7\left(c+dx\right)}{7d}+\frac{3a^4\tan^5\left(c+dx\right)}{5d}+\frac{a^4\tan^3\left(c+dx\right)}{d}+\frac{a^4\tan\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\left(a\tan^2\left(c\right)+a\right)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**4,x)
```

```
[Out] Piecewise((a**4*tan(c + d*x)**7/(7*d) + 3*a**4*tan(c + d*x)**5/(5*d) + a**4
*tan(c + d*x)**3/d + a**4*tan(c + d*x)/d, Ne(d, 0)), (x*(a*tan(c)**2 + a)**
4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(61) = 122.
time = 1.20, size = 519, normalized size = 7.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/35*(35*a^4*tan(d*x)^7*tan(c)^6 + 35*a^4*tan(d*x)^6*tan(c)^7 + 35*a^4*tan
(d*x)^7*tan(c)^4 - 105*a^4*tan(d*x)^6*tan(c)^5 - 105*a^4*tan(d*x)^5*tan(c)^
6 + 35*a^4*tan(d*x)^4*tan(c)^7 + 21*a^4*tan(d*x)^7*tan(c)^2 - 35*a^4*tan(d*
```

```
x)^6*tan(c)^3 + 315*a^4*tan(d*x)^5*tan(c)^4 + 315*a^4*tan(d*x)^4*tan(c)^5 -
 35*a^4*tan(d*x)^3*tan(c)^6 + 21*a^4*tan(d*x)^2*tan(c)^7 + 5*a^4*tan(d*x)^7
 - 7*a^4*tan(d*x)^6*tan(c) + 105*a^4*tan(d*x)^5*tan(c)^2 - 315*a^4*tan(d*x)
^4*tan(c)^3 - 315*a^4*tan(d*x)^3*tan(c)^4 + 105*a^4*tan(d*x)^2*tan(c)^5 - 7
*a^4*tan(d*x)*tan(c)^6 + 5*a^4*tan(c)^7 + 21*a^4*tan(d*x)^5 - 35*a^4*tan(d*
x)^4*tan(c) + 315*a^4*tan(d*x)^3*tan(c)^2 + 315*a^4*tan(d*x)^2*tan(c)^3 - 3
5*a^4*tan(d*x)*tan(c)^4 + 21*a^4*tan(c)^5 + 35*a^4*tan(d*x)^3 - 105*a^4*tan
(d*x)^2*tan(c) - 105*a^4*tan(d*x)*tan(c)^2 + 35*a^4*tan(c)^3 + 35*a^4*tan(d
*x) + 35*a^4*tan(c))/(d*tan(d*x)^7*tan(c)^7 - 7*d*tan(d*x)^6*tan(c)^6 + 21*
d*tan(d*x)^5*tan(c)^5 - 35*d*tan(d*x)^4*tan(c)^4 + 35*d*tan(d*x)^3*tan(c)^3
 - 21*d*tan(d*x)^2*tan(c)^2 + 7*d*tan(d*x)*tan(c) - d)
```

Mupad [B]

time = 11.52, size = 53, normalized size = 0.82

$$\frac{\frac{a^4\tan(c+d\,x)^7}{7}+\frac{3\,a^4\tan(c+d\,x)^5}{5}+a^4\tan(c+d\,x)^3+a^4\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^4,x)
[Out] (a^4*tan(c + d*x) + a^4*tan(c + d*x)^3 + (3*a^4*tan(c + d*x)^5)/5 + (a^4*ta
n(c + d*x)^7)/7)/d
```

3.180 $\int (a+a\tan^2(c+dx))^3 \, dx$

Optimal. Leaf size=50

$$\frac{a^3\tan(c+dx)}{d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan^5(c+dx)}{5d}$$

```
[Out] a^3*tan(d*x+c)/d+2/3*a^3*tan(d*x+c)^3/d+1/5*a^3*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3738, 12, 3852}

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (2*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/
(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \left(a+a\tan^2(c+dx)\right)^3\,dx = \int a^3\sec^6(c+dx)\,dx$$
$$= a^3\int \sec^6(c+dx)\,dx$$
$$= -\frac{a^3\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(c+dx)\right)}{d}$$
$$= \frac{a^3\tan(c+dx)}{d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan^5(c+dx)}{5d}$$

Mathematica [A]
time = 0.07, size = 38, normalized size = 0.76

$$\frac{a^3\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^3,x]
[Out] (a^3*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

Maple [A]
time = 0.03, size = 35, normalized size = 0.70

method	result	size
derivativedivides	$\frac{a^3\left(\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{2\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)\right)}{d}$	35
default	$\frac{a^3\left(\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{2\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)\right)}{d}$	35
norman	$\frac{a^3\tan(dx+c)}{d}+\frac{2a^3\left(\tan^3(dx+c)\right)}{3d}+\frac{a^3\left(\tan^5(dx+c)\right)}{5d}$	47
risch	$\frac{16ia^3\left(10\,e^{4i(dx+c)}+5\,e^{2i(dx+c)}+1\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
[Out] 1/d*a^3*(1/5*tan(d*x+c)^5+2/3*tan(d*x+c)^3+tan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(46) = 92.
time = 0.49, size = 102, normalized size = 2.04

$$a^3x+\frac{\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a^3}{15\,d}+\frac{\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^3}{d}-\frac{3\left(dx+c-\tan\left(dx+c\right)\right)a^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(
d*x + c))*a^3/d + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3/d - 3
*(d*x + c - tan(d*x + c))*a^3/d
```

Fricas [A]
time = 2.74, size = 43, normalized size = 0.86

$$\frac{3\,a^3\tan\left(dx+c\right)^5+10\,a^3\tan\left(dx+c\right)^3+15\,a^3\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^3*tan(d*x + c)^5 + 10*a^3*tan(d*x + c)^3 + 15*a^3*tan(d*x + c))/d
```

Sympy [A]
time = 0.15, size = 54, normalized size = 1.08

$$\begin{cases} \frac{a^3\tan^5\left(c+dx\right)}{5d}+\frac{2a^3\tan^3\left(c+dx\right)}{3d}+\frac{a^3\tan\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x\left(a\tan^2\left(c\right)+a\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*tan(c + d*x)**5/(5*d) + 2*a**3*tan(c + d*x)**3/(3*d) + a**3
*tan(c + d*x)/d, Ne(d, 0)), (x*(a*tan(c)**2 + a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(46) = 92.
time = 0.73, size = 297, normalized size = 5.94

$$-\frac{15\,a^3\tan\left(dx\right)^5\tan\left(c\right)^4+15\,a^3\tan\left(dx\right)^4\tan\left(c\right)^5+10\,a^3\tan\left(dx\right)^5\tan\left(c\right)^2-30\,a^3\tan\left(dx\right)^4\tan\left(c\right)^3-30\,a^3\tan\left(dx\right)^3\tan\left(c\right)^4+10\,a^3\tan\left(dx\right)^2\tan\left(c\right)^5+3\,a^3\tan\left(dx\right)^5-5\,a^3\tan\left(dx\right)^4\tan\left(c\right)+60\,a^3\tan\left(dx\right)^3\tan\left(c\right)^2+60\,a^3\tan\left(dx\right)^2\tan\left(c\right)^3-5\,a^3\tan\left(dx\right)\tan\left(c\right)^4+3\,a^3\tan\left(c\right)^5+10\,a^3\tan\left(dx\right)^3-30\,a^3\tan\left(dx\right)^2\tan\left(c\right)-30\,a^3\tan\left(dx\right)\tan\left(c\right)^2+10\,a^3\tan\left(c\right)^3+15\,a^3\tan\left(dx\right)+15\,a^3\tan\left(c\right)}{15\left(d\tan\left(dx\right)^5\tan\left(c\right)^5-5\,d\tan\left(dx\right)^4\tan\left(c\right)^4+10\,d\tan\left(dx\right)^3\tan\left(c\right)^3-10\,d\tan\left(dx\right)^2\tan\left(c\right)^2+5\,d\tan\left(dx\right)\tan\left(c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*a^3*tan(d*x)^5*tan(c)^4 + 15*a^3*tan(d*x)^4*tan(c)^5 + 10*a^3*tan
(d*x)^5*tan(c)^2 - 30*a^3*tan(d*x)^4*tan(c)^3 - 30*a^3*tan(d*x)^3*tan(c)^4
+ 10*a^3*tan(d*x)^2*tan(c)^5 + 3*a^3*tan(d*x)^5 - 5*a^3*tan(d*x)^4*tan(c) +
 60*a^3*tan(d*x)^3*tan(c)^2 + 60*a^3*tan(d*x)^2*tan(c)^3 - 5*a^3*tan(d*x)*t
an(c)^4 + 3*a^3*tan(c)^5 + 10*a^3*tan(d*x)^3 - 30*a^3*tan(d*x)^2*tan(c) - 3
0*a^3*tan(d*x)*tan(c)^2 + 10*a^3*tan(c)^3 + 15*a^3*tan(d*x) + 15*a^3*tan(c)
```

```
)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)
^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 11.47, size = 36, normalized size = 0.72

$$\frac{a^3 \tan(c+d\,x)\,\left(3\tan(c+d\,x)^4+10\tan(c+d\,x)^2+15\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^3,x)
[Out] (a^3*tan(c + d*x)*(10*tan(c + d*x)^2 + 3*tan(c + d*x)^4 + 15))/(15*d)
```

3.181 $\int (a+a\tan^2(c+dx))^2\,dx$

Optimal. Leaf size=32

$$\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}$$

```
[Out] a^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3738, 12, 3852}

$$\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+a\tan^2(c+dx)\right)^2\,dx &= \int a^2\sec^4(c+dx)\,dx\\
&= a^2\int \sec^4(c+dx)\,dx\\
&= -\frac{a^2\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 0.81

$$
\frac{a^2\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.78

method	result	size
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)\right)}{d}$	25
default	$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)\right)}{d}$	25
norman	$\frac{a^2\tan(dx+c)}{d}+\frac{a^2\left(\tan^3(dx+c)\right)}{3d}$	31
risch	$\frac{4ia^2\left(3\,e^{2i(dx+c)}+1\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(1/3*tan(d*x+c)^3+tan(d*x+c))
```

Maxima [A]
time = 0.49, size = 59, normalized size = 1.84

$$
a^2x+\frac{\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2}{3\,d}-\frac{2\left(dx+c-\tan\left(dx+c\right)\right)a^2}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2/d - 2*(d*x
+ c - tan(d*x + c))*a^2/d
```

Fricas [A]
time = 5.56, size = 29, normalized size = 0.91

$$\frac{a^2 \tan{(dx+c)}^3 + 3\, a^2 \tan{(dx+c)}}{3\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c))/d
```

Sympy [A]
time = 0.09, size = 37, normalized size = 1.16

$$\begin{cases} \frac{a^2 \tan^3{(c+dx)}}{3d} + \frac{a^2 \tan{(c+dx)}}{d} & \text{for } d \neq 0 \\ x(a \tan^2{(c)} + a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*tan(c + d*x)**3/(3*d) + a**2*tan(c + d*x)/d, Ne(d, 0)), (x*
(a*tan(c)**2 + a)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(30) = 60.
time = 0.61, size = 133, normalized size = 4.16

$$-\frac{3\, a^2 \tan{(dx)}^3 \tan{(c)}^2 + 3\, a^2 \tan{(dx)}^2 \tan{(c)}^3 + a^2 \tan{(dx)}^3 - 3\, a^2 \tan{(dx)}^2 \tan{(c)} - 3\, a^2 \tan{(dx)} \tan{(c)}^2 + a^2 \tan{(c)}^3 + 3\, a^2 \tan{(dx)} + 3\, a^2 \tan{(c)}}{3\, \left(d \tan{(dx)}^3 \tan{(c)}^3 - 3\, d \tan{(dx)}^2 \tan{(c)}^2 + 3\, d \tan{(dx)} \tan{(c)} - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*tan(d*x)^3*tan(c)^2 + 3*a^2*tan(d*x)^2*tan(c)^3 + a^2*tan(d*x)^
3 - 3*a^2*tan(d*x)^2*tan(c) - 3*a^2*tan(d*x)*tan(c)^2 + a^2*tan(c)^3 + 3*a^
2*tan(d*x) + 3*a^2*tan(c))/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)^2
 + 3*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 11.57, size = 24, normalized size = 0.75

$$\frac{a^2 \tan(c+d\,x)\,\left(\tan(c+d\,x)^2+3\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^2,x)
```

```
[Out] (a^2*tan(c + d*x)*(tan(c + d*x)^2 + 3))/(3*d)
```

3.182 $\int \frac{1}{a+a\tan^2(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{2a}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

```
[Out] 1/2*x/a+1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {3738, 12, 2715, 8}

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] x/(2*a) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\tan^2(c+dx)}\,dx &= \int \frac{\cos^2(c+dx)}{a}\,dx\\ &= \frac{\int \cos^2(c+dx)\,dx}{a}\\ &= \frac{\cos(c+dx)\sin(c+dx)}{2ad}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.84

$$\frac{2(c+dx)+\sin(2(c+dx))}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] (2*(c + d*x) + Sin[2*(c + d*x)])/(4*a*d)
```

Maple [A]
time = 0.06, size = 38, normalized size = 1.23

method	result	size
risch	$\frac{x}{2a}+\frac{\sin(2dx+2c)}{4ad}$	25
derivativedivides	$\frac{\frac{\tan(dx+c)}{2+2\left(\tan^2(dx+c)\right)}+\frac{\arctan(\tan(dx+c))}{2}}{da}$	38
default	$\frac{\frac{\tan(dx+c)}{2+2\left(\tan^2(dx+c)\right)}+\frac{\arctan(\tan(dx+c))}{2}}{da}$	38
norman	$\frac{\frac{x}{2a}+\frac{\tan(dx+c)}{2ad}+\frac{x\left(\tan^2(dx+c)\right)}{2a}}{1+\tan^2(dx+c)}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/2*tan(d*x+c)/(1+tan(d*x+c)^2)+1/2*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 36, normalized size = 1.16

$$\frac{\frac{dx+c}{a}+\frac{\tan(dx+c)}{a\tan(dx+c)^2+a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)/a + tan(d*x + c)/(a*tan(d*x + c)^2 + a))/d
```

Fricas [A]
time = 4.09, size = 40, normalized size = 1.29

$$\frac{dx\tan\left(dx+c\right)^{2}+dx+\tan\left(dx+c\right)}{2\left(ad\tan\left(dx+c\right)^{2}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(d*x*tan(d*x + c)^2 + d*x + tan(d*x + c))/(a*d*tan(d*x + c)^2 + a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(22) = 44$.
time = 0.28, size = 87, normalized size = 2.81

$$\begin{cases} \frac{dx\tan^{2}(c+dx)}{2ad\tan^{2}(c+dx)+2ad}+\frac{dx}{2ad\tan^{2}(c+dx)+2ad}+\frac{\tan(c+dx)}{2ad\tan^{2}(c+dx)+2ad} & \text{for } d\neq 0 \\ \frac{x}{a\tan^{2}(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*a*d) + d*x/(2*a*d
*tan(c + d*x)**2 + 2*a*d) + tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*a*d), N
e(d, 0)), (x/(a*tan(c)**2 + a), True))
```

Giac [A]
time = 0.56, size = 37, normalized size = 1.19

$$\frac{\frac{dx+c}{a}+\frac{\tan(dx+c)}{\left(\tan(dx+c)^{2}+1\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)/a + tan(d*x + c)/((tan(d*x + c)^2 + 1)*a))/d
```

Mupad [B]
time = 11.83, size = 26, normalized size = 0.84

$$\frac{\frac{\sin(2\,c+2\,d\,x)}{4\,a}+\frac{d\,x}{2\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2),x)
[Out] (sin(2*c + 2*d*x)/(4*a) + (d*x)/(2*a))/d
```

3.183 $\int \frac{1}{(a+a\tan^2(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{3x}{8a^2}+\frac{3\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}$$

```
[Out] 3/8*x/a^2+3/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3738, 12, 2715, 8}

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{3x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] (3*x)/(8*a^2) + (3*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]^3*S
in[c + d*x])/(4*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\int \frac{1}{(a+a\tan^2(c+dx))^2}\,dx = \int \frac{\cos^4(c+dx)}{a^2}\,dx$$
$$= \frac{\int \cos^4(c+dx)\,dx}{a^2}$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{3\int \cos^2(c+dx)\,dx}{4a^2}$$
$$= \frac{3\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{3\int 1\,dx}{8a^2}$$
$$= \frac{3x}{8a^2} + \frac{3\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.65

$$\frac{12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx))}{32a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] (12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])/(32*a^2*d)
```

Maple [A]
time = 0.07, size = 58, normalized size = 1.05

method	result	size
risch	$\frac{3x}{8a^2}+\frac{\sin(4dx+4c)}{32a^2d}+\frac{\sin(2dx+2c)}{4a^2d}$	42
derivativedivides	$\frac{\frac{\tan(dx+c)}{4\left(1+\tan^2(dx+c)\right)^2}+\frac{3\tan(dx+c)}{8\left(1+\tan^2(dx+c)\right)}+\frac{3\arctan(\tan(dx+c))}{8}}{d\,a^2}$	58
default	$\frac{\frac{\tan(dx+c)}{4\left(1+\tan^2(dx+c)\right)^2}+\frac{3\tan(dx+c)}{8\left(1+\tan^2(dx+c)\right)}+\frac{3\arctan(\tan(dx+c))}{8}}{d\,a^2}$	58
norman	$\frac{\frac{3x}{8a}+\frac{5\tan(dx+c)}{8ad}+\frac{3\left(\tan^3(dx+c)\right)}{8ad}+\frac{3x\left(\tan^2(dx+c)\right)}{4a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}}{a(1+\tan^2(dx+c))^2}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/4*tan(d*x+c)/(1+tan(d*x+c)^2)^2+3/8*tan(d*x+c)/(1+tan(d*x+c)^2)+
3/8*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 67, normalized size = 1.22

$$\frac{\frac{3\,\tan(dx+c)^3+5\,\tan(dx+c)}{a^2\,\tan(dx+c)^4+2\,a^2\,\tan(dx+c)^2+a^2}+\frac{3\,(dx+c)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*tan(d*x + c)^3 + 5*tan(d*x + c))/(a^2*tan(d*x + c)^4 + 2*a^2*tan(d*
x + c)^2 + a^2) + 3*(d*x + c)/a^2)/d
```

Fricas [A]
time = 3.78, size = 84, normalized size = 1.53

$$\frac{3\,dx\tan\left(dx+c\right)^4+6\,dx\tan\left(dx+c\right)^2+3\,\tan\left(dx+c\right)^3+3\,dx+5\,\tan\left(dx+c\right)}{8\,\left(a^2d\tan\left(dx+c\right)^4+2\,a^2d\tan\left(dx+c\right)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(3*d*x*tan(d*x + c)^4 + 6*d*x*tan(d*x + c)^2 + 3*tan(d*x + c)^3 + 3*d*x
 + 5*tan(d*x + c))/(a^2*d*tan(d*x + c)^4 + 2*a^2*d*tan(d*x + c)^2 + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(51) = 102$.
time = 0.42, size = 248, normalized size = 4.51

$$\begin{cases}\frac{3dx\tan^4(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d}+\frac{6dx\tan^2(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d}+\frac{3dx}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d}+\frac{3\tan^3(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d}+\frac{5\tan(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d} & \text{for } d\neq 0\\ \frac{x}{(a\tan^2(c)+a)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*d*x*tan(c + d*x)**4/(8*a**2*d*tan(c + d*x)**4 + 16*a**2*d*tan(
c + d*x)**2 + 8*a**2*d) + 6*d*x*tan(c + d*x)**2/(8*a**2*d*tan(c + d*x)**4 +
 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 3*d*x/(8*a**2*d*tan(c + d*x)**4 +
16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 3*tan(c + d*x)**3/(8*a**2*d*tan(c +
 d*x)**4 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 5*tan(c + d*x)/(8*a**2*d
*tan(c + d*x)**4 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d), Ne(d, 0)), (x/(a*
tan(c)**2 + a)**2, True))
```

Giac [A]
time = 0.62, size = 51, normalized size = 0.93

$$\frac{\frac{3\,(dx+c)}{a^2}+\frac{3\,\tan(dx+c)^3+5\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(d*x + c)/a^2 + (3*tan(d*x + c)^3 + 5*tan(d*x + c))/((tan(d*x + c)^2
 + 1)^2*a^2))/d
```

Mupad [B]
time = 11.90, size = 35, normalized size = 0.64

$$\frac{2\,\sin\left(2\,c+2\,d\,x\right)+\frac{\sin\left(4\,c+4\,d\,x\right)}{4}+3\,d\,x}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^2,x)
```

```
[Out] (2*sin(2*c + 2*d*x) + sin(4*c + 4*d*x)/4 + 3*d*x)/(8*a^2*d)
```

3.184 $\int \frac{1}{(a+a\tan^2(c+dx))^3}\,dx$

Optimal. Leaf size=79

$$\frac{5x}{16a^3}+\frac{5\cos(c+dx)\sin(c+dx)}{16a^3d}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d}+\frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d}$$

```
[Out] 5/16*x/a^3+5/16*cos(d*x+c)*sin(d*x+c)/a^3/d+5/24*cos(d*x+c)^3*sin(d*x+c)/a^
3/d+1/6*cos(d*x+c)^5*sin(d*x+c)/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3738, 12, 2715, 8}

$$\frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{16a^3d}+\frac{5x}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] (5*x)/(16*a^3) + (5*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d) + (5*Cos[c + d*x]
^3*Sin[c + d*x])/(24*a^3*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(6*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\tan^2(c+dx))^3}\,dx &= \int \frac{\cos^6(c+dx)}{a^3}\,dx\\ &= \frac{\int \cos^6(c+dx)\,dx}{a^3}\\ &= \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d}+\frac{5\int \cos^4(c+dx)\,dx}{6a^3}\\ &= \frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d}+\frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d}+\frac{5\int \cos^2(c+dx)\,dx}{8a^3}\\ &= \frac{5\cos(c+dx)\sin(c+dx)}{16a^3d}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d}+\frac{\cos^5(c+dx)\sin(c+}{6a^3d}\\ &= \frac{5x}{16a^3}+\frac{5\cos(c+dx)\sin(c+dx)}{16a^3d}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d}+\frac{\cos^5(c+dx}{6a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.58

$$\frac{60c+60dx+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))}{192a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-3),x]
[Out] (60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)
])/(192*a^3*d)
```

Maple [A]
time = 0.08, size = 78, normalized size = 0.99

method	result	size
risch	$\frac{5x}{16a^3}+\frac{\sin(6dx+6c)}{192a^3d}+\frac{3\sin(4dx+4c)}{64a^3d}+\frac{15\sin(2dx+2c)}{64a^3d}$	59
derivativedivides	$\frac{\frac{\tan(dx+c)}{6\left(1+\tan^2(dx+c)\right)^3}+\frac{5\tan(dx+c)}{24\left(1+\tan^2(dx+c)\right)^2}+\frac{5\tan(dx+c)}{16\left(1+\tan^2(dx+c)\right)}+\frac{5\arctan(\tan(dx+c))}{16}}{d\,a^3}$	78
default	$\frac{\frac{\tan(dx+c)}{6\left(1+\tan^2(dx+c)\right)^3}+\frac{5\tan(dx+c)}{24\left(1+\tan^2(dx+c)\right)^2}+\frac{5\tan(dx+c)}{16\left(1+\tan^2(dx+c)\right)}+\frac{5\arctan(\tan(dx+c))}{16}}{d\,a^3}$	78
norman	$\frac{\frac{5x}{16a}+\frac{11\tan(dx+c)}{16ad}+\frac{5\left(\tan^3(dx+c)\right)}{6ad}+\frac{5\left(\tan^5(dx+c)\right)}{16ad}+\frac{15x\left(\tan^2(dx+c)\right)}{16a}+\frac{15x\left(\tan^4(dx+c)\right)}{16a}+\frac{5x\left(\tan^6(dx+c)\right)}{16a}}{a^2\left(1+\tan^2(dx+c)\right)^3}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/6*tan(d*x+c)/(1+tan(d*x+c)^2)^3+5/24*tan(d*x+c)/(1+tan(d*x+c)^2)
^2+5/16*tan(d*x+c)/(1+tan(d*x+c)^2)+5/16*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 90, normalized size = 1.14

$$\frac{\frac{15\tan(dx+c)^5+40\tan(dx+c)^3+33\tan(dx+c)}{a^3\tan(dx+c)^6+3a^3\tan(dx+c)^4+3a^3\tan(dx+c)^2+a^3}+\frac{15(dx+c)}{a^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/48*((15*tan(d*x + c)^5 + 40*tan(d*x + c)^3 + 33*tan(d*x + c))/(a^3*tan(d*
x + c)^6 + 3*a^3*tan(d*x + c)^4 + 3*a^3*tan(d*x + c)^2 + a^3) + 15*(d*x + c
)/a^3)/d
```

Fricas [A]
time = 3.02, size = 120, normalized size = 1.52

$$\frac{15\,dx\tan\left(dx+c\right)^6+45\,dx\tan\left(dx+c\right)^4+15\,\tan\left(dx+c\right)^5+45\,dx\tan\left(dx+c\right)^2+40\,\tan\left(dx+c\right)^3+15\,dx+33\,\tan\left(dx+c\right)}{48\,\left(a^3d\tan\left(dx+c\right)^6+3\,a^3d\tan\left(dx+c\right)^4+3\,a^3d\tan\left(dx+c\right)^2+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(15*d*x*tan(d*x + c)^6 + 45*d*x*tan(d*x + c)^4 + 15*tan(d*x + c)^5 + 4
5*d*x*tan(d*x + c)^2 + 40*tan(d*x + c)^3 + 15*d*x + 33*tan(d*x + c))/(a^3*d
*tan(d*x + c)^6 + 3*a^3*d*tan(d*x + c)^4 + 3*a^3*d*tan(d*x + c)^2 + a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(75) = 150$.
time = 0.56, size = 454, normalized size = 5.75

$$\begin{cases} \frac{15dx\tan^6(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d} + \frac{45dx\tan^4(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d} + \frac{45dx\tan^2(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d} + \frac{15dx}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d} + \frac{15\tan^5(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d} + \frac{40\tan^3(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d} + \frac{33\tan(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d} & \text{for}\: d \neq 0 \\ \frac{x}{\left(a\tan^2(c)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((15*d*x*tan(c + d*x)**6/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*t
an(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*d) + 45*d*x*tan(c + d
*x)**4/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d
*tan(c + d*x)**2 + 48*a**3*d) + 45*d*x*tan(c + d*x)**2/(48*a**3*d*tan(c + d
*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*
d) + 15*d*x/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a
**3*d*tan(c + d*x)**2 + 48*a**3*d) + 15*tan(c + d*x)**5/(48*a**3*d*tan(c +
d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3
```

```
*d) + 40*tan(c + d*x)**3/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*
x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*d) + 33*tan(c + d*x)/(48*a**3*
d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2
 + 48*a**3*d), Ne(d, 0)), (x/(a*tan(c)**2 + a)**3, True))
```

Giac [A]
time = 0.65, size = 61, normalized size = 0.77

$$\frac{\frac{15\,(dx+c)}{a^3}+\frac{15\,\tan(dx+c)^5+40\,\tan(dx+c)^3+33\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^3a^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(15*(d*x + c)/a^3 + (15*tan(d*x + c)^5 + 40*tan(d*x + c)^3 + 33*tan(d*
x + c))/((tan(d*x + c)^2 + 1)^3*a^3))/d
```

Mupad [B]
time = 11.93, size = 51, normalized size = 0.65

$$\frac{5\,x}{16\,a^3}+\frac{\cos\left(c+d\,x\right)^6\,\left(\frac{5\,\tan(c+d\,x)^5}{16}+\frac{5\,\tan(c+d\,x)^3}{6}+\frac{11\,\tan(c+d\,x)}{16}\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^3,x)
```

```
[Out] (5*x)/(16*a^3) + (cos(c + d*x)^6*((11*tan(c + d*x))/16 + (5*tan(c + d*x)^3)
/6 + (5*tan(c + d*x)^5)/16))/(a^3*d)
```

3.185 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=74

$$-\frac{(a-b)\log(\cos(e+fx))}{f}-\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\tan^4(e+fx)}{4f}+\frac{b\tan^6(e+fx)}{6f}$$

```
[Out] -(a-b)*ln(cos(f*x+e))/f-1/2*(a-b)*tan(f*x+e)^2/f+1/4*(a-b)*tan(f*x+e)^4/f+1
/6*b*tan(f*x+e)^6/f
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3712, 3554, 3556}

$$\frac{(a-b)\tan^4(e+fx)}{4f}-\frac{(a-b)\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Log[Cos[e + f*x]])/f) - ((a - b)*Tan[e + f*x]^2)/(2*f) + ((a - b
)*Tan[e + f*x]^4)/(4*f) + (b*Tan[e + f*x]^6)/(6*f)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan^6(e+fx)}{6f}+(a-b)\int \tan^5(e+fx)\,dx$$
$$= \frac{(a-b)\tan^4(e+fx)}{4f}+\frac{b\tan^6(e+fx)}{6f}+(-a+b)\int \tan^3(e+f$$
$$= -\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\tan^4(e+fx)}{4f}+\frac{b\tan^6(e+fx)}{6f}$$
$$= -\frac{(a-b)\log(\cos(e+fx))}{f}-\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\tan}{4}$$

Mathematica [**A**]
time = 0.18, size = 63, normalized size = 0.85

$$\frac{12(-a+b)\log(\cos(e+fx))-6(a-b)\tan^2(e+fx)+3(a-b)\tan^4(e+fx)+2b\tan^6(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
[Out] (12*(-a + b)*Log[Cos[e + f*x]] - 6*(a - b)*Tan[e + f*x]^2 + 3*(a - b)*Tan[e
 + f*x]^4 + 2*b*Tan[e + f*x]^6)/(12*f)
```

Maple [**A**]
time = 0.08, size = 79, normalized size = 1.07

method	result
norman	$\frac{b\left(\tan^6(fx+e)\right)}{6f}-\frac{(a-b)\left(\tan^2(fx+e)\right)}{2f}+\frac{(a-b)\left(\tan^4(fx+e)\right)}{4f}+\frac{(a-b)\ln\left(1+\tan^2(fx+e)\right)}{2f}$
derivativedivides	$\frac{\frac{b\left(\tan^6(fx+e)\right)}{6}+\frac{a\left(\tan^4(fx+e)\right)}{4}-\frac{b\left(\tan^4(fx+e)\right)}{4}-\frac{a\left(\tan^2(fx+e)\right)}{2}+\frac{b\left(\tan^2(fx+e)\right)}{2}+\frac{(a-b)\ln\left(1+\tan^2(fx+e)\right)}{2}}{f}$
default	$\frac{\frac{b\left(\tan^6(fx+e)\right)}{6}+\frac{a\left(\tan^4(fx+e)\right)}{4}-\frac{b\left(\tan^4(fx+e)\right)}{4}-\frac{a\left(\tan^2(fx+e)\right)}{2}+\frac{b\left(\tan^2(fx+e)\right)}{2}+\frac{(a-b)\ln\left(1+\tan^2(fx+e)\right)}{2}}{f}$
risch	$ixa-ixb+\frac{2iae}{f}-\frac{2ibe}{f}-\frac{2\left(6a\,e^{10i(fx+e)}-9b\,e^{10i(fx+e)}+18a\,e^{8i(fx+e)}-18b\,e^{8i(fx+e)}+24a\,e^{6i(fx+e)}-34b\,e^{6i}\right.}{3f\left(e^{2i(fx+e)}+1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/6*b*tan(f*x+e)^6+1/4*a*tan(f*x+e)^4-1/4*b*tan(f*x+e)^4-1/2*a*tan(f*x
+e)^2+1/2*b*tan(f*x+e)^2+1/2*(a-b)*ln(1+tan(f*x+e)^2))
```

Maxima [A]
time = 0.28, size = 105, normalized size = 1.42

$$-\frac{6\,(a-b)\log\left(\sin\left(fx+e\right)^{2}-1\right)-\frac{6\,(2\,a-3\,b)\sin(fx+e)^{4}-3\,(7\,a-9\,b)\sin(fx+e)^{2}+9\,a-11\,b}{\sin(fx+e)^{6}-3\,\sin(fx+e)^{4}+3\,\sin(fx+e)^{2}-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*(a - b)*log(sin(f*x + e)^2 - 1) - (6*(2*a - 3*b)*sin(f*x + e)^4 -
3*(7*a - 9*b)*sin(f*x + e)^2 + 9*a - 11*b)/(sin(f*x + e)^6 - 3*sin(f*x + e)
^4 + 3*sin(f*x + e)^2 - 1))/f
```

Fricas [A]
time = 1.43, size = 71, normalized size = 0.96

$$\frac{2\,b\tan\left(fx+e\right)^{6}+3\,(a-b)\tan\left(fx+e\right)^{4}-6\,(a-b)\tan\left(fx+e\right)^{2}-6\,(a-b)\log\left(\frac{1}{\tan(fx+e)^{2}+1}\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b*tan(f*x + e)^6 + 3*(a - b)*tan(f*x + e)^4 - 6*(a - b)*tan(f*x + e
)^2 - 6*(a - b)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [A]
time = 0.18, size = 116, normalized size = 1.57

$$\begin{cases}\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{4}\left(e+fx\right)}{4f}-\frac{a\tan^{2}\left(e+fx\right)}{2f}-\frac{b\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b\tan^{6}\left(e+fx\right)}{6f}-\frac{b\tan^{4}\left(e+fx\right)}{4f}+\frac{b\tan^{2}\left(e+fx\right)}{2f} & \text{for}\: f\neq 0\\x(a+b\tan^{2}\left(e\right))\tan^{5}\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) - b*log(tan(e + f*x)**2 + 1)/(2*f) + b*tan(e + f*x)**6
/(6*f) - b*tan(e + f*x)**4/(4*f) + b*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(
a + b*tan(e)**2)*tan(e)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1719 vs. 2(72) = 144.
time = 3.71, size = 1719, normalized size = 23.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")

[Out] -1/12*(6*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan
(e)^6 - 6*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*ta
n(e)^6 + 9*a*tan(f*x)^6*tan(e)^6 - 11*b*tan(f*x)^6*tan(e)^6 - 36*a*log(4*(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 36*b*log(4*
(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)
^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 6*a*tan(f
*x)^6*tan(e)^4 - 6*b*tan(f*x)^6*tan(e)^4 - 42*a*tan(f*x)^5*tan(e)^5 + 54*b*
tan(f*x)^5*tan(e)^5 + 6*a*tan(f*x)^4*tan(e)^6 - 6*b*tan(f*x)^4*tan(e)^6 + 9
0*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 -
 90*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4
 - 3*a*tan(f*x)^6*tan(e)^2 + 3*b*tan(f*x)^6*tan(e)^2 - 36*a*tan(f*x)^5*tan(
e)^3 + 36*b*tan(f*x)^5*tan(e)^3 + 69*a*tan(f*x)^4*tan(e)^4 - 99*b*tan(f*x)^
4*tan(e)^4 - 36*a*tan(f*x)^3*tan(e)^5 + 36*b*tan(f*x)^3*tan(e)^5 - 3*a*tan(
f*x)^2*tan(e)^6 + 3*b*tan(f*x)^2*tan(e)^6 - 120*a*log(4*(tan(f*x)^4*tan(e)^
2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan
(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 120*b*log(4*(tan(f*x)^4*tan(
e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*
tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 2*b*tan(f*x)^6 + 6*a*tan(
f*x)^5*tan(e) - 18*b*tan(f*x)^5*tan(e) + 60*a*tan(f*x)^4*tan(e)^2 - 90*b*ta
n(f*x)^4*tan(e)^2 - 72*a*tan(f*x)^3*tan(e)^3 + 72*b*tan(f*x)^3*tan(e)^3 + 6
0*a*tan(f*x)^2*tan(e)^4 - 90*b*tan(f*x)^2*tan(e)^4 + 6*a*tan(f*x)*tan(e)^5
- 18*b*tan(f*x)*tan(e)^5 - 2*b*tan(e)^6 + 90*a*log(4*(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 90*b*log(4*(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 3*a*tan(f*x)^4 + 3*b*tan(f*x)
^4 - 36*a*tan(f*x)^3*tan(e) + 36*b*tan(f*x)^3*tan(e) + 69*a*tan(f*x)^2*tan(
e)^2 - 99*b*tan(f*x)^2*tan(e)^2 - 36*a*tan(f*x)*tan(e)^3 + 36*b*tan(f*x)*ta
n(e)^3 - 3*a*tan(e)^4 + 3*b*tan(e)^4 - 36*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 36*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 6*a*tan(f*x)^2 - 6*b*tan(f*x)^2 - 42*a*t
an(f*x)*tan(e) + 54*b*tan(f*x)*tan(e) + 6*a*tan(e)^2 - 6*b*tan(e)^2 + 6*a*l
og(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 6*b*log(4*(tan(f*x)^4*ta
n(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x
)*tan(e) + 1)/(tan(e)^2 + 1)) + 9*a - 11*b)/(f*tan(f*x)^6*tan(e)^6 - 6*f*ta
```

```
n(f*x)^5*tan(e)^5 + 15*f*tan(f*x)^4*tan(e)^4 - 20*f*tan(f*x)^3*tan(e)^3 + 1
5*f*tan(f*x)^2*tan(e)^2 - 6*f*tan(f*x)*tan(e) + f)
```

Mupad [B]
time = 11.71, size = 68, normalized size = 0.92

$$\frac{\tan(e+f\,x)^4\,\left(\frac{a}{4}-\frac{b}{4}\right)-\tan(e+f\,x)^2\,\left(\frac{a}{2}-\frac{b}{2}\right)+\frac{b\tan(e+f\,x)^6}{6}+\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)^4*(a/4 - b/4) - tan(e + f*x)^2*(a/2 - b/2) + (b*tan(e + f*x)^
6)/6 + log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

3.186 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=53

$$\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{b\tan^4(e+fx)}{4f}$$

```
[Out] (a-b)*ln(cos(f*x+e))/f+1/2*(a-b)*tan(f*x+e)^2/f+1/4*b*tan(f*x+e)^4/f
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3712, 3554, 3556}

$$\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - b)*Log[Cos[e + f*x]])/f + ((a - b)*Tan[e + f*x]^2)/(2*f) + (b*Tan[e +
 f*x]^4)/(4*f)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan^4(e+fx)}{4f}+(a-b)\int \tan^3(e+fx)\,dx$$
$$= \frac{(a-b)\tan^2(e+fx)}{2f}+\frac{b\tan^4(e+fx)}{4f}+(-a+b)\int \tan(e+fx)$$
$$= \frac{(a-b)\log(\cos(e+fx))}{f}+\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{b\tan^4(e+fx)}{4f}$$

Mathematica [A]
time = 0.12, size = 65, normalized size = 1.23

$$\frac{a(2\log(\cos(e+fx))+\tan^2(e+fx))}{2f}-\frac{b(4\log(\cos(e+fx))+2\tan^2(e+fx)-\tan^4(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
[Out] (a*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f) - (b*(4*Log[Cos[e + f*x]]
+ 2*Tan[e + f*x]^2 - Tan[e + f*x]^4))/(4*f)
```

Maple [A]
time = 0.05, size = 57, normalized size = 1.08

method	result
norman	$\frac{b(\tan^4(fx+e))}{4f}+\frac{(a-b)(\tan^2(fx+e))}{2f}-\frac{(a-b)\ln(1+\tan^2(fx+e))}{2f}$
derivativedivides	$\frac{\frac{b\left(\tan^4(fx+e)\right)}{4}+\frac{a\left(\tan^2(fx+e)\right)}{2}-\frac{b\left(\tan^2(fx+e)\right)}{2}+\frac{(-a+b)\ln\left(1+\tan^2(fx+e)\right)}{2}}{f}$
default	$\frac{\frac{b\left(\tan^4(fx+e)\right)}{4}+\frac{a\left(\tan^2(fx+e)\right)}{2}-\frac{b\left(\tan^2(fx+e)\right)}{2}+\frac{(-a+b)\ln\left(1+\tan^2(fx+e)\right)}{2}}{f}$
risch	$-ixa+ixb-\frac{2iae}{f}+\frac{2ibe}{f}+\frac{2a\,e^{6i(fx+e)}-4b\,e^{6i(fx+e)}+4a\,e^{4i(fx+e)}-4b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}-4b\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/4*b*tan(f*x+e)^4+1/2*a*tan(f*x+e)^2-1/2*b*tan(f*x+e)^2+1/2*(-a+b)*ln
(1+tan(f*x+e)^2))
```

Maxima [A]
time = 0.29, size = 74, normalized size = 1.40

$$\frac{2\,(a-b)\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{2\,(a-2\,b)\sin(fx+e)^2-2\,a+3\,b}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a - b)*log(sin(f*x + e)^2 - 1) - (2*(a - 2*b)*sin(f*x + e)^2 - 2*a
+ 3*b)/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 1.22, size = 54, normalized size = 1.02

$$\frac{b\tan\left(fx+e\right)^{4}+2\left(a-b\right)\tan\left(fx+e\right)^{2}+2\left(a-b\right)\log\left(\frac{1}{\tan(fx+e)^{2}+1}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b*tan(f*x + e)^4 + 2*(a - b)*tan(f*x + e)^2 + 2*(a - b)*log(1/(tan(f*x
 + e)^2 + 1)))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.
time = 0.12, size = 88, normalized size = 1.66

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{2}\left(e+fx\right)}{2f}+\frac{b\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b\tan^{4}\left(e+fx\right)}{4f}-\frac{b\tan^{2}\left(e+fx\right)}{2f} & \text{for}\: f\neq 0\\ x(a+b\tan^{2}\left(e\right))\tan^{3}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
log(tan(e + f*x)**2 + 1)/(2*f) + b*tan(e + f*x)**4/(4*f) - b*tan(e + f*x)**
2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(52) = 104$.
time = 1.45, size = 1071, normalized size = 20.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e
)^4 - 2*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan
```

```
(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(
e)^4 + 2*a*tan(f*x)^4*tan(e)^4 - 3*b*tan(f*x)^4*tan(e)^4 - 8*a*log(4*(tan(f
*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2
*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 8*b*log(4*(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 2*a*tan(f*x)^4
*tan(e)^2 - 2*b*tan(f*x)^4*tan(e)^2 - 4*a*tan(f*x)^3*tan(e)^3 + 8*b*tan(f*x
)^3*tan(e)^3 + 2*a*tan(f*x)^2*tan(e)^4 - 2*b*tan(f*x)^2*tan(e)^4 + 12*a*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 12*b*l
og(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + b*ta
n(f*x)^4 - 4*a*tan(f*x)^3*tan(e) + 8*b*tan(f*x)^3*tan(e) + 4*a*tan(f*x)^2*t
an(e)^2 - 4*b*tan(f*x)^2*tan(e)^2 - 4*a*tan(f*x)*tan(e)^3 + 8*b*tan(f*x)*ta
n(e)^3 + b*tan(e)^4 - 8*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)*tan(e) + 8*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)*tan(e) + 2*a*tan(f*x)^2 - 2*b*tan(f*x)^2 - 4*a*tan(f*x)*tan(e) + 8*b
*tan(f*x)*tan(e) + 2*a*tan(e)^2 - 2*b*tan(e)^2 + 2*a*log(4*(tan(f*x)^4*tan(
e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*
tan(e) + 1)/(tan(e)^2 + 1)) - 2*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1)) + 2*a - 3*b)/(f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*
f*tan(f*x)^2*tan(e)^2 - 4*f*tan(f*x)*tan(e) + f)
```

Mupad [B]
time = 11.70, size = 57, normalized size = 1.08

$$\frac{b\tan(e+f\,x)^4}{4\,f}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}+\frac{\tan(e+f\,x)^2\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2),x)
[Out] (b*tan(e + f*x)^4)/(4*f) - (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f + (tan(e
 + f*x)^2*(a/2 - b/2))/f
```

3.187 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=34

$$-\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^2(e+fx)}{2f}$$

```
[Out] -(a-b)*ln(cos(f*x+e))/f+1/2*b*tan(f*x+e)^2/f
```

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3712, 3556}

$$\frac{b\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - b)*Log[Cos[e + f*x]])/f) + (b*Tan[e + f*x]^2)/(2*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{b\tan^2(e+fx)}{2f}+(a-b)\int \tan(e+fx)\,dx\\ &= -\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^2(e+fx)}{2f}\end{aligned}$$

Mathematica [A]

time = 0.05, size = 40, normalized size = 1.18

$$-\frac{a\log(\cos(e+fx))}{f}+\frac{b\left(2\log(\cos(e+fx))+\tan^2(e+fx)\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*
f)
```

Maple [A]
time = 0.03, size = 35, normalized size = 1.03

method	result	size
derivativedivides	$\frac{\frac{b\left(\tan^2(fx+e)\right)}{2}+\frac{(a-b)\ln\left(1+\tan^2(fx+e)\right)}{2}}{f}$	35
default	$\frac{\frac{b\left(\tan^2(fx+e)\right)}{2}+\frac{(a-b)\ln\left(1+\tan^2(fx+e)\right)}{2}}{f}$	35
norman	$\frac{b(\tan^2(fx+e))}{2f}+\frac{(a-b)\ln(1+\tan^2(fx+e))}{2f}$	37
risch	$ixa-ixb+\frac{2iae}{f}-\frac{2ibe}{f}+\frac{2b\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{\ln\left(e^{2i(fx+e)}+1\right)a}{f}+\frac{\ln\left(e^{2i(fx+e)}+1\right)b}{f}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*b*tan(f*x+e)^2+1/2*(a-b)*ln(1+tan(f*x+e)^2))
```

Maxima [A]
time = 0.30, size = 39, normalized size = 1.15

$$-\frac{(a-b)\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{b}{\sin(fx+e)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f
```

Fricas [A]
time = 1.56, size = 38, normalized size = 1.12

$$\frac{b\tan\left(fx+e\right)^2-(a-b)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*tan(f*x + e)^2 - (a - b)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.
time = 0.08, size = 60, normalized size = 1.76

$$\begin{cases} \frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} - \frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b\tan^2\left(e+fx\right)}{2f} & \text{for } f \neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)\tan\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) - b*log(tan(e + f*x)**2 + 1)/(2
*f) + b*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e), True
))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(34) = 68$.
time = 0.73, size = 500, normalized size = 14.71

$$-\frac{a\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2-2\tan(fx)^3\tan(e)+\tan(fx)^2\tan(e)^2+\tan(fx)^2-2\tan(fx)\tan(e)+1\right)}{\tan(e)^2+1}\right)\tan\left(fx\right)^2\tan\left(e\right)^2 - b\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2-2\tan(fx)^3\tan(e)+\tan(fx)^2\tan(e)^2+\tan(fx)^2-2\tan(fx)\tan(e)+1\right)}{\tan(e)^2+1}\right)\tan\left(fx\right)^2\tan\left(e\right)^2 - b\tan\left(fx\right)^2\tan\left(e\right)^2 - 2a\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2-2\tan(fx)^3\tan(e)+\tan(fx)^2\tan(e)^2+\tan(fx)^2-2\tan(fx)\tan(e)+1\right)}{\tan(e)^2+1}\right)\tan\left(fx\right)\tan\left(e\right) + 2b\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2-2\tan(fx)^3\tan(e)+\tan(fx)^2\tan(e)^2+\tan(fx)^2-2\tan(fx)\tan(e)+1\right)}{\tan(e)^2+1}\right)\tan\left(fx\right)\tan\left(e\right) - b\tan\left(fx\right)^2 - b\tan\left(e\right)^2 + a\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2-2\tan(fx)^3\tan(e)+\tan(fx)^2\tan(e)^2+\tan(fx)^2-2\tan(fx)\tan(e)+1\right)}{\tan(e)^2+1}\right) - b\log\left(\frac{4\left(\tan(fx)^4\tan(e)^2-2\tan(fx)^3\tan(e)+\tan(fx)^2\tan(e)^2+\tan(fx)^2-2\tan(fx)\tan(e)+1\right)}{\tan(e)^2+1}\right) - b}{2\left(f\tan\left(fx\right)^2\tan\left(e\right)^2 - 2f\tan\left(fx\right)\tan\left(e\right) + f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)
^2 - b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^
2 - b*tan(f*x)^2*tan(e)^2 - 2*a*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)*tan(e) + 2*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
1))*tan(f*x)*tan(e) - b*tan(f*x)^2 - b*tan(e)^2 + a*log(4*(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1)/(tan(e)^2 + 1)) - b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
+ 1)) - b)/(f*tan(f*x)^2*tan(e)^2 - 2*f*tan(f*x)*tan(e) + f)
```

Mupad [B]
time = 11.75, size = 37, normalized size = 1.09

$$\frac{b\tan(e+f\,x)^2}{2\,f} + \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (b*tan(e + f*x)^2)/(2*f) + (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

3.188 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=26

$$-\frac{b\log(\cos(e+fx))}{f}+\frac{a\log(\sin(e+fx))}{f}$$

```
[Out] -b*ln(cos(f*x+e))/f+a*ln(sin(f*x+e))/f
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3706, 3556}

$$\frac{a\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((b*Log[Cos[e + f*x]])/f) + (a*Log[Sin[e + f*x]])/f
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3706

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Dist[A, Int[1/Tan[e + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x],
x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= a\int \cot(e+fx)\,dx+b\int \tan(e+fx)\,dx\\ &= -\frac{b\log(\cos(e+fx))}{f}+\frac{a\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 1.31

$$-\frac{b\log(\cos(e+fx))}{f}+\frac{a(\log(\cos(e+fx))+\log(\tan(e+fx)))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((b*Log[Cos[e + f*x]])/f) + (a*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]]))/f
```

Maple [A]
time = 0.13, size = 25, normalized size = 0.96

method	result	size
derivativedivides	$\frac{-b\ln(\cos(fx+e))+a\ln(\sin(fx+e))}{f}$	25
default	$\frac{-b\ln(\cos(fx+e))+a\ln(\sin(fx+e))}{f}$	25
norman	$\frac{a\ln(\tan(fx+e))}{f}-\frac{(a-b)\ln\left(1+\tan^2(fx+e)\right)}{2f}$	35
risch	$-ixa+ixb-\frac{2iae}{f}+\frac{2ibe}{f}+\frac{a\ln\left(e^{2i(fx+e)}-1\right)}{f}-\frac{\ln\left(e^{2i(fx+e)}+1\right)b}{f}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b*ln(cos(f*x+e))+a*ln(sin(f*x+e)))
```

Maxima [A]
time = 0.27, size = 33, normalized size = 1.27

$$-\frac{b\log\left(\sin\left(fx+e\right)^2-1\right)-a\log\left(\sin\left(fx+e\right)^2\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*log(sin(f*x + e)^2 - 1) - a*log(sin(f*x + e)^2))/f
```

Fricas [A]
time = 3.06, size = 49, normalized size = 1.88

$$\frac{a\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)-b\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) - b*log(1/(tan(f*x + e)^2 +
 1)))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(22) = 44$.
time = 0.21, size = 58, normalized size = 2.23

$$\begin{cases} -\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} & \text{for } f\neq 0 \\ x(a+b\tan^2\left(e\right))\cot\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*log(tan(e + f*x))/f + b*lo
g(tan(e + f*x)**2 + 1)/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*cot(e), True)
)
```

Giac [A]
time = 0.65, size = 34, normalized size = 1.31

$$\frac{a\log\left(\sin\left(fx+e\right)^2\right)-b\log\left(\left|\sin\left(fx+e\right)^2-1\right|\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(sin(f*x + e)^2) - b*log(abs(sin(f*x + e)^2 - 1)))/f
```

Mupad [B]
time = 11.66, size = 36, normalized size = 1.38

$$\frac{a\,\ln\left(\tan(e+f\,x)\right)}{f}-\frac{\ln\left({\tan(e+f\,x)}^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (a*log(tan(e + f*x)))/f - (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

3.189 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=34

$$-\frac{a\cot^2(e+fx)}{2f}-\frac{(a-b)\log(\sin(e+fx))}{f}$$

```
[Out] -1/2*a*cot(f*x+e)^2/f-(a-b)*ln(sin(f*x+e))/f
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3710, 12, 3556}

$$-\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/2*(a*Cot[e + f*x]^2)/f - ((a - b)*Log[Sin[e + f*x]])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= -\frac{a\cot^2(e+fx)}{2f}-\int (a-b)\cot(e+fx)\,dx\\ &= -\frac{a\cot^2(e+fx)}{2f}-(a-b)\int \cot(e+fx)\,dx\\ &= -\frac{a\cot^2(e+fx)}{2f}-\frac{(a-b)\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 56, normalized size = 1.65

$$\frac{b(\log(\cos(e+fx))+\log(\tan(e+fx)))}{f}-\frac{a(\cot^2(e+fx)+2\log(\cos(e+fx))+2\log(\tan(e+fx)))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (b*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]]))/f - (a*(Cot[e + f*x]^2 + 2*Log[
Cos[e + f*x]] + 2*Log[Tan[e + f*x]]))/(2*f)
```

Maple [A]
time = 0.13, size = 37, normalized size = 1.09

method	result	size
derivativedivides	$\frac{b\ln(\sin(fx+e))+a\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)}{f}$	37
default	$\frac{b\ln(\sin(fx+e))+a\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)}{f}$	37
norman	$-\frac{a}{2f\tan(fx+e)^2}-\frac{(a-b)\ln(\tan(fx+e))}{f}+\frac{(a-b)\ln\left(1+\tan^2(fx+e)\right)}{2f}$	54
risch	$ixa-ixb+\frac{2iae}{f}-\frac{2ibe}{f}+\frac{2a\,\mathrm{e}^{2i(fx+e)}}{f\left(\mathrm{e}^{2i(fx+e)}-1\right)^2}-\frac{a\ln\left(\mathrm{e}^{2i(fx+e)}-1\right)}{f}+\frac{\ln\left(\mathrm{e}^{2i(fx+e)}-1\right)b}{f}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*ln(sin(f*x+e))+a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e))))
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.97

$$-\frac{(a-b)\log\left(\sin(fx+e)^2\right)+\frac{a}{\sin(fx+e)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(sin(f*x + e)^2) + a/sin(f*x + e)^2)/f
```

Fricas [A]
time = 2.41, size = 66, normalized size = 1.94

$$-\frac{(a-b)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+a\tan\left(fx+e\right)^2+a}{2\,f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a*t
an(f*x + e)^2 + a)/(f*tan(f*x + e)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(27) = 54$.
time = 0.74, size = 100, normalized size = 2.94

$$\begin{cases} \tilde{\infty}ax & \text{for } (e=0 \vee e=-fx) \wedge (e=-fx \vee f=0) \\ x(a+b\tan^2{(e)})\cot^3{(e)} & \text{for } f=0 \\ \frac{a\log{(\tan^2{(e+fx)}+1)}}{2f} - \frac{a\log{(\tan{(e+fx)})}}{f} - \frac{a}{2f\tan^2{(e+fx)}} - \frac{b\log{(\tan^2{(e+fx)}+1)}}{2f} + \frac{b\log{(\tan{(e+fx)})}}{f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**3, Eq(f, 0)), (a*log(tan(e + f*x)**2 + 1)/(2*f)
 - a*log(tan(e + f*x))/f - a/(2*f*tan(e + f*x)**2) - b*log(tan(e + f*x)**2
+ 1)/(2*f) + b*log(tan(e + f*x))/f, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(34) = 68$.
time = 0.78, size = 165, normalized size = 4.85

$$-\frac{4\,(a-b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-8\,(a-b)\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+\frac{4\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1}-\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*(a - b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 8*(a -
b)*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + 4*a*(cos(f*x
```

```
 + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*
(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1))/f
```

Mupad [B]
time = 11.62, size = 54, normalized size = 1.59

$$\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}-\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a-b\right)}{f}-\frac{a\cot(e+f\,x)^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f - (log(tan(e + f*x))*(a - b))/f - (
a*cot(e + f*x)^2)/(2*f)
```

3.190 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=53

$$\frac{(a-b)\cot^2(e+fx)}{2f}-\frac{a\cot^4(e+fx)}{4f}+\frac{(a-b)\log(\sin(e+fx))}{f}$$

```
[Out] 1/2*(a-b)*cot(f*x+e)^2/f-1/4*a*cot(f*x+e)^4/f+(a-b)*ln(sin(f*x+e))/f
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3710, 12, 3554, 3556}

$$\frac{(a-b)\cot^2(e+fx)}{2f}+\frac{(a-b)\log(\sin(e+fx))}{f}-\frac{a\cot^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a - b)*Cot[e + f*x]^2)/(2*f) - (a*Cot[e + f*x]^4)/(4*f) + ((a - b)*Log[Si
n[e + f*x]])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
```

```
^2 + b^2, 0]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{a\cot^4(e+fx)}{4f} - \int (a-b)\cot^3(e+fx)\,dx$$
$$= -\frac{a\cot^4(e+fx)}{4f} - (a-b)\int \cot^3(e+fx)\,dx$$
$$= \frac{(a-b)\cot^2(e+fx)}{2f} - \frac{a\cot^4(e+fx)}{4f} - (-a+b)\int \cot(e+fx)$$
$$= \frac{(a-b)\cot^2(e+fx)}{2f} - \frac{a\cot^4(e+fx)}{4f} + \frac{(a-b)\log(\sin(e+fx))}{f}$$

Mathematica [A]
time = 0.16, size = 56, normalized size = 1.06

$$\frac{2(a-b)\cot^2(e+fx) - a\cot^4(e+fx) + 4(a-b)(\log(\cos(e+fx)) + \log(\tan(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
[Out] (2*(a - b)*Cot[e + f*x]^2 - a*Cot[e + f*x]^4 + 4*(a - b)*(Log[Cos[e + f*x]]
 + Log[Tan[e + f*x]]))/(4*f)
```

Maple [A]
time = 0.13, size = 58, normalized size = 1.09

method	result
derivativedivides	$\frac{b\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)+a\left(-\frac{\left(\cot^4(fx+e)\right)}{4}+\frac{\left(\cot^2(fx+e)\right)}{2}+\ln(\sin(fx+e))\right)}{f}$
default	$\frac{b\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)+a\left(-\frac{\left(\cot^4(fx+e)\right)}{4}+\frac{\left(\cot^2(fx+e)\right)}{2}+\ln(\sin(fx+e))\right)}{f}$
norman	$\frac{-\frac{a}{4f}+\frac{(a-b)\left(\tan^2(fx+e)\right)}{2f}}{\tan(fx+e)^4}+\frac{(a-b)\ln(\tan(fx+e))}{f}-\frac{(a-b)\ln\left(1+\tan^2(fx+e)\right)}{2f}$
risch	$-ixa+ixb-\frac{2iae}{f}+\frac{2ibe}{f}-\frac{2\left(2a\,e^{6i(fx+e)}-b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}+2b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}-b\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))+a*(-1/4*cot(f*x+e)^4+1/2*cot(f*x+
e)^2+ln(sin(f*x+e))))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 1.04

$$\frac{2\,(a-b)\log\left(\sin\left(fx+e\right)^2\right)+\frac{2\,(2\,a-b)\sin(fx+e)^2-a}{\sin(fx+e)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a - b)*log(sin(f*x + e)^2) + (2*(2*a - b)*sin(f*x + e)^2 - a)/sin(f
*x + e)^4)/f
```

Fricas [A]
time = 3.16, size = 91, normalized size = 1.72

$$\frac{2\,(a-b)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^4+(3\,a-2\,b)\tan\left(fx+e\right)^4+2\,(a-b)\tan\left(fx+e\right)^2-a}{4\,f\tan\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (3
*a - 2*b)*tan(f*x + e)^4 + 2*(a - b)*tan(f*x + e)^2 - a)/(f*tan(f*x + e)^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(42) = 84$.
time = 1.80, size = 124, normalized size = 2.34

$$\begin{cases} \tilde{\infty} ax & \text{for}\: e=0 \wedge f=0 \\ x(a+b\tan^2(e))\cot^5(e) & \text{for}\: f=0 \\ \tilde{\infty} ax & \text{for}\: e=-fx \\ -\frac{a\log(\tan^2(e+fx)+1)}{2f}+\frac{a\log(\tan(e+fx))}{f}+\frac{a}{2f\tan^2(e+fx)}-\frac{a}{4f\tan^4(e+fx)}+\frac{b\log(\tan^2(e+fx)+1)}{2f}-\frac{b\log(\tan(e+fx))}{f}-\frac{b}{2f\tan^2(e+fx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)*cot(e)**5, E
q(f, 0)), (zoo*a*x, Eq(e, -f*x)), (-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*lo
g(tan(e + f*x))/f + a/(2*f*tan(e + f*x)**2) - a/(4*f*tan(e + f*x)**4) + b*l
og(tan(e + f*x)**2 + 1)/(2*f) - b*log(tan(e + f*x))/f - b/(2*f*tan(e + f*x)
**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(52) = 104$.
time = 0.96, size = 265, normalized size = 5.00

$$\frac{32\,(a-b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-64\,(a-b)\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+\frac{12\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{48\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}-\frac{12\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(32*(a - b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 64*(a
- b)*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + 12*a*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + 48*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 48*b*(cos(f*x + e) - 1
)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2 - 12*a*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/f
```

Mupad [B]
time = 11.65, size = 74, normalized size = 1.40

$$\frac{\ln\left(\tan(e+f\,x)\right)\,(a-b)}{f}-\frac{\frac{a}{4}-\tan(e+f\,x)^2\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f\tan\left(e+f\,x\right)^4}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (log(tan(e + f*x))*(a - b))/f - (a/4 - tan(e + f*x)^2*(a/2 - b/2))/(f*tan(e
 + f*x)^4) - (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

3.191 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=80

$$-((a-b)x)+\frac{(a-b)\tan(e+fx)}{f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}$$

```
[Out] -(a-b)*x+(a-b)*tan(f*x+e)/f-1/3*(a-b)*tan(f*x+e)^3/f+1/5*(a-b)*tan(f*x+e)^5
/f+1/7*b*tan(f*x+e)^7/f
```

Rubi [A]

time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3712, 3554, 8}

$$\frac{(a-b)\tan^5(e+fx)}{5f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) + ((a - b)*Tan[e + f*x])/f - ((a - b)*Tan[e + f*x]^3)/(3*f) +
((a - b)*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan^7(e+fx)}{7f}+(a-b)\int \tan^6(e+fx)\,dx$$
$$= \frac{(a-b)\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}+(-a+b)\int \tan^4(e+fx$$
$$= -\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}$$
$$= \frac{(a-b)\tan(e+fx)}{f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan^5(e+f}{5f}$$
$$= -(a-b)x+\frac{(a-b)\tan(e+fx)}{f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-}{}$$

Mathematica [A]
time = 0.04, size = 129, normalized size = 1.61

$$-\frac{a\text{ArcTan}(\tan(e+fx))}{f}+\frac{b\text{ArcTan}(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}-\frac{b\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}-\frac{b\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (b*ArcTan[Tan[e + f*x]])/f + (a*Tan[e + f*x
])/f - (b*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*f) + (b*Tan[e + f*x]^3)/(
3*f) + (a*Tan[e + f*x]^5)/(5*f) - (b*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x
]^7)/(7*f)
```

Maple [A]
time = 0.06, size = 91, normalized size = 1.14

method	result
norman	$(-a+b)\,x+\frac{(a-b)\tan(fx+e)}{f}+\frac{b\left(\tan^7(fx+e)\right)}{7f}-\frac{(a-b)\left(\tan^3(fx+e)\right)}{3f}+\frac{(a-b)\left(\tan^5(fx+e)\right)}{5f}$
derivativedivides	$\frac{\frac{b\left(\tan^7(fx+e)\right)}{7}+\frac{a\left(\tan^5(fx+e)\right)}{5}-\frac{b\left(\tan^5(fx+e)\right)}{5}-\frac{a\left(\tan^3(fx+e)\right)}{3}+\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)-b\tan(fx+e)+(-a+b)\,\text{arc}}{f}$
default	$\frac{\frac{b\left(\tan^7(fx+e)\right)}{7}+\frac{a\left(\tan^5(fx+e)\right)}{5}-\frac{b\left(\tan^5(fx+e)\right)}{5}-\frac{a\left(\tan^3(fx+e)\right)}{3}+\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)-b\tan(fx+e)+(-a+b)\,\text{arc}}{f}$
risch	$-ax+bx+\frac{2i\left(315a\,e^{12i(fx+e)}-420b\,e^{12i(fx+e)}+1260a\,e^{10i(fx+e)}-1260b\,e^{10i(fx+e)}+2555a\,e^{8i(fx+e)}-3080b\,e^{8i(f}\right.}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/7*b*tan(f*x+e)^7+1/5*a*tan(f*x+e)^5-1/5*b*tan(f*x+e)^5-1/3*a*tan(f*x
+e)^3+1/3*b*tan(f*x+e)^3+a*tan(f*x+e)-b*tan(f*x+e)+(-a+b)*arctan(tan(f*x+e)
))
```

Maxima [A]
time = 0.49, size = 77, normalized size = 0.96

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,(a-b)\tan\left(fx+e\right)^{5}-35\,(a-b)\tan\left(fx+e\right)^{3}-105\,(fx+e)(a-b)+105\,(a-b)\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a - b)*tan(f*x + e)^5 - 35*(a - b)*tan(f*x
 + e)^3 - 105*(f*x + e)*(a - b) + 105*(a - b)*tan(f*x + e))/f
```

Fricas [A]
time = 2.92, size = 73, normalized size = 0.91

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,(a-b)\tan\left(fx+e\right)^{5}-35\,(a-b)\tan\left(fx+e\right)^{3}-105\,(a-b)fx+105\,(a-b)\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a - b)*tan(f*x + e)^5 - 35*(a - b)*tan(f*x
 + e)^3 - 105*(a - b)*f*x + 105*(a - b)*tan(f*x + e))/f
```

Sympy [A]
time = 0.21, size = 109, normalized size = 1.36

$$\begin{cases} -ax+\frac{a\tan^{5}\left(e+fx\right)}{5f}-\frac{a\tan^{3}\left(e+fx\right)}{3f}+\frac{a\tan\left(e+fx\right)}{f}+bx+\frac{b\tan^{7}\left(e+fx\right)}{7f}-\frac{b\tan^{5}\left(e+fx\right)}{5f}+\frac{b\tan^{3}\left(e+fx\right)}{3f}-\frac{b\tan\left(e+fx\right)}{f} & \text{for } f\neq 0\\ x(a+b\tan^{2}\left(e\right))\tan^{6}\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*x + a*tan(e + f*x)**5/(5*f) - a*tan(e + f*x)**3/(3*f) + a*tan
(e + f*x)/f + b*x + b*tan(e + f*x)**7/(7*f) - b*tan(e + f*x)**5/(5*f) + b*t
an(e + f*x)**3/(3*f) - b*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)*ta
n(e)**6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(78) = 156$.
time = 3.16, size = 1087, normalized size = 13.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/105*(105*a*f*x*tan(f*x)^7*tan(e)^7 - 105*b*f*x*tan(f*x)^7*tan(e)^7 - 735
*a*f*x*tan(f*x)^6*tan(e)^6 + 735*b*f*x*tan(f*x)^6*tan(e)^6 + 105*a*tan(f*x)
^7*tan(e)^6 - 105*b*tan(f*x)^7*tan(e)^6 + 105*a*tan(f*x)^6*tan(e)^7 - 105*b
*tan(f*x)^6*tan(e)^7 + 2205*a*f*x*tan(f*x)^5*tan(e)^5 - 2205*b*f*x*tan(f*x)
^5*tan(e)^5 - 35*a*tan(f*x)^7*tan(e)^4 + 35*b*tan(f*x)^7*tan(e)^4 - 735*a*t
an(f*x)^6*tan(e)^5 + 735*b*tan(f*x)^6*tan(e)^5 - 735*a*tan(f*x)^5*tan(e)^6
+ 735*b*tan(f*x)^5*tan(e)^6 - 35*a*tan(f*x)^4*tan(e)^7 + 35*b*tan(f*x)^4*ta
n(e)^7 - 3675*a*f*x*tan(f*x)^4*tan(e)^4 + 3675*b*f*x*tan(f*x)^4*tan(e)^4 +
21*a*tan(f*x)^7*tan(e)^2 - 21*b*tan(f*x)^7*tan(e)^2 + 245*a*tan(f*x)^6*tan(
e)^3 - 245*b*tan(f*x)^6*tan(e)^3 + 2205*a*tan(f*x)^5*tan(e)^4 - 2205*b*tan(
f*x)^5*tan(e)^4 + 2205*a*tan(f*x)^4*tan(e)^5 - 2205*b*tan(f*x)^4*tan(e)^5 +
 245*a*tan(f*x)^3*tan(e)^6 - 245*b*tan(f*x)^3*tan(e)^6 + 21*a*tan(f*x)^2*ta
n(e)^7 - 21*b*tan(f*x)^2*tan(e)^7 + 3675*a*f*x*tan(f*x)^3*tan(e)^3 - 3675*b
*f*x*tan(f*x)^3*tan(e)^3 + 15*b*tan(f*x)^7 - 42*a*tan(f*x)^6*tan(e) + 147*b
*tan(f*x)^6*tan(e) - 420*a*tan(f*x)^5*tan(e)^2 + 735*b*tan(f*x)^5*tan(e)^2
- 3150*a*tan(f*x)^4*tan(e)^3 + 3675*b*tan(f*x)^4*tan(e)^3 - 3150*a*tan(f*x)
^3*tan(e)^4 + 3675*b*tan(f*x)^3*tan(e)^4 - 420*a*tan(f*x)^2*tan(e)^5 + 735*
b*tan(f*x)^2*tan(e)^5 - 42*a*tan(f*x)*tan(e)^6 + 147*b*tan(f*x)*tan(e)^6 +
15*b*tan(e)^7 - 2205*a*f*x*tan(f*x)^2*tan(e)^2 + 2205*b*f*x*tan(f*x)^2*tan(
e)^2 + 21*a*tan(f*x)^5 - 21*b*tan(f*x)^5 + 245*a*tan(f*x)^4*tan(e) - 245*b*
tan(f*x)^4*tan(e) + 2205*a*tan(f*x)^3*tan(e)^2 - 2205*b*tan(f*x)^3*tan(e)^2
 + 2205*a*tan(f*x)^2*tan(e)^3 - 2205*b*tan(f*x)^2*tan(e)^3 + 245*a*tan(f*x)
*tan(e)^4 - 245*b*tan(f*x)*tan(e)^4 + 21*a*tan(e)^5 - 21*b*tan(e)^5 + 735*a
*f*x*tan(f*x)*tan(e) - 735*b*f*x*tan(f*x)*tan(e) - 35*a*tan(f*x)^3 + 35*b*t
an(f*x)^3 - 735*a*tan(f*x)^2*tan(e) + 735*b*tan(f*x)^2*tan(e) - 735*a*tan(f
*x)*tan(e)^2 + 735*b*tan(f*x)*tan(e)^2 - 35*a*tan(e)^3 + 35*b*tan(e)^3 - 10
5*a*f*x + 105*b*f*x + 105*a*tan(f*x) - 105*b*tan(f*x) + 105*a*tan(e) - 105*
b*tan(e))/(f*tan(f*x)^7*tan(e)^7 - 7*f*tan(f*x)^6*tan(e)^6 + 21*f*tan(f*x)^
5*tan(e)^5 - 35*f*tan(f*x)^4*tan(e)^4 + 35*f*tan(f*x)^3*tan(e)^3 - 21*f*tan
(f*x)^2*tan(e)^2 + 7*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 11.57, size = 70, normalized size = 0.88

$$\frac{\frac{b\tan(e+f\,x)^7}{7}+\left(\frac{a}{5}-\frac{b}{5}\right)\,\tan(e+f\,x)^5+\left(\frac{b}{3}-\frac{a}{3}\right)\,\tan(e+f\,x)^3+(a-b)\,\tan(e+f\,x)-f\,x\,(a-b)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)^5*(a/5 - b/5) - tan(e + f*x)^3*(a/3 - b/3) + tan(e + f*x)*(a
- b) + (b*tan(e + f*x)^7)/7 - f*x*(a - b))/f
```

3.192 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=60

$$(a-b)x-\frac{(a-b)\tan(e+fx)}{f}+\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

```
[Out] (a-b)*x-(a-b)*tan(f*x+e)/f+1/3*(a-b)*tan(f*x+e)^3/f+1/5*b*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3712, 3554, 8}

$$\frac{(a-b)\tan^3(e+fx)}{3f}-\frac{(a-b)\tan(e+fx)}{f}+x(a-b)+\frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (a - b)*x - ((a - b)*Tan[e + f*x])/f + ((a - b)*Tan[e + f*x]^3)/(3*f) + (b*
Tan[e + f*x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan^5(e+fx)}{5f} + (a-b)\int \tan^4(e+fx)\,dx$$
$$= \frac{(a-b)\tan^3(e+fx)}{3f} + \frac{b\tan^5(e+fx)}{5f} + (-a+b)\int \tan^2(e+fx$$
$$= -\frac{(a-b)\tan(e+fx)}{f} + \frac{(a-b)\tan^3(e+fx)}{3f} + \frac{b\tan^5(e+fx)}{5f} +$$
$$= (a-b)x - \frac{(a-b)\tan(e+fx)}{f} + \frac{(a-b)\tan^3(e+fx)}{3f} + \frac{b\tan^5(e}{5}$$

Mathematica [A]
time = 0.03, size = 97, normalized size = 1.62

$$\frac{a\text{ArcTan}(\tan(e+fx))}{f} - \frac{b\text{ArcTan}(\tan(e+fx))}{f} - \frac{a\tan(e+fx)}{f} + \frac{b\tan(e+fx)}{f} + \frac{a\tan^3(e+fx)}{3f} - \frac{b\tan^3(e+fx)}{3f} + \frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
[Out] (a*ArcTan[Tan[e + f*x]])/f - (b*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])/
f + (b*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f) - (b*Tan[e + f*x]^3)/(3*f
) + (b*Tan[e + f*x]^5)/(5*f)
```

Maple [A]
time = 0.04, size = 69, normalized size = 1.15

method	result
norman	$(a-b)\,x - \frac{(a-b)\tan(fx+e)}{f} + \frac{(a-b)\left(\tan^3(fx+e)\right)}{3f} + \frac{b\left(\tan^5(fx+e)\right)}{5f}$
derivativedivides	$\frac{\frac{b\left(\tan^5(fx+e)\right)}{5}+\frac{a\left(\tan^3(fx+e)\right)}{3}-\frac{b\left(\tan^3(fx+e)\right)}{3}-a\tan(fx+e)+b\tan(fx+e)+(a-b)\arctan(\tan(fx+e))}{f}$
default	$\frac{\frac{b\left(\tan^5(fx+e)\right)}{5}+\frac{a\left(\tan^3(fx+e)\right)}{3}-\frac{b\left(\tan^3(fx+e)\right)}{3}-a\tan(fx+e)+b\tan(fx+e)+(a-b)\arctan(\tan(fx+e))}{f}$
risch	$ax - bx - \frac{2i\left(30a\,e^{8i(fx+e)}-45b\,e^{8i(fx+e)}+90a\,e^{6i(fx+e)}-90b\,e^{6i(fx+e)}+110a\,e^{4i(fx+e)}-140b\,e^{4i(fx+e)}+70a\,e^{2i(fx}{15f\left(e^{2i(fx+e)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/5*b*tan(f*x+e)^5+1/3*a*tan(f*x+e)^3-1/3*b*tan(f*x+e)^3-a*tan(f*x+e)+
b*tan(f*x+e)+(a-b)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.49, size = 61, normalized size = 1.02

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,(a-b)\tan\left(fx+e\right)^{3}+15\,(fx+e)(a-b)-15\,(a-b)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a - b)*tan(f*x + e)^3 + 15*(f*x + e)*(a - b)
- 15*(a - b)*tan(f*x + e))/f
```

Fricas [A]
time = 2.80, size = 57, normalized size = 0.95

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,(a-b)\tan\left(fx+e\right)^{3}+15\,(a-b)fx-15\,(a-b)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a - b)*tan(f*x + e)^3 + 15*(a - b)*f*x - 15*(
a - b)*tan(f*x + e))/f
```

Sympy [A]
time = 0.15, size = 82, normalized size = 1.37

$$\begin{cases} ax+\frac{a\tan^{3}\left(e+fx\right)}{3f}-\frac{a\tan\left(e+fx\right)}{f}-bx+\frac{b\tan^{5}\left(e+fx\right)}{5f}-\frac{b\tan^{3}\left(e+fx\right)}{3f}+\frac{b\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x(a+b\tan^{2}\left(e\right))\tan^{4}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((a*x + a*tan(e + f*x)**3/(3*f) - a*tan(e + f*x)/f - b*x + b*tan(e
 + f*x)**5/(5*f) - b*tan(e + f*x)**3/(3*f) + b*tan(e + f*x)/f, Ne(f, 0)), (
x*(a + b*tan(e)**2)*tan(e)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(59) = 118.
time = 1.38, size = 633, normalized size = 10.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*a*f*x*tan(f*x)^5*tan(e)^5 - 15*b*f*x*tan(f*x)^5*tan(e)^5 - 75*a*f*
x*tan(f*x)^4*tan(e)^4 + 75*b*f*x*tan(f*x)^4*tan(e)^4 + 15*a*tan(f*x)^5*tan(
e)^4 - 15*b*tan(f*x)^5*tan(e)^4 + 15*a*tan(f*x)^4*tan(e)^5 - 15*b*tan(f*x)^
4*tan(e)^5 + 150*a*f*x*tan(f*x)^3*tan(e)^3 - 150*b*f*x*tan(f*x)^3*tan(e)^3
- 5*a*tan(f*x)^5*tan(e)^2 + 5*b*tan(f*x)^5*tan(e)^2 - 75*a*tan(f*x)^4*tan(e
)^3 + 75*b*tan(f*x)^4*tan(e)^3 - 75*a*tan(f*x)^3*tan(e)^4 + 75*b*tan(f*x)^3
*tan(e)^4 - 5*a*tan(f*x)^2*tan(e)^5 + 5*b*tan(f*x)^2*tan(e)^5 - 150*a*f*x*t
an(f*x)^2*tan(e)^2 + 150*b*f*x*tan(f*x)^2*tan(e)^2 - 3*b*tan(f*x)^5 + 10*a*
tan(f*x)^4*tan(e) - 25*b*tan(f*x)^4*tan(e) + 120*a*tan(f*x)^3*tan(e)^2 - 15
0*b*tan(f*x)^3*tan(e)^2 + 120*a*tan(f*x)^2*tan(e)^3 - 150*b*tan(f*x)^2*tan(
e)^3 + 10*a*tan(f*x)*tan(e)^4 - 25*b*tan(f*x)*tan(e)^4 - 3*b*tan(e)^5 + 75*
a*f*x*tan(f*x)*tan(e) - 75*b*f*x*tan(f*x)*tan(e) - 5*a*tan(f*x)^3 + 5*b*tan
(f*x)^3 - 75*a*tan(f*x)^2*tan(e) + 75*b*tan(f*x)^2*tan(e) - 75*a*tan(f*x)*t
an(e)^2 + 75*b*tan(f*x)*tan(e)^2 - 5*a*tan(e)^3 + 5*b*tan(e)^3 - 15*a*f*x +
 15*b*f*x + 15*a*tan(f*x) - 15*b*tan(f*x) + 15*a*tan(e) - 15*b*tan(e))/(f*t
an(f*x)^5*tan(e)^5 - 5*f*tan(f*x)^4*tan(e)^4 + 10*f*tan(f*x)^3*tan(e)^3 - 1
0*f*tan(f*x)^2*tan(e)^2 + 5*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 11.63, size = 53, normalized size = 0.88

$$\frac{\frac{b\tan(e+f\,x)^5}{5}+\left(\frac{a}{3}-\frac{b}{3}\right)\,\tan(e+f\,x)^3+(b-a)\,\tan(e+f\,x)+f\,x\,(a-b)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)^3*(a/3 - b/3) - tan(e + f*x)*(a - b) + (b*tan(e + f*x)^5)/5 +
 f*x*(a - b))/f
```

3.193 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=40

$$-((a-b)x)+\frac{(a-b)\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

```
[Out] -(a-b)*x+(a-b)*tan(f*x+e)/f+1/3*b*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3712, 3554, 8}

$$\frac{(a-b)\tan(e+fx)}{f}-x(a-b)+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
[Out] -((a - b)*x) + ((a - b)*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3712

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b
, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{b\tan^3(e+fx)}{3f}+(a-b)\int \tan^2(e+fx)\,dx\\ &= \frac{(a-b)\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}+(-a+b)\int 1\,dx\\ &= -(a-b)x+\frac{(a-b)\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 1.62

$$-\frac{a\text{ArcTan}(\tan(e+fx))}{f}+\frac{b\text{ArcTan}(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}-\frac{b\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (b*ArcTan[Tan[e + f*x]])/f + (a*Tan[e + f*x
])/f - (b*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Maple [A]
time = 0.03, size = 47, normalized size = 1.18

method	result	size
norman	$(-a+b)\,x+\frac{(a-b)\tan(fx+e)}{f}+\frac{b\left(\tan^3(fx+e)\right)}{3f}$	38
derivativedivides	$\frac{\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)-b\tan(fx+e)+(-a+b)\arctan(\tan(fx+e))}{f}$	47
default	$\frac{\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)-b\tan(fx+e)+(-a+b)\arctan(\tan(fx+e))}{f}$	47
risch	$-ax+bx+\frac{2i\left(3a\,e^{4i(fx+e)}-6b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-6b\,e^{2i(fx+e)}+3a-4b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/3*b*tan(f*x+e)^3+a*tan(f*x+e)-b*tan(f*x+e)+(-a+b)*arctan(tan(f*x+e))
)
```

Maxima [A]
time = 0.48, size = 44, normalized size = 1.10

$$\frac{b\tan\left(fx+e\right)^3-3\left(fx+e\right)(a-b)+3\left(a-b\right)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*(a - b) + 3*(a - b)*tan(f*x + e))/f
```

Fricas [A]
time = 1.94, size = 40, normalized size = 1.00

$$\frac{b\tan\left(fx+e\right)^3-3\left(a-b\right)fx+3\left(a-b\right)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(a - b)*f*x + 3*(a - b)*tan(f*x + e))/f
```

Sympy [A]
time = 0.10, size = 54, normalized size = 1.35

$$\begin{cases} -ax + \frac{a\tan{(e+fx)}}{f} + bx + \frac{b\tan^3{(e+fx)}}{3f} - \frac{b\tan{(e+fx)}}{f} & \text{for } f \neq 0 \\ x(a+b\tan^2{(e)})\tan^2{(e)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((-a*x + a*tan(e + f*x)/f + b*x + b*tan(e + f*x)**3/(3*f) - b*tan(
e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(40) = 80.
time = 0.74, size = 289, normalized size = 7.22

$$-\frac{3afx\tan{(fx)}^3\tan{(e)}^3 - 3bfx\tan{(fx)}^3\tan{(e)}^3 - 9afx\tan{(fx)}^2\tan{(e)}^2 + 9bfx\tan{(fx)}^2\tan{(e)}^2 + 3a\tan{(fx)}^3\tan{(e)}^2 - 3b\tan{(fx)}^3\tan{(e)}^2 + 3a\tan{(fx)}^2\tan{(e)}^3 - 3b\tan{(fx)}^2\tan{(e)}^3 + 9afx\tan{(fx)}\tan{(e)} - 9bfx\tan{(fx)}\tan{(e)} + b\tan{(fx)}^3 - 6a\tan{(fx)}^2\tan{(e)} + 9b\tan{(fx)}^2\tan{(e)} - 6a\tan{(fx)}\tan{(e)}^2 + 9b\tan{(fx)}\tan{(e)}^2 + b\tan{(e)}^3 - 3afx + 3bfx + 3a\tan{(fx)} - 3b\tan{(fx)} + 3a\tan{(e)} - 3b\tan{(e)}}{3\left(f\tan{(fx)}^3\tan{(e)}^3 - 3f\tan{(fx)}^2\tan{(e)}^2 + 3f\tan{(fx)}\tan{(e)} - f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*f*x*tan(f*x)^3*tan(e)^3 - 3*b*f*x*tan(f*x)^3*tan(e)^3 - 9*a*f*x*t
an(f*x)^2*tan(e)^2 + 9*b*f*x*tan(f*x)^2*tan(e)^2 + 3*a*tan(f*x)^3*tan(e)^2
- 3*b*tan(f*x)^3*tan(e)^2 + 3*a*tan(f*x)^2*tan(e)^3 - 3*b*tan(f*x)^2*tan(e)
^3 + 9*a*f*x*tan(f*x)*tan(e) - 9*b*f*x*tan(f*x)*tan(e) + b*tan(f*x)^3 - 6*a
*tan(f*x)^2*tan(e) + 9*b*tan(f*x)^2*tan(e) - 6*a*tan(f*x)*tan(e)^2 + 9*b*ta
n(f*x)*tan(e)^2 + b*tan(e)^3 - 3*a*f*x + 3*b*f*x + 3*a*tan(f*x) - 3*b*tan(f
*x) + 3*a*tan(e) - 3*b*tan(e))/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan(
e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 11.54, size = 37, normalized size = 0.92

$$\frac{\frac{b\tan(e+f\,x)^3}{3} + (a-b)\,\tan(e+f\,x) - f\,x\,(a-b)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)*(a - b) + (b*tan(e + f*x)^3)/3 - f*x*(a - b))/f
```

3.194 $\int (a+b\tan^2(e+fx))\,dx$

Optimal. Leaf size=19

$$ax-bx+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x-b*x+b*tan(f*x+e)/f
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3554, 8}

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[e + f*x]^2,x]
```

```
[Out] a*x - b*x + (b*Tan[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a+b\tan^2(e+fx))\,dx &= ax+b\int \tan^2(e+fx)\,dx\\ &= ax+\frac{b\tan(e+fx)}{f}-b\int 1\,dx\\ &= ax-bx+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.47

$$ax-\frac{b\text{ArcTan}(\tan(e+fx))}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[e + f*x]^2,x]
[Out] a*x - (b*ArcTan[Tan[e + f*x]])/f + (b*Tan[e + f*x])/f
```

Maple [A]
time = 0.00, size = 29, normalized size = 1.53

method	result	size
norman	$(a-b)\,x+\frac{b\tan(fx+e)}{f}$	20
derivativedivides	$\frac{b\tan(fx+e)+(a-b)\arctan(\tan(fx+e))}{f}$	27
default	$ax+\frac{b\tan(fx+e)}{f}-\frac{b\arctan(\tan(fx+e))}{f}$	29
risch	$ax-bx+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] a*x+b*tan(f*x+e)/f-b/f*arctan(tan(f*x+e))
```

Maxima [A]
time = 0.56, size = 25, normalized size = 1.32

$$ax-\frac{(fx+e-\tan(fx+e))b}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="maxima")
[Out] a*x - (f*x + e - tan(f*x + e))*b/f
```

Fricas [A]
time = 1.95, size = 22, normalized size = 1.16

$$\frac{(a-b)fx+b\tan(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="fricas")
[Out] ((a - b)*f*x + b*tan(f*x + e))/f
```

Sympy [A]
time = 0.06, size = 20, normalized size = 1.05

$$ax+b\left(\begin{cases}-x+\frac{\tan(e+fx)}{f} & \text{for } f\neq 0\\ x\tan^2(e) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)**2,x)
```

```
[Out] a*x + b*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(20) = 40.
time = 0.55, size = 252, normalized size = 13.26

$$ax + \frac{\left(\pi - 4\,fx\tan\left(fx\right)\tan\left(e\right) - \pi\mathrm{sgn}\left(2\,\tan\left(fx\right)^{2}\tan\left(e\right) + 2\,\tan\left(fx\right)\tan\left(e\right)^{2} - 2\,\tan\left(fx\right) - 2\,\tan\left(e\right)\right)\tan\left(fx\right)\tan\left(e\right) - \pi\tan\left(fx\right)\tan\left(e\right) + 2\,\arctan\left(\frac{\tan(fx)\tan(e)-1}{\tan(fx)+\tan(e)}\right)\tan\left(fx\right)\tan\left(e\right) + 2\,\arctan\left(\frac{\tan(fx)+\tan(e)}{\tan(fx)\tan(e)-1}\right)\tan\left(fx\right)\tan\left(e\right) + 4\,fx + \pi\mathrm{sgn}\left(2\,\tan\left(fx\right)^{2}\tan\left(e\right) + 2\,\tan\left(fx\right)\tan\left(e\right)^{2} - 2\,\tan\left(fx\right) - 2\,\tan\left(e\right)\right) - 2\,\arctan\left(\frac{\tan(fx)\tan(e)-1}{\tan(fx)+\tan(e)}\right) - 2\,\arctan\left(\frac{\tan(fx)+\tan(e)}{\tan(fx)\tan(e)-1}\right) - 4\,\tan\left(fx\right) - 4\,\tan\left(e\right)\right)b}{4\left(f\tan\left(fx\right)\tan\left(e\right) - f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + 1/4*(pi - 4*f*x*tan(f*x)*tan(e) - pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(
f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))*tan(f*x)*tan(e) - pi*tan(f*x)*tan(e)
 + 2*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e)))*tan(f*x)*tan(e) + 2*
arctan((tan(f*x) + tan(e))/(tan(f*x)*tan(e) - 1))*tan(f*x)*tan(e) + 4*f*x +
 pi*sgn(2*tan(f*x)^2*tan(e) + 2*tan(f*x)*tan(e)^2 - 2*tan(f*x) - 2*tan(e))
- 2*arctan((tan(f*x)*tan(e) - 1)/(tan(f*x) + tan(e))) - 2*arctan((tan(f*x)
+ tan(e))/(tan(f*x)*tan(e) - 1)) - 4*tan(f*x) - 4*tan(e))*b/(f*tan(f*x)*tan
(e) - f)
```

Mupad [B]
time = 11.44, size = 21, normalized size = 1.11

$$\frac{b\tan(e+f\,x)+f\,x\,(a-b)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(e + f*x)^2,x)
```

```
[Out] (b*tan(e + f*x) + f*x*(a - b))/f
```

3.195 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=21

$$-((a-b)x)-\frac{a\cot(e+fx)}{f}$$

```
[Out] -(a-b)*x-a*cot(f*x+e)/f
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3710, 8}

$$-(x(a-b))-\frac{a\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) - (a*Cot[e + f*x])/f
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= -\frac{a\cot(e+fx)}{f}+\int (-a+b)\,dx\\ &= -(a-b)x-\frac{a\cot(e+fx)}{f}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 34, normalized size = 1.62

$$bx - \frac{a\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
[Out] b*x - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f
```

Maple [A]
time = 0.09, size = 31, normalized size = 1.48

method	result	size
risch	$-ax+bx-\frac{2ia}{f\left(e^{2i(fx+e)}-1\right)}$	29
norman	$\frac{(-a+b)x\tan(fx+e)-\frac{a}{f}}{\tan(fx+e)}$	30
derivativedivides	$\frac{b(fx+e)+a(-\cot(fx+e)-fx-e)}{f}$	31
default	$\frac{b(fx+e)+a(-\cot(fx+e)-fx-e)}{f}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(b*(f*x+e)+a*(-cot(f*x+e)-f*x-e))
```

Maxima [A]
time = 0.51, size = 29, normalized size = 1.38

$$-\frac{(fx+e)(a-b)+\frac{a}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
[Out] -((f*x + e)*(a - b) + a/tan(f*x + e))/f
```

Fricas [A]
time = 1.58, size = 31, normalized size = 1.48

$$-\frac{(a-b)fx\tan(fx+e)+a}{f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -((a - b)*f*x*tan(f*x + e) + a)/(f*tan(f*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.
time = 0.43, size = 46, normalized size = 2.19

$$\begin{cases} \tilde{\infty}ax & \text{for } (e = 0 \vee e = -fx) \wedge (e = -fx \vee f = 0) \\ x(a + b\tan^2(e))\cot^2(e) & \text{for } f = 0 \\ -ax - \frac{a}{f\tan(e+fx)} + bx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**2, Eq(f, 0)), (-a*x - a/(f*tan(e + f*x)) + b*x,
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.
time = 0.67, size = 46, normalized size = 2.19

$$-\frac{2(fx + e)(a - b) - a\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + \frac{a}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)*(a - b) - a*tan(1/2*f*x + 1/2*e) + a/tan(1/2*f*x + 1/2*e)
)/f
```

Mupad [B]
time = 11.47, size = 21, normalized size = 1.00

$$-x(a - b) - \frac{a\cot(e + fx)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2),x)
```

```
[Out] - x*(a - b) - (a*cot(e + f*x))/f
```

3.196 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=39

$$(a-b)x+\frac{(a-b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}$$

```
[Out] (a-b)*x+(a-b)*cot(f*x+e)/f-1/3*a*cot(f*x+e)^3/f
```

Rubi [A]

time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3710, 12, 3554, 8}

$$\frac{(a-b)\cot(e+fx)}{f}+x(a-b)-\frac{a\cot^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (a - b)*x + ((a - b)*Cot[e + f*x])/f - (a*Cot[e + f*x]^3)/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= -\frac{a\cot^3(e+fx)}{3f}-\int (a-b)\cot^2(e+fx)\,dx\\ &= -\frac{a\cot^3(e+fx)}{3f}-(a-b)\int \cot^2(e+fx)\,dx\\ &= \frac{(a-b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}-(-a+b)\int 1\,dx\\ &= (a-b)x+\frac{(a-b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 65, normalized size = 1.67

$$-\frac{a\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(e+fx)\right)}{3f}-\frac{b\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
[Out] -1/3*(a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/f
 - (b*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f
```

Maple [A]
time = 0.11, size = 47, normalized size = 1.21

method	result	size
derivativedivides	$\frac{b(-\cot(fx+e)-fx-e)+a\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)}{f}$	47
default	$\frac{b(-\cot(fx+e)-fx-e)+a\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)}{f}$	47
norman	$\frac{(a-b)x\left(\tan^3(fx+e)\right)+\frac{(a-b)\left(\tan^2(fx+e)\right)}{f}-\frac{a}{3f}}{\tan(fx+e)^3}$	49
risch	$ax-bx+\frac{2i\left(6a\,e^{4i(fx+e)}-3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}+4a-3b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(b*(-cot(f*x+e)-f*x-e)+a*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e))
```

Maxima [A]
time = 0.50, size = 49, normalized size = 1.26

$$\frac{3\,(fx+e)(a-b)+\frac{3\,(a-b)\tan(fx+e)^2-a}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*(a - b) + (3*(a - b)*tan(f*x + e)^2 - a)/tan(f*x + e)^3)/f
```

Fricas [A]
time = 1.30, size = 52, normalized size = 1.33

$$\frac{3\,(a-b)fx\tan\left(fx+e\right)^3+3\,(a-b)\tan\left(fx+e\right)^2-a}{3\,f\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a - b)*f*x*tan(f*x + e)^3 + 3*(a - b)*tan(f*x + e)^2 - a)/(f*tan(f*
x + e)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(29) = 58.
time = 0.95, size = 70, normalized size = 1.79

$$\begin{cases} \tilde{\infty}ax & \text{for } (e=0 \vee e=-fx) \wedge (e=-fx \vee f=0) \\ x(a+b\tan^2(e))\cot^4(e) & \text{for } f=0 \\ ax+\frac{a}{f\tan(e+fx)}-\frac{a}{3f\tan^3(e+fx)}-bx-\frac{b}{f\tan(e+fx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**4, Eq(f, 0)), (a*x + a/(f*tan(e + f*x)) - a/(3*
f*tan(e + f*x)**3) - b*x - b/(f*tan(e + f*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(39) = 78.
time = 0.78, size = 106, normalized size = 2.72

$$\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+24\,(fx+e)(a-b)-15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\frac{15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*(a - b) - 15*a*tan(1/2*f*x +
1/2*e) + 12*b*tan(1/2*f*x + 1/2*e) + (15*a*tan(1/2*f*x + 1/2*e)^2 - 12*b*ta
n(1/2*f*x + 1/2*e)^2 - a)/tan(1/2*f*x + 1/2*e)^3)/f
```

Mupad [B]
time = 11.74, size = 40, normalized size = 1.03

$$x\,(a-b)-\frac{\frac{a}{3}-\tan(e+f\,x)^2\,(a-b)}{f\,\tan\left(e+f\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2),x)
```

```
[Out] x*(a - b) - (a/3 - tan(e + f*x)^2*(a - b))/(f*tan(e + f*x)^3)
```

3.197 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal. Leaf size=61

$$-((a-b)x)-\frac{(a-b)\cot(e+fx)}{f}+\frac{(a-b)\cot^3(e+fx)}{3f}-\frac{a\cot^5(e+fx)}{5f}$$

```
[Out] -(a-b)*x-(a-b)*cot(f*x+e)/f+1/3*(a-b)*cot(f*x+e)^3/f-1/5*a*cot(f*x+e)^5/f
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3710, 12, 3554, 8}

$$\frac{(a-b)\cot^3(e+fx)}{3f}-\frac{(a-b)\cot(e+fx)}{f}-x(a-b)-\frac{a\cot^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -((a - b)*x) - ((a - b)*Cot[e + f*x])/f + ((a - b)*Cot[e + f*x]^3)/(3*f) -
(a*Cot[e + f*x]^5)/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3710

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e
 + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /; F
reeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a
^2 + b^2, 0]
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{a\cot^5(e+fx)}{5f} - \int (a-b)\cot^4(e+fx)\,dx$$
$$= -\frac{a\cot^5(e+fx)}{5f} - (a-b)\int \cot^4(e+fx)\,dx$$
$$= \frac{(a-b)\cot^3(e+fx)}{3f} - \frac{a\cot^5(e+fx)}{5f} - (-a+b)\int \cot^2(e+f$$
$$= -\frac{(a-b)\cot(e+fx)}{f} + \frac{(a-b)\cot^3(e+fx)}{3f} - \frac{a\cot^5(e+fx)}{5f} -$$
$$= -(a-b)x - \frac{(a-b)\cot(e+fx)}{f} + \frac{(a-b)\cot^3(e+fx)}{3f} - \frac{a\cot$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 69, normalized size = 1.13

$$-\frac{a\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(e+fx)\right)}{5f} - \frac{b\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/5*(a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/f
 - (b*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(3*
f)
```

Maple [A]
time = 0.13, size = 67, normalized size = 1.10

method	result
derivativedivides	$\frac{b\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)+a\left(-\frac{\left(\cot^5(fx+e)\right)}{5}+\frac{\left(\cot^3(fx+e)\right)}{3}-\cot(fx+e)-fx-e\right)}{f}$
default	$\frac{b\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)+a\left(-\frac{\left(\cot^5(fx+e)\right)}{5}+\frac{\left(\cot^3(fx+e)\right)}{3}-\cot(fx+e)-fx-e\right)}{f}$
norman	$\frac{(-a+b)x\left(\tan^5(fx+e)\right)-\frac{a}{5f}+\frac{(a-b)\left(\tan^2(fx+e)\right)}{3f}-\frac{(a-b)\left(\tan^4(fx+e)\right)}{f}}{\tan(fx+e)^5}$
risch	$-ax+bx-\frac{2i\left(45a\,e^{8i(fx+e)}-30b\,e^{8i(fx+e)}-90a\,e^{6i(fx+e)}+90b\,e^{6i(fx+e)}+140a\,e^{4i(fx+e)}-110b\,e^{4i(fx+e)}-70a\,e\right.}{15f\left(e^{2i(fx+e)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)+a*(-1/5*cot(f*x+e)^5+1/3*cot(f*
x+e)^3-cot(f*x+e)-f*x-e))
```

Maxima [A]
time = 0.49, size = 65, normalized size = 1.07

$$-\frac{15\,(fx+e)(a-b)+\frac{15\,(a-b)\tan(fx+e)^4-5\,(a-b)\tan(fx+e)^2+3\,a}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*(a - b) + (15*(a - b)*tan(f*x + e)^4 - 5*(a - b)*tan(f*
x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

Fricas [A]
time = 1.64, size = 68, normalized size = 1.11

$$-\frac{15\,(a-b)fx\tan\left(fx+e\right)^5+15\,(a-b)\tan\left(fx+e\right)^4-5\,(a-b)\tan\left(fx+e\right)^2+3\,a}{15\,f\tan\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a - b)*f*x*tan(f*x + e)^5 + 15*(a - b)*tan(f*x + e)^4 - 5*(a - b
)*tan(f*x + e)^2 + 3*a)/(f*tan(f*x + e)^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(46) = 92$.
time = 2.74, size = 97, normalized size = 1.59

$$\begin{cases} \tilde{\infty}ax & \text{for}\ (e=0 \vee e=-fx) \wedge (e=-fx \vee f=0) \\ x(a+b\tan^2{(e)})\cot^6{(e)} & \text{for}\ f=0 \\ -ax-\frac{a}{f\tan{(e+fx)}}+\frac{a}{3f\tan^3{(e+fx)}}-\frac{a}{5f\tan^5{(e+fx)}}+bx+\frac{b}{f\tan{(e+fx)}}-\frac{b}{3f\tan^3{(e+fx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*a*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x))), (
x*(a + b*tan(e)**2)*cot(e)**6, Eq(f, 0)), (-a*x - a/(f*tan(e + f*x)) + a/(3
```

```
*f*tan(e + f*x)**3) - a/(5*f*tan(e + f*x)**5) + b*x + b/(f*tan(e + f*x)) -
b/(3*f*tan(e + f*x)**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(60) = 120$.
time = 1.03, size = 168, normalized size = 2.75

$$\frac{3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+20\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-480\,(fx+e)(a-b)+330\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-300\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{330\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-300\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+20\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,a}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}}}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*f*x + 1/2*e)^5 - 35*a*tan(1/2*f*x + 1/2*e)^3 + 20*b*tan(
1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*(a - b) + 330*a*tan(1/2*f*x + 1/2*e) - 3
00*b*tan(1/2*f*x + 1/2*e) - (330*a*tan(1/2*f*x + 1/2*e)^4 - 300*b*tan(1/2*f
*x + 1/2*e)^4 - 35*a*tan(1/2*f*x + 1/2*e)^2 + 20*b*tan(1/2*f*x + 1/2*e)^2 +
 3*a)/tan(1/2*f*x + 1/2*e)^5)/f
```

Mupad [B]
time = 11.94, size = 57, normalized size = 0.93

$$-x\,(a-b)-\frac{(a-b)\,\tan(e+f\,x)^4+\left(\frac{b}{3}-\frac{a}{3}\right)\,\tan(e+f\,x)^2+\frac{a}{5}}{f\tan\left(e+f\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2),x)
```

```
[Out] - x*(a - b) - (a/5 - tan(e + f*x)^2*(a/3 - b/3) + tan(e + f*x)^4*(a - b))/(
f*tan(e + f*x)^5)
```

3.198 $\int \tan^5(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx$

Optimal. Leaf size=105

$$-\frac{(a-b)^2\log(\cos(e+fx))}{f}-\frac{(a-b)^2\tan^2(e+fx)}{2f}+\frac{(a-b)^2\tan^4(e+fx)}{4f}+\frac{(2a-b)b\tan^6(e+fx)}{6f}+\frac{b^2\tan^8(}{8}$$

```
[Out] -(a-b)^2*ln(cos(f*x+e))/f-1/2*(a-b)^2*tan(f*x+e)^2/f+1/4*(a-b)^2*tan(f*x+e)
^4/f+1/6*(2*a-b)*b*tan(f*x+e)^6/f+1/8*b^2*tan(f*x+e)^8/f
```

Rubi [A]
time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$\frac{b(2a-b)\tan^6(e+fx)}{6f}+\frac{(a-b)^2\tan^4(e+fx)}{4f}-\frac{(a-b)^2\tan^2(e+fx)}{2f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^8(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Log[Cos[e + f*x]])/f) - ((a - b)^2*Tan[e + f*x]^2)/(2*f) + ((a
 - b)^2*Tan[e + f*x]^4)/(4*f) + ((2*a - b)*b*Tan[e + f*x]^6)/(6*f) + (b^2*T
an[e + f*x]^8)/(8*f)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^5\left(a+bx^2\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^2}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-(a-b)^2+(a-b)^2x+(2a-b)bx^2+b^2x^3+\frac{(a-b)^2}{1+x}\right.\right.}{2f}$$
$$= -\frac{(a-b)^2\log(\cos(e+fx))}{f} - \frac{(a-b)^2\tan^2(e+fx)}{2f} + \frac{(a-b)^2}{}$$

Mathematica [A]
time = 0.24, size = 89, normalized size = 0.85

$$\frac{-24(a-b)^2\log(\cos(e+fx))-12(a-b)^2\tan^2(e+fx)+6(a-b)^2\tan^4(e+fx)+4(2a-b)b\tan^6(e+fx)+3b^2\tan^8(e+fx)}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-24*(a - b)^2*Log[Cos[e + f*x]] - 12*(a - b)^2*Tan[e + f*x]^2 + 6*(a - b)^
2*Tan[e + f*x]^4 + 4*(2*a - b)*b*Tan[e + f*x]^6 + 3*b^2*Tan[e + f*x]^8)/(24
*f)
```

Maple [A]
time = 0.10, size = 143, normalized size = 1.36

method	result
norman	$\frac{b^2(\tan^8(fx+e))}{8f} - \frac{(a^2-2ab+b^2)(\tan^2(fx+e))}{2f} + \frac{(a^2-2ab+b^2)(\tan^4(fx+e))}{4f} + \frac{(2a-b)b(\tan^6(fx+e))}{6f} + \frac{(a^2-}{}$
derivativedivides	$\frac{\frac{b^2\left(\tan^8(fx+e)\right)}{8}+\frac{ab\left(\tan^6(fx+e)\right)}{3}-\frac{b^2\left(\tan^6(fx+e)\right)}{6}+\frac{a^2\left(\tan^4(fx+e)\right)}{4}-\frac{ab\left(\tan^4(fx+e)\right)}{2}+\frac{b^2\left(\tan^4(fx+e)\right)}{4}-\frac{a^2\left(\tan^2(fx+e\right.}{2}}{f}$
default	$\frac{\frac{b^2\left(\tan^8(fx+e)\right)}{8}+\frac{ab\left(\tan^6(fx+e)\right)}{3}-\frac{b^2\left(\tan^6(fx+e)\right)}{6}+\frac{a^2\left(\tan^4(fx+e)\right)}{4}-\frac{ab\left(\tan^4(fx+e)\right)}{2}+\frac{b^2\left(\tan^4(fx+e)\right)}{4}-\frac{a^2\left(\tan^2(fx+e\right.}{2}}{f}$
risch	$ia^2x-2iabx+ib^2x+\frac{2ia^2e}{f}-\frac{4iabe}{f}+\frac{2ib^2e}{f}-\frac{4\left(3a^2e^{14i(fx+e)}-9ab\,e^{14i(fx+e)}+6b^2e^{14i(fx+e)}+15a^2e^{12}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/8*b^2*tan(f*x+e)^8+1/3*a*b*tan(f*x+e)^6-1/6*b^2*tan(f*x+e)^6+1/4*a^2
*tan(f*x+e)^4-1/2*a*b*tan(f*x+e)^4+1/4*b^2*tan(f*x+e)^4-1/2*a^2*tan(f*x+e)^
2+a*b*tan(f*x+e)^2-1/2*b^2*tan(f*x+e)^2+1/2*(a^2-2*a*b+b^2)*ln(1+tan(f*x+e)
^2))
```

Maxima [A]
time = 0.29, size = 170, normalized size = 1.62

$$-\frac{12\,(a^2-2\,ab+b^2)\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{24\,(a^2-3\,ab+2\,b^2)\sin(fx+e)^6-6\,(11\,a^2-30\,ab+18\,b^2)\sin(fx+e)^4+4\,(15\,a^2-38\,ab+22\,b^2)\sin(fx+e)^2-18\,a^2+44\,ab-25\,b^2}{\sin(fx+e)^8-4\,\sin(fx+e)^6+6\,\sin(fx+e)^4-4\,\sin(fx+e)^2+1}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (24*(a^2 - 3*a*b +
2*b^2)*sin(f*x + e)^6 - 6*(11*a^2 - 30*a*b + 18*b^2)*sin(f*x + e)^4 + 4*(15
*a^2 - 38*a*b + 22*b^2)*sin(f*x + e)^2 - 18*a^2 + 44*a*b - 25*b^2)/(sin(f*x
 + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 0.97, size = 112, normalized size = 1.07

$$\frac{3\,b^2\tan\left(fx+e\right)^8+4\,(2\,ab-b^2)\tan\left(fx+e\right)^6+6\,(a^2-2\,ab+b^2)\tan\left(fx+e\right)^4-12\,(a^2-2\,ab+b^2)\tan\left(fx+e\right)^2-12\,(a^2-2\,ab+b^2)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*tan(f*x + e)^8 + 4*(2*a*b - b^2)*tan(f*x + e)^6 + 6*(a^2 - 2*a*
b + b^2)*tan(f*x + e)^4 - 12*(a^2 - 2*a*b + b^2)*tan(f*x + e)^2 - 12*(a^2 -
 2*a*b + b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(82) = 164$.
time = 0.27, size = 206, normalized size = 1.96

$$\begin{cases} \frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^4\left(e+fx\right)}{4f}-\frac{a^2\tan^2\left(e+fx\right)}{2f}-\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{ab\tan^6\left(e+fx\right)}{3f}-\frac{ab\tan^4\left(e+fx\right)}{2f}+\frac{ab\tan^2\left(e+fx\right)}{f}+\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b^2\tan^8\left(e+fx\right)}{8f}-\frac{b^2\tan^6\left(e+fx\right)}{6f}+\frac{b^2\tan^4\left(e+fx\right)}{4f}-\frac{b^2\tan^2\left(e+fx\right)}{2f} & \text{for}\: f\neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^5\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f)
 - a**2*tan(e + f*x)**2/(2*f) - a*b*log(tan(e + f*x)**2 + 1)/f + a*b*tan(e
+ f*x)**6/(3*f) - a*b*tan(e + f*x)**4/(2*f) + a*b*tan(e + f*x)**2/f + b**2*
log(tan(e + f*x)**2 + 1)/(2*f) + b**2*tan(e + f*x)**8/(8*f) - b**2*tan(e +
f*x)**6/(6*f) + b**2*tan(e + f*x)**4/(4*f) - b**2*tan(e + f*x)**2/(2*f), Ne
(f, 0)), (x*(a + b*tan(e)**2)**2*tan(e)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3813 vs. $2(102) = 204$.
time = 10.39, size = 3813, normalized size = 36.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2
*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^8*
tan(e)^8 - 24*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x
)^8*tan(e)^8 + 12*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ta
n(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan
(f*x)^8*tan(e)^8 + 18*a^2*tan(f*x)^8*tan(e)^8 - 44*a*b*tan(f*x)^8*tan(e)^8
+ 25*b^2*tan(f*x)^8*tan(e)^8 - 96*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(ta
n(e)^2 + 1))*tan(f*x)^7*tan(e)^7 + 192*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*t
an(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1
)/(tan(e)^2 + 1))*tan(f*x)^7*tan(e)^7 - 96*b^2*log(4*(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1)/(tan(e)^2 + 1))*tan(f*x)^7*tan(e)^7 + 12*a^2*tan(f*x)^8*tan(e)^6 - 24
*a*b*tan(f*x)^8*tan(e)^6 + 12*b^2*tan(f*x)^8*tan(e)^6 - 120*a^2*tan(f*x)^7*
tan(e)^7 + 304*a*b*tan(f*x)^7*tan(e)^7 - 176*b^2*tan(f*x)^7*tan(e)^7 + 12*a
^2*tan(f*x)^6*tan(e)^8 - 24*a*b*tan(f*x)^6*tan(e)^8 + 12*b^2*tan(f*x)^6*tan
(e)^8 + 336*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^
6*tan(e)^6 - 672*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(
f*x)^6*tan(e)^6 + 336*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^6*tan(e)^6 - 6*a^2*tan(f*x)^8*tan(e)^4 + 12*a*b*tan(f*x)^8*tan(e)
^4 - 6*b^2*tan(f*x)^8*tan(e)^4 - 96*a^2*tan(f*x)^7*tan(e)^5 + 192*a*b*tan(f
*x)^7*tan(e)^5 - 96*b^2*tan(f*x)^7*tan(e)^5 + 324*a^2*tan(f*x)^6*tan(e)^6 -
 872*a*b*tan(f*x)^6*tan(e)^6 + 520*b^2*tan(f*x)^6*tan(e)^6 - 96*a^2*tan(f*x
)^5*tan(e)^7 + 192*a*b*tan(f*x)^5*tan(e)^7 - 96*b^2*tan(f*x)^5*tan(e)^7 - 6
*a^2*tan(f*x)^4*tan(e)^8 + 12*a*b*tan(f*x)^4*tan(e)^8 - 6*b^2*tan(f*x)^4*ta
n(e)^8 - 672*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x
)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)
^5*tan(e)^5 + 1344*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)^5*tan(e)^5 - 672*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e
) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1
))*tan(f*x)^5*tan(e)^5 - 8*a*b*tan(f*x)^8*tan(e)^2 + 4*b^2*tan(f*x)^8*tan(e
```

)^2 + 24*a^2*tan(f*x)^7*tan(e)^3 - 96*a*b*tan(f*x)^7*tan(e)^3 + 48*b^2*tan(f*x)^7*tan(e)^3 + 276*a^2*tan(f*x)^6*tan(e)^4 - 672*a*b*tan(f*x)^6*tan(e)^4 + 336*b^2*tan(f*x)^6*tan(e)^4 - 504*a^2*tan(f*x)^5*tan(e)^5 + 1296*a*b*tan(f*x)^5*tan(e)^5 - 816*b^2*tan(f*x)^5*tan(e)^5 + 276*a^2*tan(f*x)^4*tan(e)^6 - 672*a*b*tan(f*x)^4*tan(e)^6 + 336*b^2*tan(f*x)^4*tan(e)^6 + 24*a^2*tan(f*x)^3*tan(e)^7 - 96*a*b*tan(f*x)^3*tan(e)^7 + 48*b^2*tan(f*x)^3*tan(e)^7 - 8*a*b*tan(f*x)^2*tan(e)^8 + 4*b^2*tan(f*x)^2*tan(e)^8 + 840*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 1680*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 840*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 3*b^2*tan(f*x)^8 + 16*a*b*tan(f*x)^7*tan(e) - 32*b^2*tan(f*x)^7*tan(e) - 36*a^2*tan(f*x)^6*tan(e)^2 + 168*a*b*tan(f*x)^6*tan(e)^2 - 168*b^2*tan(f*x)^6*tan(e)^2 - 384*a^2*tan(f*x)^5*tan(e)^3 + 1008*a*b*tan(f*x)^5*tan(e)^3 - 672*b^2*tan(f*x)^5*tan(e)^3 + 564*a^2*tan(f*x)^4*tan(e)^4 - 1368*a*b*tan(f*x)^4*tan(e)^4 + 684*b^2*tan(f*x)^4*tan(e)^4 - 384*a^2*tan(f*x)^3*tan(e)^5 + 1008*a*b*tan(f*x)^3*tan(e)^5 - 672*b^2*tan(f*x)^3*tan(e)^5 - 36*a^2*tan(f*x)^2*tan(e)^6 + 168*a*b*tan(f*x)^2*tan(e)^6 - 168*b^2*tan(f*x)^2*tan(e)^6 + 16*a*b*tan(f*x)*tan(e)^7 - 32*b^2*tan(f*x)*tan(e)^7 - 3*b^2*tan(e)^8 - 672*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 1344*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 672*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 8*a*b*tan(f*x)^6 + 4*b^2*tan(f*x)^6 + 24*a^2*tan(f*x)^5*tan(e) - 96*a*b*tan(f*x)^5*tan(e) + 48*b^2*tan(f*x)^5*tan(e) + 276*a^2*tan(f*x)^4*tan(e)^2 - 672*a*b*tan(f*x)^4*tan(e)^2 + 336*b^2*tan(f*x)^4*tan(e)^2 - 504*a^2*tan(f*x)^3*tan(e)^3 + 1296*a*b*tan(f*x)^3*tan(...

Mupad [B]
time = 11.59, size = 113, normalized size = 1.08

$$\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)+\tan(e+f\,x)^6\,\left(\frac{a\,b}{3}-\frac{b^2}{6}\right)+\frac{b^2\tan(e+f\,x)^8}{8}-\tan(e+f\,x)^2\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)+\tan(e+f\,x)^4\,\left(\frac{a^2}{4}-\frac{a\,b}{2}+\frac{b^2}{4}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^2,x)

[Out] (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2) + tan(e + f*x)^6*((a*b)/3 - b^2/6) + (b^2*tan(e + f*x)^8)/8 - tan(e + f*x)^2*(a^2/2 - a*b + b^2/2) + tan(e + f*x)^4*(a^2/4 - (a*b)/2 + b^2/4))/f

3.199 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=82

$$\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{(a-b)^2\tan^2(e+fx)}{2f}+\frac{(2a-b)b\tan^4(e+fx)}{4f}+\frac{b^2\tan^6(e+fx)}{6f}$$

```
[Out] (a-b)^2*ln(cos(f*x+e))/f+1/2*(a-b)^2*tan(f*x+e)^2/f+1/4*(2*a-b)*b*tan(f*x+e
)^4/f+1/6*b^2*tan(f*x+e)^6/f
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 78}

$$\frac{b(2a-b)\tan^4(e+fx)}{4f}+\frac{(a-b)^2\tan^2(e+fx)}{2f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a - b)^2*Log[Cos[e + f*x]])/f + ((a - b)^2*Tan[e + f*x]^2)/(2*f) + ((2*a
- b)*b*Tan[e + f*x]^4)/(4*f) + (b^2*Tan[e + f*x]^6)/(6*f)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{x^3(a+bx^2)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\frac{x(a+bx)^2}{1+x}\,dx,x,\tan^2(e+fx)\right)}{2f}$$
$$=\frac{\text{Subst}\left(\int\left((a-b)^2+(2a-b)bx+b^2x^2-\frac{(a-b)^2}{1+x}\right)\,dx,x,\tan^2(e+\right.}{2f}$$
$$=\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{(a-b)^2\tan^2(e+fx)}{2f}+\frac{(2a-b)b\,\text{ta}}{4}$$

Mathematica [A]
time = 0.19, size = 72, normalized size = 0.88

$$\frac{12(a-b)^2\log(\cos(e+fx))+6(a-b)^2\tan^2(e+fx)+3(2a-b)b\tan^4(e+fx)+2b^2\tan^6(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
[Out] (12*(a - b)^2*Log[Cos[e + f*x]] + 6*(a - b)^2*Tan[e + f*x]^2 + 3*(2*a - b)*
b*Tan[e + f*x]^4 + 2*b^2*Tan[e + f*x]^6)/(12*f)
```

Maple [A]
time = 0.07, size = 110, normalized size = 1.34

method	result
norman	$\frac{b^2(\tan^6(fx+e))}{6f}+\frac{(a^2-2ab+b^2)(\tan^2(fx+e))}{2f}+\frac{(2a-b)b(\tan^4(fx+e))}{4f}-\frac{(a^2-2ab+b^2)\ln(1+\tan^2(fx+e))}{2f}$
derivativedivides	$\frac{\frac{b^2\left(\tan^6(fx+e)\right)}{6}+\frac{ab\left(\tan^4(fx+e)\right)}{2}-\frac{b^2\left(\tan^4(fx+e)\right)}{4}+\frac{a^2\left(\tan^2(fx+e)\right)}{2}-ab(\tan^2(fx+e))+\frac{b^2\left(\tan^2(fx+e)\right)}{2}+\frac{\left(-a^2+2ab-b^2\right)}{}}{f}$
default	$\frac{\frac{b^2\left(\tan^6(fx+e)\right)}{6}+\frac{ab\left(\tan^4(fx+e)\right)}{2}-\frac{b^2\left(\tan^4(fx+e)\right)}{4}+\frac{a^2\left(\tan^2(fx+e)\right)}{2}-ab(\tan^2(fx+e))+\frac{b^2\left(\tan^2(fx+e)\right)}{2}+\frac{\left(-a^2+2ab-b^2\right)}{}}{f}$
risch	$-ia^2x+2iabx-ib^2x-\frac{2ia^2e}{f}+\frac{4iabe}{f}-\frac{2ib^2e}{f}+\frac{2a^2e^{10i(fx+e)}-8ab\,e^{10i(fx+e)}+6b^2e^{10i(fx+e)}+8a^2e^{8i(fx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/6*b^2*tan(f*x+e)^6+1/2*a*b*tan(f*x+e)^4-1/4*b^2*tan(f*x+e)^4+1/2*a^2
*tan(f*x+e)^2-a*b*tan(f*x+e)^2+1/2*b^2*tan(f*x+e)^2+1/2*(-a^2+2*a*b-b^2)*ln
(1+tan(f*x+e)^2))
```

Maxima [A]
time = 0.27, size = 133, normalized size = 1.62

$$\frac{6\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{6\left(a^2-4\,ab+3\,b^2\right)\sin(fx+e)^4-3\left(4\,a^2-14\,ab+9\,b^2\right)\sin(fx+e)^2+6\,a^2-18\,ab+11\,b^2}{\sin(fx+e)^6-3\,\sin(fx+e)^4+3\,\sin(fx+e)^2-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 4*a*b + 3*b
^2)*sin(f*x + e)^4 - 3*(4*a^2 - 14*a*b + 9*b^2)*sin(f*x + e)^2 + 6*a^2 - 18
*a*b + 11*b^2)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/
f
```

Fricas [A]
time = 0.90, size = 90, normalized size = 1.10

$$\frac{2\,b^2\tan\left(fx+e\right)^6+3\left(2\,ab-b^2\right)\tan\left(fx+e\right)^4+6\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^2+6\left(a^2-2\,ab+b^2\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*tan(f*x + e)^6 + 3*(2*a*b - b^2)*tan(f*x + e)^4 + 6*(a^2 - 2*a*
b + b^2)*tan(f*x + e)^2 + 6*(a^2 - 2*a*b + b^2)*log(1/(tan(f*x + e)^2 + 1))
)/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(65) = 130$.
time = 0.19, size = 160, normalized size = 1.95

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^2\left(e+fx\right)}{2f}+\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{ab\tan^4\left(e+fx\right)}{2f}-\frac{ab\tan^2\left(e+fx\right)}{f}-\frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b^2\tan^6\left(e+fx\right)}{6f}-\frac{b^2\tan^4\left(e+fx\right)}{4f}+\frac{b^2\tan^2\left(e+fx\right)}{2f} & \text{for}\ f\neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^3\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*f
) + a*b*log(tan(e + f*x)**2 + 1)/f + a*b*tan(e + f*x)**4/(2*f) - a*b*tan(e
+ f*x)**2/f - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*tan(e + f*x)**6/(6
```

```
*f) - b**2*tan(e + f*x)**4/(4*f) + b**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (
x*(a + b*tan(e)**2)**2*tan(e)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(80) = 160$.
time = 3.78, size = 2600, normalized size = 31.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*ta
n(e)^6 - 12*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^
6*tan(e)^6 + 6*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*
x)^6*tan(e)^6 + 6*a^2*tan(f*x)^6*tan(e)^6 - 18*a*b*tan(f*x)^6*tan(e)^6 + 11
*b^2*tan(f*x)^6*tan(e)^6 - 36*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1))*tan(f*x)^5*tan(e)^5 + 72*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*
x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(ta
n(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 36*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 6*a^2*tan(f*x)^6*tan(e)^4 - 12*a*b*t
an(f*x)^6*tan(e)^4 + 6*b^2*tan(f*x)^6*tan(e)^4 - 24*a^2*tan(f*x)^5*tan(e)^5
 + 84*a*b*tan(f*x)^5*tan(e)^5 - 54*b^2*tan(f*x)^5*tan(e)^5 + 6*a^2*tan(f*x)
^4*tan(e)^6 - 12*a*b*tan(f*x)^4*tan(e)^6 + 6*b^2*tan(f*x)^4*tan(e)^6 + 90*a
^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
 tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 -
180*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)
^4 + 90*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*ta
n(e)^4 + 6*a*b*tan(f*x)^6*tan(e)^2 - 3*b^2*tan(f*x)^6*tan(e)^2 - 24*a^2*tan
(f*x)^5*tan(e)^3 + 72*a*b*tan(f*x)^5*tan(e)^3 - 36*b^2*tan(f*x)^5*tan(e)^3
+ 42*a^2*tan(f*x)^4*tan(e)^4 - 138*a*b*tan(f*x)^4*tan(e)^4 + 99*b^2*tan(f*x
)^4*tan(e)^4 - 24*a^2*tan(f*x)^3*tan(e)^5 + 72*a*b*tan(f*x)^3*tan(e)^5 - 36
*b^2*tan(f*x)^3*tan(e)^5 + 6*a*b*tan(f*x)^2*tan(e)^6 - 3*b^2*tan(f*x)^2*tan
(e)^6 - 120*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^
3*tan(e)^3 + 240*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(
f*x)^3*tan(e)^3 - 120*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
```

```
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^3*tan(e)^3 + 2*b^2*tan(f*x)^6 - 12*a*b*tan(f*x)^5*tan(e) + 18*b^2
*tan(f*x)^5*tan(e) + 36*a^2*tan(f*x)^4*tan(e)^2 - 120*a*b*tan(f*x)^4*tan(e)
^2 + 90*b^2*tan(f*x)^4*tan(e)^2 - 48*a^2*tan(f*x)^3*tan(e)^3 + 144*a*b*tan(
f*x)^3*tan(e)^3 - 72*b^2*tan(f*x)^3*tan(e)^3 + 36*a^2*tan(f*x)^2*tan(e)^4 -
 120*a*b*tan(f*x)^2*tan(e)^4 + 90*b^2*tan(f*x)^2*tan(e)^4 - 12*a*b*tan(f*x)
*tan(e)^5 + 18*b^2*tan(f*x)*tan(e)^5 + 2*b^2*tan(e)^6 + 90*a^2*log(4*(tan(f
*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2
*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 180*a*b*log(4*(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 90*b^2*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 6*a*b*
tan(f*x)^4 - 3*b^2*tan(f*x)^4 - 24*a^2*tan(f*x)^3*tan(e) + 72*a*b*tan(f*x)^
3*tan(e) - 36*b^2*tan(f*x)^3*tan(e) + 42*a^2*tan(f*x)^2*tan(e)^2 - 138*a*b*
tan(f*x)^2*tan(e)^2 + 99*b^2*tan(f*x)^2*tan(e)^2 - 24*a^2*tan(f*x)*tan(e)^3
 + 72*a*b*tan(f*x)*tan(e)^3 - 36*b^2*tan(f*x)*tan(e)^3 + 6*a*b*tan(e)^4 - 3
*b^2*tan(e)^4 - 36*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)*tan(e) + 72*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)*tan(e) - 36*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)*tan(e) + 6*a^2*tan(f*x)^2 - 12*a*b*tan(f*x)^2 + 6*b^2*tan(f*x)^2 - 2
4*a^2*tan(f*x)*tan(e) + 84*a*b*tan(f*x)*tan(e) - 54*b^2*tan(f*x)*tan(e) + 6
*a^2*tan(e)^2 - 12*a*b*tan(e)^2 + 6*b^2*tan(e)^2 + 6*a^2*log(4*(tan(f*x)^4*
tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f
*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 12*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/
(tan(e)^2 + 1)) + 6*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) +
 6*a^2 - 18*a*b + 11*b^2)/(f*tan(f*x)^6*tan(e)^6 - 6*f*tan(f*x)^5*tan(e)^5
+ 15*f*tan(f*x)^4*tan(e)^4 - 20*f*tan(f*x)^3*tan(e)^3 + 15*f*tan(f*x)^2*tan
(e)^2 - 6*f*tan(f*x)*tan(e) + f)
```

Mupad [B]
time = 11.52, size = 97, normalized size = 1.18

$$\frac{\tan(e+f\,x)^4\,\left(\frac{a\,b}{2}-\frac{b^2}{4}\right)}{f}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}+\frac{b^2\tan(e+f\,x)^6}{6\,f}+\frac{\tan(e+f\,x)^2\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^2,x)
[Out] (tan(e + f*x)^4*((a*b)/2 - b^2/4))/f - (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*
```

```
b + b^2/2))/f + (b^2*tan(e + f*x)^6)/(6*f) + (tan(e + f*x)^2*(a^2/2 - a*b +
 b^2/2))/f
```

3.200 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=62

$$-\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{(a-b)b\tan^2(e+fx)}{2f}+\frac{\left(a+b\tan^2(e+fx)\right)^2}{4f}$$

```
[Out] -(a-b)^2*ln(cos(f*x+e))/f+1/2*(a-b)*b*tan(f*x+e)^2/f+1/4*(a+b*tan(f*x+e)^2)
^2/f
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 455, 45}

$$\frac{b(a-b)\tan^2(e+fx)}{2f}+\frac{\left(a+b\tan^2(e+fx)\right)^2}{4f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Log[Cos[e + f*x]])/f) + ((a - b)*b*Tan[e + f*x]^2)/(2*f) + (a
+ b*Tan[e + f*x]^2)^2/(4*f)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left((a-b)b+\frac{(a-b)^2}{1+x}+b(a+bx)\right)\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{(a-b)b\tan^2(e+fx)}{2f}+\frac{(a+b\tan\ldots}{4\ldots}$$

Mathematica [A]
time = 0.15, size = 54, normalized size = 0.87

$$\frac{-4(a-b)^2\log(\cos(e+fx))+2(2a-b)b\tan^2(e+fx)+b^2\tan^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
[Out] (-4*(a - b)^2*Log[Cos[e + f*x]] + 2*(2*a - b)*b*Tan[e + f*x]^2 + b^2*Tan[e
+ f*x]^4)/(4*f)
```

Maple [A]
time = 0.05, size = 67, normalized size = 1.08

method	result
norman	$\frac{b^2(\tan^4(fx+e))}{4f}+\frac{b(2a-b)(\tan^2(fx+e))}{2f}+\frac{(a^2-2ab+b^2)\ln(1+\tan^2(fx+e))}{2f}$
derivativedivides	$\frac{\frac{b^2\left(\tan^4(fx+e)\right)}{4}+ab(\tan^2(fx+e))-\frac{b^2\left(\tan^2(fx+e)\right)}{2}+\frac{\left(a^2-2ab+b^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}}{f}$
default	$\frac{\frac{b^2\left(\tan^4(fx+e)\right)}{4}+ab(\tan^2(fx+e))-\frac{b^2\left(\tan^2(fx+e)\right)}{2}+\frac{\left(a^2-2ab+b^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}}{f}$
risch	$ia^2x-2iabx+ib^2x+\frac{2ia^2e}{f}-\frac{4iabe}{f}+\frac{2ib^2e}{f}+\frac{4b\left(a\,e^{6i(fx+e)}-b\,e^{6i(fx+e)}+2a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+a\,e^{2\ldots}\right)}{f\left(e^{2i(fx+e)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(1/4*b^2*tan(f*x+e)^4+a*b*tan(f*x+e)^2-1/2*b^2*tan(f*x+e)^2+1/2*(a^2-2*
a*b+b^2)*ln(1+tan(f*x+e)^2))
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.39

$$-\frac{2\left(a^{2}-2\,ab+b^{2}\right)\log\left(\sin\left(fx+e\right)^{2}-1\right)+\frac{4\,\left(ab-b^{2}\right)\sin\left(fx+e\right)^{2}-4\,ab+3\,b^{2}}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) + (4*(a*b - b^2)*sin(f*
x + e)^2 - 4*a*b + 3*b^2)/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A]
time = 0.71, size = 67, normalized size = 1.08

$$\frac{b^{2}\tan\left(fx+e\right)^{4}+2\left(2\,ab-b^{2}\right)\tan\left(fx+e\right)^{2}-2\left(a^{2}-2\,ab+b^{2}\right)\log\left(\frac{1}{\tan\left(fx+e\right)^{2}+1}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*tan(f*x + e)^4 + 2*(2*a*b - b^2)*tan(f*x + e)^2 - 2*(a^2 - 2*a*b +
 b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.
time = 0.12, size = 112, normalized size = 1.81

$$\begin{cases} \frac{a^{2}\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}-\frac{ab\log\left(\tan^{2}\left(e+fx\right)+1\right)}{f}+\frac{ab\tan^{2}\left(e+fx\right)}{f}+\frac{b^{2}\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b^{2}\tan^{4}\left(e+fx\right)}{4f}-\frac{b^{2}\tan^{2}\left(e+fx\right)}{2f} & \text{for}\: f\neq 0 \\ x\left(a+b\tan^{2}\left(e\right)\right)^{2}\tan\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) - a*b*log(tan(e + f*x)**2 +
1)/f + a*b*tan(e + f*x)**2/f + b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*t
an(e + f*x)**4/(4*f) - b**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan
(e)**2)**2*tan(e), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(61) = 122$.
time = 1.62, size = 1510, normalized size = 24.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")

[Out] -1/4*(2*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 4*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 2*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 4*a*b*tan(f*x)^4*tan(e)^4 + 3*b^2*tan(f*x)^4*tan(e)^4 - 8*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 16*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 8*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 4*a*b*tan(f*x)^4*tan(e)^2 + 2*b^2*tan(f*x)^4*tan(e)^2 + 8*a*b*tan(f*x)^3*tan(e)^3 - 8*b^2*tan(f*x)^3*tan(e)^3 - 4*a*b*tan(f*x)^2*tan(e)^4 + 2*b^2*tan(f*x)^2*tan(e)^4 + 12*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 24*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 12*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - b^2*tan(f*x)^4 + 8*a*b*tan(f*x)^3*tan(e) - 8*b^2*tan(f*x)^3*tan(e) - 8*a*b*tan(f*x)^2*tan(e)^2 + 4*b^2*tan(f*x)^2*tan(e)^2 + 8*a*b*tan(f*x)*tan(e)^3 - 8*b^2*tan(f*x)*tan(e)^3 - b^2*tan(e)^4 - 8*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 16*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - 8*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - 4*a*b*tan(f*x)^2 + 2*b^2*tan(f*x)^2 + 8*a*b*tan(f*x)*tan(e) - 8*b^2*tan(f*x)*tan(e) - 4*a*b*tan(e)^2 + 2*b^2*tan(e)^2 + 2*a^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 4*a*b*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) + 2*b^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 4*a*b + 3*b^2)/(f*tan(f*x)^4*tan(e)^4 - 4*f*tan(f*x)^3*tan(e)^3 + 6*f*tan(f*x)^2*tan(e)^2 - 4*f*tan(f*x)*tan(e) + f)

Mupad [B]

time = 11.89, size = 68, normalized size = 1.10

$$\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}+\frac{\tan(e+f\,x)^2\,\left(a\,b-\frac{b^2}{2}\right)}{f}+\frac{b^2\tan(e+f\,x)^4}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2))/f + (tan(e + f*x)^2*(a*b -
b^2/2))/f + (b^2*tan(e + f*x)^4)/(4*f)
```

3.201 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=51

$$\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{a^2\log(\tan(e+fx))}{f}+\frac{b^2\tan^2(e+fx)}{2f}$$

```
[Out] (a-b)^2*ln(cos(f*x+e))/f+a^2*ln(tan(f*x+e))/f+1/2*b^2*tan(f*x+e)^2/f
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3751, 457, 84}

$$\frac{a^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{b^2\tan^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a - b)^2*Log[Cos[e + f*x]])/f + (a^2*Log[Tan[e + f*x]])/f + (b^2*Tan[e +
f*x]^2)/(2*f)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2}{x}-\frac{(a-b)^2}{1+x}\right)\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{(a-b)^2\log(\cos(e+fx))}{f} + \frac{a^2\log(\tan(e+fx))}{f} + \frac{b^2\tan^2(e+f}{2f}$$

Mathematica [A]
time = 0.09, size = 65, normalized size = 1.27

$$-\frac{2ab\log(\cos(e+fx))}{f} + \frac{a^2(\log(\cos(e+fx))+\log(\tan(e+fx)))}{f} + \frac{b^2(2\log(\cos(e+fx))+\tan^2(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
[Out] (-2*a*b*Log[Cos[e + f*x]])/f + (a^2*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]])
)/f + (b^2*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f)
```

Maple [A]
time = 0.15, size = 50, normalized size = 0.98

method	result
derivativedivides	$\frac{b^2\left(\frac{\left(\tan^2(fx+e)\right)}{2}+\ln(\cos(fx+e))\right)-2ab\ln(\cos(fx+e))+a^2\ln(\sin(fx+e))}{f}$
default	$\frac{b^2\left(\frac{\left(\tan^2(fx+e)\right)}{2}+\ln(\cos(fx+e))\right)-2ab\ln(\cos(fx+e))+a^2\ln(\sin(fx+e))}{f}$
norman	$\frac{b^2\left(\tan^2(fx+e)\right)}{2f}+\frac{a^2\ln(\tan(fx+e))}{f}-\frac{(a^2-2ab+b^2)\ln\left(1+\tan^2(fx+e)\right)}{2f}$
risch	$-ia^2x+2iabx-ib^2x+\frac{4iabe}{f}-\frac{2ib^2e}{f}-\frac{2ia^2e}{f}+\frac{2b^2e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2\ln\left(e^{2i(fx+e)}+1\right)ab}{f}+\frac{\ln(e^{2i(f}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/f*(b^2*(1/2*tan(f*x+e)^2+ln(cos(f*x+e)))-2*a*b*ln(cos(f*x+e))+a^2*ln(sin(
f*x+e)))
```

Maxima [A]
time = 0.27, size = 62, normalized size = 1.22

$$\frac{a^2 \log\left(\sin\left(fx+e\right)^2\right) - \left(2\,ab - b^2\right)\log\left(\sin\left(fx+e\right)^2 - 1\right) - \frac{b^2}{\sin(fx+e)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*log(sin(f*x + e)^2) - (2*a*b - b^2)*log(sin(f*x + e)^2 - 1) - b^2/
(sin(f*x + e)^2 - 1))/f
```

Fricas [A]
time = 0.67, size = 73, normalized size = 1.43

$$\frac{b^2\tan\left(fx+e\right)^2 + a^2\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right) - \left(2\,ab - b^2\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*tan(f*x + e)^2 + a^2*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) - (2
*a*b - b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.
time = 0.60, size = 97, normalized size = 1.90

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{a^2\log\left(\tan\left(e+fx\right)\right)}{f} + \frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f} - \frac{b^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{b^2\tan^2\left(e+fx\right)}{2f} & \text{for}\ f \neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)^2\cot\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*log(tan(e + f*x))/f
+ a*b*log(tan(e + f*x)**2 + 1)/f - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b*
*2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e), True))
```

Giac [A]
time = 0.90, size = 91, normalized size = 1.78

$$\frac{a^2\log\left(\sin\left(fx+e\right)^2\right) - \left(2\,ab - b^2\right)\log\left(\left|\sin\left(fx+e\right)^2 - 1\right|\right) + \frac{2\,ab\sin(fx+e)^2 - b^2\sin(fx+e)^2 - 2\,ab}{\sin(fx+e)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*log(sin(f*x + e)^2) - (2*a*b - b^2)*log(abs(sin(f*x + e)^2 - 1)) +
 (2*a*b*sin(f*x + e)^2 - b^2*sin(f*x + e)^2 - 2*a*b)/(sin(f*x + e)^2 - 1))/
f
```

Mupad [B]
time = 11.95, size = 62, normalized size = 1.22

$$\frac{b^2\tan(e+f\,x)^2}{2\,f}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}+\frac{a^2\,\ln\left(\tan(e+f\,x)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x)^2)/(2*f) - (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2)
)/f + (a^2*log(tan(e + f*x)))/f
```

3.202 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=56

$$-\frac{a^2\cot^2(e+fx)}{2f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}-\frac{a(a-2b)\log(\tan(e+fx))}{f}$$

```
[Out] -1/2*a^2*cot(f*x+e)^2/f-(a-b)^2*ln(cos(f*x+e))/f-a*(a-2*b)*ln(tan(f*x+e))/f
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^2\cot^2(e+fx)}{2f}-\frac{a(a-2b)\log(\tan(e+fx))}{f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a^2*Cot[e + f*x]^2)/f - ((a - b)^2*Log[Cos[e + f*x]])/f - (a*(a - 2*b
)*Log[Tan[e + f*x]])/f
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^3(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^2}-\frac{a(a-2b)}{x}+\frac{(a-b)^2}{1+x}\right)\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\cot^2(e+fx)}{2f}-\frac{(a-b)^2\log(\cos(e+fx))}{f}-\frac{a(a-2b)\log(}{f}$$

Mathematica [A]
time = 0.17, size = 51, normalized size = 0.91

$$-\frac{a^2\cot^2(e+fx)+2(a-b)^2\log(\cos(e+fx))+2a(a-2b)\log(\tan(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(a^2*Cot[e + f*x]^2 + 2*(a - b)^2*Log[Cos[e + f*x]] + 2*a*(a - 2*b)*Lo
g[Tan[e + f*x]])/f
```

Maple [A]
time = 0.16, size = 53, normalized size = 0.95

method	result
derivativedivides	$\frac{-b^2\ln(\cos(fx+e))+2ab\ln(\sin(fx+e))+a^2\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)}{f}$
default	$\frac{-b^2\ln(\cos(fx+e))+2ab\ln(\sin(fx+e))+a^2\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)}{f}$
norman	$-\frac{a^2}{2f\tan(fx+e)^2}+\frac{(a^2-2ab+b^2)\ln(1+\tan^2(fx+e))}{2f}-\frac{a(a-2b)\ln(\tan(fx+e))}{f}$
risch	$ia^2x-2iabx+ib^2x+\frac{2ib^2e}{f}+\frac{2ia^2e}{f}-\frac{4iabe}{f}+\frac{2a^2e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{2i(fx+e)}+1\right)b^2}{f}-\frac{a^2\ln(e^{2i(fx}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b^2*ln(cos(f*x+e))+2*a*b*ln(sin(f*x+e))+a^2*(-1/2*cot(f*x+e)^2-ln(sin
(f*x+e))))
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.96

$$-\frac{b^2\log\left(\sin\left(fx+e\right)^2-1\right)+\left(a^2-2\,ab\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{a^2}{\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - 2*a*b)*log(sin(f*x + e)^2) + a^2
/sin(f*x + e)^2)/f
```

Fricas [A]
time = 0.98, size = 100, normalized size = 1.79

$$-\frac{b^2\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+a^2\tan\left(fx+e\right)^2+\left(a^2-2\,ab\right)\log\left(\frac{\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+a^2}{2\,f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*log(1/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a^2*tan(f*x + e)^2 +
 (a^2 - 2*a*b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a^
2)/(f*tan(f*x + e)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(48) = 96$.
time = 1.80, size = 131, normalized size = 2.34

$$\begin{cases} \tilde{\infty} a^2 x & \text{for}\ (e=0 \vee e=-fx) \wedge (e=-fx \vee f=0) \\ x\left(a+b\tan^2(e)\right)^2\cot^3(e) & \text{for}\ f=0 \\ \frac{a^2\log\left(\tan^2(e+fx)+1\right)}{2f}-\frac{a^2\log\left(\tan(e+fx)\right)}{f}-\frac{a^2}{2f\tan^2(e+fx)}-\frac{ab\log\left(\tan^2(e+fx)+1\right)}{f}+\frac{2ab\log\left(\tan(e+fx)\right)}{f}+\frac{b^2\log\left(\tan^2(e+fx)+1\right)}{2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x)))
, (x*(a + b*tan(e)**2)**2*cot(e)**3, Eq(f, 0)), (a**2*log(tan(e + f*x)**2 +
 1)/(2*f) - a**2*log(tan(e + f*x))/f - a**2/(2*f*tan(e + f*x)**2) - a*b*log
(tan(e + f*x)**2 + 1)/f + 2*a*b*log(tan(e + f*x))/f + b**2*log(tan(e + f*x)
**2 + 1)/(2*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(57) = 114$.
time = 1.15, size = 167, normalized size = 2.98

$$\frac{a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-4\,b^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+4\,(a^2-2\,ab+b^2)\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f
*x + e) + 1)) - 4*b^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos
(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + 4*(a^2 - 2*a*b + b^2)*log(abs(-(c
os(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + 2)))/f
```

Mupad [B]
time = 12.09, size = 68, normalized size = 1.21

$$\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}+\frac{\ln\left(\tan(e+f\,x)\right)\,\left(2\,a\,b-a^2\right)}{f}-\frac{a^2\cot(e+f\,x)^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2))/f + (log(tan(e + f*x))*(2*a
*b - a^2))/f - (a^2*cot(e + f*x)^2)/(2*f)
```

3.203 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=76

$$\frac{a(a-2b)\cot^2(e+fx)}{2f}-\frac{a^2\cot^4(e+fx)}{4f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{(a-b)^2\log(\tan(e+fx))}{f}$$

```
[Out] 1/2*a*(a-2*b)*cot(f*x+e)^2/f-1/4*a^2*cot(f*x+e)^4/f+(a-b)^2*ln(cos(f*x+e))/
f+(a-b)^2*ln(tan(f*x+e))/f
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{a^2\cot^4(e+fx)}{4f}+\frac{a(a-2b)\cot^2(e+fx)}{2f}+\frac{(a-b)^2\log(\tan(e+fx))}{f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a*(a - 2*b)*Cot[e + f*x]^2)/(2*f) - (a^2*Cot[e + f*x]^4)/(4*f) + ((a - b)^
2*Log[Cos[e + f*x]])/f + ((a - b)^2*Log[Tan[e + f*x]])/f
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{x^5(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^3(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3} - \frac{a(a-2b)}{x^2} + \frac{(a-b)^2}{x} - \frac{(a-b)^2}{1+x}\right)\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{a(a-2b)\cot^2(e+fx)}{2f} - \frac{a^2\cot^4(e+fx)}{4f} + \frac{(a-b)^2\log(\cos(e}{f}$$

Mathematica [A]
time = 0.20, size = 61, normalized size = 0.80

$$\frac{2a(a-2b)\cot^2(e+fx) - a^2\cot^4(e+fx) + 4(a-b)^2(\log(\cos(e+fx)) + \log(\tan(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
[Out] (2*a*(a - 2*b)*Cot[e + f*x]^2 - a^2*Cot[e + f*x]^4 + 4*(a - b)^2*(Log[Cos[e
 + f*x]] + Log[Tan[e + f*x]]))/(4*f)
```

Maple [A]
time = 0.15, size = 73, normalized size = 0.96

method	result
derivativedivides	$\frac{b^2\ln(\sin(fx+e))+2ab\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)+a^2\left(-\frac{\left(\cot^4(fx+e)\right)}{4}+\frac{\left(\cot^2(fx+e)\right)}{2}+\ln(\sin(fx+e))\right)}{f}$
default	$\frac{b^2\ln(\sin(fx+e))+2ab\left(-\frac{\left(\cot^2(fx+e)\right)}{2}-\ln(\sin(fx+e))\right)+a^2\left(-\frac{\left(\cot^4(fx+e)\right)}{4}+\frac{\left(\cot^2(fx+e)\right)}{2}+\ln(\sin(fx+e))\right)}{f}$
norman	$\frac{-\frac{a^2}{4f}+\frac{a(a-2b)\left(\tan^2(fx+e)\right)}{2f}}{\tan(fx+e)^4}+\frac{(a^2-2ab+b^2)\ln(\tan(fx+e))}{f}-\frac{(a^2-2ab+b^2)\ln\left(1+\tan^2(fx+e)\right)}{2f}$
risch	$-ia^2x+2iabx-ib^2x-\frac{2ia^2e}{f}+\frac{4iabe}{f}-\frac{2ib^2e}{f}-\frac{4a\left(a\,\mathrm{e}^{6i(fx+e)}-b\,\mathrm{e}^{6i(fx+e)}-a\,\mathrm{e}^{4i(fx+e)}+2b\,\mathrm{e}^{4i(fx+e)}-\right.}{f\left(\mathrm{e}^{2i(fx+e)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*ln(sin(f*x+e))+2*a*b*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))+a^2*(-1/4*
cot(f*x+e)^4+1/2*cot(f*x+e)^2+ln(sin(f*x+e))))
```

Maxima [A]
time = 0.27, size = 64, normalized size = 0.84

$$\frac{2\,(a^2-2\,ab+b^2)\log\left(\sin\left(fx+e\right)^2\right)+\frac{4\,(a^2-ab)\sin(fx+e)^2-a^2}{\sin(fx+e)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2) + (4*(a^2 - a*b)*sin(f*x + e
)^2 - a^2)/sin(f*x + e)^4)/f
```

Fricas [A]
time = 2.17, size = 105, normalized size = 1.38

$$\frac{2\,(a^2-2\,ab+b^2)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^4+(3\,a^2-4\,ab)\tan\left(fx+e\right)^4+2\,(a^2-2\,ab)\tan\left(fx+e\right)^2-a^2}{4\,f\tan\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2 - 2*a*b + b^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x
 + e)^4 + (3*a^2 - 4*a*b)*tan(f*x + e)^4 + 2*(a^2 - 2*a*b)*tan(f*x + e)^2 -
 a^2)/(f*tan(f*x + e)^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(63) = 126$.
time = 5.31, size = 172, normalized size = 2.26

$$\begin{cases} \tilde{\infty}a^2x & \text{for}\ e=0\wedge f=0 \\ x(a+b\tan^2(e))^2\cot^5(e) & \text{for}\ f=0 \\ \tilde{\infty}a^2x & \text{for}\ e=-fx \\ -\frac{a^2\log(\tan^2(e+fx)+1)}{2f}+\frac{a^2\log(\tan(e+fx))}{f}+\frac{a^2}{2f\tan^2(e+fx)}-\frac{a^2}{4f\tan^4(e+fx)}+\frac{ab\log(\tan^2(e+fx)+1)}{f}-\frac{2ab\log(\tan(e+fx))}{f}-\frac{ab}{f\tan^2(e+fx)}-\frac{b^2\log(\tan^2(e+fx)+1)}{2f}+\frac{b^2\log(\tan(e+fx))}{f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e)
**5, Eq(f, 0)), (zoo*a**2*x, Eq(e, -f*x)), (-a**2*log(tan(e + f*x)**2 + 1)/
(2*f) + a**2*log(tan(e + f*x))/f + a**2/(2*f*tan(e + f*x)**2) - a**2/(4*f*t
an(e + f*x)**4) + a*b*log(tan(e + f*x)**2 + 1)/f - 2*a*b*log(tan(e + f*x))/
f - a*b/(f*tan(e + f*x)**2) - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*lo
g(tan(e + f*x))/f, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(76) = 152.

time = 1.56, size = 315, normalized size = 4.14

$$-\frac{\frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-32\,(a^2-2\,ab+b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+64\,(a^2-2\,ab+b^2)\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)+\frac{\left(a^2+\frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{96\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{48\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/64*(12*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 3
2*(a^2 - 2*a*b + b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) + 6
4*(a^2 - 2*a*b + b^2)*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))
+ (a^2 + 12*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 - 96*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^2*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2)/f
```

Mupad [B]

time = 12.18, size = 91, normalized size = 1.20

$$\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2-2\,a\,b+b^2\right)}{f}-\frac{\frac{a^2}{4}+\tan(e+f\,x)^2\,\left(a\,b-\frac{a^2}{2}\right)}{f\,\tan\left(e+f\,x\right)^4}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (log(tan(e + f*x))*(a^2 - 2*a*b + b^2))/f - (a^2/4 + tan(e + f*x)^2*(a*b -
a^2/2))/(f*tan(e + f*x)^4) - (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2)
)/f
```

3.204 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=113

$$-(a-b)^2x+\frac{(a-b)^2\tan(e+fx)}{f}-\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan^5(e+fx)}{5f}+\frac{(2a-b)b\tan^7(e+fx)}{7f}+b^2\cdots$$

```
[Out] -(a-b)^2*x+(a-b)^2*tan(f*x+e)/f-1/3*(a-b)^2*tan(f*x+e)^3/f+1/5*(a-b)^2*tan(
f*x+e)^5/f+1/7*(2*a-b)*b*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 472, 209}

$$\frac{b(2a-b)\tan^7(e+fx)}{7f}+\frac{(a-b)^2\tan^5(e+fx)}{5f}-\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^9(e+fx)}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) + ((a - b)^2*Tan[e + f*x])/f - ((a - b)^2*Tan[e + f*x]^3)/(3
*f) + ((a - b)^2*Tan[e + f*x]^5)/(5*f) + ((2*a - b)*b*Tan[e + f*x]^7)/(7*f)
 + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+bx^2\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left((a-b)^2-(a-b)^2x^2+(a-b)^2x^4+(2a-b)bx^6+b\right.\right.}{f}$$

$$= \frac{(a-b)^2\tan(e+fx)}{f} - \frac{(a-b)^2\tan^3(e+fx)}{3f} + \frac{(a-b)^2\tan^5(e}{5f}$$

$$= -(a-b)^2x + \frac{(a-b)^2\tan(e+fx)}{f} - \frac{(a-b)^2\tan^3(e+fx)}{3f} + $$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 243 vs. 2(113) = 226.
time = 0.06, size = 243, normalized size = 2.15

$$-\frac{a^2\text{ArcTan}(\tan(e+fx))}{f}+\frac{2ab\text{ArcTan}(\tan(e+fx))}{f}-\frac{b^2\text{ArcTan}(\tan(e+fx))}{f}+\frac{a^2\tan(e+fx)}{f}-\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{2ab\tan^3(e+fx)}{3f}-\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}-\frac{2ab\tan^5(e+fx)}{5f}+\frac{b^2\tan^5(e+fx)}{5f}+\frac{2ab\tan^7(e+fx)}{7f}-\frac{b^2\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTan[Tan[e + f*x]])/f) + (2*a*b*ArcTan[Tan[e + f*x]])/f - (b^2*Arc
Tan[Tan[e + f*x]])/f + (a^2*Tan[e + f*x])/f - (2*a*b*Tan[e + f*x])/f + (b^2
*Tan[e + f*x])/f - (a^2*Tan[e + f*x]^3)/(3*f) + (2*a*b*Tan[e + f*x]^3)/(3*f
) - (b^2*Tan[e + f*x]^3)/(3*f) + (a^2*Tan[e + f*x]^5)/(5*f) - (2*a*b*Tan[e
+ f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^5)/(5*f) + (2*a*b*Tan[e + f*x]^7)/(7*f)
 - (b^2*Tan[e + f*x]^7)/(7*f) + (b^2*Tan[e + f*x]^9)/(9*f)
```

Maple [A]
time = 0.08, size = 173, normalized size = 1.53

method	result
norman	$\left(-a^2+2ab-b^2\right)x+\frac{\left(a^2-2ab+b^2\right)\tan(fx+e)}{f}+\frac{b^2\left(\tan^9(fx+e)\right)}{9f}-\frac{\left(a^2-2ab+b^2\right)\left(\tan^3(fx+e)\right)}{3f}+\frac{(a^2-}{}$
derivativedivides	$\frac{\frac{b^2\left(\tan^9(fx+e)\right)}{9}+\frac{2ab\left(\tan^7(fx+e)\right)}{7}-\frac{b^2\left(\tan^7(fx+e)\right)}{7}+\frac{a^2\left(\tan^5(fx+e)\right)}{5}-\frac{2ab\left(\tan^5(fx+e)\right)}{5}+\frac{b^2\left(\tan^5(fx+e)\right)}{5}-\frac{a^2\left(\tan^3(fx+\right.}{3}}{}$
default	$\frac{\frac{b^2\left(\tan^9(fx+e)\right)}{9}+\frac{2ab\left(\tan^7(fx+e)\right)}{7}-\frac{b^2\left(\tan^7(fx+e)\right)}{7}+\frac{a^2\left(\tan^5(fx+e)\right)}{5}-\frac{2ab\left(\tan^5(fx+e)\right)}{5}+\frac{b^2\left(\tan^5(fx+e)\right)}{5}-\frac{a^2\left(\tan^3(fx+\right.}{3}}{}$
risch	$-x\,a^2+2xab-x\,b^2+\frac{2i\left(483a^2+563b^2+1575b^2\mathrm{e}^{16i(fx+e)}+945a^2\mathrm{e}^{16i(fx+e)}-1056ab-63000ab\,\mathrm{e}^{10i(fx+e)}+32\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(1/9*b^2*tan(f*x+e)^9+2/7*a*b*tan(f*x+e)^7-1/7*b^2*tan(f*x+e)^7+1/5*a^2*tan(f*x+e)^5-2/5*a*b*tan(f*x+e)^5+1/5*b^2*tan(f*x+e)^5-1/3*a^2*tan(f*x+e)^3+2/3*a*b*tan(f*x+e)^3-1/3*b^2*tan(f*x+e)^3+a^2*tan(f*x+e)-2*a*b*tan(f*x+e)+b^2*tan(f*x+e)+(-a^2+2*a*b-b^2)*arctan(tan(f*x+e)))

Maxima [A]
time = 0.48, size = 124, normalized size = 1.10

$$\frac{35\,b^2\tan\left(fx+e\right)^9+45\,\left(2\,ab-b^2\right)\tan\left(fx+e\right)^7+63\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^5-105\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3-315\,\left(a^2-2\,ab+b^2\right)(fx+e)+315\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")

[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b - b^2)*tan(f*x + e)^7 + 63*(a^2 - 2*a*b + b^2)*tan(f*x + e)^5 - 105*(a^2 - 2*a*b + b^2)*tan(f*x + e)^3 - 315*(a^2 - 2*a*b + b^2)*(f*x + e) + 315*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f

Fricas [A]
time = 2.57, size = 120, normalized size = 1.06

$$\frac{35\,b^2\tan\left(fx+e\right)^9+45\,\left(2\,ab-b^2\right)\tan\left(fx+e\right)^7+63\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^5-105\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3-315\,\left(a^2-2\,ab+b^2\right)fx+315\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")

[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b - b^2)*tan(f*x + e)^7 + 63*(a^2 - 2*a*b + b^2)*tan(f*x + e)^5 - 105*(a^2 - 2*a*b + b^2)*tan(f*x + e)^3 - 315*(a^2 - 2*a*b + b^2)*f*x + 315*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(88) = 176$.
time = 0.35, size = 212, normalized size = 1.88

$$\begin{cases} -a^2x+\frac{a^2\tan^5{(e+fx)}}{5f}-\frac{a^2\tan^3{(e+fx)}}{3f}+\frac{a^2\tan{(e+fx)}}{f}+2abx+\frac{2ab\tan^7{(e+fx)}}{7f}-\frac{2ab\tan^5{(e+fx)}}{5f}+\frac{2ab\tan^3{(e+fx)}}{3f}-\frac{2ab\tan{(e+fx)}}{f}-b^2x+\frac{b^2\tan^9{(e+fx)}}{9f}-\frac{b^2\tan^7{(e+fx)}}{7f}+\frac{b^2\tan^5{(e+fx)}}{5f}-\frac{b^2\tan^3{(e+fx)}}{3f}+\frac{b^2\tan{(e+fx)}}{f} & \text{for } f\neq 0 \\ x\left(a+b\tan^2{(e)}\right)^2\tan^6{(e)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)

[Out] Piecewise((-a**2*x + a**2*tan(e + f*x)**5/(5*f) - a**2*tan(e + f*x)**3/(3*f) + a**2*tan(e + f*x)/f + 2*a*b*x + 2*a*b*tan(e + f*x)**7/(7*f) - 2*a*b*tan

```
(e + f*x)**5/(5*f) + 2*a*b*tan(e + f*x)**3/(3*f) - 2*a*b*tan(e + f*x)/f - b
**2*x + b**2*tan(e + f*x)**9/(9*f) - b**2*tan(e + f*x)**7/(7*f) + b**2*tan(
e + f*x)**5/(5*f) - b**2*tan(e + f*x)**3/(3*f) + b**2*tan(e + f*x)/f, Ne(f,
 0)), (x*(a + b*tan(e)**2)**2*tan(e)**6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2619 vs. $2(110) = 220$.
time = 7.27, size = 2619, normalized size = 23.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/315*(315*a^2*f*x*tan(f*x)^9*tan(e)^9 - 630*a*b*f*x*tan(f*x)^9*tan(e)^9 +
 315*b^2*f*x*tan(f*x)^9*tan(e)^9 - 2835*a^2*f*x*tan(f*x)^8*tan(e)^8 + 5670*
a*b*f*x*tan(f*x)^8*tan(e)^8 - 2835*b^2*f*x*tan(f*x)^8*tan(e)^8 + 315*a^2*ta
n(f*x)^9*tan(e)^8 - 630*a*b*tan(f*x)^9*tan(e)^8 + 315*b^2*tan(f*x)^9*tan(e)
^8 + 315*a^2*tan(f*x)^8*tan(e)^9 - 630*a*b*tan(f*x)^8*tan(e)^9 + 315*b^2*ta
n(f*x)^8*tan(e)^9 + 11340*a^2*f*x*tan(f*x)^7*tan(e)^7 - 22680*a*b*f*x*tan(f
*x)^7*tan(e)^7 + 11340*b^2*f*x*tan(f*x)^7*tan(e)^7 - 105*a^2*tan(f*x)^9*tan
(e)^6 + 210*a*b*tan(f*x)^9*tan(e)^6 - 105*b^2*tan(f*x)^9*tan(e)^6 - 2835*a^
2*tan(f*x)^8*tan(e)^7 + 5670*a*b*tan(f*x)^8*tan(e)^7 - 2835*b^2*tan(f*x)^8*
tan(e)^7 - 2835*a^2*tan(f*x)^7*tan(e)^8 + 5670*a*b*tan(f*x)^7*tan(e)^8 - 28
35*b^2*tan(f*x)^7*tan(e)^8 - 105*a^2*tan(f*x)^6*tan(e)^9 + 210*a*b*tan(f*x)
^6*tan(e)^9 - 105*b^2*tan(f*x)^6*tan(e)^9 - 26460*a^2*f*x*tan(f*x)^6*tan(e)
^6 + 52920*a*b*f*x*tan(f*x)^6*tan(e)^6 - 26460*b^2*f*x*tan(f*x)^6*tan(e)^6
+ 63*a^2*tan(f*x)^9*tan(e)^4 - 126*a*b*tan(f*x)^9*tan(e)^4 + 63*b^2*tan(f*x
)^9*tan(e)^4 + 945*a^2*tan(f*x)^8*tan(e)^5 - 1890*a*b*tan(f*x)^8*tan(e)^5 +
 945*b^2*tan(f*x)^8*tan(e)^5 + 11340*a^2*tan(f*x)^7*tan(e)^6 - 22680*a*b*ta
n(f*x)^7*tan(e)^6 + 11340*b^2*tan(f*x)^7*tan(e)^6 + 11340*a^2*tan(f*x)^6*ta
n(e)^7 - 22680*a*b*tan(f*x)^6*tan(e)^7 + 11340*b^2*tan(f*x)^6*tan(e)^7 + 94
5*a^2*tan(f*x)^5*tan(e)^8 - 1890*a*b*tan(f*x)^5*tan(e)^8 + 945*b^2*tan(f*x)
^5*tan(e)^8 + 63*a^2*tan(f*x)^4*tan(e)^9 - 126*a*b*tan(f*x)^4*tan(e)^9 + 63
*b^2*tan(f*x)^4*tan(e)^9 + 39690*a^2*f*x*tan(f*x)^5*tan(e)^5 - 79380*a*b*f*
x*tan(f*x)^5*tan(e)^5 + 39690*b^2*f*x*tan(f*x)^5*tan(e)^5 + 90*a*b*tan(f*x)
^9*tan(e)^2 - 45*b^2*tan(f*x)^9*tan(e)^2 - 252*a^2*tan(f*x)^8*tan(e)^3 + 11
34*a*b*tan(f*x)^8*tan(e)^3 - 567*b^2*tan(f*x)^8*tan(e)^3 - 2835*a^2*tan(f*x
)^7*tan(e)^4 + 7560*a*b*tan(f*x)^7*tan(e)^4 - 3780*b^2*tan(f*x)^7*tan(e)^4
- 24885*a^2*tan(f*x)^6*tan(e)^5 + 52920*a*b*tan(f*x)^6*tan(e)^5 - 26460*b^2
*tan(f*x)^6*tan(e)^5 - 24885*a^2*tan(f*x)^5*tan(e)^6 + 52920*a*b*tan(f*x)^5
*tan(e)^6 - 26460*b^2*tan(f*x)^5*tan(e)^6 - 2835*a^2*tan(f*x)^4*tan(e)^7 +
7560*a*b*tan(f*x)^4*tan(e)^7 - 3780*b^2*tan(f*x)^4*tan(e)^7 - 252*a^2*tan(f
*x)^3*tan(e)^8 + 1134*a*b*tan(f*x)^3*tan(e)^8 - 567*b^2*tan(f*x)^3*tan(e)^8
 + 90*a*b*tan(f*x)^2*tan(e)^9 - 45*b^2*tan(f*x)^2*tan(e)^9 - 39690*a^2*f*x*
```

```
tan(f*x)^4*tan(e)^4 + 79380*a*b*f*x*tan(f*x)^4*tan(e)^4 - 39690*b^2*f*x*tan
(f*x)^4*tan(e)^4 + 35*b^2*tan(f*x)^9 - 180*a*b*tan(f*x)^8*tan(e) + 405*b^2*
tan(f*x)^8*tan(e) + 378*a^2*tan(f*x)^7*tan(e)^2 - 2016*a*b*tan(f*x)^7*tan(e
)^2 + 2268*b^2*tan(f*x)^7*tan(e)^2 + 3990*a^2*tan(f*x)^6*tan(e)^3 - 11760*a
*b*tan(f*x)^6*tan(e)^3 + 8820*b^2*tan(f*x)^6*tan(e)^3 + 32130*a^2*tan(f*x)^
5*tan(e)^4 - 70560*a*b*tan(f*x)^5*tan(e)^4 + 39690*b^2*tan(f*x)^5*tan(e)^4
+ 32130*a^2*tan(f*x)^4*tan(e)^5 - 70560*a*b*tan(f*x)^4*tan(e)^5 + 39690*b^2
*tan(f*x)^4*tan(e)^5 + 3990*a^2*tan(f*x)^3*tan(e)^6 - 11760*a*b*tan(f*x)^3*
tan(e)^6 + 8820*b^2*tan(f*x)^3*tan(e)^6 + 378*a^2*tan(f*x)^2*tan(e)^7 - 201
6*a*b*tan(f*x)^2*tan(e)^7 + 2268*b^2*tan(f*x)^2*tan(e)^7 - 180*a*b*tan(f*x)
*tan(e)^8 + 405*b^2*tan(f*x)*tan(e)^8 + 35*b^2*tan(e)^9 + 26460*a^2*f*x*tan
(f*x)^3*tan(e)^3 - 52920*a*b*f*x*tan(f*x)^3*tan(e)^3 + 26460*b^2*f*x*tan(f*
x)^3*tan(e)^3 + 90*a*b*tan(f*x)^7 - 45*b^2*tan(f*x)^7 - 252*a^2*tan(f*x)^6*
tan(e) + 1134*a*b*tan(f*x)^6*tan(e) - 567*b^2*tan(f*x)^6*tan(e) - 2835*a^2*
tan(f*x)^5*tan(e)^2 + 7560*a*b*tan(f*x)^5*tan(e)^2 - 3780*b^2*tan(f*x)^5*ta
n(e)^2 - 24885*a^2*tan(f*x)^4*tan(e)^3 + 52920*a*b*tan(f*x)^4*tan(e)^3 - 26
460*b^2*tan(f*x)^4*tan(e)^3 - 24885*a^2*tan(f*x)^3*tan(e)^4 + 52920*a*b*tan
(f*x)^3*tan(e)^4 - 26460*b^2*tan(f*x)^3*tan(e)^4 - 2835*a^2*tan(f*x)^2*tan(
e)^5 + 7560*a*b*tan(f*x)^2*tan(e)^5 - 3780*b^2*tan(f*x)^2*tan(e)^5 - 252*a^
2*tan(f*x)*tan(e)^6 + 1134*a*b*tan(f*x)*tan(e)^6 - 567*b^2*tan(f*x)*tan(e)^
6 + 90*a*b*tan(e)^7 - 45*b^2*tan(e)^7 - 11340*a^2*f*x*tan(f*x)^2*tan(e)^2 +
 22680*a*b*f*x*tan(f*x)^2*tan(e)^2 - 11340*b^2*f*x*tan(f*x)^2*tan(e)^2 + 63
*a^2*tan(f*x)^5 - 126*a*b*tan(f*x)^5 + 63*b^2*tan(f*x)^5 + 945*a^2*tan(f*x)
^4*tan(e) - 1890*a*b*tan(f*x)^4*tan(e) + 945*b^2*tan(f*x)^4*tan(e) + 11340*
a^2*tan(f*x)^3*tan(e)^2 - 22680*a*b*tan(f*x)^3*tan(e)^2 + 11340*b^2*tan(f*x
)^3*tan(e)^2 + 11340*a^2*tan(f*x)^2*tan(e)^3 - 22680*a*b*tan(f*x)^2*tan(e)^
3 + 11340*b^2*tan(f*x)^2*tan(e)^3 + 945*a^2*tan(f*x)*tan(e)^4 - 1890*a*b*ta
n(f*x)*tan(e)^4 + 945*b^2*tan(f*x)*tan(e)^4 + 63*a^2*tan(e)^5 - 126*a*b*tan
(e)^5 + 63*b^2*tan(e)^5 + 2835*a^2*f*x*tan(f*x)*tan(e) - 5670*a*b*f*x*tan(f
*x)*tan(e) + 2835*b^2*f*x*tan(f*x)*tan(e) - 105*a^2*tan(f*x)^3 + 210*a*b*ta
n(f*x)^3 - 105*b^2*tan(f*x)^3 - 2835*a^2*tan(f*x)^2*tan(e) + 5670*a*b*tan(f
*x)^2*tan(e) - 2835*b^2*tan(f*x)^2*tan(e) - 2835*a^2*tan(f*x)*tan(e)^2 + 56
70*a*b*tan(f*x)*tan(e)^2 - 2835*b^2*tan(f*x)*tan(e)^2 - 105*a^2*tan(e)^3 +
210*a*b*tan(e)^3 - 105*b^2*tan(e)^3 - 315*a^2*f*x + 630*a*b*f*x - 315*b^2*f
*x + 315*a^2*tan(f*x) - 630*a*b*tan(f*x) + 315*b^2*tan(f*x) + 315*a^2*tan(e
) - 630*a*b*tan(e) + 315*b^2*tan(e))/(f*tan(f*x...
```

Mupad [B]
time = 12.09, size = 155, normalized size = 1.37

$$\frac{\tan(e+f\,x)^7\,\left(\frac{2\,a\,b}{7}-\frac{b^2}{7}\right)}{f}-\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,{(a-b)}^2}{a^2-2\,a\,b+b^2}\right)\,{(a-b)}^2}{f}+\frac{\tan(e+f\,x)\,\left(a^2-2\,a\,b+b^2\right)}{f}+\frac{b^2\,\tan(e+f\,x)^9}{9\,f}-\frac{\tan(e+f\,x)^3\,\left(\frac{a^2}{3}-\frac{2\,a\,b}{3}+\frac{b^2}{3}\right)}{f}+\frac{\tan(e+f\,x)^5\,\left(\frac{a^2}{5}-\frac{2\,a\,b}{5}+\frac{b^2}{5}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^7*((2*a*b)/7 - b^2/7))/f - (atan((tan(e + f*x)*(a - b)^2)/(a^
2 - 2*a*b + b^2))*(a - b)^2)/f + (tan(e + f*x)*(a^2 - 2*a*b + b^2))/f + (b^
2*tan(e + f*x)^9)/(9*f) - (tan(e + f*x)^3*(a^2/3 - (2*a*b)/3 + b^2/3))/f +
(tan(e + f*x)^5*(a^2/5 - (2*a*b)/5 + b^2/5))/f
```

3.205 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=91

$$(a-b)^2x-\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(2a-b)b\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

[Out] (a-b)^2*x-(a-b)^2*tan(f*x+e)/f+1/3*(a-b)^2*tan(f*x+e)^3/f+1/5*(2*a-b)*b*tan(f*x+e)^5/f+1/7*b^2*tan(f*x+e)^7/f

Rubi [A]

time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 209}

$$\frac{b(2a-b)\tan^5(e+fx)}{5f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}-\frac{(a-b)^2\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x - ((a - b)^2*Tan[e + f*x])/f + ((a - b)^2*Tan[e + f*x]^3)/(3*f)
 + ((2*a - b)*b*Tan[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int\left(-(a-b)^2+(a-b)^2x^2+(2a-b)bx^4+b^2x^6+\frac{a^2-2a}{1+}\right.\right.}{f}$$

$$=-\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(2a-b)b\tan}{5f}$$

$$=(a-b)^2x-\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(91) = 182.
time = 0.05, size = 190, normalized size = 2.09

$$\frac{a^2\text{ArcTan}(\tan(e+fx))}{f}-\frac{2ab\text{ArcTan}(\tan(e+fx))}{f}+\frac{b^2\text{ArcTan}(\tan(e+fx))}{f}-\frac{a^2\tan(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}-\frac{b^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}-\frac{2ab\tan^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{2ab\tan^5(e+fx)}{5f}-\frac{b^2\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a^2*ArcTan[Tan[e + f*x]])/f - (2*a*b*ArcTan[Tan[e + f*x]])/f + (b^2*ArcTan
[Tan[e + f*x]])/f - (a^2*Tan[e + f*x])/f + (2*a*b*Tan[e + f*x])/f - (b^2*Ta
n[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f) - (2*a*b*Tan[e + f*x]^3)/(3*f) +
 (b^2*Tan[e + f*x]^3)/(3*f) + (2*a*b*Tan[e + f*x]^5)/(5*f) - (b^2*Tan[e + f
*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Maple [A]
time = 0.06, size = 133, normalized size = 1.46

method	result
norman	$(a^2-2ab+b^2)\,x+\frac{b^2(\tan^7(fx+e))}{7f}-\frac{(a^2-2ab+b^2)\tan(fx+e)}{f}+\frac{(a^2-2ab+b^2)(\tan^3(fx+e))}{3f}+\frac{(2a-b)b}{}$
derivativedivides	$\frac{\frac{b^2\left(\tan^7(fx+e)\right)}{7}+\frac{2ab\left(\tan^5(fx+e)\right)}{5}-\frac{b^2\left(\tan^5(fx+e)\right)}{5}+\frac{a^2\left(\tan^3(fx+e)\right)}{3}-\frac{2ab\left(\tan^3(fx+e)\right)}{3}+\frac{b^2\left(\tan^3(fx+e)\right)}{3}-a^2\tan(fx+e}{f}$
default	$\frac{\frac{b^2\left(\tan^7(fx+e)\right)}{7}+\frac{2ab\left(\tan^5(fx+e)\right)}{5}-\frac{b^2\left(\tan^5(fx+e)\right)}{5}+\frac{a^2\left(\tan^3(fx+e)\right)}{3}-\frac{2ab\left(\tan^3(fx+e)\right)}{3}+\frac{b^2\left(\tan^3(fx+e)\right)}{3}-a^2\tan(fx+e}{f}$
risch	$x\,a^2-2xab+x\,b^2-\frac{4i(105a^2e^{12i(fx+e)}-315ab\,e^{12i(fx+e)}+210b^2e^{12i(fx+e)}+525a^2e^{10i(fx+e)}-1260ab\,e^{10i(f}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/7*b^2*tan(f*x+e)^7+2/5*a*b*tan(f*x+e)^5-1/5*b^2*tan(f*x+e)^5+1/3*a^2
*tan(f*x+e)^3-2/3*a*b*tan(f*x+e)^3+1/3*b^2*tan(f*x+e)^3-a^2*tan(f*x+e)+2*a*
b*tan(f*x+e)-b^2*tan(f*x+e)+(a^2-2*a*b+b^2)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.49, size = 102, normalized size = 1.12

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\,\left(2\,ab-b^2\right)\tan\left(fx+e\right)^5+35\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3+105\,\left(a^2-2\,ab+b^2\right)\left(fx+e\right)-105\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b - b^2)*tan(f*x + e)^5 + 35*(a^2 -
2*a*b + b^2)*tan(f*x + e)^3 + 105*(a^2 - 2*a*b + b^2)*(f*x + e) - 105*(a^2
- 2*a*b + b^2)*tan(f*x + e))/f
```

Fricas [A]
time = 3.10, size = 98, normalized size = 1.08

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\,\left(2\,ab-b^2\right)\tan\left(fx+e\right)^5+35\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3+105\,\left(a^2-2\,ab+b^2\right)fx-105\,\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b - b^2)*tan(f*x + e)^5 + 35*(a^2 -
2*a*b + b^2)*tan(f*x + e)^3 + 105*(a^2 - 2*a*b + b^2)*f*x - 105*(a^2 - 2*a*
b + b^2)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(71) = 142$.
time = 0.22, size = 165, normalized size = 1.81

$$\begin{cases} a^2x+\frac{a^2\tan^3{(e+fx)}}{3f}-\frac{a^2\tan{(e+fx)}}{f}-2abx+\frac{2ab\tan^5{(e+fx)}}{5f}-\frac{2ab\tan^3{(e+fx)}}{3f}+\frac{2ab\tan{(e+fx)}}{f}+b^2x+\frac{b^2\tan^7{(e+fx)}}{7f}-\frac{b^2\tan^5{(e+fx)}}{5f}+\frac{b^2\tan^3{(e+fx)}}{3f}-\frac{b^2\tan{(e+fx)}}{f} & \text{for}\ f\neq 0 \\ x\left(a+b\tan^2{(e)}\right)^2\tan^4{(e)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + a**2*tan(e + f*x)**3/(3*f) - a**2*tan(e + f*x)/f - 2*a*
b*x + 2*a*b*tan(e + f*x)**5/(5*f) - 2*a*b*tan(e + f*x)**3/(3*f) + 2*a*b*tan
(e + f*x)/f + b**2*x + b**2*tan(e + f*x)**7/(7*f) - b**2*tan(e + f*x)**5/(5
*f) + b**2*tan(e + f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a +
b*tan(e)**2)**2*tan(e)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1684 vs. $2(89) = 178$.
time = 2.45, size = 1684, normalized size = 18.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^2*f*x*tan(f*x)^7*tan(e)^7 - 210*a*b*f*x*tan(f*x)^7*tan(e)^7 +
105*b^2*f*x*tan(f*x)^7*tan(e)^7 - 735*a^2*f*x*tan(f*x)^6*tan(e)^6 + 1470*a*
b*f*x*tan(f*x)^6*tan(e)^6 - 735*b^2*f*x*tan(f*x)^6*tan(e)^6 + 105*a^2*tan(f
*x)^7*tan(e)^6 - 210*a*b*tan(f*x)^7*tan(e)^6 + 105*b^2*tan(f*x)^7*tan(e)^6
+ 105*a^2*tan(f*x)^6*tan(e)^7 - 210*a*b*tan(f*x)^6*tan(e)^7 + 105*b^2*tan(f
*x)^6*tan(e)^7 + 2205*a^2*f*x*tan(f*x)^5*tan(e)^5 - 4410*a*b*f*x*tan(f*x)^5
*tan(e)^5 + 2205*b^2*f*x*tan(f*x)^5*tan(e)^5 - 35*a^2*tan(f*x)^7*tan(e)^4 +
 70*a*b*tan(f*x)^7*tan(e)^4 - 35*b^2*tan(f*x)^7*tan(e)^4 - 735*a^2*tan(f*x)
^6*tan(e)^5 + 1470*a*b*tan(f*x)^6*tan(e)^5 - 735*b^2*tan(f*x)^6*tan(e)^5 -
735*a^2*tan(f*x)^5*tan(e)^6 + 1470*a*b*tan(f*x)^5*tan(e)^6 - 735*b^2*tan(f*
x)^5*tan(e)^6 - 35*a^2*tan(f*x)^4*tan(e)^7 + 70*a*b*tan(f*x)^4*tan(e)^7 - 3
5*b^2*tan(f*x)^4*tan(e)^7 - 3675*a^2*f*x*tan(f*x)^4*tan(e)^4 + 7350*a*b*f*x
*tan(f*x)^4*tan(e)^4 - 3675*b^2*f*x*tan(f*x)^4*tan(e)^4 - 42*a*b*tan(f*x)^7
*tan(e)^2 + 21*b^2*tan(f*x)^7*tan(e)^2 + 140*a^2*tan(f*x)^6*tan(e)^3 - 490*
a*b*tan(f*x)^6*tan(e)^3 + 245*b^2*tan(f*x)^6*tan(e)^3 + 1995*a^2*tan(f*x)^5
*tan(e)^4 - 4410*a*b*tan(f*x)^5*tan(e)^4 + 2205*b^2*tan(f*x)^5*tan(e)^4 + 1
995*a^2*tan(f*x)^4*tan(e)^5 - 4410*a*b*tan(f*x)^4*tan(e)^5 + 2205*b^2*tan(f
*x)^4*tan(e)^5 + 140*a^2*tan(f*x)^3*tan(e)^6 - 490*a*b*tan(f*x)^3*tan(e)^6
+ 245*b^2*tan(f*x)^3*tan(e)^6 - 42*a*b*tan(f*x)^2*tan(e)^7 + 21*b^2*tan(f*x
)^2*tan(e)^7 + 3675*a^2*f*x*tan(f*x)^3*tan(e)^3 - 7350*a*b*f*x*tan(f*x)^3*t
an(e)^3 + 3675*b^2*f*x*tan(f*x)^3*tan(e)^3 - 15*b^2*tan(f*x)^7 + 84*a*b*tan
(f*x)^6*tan(e) - 147*b^2*tan(f*x)^6*tan(e) - 210*a^2*tan(f*x)^5*tan(e)^2 +
840*a*b*tan(f*x)^5*tan(e)^2 - 735*b^2*tan(f*x)^5*tan(e)^2 - 2730*a^2*tan(f*
x)^4*tan(e)^3 + 6300*a*b*tan(f*x)^4*tan(e)^3 - 3675*b^2*tan(f*x)^4*tan(e)^3
 - 2730*a^2*tan(f*x)^3*tan(e)^4 + 6300*a*b*tan(f*x)^3*tan(e)^4 - 3675*b^2*t
an(f*x)^3*tan(e)^4 - 210*a^2*tan(f*x)^2*tan(e)^5 + 840*a*b*tan(f*x)^2*tan(e
)^5 - 735*b^2*tan(f*x)^2*tan(e)^5 + 84*a*b*tan(f*x)*tan(e)^6 - 147*b^2*tan(
f*x)*tan(e)^6 - 15*b^2*tan(e)^7 - 2205*a^2*f*x*tan(f*x)^2*tan(e)^2 + 4410*a
*b*f*x*tan(f*x)^2*tan(e)^2 - 2205*b^2*f*x*tan(f*x)^2*tan(e)^2 - 42*a*b*tan(
f*x)^5 + 21*b^2*tan(f*x)^5 + 140*a^2*tan(f*x)^4*tan(e) - 490*a*b*tan(f*x)^4
*tan(e) + 245*b^2*tan(f*x)^4*tan(e) + 1995*a^2*tan(f*x)^3*tan(e)^2 - 4410*a
*b*tan(f*x)^3*tan(e)^2 + 2205*b^2*tan(f*x)^3*tan(e)^2 + 1995*a^2*tan(f*x)^2
*tan(e)^3 - 4410*a*b*tan(f*x)^2*tan(e)^3 + 2205*b^2*tan(f*x)^2*tan(e)^3 + 1
40*a^2*tan(f*x)*tan(e)^4 - 490*a*b*tan(f*x)*tan(e)^4 + 245*b^2*tan(f*x)*tan
(e)^4 - 42*a*b*tan(e)^5 + 21*b^2*tan(e)^5 + 735*a^2*f*x*tan(f*x)*tan(e) - 1
470*a*b*f*x*tan(f*x)*tan(e) + 735*b^2*f*x*tan(f*x)*tan(e) - 35*a^2*tan(f*x)
^3 + 70*a*b*tan(f*x)^3 - 35*b^2*tan(f*x)^3 - 735*a^2*tan(f*x)^2*tan(e) + 14
70*a*b*tan(f*x)^2*tan(e) - 735*b^2*tan(f*x)^2*tan(e) - 735*a^2*tan(f*x)*tan
(e)^2 + 1470*a*b*tan(f*x)*tan(e)^2 - 735*b^2*tan(f*x)*tan(e)^2 - 35*a^2*tan
(e)^3 + 70*a*b*tan(e)^3 - 35*b^2*tan(e)^3 - 105*a^2*f*x + 210*a*b*f*x - 105
*b^2*f*x + 105*a^2*tan(f*x) - 210*a*b*tan(f*x) + 105*b^2*tan(f*x) + 105*a^2
```

```
*tan(e) - 210*a*b*tan(e) + 105*b^2*tan(e))/(f*tan(f*x)^7*tan(e)^7 - 7*f*tan
(f*x)^6*tan(e)^6 + 21*f*tan(f*x)^5*tan(e)^5 - 35*f*tan(f*x)^4*tan(e)^4 + 35
*f*tan(f*x)^3*tan(e)^3 - 21*f*tan(f*x)^2*tan(e)^2 + 7*f*tan(f*x)*tan(e) - f
)
```

Mupad [B]
time = 12.13, size = 127, normalized size = 1.40

$$\frac{\operatorname{atan}\left(\frac{\tan(e+f\,x)\,{(a-b)}^2}{a^2-2\,a\,b+b^2}\right)\,{(a-b)}^2}{f}+\frac{{\tan(e+f\,x)}^5\,\left(\frac{2\,a\,b}{5}-\frac{b^2}{5}\right)}{f}-\frac{\tan(e+f\,x)\,\left(a^2-2\,a\,b+b^2\right)}{f}+\frac{b^2\,{\tan(e+f\,x)}^7}{7\,f}+\frac{{\tan(e+f\,x)}^3\,\left(\frac{a^2}{3}-\frac{2\,a\,b}{3}+\frac{b^2}{3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*b + b^2))*(a - b)^2)/f + (tan(e +
 f*x)^5*((2*a*b)/5 - b^2/5))/f - (tan(e + f*x)*(a^2 - 2*a*b + b^2))/f + (b^
2*tan(e + f*x)^7)/(7*f) + (tan(e + f*x)^3*(a^2/3 - (2*a*b)/3 + b^2/3))/f
```

3.206 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=69

$$-(a-b)^2x+\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(2a-b)b\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] -(a-b)^2*x+(a-b)^2*tan(f*x+e)/f+1/3*(2*a-b)*b*tan(f*x+e)^3/f+1/5*b^2*tan(f*
x+e)^5/f
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 472, 209}

$$\frac{b(2a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)^2\tan(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) + ((a - b)^2*Tan[e + f*x])/f + ((2*a - b)*b*Tan[e + f*x]^3)/
(3*f) + (b^2*Tan[e + f*x]^5)/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left((a-b)^2+(2a-b)bx^2+b^2x^4+\frac{-a^2+2ab-b^2}{1+x^2}\right)\,dx, x, \tan\right.}{f}$$

$$= \frac{(a-b)^2\tan(e+fx)}{f}+\frac{(2a-b)b\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx}{5f}$$

$$= -(a-b)^2x+\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(2a-b)b\tan^3(e+fx)}{3f}+$$

Mathematica [A]

time = 0.04, size = 137, normalized size = 1.99

$$-\frac{a^2\text{ArcTan}(\tan(e+fx))}{f}+\frac{2ab\text{ArcTan}(\tan(e+fx))}{f}-\frac{b^2\text{ArcTan}(\tan(e+fx))}{f}+\frac{a^2\tan(e+fx)}{f}-\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}+\frac{2ab\tan^3(e+fx)}{3f}-\frac{b^2\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTan[Tan[e + f*x]])/f) + (2*a*b*ArcTan[Tan[e + f*x]])/f - (b^2*Arc
Tan[Tan[e + f*x]])/f + (a^2*Tan[e + f*x])/f - (2*a*b*Tan[e + f*x])/f + (b^2
*Tan[e + f*x])/f + (2*a*b*Tan[e + f*x]^3)/(3*f) - (b^2*Tan[e + f*x]^3)/(3*f
) + (b^2*Tan[e + f*x]^5)/(5*f)
```

Maple [A]

time = 0.05, size = 97, normalized size = 1.41

method	result
norman	$\left(-a^2+2ab-b^2\right)x+\frac{(a^2-2ab+b^2)\tan(fx+e)}{f}+\frac{b^2\left(\tan^5(fx+e)\right)}{5f}+\frac{(2a-b)b\left(\tan^3(fx+e)\right)}{3f}$
derivativedivides	$\frac{\frac{b^2\left(\tan^5(fx+e)\right)}{5}+\frac{2ab\left(\tan^3(fx+e)\right)}{3}-\frac{b^2\left(\tan^3(fx+e)\right)}{3}+a^2\tan(fx+e)-2ab\tan(fx+e)+b^2\tan(fx+e)+\left(-a^2+2ab-b^2\right)\text{arcta}}{f}$
default	$\frac{\frac{b^2\left(\tan^5(fx+e)\right)}{5}+\frac{2ab\left(\tan^3(fx+e)\right)}{3}-\frac{b^2\left(\tan^3(fx+e)\right)}{3}+a^2\tan(fx+e)-2ab\tan(fx+e)+b^2\tan(fx+e)+\left(-a^2+2ab-b^2\right)\text{arcta}}{f}$
risch	$-x\,a^2+2xab-x\,b^2+\frac{2i\left(15a^2e^{8i(fx+e)}-60ab\,e^{8i(fx+e)}+45b^2e^{8i(fx+e)}+60a^2e^{6i(fx+e)}-180ab\,e^{6i(fx+e)}+90b^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/5*b^2*tan(f*x+e)^5+2/3*a*b*tan(f*x+e)^3-1/3*b^2*tan(f*x+e)^3+a^2*tan
(f*x+e)-2*a*b*tan(f*x+e)+b^2*tan(f*x+e)+(-a^2+2*a*b-b^2)*arctan(tan(f*x+e))
)
```

Maxima [A]
time = 0.49, size = 80, normalized size = 1.16

$$\frac{3\,b^2\tan\left(fx+e\right)^5+5\,(2\,ab-b^2)\tan\left(fx+e\right)^3-15\,(a^2-2\,ab+b^2)(fx+e)+15\,(a^2-2\,ab+b^2)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b - b^2)*tan(f*x + e)^3 - 15*(a^2 - 2*a
*b + b^2)*(f*x + e) + 15*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Fricas [A]
time = 2.37, size = 76, normalized size = 1.10

$$\frac{3\,b^2\tan\left(fx+e\right)^5+5\,(2\,ab-b^2)\tan\left(fx+e\right)^3-15\,(a^2-2\,ab+b^2)fx+15\,(a^2-2\,ab+b^2)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b - b^2)*tan(f*x + e)^3 - 15*(a^2 - 2*a
*b + b^2)*f*x + 15*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(54) = 108$.
time = 0.15, size = 117, normalized size = 1.70

$$\begin{cases} -a^2x+\frac{a^2\tan\left(e+fx\right)}{f}+2abx+\frac{2ab\tan^3\left(e+fx\right)}{3f}-\frac{2ab\tan\left(e+fx\right)}{f}-b^2x+\frac{b^2\tan^5\left(e+fx\right)}{5f}-\frac{b^2\tan^3\left(e+fx\right)}{3f}+\frac{b^2\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0 \\ x(a+b\tan^2\left(e\right))^2\tan^2\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((-a**2*x + a**2*tan(e + f*x)/f + 2*a*b*x + 2*a*b*tan(e + f*x)**3/
(3*f) - 2*a*b*tan(e + f*x)/f - b**2*x + b**2*tan(e + f*x)**5/(5*f) - b**2*t
an(e + f*x)**3/(3*f) + b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)
**2*tan(e)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(68) = 136$.
time = 1.27, size = 937, normalized size = 13.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/15*(15*a^2*f*x*tan(f*x)^5*tan(e)^5 - 30*a*b*f*x*tan(f*x)^5*tan(e)^5 + 15
*b^2*f*x*tan(f*x)^5*tan(e)^5 - 75*a^2*f*x*tan(f*x)^4*tan(e)^4 + 150*a*b*f*x
*tan(f*x)^4*tan(e)^4 - 75*b^2*f*x*tan(f*x)^4*tan(e)^4 + 15*a^2*tan(f*x)^5*t
an(e)^4 - 30*a*b*tan(f*x)^5*tan(e)^4 + 15*b^2*tan(f*x)^5*tan(e)^4 + 15*a^2*
tan(f*x)^4*tan(e)^5 - 30*a*b*tan(f*x)^4*tan(e)^5 + 15*b^2*tan(f*x)^4*tan(e)
^5 + 150*a^2*f*x*tan(f*x)^3*tan(e)^3 - 300*a*b*f*x*tan(f*x)^3*tan(e)^3 + 15
0*b^2*f*x*tan(f*x)^3*tan(e)^3 + 10*a*b*tan(f*x)^5*tan(e)^2 - 5*b^2*tan(f*x)
^5*tan(e)^2 - 60*a^2*tan(f*x)^4*tan(e)^3 + 150*a*b*tan(f*x)^4*tan(e)^3 - 75
*b^2*tan(f*x)^4*tan(e)^3 - 60*a^2*tan(f*x)^3*tan(e)^4 + 150*a*b*tan(f*x)^3*
tan(e)^4 - 75*b^2*tan(f*x)^3*tan(e)^4 + 10*a*b*tan(f*x)^2*tan(e)^5 - 5*b^2*
tan(f*x)^2*tan(e)^5 - 150*a^2*f*x*tan(f*x)^2*tan(e)^2 + 300*a*b*f*x*tan(f*x
)^2*tan(e)^2 - 150*b^2*f*x*tan(f*x)^2*tan(e)^2 + 3*b^2*tan(f*x)^5 - 20*a*b*
tan(f*x)^4*tan(e) + 25*b^2*tan(f*x)^4*tan(e) + 90*a^2*tan(f*x)^3*tan(e)^2 -
 240*a*b*tan(f*x)^3*tan(e)^2 + 150*b^2*tan(f*x)^3*tan(e)^2 + 90*a^2*tan(f*x
)^2*tan(e)^3 - 240*a*b*tan(f*x)^2*tan(e)^3 + 150*b^2*tan(f*x)^2*tan(e)^3 -
20*a*b*tan(f*x)*tan(e)^4 + 25*b^2*tan(f*x)*tan(e)^4 + 3*b^2*tan(e)^5 + 75*a
^2*f*x*tan(f*x)*tan(e) - 150*a*b*f*x*tan(f*x)*tan(e) + 75*b^2*f*x*tan(f*x)*
tan(e) + 10*a*b*tan(f*x)^3 - 5*b^2*tan(f*x)^3 - 60*a^2*tan(f*x)^2*tan(e) +
150*a*b*tan(f*x)^2*tan(e) - 75*b^2*tan(f*x)^2*tan(e) - 60*a^2*tan(f*x)*tan(
e)^2 + 150*a*b*tan(f*x)*tan(e)^2 - 75*b^2*tan(f*x)*tan(e)^2 + 10*a*b*tan(e)
^3 - 5*b^2*tan(e)^3 - 15*a^2*f*x + 30*a*b*f*x - 15*b^2*f*x + 15*a^2*tan(f*x
) - 30*a*b*tan(f*x) + 15*b^2*tan(f*x) + 15*a^2*tan(e) - 30*a*b*tan(e) + 15*
b^2*tan(e))/(f*tan(f*x)^5*tan(e)^5 - 5*f*tan(f*x)^4*tan(e)^4 + 10*f*tan(f*x
)^3*tan(e)^3 - 10*f*tan(f*x)^2*tan(e)^2 + 5*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 11.67, size = 100, normalized size = 1.45

$$\frac{\tan(e+f\,x)^3\,\left(\frac{2\,a\,b}{3}-\frac{b^2}{3}\right)}{f}-\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,(a-b)^2}{a^2-2\,a\,b+b^2}\right)\,(a-b)^2}{f}+\frac{\tan(e+f\,x)\,\left(a^2-2\,a\,b+b^2\right)}{f}+\frac{b^2\,\tan(e+f\,x)^5}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)^3*((2*a*b)/3 - b^2/3))/f - (atan((tan(e + f*x)*(a - b)^2)/(a^
2 - 2*a*b + b^2))*(a - b)^2)/f + (tan(e + f*x)*(a^2 - 2*a*b + b^2))/f + (b^
2*tan(e + f*x)^5)/(5*f)
```

3.207 $\int (a+b\tan^2(e+fx))^2 \, dx$

Optimal. Leaf size=46

$$(a-b)^2x+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] (a-b)^2*x+(2*a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 398, 209}

$$\frac{b(2a-b)\tan(e+fx)}{f}+x(a-b)^2+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e\right.}{f}$$
$$= (a-b)^2x+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.38, size = 73, normalized size = 1.59

$$\frac{\tan(e+fx)\left(\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(e+fx)}\right)}{\sqrt{-\tan^2(e+fx)}}+b(6a-b(3-\tan^2(e+fx)))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (Tan[e + f*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[e + f*x]^2]])/Sqrt[-Tan[e + f
*x]^2] + b*(6*a - b*(3 - Tan[e + f*x]^2))))/(3*f)
```

Maple [A]
time = 0.00, size = 59, normalized size = 1.28

method	result	size
norman	$\left(a^2-2ab+b^2\right)x+\frac{(2a-b)b\tan(fx+e)}{f}+\frac{b^2\left(\tan^3(fx+e)\right)}{3f}$	49
derivativedivides	$\frac{\frac{b^2\left(\tan^3(fx+e)\right)}{3}+2ab\tan(fx+e)-b^2\tan(fx+e)+\left(a^2-2ab+b^2\right)\arctan(\tan(fx+e))}{f}$	59
default	$\frac{\frac{b^2\left(\tan^3(fx+e)\right)}{3}+2ab\tan(fx+e)-b^2\tan(fx+e)+\left(a^2-2ab+b^2\right)\arctan(\tan(fx+e))}{f}$	59
risch	$x\,a^2-2xab+x\,b^2-\frac{4ib\left(-3a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}+3b\,e^{2i(fx+e)}-3a+2b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/f*(1/3*b^2*tan(f*x+e)^3+2*a*b*tan(f*x+e)-b^2*tan(f*x+e)+(a^2-2*a*b+b^2)*a
rctan(tan(f*x+e)))

Maxima [A]
time = 0.49, size = 63, normalized size = 1.37

$$a^2x - \frac{2\,(fx+e-\tan{(fx+e)})ab}{f} + \frac{\left(\tan{(fx+e)}^3 + 3\,fx + 3\,e - 3\,\tan{(fx+e)}\right)b^2}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")

[Out] a^2*x - 2*(f*x + e - tan(f*x + e))*a*b/f + 1/3*(tan(f*x + e)^3 + 3*f*x + 3*
e - 3*tan(f*x + e))*b^2/f

Fricas [A]
time = 4.13, size = 53, normalized size = 1.15

$$\frac{b^2\tan{(fx+e)}^3 + 3\,(a^2-2\,ab+b^2)fx + 3\,(2\,ab-b^2)\tan{(fx+e)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")

[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*x + 3*(2*a*b - b^2)*tan(f
*x + e))/f

Sympy [A]
time = 0.10, size = 68, normalized size = 1.48

$$\begin{cases} a^2x - 2abx + \frac{2ab\tan{(e+fx)}}{f} + b^2x + \frac{b^2\tan^3{(e+fx)}}{3f} - \frac{b^2\tan{(e+fx)}}{f} & \text{for}\ f \neq 0 \\ x(a+b\tan^2{(e)})^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e)**2)**2,x)

[Out] Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(e + f*x)/f + b**2*x + b**2*tan(e +
f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)**2, Tr
ue))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(46) = 92.
time = 0.64, size = 382, normalized size = 8.30

$$\frac{3\,a^2fx\tan{(fx)}^3\tan{(e)}^3 - 6\,abfx\tan{(fx)}^3\tan{(e)}^3 + 3\,b^2fx\tan{(fx)}^3\tan{(e)}^3 - 9\,a^2fx\tan{(fx)}^2\tan{(e)}^2 + 18\,abfx\tan{(fx)}^2\tan{(e)}^2 - 9\,b^2fx\tan{(fx)}^2\tan{(e)}^2 - 6\,ab\tan{(fx)}^3\tan{(e)}^2 + 3\,b^2\tan{(fx)}^3\tan{(e)}^2 - 6\,ab\tan{(fx)}^2\tan{(e)}^3 + 3\,b^2\tan{(fx)}^2\tan{(e)}^3 + 9\,a^2fx\tan{(fx)}\tan{(e)} - 18\,abfx\tan{(fx)}\tan{(e)} + 9\,b^2fx\tan{(fx)}\tan{(e)} - b^2\tan{(fx)}^3 + 12\,ab\tan{(fx)}^2\tan{(e)} - 9\,b^2\tan{(fx)}^2\tan{(e)} + 12\,ab\tan{(fx)}\tan{(e)}^2 - 9\,b^2\tan{(fx)}\tan{(e)}^2 - b^2\tan{(e)}^3 - 3\,a^2fx + 6\,abfx - 3\,b^2fx - 6\,ab\tan{(fx)} + 3\,b^2\tan{(fx)} - 6\,ab\tan{(e)} + 3\,b^2\tan{(e)}}{3\,\left(f\tan{(fx)}^3\tan{(e)}^3 - 3\,f\tan{(fx)}^2\tan{(e)}^2 + 3\,f\tan{(fx)}\tan{(e)} - f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*f*x*tan(f*x)^3*tan(e)^3 - 6*a*b*f*x*tan(f*x)^3*tan(e)^3 + 3*b^2*
f*x*tan(f*x)^3*tan(e)^3 - 9*a^2*f*x*tan(f*x)^2*tan(e)^2 + 18*a*b*f*x*tan(f*
x)^2*tan(e)^2 - 9*b^2*f*x*tan(f*x)^2*tan(e)^2 - 6*a*b*tan(f*x)^3*tan(e)^2 +
 3*b^2*tan(f*x)^3*tan(e)^2 - 6*a*b*tan(f*x)^2*tan(e)^3 + 3*b^2*tan(f*x)^2*t
an(e)^3 + 9*a^2*f*x*tan(f*x)*tan(e) - 18*a*b*f*x*tan(f*x)*tan(e) + 9*b^2*f*
x*tan(f*x)*tan(e) - b^2*tan(f*x)^3 + 12*a*b*tan(f*x)^2*tan(e) - 9*b^2*tan(f
*x)^2*tan(e) + 12*a*b*tan(f*x)*tan(e)^2 - 9*b^2*tan(f*x)*tan(e)^2 - b^2*tan
(e)^3 - 3*a^2*f*x + 6*a*b*f*x - 3*b^2*f*x - 6*a*b*tan(f*x) + 3*b^2*tan(f*x)
 - 6*a*b*tan(e) + 3*b^2*tan(e))/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*tan
(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 12.02, size = 76, normalized size = 1.65

$$\frac{\tan(e+f\,x)\,\left(2\,a\,b-b^2\right)}{f}+\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,{(a-b)}^2}{a^2-2\,a\,b+b^2}\right)\,{(a-b)}^2}{f}+\frac{b^2\,{\tan(e+f\,x)}^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (tan(e + f*x)*(2*a*b - b^2))/f + (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*
b + b^2))*(a - b)^2)/f + (b^2*tan(e + f*x)^3)/(3*f)
```

3.208 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=38

$$-(a-b)^2x-\frac{a^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] -(a-b)^2*x-a^2*cot(f*x+e)/f+b^2*tan(f*x+e)/f
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3751, 472, 209}

$$-\frac{a^2\cot(e+fx)}{f}-x(a-b)^2+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) - (a^2*Cot[e + f*x])/f + (b^2*Tan[e + f*x])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^2(1+x^2)} \, dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2}{x^2}-\frac{(a-b)^2}{1+x^2}\right) \, dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}-\frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x,$$
$$= -(a-b)^2x-\frac{a^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 66, normalized size = 1.74

$$2abx-\frac{b^2\text{ArcTan}(\tan(e+fx))}{f}-\frac{a^2\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f}+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] 2*a*b*x - (b^2*ArcTan[Tan[e + f*x]])/f - (a^2*Cot[e + f*x]*Hypergeometric2F
1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f + (b^2*Tan[e + f*x])/f
```

Maple [A]
time = 0.11, size = 53, normalized size = 1.39

method	result	size
derivativedivides	$\frac{b^2(\tan(fx+e)-fx-e)+2ab(fx+e)+a^2(-\cot(fx+e)-fx-e)}{f}$	53
default	$\frac{b^2(\tan(fx+e)-fx-e)+2ab(fx+e)+a^2(-\cot(fx+e)-fx-e)}{f}$	53
norman	$\frac{\frac{b^2\left(\tan^2(fx+e)\right)}{f}+(-a^2+2ab-b^2)x\tan(fx+e)-\frac{a^2}{f}}{\tan(fx+e)}$	57
risch	$-x\,a^2+2xab-x\,b^2-\frac{2i\left(a^2e^{2i(fx+e)}-b^2e^{2i(fx+e)}+a^2+b^2\right)}{f\left(e^{2i(fx+e)}-1\right)\left(e^{2i(fx+e)}+1\right)}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(tan(f*x+e)-f*x-e)+2*a*b*(f*x+e)+a^2*(-cot(f*x+e)-f*x-e))
```

Maxima [A]
time = 0.48, size = 49, normalized size = 1.29

$$\frac{b^2\tan(fx+e)-(a^2-2\,ab+b^2)(fx+e)-\frac{a^2}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] (b^2*tan(f*x + e) - (a^2 - 2*a*b + b^2)*(f*x + e) - a^2/tan(f*x + e))/f
```

Fricas [A]
time = 3.42, size = 53, normalized size = 1.39

$$-\frac{(a^2-2\,ab+b^2)fx\tan(fx+e)-b^2\tan(fx+e)^2+a^2}{f\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -((a^2 - 2*a*b + b^2)*f*x*tan(f*x + e) - b^2*tan(f*x + e)^2 + a^2)/(f*tan(f
*x + e))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.
time = 0.98, size = 73, normalized size = 1.92

$$\begin{cases} \tilde{\infty}a^2x & \text{for } (e=0\vee e=-fx)\wedge(e=-fx\vee f=0) \\ x\left(a+b\tan^2(e)\right)^2\cot^2(e) & \text{for } f=0 \\ -a^2x-\frac{a^2}{f\tan(e+fx)}+2abx-b^2x+\frac{b^2\tan(e+fx)}{f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x)))
, (x*(a + b*tan(e)**2)**2*cot(e)**2, Eq(f, 0)), (-a**2*x - a**2/(f*tan(e +
f*x)) + 2*a*b*x - b**2*x + b**2*tan(e + f*x)/f, True))
```

Giac [A]
time = 0.98, size = 49, normalized size = 1.29

$$\frac{b^2\tan(fx+e)-(a^2-2\,ab+b^2)(fx+e)-\frac{a^2}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] (b^2*tan(f*x + e) - (a^2 - 2*a*b + b^2)*(f*x + e) - a^2/tan(f*x + e))/f
```

Mupad [B]
time = 12.02, size = 70, normalized size = 1.84

$$\frac{b^2\tan(e+f\,x)}{f}-\frac{\operatorname{atan}\left(\frac{\tan(e+f\,x)\,(a-b)^2}{a^2-2\,a\,b+b^2}\right)\,(a-b)^2}{f}-\frac{a^2}{f\tan\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (b^2*tan(e + f*x))/f - (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*b + b^2))*
(a - b)^2)/f - a^2/(f*tan(e + f*x))
```

3.209 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=44

$$(a-b)^2x+\frac{a(a-2b)\cot(e+fx)}{f}-\frac{a^2\cot^3(e+fx)}{3f}$$

```
[Out] (a-b)^2*x+a*(a-2*b)*cot(f*x+e)/f-1/3*a^2*cot(f*x+e)^3/f
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3751, 472, 209}

$$-\frac{a^2\cot^3(e+fx)}{3f}+\frac{a(a-2b)\cot(e+fx)}{f}+x(a-b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + (a*(a - 2*b)*Cot[e + f*x])/f - (a^2*Cot[e + f*x]^3)/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4} - \frac{a(a-2b)}{x^2} + \frac{(a-b)^2}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a(a-2b)\cot(e+fx)}{f} - \frac{a^2\cot^3(e+fx)}{3f} + \frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{f}$$

$$= (a-b)^2x + \frac{a(a-2b)\cot(e+fx)}{f} - \frac{a^2\cot^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.85, size = 71, normalized size = 1.61

$$-\frac{\cot(e+fx)\left(a(-3a+6b+a\cot^2(e+fx)) + 3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(e+fx)}\right)\sqrt{-\tan^2(e+fx)}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/3*(Cot[e + f*x]*(a*(-3*a + 6*b + a*Cot[e + f*x]^2) + 3*(a - b)^2*ArcTanh
[Sqrt[-Tan[e + f*x]^2]]*Sqrt[-Tan[e + f*x]^2]))/f
```

Maple [A]
time = 0.13, size = 60, normalized size = 1.36

method	result	size
norman	$\frac{(a^2-2ab+b^2)x(\tan^3(fx+e))+\frac{a(a-2b)\left(\tan^2(fx+e)\right)}{f}-\frac{a^2}{3f}}{\tan(fx+e)^3}$	58
derivativedivides	$\frac{b^2(fx+e)+2ab(-\cot(fx+e)-fx-e)+a^2\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)}{f}$	60
default	$\frac{b^2(fx+e)+2ab(-\cot(fx+e)-fx-e)+a^2\left(-\frac{\left(\cot^3(fx+e)\right)}{3}+\cot(fx+e)+fx+e\right)}{f}$	60
risch	$x\,a^2-2xab+x\,b^2+\frac{4ia\left(3a\,e^{4i(fx+e)}-3b\,e^{4i(fx+e)}-3a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}+2a-3b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(f*x+e)+2*a*b*(-cot(f*x+e)-f*x-e)+a^2*(-1/3*cot(f*x+e)^3+cot(f*x+e
)+f*x+e))
```

Maxima [A]
time = 0.50, size = 60, normalized size = 1.36

$$\frac{3\,(a^2-2\,ab+b^2)(fx+e)+\frac{3\,(a^2-2\,ab)\,\tan(fx+e)^2-a^2}{\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2 - 2*a*b + b^2)*(f*x + e) + (3*(a^2 - 2*a*b)*tan(f*x + e)^2 - a^
2)/tan(f*x + e)^3)/f
```

Fricas [A]
time = 5.89, size = 63, normalized size = 1.43

$$\frac{3\,(a^2-2\,ab+b^2)fx\tan\left(fx+e\right)^3+3\,(a^2-2\,ab)\tan\left(fx+e\right)^2-a^2}{3\,f\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2 - 2*a*b + b^2)*f*x*tan(f*x + e)^3 + 3*(a^2 - 2*a*b)*tan(f*x + e
)^2 - a^2)/(f*tan(f*x + e)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(36) = 72.
time = 2.70, size = 90, normalized size = 2.05

$$\begin{cases} \tilde{\infty}a^2x & \text{for } (e=0\vee e=-fx)\wedge(e=-fx\vee f=0)\\ x(a+b\tan^2(e))^2\cot^4(e) & \text{for } f=0\\ a^2x+\frac{a^2}{f\tan(e+fx)}-\frac{a^2}{3f\tan^3(e+fx)}-2abx-\frac{2ab}{f\tan(e+fx)}+b^2x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x)))
, (x*(a + b*tan(e)**2)**2*cot(e)**4, Eq(f, 0)), (a**2*x + a**2/(f*tan(e + f
*x)) - a**2/(3*f*tan(e + f*x)**3) - 2*a*b*x - 2*a*b/(f*tan(e + f*x)) + b**2
*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(44) = 88.
time = 1.26, size = 122, normalized size = 2.77

$$\frac{a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\,(a^2-2\,ab+b^2)(fx+e)+\frac{15\,a^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-24\,ab\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-a^2}{\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 - 15*a^2*tan(1/2*f*x + 1/2*e) + 24*a*b*tan
(1/2*f*x + 1/2*e) + 24*(a^2 - 2*a*b + b^2)*(f*x + e) + (15*a^2*tan(1/2*f*x
+ 1/2*e)^2 - 24*a*b*tan(1/2*f*x + 1/2*e)^2 - a^2)/tan(1/2*f*x + 1/2*e)^3)/f
```

Mupad [B]
time = 11.61, size = 58, normalized size = 1.32

$$a^2\,x+b^2\,x+\frac{a^2\cot(e+f\,x)}{f}-2\,a\,b\,x-\frac{a^2\cot(e+f\,x)^3}{3\,f}-\frac{2\,a\,b\cot(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] a^2*x + b^2*x + (a^2*cot(e + f*x))/f - 2*a*b*x - (a^2*cot(e + f*x)^3)/(3*f)
 - (2*a*b*cot(e + f*x))/f
```

3.210 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=68

$$-(a-b)^2x-\frac{(a-b)^2\cot(e+fx)}{f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-\frac{a^2\cot^5(e+fx)}{5f}$$

```
[Out] -(a-b)^2*x-(a-b)^2*cot(f*x+e)/f+1/3*a*(a-2*b)*cot(f*x+e)^3/f-1/5*a^2*cot(f*
x+e)^5/f
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 472, 209}

$$-\frac{a^2\cot^5(e+fx)}{5f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-\frac{(a-b)^2\cot(e+fx)}{f}-x(a-b)^2$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -((a - b)^2*x) - ((a - b)^2*Cot[e + f*x])/f + (a*(a - 2*b)*Cot[e + f*x]^3)/
(3*f) - (a^2*Cot[e + f*x]^5)/(5*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^6}-\frac{a(a-2b)}{x^4}+\frac{(a-b)^2}{x^2}-\frac{(a-b)^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a-b)^2\cot(e+fx)}{f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-\frac{a^2\cot^5(e+f$$
$$= -(a-b)^2x-\frac{(a-b)^2\cot(e+fx)}{f}+\frac{a(a-2b)\cot^3(e+fx)}{3f}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 104, normalized size = 1.53

$$-\frac{a^2\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(e+fx)\right)}{5f}-\frac{2ab\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(e+fx)\right)}{3f}-\frac{b^2\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/5*(a^2*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])
/f - (2*a*b*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2
])/(3*f) - (b^2*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^
2])/f
```

Maple [A]
time = 0.15, size = 91, normalized size = 1.34

method	result
norman	$\frac{(-a^2+2ab-b^2)x(\tan^5(fx+e))-\frac{a^2}{5f}-\frac{(a^2-2ab+b^2)(\tan^4(fx+e))}{f}+\frac{a(a-2b)(\tan^2(fx+e))}{3f}}{\tan(fx+e)^5}$
derivativedivides	$\frac{b^2(-\cot(fx+e)-fx-e)+2ab\left(-\frac{(\cot^3(fx+e))}{3}+\cot(fx+e)+fx+e\right)+a^2\left(-\frac{(\cot^5(fx+e))}{5}+\frac{(\cot^3(fx+e))}{3}-\cot(fx+e)-f\right.}{f}$
default	$\frac{b^2(-\cot(fx+e)-fx-e)+2ab\left(-\frac{(\cot^3(fx+e))}{3}+\cot(fx+e)+fx+e\right)+a^2\left(-\frac{(\cot^5(fx+e))}{5}+\frac{(\cot^3(fx+e))}{3}-\cot(fx+e)-f\right.}{f}$
risch	$-x\,a^2+2xab-x\,b^2-\frac{2i(45a^2e^{8i(fx+e)}-60ab\,e^{8i(fx+e)}+15b^2e^{8i(fx+e)}-90a^2e^{6i(fx+e)}+180ab\,e^{6i(fx+e)}-60b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(-cot(f*x+e)-f*x-e)+2*a*b*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)+a^2
*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e))
```

Maxima [A]
time = 0.48, size = 82, normalized size = 1.21

$$-\frac{15\,(a^2-2\,ab+b^2)(fx+e)+\frac{15\,(a^2-2\,ab+b^2)\tan(fx+e)^4-5\,(a^2-2\,ab)\tan(fx+e)^2+3\,a^2}{\tan(fx+e)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^2 - 2*a*b + b^2)*(f*x + e) + (15*(a^2 - 2*a*b + b^2)*tan(f*x +
 e)^4 - 5*(a^2 - 2*a*b)*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e)^5)/f
```

Fricas [A]
time = 4.20, size = 85, normalized size = 1.25

$$-\frac{15\,(a^2-2\,ab+b^2)fx\tan\left(fx+e\right)^5+15\,(a^2-2\,ab+b^2)\tan\left(fx+e\right)^4-5\,(a^2-2\,ab)\tan\left(fx+e\right)^2+3\,a^2}{15\,f\tan\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a^2 - 2*a*b + b^2)*f*x*tan(f*x + e)^5 + 15*(a^2 - 2*a*b + b^2)*t
an(f*x + e)^4 - 5*(a^2 - 2*a*b)*tan(f*x + e)^2 + 3*a^2)/(f*tan(f*x + e)^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(53) = 106$.
time = 6.55, size = 134, normalized size = 1.97

$$\begin{cases} \tilde{\infty}a^2x & \text{for } (e=0 \vee e=-fx) \wedge (e=-fx \vee f=0) \\ x(a+b\tan^2(e))^2\cot^6(e) & \text{for } f=0 \\ -a^2x-\frac{a^2}{f\tan(e+fx)}+\frac{a^2}{3f\tan^3(e+fx)}-\frac{a^2}{5f\tan^5(e+fx)}+2abx+\frac{2ab}{f\tan(e+fx)}-\frac{2ab}{3f\tan^3(e+fx)}-b^2x-\frac{b^2}{f\tan(e+fx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*a**2*x, (Eq(e, 0) | Eq(e, -f*x)) & (Eq(f, 0) | Eq(e, -f*x)))
, (x*(a + b*tan(e)**2)**2*cot(e)**6, Eq(f, 0)), (-a**2*x - a**2/(f*tan(e +
f*x)) + a**2/(3*f*tan(e + f*x)**3) - a**2/(5*f*tan(e + f*x)**5) + 2*a*b*x +
 2*a*b/(f*tan(e + f*x)) - 2*a*b/(3*f*tan(e + f*x)**3) - b**2*x - b**2/(f*ta
n(e + f*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(67) = 134.
time = 1.76, size = 222, normalized size = 3.26

$$\frac{3\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-35\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+330\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-600\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+240\,b^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-480\,(a^{2}-2\,ab+b^{2})(fx+e)-\frac{330\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-600\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+240\,b^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-35\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+40\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,a^{2}}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}}}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*f*x + 1/2*e)^5 - 35*a^2*tan(1/2*f*x + 1/2*e)^3 + 40*a*
b*tan(1/2*f*x + 1/2*e)^3 + 330*a^2*tan(1/2*f*x + 1/2*e) - 600*a*b*tan(1/2*f
*x + 1/2*e) + 240*b^2*tan(1/2*f*x + 1/2*e) - 480*(a^2 - 2*a*b + b^2)*(f*x +
 e) - (330*a^2*tan(1/2*f*x + 1/2*e)^4 - 600*a*b*tan(1/2*f*x + 1/2*e)^4 + 24
0*b^2*tan(1/2*f*x + 1/2*e)^4 - 35*a^2*tan(1/2*f*x + 1/2*e)^2 + 40*a*b*tan(1
/2*f*x + 1/2*e)^2 + 3*a^2)/tan(1/2*f*x + 1/2*e)^5)/f
```

Mupad [B]
time = 11.52, size = 76, normalized size = 1.12

$$2\,a\,b\,x-b^2\,x-\frac{\cot(e+f\,x)^5\,\left(\tan(e+f\,x)^4\,\left(a^2-2\,a\,b+b^2\right)+\frac{a^2}{5}+\tan(e+f\,x)^2\,\left(\frac{2\,a\,b}{3}-\frac{a^2}{3}\right)\right)}{f}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] 2*a*b*x - b^2*x - (cot(e + f*x)^5*(tan(e + f*x)^4*(a^2 - 2*a*b + b^2) + a^2
/5 + tan(e + f*x)^2*((2*a*b)/3 - a^2/3)))/f - a^2*x
```

3.211 $\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=71

$$-\frac{\log(\cos(e+fx))}{(a-b)f}-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)b^2f}+\frac{\tan^2(e+fx)}{2bf}$$

```
[Out] -ln(cos(f*x+e))/(a-b)/f-1/2*a^2*ln(a+b*tan(f*x+e)^2)/(a-b)/b^2/f+1/2*tan(f*
x+e)^2/b/f
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 84}

$$-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)}-\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\tan^2(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(Log[Cos[e + f*x]]/((a - b)*f)) - (a^2*Log[a + b*Tan[e + f*x]^2])/(2*(a -
b)*b^2*f) + Tan[e + f*x]^2/(2*b*f)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b}+\frac{1}{(a-b)(1+x)}-\frac{a^2}{(a-b)b(a+bx)}\right)\,dx, x, \tan^2(e+fx)\right)}{2f}\\ &= -\frac{\log(\cos(e+fx))}{(a-b)f}-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)b^2f}+\frac{\tan^2(e+fx)}{2bf}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 64, normalized size = 0.90

$$\frac{-\frac{2\log(\cos(e+fx))}{a-b}-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{(a-b)b^2}+\frac{\tan^2(e+fx)}{b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((-2*Log[Cos[e + f*x]])/(a - b) - (a^2*Log[a + b*Tan[e + f*x]^2])/((a - b)*
b^2) + Tan[e + f*x]^2/b)/(2*f)
```

Maple [A]
time = 0.11, size = 67, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{\tan^2(fx+e)}{2b}+\frac{\ln\left(1+\tan^2(fx+e)\right)}{2a-2b}-\frac{a^2\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{2b^2(a-b)}}{f}$
default	$\frac{\frac{\tan^2(fx+e)}{2b}+\frac{\ln\left(1+\tan^2(fx+e)\right)}{2a-2b}-\frac{a^2\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{2b^2(a-b)}}{f}$
norman	$\frac{\tan^2(fx+e)}{2bf}+\frac{\ln(1+\tan^2(fx+e))}{2f(a-b)}-\frac{a^2\ln(a+b(\tan^2(fx+e)))}{2(a-b)b^2f}$
risch	$-\frac{ix}{a-b}-\frac{2iax}{b^2}-\frac{2iae}{b^2f}-\frac{2ix}{b}-\frac{2ie}{bf}+\frac{2ia^2x}{(a-b)b^2}+\frac{2ia^2e}{(a-b)b^2f}+\frac{2\,e^{2i(fx+e)}}{fb\left(e^{2i(fx+e)}+1\right)^2}+\frac{\ln\left(e^{2i(fx+e)}+1\right)a}{b^2f}+\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

[Out] 1/f*(1/2*tan(f*x+e)^2/b+1/2/(a-b)*ln(1+tan(f*x+e)^2)-1/2*a^2/b^2/(a-b)*ln(a+b*tan(f*x+e)^2))

Maxima [A]
time = 0.27, size = 79, normalized size = 1.11

$$-\frac{\frac{a^2\log\left(-(a-b)\sin(fx+e)^2+a\right)}{ab^2-b^3}-\frac{(a+b)\log\left(\sin(fx+e)^2-1\right)}{b^2}+\frac{1}{b\sin(fx+e)^2-b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")

[Out] -1/2*(a^2*log(-(a - b)*sin(f*x + e)^2 + a)/(a*b^2 - b^3) - (a + b)*log(sin(f*x + e)^2 - 1)/b^2 + 1/(b*sin(f*x + e)^2 - b))/f

Fricas [A]
time = 2.51, size = 96, normalized size = 1.35

$$-\frac{a^2\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)-(ab-b^2)\tan(fx+e)^2-(a^2-b^2)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2(ab^2-b^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")

[Out] -1/2*(a^2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) - (a*b - b^2)*tan(f*x + e)^2 - (a^2 - b^2)*log(1/(tan(f*x + e)^2 + 1)))/((a*b^2 - b^3)*f)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(53) = 106$.
time = 9.00, size = 338, normalized size = 4.76

$$\begin{cases} \tilde{\infty}x\tan^3(e) & \text{for } a=0\wedge b=0\wedge f=0 \\ \frac{\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}}{a} & \text{for } b=0 \\ -\frac{2\log(\tan^2(e+fx)+1)\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}-\frac{2\log(\tan^2(e+fx)+1)}{2bf\tan^2(e+fx)+2bf}+\frac{\tan^4(e+fx)}{2bf\tan^2(e+fx)+2bf}-\frac{2}{2bf\tan^2(e+fx)+2bf} & \text{for } a=b \\ \frac{x\tan^5(e)}{a+b\tan^2(e)} & \text{for } f=0 \\ -\frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2ab^2f-2b^3f}-\frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2ab^2f-2b^3f}+\frac{ab\tan^2(e+fx)}{2ab^2f-2b^3f}+\frac{b^2\log(\tan^2(e+fx)+1)}{2ab^2f-2b^3f}-\frac{b^2\tan^2(e+fx)}{2ab^2f-2b^3f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2),x)

[Out] Piecewise((zoo*x*tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(e + f*x)**2/(2*f))/a, Eq(b, 0)), (-2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2

```
+ 2*b*f) - 2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan
(e + f*x)**4/(2*b*f*tan(e + f*x)**2 + 2*b*f) - 2/(2*b*f*tan(e + f*x)**2 + 2
*b*f), Eq(a, b)), (x*tan(e)**5/(a + b*tan(e)**2), Eq(f, 0)), (-a**2*log(-sq
rt(-a/b) + tan(e + f*x))/(2*a*b**2*f - 2*b**3*f) - a**2*log(sqrt(-a/b) + ta
n(e + f*x))/(2*a*b**2*f - 2*b**3*f) + a*b*tan(e + f*x)**2/(2*a*b**2*f - 2*b
**3*f) + b**2*log(tan(e + f*x)**2 + 1)/(2*a*b**2*f - 2*b**3*f) - b**2*tan(e
 + f*x)**2/(2*a*b**2*f - 2*b**3*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(70) = 140$.
time = 1.59, size = 341, normalized size = 4.80

$$-\frac{\frac{a^3\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+4b\right|\right)}{a^2b^2-ab^3}-\frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{a-b}-\frac{(a+b)\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)}{b^2}+\frac{a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2a+6b}{b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a^3*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e)
 - 1)/(cos(f*x + e) + 1)) - 2*a + 4*b))/(a^2*b^2 - a*b^3) - log(abs(-(cos(f
*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2
))/(a - b) - (a + b)*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(
f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/b^2 + (a*((cos(f*x + e) + 1)/(cos(f*
x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + b*((cos(f*x + e) + 1
)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a + 6*b)/
(b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + 2)))/f
```

Mupad [B]
time = 11.86, size = 74, normalized size = 1.04

$$\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,\left(a-b\right)}+\frac{\tan(e+f\,x)^2}{2\,b\,f}-\frac{a^2\,\ln\left(b\tan(e+f\,x)^2+a\right)}{2\,f\,\left(a\,b^2-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) + tan(e + f*x)^2/(2*b*f) - (a^2*log(a
 + b*tan(e + f*x)^2))/(2*f*(a*b^2 - b^3))
```

3.212 $\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{\log(\cos(e+fx))}{(a-b)f}+\frac{a\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)bf}$$

[Out] ln(cos(f*x+e))/(a-b)/f+1/2*a*ln(a+b*tan(f*x+e)^2)/(a-b)/b/f

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 78}

$$\frac{a\log\left(a+b\tan^2(e+fx)\right)}{2bf(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]

[Out] Log[Cos[e + f*x]]/((a - b)*f) + (a*Log[a + b*Tan[e + f*x]^2])/(2*(a - b)*b*f)

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)}\,dx,x,\tan^2(e+fx)\right)}{2f}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)(1+x)}+\frac{a}{(a-b)(a+bx)}\right)\,dx,x,\tan^2(e+fx)\right)}{2f}\\ &= \frac{\log(\cos(e+fx))}{(a-b)f}+\frac{a\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)bf}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.82

$$\frac{2b\log(\cos(e+fx))+a\log\left(a+b\tan^2(e+fx)\right)}{2abf-2b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
[Out] (2*b*Log[Cos[e + f*x]] + a*Log[a + b*Tan[e + f*x]^2])/(2*a*b*f - 2*b^2*f)
```

Maple [A]
time = 0.09, size = 52, normalized size = 1.04

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)}+\frac{a\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{2(a-b)b}}{f}$	52
default	$\frac{-\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)}+\frac{a\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{2(a-b)b}}{f}$	52
norman	$-\frac{\ln(1+\tan^2(fx+e))}{2f(a-b)}+\frac{a\ln(a+b(\tan^2(fx+e)))}{2(a-b)bf}$	54
risch	$\frac{ix}{a-b}+\frac{2ix}{b}+\frac{2ie}{bf}-\frac{2iax}{b(a-b)}-\frac{2iae}{fb(a-b)}-\frac{\ln(e^{2i(fx+e)}+1)}{bf}+\frac{a\ln\left(e^{4i(fx+e)}+\frac{2(a+b)e^{2i(fx+e)}}{a-b}+1\right)}{2fb(a-b)}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/(a-b)*ln(1+tan(f*x+e)^2)+1/2*a/(a-b)/b*ln(a+b*tan(f*x+e)^2))
```

Maxima [A]
time = 0.27, size = 55, normalized size = 1.10

$$\frac{\frac{a\log\left(-(a-b)\sin(fx+e)^2+a\right)}{ab-b^2}-\frac{\log\left(\sin(fx+e)^2-1\right)}{b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*log(-(a - b)*sin(f*x + e)^2 + a)/(a*b - b^2) - log(sin(f*x + e)^2 -
1)/b)/f
```

Fricas [A]
time = 2.34, size = 68, normalized size = 1.36

$$\frac{a\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)-(a-b)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2(ab-b^2)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) - (a - b)*log(1/(ta
n(f*x + e)^2 + 1)))/((a*b - b^2)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(36) = 72.
time = 1.93, size = 230, normalized size = 4.60

$$\begin{cases} \tilde{\infty}x\tan(e) & \text{for } a=0\wedge b=0\wedge f=0 \\ \frac{-\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\tan^2(e+fx)}{2f}}{a} & \text{for } b=0 \\ \frac{\log(\tan^2(e+fx)+1)\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}+\frac{\log(\tan^2(e+fx)+1)}{2bf\tan^2(e+fx)+2bf}+\frac{1}{2bf\tan^2(e+fx)+2bf} & \text{for } a=b \\ \frac{x\tan^3(e)}{a+b\tan^2(e)} & \text{for } f=0 \\ \frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2abf-2b^2f}+\frac{a\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2abf-2b^2f}-\frac{b\log(\tan^2(e+fx)+1)}{2abf-2b^2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(tan(e + f*
x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a, Eq(b, 0)), (log(tan(e + f*x)**
```

```
2 + 1)*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) + log(tan(e + f*x)**
2 + 1)/(2*b*f*tan(e + f*x)**2 + 2*b*f) + 1/(2*b*f*tan(e + f*x)**2 + 2*b*f),
 Eq(a, b)), (x*tan(e)**3/(a + b*tan(e)**2), Eq(f, 0)), (a*log(-sqrt(-a/b) +
 tan(e + f*x))/(2*a*b*f - 2*b**2*f) + a*log(sqrt(-a/b) + tan(e + f*x))/(2*a
*b*f - 2*b**2*f) - b*log(tan(e + f*x)**2 + 1)/(2*a*b*f - 2*b**2*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(50) = 100$.
time = 0.84, size = 189, normalized size = 3.78

$$\frac{\frac{a^{2}\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2\,a+4\,b\right|\right)}{a^{2}b-ab^{2}}-\frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{a-b}-\frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e)
- 1)/(cos(f*x + e) + 1)) - 2*a + 4*b))/(a^2*b - a*b^2) - log(abs(-(cos(f*x
+ e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/
(a - b) - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
1)/(cos(f*x + e) + 1) - 2))/b)/f
```

Mupad [B]
time = 11.78, size = 54, normalized size = 1.08

$$\frac{a\,\ln\left(b\tan(e+f\,x)^{2}+a\right)}{2\,f\,\left(a\,b-b^{2}\right)}-\frac{\ln\left(\tan(e+f\,x)^{2}+1\right)}{2\,f\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (a*log(a + b*tan(e + f*x)^2))/(2*f*(a*b - b^2)) - log(tan(e + f*x)^2 + 1)/(
2*f*(a - b))
```

3.213 $\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=36

$$-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)f}$$

```
[Out] -1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)/f
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3751, 455, 36, 31}

$$-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/2*Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/((a - b)*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \tan^2(e+fx)\right)}{2(a-b)f} \\
&= -\frac{\log(\cos(e+fx))}{(a-b)f} - \frac{\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)f}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 1.03

$$-\frac{2\log(\cos(e+fx)) + \log\left(a+b\tan^2(e+fx)\right)}{2(a-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2),x]
[Out] -1/2*(2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2])/((a - b)*f)
```

Maple [A]
time = 0.06, size = 48, normalized size = 1.33

method	result	size
derivativedivides	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2a-2b}-\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{2(a-b)}}{f}$	48
default	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2a-2b}-\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{2(a-b)}}{f}$	48
norman	$\frac{\ln(1+\tan^2(fx+e))}{2f(a-b)}-\frac{\ln(a+b(\tan^2(fx+e)))}{2f(a-b)}$	50
risch	$\frac{ix}{a-b}+\frac{2ie}{f(a-b)}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+b)e^{2i(fx+e)}}{a-b}+1\right)}{2f(a-b)}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
[Out] 1/f*(1/2/(a-b)*ln(1+tan(f*x+e)^2)-1/2/(a-b)*ln(a+b*tan(f*x+e)^2))
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.86

$$-\frac{\log\left(-(a-b)\sin\left(fx+e\right)^2+a\right)}{2\left(a-b\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
[Out] -1/2*log(-(a - b)*sin(f*x + e)^2 + a)/((a - b)*f)
```

Fricas [A]
time = 4.85, size = 40, normalized size = 1.11

$$-\frac{\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{2\left(a-b\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
[Out] -1/2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))/((a - b)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(29) = 58$.
time = 1.13, size = 133, normalized size = 3.69

$$\begin{cases} \frac{\tilde{\infty}x}{\tan\left(e\right)} & \text{for}\ a=0\wedge b=0\wedge f=0 \\ \frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2af} & \text{for}\ b=0 \\ -\frac{1}{2bf\tan^2\left(e+fx\right)+2bf} & \text{for}\ a=b \\ \frac{x\tan\left(e\right)}{a+b\tan^2\left(e\right)} & \text{for}\ f=0 \\ -\frac{\log\left(-\sqrt{-\frac{a}{b}}+\tan\left(e+fx\right)\right)}{2af-2bf}-\frac{\log\left(\sqrt{-\frac{a}{b}}+\tan\left(e+fx\right)\right)}{2af-2bf}+\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2af-2bf} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2),x)
[Out] Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e + f*x)
**2 + 1)/(2*a*f), Eq(b, 0)), (-1/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b))
, (x*tan(e)/(a + b*tan(e)**2), Eq(f, 0)), (-log(-sqrt(-a/b) + tan(e + f*x))
```

```
/(2*a*f - 2*b*f) - log(sqrt(-a/b) + tan(e + f*x))/(2*a*f - 2*b*f) + log(tan
(e + f*x)**2 + 1)/(2*a*f - 2*b*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(36) = 72.
time = 0.65, size = 122, normalized size = 3.39

$$-\frac{\frac{\log\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a-b}-\frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a-b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a
- b) - 2*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(a - b))/f
```

Mupad [B]
time = 11.85, size = 66, normalized size = 1.83

$$-\frac{\mathrm{atan}\left(\frac{a\,\tan(e+f\,x)^2\,1\mathrm{i}-b\,\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a+a\,\tan(e+f\,x)^2+b\,\tan(e+f\,x)^2}\right)\,1\mathrm{i}}{f\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2),x)
```

```
[Out] -(atan((a*tan(e + f*x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2
+ b*tan(e + f*x)^2))*1i)/(f*(a - b))
```

3.214 $\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{\log(\cos(e+fx))}{(a-b)f}+\frac{\log(\tan(e+fx))}{af}+\frac{b\log\left(a+b\tan^2(e+fx)\right)}{2a(a-b)f}$$

```
[Out] ln(cos(f*x+e))/(a-b)/f+ln(tan(f*x+e))/a/f+1/2*b*ln(a+b*tan(f*x+e)^2)/a/(a-b
)/f
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 457, 84}

$$\frac{b\log\left(a+b\tan^2(e+fx)\right)}{2af(a-b)}+\frac{\log(\cos(e+fx))}{f(a-b)}+\frac{\log(\tan(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)*f) + Log[Tan[e + f*x]]/(a*f) + (b*Log[a + b*Tan[
e + f*x]^2])/(2*a*(a - b)*f)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{ax} - \frac{1}{(a-b)(1+x)} + \frac{b^2}{a(a-b)(a+bx)}\right)\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{\log(\tan(e+fx))}{af} + \frac{b\log\left(a+b\tan^2(e+fx)\right)}{2a(a-b)f}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.89

$$\frac{2a\log(\cos(e+fx)) + 2(a-b)\log(\tan(e+fx)) + b\log\left(a+b\tan^2(e+fx)\right)}{2a(a-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (2*a*Log[Cos[e + f*x]] + 2*(a - b)*Log[Tan[e + f*x]] + b*Log[a + b*Tan[e + 
f*x]^2])/(2*a*(a - b)*f)
```

Maple [A]
time = 0.21, size = 71, normalized size = 1.11

method	result	size
norman	$\frac{\ln(\tan(fx+e))}{af} - \frac{\ln(1+\tan^2(fx+e))}{2f(a-b)} + \frac{b\ln(a+b(\tan^2(fx+e)))}{2a(a-b)f}$	68
derivativedivides	$\frac{\frac{\ln(\cos(fx+e)+1)}{2a} + \frac{\ln(\cos(fx+e)-1)}{2a} + \frac{b\ln\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}{2a(a-b)}}{f}$	71
default	$\frac{\frac{\ln(\cos(fx+e)+1)}{2a} + \frac{\ln(\cos(fx+e)-1)}{2a} + \frac{b\ln\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)}{2a(a-b)}}{f}$	71
risch	$\frac{ix}{a-b} - \frac{2ix}{a} - \frac{2ie}{af} - \frac{2ibx}{a(a-b)} - \frac{2ibe}{af(a-b)} + \frac{\ln(e^{2i(fx+e)}-1)}{af} + \frac{b\ln\left(e^{4i(fx+e)} + \frac{2(a+b)e^{2i(fx+e)}}{a-b} + 1\right)}{2af(a-b)}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/a*ln(cos(f*x+e)+1)+1/2/a*ln(cos(f*x+e)-1)+1/2*b/a/(a-b)*ln(a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b))
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.80

$$\frac{\frac{b\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^2-ab}+\frac{\log\left(\sin(fx+e)^2\right)}{a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - a*b) + log(sin(f*x + e)^2)/a
)/f
```

Fricas [A]
time = 4.73, size = 76, normalized size = 1.19

$$\frac{(a-b)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)+b\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{2\,(a^2-ab)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + b*log((b*tan(f*x +
e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^2 - a*b)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(48) = 96$.
time = 4.21, size = 388, normalized size = 6.06

$$\begin{cases} \frac{\tilde{\infty}x\cot(e)}{\tan^2(e)} & \text{for}\ a=0\wedge b=0\wedge f=0 \\ \frac{\frac{\log\left(\tan^2(e+fx)+1\right)}{2f}-\frac{\log(\tan(e+fx))}{f}-\frac{1}{2f\tan^2(e+fx)}}{b} & \text{for}\ a=0 \\ -\frac{\log(\tan^2(e+fx)+1)\tan^2(e+fx)}{2af\tan^2(e+fx)+2af}-\frac{\log(\tan^2(e+fx)+1)}{2af\tan^2(e+fx)+2af}+\frac{2\log(\tan(e+fx))\tan^2(e+fx)}{2af\tan^2(e+fx)+2af}+\frac{2\log(\tan(e+fx))}{2af\tan^2(e+fx)+2af}+\frac{1}{2af\tan^2(e+fx)+2af} & \text{for}\ a=b \\ \frac{x\cot(e)}{a+b\tan^2(e)} & \text{for}\ f=0 \\ \frac{-\frac{\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{\log(\tan(e+fx))}{f}}{a} & \text{for}\ b=0 \\ -\frac{a\log(\tan^2(e+fx)+1)}{2a^2f-2abf}+\frac{2a\log(\tan(e+fx))}{2a^2f-2abf}+\frac{b\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2a^2f-2abf}+\frac{b\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2a^2f-2abf}-\frac{2b\log(\tan(e+fx))}{2a^2f-2abf} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*cot(e)/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((log(t
an(e + f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2))/
```

```
b, Eq(a, 0)), (-log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a*f*tan(e + f*x
)**2 + 2*a*f) - log(tan(e + f*x)**2 + 1)/(2*a*f*tan(e + f*x)**2 + 2*a*f) +
2*log(tan(e + f*x))*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**2 + 2*a*f) + 2*log
(tan(e + f*x))/(2*a*f*tan(e + f*x)**2 + 2*a*f) + 1/(2*a*f*tan(e + f*x)**2 +
 2*a*f), Eq(a, b)), (x*cot(e)/(a + b*tan(e)**2), Eq(f, 0)), ((-log(tan(e +
f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f)/a, Eq(b, 0)), (-a*log(tan(e + f*x
)**2 + 1)/(2*a**2*f - 2*a*b*f) + 2*a*log(tan(e + f*x))/(2*a**2*f - 2*a*b*f)
 + b*log(-sqrt(-a/b) + tan(e + f*x))/(2*a**2*f - 2*a*b*f) + b*log(sqrt(-a/b
) + tan(e + f*x))/(2*a**2*f - 2*a*b*f) - 2*b*log(tan(e + f*x))/(2*a**2*f -
2*a*b*f), True))
```

Giac [A]
time = 0.72, size = 59, normalized size = 0.92

$$\frac{\frac{b\log\left(\left|-a\sin(fx+e)^2+b\sin(fx+e)^2+a\right|\right)}{a^2-ab}+\frac{\log\left(\sin(fx+e)^2\right)}{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(abs(-a*sin(f*x + e)^2 + b*sin(f*x + e)^2 + a))/(a^2 - a*b) + log
(sin(f*x + e)^2)/a)/f
```

Mupad [B]
time = 11.71, size = 68, normalized size = 1.06

$$\frac{\ln\left(\tan(e+f\,x)\right)}{a\,f}-\frac{\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,f\,\left(a-b\right)}-\frac{b\,\ln\left(b\,{\tan(e+f\,x)}^2+a\right)}{2\,f\,\left(a\,b-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x))/(a*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) - (b*log(a
+ b*tan(e + f*x)^2))/(2*f*(a*b - a^2))
```

3.215 $\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=89

$$-\frac{\cot^2(e+fx)}{2af}-\frac{\log(\cos(e+fx))}{(a-b)f}-\frac{(a+b)\log(\tan(e+fx))}{a^2f}-\frac{b^2\log\left(a+b\tan^2(e+fx)\right)}{2a^2(a-b)f}$$

```
[Out] -1/2*cot(f*x+e)^2/a/f-ln(cos(f*x+e))/(a-b)/f-(a+b)*ln(tan(f*x+e))/a^2/f-1/2
*b^2*ln(a+b*tan(f*x+e)^2)/a^2/(a-b)/f
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 84}

$$-\frac{b^2\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)}-\frac{(a+b)\log(\tan(e+fx))}{a^2f}-\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^2(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/2*Cot[e + f*x]^2/(a*f) - Log[Cos[e + f*x]]/((a - b)*f) - ((a + b)*Log[Ta
n[e + f*x]])/(a^2*f) - (b^2*Log[a + b*Tan[e + f*x]^2])/(2*a^2*(a - b)*f)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{-a-b}{a^2x}+\frac{1}{(a-b)(1+x)}-\frac{b^3}{a^2(a-b)(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\cot^2(e+fx)}{2af}-\frac{\log(\cos(e+fx))}{(a-b)f}-\frac{(a+b)\log(\tan(e+fx))}{a^2f}-\frac{b^2\log(a+b\tan}{2a^2(a-}$$

Mathematica [A]
time = 0.18, size = 63, normalized size = 0.71

$$-\frac{\frac{\cot^2(e+fx)}{a}+\frac{b^2\log(b+a\cot^2(e+fx))}{a^2(a-b)}+\frac{2\log(\sin(e+fx))}{a-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/2*(Cot[e + f*x]^2/a + (b^2*Log[b + a*Cot[e + f*x]^2])/(a^2*(a - b)) + (2
*Log[Sin[e + f*x]])/(a - b))/f
```

Maple [A]
time = 0.24, size = 117, normalized size = 1.31

method	result
norman	$-\frac{1}{2af\tan(fx+e)^2}+\frac{\ln(1+\tan^2(fx+e))}{2f(a-b)}-\frac{(a+b)\ln(\tan(fx+e))}{a^2f}-\frac{b^2\ln(a+b(\tan^2(fx+e)))}{2a^2(a-b)f}$
derivativedivides	$\frac{-\frac{1}{4a(\cos(fx+e)+1)}+\frac{(-a-b)\ln(\cos(fx+e)+1)}{2a^2}+\frac{1}{4a(\cos(fx+e)-1)}+\frac{(-a-b)\ln(\cos(fx+e)-1)}{2a^2}-\frac{b^2\ln\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)}{2a^2(a-b)}}{f}$
default	$\frac{-\frac{1}{4a(\cos(fx+e)+1)}+\frac{(-a-b)\ln(\cos(fx+e)+1)}{2a^2}+\frac{1}{4a(\cos(fx+e)-1)}+\frac{(-a-b)\ln(\cos(fx+e)-1)}{2a^2}-\frac{b^2\ln\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)}{2a^2(a-b)}}{f}$
risch	$-\frac{ix}{a-b}+\frac{2ix}{a}+\frac{2ie}{af}+\frac{2ibx}{a^2}+\frac{2ibe}{a^2f}+\frac{2ib^2x}{a^2(a-b)}+\frac{2ib^2e}{a^2f(a-b)}+\frac{2\,e^{2i(fx+e)}}{fa(e^{2i(fx+e)}-1)^2}-\frac{\ln(e^{2i(fx+e)}-1)}{af}-\frac{\ln(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/4/a/(cos(f*x+e)+1)+1/2*(-a-b)/a^2*ln(cos(f*x+e)+1)+1/4/a/(cos(f*x+e
)-1)+1/2*(-a-b)/a^2*ln(cos(f*x+e)-1)-1/2*b^2/a^2/(a-b)*ln(a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b))
```

Maxima [A]
time = 0.27, size = 71, normalized size = 0.80

$$-\frac{\frac{b^2\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^3-a^2b}+\frac{(a+b)\log\left(\sin(fx+e)^2\right)}{a^2}+\frac{1}{a\sin(fx+e)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - a^2*b) + (a + b)*log(sin(
f*x + e)^2)/a^2 + 1/(a*sin(f*x + e)^2))/f
```

Fricas [A]
time = 5.22, size = 136, normalized size = 1.53

$$-\frac{b^2\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+(a^2-b^2)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+(a^2-ab)\tan\left(fx+e\right)^2+a^2-ab}{2\left(a^3-a^2b\right)f\tan\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 +
 (a^2 - b^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (a^2
 - a*b)*tan(f*x + e)^2 + a^2 - a*b)/((a^3 - a^2*b)*f*tan(f*x + e)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 733 vs. 2(71) = 142.
time = 16.26, size = 733, normalized size = 8.24

$$\begin{cases} \tilde{\infty}x & \text{for}\ a=0\wedge b=0\wedge e=0\wedge f=0 \\ \frac{\frac{\log\left(\tan^2(e+fx)+1\right)}{2f}-\frac{\log\left(\tan(e+fx)\right)}{f}-\frac{1}{2f\tan^2(e+fx)}}{a} & \text{for}\ b=0 \\ \frac{-\frac{\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{\log\left(\tan(e+fx)\right)}{f}+\frac{1}{2f\tan^2(e+fx)}-\frac{1}{4f\tan^4(e+fx)}}{b} & \text{for}\ a=0 \\ \frac{2\log\left(\tan^2(e+fx)+1\right)\tan^4(e+fx)}{2af\tan^4(e+fx)+2af\tan^2(e+fx)}+\frac{2\log\left(\tan^2(e+fx)+1\right)\tan^2(e+fx)}{2af\tan^4(e+fx)+2af\tan^2(e+fx)}-\frac{4\log\left(\tan(e+fx)\right)\tan^4(e+fx)}{2af\tan^4(e+fx)+2af\tan^2(e+fx)}-\frac{4\log\left(\tan(e+fx)\right)\tan^2(e+fx)}{2af\tan^4(e+fx)+2af\tan^2(e+fx)}-\frac{2\tan^2(e+fx)}{2af\tan^4(e+fx)+2af\tan^2(e+fx)}-\frac{1}{2af\tan^4(e+fx)+2af\tan^2(e+fx)} & \text{for}\ a=b \\ \frac{\tilde{\infty}x}{a} & \text{for}\ e=-fx \\ \frac{x\cot^3(e)}{a+b\tan^2(e)} & \text{for}\ f=0 \\ \frac{a^2\log\left(\tan^2(e+fx)+1\right)\tan^2(e+fx)}{2a^3f\tan^2(e+fx)-2a^2bf\tan^2(e+fx)}-\frac{2a^2\log\left(\tan(e+fx)\right)\tan^2(e+fx)}{2a^3f\tan^2(e+fx)-2a^2bf\tan^2(e+fx)}-\frac{a^2}{2a^3f\tan^2(e+fx)-2a^2bf\tan^2(e+fx)}+\frac{ab}{2a^3f\tan^2(e+fx)-2a^2bf\tan^2(e+fx)}-\frac{b^2\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)\tan^2(e+fx)}{2a^3f\tan^2(e+fx)-2a^2bf\tan^2(e+fx)}-\frac{b^2\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)\tan^2(e+fx)}{2a^3f\tan^2(e+fx)-2a^2bf\tan^2(e+fx)}+\frac{2b^2\log\left(\tan(e+fx)\right)\tan^2(e+fx)}{2a^3f\tan^2(e+fx)-2a^2bf\tan^2(e+fx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((log(tan(e +
 f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2))/a, Eq(
b, 0)), ((-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f + 1/(2*f*ta
n(e + f*x)**2) - 1/(4*f*tan(e + f*x)**4))/b, Eq(a, 0)), (2*log(tan(e + f*x)
```

```
**2 + 1)*tan(e + f*x)**4/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x)**2) +
2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**4 + 2*a*f*t
an(e + f*x)**2) - 4*log(tan(e + f*x))*tan(e + f*x)**4/(2*a*f*tan(e + f*x)**
4 + 2*a*f*tan(e + f*x)**2) - 4*log(tan(e + f*x))*tan(e + f*x)**2/(2*a*f*tan
(e + f*x)**4 + 2*a*f*tan(e + f*x)**2) - 2*tan(e + f*x)**2/(2*a*f*tan(e + f*
x)**4 + 2*a*f*tan(e + f*x)**2) - 1/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f
*x)**2), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)), (x*cot(e)**3/(a + b*tan(e)**2),
 Eq(f, 0)), (a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**3*f*tan(e
+ f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - 2*a**2*log(tan(e + f*x))*tan(e +
f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - a**2/(2*a
**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) + a*b/(2*a**3*f*tan(e +
 f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - b**2*log(-sqrt(-a/b) + tan(e + f*x
))*tan(e + f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2)
- b**2*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*f*tan(e + f*x
)**2 - 2*a**2*b*f*tan(e + f*x)**2) + 2*b**2*log(tan(e + f*x))*tan(e + f*x)*
*2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(89) = 178.
time = 0.91, size = 263, normalized size = 2.96

$$-\frac{\frac{4\,b^2\log\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3-a^2b}+\frac{4\,(a+b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2}-\frac{8\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a-b}-\frac{\left(a+\frac{4\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{a^2(\cos(fx+e)-1)}-\frac{\cos(fx+e)-1}{a(\cos(fx+e)+1)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2)/(a^3 - a^2*b) + 4*(a + b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) +
1))/a^2 - 8*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(a - b) -
(a + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1))*(cos(f*x + e) + 1)/(a^2*(cos(f*x + e) - 1)) - (cos(f*x + e
) - 1)/(a*(cos(f*x + e) + 1)))/f
```

Mupad [B]
time = 11.60, size = 89, normalized size = 1.00

$$\frac{\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,f\,\left(a-b\right)}-\frac{{\cot(e+f\,x)}^2}{2\,a\,f}-\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a+b\right)}{a^2\,f}-\frac{b^2\,\ln\left(b\,{\tan(e+f\,x)}^2+a\right)}{2\,a^2\,f\,\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2),x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) - cot(e + f*x)^2/(2*a*f) - (log(tan(e
 + f*x))*(a + b))/(a^2*f) - (b^2*log(a + b*tan(e + f*x)^2))/(2*a^2*f*(a - b
))
```

3.216 $\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=115

$$\frac{(a+b)\cot^2(e+fx)}{2a^2f}-\frac{\cot^4(e+fx)}{4af}+\frac{\log(\cos(e+fx))}{(a-b)f}+\frac{(a^2+ab+b^2)\log(\tan(e+fx))}{a^3f}+\frac{b^3\log\left(a+b\tan^2(e+fx)\right)}{2a^3(a-b)f}$$

```
[Out] 1/2*(a+b)*cot(f*x+e)^2/a^2/f-1/4*cot(f*x+e)^4/a/f+ln(cos(f*x+e))/(a-b)/f+(a
^2+a*b+b^2)*ln(tan(f*x+e))/a^3/f+1/2*b^3*ln(a+b*tan(f*x+e)^2)/a^3/(a-b)/f
```

Rubi [A]

time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 84}

$$\frac{b^3\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)}+\frac{(a+b)\cot^2(e+fx)}{2a^2f}+\frac{(a^2+ab+b^2)\log(\tan(e+fx))}{a^3f}+\frac{\log(\cos(e+fx))}{f(a-b)}-\frac{\cot^4(e+fx)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] ((a + b)*Cot[e + f*x]^2)/(2*a^2*f) - Cot[e + f*x]^4/(4*a*f) + Log[Cos[e + f
*x]]/((a - b)*f) + ((a^2 + a*b + b^2)*Log[Tan[e + f*x]])/(a^3*f) + (b^3*Log
[a + b*Tan[e + f*x]^2])/(2*a^3*(a - b)*f)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} + \frac{-a-b}{a^2x^2} + \frac{a^2+ab+b^2}{a^3x} - \frac{1}{(a-b)(1+x)} + \frac{b^4}{a^3(a-b)(a+bx)}\right) dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{(a+b)\cot^2(e+fx)}{2a^2f} - \frac{\cot^4(e+fx)}{4af} + \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{(a^2+ab+b^2)\log(\tan}{a^3f}$$

Mathematica [A]
time = 0.24, size = 83, normalized size = 0.72

$$-\frac{-\frac{(a+b)\cot^2(e+fx)}{a^2} + \frac{\cot^4(e+fx)}{2a} - \frac{b^3\log(b+a\cot^2(e+fx))}{a^3(a-b)} - \frac{2\log(\sin(e+fx))}{a-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -1/2*(-(((a + b)*Cot[e + f*x]^2)/a^2) + Cot[e + f*x]^4/(2*a) - (b^3*Log[b +
 a*Cot[e + f*x]^2])/(a^3*(a - b)) - (2*Log[Sin[e + f*x]])/(a - b))/f
```

Maple [A]
time = 0.26, size = 167, normalized size = 1.45

method	result
norman	$\frac{-\frac{1}{4af}+\frac{(a+b)\left(\tan^2(fx+e)\right)}{2a^2f}}{\tan(fx+e)^4} + \frac{(a^2+ab+b^2)\ln(\tan(fx+e))}{a^3f} - \frac{\ln(1+\tan^2(fx+e))}{2f(a-b)} + \frac{b^3\ln(a+b(\tan^2(fx+e)))}{2a^3(a-b)f}$
derivativedivides	$\frac{-\frac{1}{16a(\cos(fx+e)+1)^2} - \frac{-7a-4b}{16a^2(\cos(fx+e)+1)} + \frac{\left(a^2+ab+b^2\right)\ln(\cos(fx+e)+1)}{2a^3} - \frac{1}{16a(\cos(fx+e)-1)^2} - \frac{7a+4b}{16a^2(\cos(fx+e)-1)} + \frac{\left(a^2+ab+\right.}{}}{f}$
default	$\frac{-\frac{1}{16a(\cos(fx+e)+1)^2} - \frac{-7a-4b}{16a^2(\cos(fx+e)+1)} + \frac{\left(a^2+ab+b^2\right)\ln(\cos(fx+e)+1)}{2a^3} - \frac{1}{16a(\cos(fx+e)-1)^2} - \frac{7a+4b}{16a^2(\cos(fx+e)-1)} + \frac{\left(a^2+ab+\right.}{}}{f}$
risch	$\frac{ix}{a-b} - \frac{2ix}{a} - \frac{2ie}{af} - \frac{2ibx}{a^2} - \frac{2ibe}{a^2f} - \frac{2ib^2x}{a^3} - \frac{2ib^2e}{a^3f} - \frac{2ib^3x}{(a-b)a^3} - \frac{2ib^3e}{(a-b)a^3f} - \frac{2(2a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/16/a/(cos(f*x+e)+1)^2-1/16*(-7*a-4*b)/a^2/(cos(f*x+e)+1)+1/2*(a^2+a
*b+b^2)/a^3*ln(cos(f*x+e)+1)-1/16/a/(cos(f*x+e)-1)^2-1/16*(7*a+4*b)/a^2/(co
s(f*x+e)-1)+1/2*(a^2+a*b+b^2)/a^3*ln(cos(f*x+e)-1)+1/2*b^3/a^3/(a-b)*ln(a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b))
```

Maxima [A]
time = 0.28, size = 100, normalized size = 0.87

$$\frac{\frac{2\,b^3\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^4-a^3b}+\frac{2\left(a^2+ab+b^2\right)\log\left(\sin(fx+e)^2\right)}{a^3}+\frac{2\,(2\,a+b)\sin(fx+e)^2-a}{a^2\sin(fx+e)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^3*log(-(a - b)*sin(f*x + e)^2 + a)/(a^4 - a^3*b) + 2*(a^2 + a*b +
b^2)*log(sin(f*x + e)^2)/a^3 + (2*(2*a + b)*sin(f*x + e)^2 - a)/(a^2*sin(f*
x + e)^4))/f
```

Fricas [A]
time = 3.36, size = 172, normalized size = 1.50

$$\frac{2\,b^3\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^4+2\left(a^3-b^3\right)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^4+\left(3\,a^3-a^2b-2\,ab^2\right)\tan\left(fx+e\right)^4-a^3+a^2b+2\left(a^3-ab^2\right)\tan\left(fx+e\right)^2}{4\left(a^4-a^3b\right)f\tan\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4
+ 2*(a^3 - b^3)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (
3*a^3 - a^2*b - 2*a*b^2)*tan(f*x + e)^4 - a^3 + a^2*b + 2*(a^3 - a*b^2)*tan
(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(114) = 228.
time = 1.17, size = 408, normalized size = 3.55

$$\frac{\frac{32\,b^3\log\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4-a^3b}-\frac{64\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a-b}-\frac{\frac{12\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2}+\frac{32\left(a^2+ab+b^2\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3}-\frac{\left(a^2+\frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{48\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{48\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{a^3(\cos(fx+e)-1)^2}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")

[Out] 1/64*(32*b^3*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 - a^3*b) - 64*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(a - b) - (12*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 + 32*(a^2 + a*b + b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^3 - (a^2 + 12*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(a^3*(cos(f*x + e) - 1)^2))/f

Mupad [B]
time = 11.78, size = 118, normalized size = 1.03

$$\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+a\,b+b^2\right)}{a^3\,f}-\frac{\ln\left({\tan(e+f\,x)}^2+1\right)}{2\,f\,\left(a-b\right)}-\frac{b^3\,\ln\left(b\,{\tan(e+f\,x)}^2+a\right)}{f\,\left(2\,a^3\,b-2\,a^4\right)}-\frac{{\cot(e+f\,x)}^4\,\left(\frac{1}{4\,a}-\frac{{\tan(e+f\,x)}^2\,\left(a+b\right)}{2\,a^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2),x)

[Out] (log(tan(e + f*x))*(a*b + a^2 + b^2))/(a^3*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) - (b^3*log(a + b*tan(e + f*x)^2))/(f*(2*a^3*b - 2*a^4)) - (cot(e + f*x)^4*(1/(4*a) - (tan(e + f*x)^2*(a + b))/(2*a^2)))/f

3.217 $\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$-\frac{x}{a-b}+\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{5/2}f}-\frac{(a+b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] -x/(a-b)+a^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)/b^(5/2)/f-(a+b)*t
an(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Rubi [A]
time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 490, 596, 536, 209, 211}

$$\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}f(a-b)}-\frac{(a+b)\tan(e+fx)}{b^2f}-\frac{x}{a-b}+\frac{\tan^3(e+fx)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (a^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*b^
(5/2)*f) - ((a + b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{x^2(3a+3(a+b)x^2)}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{3bf}\\
&= -\frac{(a+b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} + \frac{\text{Subst}\left(\int \frac{3a(a+b)+3(a^2+ab+b^2)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(\right.}{3b^2f}\\
&= -\frac{(a+b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} + \frac{a^3\text{S}}{}\\
&= -\frac{x}{a-b} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{5/2}f} - \frac{(a+b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf}
\end{aligned}$$

Mathematica [A]
time = 0.57, size = 92, normalized size = 1.08

$$\frac{-3a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\left(3b^2(e+fx)+(a-b)\left(3a+4b-b\sec^2(e+fx)\right)\tan(e+fx)\right)}{3b^{5/2}(-a+b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*a^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[b]*(3*b^2*(e + f*
x) + (a - b)*(3*a + 4*b - b*Sec[e + f*x]^2)*Tan[e + f*x]))/(3*b^(5/2)*(-a +
 b)*f)
```

Maple [A]
time = 0.19, size = 88, normalized size = 1.04

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)+b\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{a^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{b^2(a-b)\sqrt{ab}}}{f}$
default	$\frac{-\frac{-\frac{b\left(\tan^3(fx+e)\right)}{3}+a\tan(fx+e)+b\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{a^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{b^2(a-b)\sqrt{ab}}}{f}$
risch	$-\frac{x}{a-b}-\frac{2i\left(3a\,e^{4i(fx+e)}+6b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}+3a+4b\right)}{3f\,b^2\left(e^{2i(fx+e)}+1\right)^3}-\frac{\sqrt{-ab}\,a^2\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-a}}{a-}\right.}{2b^3(a-b)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)+b*tan(f*x+e))-1/(a-b)*arctan(
tan(f*x+e))+1/b^2*a^3/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 87, normalized size = 1.02

$$\frac{\frac{3\,a^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(ab^2-b^3)\sqrt{ab}}-\frac{3\,(fx+e)}{a-b}+\frac{b\tan(fx+e)^3-3\,(a+b)\tan(fx+e)}{b^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

[Out] 1/3*(3*a^3*arctan(b*tan(f*x + e)/sqrt(a*b))/((a*b^2 - b^3)*sqrt(a*b)) - 3*(
f*x + e)/(a - b) + (b*tan(f*x + e)^3 - 3*(a + b)*tan(f*x + e))/b^2)/f

Fricas [A]
time = 4.95, size = 290, normalized size = 3.41

$$\left[-\frac{12 b^{2} f x-4\left(a b-b^{2}\right) \tan \left(f x+e\right)^{3}+3 a^{2} \sqrt{-\frac{a}{b}} \log \left(\frac{b^{2} \tan \left(f x+e\right)^{4}-6 a b \tan \left(f x+e\right)^{2}+a^{2}-4\left(b^{2} \tan \left(f x+e\right)^{3}-a b \tan \left(f x+e\right)\right) \sqrt{-\frac{a}{b}}}{b^{2} \tan \left(f x+e\right)^{4}+2 a b \tan \left(f x+e\right)^{2}+a^{2}}\right)+12\left(a^{2}-b^{2}\right) \tan \left(f x+e\right)}{12\left(a b^{2}-b^{3}\right) f},-\frac{6 b^{2} f x-2\left(a b-b^{2}\right) \tan \left(f x+e\right)^{3}-3 a^{2} \sqrt{\frac{a}{b}} \arctan \left(\frac{\left(b \tan \left(f x+e\right)^{2}-a\right) \sqrt{\frac{a}{b}}}{2 a \tan \left(f x+e\right)}\right)+6\left(a^{2}-b^{2}\right) \tan \left(f x+e\right)}{6\left(a b^{2}-b^{3}\right) f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")

[Out] [-1/12*(12*b^2*f*x - 4*(a*b - b^2)*tan(f*x + e)^3 + 3*a^2*sqrt(-a/b)*log((b
^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b^2*tan(f*x + e)^3 - a*
b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^
2)) + 12*(a^2 - b^2)*tan(f*x + e))/((a*b^2 - b^3)*f), -1/6*(6*b^2*f*x - 2*(
a*b - b^2)*tan(f*x + e)^3 - 3*a^2*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 -
a)*sqrt(a/b)/(a*tan(f*x + e))) + 6*(a^2 - b^2)*tan(f*x + e))/((a*b^2 - b^3)
*f)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(66) = 132.
time = 19.89, size = 595, normalized size = 7.00

$$\begin{cases} \tilde{\infty} x \tan^{4}{\left(e \right)} & \text{for}\: a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{- x + \frac{\tan^{5}{\left(e + f x \right)}}{5 f} - \frac{\tan^{3}{\left(e + f x \right)}}{3 f} + \frac{\tan{\left(e + f x \right)}}{f}}{a} & \text{for}\: b = 0 \\ \frac{x + \frac{\tan^{3}{\left(e + f x \right)}}{3 f} - \frac{\tan{\left(e + f x \right)}}{f}}{b} & \text{for}\: a = 0 \\ \frac{15 f x \tan^{2}{\left(e + f x \right)}}{6 b f \tan^{2}{\left(e + f x \right)} + 6 b f} + \frac{15 f x}{6 b f \tan^{2}{\left(e + f x \right)} + 6 b f} + \frac{2 \tan^{5}{\left(e + f x \right)}}{6 b f \tan^{2}{\left(e + f x \right)} + 6 b f} - \frac{10 \tan^{3}{\left(e + f x \right)}}{6 b f \tan^{2}{\left(e + f x \right)} + 6 b f} - \frac{15 \tan{\left(e + f x \right)}}{6 b f \tan^{2}{\left(e + f x \right)} + 6 b f} & \text{for}\: a = b \\ \frac{x \tan^{6}{\left(e \right)}}{a + b \tan^{2}{\left(e \right)}} & \text{for}\: f = 0 \\ \frac{3 a^{3} \log{\left(- \sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)}}{6 a b^{3} f \sqrt{- \frac{a}{b}} - 6 b^{4} f \sqrt{- \frac{a}{b}}} - \frac{3 a^{3} \log{\left(\sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)}}{6 a b^{3} f \sqrt{- \frac{a}{b}} - 6 b^{4} f \sqrt{- \frac{a}{b}}} - \frac{6 a^{2} b \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}}{6 a b^{3} f \sqrt{- \frac{a}{b}} - 6 b^{4} f \sqrt{- \frac{a}{b}}} + \frac{2 a b^{2} \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + f x \right)}}{6 a b^{3} f \sqrt{- \frac{a}{b}} - 6 b^{4} f \sqrt{- \frac{a}{b}}} - \frac{6 b^{3} f x \sqrt{- \frac{a}{b}}}{6 a b^{3} f \sqrt{- \frac{a}{b}} - 6 b^{4} f \sqrt{- \frac{a}{b}}} - \frac{2 b^{3} \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + f x \right)}}{6 a b^{3} f \sqrt{- \frac{a}{b}} - 6 b^{4} f \sqrt{- \frac{a}{b}}} + \frac{6 b^{3} \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}}{6 a b^{3} f \sqrt{- \frac{a}{b}} - 6 b^{4} f \sqrt{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2),x)

[Out] Piecewise((zoo*x*tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e +
 f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a, Eq(b, 0)), ((x
+ tan(e + f*x)**3/(3*f) - tan(e + f*x)/f)/b, Eq(a, 0)), (15*f*x*tan(e + f*x
)**2/(6*b*f*tan(e + f*x)**2 + 6*b*f) + 15*f*x/(6*b*f*tan(e + f*x)**2 + 6*b*
f) + 2*tan(e + f*x)**5/(6*b*f*tan(e + f*x)**2 + 6*b*f) - 10*tan(e + f*x)**3
/(6*b*f*tan(e + f*x)**2 + 6*b*f) - 15*tan(e + f*x)/(6*b*f*tan(e + f*x)**2 +
 6*b*f), Eq(a, b)), (x*tan(e)**6/(a + b*tan(e)**2), Eq(f, 0)), (3*a**3*log(
-sqrt(-a/b) + tan(e + f*x))/(6*a*b**3*f*sqrt(-a/b) - 6*b**4*f*sqrt(-a/b)) -
 3*a**3*log(sqrt(-a/b) + tan(e + f*x))/(6*a*b**3*f*sqrt(-a/b) - 6*b**4*f*sq
rt(-a/b)) - 6*a**2*b*sqrt(-a/b)*tan(e + f*x)/(6*a*b**3*f*sqrt(-a/b) - 6*b**
4*f*sqrt(-a/b)) + 2*a*b**2*sqrt(-a/b)*tan(e + f*x)**3/(6*a*b**3*f*sqrt(-a/b
) - 6*b**4*f*sqrt(-a/b)) - 6*b**3*f*x*sqrt(-a/b)/(6*a*b**3*f*sqrt(-a/b) - 6

```
*b**4*f*sqrt(-a/b)) - 2*b**3*sqrt(-a/b)*tan(e + f*x)**3/(6*a*b**3*f*sqrt(-a
/b) - 6*b**4*f*sqrt(-a/b)) + 6*b**3*sqrt(-a/b)*tan(e + f*x)/(6*a*b**3*f*sqr
t(-a/b) - 6*b**4*f*sqrt(-a/b)), True))
```

Giac [A]
time = 2.12, size = 118, normalized size = 1.39

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)a^3}{(ab^2-b^3)\sqrt{ab}}-\frac{3\,(fx+e)}{a-b}+\frac{b^2\tan(fx+e)^3-3\,ab\tan(fx+e)-3\,b^2\tan(fx+e)}{b^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b)))*a^3/((a*b^2 - b^3)*sqrt(a*b)) - 3*(f*x + e)/(a - b) + (b^2*tan(f*x + e
)^3 - 3*a*b*tan(f*x + e) - 3*b^2*tan(f*x + e))/b^3)/f
```

Mupad [B]
time = 12.00, size = 1310, normalized size = 15.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2),x)
```

```
[Out] tan(e + f*x)^3/(3*b*f) + (2*atan((((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a
^4*b^3)/b^3 - (tan(e + f*x)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)/(
b^3*(2*a - 2*b)))*1i)/(2*a - 2*b) + (2*tan(e + f*x)*(a^6 + b^6))/b^3)/(2*a
- 2*b) - ((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*
x)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)/(b^3*(2*a - 2*b)))*1i)/(2*
a - 2*b) - (2*tan(e + f*x)*(a^6 + b^6))/b^3)/(2*a - 2*b))/((((((4*a*b^6 - 4
*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(4*a*b^7 - 4*b^8 + 4*
a^2*b^6 - 4*a^3*b^5)*2i)/(b^3*(2*a - 2*b)))*1i)/(2*a - 2*b) + (2*tan(e + f*
x)*(a^6 + b^6))/b^3)*1i)/(2*a - 2*b) - (2*(a^4*b + a^5 + a^3*b^2))/b^3 + ((
(((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(4*a*b^
7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)/(b^3*(2*a - 2*b)))*1i)/(2*a - 2*b) -
 (2*tan(e + f*x)*(a^6 + b^6))/b^3)*1i)/(2*a - 2*b))))/(f*(2*a - 2*b)) - (ta
n(e + f*x)*(a + b))/(b^2*f) - (atan(((((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 +
 4*a^4*b^3)/b^3 + (tan(e + f*x)*(-a^5*b^5)^(1/2)*(4*a*b^7 - 4*b^8 + 4*a^2*b
^6 - 4*a^3*b^5))/(b^3*(a*b^5 - b^6)))*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)) -
 (2*tan(e + f*x)*(a^6 + b^6))/b^3)*(-a^5*b^5)^(1/2)*1i)/(2*(a*b^5 - b^6)) -
 (((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(-a^
5*b^5)^(1/2)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5))/(b^3*(a*b^5 - b^6))
```

```
)*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)) + (2*tan(e + f*x)*(a^6 + b^6))/b^3)*(
-a^5*b^5)^(1/2)*1i)/(2*(a*b^5 - b^6)))/((((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^
4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(-a^5*b^5)^(1/2)*(4*a*b^7 - 4*b^8 + 4*a^
2*b^6 - 4*a^3*b^5))/(b^3*(a*b^5 - b^6)))*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)
) - (2*tan(e + f*x)*(a^6 + b^6))/b^3)*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)) -
 (2*(a^4*b + a^5 + a^3*b^2))/b^3 + (((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4
*a^4*b^3)/b^3 - (tan(e + f*x)*(-a^5*b^5)^(1/2)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6
 - 4*a^3*b^5))/(b^3*(a*b^5 - b^6)))*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)) + (
2*tan(e + f*x)*(a^6 + b^6))/b^3)*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6))))*(-a^
5*b^5)^(1/2)*1i)/(f*(a*b^5 - b^6))
```

3.218 $\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=63

$$\frac{x}{a-b}-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{3/2}f}+\frac{\tan(e+fx)}{bf}$$

```
[Out] x/(a-b)-a^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)/b^(3/2)/f+tan(f*x+
e)/b/f
```

Rubi [A]
time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3751, 490, 536, 209, 211}

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}f(a-b)}+\frac{x}{a-b}+\frac{\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] x/(a - b) - (a^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*b^(3/
2)*f) + Tan[e + f*x]/(b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{bf} \\
&= \frac{\tan(e+fx)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} - \frac{a^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e\right.}{(a-b)bf} \\
&= \frac{x}{a-b} - \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{3/2}f} + \frac{\tan(e+fx)}{bf}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 70, normalized size = 1.11

$$
\frac{e+fx}{(a-b)f} - \frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{3/2}f} + \frac{\tan(e+fx)}{bf}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (e + f*x)/((a - b)*f) - (a^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((
a - b)*b^(3/2)*f) + Tan[e + f*x]/(b*f)
```

Maple [A]
time = 0.17, size = 65, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a-b}-\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{b(a-b)\sqrt{ab}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a-b}-\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{b(a-b)\sqrt{ab}}}{f}$
risch	$\frac{x}{a-b}+\frac{2i}{fb\left(e^{2i(fx+e)}+1\right)}+\frac{\sqrt{-ab}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2b^2(a-b)f}-\frac{\sqrt{-ab}\,a\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}}{a-b}\right.}{2b^2(a-b)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/b*tan(f*x+e)+1/(a-b)*arctan(tan(f*x+e))-1/b*a^2/(a-b)/(a*b)^(1/2)*ar
ctan(b*tan(f*x+e)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 68, normalized size = 1.08

$$-\frac{\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(ab-b^2)\sqrt{ab}}-\frac{fx+e}{a-b}-\frac{\tan(fx+e)}{b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(a^2*arctan(b*tan(f*x + e)/sqrt(a*b))/((a*b - b^2)*sqrt(a*b)) - (f*x + e)/
(a - b) - tan(f*x + e)/b)/f
```

Fricas [A]
time = 3.31, size = 230, normalized size = 3.65

$$\left[\frac{4\,bfx-a\sqrt{-\frac{a}{b}}\,\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\left(b^2\tan(fx+e)^3-ab\tan(fx+e)\right)\sqrt{-\frac{a}{b}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)+4\,(a-b)\tan\left(fx+e\right)}{4\,(ab-b^2)f},\ \frac{2\,bfx-a\sqrt{\frac{a}{b}}\,\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{a}{b}}}{2\,a\tan(fx+e)}\right)+2\,(a-b)\tan\left(fx+e\right)}{2\,(ab-b^2)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*f*x - a*sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2
 + a^2 + 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x
 + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) + 4*(a - b)*tan(f*x + e))/((a*b - b^
2)*f), 1/2*(2*b*f*x - a*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/
b)/(a*tan(f*x + e))) + 2*(a - b)*tan(f*x + e))/((a*b - b^2)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(48) = 96$.
time = 3.71, size = 427, normalized size = 6.78

$$\begin{cases} \tilde{\infty} x \tan^{2}{\left(e \right)} & \text{for}\: a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{x + \frac{\tan^{3}{\left(e + f x \right)}}{3 f} - \frac{\tan{\left(e + f x \right)}}{f}}{a} & \text{for}\: b = 0 \\ \frac{- x + \frac{\tan{\left(e + f x \right)}}{f}}{b} & \text{for}\: a = 0 \\ - \frac{3 f x \tan^{2}{\left(e + f x \right)}}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} - \frac{3 f x}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} + \frac{2 \tan^{3}{\left(e + f x \right)}}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} + \frac{3 \tan{\left(e + f x \right)}}{2 b f \tan^{2}{\left(e + f x \right)} + 2 b f} & \text{for}\: a = b \\ \frac{x \tan^{4}{\left(e \right)}}{a + b \tan^{2}{\left(e \right)}} & \text{for}\: f = 0 \\ - \frac{a^{2} \log{\left(- \sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)}}{2 a b^{2} f \sqrt{- \frac{a}{b}} - 2 b^{3} f \sqrt{- \frac{a}{b}}} + \frac{a^{2} \log{\left(\sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)}}{2 a b^{2} f \sqrt{- \frac{a}{b}} - 2 b^{3} f \sqrt{- \frac{a}{b}}} + \frac{2 a b \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}}{2 a b^{2} f \sqrt{- \frac{a}{b}} - 2 b^{3} f \sqrt{- \frac{a}{b}}} + \frac{2 b^{2} f x \sqrt{- \frac{a}{b}}}{2 a b^{2} f \sqrt{- \frac{a}{b}} - 2 b^{3} f \sqrt{- \frac{a}{b}}} - \frac{2 b^{2} \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}}{2 a b^{2} f \sqrt{- \frac{a}{b}} - 2 b^{3} f \sqrt{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((x + tan(e +
f*x)**3/(3*f) - tan(e + f*x)/f)/a, Eq(b, 0)), ((-x + tan(e + f*x)/f)/b, Eq(
a, 0)), (-3*f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) - 3*f*x/(2*
b*f*tan(e + f*x)**2 + 2*b*f) + 2*tan(e + f*x)**3/(2*b*f*tan(e + f*x)**2 + 2
*b*f) + 3*tan(e + f*x)/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), (x*tan(e
)**4/(a + b*tan(e)**2), Eq(f, 0)), (-a**2*log(-sqrt(-a/b) + tan(e + f*x))/(
2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)) + a**2*log(sqrt(-a/b) + tan(e
+ f*x))/(2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)) + 2*a*b*sqrt(-a/b)*ta
n(e + f*x)/(2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)) + 2*b**2*f*x*sqrt(
-a/b)/(2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)) - 2*b**2*sqrt(-a/b)*tan
(e + f*x)/(2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)), True))
```

Giac [A]
time = 1.07, size = 86, normalized size = 1.37

$$-\frac{\frac{\left(\pi \left\lfloor \frac{f x + e}{\pi} + \frac{1}{2} \right\rfloor \mathrm{sgn}\left(b\right) + \arctan\left(\frac{b \tan\left(f x + e\right)}{\sqrt{a b}}\right)\right)a^{2}}{\left(a b - b^{2}\right)\sqrt{a b}} - \frac{f x + e}{a - b} - \frac{\tan\left(f x + e\right)}{b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*
a^2/((a*b - b^2)*sqrt(a*b)) - (f*x + e)/(a - b) - tan(f*x + e)/b)/f

Mupad [B]
time = 12.18, size = 1212, normalized size = 19.24

$$\frac{\tan(e+fx)}{bf}-\frac{2\,\mathrm{atan}\left(\frac{\frac{\frac{2\tan(e+fx)\left(a^4+b^4\right)}{b}+\frac{\left(\frac{4a^3b^2-8a^2b^3+4ab^4}{b}-\frac{\tan(e+fx)\left(-4a^3b^3+4a^2b^4+4ab^5-4b^6\right)2i}{b\,(2a-2b)}\right)1i}{2a-2b}}{2a-2b}-\frac{-\frac{2\tan(e+fx)\left(a^4+b^4\right)}{b}+\frac{\left(\frac{4a^3b^2-8a^2b^3+4ab^4}{b}+\frac{\tan(e+fx)\left(-4a^3b^3+4a^2b^4+4ab^5-4b^6\right)2i}{b\,(2a-2b)}\right)1i}{2a-2b}}{2a-2b}}{-\frac{2\left(a^3+ba^2\right)}{b}+\frac{\left(\frac{2\tan(e+fx)\left(a^4+b^4\right)}{b}+\frac{\left(\frac{4a^3b^2-8a^2b^3+4ab^4}{b}-\frac{\tan(e+fx)\left(-4a^3b^3+4a^2b^4+4ab^5-4b^6\right)2i}{b\,(2a-2b)}\right)1i}{2a-2b}\right)1i}{2a-2b}+\frac{\left(-\frac{2\tan(e+fx)\left(a^4+b^4\right)}{b}+\frac{\left(\frac{4a^3b^2-8a^2b^3+4ab^4}{b}+\frac{\tan(e+fx)\left(-4a^3b^3+4a^2b^4+4ab^5-4b^6\right)2i}{b\,(2a-2b)}\right)1i}{2a-2b}\right)1i}{2a-2b}}\right)}{f\,(2a-2b)}+\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-a^3b^3}\left(\frac{\left(\frac{4a^3b^2-8a^2b^3+4ab^4}{b}+\frac{\tan(e+fx)\sqrt{-a^3b^3}\left(-4a^3b^3+4a^2b^4+4ab^5-4b^6\right)}{b\left(ab^3-b^4\right)}\right)\sqrt{-a^3b^3}}{2\left(ab^3-b^4\right)}-\frac{2\tan(e+fx)\left(a^4+b^4\right)}{b}\right)1i}{2\left(ab^3-b^4\right)}-\frac{\sqrt{-a^3b^3}\left(\frac{\left(\frac{4a^3b^2-8a^2b^3+4ab^4}{b}-\frac{\tan(e+fx)\sqrt{-a^3b^3}\left(-4a^3b^3+4a^2b^4+4ab^5-4b^6\right)}{b\left(ab^3-b^4\right)}\right)\sqrt{-a^3b^3}}{2\left(ab^3-b^4\right)}+\frac{2\tan(e+fx)\left(a^4+b^4\right)}{b}\right)1i}{2\left(ab^3-b^4\right)}}{\frac{\sqrt{-a^3b^3}\left(\frac{\left(\frac{4a^3b^2-8a^2b^3+4ab^4}{b}+\frac{\tan(e+fx)\sqrt{-a^3b^3}\left(-4a^3b^3+4a^2b^4+4ab^5-4b^6\right)}{b\left(ab^3-b^4\right)}\right)\sqrt{-a^3b^3}}{2\left(ab^3-b^4\right)}-\frac{2\tan(e+fx)\left(a^4+b^4\right)}{b}\right)}{2\left(ab^3-b^4\right)}-\frac{2\left(a^3+ba^2\right)}{b}+\frac{\sqrt{-a^3b^3}\left(\frac{\left(\frac{4a^3b^2-8a^2b^3+4ab^4}{b}-\frac{\tan(e+fx)\sqrt{-a^3b^3}\left(-4a^3b^3+4a^2b^4+4ab^5-4b^6\right)}{b\left(ab^3-b^4\right)}\right)\sqrt{-a^3b^3}}{2\left(ab^3-b^4\right)}+\frac{2\tan(e+fx)\left(a^4+b^4\right)}{b}\right)}{2\left(ab^3-b^4\right)}}\right)\sqrt{-a^3b^3}\,1i}{f\left(ab^3-b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2),x)

[Out] tan(e + f*x)/(b*f) - (2*atan((((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (tan
(e + f*x)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i
)/(2*a - 2*b) + (2*tan(e + f*x)*(a^4 + b^4))/b)/(2*a - 2*b) - ((((4*a*b^4 -
 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*
a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i)/(2*a - 2*b) - (2*tan(e + f*x)*(a^4 + b^4)
)/b)/(2*a - 2*b))/((((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (tan(e + f*x)*
(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i)/(2*a - 2
*b) + (2*tan(e + f*x)*(a^4 + b^4))/b)*1i)/(2*a - 2*b) - (2*(a^2*b + a^3))/b
 + (((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)*(4*a*b^5 - 4*b^6
 + 4*a^2*b^4 - 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i)/(2*a - 2*b) - (2*tan(e +
 f*x)*(a^4 + b^4))/b)*1i)/(2*a - 2*b))))/(f*(2*a - 2*b)) + (atan((((-a^3*b^
3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)*(-a^3*b^3)^
(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3))/(b*(a*b^3 - b^4)))*(-a^3*b
^3)^(1/2))/(2*(a*b^3 - b^4)) - (2*tan(e + f*x)*(a^4 + b^4))/b)*1i)/(2*(a*b^
3 - b^4)) - ((-a^3*b^3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (tan
(e + f*x)*(-a^3*b^3)^(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3))/(b*(a
b^3 - b^4)))(-a^3*b^3)^(1/2))/(2*(a*b^3 - b^4)) + (2*tan(e + f*x)*(a^4 +
b^4))/b)*1i)/(2*(a*b^3 - b^4)))/(((-a^3*b^3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3
+ 4*a^3*b^2)/b + (tan(e + f*x)*(-a^3*b^3)^(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^
4 - 4*a^3*b^3))/(b*(a*b^3 - b^4)))*(-a^3*b^3)^(1/2))/(2*(a*b^3 - b^4)) - (2
*tan(e + f*x)*(a^4 + b^4))/b))/(2*(a*b^3 - b^4)) - (2*(a^2*b + a^3))/b + ((
-a^3*b^3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (tan(e + f*x)*(-a^
3*b^3)^(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3))/(b*(a*b^3 - b^4)))*
(-a^3*b^3)^(1/2))/(2*(a*b^3 - b^4)) + (2*tan(e + f*x)*(a^4 + b^4))/b))/(2*(
a*b^3 - b^4))))*(-a^3*b^3)^(1/2)*1i)/(f*(a*b^3 - b^4))

3.219 $\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$-\frac{x}{a-b}+\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}\,f}$$

```
[Out] -x/(a-b)+arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*a^(1/2)/(a-b)/f/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3751, 492, 209, 211}

$$\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}\,f(a-b)}-\frac{x}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*Sq
rt[b]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} + \frac{a\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f}$$
$$= -\frac{x}{a-b} + \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}\,f}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.98

$$\frac{\text{ArcTan}(\tan(e+fx)) - \frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}}}{-af+bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (ArcTan[Tan[e + f*x]] - (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sq
rt[b])/(-(a*f) + b*f)
```

Maple [A]
time = 0.14, size = 50, normalized size = 1.00

method	result	size
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50

default	$\frac{-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
risch	$-\frac{x}{a-b}-\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2b(a-b)f}+\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2b(a-b)f}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a-b)*arctan(tan(f*x+e))+a/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a
*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 49, normalized size = 0.98

$$\frac{\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}-\frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (a*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b)
)/f
```

Fricas [A]
time = 2.60, size = 189, normalized size = 3.78

$$\left[-\frac{4\,fx+\sqrt{-\frac{a}{b}}\,\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2-4\left(b^2\tan(fx+e)^3-ab\tan(fx+e)\right)\sqrt{-\frac{a}{b}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{4\,(a-b)f},\ -\frac{2\,fx-\sqrt{\frac{a}{b}}\,\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{a}{b}}}{2\,a\tan(fx+e)}\right)}{2\,(a-b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*f*x + sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), -1/2*(2*f*x - sqrt(a/b)*a
rctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x + e))))/((a - b)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(37) = 74$.

time = 1.24, size = 252, normalized size = 5.04

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf} + \frac{fx}{2bf\tan^2(e+fx)+2bf} - \frac{\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} & \text{for } a = b \\ \frac{x\tan^2(e)}{a+b\tan^2(e)} & \text{for } f = 0 \\ \frac{-x+\frac{\tan(e+fx)}{f}}{a} & \text{for } b = 0 \\ \frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2abf\sqrt{-\frac{a}{b}}-2b^2f\sqrt{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2abf\sqrt{-\frac{a}{b}}-2b^2f\sqrt{-\frac{a}{b}}} - \frac{2bfx\sqrt{-\frac{a}{b}}}{2abf\sqrt{-\frac{a}{b}}-2b^2f\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/b, Eq(a, 0)), (f*x*ta
n(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) + f*x/(2*b*f*tan(e + f*x)**2
+ 2*b*f) - tan(e + f*x)/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), (x*tan(
e)**2/(a + b*tan(e)**2), Eq(f, 0)), ((-x + tan(e + f*x)/f)/a, Eq(b, 0)), (a
*log(-sqrt(-a/b) + tan(e + f*x))/(2*a*b*f*sqrt(-a/b) - 2*b**2*f*sqrt(-a/b))
 - a*log(sqrt(-a/b) + tan(e + f*x))/(2*a*b*f*sqrt(-a/b) - 2*b**2*f*sqrt(-a/
b)) - 2*b*f*x*sqrt(-a/b)/(2*a*b*f*sqrt(-a/b) - 2*b**2*f*sqrt(-a/b)), True))
```

Giac [A]

time = 0.75, size = 67, normalized size = 1.34

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)a}{\sqrt{ab}\,(a-b)} - \frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*a
/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b))/f
```

Mupad [B]

time = 11.57, size = 135, normalized size = 2.70

$$-\frac{2\,\mathrm{atan}\left(\frac{\tan(e+f\,x)\,\left(2\,a^2\,b+2\,b^3\right)+\frac{\tan(e+f\,x)\,\left(-8\,a^3\,b^2+8\,a^2\,b^3+8\,a\,b^4-8\,b^5\right)}{(2\,a-2\,b)^2}}{a\,b\,(2\,a-2\,b)}\right)}{f\,\left(2\,a-2\,b\right)} - \frac{\mathrm{atanh}\left(\frac{\tan(e+f\,x)\,\sqrt{-a\,b}}{a}\right)\,\sqrt{-a\,b}}{f\,\left(a\,b-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2),x)
```

```
[Out] - (2*atan((tan(e + f*x)*(2*a^2*b + 2*b^3) + (tan(e + f*x)*(8*a*b^4 - 8*b^5
+ 8*a^2*b^3 - 8*a^3*b^2))/(2*a - 2*b)^2)/(a*b*(2*a - 2*b))))/(f*(2*a - 2*b)
) - (atanh((tan(e + f*x)*(-a*b)^(1/2))/a)*(-a*b)^(1/2))/(f*(a*b - b^2))
```

3.220 $\int \frac{1}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)f}$$

```
[Out] x/(a-b)-arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/(a-b)/f/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3741, 3756, 211}

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3741

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \frac{x}{a-b} - \frac{b\int \frac{\sec^2(e+fx)}{a+b\tan^2(e+fx)}\,dx}{a-b}$$
$$= \frac{x}{a-b} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f}$$
$$= \frac{x}{a-b} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)f}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.98

$$\frac{\text{ArcTan}(\tan(e+fx)) - \frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}}{af-bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[e + f*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sq
rt[a])/(a*f - b*f)
```

Maple [A]
time = 0.00, size = 50, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
default	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)f}-\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)f}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)*arctan(tan(f*x+e))-1/(a-b)*b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(
a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 50, normalized size = 1.00

$$-\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}-\frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b
))/f
```

Fricas [A]
time = 2.78, size = 190, normalized size = 3.80

$$\left[\frac{4\,fx-\sqrt{-\frac{b}{a}}\,\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\,\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{4\,(a-b)f},\ \frac{2\,fx-\sqrt{\frac{b}{a}}\,\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)}{2\,(a-b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x - sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a
^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e
)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), 1/2*(2*f*x - sqrt(b/a)*arc
tan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a - b)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(37) = 74.
time = 1.23, size = 240, normalized size = 4.80

$$\begin{cases} \frac{\tilde{\infty}x}{\tan^2(e)} & \text{for } a=0\wedge b=0\wedge f=0 \\ \frac{-x-\frac{1}{f\tan(e+fx)}}{b} & \text{for } a=0 \\ \frac{fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}+\frac{fx}{2bf\tan^2(e+fx)+2bf}+\frac{\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} & \text{for } a=b \\ \frac{x}{a+b\tan^2(e)} & \text{for } f=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{2fx\sqrt{-\frac{a}{b}}}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}}-\frac{\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}}+\frac{\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x - 1/(f*ta
n(e + f*x)))/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*
b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan(e + f*x)/(2*b*f*tan(e + f*
x)**2 + 2*b*f), Eq(a, b)), (x/(a + b*tan(e)**2), Eq(f, 0)), (x/a, Eq(b, 0))
, (2*f*x*sqrt(-a/b)/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)) - log(-sqrt(-a/b)
 + tan(e + f*x))/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)) + log(sqrt(-a/b) + t
an(e + f*x))/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)), True))
```

Giac [A]
time = 0.55, size = 68, normalized size = 1.36

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,(a-b)}-\frac{fx+e}{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*
b/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b))/f
```

Mupad [B]
time = 12.30, size = 948, normalized size = 18.96

$$-\frac{\mathrm{atan}\left(\frac{\frac{-4b^3\tan(e+fx)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}}{2a-2b}+\frac{-4b^3\tan(e+fx)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}}{2a-2b}}{\frac{\left(-4b^3\tan(e+fx)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}\right)1i}{2a-2b}-\frac{\left(-4b^3\tan(e+fx)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(e+fx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1i}{2a-2b}\right)1i}{2a-2b}\right)1i}{2a-2b}}\right)}{f\,(a-b)}+\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-ab}\left(2b^3\tan(e+fx)-\frac{\sqrt{-ab}\left(2b^4-4ab^3+2a^2b^2+\frac{\tan(e+fx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4\left(ab-a^2\right)}\right)}{2\left(ab-a^2\right)}\right)1i}{ab-a^2}+\frac{\sqrt{-ab}\left(2b^3\tan(e+fx)-\frac{\sqrt{-ab}\left(4ab^3-2b^4-2a^2b^2+\frac{\tan(e+fx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4\left(ab-a^2\right)}\right)}{2\left(ab-a^2\right)}\right)1i}{ab-a^2}}{\frac{\sqrt{-ab}\left(2b^3\tan(e+fx)-\frac{\sqrt{-ab}\left(2b^4-4ab^3+2a^2b^2+\frac{\tan(e+fx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4\left(ab-a^2\right)}\right)}{2\left(ab-a^2\right)}\right)}{ab-a^2}-\frac{\sqrt{-ab}\left(2b^3\tan(e+fx)-\frac{\sqrt{-ab}\left(4ab^3-2b^4-2a^2b^2+\frac{\tan(e+fx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4\left(ab-a^2\right)}\right)}{2\left(ab-a^2\right)}\right)}{ab-a^2}}\right)\sqrt{-ab}\,1i}{a\,f\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 +
 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^
3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2) + ((-a*b)^(1/2)*
(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan(e +
 f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2
))))/(2*(a*b - a^2)))*1i)/(a*b - a^2))/(((-a*b)^(1/2)*(2*b^3*tan(e + f*x) -
 ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 + 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8
*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))
)/(a*b - a^2) - ((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3
- 2*b^4 - 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b
^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2))))/(a*b - a^2)))*(-a*b)^(
```

1/2)*1i)/(a*f*(a - b)) - atan(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))/(2*a - 2*b) + (((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))/(2*a - 2*b))/(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b) - ((((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b)))/(f*(a - b))

3.221 $\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=64

$$-\frac{x}{a-b}+\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)f}-\frac{\cot(e+fx)}{af}$$

```
[Out] -x/(a-b)+b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(3/2)/(a-b)/f-cot(f*x
+e)/a/f
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3751, 491, 536, 209, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f(a-b)}-\frac{x}{a-b}-\frac{\cot(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*(a
 - b)*f) - Cot[e + f*x]/(a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{af} + \frac{\text{Subst}\left(\int \frac{-a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{\cot(e+fx)}{af} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} + \frac{b^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan\right.}{a(a-b)f}$$

$$= -\frac{x}{a-b} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)f} - \frac{\cot(e+fx)}{af}$$

Mathematica [A]
time = 0.18, size = 68, normalized size = 1.06

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right) - \sqrt{a}\,(a(e+fx)+(a-b)\cot(e+fx))}{a^{3/2}(a-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*(a*(e + f*x) + (a - b)*Cot[e + f*x]))/(a^(3/2)*(a - b)*f)

Maple [A]
time = 0.27, size = 68, normalized size = 1.06

method	result
derivativedivides	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a(a-b)\sqrt{ab}}-\frac{1}{a\tan(fx+e)}-\frac{\arctan(\tan(fx+e))}{a-b}}{f}$
default	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a(a-b)\sqrt{ab}}-\frac{1}{a\tan(fx+e)}-\frac{\arctan(\tan(fx+e))}{a-b}}{f}$
risch	$-\frac{x}{a-b}-\frac{2i}{fa\left(e^{2i(fx+e)}-1\right)}-\frac{\sqrt{-ab}\, b\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a^2(a-b)f}+\frac{\sqrt{-ab}\, b\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}\ldots}{a-b}\right.}{2a^2(a-b)f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)

[Out] 1/f*(1/a*b^2/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/a/tan(f*x+e)-1/(a-b)*arctan(tan(f*x+e)))

Maxima [A]
time = 0.51, size = 68, normalized size = 1.06

$$\frac{\frac{b^2 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2-ab)\sqrt{ab}}-\frac{fx+e}{a-b}-\frac{1}{a\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")

[Out] (b^2*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2 - a*b)*sqrt(a*b)) - (f*x + e)/(a - b) - 1/(a*tan(f*x + e)))/f

Fricas [A]
time = 1.88, size = 257, normalized size = 4.02

$$\left[-\frac{4\,afx\tan\left(fx+e\right)+b\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2-4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)\tan\left(fx+e\right)+4\,a-4\,b}{4\left(a^2-ab\right)f\tan\left(fx+e\right)}, -\frac{2\,afx\tan\left(fx+e\right)-b\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)\tan\left(fx+e\right)+2\,a-2\,b}{2\left(a^2-ab\right)f\tan\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*f*x*tan(f*x + e) + b*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*
tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a)
)/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e) + 4*a - 4
*b)/((a^2 - a*b)*f*tan(f*x + e)), -1/2*(2*a*f*x*tan(f*x + e) - b*sqrt(b/a)*
arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e)))*tan(f*x + e)
+ 2*a - 2*b)/((a^2 - a*b)*f*tan(f*x + e))]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(48) = 96$.
time = 8.01, size = 522, normalized size = 8.16

$$\begin{cases} \tilde{\infty} x & \text{for}\: a = 0 \wedge b = 0 \wedge e = 0 \wedge f = 0 \\ \frac{- x - \frac{\cot{\left(e + f x \right)}}{f}}{a} & \text{for}\: b = 0 \\ \frac{x + \frac{1}{f \tan{\left(e + f x \right)}} - \frac{1}{3 f \tan^{3}{\left(e + f x \right)}}}{b} & \text{for}\: a = 0 \\ - \frac{3 f x \tan^{3}{\left(e + f x \right)}}{2 b f \tan^{3}{\left(e + f x \right)} + 2 b f \tan{\left(e + f x \right)}} - \frac{3 f x \tan{\left(e + f x \right)}}{2 b f \tan^{3}{\left(e + f x \right)} + 2 b f \tan{\left(e + f x \right)}} - \frac{3 \tan^{2}{\left(e + f x \right)}}{2 b f \tan^{3}{\left(e + f x \right)} + 2 b f \tan{\left(e + f x \right)}} - \frac{2}{2 b f \tan^{3}{\left(e + f x \right)} + 2 b f \tan{\left(e + f x \right)}} & \text{for}\: a = b \\ \frac{\tilde{\infty} x}{a} & \text{for}\: e = - f x \\ \frac{x \cot^{2}{\left(e \right)}}{a + b \tan^{2}{\left(e \right)}} & \text{for}\: f = 0 \\ - \frac{2 a f x \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}}{2 a^{2} f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)} - 2 a b f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}} - \frac{2 a \sqrt{- \frac{a}{b}}}{2 a^{2} f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)} - 2 a b f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}} + \frac{2 b \sqrt{- \frac{a}{b}}}{2 a^{2} f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)} - 2 a b f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}} + \frac{b \log{\left(- \sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)} \tan{\left(e + f x \right)}}{2 a^{2} f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)} - 2 a b f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}} - \frac{b \log{\left(\sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)} \tan{\left(e + f x \right)}}{2 a^{2} f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)} - 2 a b f \sqrt{- \frac{a}{b}} \tan{\left(e + f x \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-x - cot(e
+ f*x)/f)/a, Eq(b, 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3))
/b, Eq(a, 0)), (-3*f*x*tan(e + f*x)**3/(2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e
 + f*x)) - 3*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e + f*x))
- 3*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e + f*x)) - 2/(2*b*f
*tan(e + f*x)**3 + 2*b*f*tan(e + f*x)), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)),
(x*cot(e)**2/(a + b*tan(e)**2), Eq(f, 0)), (-2*a*f*x*sqrt(-a/b)*tan(e + f*x
)/(2*a**2*f*sqrt(-a/b)*tan(e + f*x) - 2*a*b*f*sqrt(-a/b)*tan(e + f*x)) - 2*
a*sqrt(-a/b)/(2*a**2*f*sqrt(-a/b)*tan(e + f*x) - 2*a*b*f*sqrt(-a/b)*tan(e +
 f*x)) + 2*b*sqrt(-a/b)/(2*a**2*f*sqrt(-a/b)*tan(e + f*x) - 2*a*b*f*sqrt(-a
/b)*tan(e + f*x)) + b*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)/(2*a**2*
f*sqrt(-a/b)*tan(e + f*x) - 2*a*b*f*sqrt(-a/b)*tan(e + f*x)) - b*log(sqrt(-
a/b) + tan(e + f*x))*tan(e + f*x)/(2*a**2*f*sqrt(-a/b)*tan(e + f*x) - 2*a*b
*f*sqrt(-a/b)*tan(e + f*x)), True))
```

Giac [A]
time = 0.83, size = 86, normalized size = 1.34

$$\frac{\frac{\left(\pi \left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b^2}{(a^2-ab)\sqrt{ab}}-\frac{fx+e}{a-b}-\frac{1}{a\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))*b
^2/((a^2 - a*b)*sqrt(a*b)) - (f*x + e)/(a - b) - 1/(a*tan(f*x + e)))/f
```

Mupad [B]
time = 11.77, size = 438, normalized size = 6.84

$$\frac{a^2\,b-a^3}{f\,\left(a^4\,\mathrm{tan}\left(e+f\,x\right)-a^3\,b\,\mathrm{tan}\left(e+f\,x\right)\right)}+\frac{\mathrm{atan}\left(\frac{a^6\,b\,\mathrm{tan}\left(e+f\,x\right)\,\sqrt{-a^3\,b^3}\,1{}\mathrm{i}-a^3\,b^4\,\mathrm{tan}\left(e+f\,x\right)\,\sqrt{-a^3\,b^3}\,1{}\mathrm{i}}{a^5\,b^5-a^8\,b^2}\right)\,\sqrt{-a^3\,b^3}\,1{}\mathrm{i}-a^3\,\mathrm{atan}\left(\frac{\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(2\,a^5\,b^3+2\,a^3\,b^5\right)+\frac{\left(4\,a^5\,b^4-4\,a^4\,b^5+4\,a^6\,b^3-4\,a^7\,b^2+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(8\,a^8\,b^2-8\,a^7\,b^3-8\,a^6\,b^4+8\,a^5\,b^5\right)\,1{}\mathrm{i}}{2\,a-2\,b}\right)\,1{}\mathrm{i}}{2\,a-2\,b}}{2\,a-2\,b}+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(2\,a^5\,b^3+2\,a^3\,b^5\right)+\frac{\left(4\,a^4\,b^5-4\,a^5\,b^4-4\,a^6\,b^3+4\,a^7\,b^2+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(8\,a^8\,b^2-8\,a^7\,b^3-8\,a^6\,b^4+8\,a^5\,b^5\right)\,1{}\mathrm{i}}{2\,a-2\,b}\right)\,1{}\mathrm{i}}{2\,a-2\,b}}{2\,a-2\,b}}{2\,a^5\,b^2+2\,a^4\,b^3+2\,a^3\,b^4}\right)}{f\,\left(a^3\,b-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (a^2*b - a^3)/(f*(a^4*tan(e + f*x) - a^3*b*tan(e + f*x))) + (atan((a^6*b*ta
n(e + f*x)*(-a^3*b^3)^(1/2)*1i - a^3*b^4*tan(e + f*x)*(-a^3*b^3)^(1/2)*1i)/
(a^5*b^5 - a^8*b^2))*(-a^3*b^3)^(1/2)*1i - a^3*atan(((((4*a^5*b^4 - 4*a^4*b
^5 + 4*a^6*b^3 - 4*a^7*b^2 + (tan(e + f*x)*(8*a^5*b^5 - 8*a^6*b^4 - 8*a^7*b
^3 + 8*a^8*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^3*b^5
+ 2*a^5*b^3))/(2*a - 2*b) + (((4*a^4*b^5 - 4*a^5*b^4 - 4*a^6*b^3 + 4*a^7*b^
2 + (tan(e + f*x)*(8*a^5*b^5 - 8*a^6*b^4 - 8*a^7*b^3 + 8*a^8*b^2)*1i)/(2*a
- 2*b))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^3*b^5 + 2*a^5*b^3))/(2*a - 2*b)
)/(2*a^3*b^4 + 2*a^4*b^3 + 2*a^5*b^2)))/(f*(a^3*b - a^4))
```

3.222 $\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$\frac{x}{a-b}-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)f}+\frac{(a+b)\cot(e+fx)}{a^2f}-\frac{\cot^3(e+fx)}{3af}$$

```
[Out] x/(a-b)-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(5/2)/(a-b)/f+(a+b)*co
t(f*x+e)/a^2/f-1/3*cot(f*x+e)^3/a/f
```

Rubi [A]
time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3751, 491, 597, 536, 209, 211}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f(a-b)}+\frac{(a+b)\cot(e+fx)}{a^2f}+\frac{x}{a-b}-\frac{\cot^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] x/(a - b) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*(a -
b)*f) + ((a + b)*Cot[e + f*x])/(a^2*f) - Cot[e + f*x]^3/(3*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{\cot^3(e+fx)}{3af} + \frac{\text{Subst}\left(\int \frac{-3(a+b)-3bx^2}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{3af} \\
&= \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af} - \frac{\text{Subst}\left(\int \frac{-3(a^2+ab+b^2)-3b(a+b)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e\right.}{3a^2f} \\
&= \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)f} - \frac{b^3\text{Sub}}{} \\
&= \frac{x}{a-b} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)f} + \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af}
\end{aligned}$$

Mathematica [A]
time = 0.48, size = 92, normalized size = 1.10

$$\frac{-3b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}\left(3a^2(e+fx)-(a-b)\cot(e+fx)\left(-4a-3b+a\csc^2(e+fx)\right)\right)}{3a^{5/2}(a-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (-3*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(3*a^2*(e + f*
x) - (a - b)*Cot[e + f*x]*(-4*a - 3*b + a*Csc[e + f*x]^2)))/(3*a^(5/2)*(a -
 b)*f)
```

Maple [A]
time = 0.31, size = 88, normalized size = 1.05

method	result
derivativedivides	$\frac{-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^2(a-b)\sqrt{ab}}-\frac{1}{3a\tan(fx+e)^3}-\frac{-a-b}{a^2\tan(fx+e)}+\frac{\arctan(\tan(fx+e))}{a-b}}{f}$
default	$\frac{-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^2(a-b)\sqrt{ab}}-\frac{1}{3a\tan(fx+e)^3}-\frac{-a-b}{a^2\tan(fx+e)}+\frac{\arctan(\tan(fx+e))}{a-b}}{f}$
risch	$\frac{x}{a-b}+\frac{2i\left(6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}-6b\,e^{2i(fx+e)}+4a+3b\right)}{3f\,a^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-ab}\,b^2\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}}{a-b}\right.}{2a^3(a-b)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/a^2*b^3/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/3/a/ta
n(f*x+e)^3-(-a-b)/a^2/tan(f*x+e)+1/(a-b)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.51, size = 90, normalized size = 1.07

$$-\frac{\frac{3\,b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-a^2b)\sqrt{ab}}-\frac{3\,(fx+e)}{a-b}-\frac{3\,(a+b)\tan(fx+e)^2-a}{a^2\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

[Out] -1/3*(3*b^3*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - a^2*b)*sqrt(a*b)) - 3*(f*x + e)/(a - b) - (3*(a + b)*tan(f*x + e)^2 - a)/(a^2*tan(f*x + e)^3))/f

Fricas [A]
time = 2.79, size = 324, normalized size = 3.86

$$\left[\frac{12\,a^{2}fx\tan\left(fx+e\right)^{3}-3\,b^{2}\sqrt{-\frac{b}{a}}\log\left(\frac{b^{2}\tan(fx+e)^{4}-6\,ab\tan(fx+e)^{2}+a^{2}+4\left(ab\tan(fx+e)^{3}-a^{2}\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^{2}\tan(fx+e)^{4}+2\,ab\tan(fx+e)^{2}+a^{2}}\right)\tan\left(fx+e\right)^{3}+12\left(a^{2}-b^{2}\right)\tan\left(fx+e\right)^{2}-4\,a^{2}+4\,ab}{12\left(a^{3}-a^{2}b\right)f\tan\left(fx+e\right)^{3}},\ \frac{6\,a^{2}fx\tan\left(fx+e\right)^{3}-3\,b^{2}\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^{2}-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)\tan\left(fx+e\right)^{3}+6\left(a^{2}-b^{2}\right)\tan\left(fx+e\right)^{2}-2\,a^{2}+2\,ab}{6\left(a^{3}-a^{2}b\right)f\tan\left(fx+e\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")

[Out] [1/12*(12*a^2*f*x*tan(f*x + e)^3 - 3*b^2*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e)^3 + 12*(a^2 - b^2)*tan(f*x + e)^2 - 4*a^2 + 4*a*b)/((a^3 - a^2*b)*f*tan(f*x + e)^3), 1/6*(6*a^2*f*x*tan(f*x + e)^3 - 3*b^2*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e)))*tan(f*x + e)^3 + 6*(a^2 - b^2)*tan(f*x + e)^2 - 2*a^2 + 2*a*b)/((a^3 - a^2*b)*f*tan(f*x + e)^3)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(66) = 132$.
time = 87.96, size = 775, normalized size = 9.23

$$\begin{cases} \tilde{\infty} x & \text{for}\: a = 0 \wedge b = 0 \wedge e = 0 \wedge f = 0 \\ \frac{x - \frac{\cot^{3}{\left(e + fx \right)}}{3f} + \frac{\cot{\left(e + fx \right)}}{f}}{a} & \text{for}\: b = 0 \\ \frac{- x - \frac{1}{f \tan{\left(e + fx \right)}} + \frac{1}{3f \tan^{3}{\left(e + fx \right)}} - \frac{1}{5f \tan^{5}{\left(e + fx \right)}}}{b} & \text{for}\: a = 0 \\ \frac{15fx \tan^{5}{\left(e + fx \right)}}{6bf \tan^{5}{\left(e + fx \right)} + 6bf \tan^{3}{\left(e + fx \right)}} + \frac{15fx \tan^{3}{\left(e + fx \right)}}{6bf \tan^{5}{\left(e + fx \right)} + 6bf \tan^{3}{\left(e + fx \right)}} + \frac{15 \tan^{4}{\left(e + fx \right)}}{6bf \tan^{5}{\left(e + fx \right)} + 6bf \tan^{3}{\left(e + fx \right)}} + \frac{10 \tan^{2}{\left(e + fx \right)}}{6bf \tan^{5}{\left(e + fx \right)} + 6bf \tan^{3}{\left(e + fx \right)}} - \frac{2}{6bf \tan^{5}{\left(e + fx \right)} + 6bf \tan^{3}{\left(e + fx \right)}} & \text{for}\: a = b \\ \frac{\tilde{\infty} x}{a} & \text{for}\: e = - fx \\ \frac{x \cot^{4}{\left(e \right)}}{a + b \tan^{2}{\left(e \right)}} & \text{for}\: f = 0 \\ \frac{6a^{2} fx \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)}}{6a^{3} f \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)} - 6a^{2} bf \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)}} + \frac{6a^{2} \sqrt{- \frac{a}{b}} \tan^{2}{\left(e + fx \right)}}{6a^{3} f \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)} - 6a^{2} bf \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)}} - \frac{2a^{2} \sqrt{- \frac{a}{b}}}{6a^{3} f \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)} - 6a^{2} bf \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)}} + \frac{2ab \sqrt{- \frac{a}{b}}}{6a^{3} f \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)} - 6a^{2} bf \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)}} - \frac{6b^{2} \sqrt{- \frac{a}{b}} \tan^{2}{\left(e + fx \right)}}{6a^{3} f \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)} - 6a^{2} bf \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)}} - \frac{3b^{2} \log{\left(- \sqrt{- \frac{a}{b}} + \tan{\left(e + fx \right)} \right)} \tan^{3}{\left(e + fx \right)}}{6a^{3} f \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)} - 6a^{2} bf \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)}} + \frac{3b^{2} \log{\left(\sqrt{- \frac{a}{b}} + \tan{\left(e + fx \right)} \right)} \tan^{3}{\left(e + fx \right)}}{6a^{3} f \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)} - 6a^{2} bf \sqrt{- \frac{a}{b}} \tan^{3}{\left(e + fx \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2),x)

[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((x - cot(e + f*x)**3/(3*f) + cot(e + f*x)/f)/a, Eq(b, 0)), ((-x - 1/(f*tan(e + f*x)) + 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e + f*x)**5))/b, Eq(a, 0)), (15*f*x*tan(e + f*x)**5/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) + 15*f*x*tan(e + f*x)**3/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) + 15*tan(e + f*x)**4/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) + 10*tan(e + f*x)**2/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) - 2/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)), (x*cot(e)**4/(a + b*tan(e)**2), Eq(f, 0)), (6*a**2*f*x*sqrt(-a/b)*tan(e + f*x)**3/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x)**3) + 6*a**2*sqrt(-a/b)*tan(e + f*x)**2/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x)**3) - 2*a**2*sqrt(-a/b)/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x)**3) + 2*a*b*sqrt(-a/b)/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x)

```
**3) - 6*b**2*sqrt(-a/b)*tan(e + f*x)**2/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)*
*3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x)**3) - 3*b**2*log(-sqrt(-a/b) + tan(
e + f*x))*tan(e + f*x)**3/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*b*f
*sqrt(-a/b)*tan(e + f*x)**3) + 3*b**2*log(sqrt(-a/b) + tan(e + f*x))*tan(e
+ f*x)**3/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(
e + f*x)**3), True))
```

Giac [A]
time = 1.00, size = 118, normalized size = 1.40

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b^3}{(a^3-a^2b)\sqrt{ab}}-\frac{3\,(fx+e)}{a-b}-\frac{3\,a\tan(fx+e)^2+3\,b\tan(fx+e)^2-a}{a^2\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b)))*b^3/((a^3 - a^2*b)*sqrt(a*b)) - 3*(f*x + e)/(a - b) - (3*a*tan(f*x +
e)^2 + 3*b*tan(f*x + e)^2 - a)/(a^2*tan(f*x + e)^3))/f
```

Mupad [B]
time = 12.20, size = 484, normalized size = 5.76

$$\frac{a^4\,b+{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(3\,a^5-3\,a^3\,b^2\right)-a^5}{f\,\left(3\,a^6\,{\mathrm{tan}\left(e+f\,x\right)}^3-3\,a^5\,b\,{\mathrm{tan}\left(e+f\,x\right)}^3\right)}-\frac{\mathrm{atan}\left(\frac{a^{10}\,b\,\mathrm{tan}\left(e+f\,x\right)\,\sqrt{-a^5\,b^5}\,1{}\mathrm{i}-a^5\,b^6\,\mathrm{tan}\left(e+f\,x\right)\,\sqrt{-a^5\,b^5}\,1{}\mathrm{i}}{a^8\,b^8-a^{13}\,b^3}\right)\,\sqrt{-a^5\,b^5}\,3{}\mathrm{i}-3\,a^5\,\mathrm{atan}\left(\frac{\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(2\,a^{10}\,b^3+2\,a^6\,b^7\right)+\frac{\left(4\,a^9\,b^5-4\,a^8\,b^6+4\,a^{11}\,b^3-4\,a^{12}\,b^2+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(8\,a^{13}\,b^2-8\,a^{12}\,b^3-8\,a^{11}\,b^4+8\,a^{10}\,b^5\right)\,1{}\mathrm{i}}{2\,a-2\,b}\right)\,1{}\mathrm{i}}{2\,a-2\,b}}{2\,a-2\,b}+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(2\,a^{10}\,b^3+2\,a^6\,b^7\right)+\frac{\left(4\,a^8\,b^6-4\,a^9\,b^5-4\,a^{11}\,b^3+4\,a^{12}\,b^2+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(8\,a^{13}\,b^2-8\,a^{12}\,b^3-8\,a^{11}\,b^4+8\,a^{10}\,b^5\right)\,1{}\mathrm{i}}{2\,a-2\,b}\right)\,1{}\mathrm{i}}{2\,a-2\,b}}{2\,a-2\,b}}{2\,a^{10}\,b^2+2\,a^9\,b^3+2\,a^8\,b^4+2\,a^7\,b^5+2\,a^6\,b^6}\right)}{f\,\left(3\,a^5\,b-3\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (a^4*b + tan(e + f*x)^2*(3*a^5 - 3*a^3*b^2) - a^5)/(f*(3*a^6*tan(e + f*x)^3
 - 3*a^5*b*tan(e + f*x)^3)) - (atan((a^10*b*tan(e + f*x)*(-a^5*b^5)^(1/2)*1
i - a^5*b^6*tan(e + f*x)*(-a^5*b^5)^(1/2)*1i)/(a^8*b^8 - a^13*b^3))*(-a^5*b
^5)^(1/2)*3i - 3*a^5*atan(((((4*a^9*b^5 - 4*a^8*b^6 + 4*a^11*b^3 - 4*a^12*b
^2 + (tan(e + f*x)*(8*a^10*b^5 - 8*a^11*b^4 - 8*a^12*b^3 + 8*a^13*b^2)*1i)/
(2*a - 2*b))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^6*b^7 + 2*a^10*b^3))/(2*a
- 2*b) + (((4*a^8*b^6 - 4*a^9*b^5 - 4*a^11*b^3 + 4*a^12*b^2 + (tan(e + f*x)
*(8*a^10*b^5 - 8*a^11*b^4 - 8*a^12*b^3 + 8*a^13*b^2)*1i)/(2*a - 2*b))*1i)/(
2*a - 2*b) + tan(e + f*x)*(2*a^6*b^7 + 2*a^10*b^3))/(2*a - 2*b))/(2*a^6*b^6
 + 2*a^7*b^5 + 2*a^8*b^4 + 2*a^9*b^3 + 2*a^10*b^2)))/(f*(3*a^5*b - 3*a^6))
```

3.223 $\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=113

$$-\frac{x}{a-b}+\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)f}-\frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f}+\frac{(a+b)\cot^3(e+fx)}{3a^2f}-\frac{\cot^5(e+fx)}{5af}$$

```
[Out] -x/(a-b)+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(7/2)/(a-b)/f-(a^2+a*
b+b^2)*cot(f*x+e)/a^3/f+1/3*(a+b)*cot(f*x+e)^3/a^2/f-1/5*cot(f*x+e)^5/a/f
```

Rubi [A]
time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 491, 597, 536, 209, 211}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f(a-b)}+\frac{(a+b)\cot^3(e+fx)}{3a^2f}-\frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f}-\frac{x}{a-b}-\frac{\cot^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] -(x/(a - b)) + (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(a
 - b)*f) - ((a^2 + a*b + b^2)*Cot[e + f*x])/(a^3*f) + ((a + b)*Cot[e + f*x]
^3)/(3*a^2*f) - Cot[e + f*x]^5/(5*a*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5af} + \frac{\text{Subst}\left(\int \frac{-5(a+b)-5bx^2}{x^4(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{5af}$$

$$= \frac{(a+b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} - \frac{\text{Subst}\left(\int \frac{-15(a^2+ab+b^2)-15b(a+b)x^2}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan\right.}{15a^2f}$$

$$= -\frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x}{a-b} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)f} - \frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3}{3a^2}$$

Mathematica [A]

time = 1.28, size = 121, normalized size = 1.07

$$\frac{15b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) + \sqrt{a}\left(-15a^3(e+fx) - (a-b)\cot(e+fx)\left(23a^2+20ab+15b^2-a(11a+5b)\csc^2(e+fx)+3a^2\csc^4(e+fx)\right)\right)}{15a^{7/2}(a-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

```
[Out] (15*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-15*a^3*(e +
f*x) - (a - b)*Cot[e + f*x]*(23*a^2 + 20*a*b + 15*b^2 - a*(11*a + 5*b)*Csc[
e + f*x]^2 + 3*a^2*Csc[e + f*x]^4)))/(15*a^(7/2)*(a - b)*f)
```

Maple [A]

time = 0.34, size = 111, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^3(a-b)\sqrt{ab}} - \frac{1}{5a\tan(fx+e)^5} - \frac{-a-b}{3a^2\tan(fx+e)^3} - \frac{a^2+ab+b^2}{a^3\tan(fx+e)} - \frac{\arctan(\tan(fx+e))}{a-b}}{f}$
default	$\frac{\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^3(a-b)\sqrt{ab}} - \frac{1}{5a\tan(fx+e)^5} - \frac{-a-b}{3a^2\tan(fx+e)^3} - \frac{a^2+ab+b^2}{a^3\tan(fx+e)} - \frac{\arctan(\tan(fx+e))}{a-b}}{f}$

risch	$-\frac{x}{a-b}-\frac{2i\left(45a^{2}\mathrm{e}^{8i(fx+e)}+30ab\,\mathrm{e}^{8i(fx+e)}+15b^{2}\mathrm{e}^{8i(fx+e)}-90a^{2}\mathrm{e}^{6i(fx+e)}-90ab\,\mathrm{e}^{6i(fx+e)}-60b^{2}\mathrm{e}^{6i(fx+e)}+140a^{2}\mathrm{e}^{4i}\right.}{15f\,a^{3}\left(\mathrm{e}^{2i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/a^3*b^4/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/5/a/tan
(f*x+e)^5-1/3*(-a-b)/a^2/tan(f*x+e)^3-(a^2+a*b+b^2)/a^3/tan(f*x+e)-1/(a-b)*
arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.49, size = 117, normalized size = 1.04

$$\frac{\frac{15\,b^{4}\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^{4}-a^{3}b)\sqrt{ab}}-\frac{15\,(fx+e)}{a-b}-\frac{15\,(a^{2}+ab+b^{2})\tan(fx+e)^{4}-5\,(a^{2}+ab)\tan(fx+e)^{2}+3\,a^{2}}{a^{3}\tan(fx+e)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b^4*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - a^3*b)*sqrt(a*b)) - 1
5*(f*x + e)/(a - b) - (15*(a^2 + a*b + b^2)*tan(f*x + e)^4 - 5*(a^2 + a*b)*
tan(f*x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

Fricas [A]
time = 4.31, size = 370, normalized size = 3.27

$$\left[-\frac{60\,a^{3}fx\tan(fx+e)^{5}+15\,b^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{b^{2}\tan(fx+e)^{4}-6\,ab\tan(fx+e)^{2}+a^{2}-4\,\left(ab\tan(fx+e)^{3}-a^{2}\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^{2}\tan(fx+e)^{4}+2\,ab\tan(fx+e)^{2}+a^{2}}\right)\tan(fx+e)^{5}+60\,(a^{3}-b^{3})\tan(fx+e)^{4}+12\,a^{3}-12\,a^{2}b-20\,(a^{3}-ab^{2})\tan(fx+e)^{2}}{60\,(a^{4}-a^{3}b)f\tan(fx+e)^{5}},\ -\frac{30\,a^{3}fx\tan(fx+e)^{5}-15\,b^{3}\sqrt{\frac{b}{a}}\arctan\left(\frac{(b\tan(fx+e)^{2}-a)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)\tan(fx+e)^{5}+30\,(a^{3}-b^{3})\tan(fx+e)^{4}+6\,a^{3}-6\,a^{2}b-10\,(a^{3}-ab^{2})\tan(fx+e)^{2}}{30\,(a^{4}-a^{3}b)f\tan(fx+e)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(60*a^3*f*x*tan(f*x + e)^5 + 15*b^3*sqrt(-b/a)*log((b^2*tan(f*x + e)
^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))
*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e
)^5 + 60*(a^3 - b^3)*tan(f*x + e)^4 + 12*a^3 - 12*a^2*b - 20*(a^3 - a*b^2)*
tan(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^5), -1/30*(30*a^3*f*x*tan(f*x
 + e)^5 - 15*b^3*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*t
an(f*x + e)))*tan(f*x + e)^5 + 30*(a^3 - b^3)*tan(f*x + e)^4 + 6*a^3 - 6*a^
2*b - 10*(a^3 - a*b^2)*tan(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.13, size = 164, normalized size = 1.45

$$\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)b^4}{(a^4-a^3b)\sqrt{ab}}-\frac{15(fx+e)}{a-b}-\frac{15a^2\tan(fx+e)^4+15ab\tan(fx+e)^4+15b^2\tan(fx+e)^4-5a^2\tan(fx+e)^2-5ab\tan(fx+e)^2+3a^2}{a^3\tan(fx+e)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b)))*b^4/((a^4 - a^3*b)*sqrt(a*b)) - 15*(f*x + e)/(a - b) - (15*a^2*tan(f
*x + e)^4 + 15*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 - 5*a^2*tan(f*x +
 e)^2 - 5*a*b*tan(f*x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

Mupad [B]
time = 13.74, size = 524, normalized size = 4.64

$$\frac{\mathrm{atan}\left(\frac{a^{14}b\tan(e+fx)\sqrt{-a^7b^7}\,1i-a^7b^8\tan(e+fx)\sqrt{-a^7b^7}\,1i}{a^{11}b^{11}-a^{18}b^4}\right)\sqrt{-a^7b^7}\,15i-15a^7\,\mathrm{atan}\left(\frac{\frac{\tan(e+fx)\left(2a^9b^9+2a^{15}b^3\right)+\frac{\left(4a^{13}b^6-4a^{12}b^7+4a^{16}b^3-4a^{17}b^2+\frac{\tan(e+fx)\left(8a^{15}b^5-8a^{16}b^4-8a^{17}b^3+8a^{18}b^2\right)1i}{2a-2b}\right)1i}{2a-2b}}{2a-2b}+\frac{\tan(e+fx)\left(2a^9b^9+2a^{15}b^3\right)+\frac{\left(4a^{12}b^7-4a^{13}b^6-4a^{16}b^3+4a^{17}b^2+\frac{\tan(e+fx)\left(8a^{15}b^5-8a^{16}b^4-8a^{17}b^3+8a^{18}b^2\right)1i}{2a-2b}\right)1i}{2a-2b}}{2a-2b}}{2a^{15}b^2+2a^{14}b^3+2a^{13}b^4+2a^{12}b^5+2a^{11}b^6+2a^{10}b^7+2a^9b^8}\right)}{f\left(15a^7b-15a^8\right)}+\frac{3a^6b+\tan(e+fx)^2\left(5a^7-5a^5b^2\right)-\tan(e+fx)^4\left(15a^7-15a^4b^3\right)-3a^7}{f\left(15a^8\tan(e+fx)^5-15a^7b\tan(e+fx)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2),x)
```

```
[Out] (atan((a^14*b*tan(e + f*x)*(-a^7*b^7)^(1/2)*1i - a^7*b^8*tan(e + f*x)*(-a^7
*b^7)^(1/2)*1i)/(a^11*b^11 - a^18*b^4))*(-a^7*b^7)^(1/2)*15i - 15*a^7*atan(
((((4*a^13*b^6 - 4*a^12*b^7 + 4*a^16*b^3 - 4*a^17*b^2 + (tan(e + f*x)*(8*a^
15*b^5 - 8*a^16*b^4 - 8*a^17*b^3 + 8*a^18*b^2)*1i)/(2*a - 2*b))*1i)/(2*a -
2*b) + tan(e + f*x)*(2*a^9*b^9 + 2*a^15*b^3))/(2*a - 2*b) + (((4*a^12*b^7 -
 4*a^13*b^6 - 4*a^16*b^3 + 4*a^17*b^2 + (tan(e + f*x)*(8*a^15*b^5 - 8*a^16*
b^4 - 8*a^17*b^3 + 8*a^18*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) + tan(e + f
*x)*(2*a^9*b^9 + 2*a^15*b^3))/(2*a - 2*b))/(2*a^9*b^8 + 2*a^10*b^7 + 2*a^11
*b^6 + 2*a^12*b^5 + 2*a^13*b^4 + 2*a^14*b^3 + 2*a^15*b^2)))/(f*(15*a^7*b -
15*a^8)) + (3*a^6*b + tan(e + f*x)^2*(5*a^7 - 5*a^5*b^2) - tan(e + f*x)^4*(
15*a^7 - 15*a^4*b^3) - 3*a^7)/(f*(15*a^8*tan(e + f*x)^5 - 15*a^7*b*tan(e +
f*x)^5))
```

3.224 $\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal. Leaf size=90

$$-\frac{\log(\cos(e+fx))}{(a-b)^2f}+\frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)^2b^2f}+\frac{a^2}{2(a-b)b^2f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -ln(cos(f*x+e))/(a-b)^2/f+1/2*a*(a-2*b)*ln(a+b*tan(f*x+e)^2)/(a-b)^2/b^2/f+
1/2*a^2/(a-b)/b^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$\frac{a^2}{2b^2f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2b^2f(a-b)^2}-\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(Log[Cos[e + f*x]]/((a - b)^2*f)) + (a*(a - 2*b)*Log[a + b*Tan[e + f*x]^2]
)/(2*(a - b)^2*b^2*f) + a^2/(2*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

`alQ[n]))`

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a-b)^2(1+x)} - \frac{a^2}{(a-b)b(a+bx)^2} + \frac{a(a-2b)}{(a-b)^2b(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\log(\cos(e+fx))}{(a-b)^2f} + \frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)^2b^2f} + \frac{a^2}{2(a-b)b^2f\left(a+b\tan\right.}$$

Mathematica [A]
time = 0.49, size = 73, normalized size = 0.81

$$\frac{-2\log(\cos(e+fx)) + \frac{a(a-2b)\log\left(a+b\tan^2(e+fx)\right)}{b^2} + \frac{a^2(a-b)}{b^2\left(a+b\tan^2(e+fx)\right)}}{2(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
[Out] (-2*Log[Cos[e + f*x]] + (a*(a - 2*b)*Log[a + b*Tan[e + f*x]^2])/b^2 + (a^2*
(a - b))/(b^2*(a + b*Tan[e + f*x]^2)))/(2*(a - b)^2*f)
```

Maple [A]
time = 0.14, size = 83, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^2}+\frac{a\left(\frac{(a-2b)\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{b^2}+\frac{a(a-b)}{b^2\left(a+b\left(\tan^2(fx+e)\right)\right)}\right)}{2(a-b)^2}}{f}$
default	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^2}+\frac{a\left(\frac{(a-2b)\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{b^2}+\frac{a(a-b)}{b^2\left(a+b\left(\tan^2(fx+e)\right)\right)}\right)}{2(a-b)^2}}{f}$
norman	$\frac{a^2}{2(a-b)b^2f(a+b(\tan^2(fx+e)))}+\frac{\ln(1+\tan^2(fx+e))}{2f(a^2-2ab+b^2)}+\frac{a(a-2b)\ln(a+b(\tan^2(fx+e)))}{2b^2f(a^2-2ab+b^2)}$

risch	$-\frac{ix}{a^2-2ab+b^2}+\frac{2ix}{b^2}+\frac{2ie}{b^2f}-\frac{2ia^2x}{b^2(a^2-2ab+b^2)}-\frac{2ia^2e}{b^2f(a^2-2ab+b^2)}+\frac{4iax}{b(a^2-2ab+b^2)}+\frac{4iae}{bf(a^2-2ab+b^2)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a-b)^2*ln(1+tan(f*x+e)^2)+1/2*a/(a-b)^2*((a-2*b)/b^2*ln(a+b*tan(f
*x+e)^2)+a*(a-b)/b^2/(a+b*tan(f*x+e)^2)))
```

Maxima [A]
time = 0.28, size = 131, normalized size = 1.46

$$-\frac{\frac{a^2}{a^3b-2\,a^2b^2+ab^3-(a^3b-3\,a^2b^2+3\,ab^3-b^4)\sin(fx+e)^2}-\frac{(a^2-2\,ab)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^2b^2-2\,ab^3+b^4}+\frac{\log\left(\sin(fx+e)^2-1\right)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2/(a^3*b - 2*a^2*b^2 + a*b^3 - (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*
sin(f*x + e)^2) - (a^2 - 2*a*b)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^2*b^2 -
 2*a*b^3 + b^4) + log(sin(f*x + e)^2 - 1)/b^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(89) = 178.
time = 2.72, size = 193, normalized size = 2.14

$$-\frac{a^2b\tan(fx+e)^2+a^2b-\left(a^3-2\,a^2b+(a^2b-2\,ab^2)\tan(fx+e)^2\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)+\left(a^3-2\,a^2b+ab^2+(a^2b-2\,ab^2+b^3)\tan(fx+e)^2\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\left((a^2b^3-2\,ab^4+b^5)f\tan(fx+e)^2+(a^3b^2-2\,a^2b^3+ab^4)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*tan(f*x + e)^2 + a^2*b - (a^3 - 2*a^2*b + (a^2*b - 2*a*b^2)*tan
(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) + (a^3 - 2*a^
2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*log(1/(tan(f*x + e)^2
 + 1)))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3
+ a*b^4)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. 2(71) = 142.
time = 32.77, size = 1542, normalized size = 17.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
[Out] Piecewise((zoo*x*tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (2*log(tan(e + f*
x)**2 + 1)*tan(e + f*x)**4/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x
)**2 + 4*b**2*f) + 4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*b**2*f*tan
(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 2*log(tan(e + f*x)**2
 + 1)/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 4*
tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b*
*2*f) + 3/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f),
 Eq(a, b)), ((log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(
e + f*x)**2/(2*f))/a**2, Eq(b, 0)), (x*tan(e)**5/(a + b*tan(e)**2)**2, Eq(f
, 0)), (a**3*log(-sqrt(-a/b) + tan(e + f*x))/(2*a**3*b**2*f + 2*a**2*b**3*f
*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f
+ 2*b**5*f*tan(e + f*x)**2) + a**3*log(sqrt(-a/b) + tan(e + f*x))/(2*a**3*b
**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e +
f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**3/(2*a**3*b**2*f + 2*
a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 +
2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**2*b*log(-sqrt(-a/b) + tan(e + f
*x))*tan(e + f*x)**2/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**
2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)*
*2) - 2*a**2*b*log(-sqrt(-a/b) + tan(e + f*x))/(2*a**3*b**2*f + 2*a**2*b**3
*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*
f + 2*b**5*f*tan(e + f*x)**2) + a**2*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e
 + f*x)**2/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f -
 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) - 2*a*
*2*b*log(sqrt(-a/b) + tan(e + f*x))/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e +
f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*
f*tan(e + f*x)**2) - a**2*b/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2
- 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e
+ f*x)**2) - 2*a*b**2*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a*
*3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(
e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) - 2*a*b**2*log(sqrt(-a
/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e +
f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*
f*tan(e + f*x)**2) + a*b**2*log(tan(e + f*x)**2 + 1)/(2*a**3*b**2*f + 2*a**
2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a
*b**4*f + 2*b**5*f*tan(e + f*x)**2) + b**3*log(tan(e + f*x)**2 + 1)*tan(e +
 f*x)**2/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4
*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(89) = 178.

time = 1.75, size = 399, normalized size = 4.43

$$\frac{\frac{(a^3-2a^2b)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+4b\right|\right)}{a^3b^2-2a^2b^3+ab^4}+\frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{a^2-2ab+b^2}-\frac{a^3\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a^2b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2a^3-12a^2b+12ab^2}{(a^2b^2-2ab^3+b^4)\left(a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2a-4b\right)}-\frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)}{b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 - 2*a^2*b)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (c
os(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 4*b))/(a^3*b^2 - 2*a^2*b^3 + a
*b^4) + log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 2))/(a^2 - 2*a*b + b^2) - (a^3*((cos(f*x + e) + 1)/(c
os(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a^2*b*((cos(f
*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) +
2*a^3 - 12*a^2*b + 12*a*b^2)/((a^2*b^2 - 2*a*b^3 + b^4)*(a*((cos(f*x + e) +
 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a - 4*b
)) - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(c
os(f*x + e) + 1) - 2))/b^2)/f
```

Mupad [B]
time = 11.59, size = 90, normalized size = 1.00

$$\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,{\left(a-b\right)}^2}+\frac{a^2}{2\,b^2\,f\,\left(b\tan\left(e+f\,x\right)^2+a\right)\,\left(a-b\right)}+\frac{a\,\ln\left(b\tan(e+f\,x)^2+a\right)\,\left(a-2\,b\right)}{2\,b^2\,f\,{\left(a-b\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) + a^2/(2*b^2*f*(a + b*tan(e + f*x)^
2)*(a - b)) + (a*log(a + b*tan(e + f*x)^2)*(a - 2*b))/(2*b^2*f*(a - b)^2)
```

3.225 $\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal. Leaf size=69

$$\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^2f}-\frac{a}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] 1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^2/f-1/2*a/(a-b)/b/f/(a+b*tan(f*
x+e)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3751, 457, 78}

$$\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}-\frac{a}{2bf(a-b)\left(a+b\tan^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^2*f) - a/(2*(a - b)*b*f
*(a + b*Tan[e + f*x]^2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)^2(1+x)}+\frac{a}{(a-b)(a+bx)^2}+\frac{b}{(a-b)^2(a+bx)}\right)dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^2f} - \frac{a}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 61, normalized size = 0.88

$$\frac{2\log(\cos(e+fx))+\log\left(a+b\tan^2(e+fx)\right)+\frac{a(-a+b)}{b(a+b\tan^2(e+fx))}}{2(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
[Out] (2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2] + (a*(-a + b))/(b*(a + b*T
an[e + f*x]^2)))/(2*(a - b)^2*f)
```

Maple [A]
time = 0.10, size = 74, normalized size = 1.07

method	result
derivativedivides	$\frac{-\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^2}+\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)-\frac{a(a-b)}{b\left(a+b\left(\tan^2(fx+e)\right)\right)}}{2(a-b)^2}}{f}$
default	$\frac{-\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^2}+\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)-\frac{a(a-b)}{b\left(a+b\left(\tan^2(fx+e)\right)\right)}}{2(a-b)^2}}{f}$
norman	$\frac{\tan^2(fx+e)}{2f(a-b)(a+b(\tan^2(fx+e)))}-\frac{\ln(1+\tan^2(fx+e))}{2f(a^2-2ab+b^2)}+\frac{\ln(a+b(\tan^2(fx+e)))}{2f(a^2-2ab+b^2)}$

risch	$-\frac{ix}{a^2-2ab+b^2}-\frac{2ie}{f(a^2-2ab+b^2)}+\frac{2a\,\mathrm{e}^{2i(fx+e)}}{f(a-b)^2\left(a\,\mathrm{e}^{4i(fx+e)}-b\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{2i(fx+e)}+a-b\right)}+\frac{\ln\left(\mathrm{e}^{4i(fx}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/(a-b)^2*ln(1+tan(f*x+e)^2)+1/2/(a-b)^2*(ln(a+b*tan(f*x+e)^2)-a*(a
-b)/b/(a+b*tan(f*x+e)^2)))
```

Maxima [A]
time = 0.28, size = 90, normalized size = 1.30

$$\frac{\frac{a}{a^3-2\,a^2b+ab^2-(a^3-3\,a^2b+3\,ab^2-b^3)\sin(fx+e)^2}+\frac{\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a/(a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sin(f*x + e
)^2) + log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - 2*a*b + b^2))/f
```

Fricas [A]
time = 3.35, size = 103, normalized size = 1.49

$$\frac{a\tan\left(fx+e\right)^2+\left(b\tan\left(fx+e\right)^2+a\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)+a}{2\left((a^2b-2\,ab^2+b^3)f\tan\left(fx+e\right)^2+(a^3-2\,a^2b+ab^2)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*tan(f*x + e)^2 + (b*tan(f*x + e)^2 + a)*log((b*tan(f*x + e)^2 + a)/(
tan(f*x + e)^2 + 1)) + a)/((a^2*b - 2*a*b^2 + b^3)*f*tan(f*x + e)^2 + (a^3
- 2*a^2*b + a*b^2)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(51) = 102$.
time = 14.37, size = 910, normalized size = 13.19

$$\begin{cases} \frac{\tilde{\infty} x}{\tan(e)} & \text{for}\ a=0 \wedge b=0 \wedge f=0 \\ \frac{-\frac{\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{\tan^2(e+fx)}{2f}}{a^2} & \text{for}\ b=0 \\ -\frac{2\tan^2(e+fx)}{4b^2f\tan^4(e+fx)+8b^2f\tan^2(e+fx)+4b^2f}-\frac{1}{4b^2f\tan^4(e+fx)+8b^2f\tan^2(e+fx)+4b^2f} & \text{for}\ a=b \\ \frac{x\tan^3(e)}{\left(a+b\tan^2(e)\right)^2} & \text{for}\ f=0 \\ -\frac{a^2}{2a^3bf+2a^2b^2f\tan^2(e+fx)-4a^2b^2f-4ab^3f\tan^2(e+fx)+2ab^3f+2b^4f\tan^2(e+fx)}+\frac{ab\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2a^3bf+2a^2b^2f\tan^2(e+fx)-4a^2b^2f-4ab^3f\tan^2(e+fx)+2ab^3f+2b^4f\tan^2(e+fx)}+\frac{ab\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2a^3bf+2a^2b^2f\tan^2(e+fx)-4a^2b^2f-4ab^3f\tan^2(e+fx)+2ab^3f+2b^4f\tan^2(e+fx)}-\frac{ab\log\left(\tan^2(e+fx)+1\right)}{2a^3bf+2a^2b^2f\tan^2(e+fx)-4a^2b^2f-4ab^3f\tan^2(e+fx)+2ab^3f+2b^4f\tan^2(e+fx)}+\frac{ab}{2a^3bf+2a^2b^2f\tan^2(e+fx)-4a^2b^2f-4ab^3f\tan^2(e+fx)+2ab^3f+2b^4f\tan^2(e+fx)}+\frac{b^2\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)\tan^2(e+fx)}{2a^3bf+2a^2b^2f\tan^2(e+fx)-4a^2b^2f-4ab^3f\tan^2(e+fx)+2ab^3f+2b^4f\tan^2(e+fx)}+\frac{b^2\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)\tan^2(e+fx)}{2a^3bf+2a^2b^2f\tan^2(e+fx)-4a^2b^2f-4ab^3f\tan^2(e+fx)+2ab^3f+2b^4f\tan^2(e+fx)}-\frac{b^2\log\left(\tan^2(e+fx)+1\right)\tan^2(e+fx)}{2a^3bf+2a^2b^2f\tan^2(e+fx)-4a^2b^2f-4ab^3f\tan^2(e+fx)+2ab^3f+2b^4f\tan^2(e+fx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(tan(e + f*
x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a**2, Eq(b, 0)), (-2*tan(e + f*x)
**2/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) - 1/(4
*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b)),
(x*tan(e)**3/(a + b*tan(e)**2)**2, Eq(f, 0)), (-a**2/(2*a**3*b*f + 2*a**2*b
**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b*
*3*f + 2*b**4*f*tan(e + f*x)**2) + a*b*log(-sqrt(-a/b) + tan(e + f*x))/(2*a
**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e
+ f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) + a*b*log(sqrt(-a/b) + t
an(e + f*x))/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f -
4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) - a*b*l
og(tan(e + f*x)**2 + 1)/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**
2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)*
*2) + a*b/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a
*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) + b**2*log
(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b*f + 2*a**2*b**2*f*ta
n(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2
*b**4*f*tan(e + f*x)**2) + b**2*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)
**2/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*
f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) - b**2*log(tan(e
 + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2
 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e
 + f*x)**2), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(68) = 136$.

time = 0.95, size = 295, normalized size = 4.28

$$\frac{\frac{\log\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2-2\,ab+b^2}-\frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2-2\,ab+b^2}-\frac{a+\frac{6\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{(a^2-2\,ab+b^2)\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^2
 - 2*a*b + b^2) - 2*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(a
^2 - 2*a*b + b^2) - (a + 6*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2)/((a^2 - 2*a*b + b^2)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2)))/f
```

Mupad [**B**]

time = 11.60, size = 270, normalized size = 3.91

$$-\frac{\frac{a^2\cos(e+f\,x)^2}{2}+b^2\sin\left(e+f\,x\right)^2\,\mathrm{atan}\left(\frac{a\sin(e+f\,x)^2-b\sin(e+f\,x)^2}{a\cos(e+f\,x)^2\,2\mathrm{i}+a\sin(e+f\,x)^2\,1\mathrm{i}+b\sin(e+f\,x)^2\,1\mathrm{i}}\right)\,1\mathrm{i}-\frac{a\,b\cos(e+f\,x)^2}{2}+a\,b\cos\left(e+f\,x\right)^2\,\mathrm{atan}\left(\frac{a\sin(e+f\,x)^2-b\sin(e+f\,x)^2}{a\cos(e+f\,x)^2\,2\mathrm{i}+a\sin(e+f\,x)^2\,1\mathrm{i}+b\sin(e+f\,x)^2\,1\mathrm{i}}\right)\,1\mathrm{i}}{f\,\left(a^3\,b\cos\left(e+f\,x\right)^2-2\,a^2\,b^2\cos\left(e+f\,x\right)^2+a^2\,b^2\sin\left(e+f\,x\right)^2+a\,b^3\cos\left(e+f\,x\right)^2-2\,a\,b^3\sin\left(e+f\,x\right)^2+b^4\sin\left(e+f\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] -((a^2*cos(e + f*x)^2)/2 + b^2*sin(e + f*x)^2*atan((a*sin(e + f*x)^2 - b*si
n(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2
*1i))*1i - (a*b*cos(e + f*x)^2)/2 + a*b*cos(e + f*x)^2*atan((a*sin(e + f*x)
^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e
 + f*x)^2*1i))*1i)/(f*(b^4*sin(e + f*x)^2 + a*b^3*cos(e + f*x)^2 + a^3*b*co
s(e + f*x)^2 - 2*a*b^3*sin(e + f*x)^2 - 2*a^2*b^2*cos(e + f*x)^2 + a^2*b^2*
sin(e + f*x)^2))
```

3.226 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^2f}+\frac{1}{2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^2/f+1/2/(a-b)/f/(a+b*tan(f*x+e
)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 455, 46}

$$\frac{1}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/((a - b)^2*f) + 1/(2*(a - b)*
f*(a + b*Tan[e + f*x]^2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2(1+x)} - \frac{b}{(a-b)(a+bx)^2} - \frac{b}{(a-b)^2(a+bx)}\right)\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^2f} + \frac{1}{2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

Mathematica [A]
time = 0.44, size = 57, normalized size = 0.88

$$-\frac{2\log(\cos(e+fx)) + \log\left(a+b\tan^2(e+fx)\right) + \frac{-a+b}{a+b\tan^2(e+fx)}}{2(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
[Out] -1/2*(2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2] + (-a + b)/(a + b*Tan
[e + f*x]^2))/((a - b)^2*f)
```

Maple [A]
time = 0.10, size = 78, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^2} - \frac{b\left(\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{b} - \frac{a-b}{b\left(a+b\left(\tan^2(fx+e)\right)\right)}\right)}{2(a-b)^2}}{f}$
default	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^2} - \frac{b\left(\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{b} - \frac{a-b}{b\left(a+b\left(\tan^2(fx+e)\right)\right)}\right)}{2(a-b)^2}}{f}$
norman	$-\frac{b\left(\tan^2(fx+e)\right)}{2af(a-b)\left(a+b\left(\tan^2(fx+e)\right)\right)} + \frac{\ln\left(1+\tan^2(fx+e)\right)}{2f\left(a^2-2ab+b^2\right)} - \frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{2f\left(a^2-2ab+b^2\right)}$

risch	$\frac{ix}{a^2-2ab+b^2}+\frac{2ie}{f(a^2-2ab+b^2)}+\frac{2b\,\mathrm{e}^{2i(fx+e)}}{f(-a+b)^2\left(-a\,\mathrm{e}^{4i(fx+e)}+b\,\mathrm{e}^{4i(fx+e)}-2a\,\mathrm{e}^{2i(fx+e)}-2b\,\mathrm{e}^{2i(fx+e)}-a+b\right)}-\frac{\ln\left(\mathrm{e}^{4i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a-b)^2*ln(1+tan(f*x+e)^2)-1/2/(a-b)^2*b*(1/b*ln(a+b*tan(f*x+e)^2)
-(a-b)/b/(a+b*tan(f*x+e)^2)))
```

Maxima [A]
time = 0.28, size = 90, normalized size = 1.38

$$-\frac{\frac{b}{a^3-2\,a^2b+ab^2-(a^3-3\,a^2b+3\,ab^2-b^3)\sin(fx+e)^2}+\frac{\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b/(a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sin(f*x +
e)^2) + log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - 2*a*b + b^2))/f
```

Fricas [A]
time = 2.02, size = 103, normalized size = 1.58

$$-\frac{b\tan\left(fx+e\right)^2+\left(b\tan\left(fx+e\right)^2+a\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)+b}{2\left((a^2b-2\,ab^2+b^3)f\tan\left(fx+e\right)^2+(a^3-2\,a^2b+ab^2)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*tan(f*x + e)^2 + (b*tan(f*x + e)^2 + a)*log((b*tan(f*x + e)^2 + a)/
(tan(f*x + e)^2 + 1)) + b)/((a^2*b - 2*a*b^2 + b^3)*f*tan(f*x + e)^2 + (a^3
 - 2*a^2*b + a*b^2)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 796 vs. 2(49) = 98.
time = 14.27, size = 796, normalized size = 12.25

$$\begin{cases} \frac{\tilde{\infty} x}{\tan^{3}{\left(e \right)}} & \text{for}\: a = 0 \wedge b = 0 \wedge f = 0 \\ \frac{\log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{2 a^{2} f} & \text{for}\: b = 0 \\ - \frac{1}{4 b^{2} f \tan^{4}{\left(e + f x \right)} + 8 b^{2} f \tan^{2}{\left(e + f x \right)} + 4 b^{2} f} & \text{for}\: a = b \\ \frac{x \tan{\left(e \right)}}{\left(a + b \tan^{2}{\left(e \right)}\right)^{2}} & \text{for}\: f = 0 \\ - \frac{a \log{\left(- \sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)}}{2 a^{3} f + 2 a^{2} b f \tan^{2}{\left(e + f x \right)} - 4 a^{2} b f - 4 a b^{2} f \tan^{2}{\left(e + f x \right)} + 2 a b^{2} f + 2 b^{3} f \tan^{2}{\left(e + f x \right)}} - \frac{a \log{\left(\sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)}}{2 a^{3} f + 2 a^{2} b f \tan^{2}{\left(e + f x \right)} - 4 a^{2} b f - 4 a b^{2} f \tan^{2}{\left(e + f x \right)} + 2 a b^{2} f + 2 b^{3} f \tan^{2}{\left(e + f x \right)}} + \frac{a \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{2 a^{3} f + 2 a^{2} b f \tan^{2}{\left(e + f x \right)} - 4 a^{2} b f - 4 a b^{2} f \tan^{2}{\left(e + f x \right)} + 2 a b^{2} f + 2 b^{3} f \tan^{2}{\left(e + f x \right)}} + \frac{a}{2 a^{3} f + 2 a^{2} b f \tan^{2}{\left(e + f x \right)} - 4 a^{2} b f - 4 a b^{2} f \tan^{2}{\left(e + f x \right)} + 2 a b^{2} f + 2 b^{3} f \tan^{2}{\left(e + f x \right)}} - \frac{b \log{\left(- \sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)} \tan^{2}{\left(e + f x \right)}}{2 a^{3} f + 2 a^{2} b f \tan^{2}{\left(e + f x \right)} - 4 a^{2} b f - 4 a b^{2} f \tan^{2}{\left(e + f x \right)} + 2 a b^{2} f + 2 b^{3} f \tan^{2}{\left(e + f x \right)}} - \frac{b \log{\left(\sqrt{- \frac{a}{b}} + \tan{\left(e + f x \right)} \right)} \tan^{2}{\left(e + f x \right)}}{2 a^{3} f + 2 a^{2} b f \tan^{2}{\left(e + f x \right)} - 4 a^{2} b f - 4 a b^{2} f \tan^{2}{\left(e + f x \right)} + 2 a b^{2} f + 2 b^{3} f \tan^{2}{\left(e + f x \right)}} + \frac{b \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)} \tan^{2}{\left(e + f x \right)}}{2 a^{3} f + 2 a^{2} b f \tan^{2}{\left(e + f x \right)} - 4 a^{2} b f - 4 a b^{2} f \tan^{2}{\left(e + f x \right)} + 2 a b^{2} f + 2 b^{3} f \tan^{2}{\left(e + f x \right)}} - \frac{b}{2 a^{3} f + 2 a^{2} b f \tan^{2}{\left(e + f x \right)} - 4 a^{2} b f - 4 a b^{2} f \tan^{2}{\left(e + f x \right)} + 2 a b^{2} f + 2 b^{3} f \tan^{2}{\left(e + f x \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e + f
*x)**2 + 1)/(2*a**2*f), Eq(b, 0)), (-1/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f
*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b)), (x*tan(e)/(a + b*tan(e)**2)**2, Eq
(f, 0)), (-a*log(-sqrt(-a/b) + tan(e + f*x))/(2*a**3*f + 2*a**2*b*f*tan(e +
 f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*
tan(e + f*x)**2) - a*log(sqrt(-a/b) + tan(e + f*x))/(2*a**3*f + 2*a**2*b*f*
tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*
b**3*f*tan(e + f*x)**2) + a*log(tan(e + f*x)**2 + 1)/(2*a**3*f + 2*a**2*b*f
*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2
*b**3*f*tan(e + f*x)**2) + a/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**
2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2)
 - b*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*f + 2*a**2*b*f
*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2
*b**3*f*tan(e + f*x)**2) - b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2
/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f
*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) + b*log(tan(e + f*x)**2 + 1
)*tan(e + f*x)**2/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a
*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) - b/(2*a**
3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2
+ 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(64) = 128$.

time = 0.73, size = 307, normalized size = 4.72

$$-\frac{\frac{\log\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2-2\,ab+b^2}-\frac{2\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2-2\,ab+b^2}-\frac{a^2+\frac{2\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{(a^3-2\,a^2b+ab^2)\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^
2 - 2*a*b + b^2) - 2*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(
a^2 - 2*a*b + b^2) - (a^2 + 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4
*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2)/((a^3 - 2*a^2*b + a*b^2)*(a + 2*a*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2)))/f
```

Mupad [B]

time = 11.64, size = 195, normalized size = 3.00

$$-\frac{b\left(1+\tan(e+f\,x)^2\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)^2\,1\mathrm{i}-b\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a+a\tan(e+f\,x)^2+b\tan(e+f\,x)^2}\right)\,2\mathrm{i}\right)+a\left(-1+\mathrm{atan}\left(\frac{a\tan(e+f\,x)^2\,1\mathrm{i}-b\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a+a\tan(e+f\,x)^2+b\tan(e+f\,x)^2}\right)\,2\mathrm{i}\right)}{f\,\left(2\,a^3+2\,a^2\,b\tan\left(e+f\,x\right)^2-4\,a^2\,b-4\,a\,b^2\tan\left(e+f\,x\right)^2+2\,a\,b^2+2\,b^3\tan\left(e+f\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] -(b*(tan(e + f*x)^2*atan((a*tan(e + f*x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a +
 a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*2i + 1) + a*(atan((a*tan(e + f*x)^2*
1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*2i -
 1))/(f*(2*a*b^2 - 4*a^2*b + 2*a^3 + 2*b^3*tan(e + f*x)^2 - 4*a*b^2*tan(e +
 f*x)^2 + 2*a^2*b*tan(e + f*x)^2))
```

3.227 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{\log(\cos(e+fx))}{(a-b)^2f}+\frac{\log(\tan(e+fx))}{a^2f}+\frac{(2a-b)b\log\left(a+b\tan^2(e+fx)\right)}{2a^2(a-b)^2f}-\frac{b}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] ln(cos(f*x+e))/(a-b)^2/f+ln(tan(f*x+e))/a^2/f+1/2*(2*a-b)*b*ln(a+b*tan(f*x+
e)^2)/a^2/(a-b)^2/f-1/2*b/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 457, 84}

$$\frac{b(2a-b)\log\left(a+b\tan^2(e+fx)\right)}{2a^2f(a-b)^2}+\frac{\log(\tan(e+fx))}{a^2f}-\frac{b}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)^2*f) + Log[Tan[e + f*x]]/(a^2*f) + ((2*a - b)*b*
Log[a + b*Tan[e + f*x]^2])/(2*a^2*(a - b)^2*f) - b/(2*a*(a - b)*f*(a + b*Ta
n[e + f*x]^2))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{(a-b)^2(1+x)} + \frac{b^2}{a(a-b)(a+bx)^2} + \frac{(2a-b)b^2}{a^2(a-b)^2(a+bx)}\right) dx, x, \tan^2(e+f\ldots}{2f}$$

$$= \frac{\log(\cos(e+fx))}{(a-b)^2f} + \frac{\log(\tan(e+fx))}{a^2f} + \frac{(2a-b)b\log\left(a+b\tan^2(e+fx)\right)}{2a^2(a-b)^2f} - \ldots$$

Mathematica [A]
time = 1.42, size = 90, normalized size = 0.87

$$\frac{\frac{2\log(\cos(e+fx))}{(a-b)^2} + \frac{2\log(\tan(e+fx)) + \frac{b\left((2a-b)\log\left(a+b\tan^2(e+fx)\right) + \frac{a(-a+b)}{a+b\tan^2(e+fx)}\right)}{(a-b)^2}}{a^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((2*Log[Cos[e + f*x]])/(a - b)^2 + (2*Log[Tan[e + f*x]] + (b*((2*a - b)*Log
[a + b*Tan[e + f*x]^2] + (a*(-a + b))/(a + b*Tan[e + f*x]^2)))/(a - b)^2)/a
^2)/(2*f)
```

Maple [A]
time = 0.30, size = 115, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{\ln(\cos(fx+e)+1)}{2a^2} + \frac{\ln(\cos(fx+e)-1)}{2a^2} + \frac{b\left(\frac{ab}{(a-b)^2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)} + \frac{(2a-b)\ln\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b\ldots}{(a-b)^2}\right.}{2a^2}}{f}$
default	$\frac{\frac{\ln(\cos(fx+e)+1)}{2a^2} + \frac{\ln(\cos(fx+e)-1)}{2a^2} + \frac{b\left(\frac{ab}{(a-b)^2\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)} + \frac{(2a-b)\ln\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b\ldots}{(a-b)^2}\right.}{2a^2}}{f}$
norman	$\frac{b^2\left(\tan^2(fx+e)\right)}{2a^2f(a-b)\left(a+b\left(\tan^2(fx+e)\right)\right)} + \frac{\ln(\tan(fx+e))}{a^2f} - \frac{\ln\left(1+\tan^2(fx+e)\right)}{2f(a^2-2ab+b^2)} + \frac{b(2a-b)\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{2a^2f(a^2-2ab+b^2)}$
risch	$\frac{ix}{a^2-2ab+b^2} - \frac{2ix}{a^2} - \frac{2ie}{a^2f} - \frac{4ibx}{a(a^2-2ab+b^2)} - \frac{4ibe}{af(a^2-2ab+b^2)} + \frac{2ib^2x}{a^2(a^2-2ab+b^2)} + \frac{2ib^2e}{a^2f(a^2-2ab+b^2)} - \ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/a^2*ln(cos(f*x+e)+1)+1/2/a^2*ln(cos(f*x+e)-1)+1/2*b/a^2*(a*b/(a-b)
^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+(2*a-b)/(a-b)^2*ln(a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)))
```

Maxima [A]
time = 0.29, size = 127, normalized size = 1.23

$$\frac{\frac{b^2}{a^4-2a^3b+a^2b^2-(a^4-3a^3b+3a^2b^2-ab^3)\sin(fx+e)^2}+\frac{(2ab-b^2)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^4-2a^3b+a^2b^2}+\frac{\log\left(\sin(fx+e)^2\right)}{a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2/(a^4 - 2*a^3*b + a^2*b^2 - (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sin
(f*x + e)^2) + (2*a*b - b^2)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^4 - 2*a^3*
b + a^2*b^2) + log(sin(f*x + e)^2)/a^2)/f
```

Fricas [A]
time = 1.19, size = 205, normalized size = 1.99

$$\frac{ab^2\tan(fx+e)^2+ab^2+\left(a^3-2a^2b+ab^2+(a^2b-2ab^2+b^3)\tan(fx+e)^2\right)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)+\left(2a^2b-ab^2+(2ab^2-b^3)\tan(fx+e)^2\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{2\left((a^4b-2a^3b^2+a^2b^3)f\tan(fx+e)^2+(a^5-2a^4b+a^3b^2)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*tan(f*x + e)^2 + a*b^2 + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b
^2 + b^3)*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + (2*a^2
*b - a*b^2 + (2*a*b^2 - b^3)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(ta
n(f*x + e)^2 + 1)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5
- 2*a^4*b + a^3*b^2)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2341 vs. 2(80) = 160.
time = 104.09, size = 2341, normalized size = 22.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x*cot(e)/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(
tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f)/a**2, Eq(b, 0)), ((-log(t
an(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f + 1/(2*f*tan(e + f*x)**2) -
 1/(4*f*tan(e + f*x)**4))/b**2, Eq(a, 0)), (-2*log(tan(e + f*x)**2 + 1)*tan
(e + f*x)**4/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*
f) - 4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**4 +
 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) - 2*log(tan(e + f*x)**2 + 1)/(4*b**2*
f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 4*log(tan(e + f*
x))*tan(e + f*x)**4/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 +
4*b**2*f) + 8*log(tan(e + f*x))*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**4 +
 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 4*log(tan(e + f*x))/(4*b**2*f*tan(e
 + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 2*tan(e + f*x)**2/(4*b*
*2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 3/(4*b**2*f*t
an(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b)), (x*cot(e)
/(a + b*tan(e)**2)**2, Eq(f, 0)), (-a**3*log(tan(e + f*x)**2 + 1)/(2*a**5*f
 + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2
+ 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2) + 2*a**3*log(tan(e + f*x))
/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e
+ f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2) + 2*a**2*b*log(-
sqrt(-a/b) + tan(e + f*x))/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*
b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e +
 f*x)**2) + 2*a**2*b*log(sqrt(-a/b) + tan(e + f*x))/(2*a**5*f + 2*a**4*b*f*
tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*
f + 2*a**2*b**3*f*tan(e + f*x)**2) - a**2*b*log(tan(e + f*x)**2 + 1)*tan(e
+ f*x)**2/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2
*f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2) + 2*a**
2*b*log(tan(e + f*x))*tan(e + f*x)**2/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**
2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**
3*f*tan(e + f*x)**2) - 4*a**2*b*log(tan(e + f*x))/(2*a**5*f + 2*a**4*b*f*ta
n(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f
+ 2*a**2*b**3*f*tan(e + f*x)**2) - a**2*b/(2*a**5*f + 2*a**4*b*f*tan(e + f*
x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2
*b**3*f*tan(e + f*x)**2) + 2*a*b**2*log(-sqrt(-a/b) + tan(e + f*x))*tan(e +
 f*x)**2/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*
f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2) - a*b**2
*log(-sqrt(-a/b) + tan(e + f*x))/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4
*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*t
an(e + f*x)**2) + 2*a*b**2*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(
2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e +
f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2) - a*b**2*log(sqrt(
-a/b) + tan(e + f*x))/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f -
 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)
**2) - 4*a*b**2*log(tan(e + f*x))*tan(e + f*x)**2/(2*a**5*f + 2*a**4*b*f*ta
n(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f
+ 2*a**2*b**3*f*tan(e + f*x)**2) + 2*a*b**2*log(tan(e + f*x))/(2*a**5*f + 2
```

```
*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*
a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2) + a*b**2/(2*a**5*f + 2*a**4*b*
f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**
2*f + 2*a**2*b**3*f*tan(e + f*x)**2) - b**3*log(-sqrt(-a/b) + tan(e + f*x))
*tan(e + f*x)**2/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a*
*3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2)
- b**3*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**5*f + 2*a**4*b*
f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**
2*f + 2*a**2*b**3*f*tan(e + f*x)**2) + 2*b**3*log(tan(e + f*x))*tan(e + f*x
)**2/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f - 4*a**3*b**2*f*ta
n(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2), True))
```

Giac [A]
time = 1.00, size = 152, normalized size = 1.48

$$\frac{\frac{(2\,ab-b^2)\log\left(\left|-a\sin(fx+e)^2+b\sin(fx+e)^2+a\right|\right)}{a^4-2\,a^3b+a^2b^2}-\frac{2\,ab\sin(fx+e)^2-b^2\sin(fx+e)^2-2\,ab}{(a^3-a^2b)\left(a\sin(fx+e)^2-b\sin(fx+e)^2-a\right)}+\frac{\log\left(\sin(fx+e)^2\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a*b - b^2)*log(abs(-a*sin(f*x + e)^2 + b*sin(f*x + e)^2 + a))/(a^4
- 2*a^3*b + a^2*b^2) - (2*a*b*sin(f*x + e)^2 - b^2*sin(f*x + e)^2 - 2*a*b)/
((a^3 - a^2*b)*(a*sin(f*x + e)^2 - b*sin(f*x + e)^2 - a)) + log(sin(f*x + e
)^2)/a^2)/f
```

Mupad [B]
time = 11.73, size = 104, normalized size = 1.01

$$\frac{\ln\left(\tan(e+f\,x)\right)}{a^2\,f}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^2}-\frac{b}{2\,a\,f\,\left(b\tan\left(e+f\,x\right)^2+a\right)\,(a-b)}+\frac{b\,\ln\left(b\tan(e+f\,x)^2+a\right)\,(2\,a-b)}{2\,a^2\,f\,(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] log(tan(e + f*x))/(a^2*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) - b/(2*
a*f*(a + b*tan(e + f*x)^2)*(a - b)) + (b*log(a + b*tan(e + f*x)^2)*(2*a - b
))/(2*a^2*f*(a - b)^2)
```

3.228 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=132

$$-\frac{\cot^2(e+fx)}{2a^2f}-\frac{\log(\cos(e+fx))}{(a-b)^2f}-\frac{(a+2b)\log(\tan(e+fx))}{a^3f}-\frac{(3a-2b)b^2\log\left(a+b\tan^2(e+fx)\right)}{2a^3(a-b)^2f}+\frac{}{2a^2(a}$$

```
[Out] -1/2*cot(f*x+e)^2/a^2/f-ln(cos(f*x+e))/(a-b)^2/f-(a+2*b)*ln(tan(f*x+e))/a^3
/f-1/2*(3*a-2*b)*b^2*ln(a+b*tan(f*x+e)^2)/a^3/(a-b)^2/f+1/2*b^2/a^2/(a-b)/f
/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{b^2(3a-2b)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^2}-\frac{(a+2b)\log(\tan(e+fx))}{a^3f}+\frac{b^2}{2a^2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{\cot^2(e+fx)}{2a^2f}-\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*Cot[e + f*x]^2/(a^2*f) - Log[Cos[e + f*x]]/((a - b)^2*f) - ((a + 2*b)*
Log[Tan[e + f*x]])/(a^3*f) - ((3*a - 2*b)*b^2*Log[a + b*Tan[e + f*x]^2])/(2
*a^3*(a - b)^2*f) + b^2/(2*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^2}+\frac{-a-2b}{a^3x}+\frac{1}{(a-b)^2(1+x)}-\frac{b^3}{a^2(a-b)(a+bx)^2}-\frac{(3a-2b)b^3}{a^3(a-b)^2(a+bx)}\right)dx, x, \tan\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2a^2f}-\frac{\log(\cos(e+fx))}{(a-b)^2f}-\frac{(a+2b)\log(\tan(e+fx))}{a^3f}-\frac{(3a-2b)b^2}{}$$

Mathematica [A]
time = 0.58, size = 98, normalized size = 0.74

$$-\frac{\frac{\cot^2(e+fx)}{a^2}+\frac{b^3}{a^3(a-b)\left(b+a\cot^2(e+fx)\right)}+\frac{(3a-2b)b^2\log\left(b+a\cot^2(e+fx)\right)}{a^3(a-b)^2}+\frac{2\log(\sin(e+fx))}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(Cot[e + f*x]^2/a^2 + b^3/(a^3*(a - b)*(b + a*Cot[e + f*x]^2)) + ((3*a
 - 2*b)*b^2*Log[b + a*Cot[e + f*x]^2])/(a^3*(a - b)^2) + (2*Log[Sin[e + f*x
]])/(a - b)^2)/f
```

Maple [A]
time = 0.33, size = 161, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{1}{4a^2(\cos(fx+e)+1)}+\frac{(-a-2b)\ln(\cos(fx+e)+1)}{2a^3}+\frac{1}{4a^2(\cos(fx+e)-1)}+\frac{(-a-2b)\ln(\cos(fx+e)-1)}{2a^3}-b^2\left(\frac{ab}{(a-b)^2\left(a\left(\cos^2(fx+e)\right)-(\right.}\right.}{f}$
default	$\frac{-\frac{1}{4a^2(\cos(fx+e)+1)}+\frac{(-a-2b)\ln(\cos(fx+e)+1)}{2a^3}+\frac{1}{4a^2(\cos(fx+e)-1)}+\frac{(-a-2b)\ln(\cos(fx+e)-1)}{2a^3}-b^2\left(\frac{ab}{(a-b)^2\left(a\left(\cos^2(fx+e)\right)-(\right.}\right.}{f}$

norman	$\frac{-\frac{1}{2af}+\frac{\left(-a\,b^{2}+2b^{3}\right)\left(\tan^{2}(fx+e)\right)}{2a^{2}fb(a-b)}}{\tan(fx+e)^{2}(a+b(\tan^{2}(fx+e)))}+\frac{\ln(1+\tan^{2}(fx+e))}{2f(a^{2}-2ab+b^{2})}-\frac{(a+2b)\ln(\tan(fx+e))}{a^{3}f}-\frac{b^{2}(3a-2b)\ln\left(a+b\left(\tan^{2}(fx+e\right.\right.}{2a^{3}f(a^{2}-2ab+b^{2})}$
risch	$-\frac{ix}{a^{2}-2ab+b^{2}}+\frac{2ix}{a^{2}}+\frac{2ie}{a^{2}f}+\frac{4ibx}{a^{3}}+\frac{4ibe}{a^{3}f}+\frac{6ib^{2}x}{a^{2}(a^{2}-2ab+b^{2})}+\frac{6ib^{2}e}{a^{2}f(a^{2}-2ab+b^{2})}-\frac{4ib^{3}x}{a^{3}(a^{2}-2ab+b^{2})}-\frac{}{a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/4/a^2/(cos(f*x+e)+1)+1/2*(-a-2*b)/a^3*ln(cos(f*x+e)+1)+1/4/a^2/(cos
(f*x+e)-1)+1/2*(-a-2*b)/a^3*ln(cos(f*x+e)-1)-1/2*b^2/a^3*(a*b/(a-b)^2/(a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)+(3*a-2*b)/(a-b)^2*ln(a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)))
```

Maxima [A]
time = 0.29, size = 192, normalized size = 1.45

$$-\frac{\frac{\left(3\,ab^{2}-2\,b^{3}\right)\log\left(-(a-b)\sin(fx+e)^{2}+a\right)}{a^{5}-2\,a^{4}b+a^{3}b^{2}}-\frac{a^{3}-2\,a^{2}b+ab^{2}-\left(a^{3}-3\,a^{2}b+3\,ab^{2}-2\,b^{3}\right)\sin(fx+e)^{2}}{\left(a^{5}-3\,a^{4}b+3\,a^{3}b^{2}-a^{2}b^{3}\right)\sin(fx+e)^{4}-\left(a^{5}-2\,a^{4}b+a^{3}b^{2}\right)\sin(fx+e)^{2}}+\frac{(a+2\,b)\log\left(\sin(fx+e)^{2}\right)}{a^{3}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a*b^2 - 2*b^3)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^5 - 2*a^4*b + a
^3*b^2) - (a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - 2*b^3)*sin(f*
x + e)^2)/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sin(f*x + e)^4 - (a^5 - 2*
a^4*b + a^3*b^2)*sin(f*x + e)^2) + (a + 2*b)*log(sin(f*x + e)^2)/a^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(131) = 262.
time = 1.48, size = 304, normalized size = 2.30

$$-\frac{\left(a^{3}b-2\,a^{2}b^{2}+2\,ab^{3}\right)\tan(fx+e)^{4}+a^{4}-2\,a^{3}b+a^{2}b^{2}+\left(a^{4}-a^{3}b-a^{2}b^{2}+2\,ab^{3}\right)\tan(fx+e)^{2}+\left(\left(a^{3}b-3\,ab^{3}+2\,b^{4}\right)\tan(fx+e)^{4}+\left(a^{4}-3\,a^{2}b^{2}+2\,ab^{3}\right)\tan(fx+e)^{2}\right)\log\left(\frac{\tan(fx+e)^{2}}{\tan(fx+e)^{2}+1}\right)+\left(\left(3\,ab^{3}-2\,b^{4}\right)\tan(fx+e)^{4}+\left(3\,a^{2}b^{2}-2\,ab^{3}\right)\tan(fx+e)^{2}\right)\log\left(\frac{b\tan(fx+e)^{2}+a}{\tan(fx+e)^{2}+1}\right)}{2\left(\left(a^{5}b-2\,a^{4}b^{2}+a^{3}b^{3}\right)f\tan(fx+e)^{4}+\left(a^{6}-2\,a^{5}b+a^{4}b^{2}\right)f\tan(fx+e)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^3*b - 2*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^4 + a^4 - 2*a^3*b + a^2*b^
2 + (a^4 - a^3*b - a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2 + ((a^3*b - 3*a*b^3 +
2*b^4)*tan(f*x + e)^4 + (a^4 - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*log(tan
(f*x + e)^2/(tan(f*x + e)^2 + 1)) + ((3*a*b^3 - 2*b^4)*tan(f*x + e)^4 + (3*
a^2*b^2 - 2*a*b^3)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)
^2 + 1)))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^6 - 2*a^5*b
+ a^4*b^2)*f*tan(f*x + e)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3526 vs. 2(107) = 214.
time = 129.01, size = 3526, normalized size = 26.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((log(tan(e +
 f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2))/a**2,
Eq(b, 0)), ((log(tan(e + f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*
tan(e + f*x)**2) + 1/(4*f*tan(e + f*x)**4) - 1/(6*f*tan(e + f*x)**6))/b**2,
 Eq(a, 0)), (6*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**6/(4*b**2*f*tan(e + f
*x)**6 + 8*b**2*f*tan(e + f*x)**4 + 4*b**2*f*tan(e + f*x)**2) + 12*log(tan(
e + f*x)**2 + 1)*tan(e + f*x)**4/(4*b**2*f*tan(e + f*x)**6 + 8*b**2*f*tan(e
 + f*x)**4 + 4*b**2*f*tan(e + f*x)**2) + 6*log(tan(e + f*x)**2 + 1)*tan(e +
 f*x)**2/(4*b**2*f*tan(e + f*x)**6 + 8*b**2*f*tan(e + f*x)**4 + 4*b**2*f*ta
n(e + f*x)**2) - 12*log(tan(e + f*x))*tan(e + f*x)**6/(4*b**2*f*tan(e + f*x
)**6 + 8*b**2*f*tan(e + f*x)**4 + 4*b**2*f*tan(e + f*x)**2) - 24*log(tan(e
+ f*x))*tan(e + f*x)**4/(4*b**2*f*tan(e + f*x)**6 + 8*b**2*f*tan(e + f*x)**
4 + 4*b**2*f*tan(e + f*x)**2) - 12*log(tan(e + f*x))*tan(e + f*x)**2/(4*b**
2*f*tan(e + f*x)**6 + 8*b**2*f*tan(e + f*x)**4 + 4*b**2*f*tan(e + f*x)**2)
- 6*tan(e + f*x)**4/(4*b**2*f*tan(e + f*x)**6 + 8*b**2*f*tan(e + f*x)**4 +
4*b**2*f*tan(e + f*x)**2) - 9*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**6 + 8
*b**2*f*tan(e + f*x)**4 + 4*b**2*f*tan(e + f*x)**2) - 2/(4*b**2*f*tan(e + f
*x)**6 + 8*b**2*f*tan(e + f*x)**4 + 4*b**2*f*tan(e + f*x)**2), Eq(a, b)), (
zoo*x/a**2, Eq(e, -f*x)), (x*cot(e)**3/(a + b*tan(e)**2)**2, Eq(f, 0)), (a*
*4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**6*f*tan(e + f*x)**2 + 2*a
**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f*x)**2 - 4*a**4*b**2*f*tan(e
+ f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*a**3*b**3*f*tan(e + f*x)**4)
- 2*a**4*log(tan(e + f*x))*tan(e + f*x)**2/(2*a**6*f*tan(e + f*x)**2 + 2*a*
*5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f*x)**2 - 4*a**4*b**2*f*tan(e +
 f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*a**3*b**3*f*tan(e + f*x)**4) -
 a**4/(2*a**6*f*tan(e + f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*t
an(e + f*x)**2 - 4*a**4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)
**2 + 2*a**3*b**3*f*tan(e + f*x)**4) + a**3*b*log(tan(e + f*x)**2 + 1)*tan(
e + f*x)**4/(2*a**6*f*tan(e + f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5
*b*f*tan(e + f*x)**2 - 4*a**4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e
+ f*x)**2 + 2*a**3*b**3*f*tan(e + f*x)**4) - 2*a**3*b*log(tan(e + f*x))*tan
(e + f*x)**4/(2*a**6*f*tan(e + f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**
5*b*f*tan(e + f*x)**2 - 4*a**4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e
 + f*x)**2 + 2*a**3*b**3*f*tan(e + f*x)**4) - a**3*b*tan(e + f*x)**2/(2*a**
6*f*tan(e + f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f*x)*
*2 - 4*a**4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*a**3
```

```
*b**3*f*tan(e + f*x)**4) + 2*a**3*b/(2*a**6*f*tan(e + f*x)**2 + 2*a**5*b*f*
tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f*x)**2 - 4*a**4*b**2*f*tan(e + f*x)**
4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*a**3*b**3*f*tan(e + f*x)**4) - 3*a**2
*b**2*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**6*f*tan(e + f*x
)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f*x)**2 - 4*a**4*b**
2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*a**3*b**3*f*tan(e +
 f*x)**4) - 3*a**2*b**2*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a
**6*f*tan(e + f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f*x
)**2 - 4*a**4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*a*
*3*b**3*f*tan(e + f*x)**4) + 6*a**2*b**2*log(tan(e + f*x))*tan(e + f*x)**2/
(2*a**6*f*tan(e + f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e +
 f*x)**2 - 4*a**4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 +
2*a**3*b**3*f*tan(e + f*x)**4) + 3*a**2*b**2*tan(e + f*x)**2/(2*a**6*f*tan(
e + f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f*x)**2 - 4*a
**4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*a**3*b**3*f*
tan(e + f*x)**4) - a**2*b**2/(2*a**6*f*tan(e + f*x)**2 + 2*a**5*b*f*tan(e +
 f*x)**4 - 4*a**5*b*f*tan(e + f*x)**2 - 4*a**4*b**2*f*tan(e + f*x)**4 + 2*a
**4*b**2*f*tan(e + f*x)**2 + 2*a**3*b**3*f*tan(e + f*x)**4) - 3*a*b**3*log(
-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**4/(2*a**6*f*tan(e + f*x)**2 + 2*a
**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f*x)**2 - 4*a**4*b**2*f*tan(e
+ f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*a**3*b**3*f*tan(e + f*x)**4)
+ 2*a*b**3*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**6*f*tan(e
+ f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f*x)**2 - 4*a**
4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*a**3*b**3*f*ta
n(e + f*x)**4) - 3*a*b**3*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**4/(2
*a**6*f*tan(e + f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5*b*f*tan(e + f
*x)**2 - 4*a**4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e + f*x)**2 + 2*
a**3*b**3*f*tan(e + f*x)**4) + 2*a*b**3*log(sqrt(-a/b) + tan(e + f*x))*tan(
e + f*x)**2/(2*a**6*f*tan(e + f*x)**2 + 2*a**5*b*f*tan(e + f*x)**4 - 4*a**5
*b*f*tan(e + f*x)**2 - 4*a**4*b**2*f*tan(e + f*x)**4 + 2*a**4*b**2*f*tan(e
+ f*x)**2 + 2*a**3*b**3*f*tan(e + f*x)**4) + 6*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 684 vs. 2(131) = 262.
time = 1.11, size = 684, normalized size = 5.18

$$-\frac{\frac{12\,(3\,ab^2-2\,b^3)\log\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^5-2\,a^4b+a^3b^2}-\frac{24\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2-2\,ab+b^2}-\frac{3\,a^4-6\,a^3b+3\,a^2b^2+\frac{10\,a^4(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{24\,a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{42\,a^2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{20\,ab^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{11\,a^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{22\,a^3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{27\,a^2b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{16\,ab^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{16\,b^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{4\,a^4(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{12\,a^2b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{8\,ab^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}}{(a^5-2\,a^4b+a^3b^2)\left(\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{4\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}\right)}+\frac{12\,(a+2\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3}-\frac{3\,(\cos(fx+e)-1)}{a^2(\cos(fx+e)+1)}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(3*a*b^2 - 2*b^3)*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2)/(a^5 - 2*a^4*b + a^3*b^2) - 24*log(abs(-(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 1))/(a^2 - 2*a*b + b^2) - (3*a^4 - 6*a^3*b + 3*a^2*
```

```
b^2 + 10*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a^3*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 42*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 - 20*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 11*a^4*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 22*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + 27*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 16*a*b^3*(cos(
f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 16*b^4*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + 4*a^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 12*a^2*b^2
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 8*a*b^3*(cos(f*x + e) - 1)^3/(
cos(f*x + e) + 1)^3)/((a^5 - 2*a^4*b + a^3*b^2)*(a*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 2*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 4*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
 1)^3)) + 12*(a + 2*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^
3 - 3*(cos(f*x + e) - 1)/(a^2*(cos(f*x + e) + 1)))/f
```

Mupad [B]
time = 11.88, size = 144, normalized size = 1.09

$$\frac{\ln\left(b\tan(e+f\,x)^2+a\right)\,\left(\frac{b}{a^3}+\frac{1}{2\,a^2}-\frac{1}{2\,(a-b)^2}\right)}{f}-\frac{\frac{1}{2\,a}+\frac{\tan(e+f\,x)^2\,(a\,b-2\,b^2)}{2\,a^2\,(a-b)}}{f\,\left(b\tan\left(e+f\,x\right)^4+a\tan\left(e+f\,x\right)^2\right)}+\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^2}-\frac{\ln\left(\tan(e+f\,x)\right)\,(a+2\,b)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (log(a + b*tan(e + f*x)^2)*(b/a^3 + 1/(2*a^2) - 1/(2*(a - b)^2)))/f - (1/(2
*a) + (tan(e + f*x)^2*(a*b - 2*b^2))/(2*a^2*(a - b)))/(f*(a*tan(e + f*x)^2
+ b*tan(e + f*x)^4)) + log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) - (log(tan(e
 + f*x))*(a + 2*b))/(a^3*f)
```

3.229 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=161

$$\frac{(a+2b)\cot^2(e+fx)}{2a^3f}-\frac{\cot^4(e+fx)}{4a^2f}+\frac{\log(\cos(e+fx))}{(a-b)^2f}+\frac{(a^2+2ab+3b^2)\log(\tan(e+fx))}{a^4f}+\frac{(4a-3b)b^3\,\mathrm{lo}}{2a}$$

```
[Out] 1/2*(a+2*b)*cot(f*x+e)^2/a^3/f-1/4*cot(f*x+e)^4/a^2/f+ln(cos(f*x+e))/(a-b)^
2/f+(a^2+2*a*b+3*b^2)*ln(tan(f*x+e))/a^4/f+1/2*(4*a-3*b)*b^3*ln(a+b*tan(f*x
+e)^2)/a^4/(a-b)^2/f-1/2*b^3/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$\frac{b^3(4a-3b)\log\left(a+b\tan^2(e+fx)\right)}{2a^4f(a-b)^2}-\frac{b^3}{2a^3f(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{(a+2b)\cot^2(e+fx)}{2a^3f}-\frac{\cot^4(e+fx)}{4a^2f}+\frac{(a^2+2ab+3b^2)\log(\tan(e+fx))}{a^4f}+\frac{\log(\cos(e+fx))}{f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b)*Cot[e + f*x]^2)/(2*a^3*f) - Cot[e + f*x]^4/(4*a^2*f) + Log[Cos[e
 + f*x]]/((a - b)^2*f) + ((a^2 + 2*a*b + 3*b^2)*Log[Tan[e + f*x]])/(a^4*f)
+ ((4*a - 3*b)*b^3*Log[a + b*Tan[e + f*x]^2])/(2*a^4*(a - b)^2*f) - b^3/(2*
a^3*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^2}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^3} + \frac{-a-2b}{a^3x^2} + \frac{a^2+2ab+3b^2}{a^4x} - \frac{1}{(a-b)^2(1+x)} + \frac{b^4}{a^3(a-b)(a+bx)^2} + \frac{(4a-3b)b^4}{a^4(a-b)^2(a+b}\right.\right.}{2f}$$

$$= \frac{(a+2b)\cot^2(e+fx)}{2a^3f} - \frac{\cot^4(e+fx)}{4a^2f} + \frac{\log(\cos(e+fx))}{(a-b)^2f} + \frac{(a^2+2ab+3b^2)\,\text{l}}{a^4}$$

Mathematica [A]

time = 0.71, size = 121, normalized size = 0.75

$$-\frac{-\frac{(a+2b)\cot^2(e+fx)}{a^3} + \frac{\cot^4(e+fx)}{2a^2} - \frac{b^4}{a^4(a-b)(b+a\cot^2(e+fx))} - \frac{(4a-3b)b^3\log\left(b+a\cot^2(e+fx)\right)}{a^4(a-b)^2} - \frac{2\log(\sin(e+fx))}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -1/2*(-(((a + 2*b)*Cot[e + f*x]^2)/a^3) + Cot[e + f*x]^4/(2*a^2) - b^4/(a^4
*(a - b)*(b + a*Cot[e + f*x]^2)) - ((4*a - 3*b)*b^3*Log[b + a*Cot[e + f*x]^
2])/(a^4*(a - b)^2) - (2*Log[Sin[e + f*x]])/(a - b)^2)/f
```

Maple [A]

time = 0.37, size = 217, normalized size = 1.35

method	result
norman	$\frac{-\frac{1}{4af}+\frac{(2a+3b)\left(\tan^2(fx+e)\right)}{4a^2f}+\frac{\left(-a^2b-a\,b^2+3b^3\right)b\left(\tan^6(fx+e)\right)}{2a^4f(a-b)}}{\tan(fx+e)^4\left(a+b\left(\tan^2(fx+e)\right)\right)} + \frac{(a^2+2ab+3b^2)\ln(\tan(fx+e))}{a^4f} - \frac{\ln\left(1+\tan^2(fx+e)\right)}{2f(a^2-2ab+b^2)}$
derivativedivides	$\frac{-\frac{1}{16a^2(\cos(fx+e)+1)^2}-\frac{-7a-8b}{16a^3(\cos(fx+e)+1)}+\frac{\left(a^2+2ab+3b^2\right)\ln(\cos(fx+e)+1)}{2a^4}-\frac{1}{16a^2(\cos(fx+e)-1)^2}-\frac{7a+8b}{16a^3(\cos(fx+e)-1)}+\frac{\left(a^2\right.}{}}{f}$

default	$-\frac{1}{16a^2(\cos(fx+e)+1)^2}-\frac{-7a-8b}{16a^3(\cos(fx+e)+1)}+\frac{(a^2+2ab+3b^2)\ln(\cos(fx+e)+1)}{2a^4}-\frac{1}{16a^2(\cos(fx+e)-1)^2}-\frac{7a+8b}{16a^3(\cos(fx+e)-1)}+\frac{(a$
risch	$-\frac{6ib^2x}{a^4}-\frac{6ib^2e}{a^4f}-\frac{2ie}{a^2f}-\frac{4ibe}{a^3f}-\frac{2ix}{a^2}-\frac{8ib^3e}{a^3f(a^2-2ab+b^2)}+\frac{6ib^4x}{a^4(a^2-2ab+b^2)}+\frac{ix}{a^2-2ab+b^2}-\frac{8ib^3x}{a^3(a^2-2ab+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/16/a^2/(cos(f*x+e)+1)^2-1/16*(-7*a-8*b)/a^3/(cos(f*x+e)+1)+1/2*(a^2
+2*a*b+3*b^2)/a^4*ln(cos(f*x+e)+1)-1/16/a^2/(cos(f*x+e)-1)^2-1/16*(7*a+8*b)
/a^3/(cos(f*x+e)-1)+1/2*(a^2+2*a*b+3*b^2)/a^4*ln(cos(f*x+e)-1)+1/2*b^3/a^4*
(a*b/(a-b)^2/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+(4*a-3*b)/(a-b)^2*ln(a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)))
```

Maxima [A]
time = 0.30, size = 242, normalized size = 1.50

$$\frac{\frac{2\,(4\,ab^3-3\,b^4)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^6-2\,a^5b+a^4b^2}+\frac{2\,(2\,a^4-4\,a^3b+4\,ab^3-3\,b^4)\sin(fx+e)^4+a^4-2\,a^3b+a^2b^2-(5\,a^4-7\,a^3b-a^2b^2+3\,ab^3)\sin(fx+e)^2}{(a^6-3\,a^5b+3\,a^4b^2-a^3b^3)\sin(fx+e)^6-(a^6-2\,a^5b+a^4b^2)\sin(fx+e)^4}+\frac{2\,(a^2+2\,ab+3\,b^2)\log\left(\sin(fx+e)^2\right)}{a^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(4*a*b^3 - 3*b^4)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^6 - 2*a^5*b +
a^4*b^2) + (2*(2*a^4 - 4*a^3*b + 4*a*b^3 - 3*b^4)*sin(f*x + e)^4 + a^4 - 2*
a^3*b + a^2*b^2 - (5*a^4 - 7*a^3*b - a^2*b^2 + 3*a*b^3)*sin(f*x + e)^2)/((a
^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*sin(f*x + e)^6 - (a^6 - 2*a^5*b + a^4*b
^2)*sin(f*x + e)^4) + 2*(a^2 + 2*a*b + 3*b^2)*log(sin(f*x + e)^2)/a^4)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(159) = 318$.
time = 1.30, size = 360, normalized size = 2.24

$$\frac{(3\,a^4b-2\,a^3b^2-5\,a^2b^3+6\,ab^4)\tan(fx+e)^6-a^5+2\,a^4b-a^3b^2+(3\,a^5-5\,a^3b^2-2\,a^2b^3+6\,ab^4)\tan(fx+e)^4+(2\,a^5-a^4b-4\,a^3b^2+3\,a^2b^3)\tan(fx+e)^2+2\left((a^4b-4\,ab^4+3\,b^5)\tan(fx+e)^6+(a^5-4\,a^2b^3+3\,ab^4)\tan(fx+e)^4\right)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)+2\left((4\,ab^4-3\,b^5)\tan(fx+e)^6+(4\,a^2b^3-3\,ab^4)\tan(fx+e)^4\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{4\left((a^6b-2\,a^5b^2+a^4b^3)f\tan(fx+e)^6+(a^7-2\,a^6b+a^5b^2)f\tan(fx+e)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^4*b - 2*a^3*b^2 - 5*a^2*b^3 + 6*a*b^4)*tan(f*x + e)^6 - a^5 + 2*a
^4*b - a^3*b^2 + (3*a^5 - 5*a^3*b^2 - 2*a^2*b^3 + 6*a*b^4)*tan(f*x + e)^4 +
 (2*a^5 - a^4*b - 4*a^3*b^2 + 3*a^2*b^3)*tan(f*x + e)^2 + 2*((a^4*b - 4*a*b
^4 + 3*b^5)*tan(f*x + e)^6 + (a^5 - 4*a^2*b^3 + 3*a*b^4)*tan(f*x + e)^4)*lo
g(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + 2*((4*a*b^4 - 3*b^5)*tan(f*x + e)^
```

```
6 + (4*a^2*b^3 - 3*a*b^4)*tan(f*x + e)^4)*log((b*tan(f*x + e)^2 + a)/(tan(f
*x + e)^2 + 1)))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2
*a^6*b + a^5*b^2)*f*tan(f*x + e)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(159) = 318.
time = 1.11, size = 685, normalized size = 4.25

$$\frac{\frac{32\,(4\,ab^3-3\,b^4)\log\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^6-2\,a^5b+a^4b^2}-\frac{64\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2-2\,ab+b^2}-\frac{32\left(4\,a^2b^3-3\,ab^4+\frac{8\,a^2b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{18\,ab^4(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,b^5(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,a^2b^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3\,ab^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{(a^6-2\,a^5b+a^4b^2)\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}+\frac{32\,(a^2+2\,ab+3\,b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4}-\frac{\frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{16\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^4}-\frac{\left(a^2+\frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{16\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{96\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{144\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{a^4(\cos(fx+e)-1)^2}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(32*(4*a*b^3 - 3*b^4)*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2)/(a^6 - 2*a^5*b + a^4*b^2) - 64*log(abs(-(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 1))/(a^2 - 2*a*b + b^2) - 32*(4*a^2*b^3 - 3*a*b^4 +
8*a^2*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 18*a*b^4*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 8*b^5*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*a^2
*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^4*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2)/((a^6 - 2*a^5*b + a^4*b^2)*(a + 2*a*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) + 32*(a^2 + 2*a*b + 3*b^2)*log(abs
(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^4 - (12*a^2*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 16*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a^2*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^4 - (a^2 + 12*a^2*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + 16*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48
*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 96*a*b*(cos(f*x + e) - 1)^
2/(cos(f*x + e) + 1)^2 + 144*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)
*(cos(f*x + e) + 1)^2/(a^4*(cos(f*x + e) - 1)^2))/f
```

Mupad [**B**]
time = 12.37, size = 191, normalized size = 1.19

$$\frac{\frac{\tan(e+f\,x)^2\,(2\,a+3\,b)}{4\,a^2}-\frac{1}{4\,a}+\frac{\tan(e+f\,x)^4\,(a^2\,b+a\,b^2-3\,b^3)}{2\,a^3\,(a-b)}}{f\,\left(b\tan\left(e+f\,x\right)^6+a\tan\left(e+f\,x\right)^4\right)}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,{(a-b)}^2}+\frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+2\,a\,b+3\,b^2\right)}{a^4\,f}+\frac{\ln\left(b\tan(e+f\,x)^2+a\right)\,\left(4\,a\,b^3-3\,b^4\right)}{f\,\left(2\,a^6-4\,a^5\,b+2\,a^4\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] ((tan(e + f*x)^2*(2*a + 3*b))/(4*a^2) - 1/(4*a) + (tan(e + f*x)^4*(a*b^2 +
a^2*b - 3*b^3))/(2*a^3*(a - b)))/(f*(a*tan(e + f*x)^4 + b*tan(e + f*x)^6))
- log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) + (log(tan(e + f*x))*(2*a*b + a^2
 + 3*b^2))/(a^4*f) + (log(a + b*tan(e + f*x)^2)*(4*a*b^3 - 3*b^4))/(f*(2*a^
6 - 4*a^5*b + 2*a^4*b^2))
```

3.230 $\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal. Leaf size=130

$$-\frac{x}{(a-b)^2}-\frac{a^{3/2}(3a-5b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{5/2}f}+\frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f}-\frac{a\tan^3(e+fx)}{2(a-b)bf\,(a+b\tan^2(e+fx))}$$

```
[Out] -x/(a-b)^2-1/2*a^(3/2)*(3*a-5*b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)^2
/b^(5/2)/f+1/2*(3*a-2*b)*tan(f*x+e)/(a-b)/b^2/f-1/2*a*tan(f*x+e)^3/(a-b)/b/
f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 481, 596, 536, 209, 211}

$$-\frac{a^{3/2}(3a-5b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2b^{5/2}f(a-b)^2}+\frac{(3a-2b)\tan(e+fx)}{2b^2f(a-b)}-\frac{a\tan^3(e+fx)}{2bf(a-b)\,(a+b\tan^2(e+fx))}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) - (a^(3/2)*(3*a - 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*(a - b)^2*b^(5/2)*f) + ((3*a - 2*b)*Tan[e + f*x])/(2*(a - b)*b^2*f) -
 (a*Tan[e + f*x]^3)/(2*(a - b)*b*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{2(a-b)bf\,(a+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(3a-2b)x^2)}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx\right.}{2(a-b)bf}$$

$$= \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} - \frac{a\tan^3(e+fx)}{2(a-b)bf\,(a+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{a(3a-2b)+}{(1+}\right.}{}$$

$$= \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} - \frac{a\tan^3(e+fx)}{2(a-b)bf\,(a+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.}{(a-b}$$

$$= -\frac{x}{(a-b)^2} - \frac{a^{3/2}(3a-5b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{5/2}f} + \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} -$$

Mathematica [A]
time = 0.87, size = 118, normalized size = 0.91

$$\frac{-\frac{2(e+fx)}{(a-b)^2} - \frac{a^{3/2}(3a-5b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^2b^{5/2}} + \frac{a^2\sin(2(e+fx))}{(a-b)b^2(a+b+(a-b)\cos(2(e+fx)))} + \frac{2\tan(e+fx)}{b^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
[Out] ((-2*(e + f*x))/(a - b)^2 - (a^(3/2)*(3*a - 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/((a - b)^2*b^(5/2)) + (a^2*Sin[2*(e + f*x)])/((a - b)*b^2*(a
+ b + (a - b)*Cos[2*(e + f*x)])) + (2*Tan[e + f*x])/b^2)/(2*f)
```

Maple [A]
time = 0.27, size = 104, normalized size = 0.80

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b^2} - \frac{\arctan(\tan(fx+e))}{(a-b)^2} - \frac{a^2\left(\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)} + \frac{(3a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^2b^2}}{f}$

default	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{(a-b)^2}-\frac{a^2\left(\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(3a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^2b^2}}{f}$
risch	$-\frac{x}{a^2-2ab+b^2}+\frac{i\left(3a^3 e^{4i(fx+e)}-5a^2b\,e^{4i(fx+e)}+6a\,b^2e^{4i(fx+e)}-2b^3e^{4i(fx+e)}+6a^3e^{2i(fx+e)}-4a^2b\,e^{2i(fx+e)}-4a\,b^2\right.}{f\,b^2(a-b)^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/b^2*tan(f*x+e)-1/(a-b)^2*arctan(tan(f*x+e))-a^2/(a-b)^2/b^2*((-1/2*a
+1/2*b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(3*a-5*b)/(a*b)^(1/2)*arctan(b*ta
n(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.50, size = 140, normalized size = 1.08

$$\frac{\frac{a^2\tan(fx+e)}{a^2b^2-ab^3+(ab^3-b^4)\tan(fx+e)^2}-\frac{(3\,a^3-5\,a^2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2b^2-2\,ab^3+b^4)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}+\frac{2\,\tan(fx+e)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/(a^2*b^2 - a*b^3 + (a*b^3 - b^4)*tan(f*x + e)^2) - (3
*a^3 - 5*a^2*b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*b^2 - 2*a*b^3 + b^4)
*sqrt(a*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2) + 2*tan(f*x + e)/b^2)/f
```

Fricas [A]
time = 1.57, size = 492, normalized size = 3.78

$$\left[-\frac{8\,b^3fx\tan(fx+e)^2+8\,ab^2fx-8\,(a^2b-2\,ab^2+b^3)\tan(fx+e)^3+(3\,a^3-5\,a^2b+(3\,a^2b-5\,ab^2)\tan(fx+e)^2)\sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\left(b^2\tan(fx+e)^3-ab\tan(fx+e)\right)\sqrt{-\frac{a}{b}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)-4\,(3\,a^3-5\,a^2b+2\,ab^2)\tan(fx+e)}{8\left((a^2b^3-2\,ab^4+b^5)f\tan(fx+e)^2+(a^3b^2-2\,a^2b^3+ab^4)f\right)},\ -\frac{4\,b^3fx\tan(fx+e)^2+4\,ab^2fx-4\,(a^2b-2\,ab^2+b^3)\tan(fx+e)^3+(3\,a^3-5\,a^2b+(3\,a^2b-5\,ab^2)\tan(fx+e)^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{a}{b}}}{2\,a\tan(fx+e)}\right)-2\,(3\,a^3-5\,a^2b+2\,ab^2)\tan(fx+e)}{4\left((a^2b^3-2\,ab^4+b^5)f\tan(fx+e)^2+(a^3b^2-2\,a^2b^3+ab^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*b^3*f*x*tan(f*x + e)^2 + 8*a*b^2*f*x - 8*(a^2*b - 2*a*b^2 + b^3)*t
an(f*x + e)^3 + (3*a^3 - 5*a^2*b + (3*a^2*b - 5*a*b^2)*tan(f*x + e)^2)*sqrt
(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(b^2*tan(f*
x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*
x + e)^2 + a^2)) - 4*(3*a^3 - 5*a^2*b + 2*a*b^2)*tan(f*x + e))/((a^2*b^3 -
```

```
2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), -1/4*(4
*b^3*f*x*tan(f*x + e)^2 + 4*a*b^2*f*x - 4*(a^2*b - 2*a*b^2 + b^3)*tan(f*x +
 e)^3 + (3*a^3 - 5*a^2*b + (3*a^2*b - 5*a*b^2)*tan(f*x + e)^2)*sqrt(a/b)*ar
ctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x + e))) - 2*(3*a^3 - 5*
a^2*b + 2*a*b^2)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2
+ (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2859 vs. $2(102) = 204$.
time = 43.52, size = 2859, normalized size = 21.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x*tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e +
 f*x)/f)/b**2, Eq(a, 0)), (-15*f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**
4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) - 30*f*x*tan(e + f*x)**2/(8*b**2*
f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) - 15*f*x/(8*b**2*
f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 8*tan(e + f*x)*
*5/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 25*t
an(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b*
*2*f) + 15*tan(e + f*x)/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)*
*2 + 8*b**2*f), Eq(a, b)), (x*tan(e)**6/(a + b*tan(e)**2)**2, Eq(f, 0)), ((
-x + tan(e + f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a**2,
Eq(b, 0)), (-3*a**4*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**3*b**3*f*sqrt(-a/
b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) -
8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sq
rt(-a/b)*tan(e + f*x)**2) + 3*a**4*log(sqrt(-a/b) + tan(e + f*x))/(4*a**3*b
**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f
*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b)
 + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) + 6*a**3*b*sqrt(-a/b)*tan(e + f*x)/
(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a*
*2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*s
qrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) - 3*a**3*b*log(-sqrt(-a/b)
 + tan(e + f*x))*tan(e + f*x)**2/(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*
sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/
b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*
x)**2) + 5*a**3*b*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**3*b**3*f*sqrt(-a/b)
 + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8*
a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqrt
(-a/b)*tan(e + f*x)**2) + 3*a**3*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f
*x)**2/(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2
 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b
```

```
**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) - 5*a**3*b*log(sqrt
(-a/b) + tan(e + f*x))/(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)
*tan(e + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e +
 f*x)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) + 4
*a**2*b**2*sqrt(-a/b)*tan(e + f*x)**3/(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b*
*4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqr
t(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e
 + f*x)**2) - 10*a**2*b**2*sqrt(-a/b)*tan(e + f*x)/(4*a**3*b**3*f*sqrt(-a/b
) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8
*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqr
t(-a/b)*tan(e + f*x)**2) + 5*a**2*b**2*log(-sqrt(-a/b) + tan(e + f*x))*tan(
e + f*x)**2/(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*
x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 +
4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) - 5*a**2*b**2*
log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**3*b**3*f*sqrt(-a/b) +
4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b
**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a
/b)*tan(e + f*x)**2) - 4*a*b**3*f*x*sqrt(-a/b)/(4*a**3*b**3*f*sqrt(-a/b) +
4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b
**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a
/b)*tan(e + f*x)**2) - 8*a*b**3*sqrt(-a/b)*tan(e + f*x)**3/(4*a**3*b**3*f*s
qrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f*sqrt(-
a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b*
*6*f*sqrt(-a/b)*tan(e + f*x)**2) + 4*a*b**3*sqrt(-a/b)*tan(e + f*x)/(4*a**3
*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4
*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/
b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) - 4*b**4*f*x*sqrt(-a/b)*tan(e + f
*x)**2/(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2
 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b
**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) + 4*b**4*sqrt(-a/b)
*tan(e + f*x)**3/(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e
 + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)*
*2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2), True))
```

Giac [A]

time = 2.21, size = 149, normalized size = 1.15

$$\frac{\frac{a^2\tan(fx+e)}{(ab^2-b^3)\left(b\tan(fx+e)^2+a\right)}-\frac{(3a^3-5a^2b)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^2b^2-2ab^3+b^4)\sqrt{ab}}-\frac{2(fx+e)}{a^2-2ab+b^2}+\frac{2\tan(fx+e)}{b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/((a*b^2 - b^3)*(b*tan(f*x + e)^2 + a)) - (3*a^3 - 5*a
^2*b)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
```

```
)))/((a^2*b^2 - 2*a*b^3 + b^4)*sqrt(a*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2)
 + 2*tan(f*x + e)/b^2)/f
```

Mupad [B]
time = 13.01, size = 2581, normalized size = 19.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] (2*atan((((((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 -
6*a^6*b^3)*1i)/(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(16*b^
10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))/(2*(b^5
 - 2*a*b^4 + a^2*b^3)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (
tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2))/(2*(b^5 - 2*a*b^4 + a
^2*b^3)))/(2*a^2 - 4*a*b + 2*b^2) - ((((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 -
 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)*1i)/(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*
b^3) + (tan(e + f*x)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4
*b^6 + 16*a^5*b^5))/(2*(b^5 - 2*a*b^4 + a^2*b^3)*(2*a^2 - 4*a*b + 2*b^2)))/
(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*
b^2))/(2*(b^5 - 2*a*b^4 + a^2*b^3)))/(2*a^2 - 4*a*b + 2*b^2))/((((((4*a*b^8
 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)*1i)/(3*a*
b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(16*b^10 - 48*a*b^9 + 32*a
^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))/(2*(b^5 - 2*a*b^4 + a^2*b^3
)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(9*
a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2)*1i)/(2*(b^5 - 2*a*b^4 + a^2*b^3)))/(2*
a^2 - 4*a*b + 2*b^2) - ((9*a^5)/2 - (21*a^4*b)/2 + 5*a^2*b^3 + 2*a^3*b^2)/(
3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) + (((((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b
^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)*1i)/(3*a*b^5 - b^6 - 3*a^2*b^4 +
a^3*b^3) + (tan(e + f*x)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48
*a^4*b^6 + 16*a^5*b^5))/(2*(b^5 - 2*a*b^4 + a^2*b^3)*(2*a^2 - 4*a*b + 2*b^2
)))*1i)/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 +
 25*a^4*b^2)*1i)/(2*(b^5 - 2*a*b^4 + a^2*b^3)))/(2*a^2 - 4*a*b + 2*b^2))))/
(f*(2*a^2 - 4*a*b + 2*b^2)) + tan(e + f*x)/(b^2*f) + (a^2*tan(e + f*x))/(2*
f*(a - b)*(a*b^2 + b^3*tan(e + f*x)^2)) - (atan((((3*a - 5*b)*(-a^3*b^5)^(1
/2)*((tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2))/(2*(b^5 - 2*a*b
^4 + a^2*b^3)) - (((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5
*b^4 - 6*a^6*b^3)/(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(3*
a - 5*b)*(-a^3*b^5)^(1/2)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 4
8*a^4*b^6 + 16*a^5*b^5))/(8*(b^5 - 2*a*b^4 + a^2*b^3)*(b^7 - 2*a*b^6 + a^2*
b^5)))*(3*a - 5*b)*(-a^3*b^5)^(1/2))/(4*(b^7 - 2*a*b^6 + a^2*b^5)))*1i)/(4*
(b^7 - 2*a*b^6 + a^2*b^5)) + ((3*a - 5*b)*(-a^3*b^5)^(1/2)*((tan(e + f*x)*(
9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2))/(2*(b^5 - 2*a*b^4 + a^2*b^3)) + (((
4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)/(3
```

```
*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) + (tan(e + f*x)*(3*a - 5*b)*(-a^3*b^5)^
(1/2)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b
^5))/(8*(b^5 - 2*a*b^4 + a^2*b^3)*(b^7 - 2*a*b^6 + a^2*b^5)))*(3*a - 5*b)*(
-a^3*b^5)^(1/2))/(4*(b^7 - 2*a*b^6 + a^2*b^5)))*1i)/(4*(b^7 - 2*a*b^6 + a^2
*b^5)))/(((9*a^5)/2 - (21*a^4*b)/2 + 5*a^2*b^3 + 2*a^3*b^2)/(3*a*b^5 - b^6
- 3*a^2*b^4 + a^3*b^3) + ((3*a - 5*b)*(-a^3*b^5)^(1/2)*((tan(e + f*x)*(9*a^
6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2))/(2*(b^5 - 2*a*b^4 + a^2*b^3)) - (((4*a*
b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)/(3*a*b
^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(3*a - 5*b)*(-a^3*b^5)^(1/2
)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))
/(8*(b^5 - 2*a*b^4 + a^2*b^3)*(b^7 - 2*a*b^6 + a^2*b^5)))*(3*a - 5*b)*(-a^3
*b^5)^(1/2))/(4*(b^7 - 2*a*b^6 + a^2*b^5))))/(4*(b^7 - 2*a*b^6 + a^2*b^5))
- ((3*a - 5*b)*(-a^3*b^5)^(1/2)*((tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 +
25*a^4*b^2))/(2*(b^5 - 2*a*b^4 + a^2*b^3)) + (((4*a*b^8 - 22*a^2*b^7 + 48*a
^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)/(3*a*b^5 - b^6 - 3*a^2*b^4 +
a^3*b^3) + (tan(e + f*x)*(3*a - 5*b)*(-a^3*b^5)^(1/2)*(16*b^10 - 48*a*b^9 +
 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))/(8*(b^5 - 2*a*b^4 + a^
2*b^3)*(b^7 - 2*a*b^6 + a^2*b^5)))*(3*a - 5*b)*(-a^3*b^5)^(1/2))/(4*(b^7 -
2*a*b^6 + a^2*b^5))))/(4*(b^7 - 2*a*b^6 + a^2*b^5))))*(3*a - 5*b)*(-a^3*b^5
)^(1/2)*1i)/(2*f*(b^7 - 2*a*b^6 + a^2*b^5))
```

3.231 $\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^2} dx$

Optimal. Leaf size=95

$$\frac{x}{(a-b)^2}+\frac{\sqrt{a}\,(a-3b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{3/2}f}-\frac{a\tan(e+fx)}{2(a-b)bf\,(a+b\tan^2(e+fx))}$$

```
[Out] x/(a-b)^2+1/2*(a-3*b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*a^(1/2)/(a-b)^2/b^
(3/2)/f-1/2*a*tan(f*x+e)/(a-b)/b/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 481, 536, 209, 211}

$$\frac{\sqrt{a}\,(a-3b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2b^{3/2}f(a-b)^2}-\frac{a\tan(e+fx)}{2bf(a-b)\,(a+b\tan^2(e+fx))}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] x/(a - b)^2 + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*(a - b)^2*b^(3/2)*f) - (a*Tan[e + f*x])/(2*(a - b)*b*f*(a + b*Tan[e + f*x]
^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{a\tan(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{a+(a-2b)x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right.}{2(a-b)bf}\\ &= -\frac{a\tan(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+fx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2f}+\frac{(a(}{}\\ &= \frac{x}{(a-b)^2}+\frac{\sqrt{a}\,(a-3b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{3/2}f}-\frac{a\tan(e+fx)}{2(a-b)bf\left(a+b\tan^2(e+\right.}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 94, normalized size = 0.99

$$\frac{2(e+fx)+\frac{\sqrt{a}\,(a-3b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}}-\frac{a(a-b)\sin(2(e+fx))}{b(a+b+(a-b)\cos(2(e+fx)))}}{2(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (2*(e + f*x) + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/b
^(3/2) - (a*(a - b)*Sin[2*(e + f*x)])/(b*(a + b + (a - b)*Cos[2*(e + f*x)])
))/(2*(a - b)^2*f)
```

Maple [A]
time = 0.22, size = 90, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^2}+\frac{a\left(-\frac{(a-b)\tan(fx+e)}{2b\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{(a-3b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a-b)^2}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^2}+\frac{a\left(-\frac{(a-b)\tan(fx+e)}{2b\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{(a-3b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a-b)^2}}{f}$
risch	$\frac{x}{a^2-2ab+b^2}-\frac{ia\left(a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}+a-b\right)}{fb(a-b)^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)}+\frac{\sqrt{-ab}\,a\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{}}{}\right.}{4b^2(a-b)^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^2*arctan(tan(f*x+e))+a/(a-b)^2*(-1/2*(a-b)/b*tan(f*x+e)/(a+b*t
an(f*x+e)^2)+1/2*(a-3*b)/b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.51, size = 118, normalized size = 1.24

$$-\frac{\frac{a\tan(fx+e)}{a^2b-ab^2+(ab^2-b^3)\tan(fx+e)^2}-\frac{(a^2-3\,ab)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2b-2\,ab^2+b^3)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*tan(f*x + e)/(a^2*b - a*b^2 + (a*b^2 - b^3)*tan(f*x + e)^2) - (a^2
- 3*a*b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*b - 2*a*b^2 + b^3)*sqrt(a*b
)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2))/f
```

Fricas [A]
time = 1.87, size = 397, normalized size = 4.18

$$\left[\frac{8\,b^{2}fx\tan\left(fx+e\right)^{2}+8\,abfx-\left(\left(ab-3\,b^{2}\right)\tan\left(fx+e\right)^{2}+a^{2}-3\,ab\right)\sqrt{-\frac{a}{b}}\log\left(\frac{b^{2}\tan(fx+e)^{4}-6\,ab\tan(fx+e)^{2}+a^{2}-4\,\left(b^{2}\tan(fx+e)^{3}-ab\tan(fx+e)\right)\sqrt{-\frac{a}{b}}}{b^{2}\tan(fx+e)^{4}+2\,ab\tan(fx+e)^{2}+a^{2}}\right)-4\,\left(a^{2}-ab\right)\tan\left(fx+e\right)}{8\,\left(\left(a^{2}b^{2}-2\,ab^{3}+b^{4}\right)f\tan\left(fx+e\right)^{2}+\left(a^{3}b-2\,a^{2}b^{2}+ab^{3}\right)f\right)},\ \frac{4\,b^{2}fx\tan\left(fx+e\right)^{2}+4\,abfx+\left(\left(ab-3\,b^{2}\right)\tan\left(fx+e\right)^{2}+a^{2}-3\,ab\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{\left(b\tan(fx+e)^{2}-a\right)\sqrt{\frac{a}{b}}}{2\,a\tan(fx+e)}\right)-2\,\left(a^{2}-ab\right)\tan\left(fx+e\right)}{4\,\left(\left(a^{2}b^{2}-2\,ab^{3}+b^{4}\right)f\tan\left(fx+e\right)^{2}+\left(a^{3}b-2\,a^{2}b^{2}+ab^{3}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*b^2*f*x*tan(f*x + e)^2 + 8*a*b*f*x - ((a*b - 3*b^2)*tan(f*x + e)^2
+ a^2 - 3*a*b)*sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^2 - a*b)*tan(f*x + e))/((a^2*b^2
 - 2*a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f), 1/4*(4
*b^2*f*x*tan(f*x + e)^2 + 4*a*b*f*x + ((a*b - 3*b^2)*tan(f*x + e)^2 + a^2 -
 3*a*b)*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x +
e))) - 2*(a^2 - a*b)*tan(f*x + e))/((a^2*b^2 - 2*a*b^3 + b^4)*f*tan(f*x + e
)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2157 vs. $2(78) = 156$.
time = 15.29, size = 2157, normalized size = 22.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((x + tan(e + f*x)**3/(3
*f) - tan(e + f*x)/f)/a**2, Eq(b, 0)), (x/b**2, Eq(a, 0)), (3*f*x*tan(e + f
*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) +
6*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2
 + 8*b**2*f) + 3*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2
+ 8*b**2*f) - 5*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e
 + f*x)**2 + 8*b**2*f) - 3*tan(e + f*x)/(8*b**2*f*tan(e + f*x)**4 + 16*b**2
*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x*tan(e)**4/(a + b*tan(e)**2)**
2, Eq(f, 0)), (a**3*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**3*b**2*f*sqrt(-a/
b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) -
8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**4*f*sqrt(-a/b) + 4*b**5*f*sq
rt(-a/b)*tan(e + f*x)**2) - a**3*log(sqrt(-a/b) + tan(e + f*x))/(4*a**3*b**
2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**3*f*s
qrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**4*f*sqrt(-a/b) +
 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2) - 2*a**2*b*sqrt(-a/b)*tan(e + f*x)/(4
*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2
*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**4*f*sqr
```

```
t(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2) + a**2*b*log(-sqrt(-a/b) + t
an(e + f*x))*tan(e + f*x)**2/(4*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt
(-a/b)*tan(e + f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*t
an(e + f*x)**2 + 4*a*b**4*f*sqrt(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**
2) - 3*a**2*b*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**3*b**2*f*sqrt(-a/b) + 4
*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) - 8*a*b*
*4*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**4*f*sqrt(-a/b) + 4*b**5*f*sqrt(-a/
b)*tan(e + f*x)**2) - a**2*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2
/(4*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a
**2*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**4*f*
sqrt(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2) + 3*a**2*b*log(sqrt(-a/b)
 + tan(e + f*x))/(4*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e
 + f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)*
*2 + 4*a*b**4*f*sqrt(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2) + 4*a*b**
2*f*x*sqrt(-a/b)/(4*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e
 + f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)*
*2 + 4*a*b**4*f*sqrt(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2) + 2*a*b**
2*sqrt(-a/b)*tan(e + f*x)/(4*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a
/b)*tan(e + f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(
e + f*x)**2 + 4*a*b**4*f*sqrt(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2)
- 3*a*b**2*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**3*b**2*f*s
qrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**3*f*sqrt(-
a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**4*f*sqrt(-a/b) + 4*b*
*5*f*sqrt(-a/b)*tan(e + f*x)**2) + 3*a*b**2*log(sqrt(-a/b) + tan(e + f*x))*
tan(e + f*x)**2/(4*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e
+ f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)**
2 + 4*a*b**4*f*sqrt(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2) + 4*b**3*f
*x*sqrt(-a/b)*tan(e + f*x)**2/(4*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqr
t(-a/b)*tan(e + f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*
tan(e + f*x)**2 + 4*a*b**4*f*sqrt(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)*
*2), True))
```

Giac [A]
time = 1.15, size = 127, normalized size = 1.34

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(a^2-3\,ab)}{(a^2b-2\,ab^2+b^3)\sqrt{ab}}+\frac{2\,(fx+e)}{a^2-2\,ab+b^2}-\frac{a\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)(ab-b^2)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(a^2 - 3*a*b)/((a^2*b - 2*a*b^2 + b^3)*sqrt(a*b)) + 2*(f*x + e)/(a^2 - 2
*a*b + b^2) - a*tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a*b - b^2)))/f
```

Mupad [B]
time = 13.52, size = 2358, normalized size = 24.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^2,x)
[Out] (2*atan((((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)*1i)/
(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) - (tan(e + f*x)*(16*b^8 - 48*a*b^7 + 32
*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2*a*b^2 + b^3
)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(a^4 -
6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)))/(2*a^2 - 4*a*b +
 2*b^2) - ((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)*1i)
/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e + f*x)*(16*b^8 - 48*a*b^7 + 3
2*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2*a*b^2 + b^
3)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(a^4 -
 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)))/(2*a^2 - 4*a*b
+ 2*b^2))/((((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)*1
i)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) - (tan(e + f*x)*(16*b^8 - 48*a*b^7 +
 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2*a*b^2 +
b^3)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*
(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2)*1i)/(2*(a^2*b - 2*a*b^2 + b^3)))/(2*a^2
 - 4*a*b + 2*b^2) + (((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a
^5*b^2)*1i)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e + f*x)*(16*b^8 - 4
8*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2
*a*b^2 + b^3)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) - (tan(
e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2)*1i)/(2*(a^2*b - 2*a*b^2 + b^3)
))/(2*a^2 - 4*a*b + 2*b^2) + (3*a*b^2 - (5*a^2*b)/2 + a^3/2)/(3*a*b^3 + a^3
*b - b^4 - 3*a^2*b^2))))/(f*(2*a^2 - 4*a*b + 2*b^2)) + (atan(((((tan(e + f*
x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)) - (((2*
a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)/(3*a*b^3 + a^3*b -
b^4 - 3*a^2*b^2) - (tan(e + f*x)*(a - 3*b)*(-a*b^3)^(1/2)*(16*b^8 - 48*a*b^
7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(8*(a^2*b - 2*a*b^2
 + b^3)*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/2))/(4*(b^5 - 2*a
*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/2)*1i)/(4*(b^5 - 2*a*b^4 + a^2*b^3)
) + (((tan(e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^
2 + b^3)) + (((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)/(3
*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e + f*x)*(a - 3*b)*(-a*b^3)^(1/2)*
(16*b^8 - 48*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(8
*(a^2*b - 2*a*b^2 + b^3)*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/
2))/(4*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/2)*1i)/(4*(b^5 - 2
*a*b^4 + a^2*b^3)))/((3*a*b^2 - (5*a^2*b)/2 + a^3/2)/(3*a*b^3 + a^3*b - b^4
 - 3*a^2*b^2) - (((tan(e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^
2*b - 2*a*b^2 + b^3)) - (((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2
```

```
*a^5*b^2)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) - (tan(e + f*x)*(a - 3*b)*(-a
*b^3)^(1/2)*(16*b^8 - 48*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*
a^5*b^3))/(8*(a^2*b - 2*a*b^2 + b^3)*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*
(-a*b^3)^(1/2))/(4*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*b^3)^(1/2))/(4
*(b^5 - 2*a*b^4 + a^2*b^3)) + (((tan(e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^
2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)) + (((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 -
 8*a^4*b^3 + 2*a^5*b^2)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e + f*x)
*(a - 3*b)*(-a*b^3)^(1/2)*(16*b^8 - 48*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48
*a^4*b^4 + 16*a^5*b^3))/(8*(a^2*b - 2*a*b^2 + b^3)*(b^5 - 2*a*b^4 + a^2*b^3
)))*(a - 3*b)*(-a*b^3)^(1/2))/(4*(b^5 - 2*a*b^4 + a^2*b^3)))*(a - 3*b)*(-a*
b^3)^(1/2))/(4*(b^5 - 2*a*b^4 + a^2*b^3))))*(a - 3*b)*(-a*b^3)^(1/2)*1i)/(2
*f*(b^5 - 2*a*b^4 + a^2*b^3)) - (a*tan(e + f*x))/(2*b*f*(a + b*tan(e + f*x)
^2)*(a - b))
```

3.232 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{x}{(a-b)^2}+\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a-b)^2\sqrt{b}\,f}+\frac{\tan(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -x/(a-b)^2+1/2*(a+b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)^2/f/a^(1/2)/b
^(1/2)+1/2*tan(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 482, 536, 209, 211}

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\,f(a-b)^2}+\frac{\tan(e+fx)}{2f(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) + ((a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2*Sqrt[a
]*(a - b)^2*Sqrt[b]*f) + Tan[e + f*x]/(2*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2(a-b)f} \\
&= \frac{\tan(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2f} + \frac{(a+b)\text{S}}{} \\
&= -\frac{x}{(a-b)^2} + \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a-b)^2\sqrt{b}\,f} + \frac{\tan(e+fx)}{2(a-b)f\,(a+b\tan^2(e+fx))}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 87, normalized size = 0.97

$$\frac{-2(e+fx) + \frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{(a-b)\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))}}{2(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (-2*(e + f*x) + ((a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*S
qrt[b]) + ((a - b)*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]))/(2
*(a - b)^2*f)
```

Maple [A]
time = 0.21, size = 83, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^2}+\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a-b)^2}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^2}+\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a-b)^2}}{f}$
risch	$-\frac{x}{a^2-2ab+b^2}+\frac{i\left(a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}+a-b\right)}{f(a-b)^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)}+\frac{\ln\left(e^{2i(fx+e)}-\frac{2iab-\sqrt{-a\cdots}}{(a-b\cdots}\right.}{4\sqrt{-ab}\,(a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a-b)^2*arctan(tan(f*x+e))+1/(a-b)^2*((1/2*a-1/2*b)*tan(f*x+e)/(a+b
*tan(f*x+e)^2)+1/2*(a+b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.49, size = 101, normalized size = 1.12

$$\frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}+\frac{\tan(fx+e)}{(ab-b^2)\tan(fx+e)^2+a^2-ab}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a + b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2 - 2*a*b + b^2)*sqrt(a*b
)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2) + tan(f*x + e)/((a*b - b^2)*tan(f*x +
e)^2 + a^2 - a*b))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(81) = 162.
time = 2.54, size = 409, normalized size = 4.54

$$\left[-\frac{8\,ab^2fx\tan(fx+e)^2+8\,a^2bfx+\left((ab+b^2)\tan(fx+e)^2+a^2+ab\right)\sqrt{-ab}\,\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2-4\left(b\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{-ab}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)-4\,(a^2b-ab^2)\tan(fx+e)}{8\left((a^3b^2-2\,a^2b^3+ab^4)f\tan(fx+e)^2+(a^4b-2\,a^3b^2+a^2b^3)f\right)},\ -\frac{4\,ab^2fx\tan(fx+e)^2+4\,a^2bfx-\left((ab+b^2)\tan(fx+e)^2+a^2+ab\right)\sqrt{ab}\,\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{ab}}{2\,ab\tan(fx+e)}\right)-2\,(a^2b-ab^2)\tan(fx+e)}{4\left((a^3b^2-2\,a^2b^3+ab^4)f\tan(fx+e)^2+(a^4b-2\,a^3b^2+a^2b^3)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")

[Out] [-1/8*(8*a*b^2*f*x*tan(f*x + e)^2 + 8*a^2*b*f*x + ((a*b + b^2)*tan(f*x + e)^2 + a^2 + a*b)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^2*b - a*b^2)*tan(f*x + e))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*f*tan(f*x + e)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*f), -1/4*(4*a*b^2*f*x*tan(f*x + e)^2 + 4*a^2*b*f*x - ((a*b + b^2)*tan(f*x + e)^2 + a^2 + a*b)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)/(a*b*tan(f*x + e))) - 2*(a^2*b - a*b^2)*tan(f*x + e))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*f*tan(f*x + e)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*f)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2113 vs. $2(73) = 146$.
time = 15.32, size = 2113, normalized size = 23.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)

[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x - 1/(f*tan(e + f*x)))/b**2, Eq(a, 0)), (f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 2*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) - tan(e + f*x)/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x*tan(e)**2/(a + b*tan(e)**2)**2, Eq(f, 0)), ((-x + tan(e + f*x)/f)/a**2, Eq(b, 0)), (a**2*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) - a**2*log(sqrt(-a/b) + tan(e + f*x))/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) - 4*a*b*f*x*sqrt(-a/b)/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) + 2*a*b*sqrt(-a/b)*tan(e + f*x)/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) + a*b*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*

```
b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) + a*b*log(-sqrt(-a
/b) + tan(e + f*x))/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e
 + f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)*
*2 + 4*a*b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) - a*b*log
(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**3*b*f*sqrt(-a/b) + 4*a**2
*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*
sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(-a/b)*ta
n(e + f*x)**2) - a*b*log(sqrt(-a/b) + tan(e + f*x))/(4*a**3*b*f*sqrt(-a/b)
+ 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a
*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(
-a/b)*tan(e + f*x)**2) - 4*b**2*f*x*sqrt(-a/b)*tan(e + f*x)**2/(4*a**3*b*f*
sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqrt(
-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) + 4*b
**4*f*sqrt(-a/b)*tan(e + f*x)**2) - 2*b**2*sqrt(-a/b)*tan(e + f*x)/(4*a**3*
b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*s
qrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) +
 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) + b**2*log(-sqrt(-a/b) + tan(e + f*x)
)*tan(e + f*x)**2/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e +
 f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2
 + 4*a*b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) - b**2*log(
sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*
b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*s
qrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) + 4*b**4*f*sqrt(-a/b)*tan
(e + f*x)**2), True))
```

Giac [A]
time = 0.85, size = 112, normalized size = 1.24

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(a+b)}{(a^2-2\,ab+b^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)(a-b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(a + b)/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2
) + tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a - b)))/f
```

Mupad [B]
time = 13.04, size = 2136, normalized size = 23.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^2,x)

[Out] tan(e + f*x)/(2*f*(a + b*tan(e + f*x)^2)*(a - b)) - (2*atan((((((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)*1i)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (tan(e + f*x)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b + b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)*1i)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(e + f*x)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b + b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)*1i)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (tan(e + f*x)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b + b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3)*1i)/(2*(a^2 - 2*a*b + b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (((((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)*1i)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(e + f*x)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b + b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3)*1i)/(2*(a^2 - 2*a*b + b^2)))/(2*a^2 - 4*a*b + 2*b^2) + ((a*b)/2 + b^2/2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3))))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((-a*b)^(1/2)*(a + b)*((tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b^2)) - (((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(8*(a^2 - 2*a*b + b^2)*(a*b^3 + a^3*b - 2*a^2*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a*b^3 + a^3*b - 2*a^2*b^2)))*1i)/(4*(a*b^3 + a^3*b - 2*a^2*b^2)) + ((-a*b)^(1/2)*(a + b)*((tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b^2)) + (((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(8*(a^2 - 2*a*b + b^2)*(a*b^3 + a^3*b - 2*a^2*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a*b^3 + a^3*b - 2*a^2*b^2)))*1i)/(4*(a*b^3 + a^3*b - 2*a^2*b^2)))/(((a*b)/2 + b^2/2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - ((-a*b)^(1/2)*(a + b)*((tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b^2)) - (((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(8*(a^2 - 2*a*b + b^2)*(a*b^3 + a^3*b - 2*a^2*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a*b^3 + a^3*b - 2*a^2*b^2))))/(4*(a*b^3 + a^3*b - 2*a^2*b^2)) + ((-a*b)^(1/2)*(a + b)*((tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b^2)) + (((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(e + f*x)*(-a*b)^(1/2)*(a + b)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(8*(a^2 - 2*a*b

```
 + b^2)*(a*b^3 + a^3*b - 2*a^2*b^2)))*(-a*b)^(1/2)*(a + b))/(4*(a*b^3 + a^3
*b - 2*a^2*b^2))))/(4*(a*b^3 + a^3*b - 2*a^2*b^2))))*(-a*b)^(1/2)*(a + b)*1
i)/(2*f*(a*b^3 + a^3*b - 2*a^2*b^2))
```

3.233 $\int \frac{1}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{x}{(a-b)^2}-\frac{(3a-b)\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2f}-\frac{b\tan(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))}$$

```
[Out] x/(a-b)^2-1/2*(3*a-b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1/2)/a^(3/2)/(a
-b)^2/f-1/2*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3742, 425, 536, 209, 211}

$$-\frac{\sqrt{b}\,(3a-b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}f(a-b)^2}-\frac{b\tan(e+fx)}{2af(a-b)\,(a+b\tan^2(e+fx))}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*f) - (b*Tan[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]
^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right.}{2a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{(a-b)^2f} - \frac{((3}{}$$

$$= \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2f} - \frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+\right.}$$

Mathematica [A]
time = 0.71, size = 88, normalized size = 0.91

$$\frac{2\text{ArcTan}(\tan(e+fx)) + \frac{\sqrt{b}\,(-3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(-a+b)\tan(e+fx)}{a(a+b\tan^2(e+fx))}}{2(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-2),x]
```

```
[Out] (2*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[e + f*x])/(a*(a + b*Tan[e + f*x]^2)))/
(2*(a - b)^2*f)
```

Maple [A]

time = 0.00, size = 93, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^2}-\frac{b\left(\frac{(a-b)\tan(fx+e)}{2a\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^2}-\frac{b\left(\frac{(a-b)\tan(fx+e)}{2a\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}}{f}$
risch	$\frac{x}{a^2-2ab+b^2}+\frac{ib\left(a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}+a-b\right)}{fa(-a+b)^2\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\right.}{4a(a-b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^2*arctan(tan(f*x+e))-1/(a-b)^2*b*(1/2/a*(a-b)*tan(f*x+e)/(a+b*
tan(f*x+e)^2)+1/2*(3*a-b)/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]

time = 0.50, size = 118, normalized size = 1.22

$$-\frac{\frac{b\tan(fx+e)}{a^3-a^2b+(a^2b-ab^2)\tan(fx+e)^2}+\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(f*x + e)/(a^3 - a^2*b + (a^2*b - a*b^2)*tan(f*x + e)^2) + (3*a*
b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b
)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2))/f
```

Fricas [A]

time = 2.96, size = 406, normalized size = 4.19

$$\left[\frac{8\,abfx\tan(fx+e)^2+8\,a^2fx-\left((3\,ab-b^2)\tan(fx+e)^2+3\,a^2-ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)-4\,(ab-b^2)\tan(fx+e)}{8\left((a^3b-2\,a^2b^2+ab^3)f\tan(fx+e)^2+(a^4-2\,a^3b+a^2b^2)f\right)},\ \frac{4\,abfx\tan(fx+e)^2+4\,a^2fx-\left((3\,ab-b^2)\tan(fx+e)^2+3\,a^2-ab\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)-2\,(ab-b^2)\tan(fx+e)}{4\left((a^3b-2\,a^2b^2+ab^3)f\tan(fx+e)^2+(a^4-2\,a^3b+a^2b^2)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*f*x*tan(f*x + e)^2 + 8*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2
+ 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a*b - b^2)*tan(f*x + e))/((a^3*b -
 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), 1/4*(4
*a*b*f*x*tan(f*x + e)^2 + 4*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2 + 3*a^2
 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x +
e))) - 2*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x +
 e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. $2(78) = 156$.
time = 14.93, size = 2125, normalized size = 21.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**2, Eq(b,
 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b**2, Eq(a, 0)),
(3*f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**
2 + 8*b**2*f) + 6*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f
*tan(e + f*x)**2 + 8*b**2*f) + 3*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*
tan(e + f*x)**2 + 8*b**2*f) + 3*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 +
 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 5*tan(e + f*x)/(8*b**2*f*tan(e + f
*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x/(a + b*tan(e)
**2)**2, Eq(f, 0)), (4*a**2*f*x*sqrt(-a/b)/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*
f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-
a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan
(e + f*x)**2) - 3*a**2*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**4*f*sqrt(-a/b)
 + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b
**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sq
rt(-a/b)*tan(e + f*x)**2) + 3*a**2*log(sqrt(-a/b) + tan(e + f*x))/(4*a**4*f
*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b)
 - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*
a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + 4*a*b*f*x*sqrt(-a/b)*tan(e + f*x)**2
/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*
sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(
-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) - 2*a*b*sqrt(-a/b)*tan(e + f
*x)/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b
*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sq
```

```
rt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) - 3*a*b*log(-sqrt(-a/b) +
 tan(e + f*x))*tan(e + f*x)**2/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)
*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e +
 f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2
) + a*b*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*s
qrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b
)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e
+ f*x)**2) + 3*a*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**4*f
*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b)
 - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*
a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) - a*b*log(sqrt(-a/b) + tan(e + f*x))/(
4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sq
rt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a
/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + 2*b**2*sqrt(-a/b)*tan(e + f*
x)/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*
f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqr
t(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + b**2*log(-sqrt(-a/b) + t
an(e + f*x))*tan(e + f*x)**2/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*t
an(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f
*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2)
- b**2*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**4*f*sqrt(-a/b)
+ 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b*
*2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqr
t(-a/b)*tan(e + f*x)**2), True))
```

Giac [A]
time = 0.59, size = 127, normalized size = 1.31

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)(a^2-ab)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(f*x + e)/(a^2 -
2*a*b + b^2) + b*tan(f*x + e)/((b*tan(f*x + e)^2 + a)*(a^2 - a*b)))/f
```

Mupad [B]
time = 13.52, size = 2489, normalized size = 25.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^2,x)

[Out] (2*atan((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 -
4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^
2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2
*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2
) + (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2
)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^
4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) +
 (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*
b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(
2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^
4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*a*b^7 - 12*a^2*b^
6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 +
a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5
+ 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^
2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(b^5 - 6*a
*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^
2) + (((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a
^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(e + f*x)*(16*a^2*b
^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a
^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^
2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2
*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((3*a*b^3)/2 - b^4/2)/(3*a^4*b - a^5 + a^
2*b^3 - 3*a^3*b^2))))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((-a^3*b)^(1/2)*
((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2))
- (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^
2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(3*
a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16
*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b
)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2
*a^4*b + a^3*b^2)) + ((-a^3*b)^(1/2)*((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2
*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5
 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^
2) + (tan(e + f*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a
^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2
)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b +
 a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2*a^4*b + a^3*b^2)))/((((3*a*b^3)/2 - b^
4/2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + ((-a^3*b)^(1/2)*((tan(e + f*x)
*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) - (((2*a*b^7 -
 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b -
a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2
*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*
(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a -
 b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)
```

```
) - ((-a^3*b)^(1/2)*((tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 -
2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 1
8*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(e + f*x
)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b
^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b
+ a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a
- b))/(4*(a^5 - 2*a^4*b + a^3*b^2))))*(-a^3*b)^(1/2)*(3*a - b)*1i)/(2*f*(a^
5 - 2*a^4*b + a^3*b^2)) - (b*tan(e + f*x))/(2*a*f*(a + b*tan(e + f*x)^2)*(a
 - b))
```

3.234 $\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal. Leaf size=128

$$-\frac{x}{(a-b)^2}+\frac{(5a-3b)b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^2f}-\frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f}-\frac{b\cot(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))}$$

```
[Out] -x/(a-b)^2+1/2*(5*a-3*b)*b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(5/2)
/(a-b)^2/f-1/2*(2*a-3*b)*cot(f*x+e)/a^2/(a-b)/f-1/2*b*cot(f*x+e)/a/(a-b)/f/
(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 483, 597, 536, 209, 211}

$$\frac{b^{3/2}(5a-3b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f(a-b)^2}-\frac{(2a-3b)\cot(e+fx)}{2a^2f(a-b)}-\frac{b\cot(e+fx)}{2af(a-b)\,(a+b\tan^2(e+fx))}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) + ((5*a - 3*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*a^(5/2)*(a - b)^2*f) - ((2*a - 3*b)*Cot[e + f*x])/(2*a^2*(a - b)*f) -
 (b*Cot[e + f*x])/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2a-3b-3bx^2}{x^2(1+x^2)(a+bx^2)}\,dx, x, \tan(e+f$$

$$= -\frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} - \frac{b\cot(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{2a^2+2c}{(}\right.}{}$$

$$= -\frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} - \frac{b\cot(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,c\right.}{(a}$$

$$= -\frac{x}{(a-b)^2} + \frac{(5a-3b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^2f} - \frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} -$$

Mathematica [A]
time = 2.14, size = 117, normalized size = 0.91

$$\frac{\frac{(5a-3b)b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^2} - \frac{2\cot(e+fx)}{a^2} + \frac{-2(e+fx)+\frac{(a-b)b^2\sin(2(e+fx))}{a^2(a+b+(a-b)\cos(2(e+fx)))}}{(a-b)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] (((5*a - 3*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*(a -
 b)^2) - (2*Cot[e + f*x])/a^2 + (-2*(e + f*x) + ((a - b)*b^2*Sin[2*(e + f*x
)])/(a^2*(a + b + (a - b)*Cos[2*(e + f*x)])))/(a - b)^2)/(2*f)
```

Maple [A]
time = 0.38, size = 106, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{b^2\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(5a-3b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2(a-b)^2}-\frac{1}{a^2\tan(fx+e)}-\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$

default	$\dfrac{\dfrac{b^2\left(\dfrac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\dfrac{(5a-3b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2(a-b)^2}-\dfrac{1}{a^2\tan(fx+e)}-\dfrac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
risch	$-\dfrac{x}{a^2-2ab+b^2}-\dfrac{i\left(2a^3e^{4i(fx+e)}-6a^2b\,e^{4i(fx+e)}+5a\,b^2e^{4i(fx+e)}-3b^3e^{4i(fx+e)}+4a^3e^{2i(fx+e)}-4a^2b\,e^{2i(fx+e)}-4a\,b^2e^{2}\right.}{f\,a^2(a-b)^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2/a^2/(a-b)^2*((1/2*a-1/2*b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(5*a-
3*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/a^2/tan(f*x+e)-1/(a-b)
^2*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.50, size = 156, normalized size = 1.22

$$\frac{\frac{(5\,ab^2-3\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^4-2\,a^3b+a^2b^2)\sqrt{ab}}-\frac{(2\,ab-3\,b^2)\tan(fx+e)^2+2\,a^2-2\,ab}{(a^3b-a^2b^2)\tan(fx+e)^3+(a^4-a^3b)\tan(fx+e)}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((5*a*b^2 - 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 2*a^3*b + a
^2*b^2)*sqrt(a*b)) - ((2*a*b - 3*b^2)*tan(f*x + e)^2 + 2*a^2 - 2*a*b)/((a^3
*b - a^2*b^2)*tan(f*x + e)^3 + (a^4 - a^3*b)*tan(f*x + e)) - 2*(f*x + e)/(a
^2 - 2*a*b + b^2))/f
```

Fricas [A]
time = 3.62, size = 525, normalized size = 4.10

$$\left[-\frac{8\,a^2bfx\tan(fx+e)^3+8\,a^3fx\tan(fx+e)+8\,a^3-16\,a^2b+8\,ab^2+4\,(2\,a^2b-5\,ab^2+3\,b^3)\tan(fx+e)^2+((5\,ab^2-3\,b^3)\tan(fx+e)^3+(5\,a^2b-3\,ab^2)\tan(fx+e))\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2-4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{8\,((a^4b-2\,a^3b^2+a^2b^3)f\tan(fx+e)^3+(a^5-2\,a^4b+a^3b^2)f\tan(fx+e))},-\frac{4\,a^2bfx\tan(fx+e)^3+4\,a^3fx\tan(fx+e)+4\,a^3-8\,a^2b+4\,ab^2+2\,(2\,a^2b-5\,ab^2+3\,b^3)\tan(fx+e)^2-((5\,ab^2-3\,b^3)\tan(fx+e)^3+(5\,a^2b-3\,ab^2)\tan(fx+e))\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)}{4\,((a^4b-2\,a^3b^2+a^2b^3)f\tan(fx+e)^3+(a^5-2\,a^4b+a^3b^2)f\tan(fx+e))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a^2*b*f*x*tan(f*x + e)^3 + 8*a^3*f*x*tan(f*x + e) + 8*a^3 - 16*a^2
*b + 8*a*b^2 + 4*(2*a^2*b - 5*a*b^2 + 3*b^3)*tan(f*x + e)^2 + ((5*a*b^2 - 3
*b^3)*tan(f*x + e)^3 + (5*a^2*b - 3*a*b^2)*tan(f*x + e))*sqrt(-b/a)*log((b^
2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2
*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2
```

```
)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*
b^2)*f*tan(f*x + e)), -1/4*(4*a^2*b*f*x*tan(f*x + e)^3 + 4*a^3*f*x*tan(f*x
+ e) + 4*a^3 - 8*a^2*b + 4*a*b^2 + 2*(2*a^2*b - 5*a*b^2 + 3*b^3)*tan(f*x +
e)^2 - ((5*a*b^2 - 3*b^3)*tan(f*x + e)^3 + (5*a^2*b - 3*a*b^2)*tan(f*x + e)
)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/
((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)
*f*tan(f*x + e))]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3245 vs. $2(102) = 204$.
time = 141.13, size = 3245, normalized size = 25.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-x - cot(e
+ f*x)/f)/a**2, Eq(b, 0)), ((-x - 1/(f*tan(e + f*x)) + 1/(3*f*tan(e + f*x)*
*3) - 1/(5*f*tan(e + f*x)**5))/b**2, Eq(a, 0)), (-15*f*x*tan(e + f*x)**5/(8
*b**2*f*tan(e + f*x)**5 + 16*b**2*f*tan(e + f*x)**3 + 8*b**2*f*tan(e + f*x)
) - 30*f*x*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**5 + 16*b**2*f*tan(e + f*
x)**3 + 8*b**2*f*tan(e + f*x)) - 15*f*x*tan(e + f*x)/(8*b**2*f*tan(e + f*x)
**5 + 16*b**2*f*tan(e + f*x)**3 + 8*b**2*f*tan(e + f*x)) - 15*tan(e + f*x)*
*4/(8*b**2*f*tan(e + f*x)**5 + 16*b**2*f*tan(e + f*x)**3 + 8*b**2*f*tan(e +
 f*x)) - 25*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**5 + 16*b**2*f*tan(e + f
*x)**3 + 8*b**2*f*tan(e + f*x)) - 8/(8*b**2*f*tan(e + f*x)**5 + 16*b**2*f*t
an(e + f*x)**3 + 8*b**2*f*tan(e + f*x)), Eq(a, b)), (zoo*x/a**2, Eq(e, -f*x
)), (x*cot(e)**2/(a + b*tan(e)**2)**2, Eq(f, 0)), (-4*a**3*f*x*sqrt(-a/b)*t
an(e + f*x)/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqrt(-a/b)*tan(e
 + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2*f*sqrt(-a/b)*
tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a**2*b**3*f*sqr
t(-a/b)*tan(e + f*x)**3) - 4*a**3*sqrt(-a/b)/(4*a**5*f*sqrt(-a/b)*tan(e + f
*x) + 4*a**4*b*f*sqrt(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e +
 f*x) - 8*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)
*tan(e + f*x) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3) - 4*a**2*b*f*x*sq
rt(-a/b)*tan(e + f*x)**3/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqr
t(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2*
f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a*
*2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3) - 4*a**2*b*sqrt(-a/b)*tan(e + f*x)**2
/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqrt(-a/b)*tan(e + f*x)**3
- 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x
)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a**2*b**3*f*sqrt(-a/b)*tan
(e + f*x)**3) + 8*a**2*b*sqrt(-a/b)/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a
**4*b*f*sqrt(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8
```

```
*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e +
f*x) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3) + 5*a**2*b*log(-sqrt(-a/b)
 + tan(e + f*x))*tan(e + f*x)/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*
f*sqrt(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*
b**2*f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) +
 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3) - 5*a**2*b*log(sqrt(-a/b) + tan(
e + f*x))*tan(e + f*x)/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqrt(
-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2*f*
sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a**2
*b**3*f*sqrt(-a/b)*tan(e + f*x)**3) + 10*a*b**2*sqrt(-a/b)*tan(e + f*x)**2/
(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqrt(-a/b)*tan(e + f*x)**3 -
 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x)
**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a**2*b**3*f*sqrt(-a/b)*tan(
e + f*x)**3) - 4*a*b**2*sqrt(-a/b)/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a*
*4*b*f*sqrt(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*
a**3*b**2*f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f
*x) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3) + 5*a*b**2*log(-sqrt(-a/b)
+ tan(e + f*x))*tan(e + f*x)**3/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*
b*f*sqrt(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**
3*b**2*f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x)
 + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3) - 3*a*b**2*log(-sqrt(-a/b) + t
an(e + f*x))*tan(e + f*x)/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sq
rt(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2
*f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a
**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3) - 5*a*b**2*log(sqrt(-a/b) + tan(e +
f*x))*tan(e + f*x)**3/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqrt(-
a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2*f*s
qrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a**2*
b**3*f*sqrt(-a/b)*tan(e + f*x)**3) + 3*a*b**2*log(sqrt(-a/b) + tan(e + f*x)
)*tan(e + f*x)/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqrt(-a/b)*ta
n(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2*f*sqrt(-a/
b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a**2*b**3*f*
sqrt(-a/b)*tan(e + f*x)**3) - 6*b**3*sqrt(-a/b)*tan(e + f*x)**2/(4*a**5*f*s
qrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqrt(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f
*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**
3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3
) - 3*b**3*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**3/(4*a**5*f*sqrt(-
a/b)*tan(e + f*x) + 4*a**4*b*f*sqrt(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt
(-a/b)*tan(e + f*x) - 8*a**3*b**2*f*sqrt(-a/b)*...
```

Giac [A]

time = 1.15, size = 171, normalized size = 1.34

$$\frac{\frac{(5\,ab^2-3\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^4-2\,a^3b+a^2b^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}-\frac{2\,ab\tan(fx+e)^2-3\,b^2\tan(fx+e)^2+2\,a^2-2\,ab}{\left(b\tan(fx+e)^3+a\tan(fx+e)\right)(a^3-a^2b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 - 3*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b)))/((a^4 - 2*a^3*b + a^2*b^2)*sqrt(a*b)) - 2*(f*x + e)/(a
^2 - 2*a*b + b^2) - (2*a*b*tan(f*x + e)^2 - 3*b^2*tan(f*x + e)^2 + 2*a^2 -
2*a*b)/((b*tan(f*x + e)^3 + a*tan(f*x + e))*(a^3 - a^2*b)))/f
```

Mupad [B]
time = 14.23, size = 2674, normalized size = 20.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] - (1/a + (tan(e + f*x)^2*(2*a*b - 3*b^2))/(2*a^2*(a - b)))/(f*(a*tan(e + f*
x) + b*tan(e + f*x)^3)) - (2*atan(((((1280*a^9*b^9 - 192*a^8*b^10 - 3520*a^
10*b^8 + 4992*a^11*b^7 - 3520*a^12*b^6 + 512*a^13*b^5 + 960*a^14*b^4 - 640*
a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(256*a^10*b^10 - 1536*a^11*b^9 + 35
84*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3
 - 256*a^18*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2) +
 tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1
744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3))/(2*a^2 - 4*a*b +
 2*b^2) + (((192*a^8*b^10 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^11*b^7 +
3520*a^12*b^6 - 512*a^13*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*a^16*b^2 +
 (tan(e + f*x)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b
^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2)*1i)/(2*a
^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2) + tan(e + f*x)*(144*a^6*b^
10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a^11*b
^5 + 192*a^12*b^4 - 64*a^13*b^3))/(2*a^2 - 4*a*b + 2*b^2))/(((((192*a^8*b^1
0 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^11*b^7 + 3520*a^12*b^6 - 512*a^13
*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*a^16*b^2 + (tan(e + f*x)*(256*a^10
*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 358
4*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)
/(2*a^2 - 4*a*b + 2*b^2) + tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*
a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a
^13*b^3))*1i)/(2*a^2 - 4*a*b + 2*b^2) - ((((1280*a^9*b^9 - 192*a^8*b^10 - 3
```

520*a^10*b^8 + 4992*a^11*b^7 - 3520*a^12*b^6 + 512*a^13*b^5 + 960*a^14*b^4 - 640*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2) + tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3))*1i)/(2*a^2 - 4*a*b + 2*b^2) + 144*a^6*b^8 - 624*a^7*b^7 + 976*a^8*b^6 - 656*a^9*b^5 + 160*a^10*b^4)))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3) + ((5*a - 3*b)*(-a^5*b^3)^(1/2)*(1280*a^9*b^9 - 192*a^8*b^10 - 3520*a^10*b^8 + 4992*a^11*b^7 - 3520*a^12*b^6 + 512*a^13*b^5 + 960*a^14*b^4 - 640*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(5*a - 3*b)*(-a^5*b^3)^(1/2)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))/(4*(a^7 - 2*a^6*b + a^5*b^2)))*(5*a - 3*b)*(-a^5*b^3)^(1/2)*1i)/(4*(a^7 - 2*a^6*b + a^5*b^2)) + ((tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3) + ((5*a - 3*b)*(-a^5*b^3)^(1/2)*(192*a^8*b^10 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^11*b^7 + 3520*a^12*b^6 - 512*a^13*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*a^16*b^2 + (tan(e + f*x)*(5*a - 3*b)*(-a^5*b^3)^(1/2)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))/(4*(a^7 - 2*a^6*b + a^5*b^2)))*(5*a - 3*b)*(-a^5*b^3)^(1/2)*1i)/(4*(a^7 - 2*a^6*b + a^5*b^2)))/(144*a^6*b^8 - 624*a^7*b^7 + 976*a^8*b^6 - 656*a^9*b^5 + 160*a^10*b^4 - ((tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3) + ((5*a - 3*b)*(-a^5*b^3)^(1/2)*(1280*a^9*b^9 - 192*a^8*b^10 - 3520*a^10*b^8 + 4992*a^11*b^7 - 3520*a^12*b^6 + 512*a^13*b^5 + 960*a^14*b^4 - 640*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(5*a - 3*b)*(-a^5*b^3)^(1/2)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))/(4*(a^7 - 2*a^6*b + a^5*b^2)))*(5*a - 3*b)*(-a^5*b^3)^(1/2))/(4*(a^7 - 2*a^6*b + a^5*b^2)) + ((tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3) + ((5*a - 3*b)*(-a^5*b^3)^(1/2)*(192*a^8*b^10 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^11*b^7 + 3520*a^12*b^6 - 512*a^13*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*a^16*b^2 + (tan(e + f*x)*(5*a - 3*b)*(-a^5*b^3)^(1/2)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))/(4*(a^7 - 2*a^6*b + a^5*b^2)))*(5*a - 3*b)*(-a^5*b^3)^(1/2))/(4*(a^7 - 2*a^6*b + a^5*b^2))))*(5*a - 3*b)*(-a^5*b^3)^(1/2)*1i)/(2*f*(a^7 - 2*a^6*b + a^5*b^2))

3.235 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=169

$$\frac{x}{(a-b)^2}-\frac{(7a-5b)b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^2f}+\frac{(2a^2+2ab-5b^2)\cot(e+fx)}{2a^3(a-b)f}-\frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f}-2$$

```
[Out] x/(a-b)^2-1/2*(7*a-5*b)*b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(7/2)/
(a-b)^2/f+1/2*(2*a^2+2*a*b-5*b^2)*cot(f*x+e)/a^3/(a-b)/f-1/6*(2*a-5*b)*cot(
f*x+e)^3/a^2/(a-b)/f-1/2*b*cot(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 483, 597, 536, 209, 211}

$$-\frac{b^{5/2}(7a-5b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f(a-b)^2}-\frac{(2a-5b)\cot^3(e+fx)}{6a^2f(a-b)}+\frac{(2a^2+2ab-5b^2)\cot(e+fx)}{2a^3f(a-b)}-\frac{b\cot^3(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] x/(a - b)^2 - ((7*a - 5*b)*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/
(2*a^(7/2)*(a - b)^2*f) + ((2*a^2 + 2*a*b - 5*b^2)*Cot[e + f*x])/(2*a^3*(a
- b)*f) - ((2*a - 5*b)*Cot[e + f*x]^3)/(6*a^2*(a - b)*f) - (b*Cot[e + f*x]^
3)/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2a-5b-5bx^2}{x^4(1+x^2)(a+bx^2)}\,dx, x, \tan(e+f\right.}{2a(a-b)f}$$

$$= -\frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{3(2a^2\right.}{}$$

$$= \frac{(2a^2+2ab-5b^2)\cot(e+fx)}{2a^3(a-b)f} - \frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e}{2a(a-b)f\,(a+b}$$

$$= \frac{(2a^2+2ab-5b^2)\cot(e+fx)}{2a^3(a-b)f} - \frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e}{2a(a-b)f\,(a+b}$$

$$= \frac{x}{(a-b)^2} - \frac{(7a-5b)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^2f} + \frac{(2a^2+2ab-5b^2)\cot(e+}{2a^3(a-b)f}$$

Mathematica [A]
time = 2.41, size = 137, normalized size = 0.81

$$\frac{\frac{3b^{5/2}(-7a+5b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)^2} - \frac{2\cot(e+fx)(-4a-6b+a\csc^2(e+fx))}{a^3} + \frac{3\left(2(e+fx)-\frac{(a-b)b^3\sin(2(e+fx))}{a^3(a+b+(a-b)\cos(2(e+fx)))}\right)}{(a-b)^2}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((3*b^(5/2)*(-7*a + 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(
a - b)^2) - (2*Cot[e + f*x]*(-4*a - 6*b + a*Csc[e + f*x]^2))/a^3 + (3*(2*(e
 + f*x) - ((a - b)*b^3*Sin[2*(e + f*x)])/(a^3*(a + b + (a - b)*Cos[2*(e + f
*x)]))))/(a - b)^2)/(6*f)
```

Maple [A]
time = 0.42, size = 126, normalized size = 0.75

method	result

derivativedivides	$\frac{-\frac{b^3\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(7a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3(a-b)^2}-\frac{1}{3a^2\tan(fx+e)^3}-\frac{-a-2b}{a^3\tan(fx+e)}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
default	$\frac{-\frac{b^3\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(7a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3(a-b)^2}-\frac{1}{3a^2\tan(fx+e)^3}-\frac{-a-2b}{a^3\tan(fx+e)}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
risch	$\frac{x}{a^2-2ab+b^2}+\frac{i(12a^4e^{8i(fx+e)}-24a^3b\,e^{8i(fx+e)}+21ab^3e^{8i(fx+e)}-15b^4e^{8i(fx+e)}+12a^4e^{6i(fx+e)}+12a^3b\,e^{6i(fx+e)}-12}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b^3/a^3/(a-b)^2*((1/2*a-1/2*b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(7*a
-5*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/3/a^2/tan(f*x+e)^3-(-
a-2*b)/a^3/tan(f*x+e)+1/(a-b)^2*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.50, size = 199, normalized size = 1.18

$$-\frac{\frac{3\,(7\,ab^3-5\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^5-2\,a^4b+a^3b^2)\sqrt{ab}}-\frac{3\,(2\,a^2b+2\,ab^2-5\,b^3)\tan(fx+e)^4-2\,a^3+2\,a^2b+2\,(3\,a^3+2\,a^2b-5\,ab^2)\tan(fx+e)^2}{(a^4b-a^3b^2)\tan(fx+e)^5+(a^5-a^4b)\tan(fx+e)^3}-\frac{6\,(fx+e)}{a^2-2\,ab+b^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(7*a*b^3 - 5*b^4)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5 - 2*a^4*b
+ a^3*b^2)*sqrt(a*b)) - (3*(2*a^2*b + 2*a*b^2 - 5*b^3)*tan(f*x + e)^4 - 2*a
^3 + 2*a^2*b + 2*(3*a^3 + 2*a^2*b - 5*a*b^2)*tan(f*x + e)^2)/((a^4*b - a^3*
b^2)*tan(f*x + e)^5 + (a^5 - a^4*b)*tan(f*x + e)^3) - 6*(f*x + e)/(a^2 - 2*
a*b + b^2))/f
```

Fricas [A]
time = 4.79, size = 620, normalized size = 3.67

$$\left[\frac{24\,a^3bfx\tan(fx+e)^5+24\,a^4fx\tan(fx+e)^3+12\,(2\,a^3b-7\,ab^3+5\,b^4)\tan(fx+e)^4-8\,a^4+16\,a^3b-8\,a^2b^2+8\,(3\,a^4-a^3b-7\,a^2b^2+5\,ab^3)\tan(fx+e)^2-3\,((7\,ab^3-5\,b^4)\tan(fx+e)^5+(7\,a^2b^2-5\,ab^3)\tan(fx+e)^3)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\,(ab\tan(fx+e)^3-a^2\tan(fx+e))\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{24\,((a^5b-2\,a^4b^2+a^3b^3)f\tan(fx+e)^5+(a^6-2\,a^5b+a^4b^2)f\tan(fx+e)^3)}, \frac{12\,a^3bfx\tan(fx+e)^5+12\,a^4fx\tan(fx+e)^3+6\,(2\,a^3b-7\,ab^3+5\,b^4)\tan(fx+e)^4-4\,a^4+8\,a^3b-4\,a^2b^2+4\,(3\,a^4-a^3b-7\,a^2b^2+5\,ab^3)\tan(fx+e)^2-3\,((7\,ab^3-5\,b^4)\tan(fx+e)^5+(7\,a^2b^2-5\,ab^3)\tan(fx+e)^3)\sqrt{\frac{b}{a}}\arctan\left(\frac{(b\tan(fx+e)^2-a)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)}{12\,((a^5b-2\,a^4b^2+a^3b^3)f\tan(fx+e)^5+(a^6-2\,a^5b+a^4b^2)f\tan(fx+e)^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(24*a^3*b*f*x*tan(f*x + e)^5 + 24*a^4*f*x*tan(f*x + e)^3 + 12*(2*a^3*
b - 7*a*b^3 + 5*b^4)*tan(f*x + e)^4 - 8*a^4 + 16*a^3*b - 8*a^2*b^2 + 8*(3*a
^4 - a^3*b - 7*a^2*b^2 + 5*a*b^3)*tan(f*x + e)^2 - 3*((7*a*b^3 - 5*b^4)*tan
(f*x + e)^5 + (7*a^2*b^2 - 5*a*b^3)*tan(f*x + e)^3)*sqrt(-b/a)*log((b^2*tan
(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(
f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/(
(a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*
f*tan(f*x + e)^3), 1/12*(12*a^3*b*f*x*tan(f*x + e)^5 + 12*a^4*f*x*tan(f*x +
 e)^3 + 6*(2*a^3*b - 7*a*b^3 + 5*b^4)*tan(f*x + e)^4 - 4*a^4 + 8*a^3*b - 4*
a^2*b^2 + 4*(3*a^4 - a^3*b - 7*a^2*b^2 + 5*a*b^3)*tan(f*x + e)^2 - 3*((7*a*
b^3 - 5*b^4)*tan(f*x + e)^5 + (7*a^2*b^2 - 5*a*b^3)*tan(f*x + e)^3)*sqrt(b/
a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^5*b -
 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*
x + e)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.97, size = 179, normalized size = 1.06

$$-\frac{\frac{3\,b^{3}\tan\left(fx+e\right)}{\left(a^{4}-a^{3}b\right)\left(b\tan\left(fx+e\right)^{2}+a\right)}+\frac{3\left(7\,ab^{3}-5\,b^{4}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab}}\right)\right)}{\left(a^{5}-2\,a^{4}b+a^{3}b^{2}\right)\sqrt{ab}}-\frac{6\left(fx+e\right)}{a^{2}-2\,ab+b^{2}}-\frac{2\left(3\,a\tan\left(fx+e\right)^{2}+6\,b\tan\left(fx+e\right)^{2}-a\right)}{a^{3}\tan\left(fx+e\right)^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^3*tan(f*x + e)/((a^4 - a^3*b)*(b*tan(f*x + e)^2 + a)) + 3*(7*a*b^
3 - 5*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqr
t(a*b)))/((a^5 - 2*a^4*b + a^3*b^2)*sqrt(a*b)) - 6*(f*x + e)/(a^2 - 2*a*b +
 b^2) - 2*(3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 - a)/(a^3*tan(f*x + e)^3
))/f
```

Mupad [**B**]
time = 15.75, size = 2000, normalized size = 11.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^2,x)

[Out] (2*atan((2*tan(e + f*x)*(2320*a^10*b^11 - 400*a^9*b^12 - 5344*a^11*b^10 + 6112*a^12*b^9 - 3472*a^13*b^8 + 784*a^14*b^7 - 64*a^15*b^6 + 192*a^16*b^5 - 192*a^17*b^4 + 64*a^18*b^3 + (256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*b^8 - 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*a^23*b^2)/(2*a^2 - 4*a*b + 2*b^2)^2))/((2*a^2 - 4*a*b + 2*b^2)*((2*(320*a^12*b^11 - 2048*a^13*b^10 + 5440*a^14*b^9 - 7680*a^15*b^8 + 6208*a^16*b^7 - 3200*a^17*b^6 + 1728*a^18*b^5 - 1280*a^19*b^4 + 640*a^20*b^3 - 128*a^21*b^2))/(2*a^2 - 4*a*b + 2*b^2)^2 - 400*a^9*b^10 + 1520*a^10*b^9 - 1904*a^11*b^8 + 624*a^12*b^7 + 384*a^13*b^6 - 224*a^14*b^5))))/(f*(2*a^2 - 4*a*b + 2*b^2)) + ((tan(e + f*x)^2*(3*a + 5*b))/(3*a^2) - 1/(3*a) + (tan(e + f*x)^4*(2*a*b^2 + 2*a^2*b - 5*b^3))/(2*a^3*(a - b)))/(f*(a*tan(e + f*x)^3 + b*tan(e + f*x)^5)) + (atan((((tan(e + f*x)*(400*a^9*b^12 - 2320*a^10*b^11 + 5344*a^11*b^10 - 6112*a^12*b^9 + 3472*a^13*b^8 - 784*a^14*b^7 + 64*a^15*b^6 - 192*a^16*b^5 + 192*a^17*b^4 - 64*a^18*b^3) + ((7*a - 5*b)*(-a^7*b^5)^(1/2)*(2048*a^13*b^10 - 320*a^12*b^11 - 5440*a^14*b^9 + 7680*a^15*b^8 - 6208*a^16*b^7 + 3200*a^17*b^6 - 1728*a^18*b^5 + 1280*a^19*b^4 - 640*a^20*b^3 + 128*a^21*b^2 + (tan(e + f*x)*(7*a - 5*b)*(-a^7*b^5)^(1/2)*(256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*b^8 - 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*a^23*b^2))/(4*(a^9 - 2*a^8*b + a^7*b^2))))/(4*(a^9 - 2*a^8*b + a^7*b^2)))*(7*a - 5*b)*(-a^7*b^5)^(1/2)*1i)/(4*(a^9 - 2*a^8*b + a^7*b^2)) + ((tan(e + f*x)*(400*a^9*b^12 - 2320*a^10*b^11 + 5344*a^11*b^10 - 6112*a^12*b^9 + 3472*a^13*b^8 - 784*a^14*b^7 + 64*a^15*b^6 - 192*a^16*b^5 + 192*a^17*b^4 - 64*a^18*b^3) + ((7*a - 5*b)*(-a^7*b^5)^(1/2)*(320*a^12*b^11 - 2048*a^13*b^10 + 5440*a^14*b^9 - 7680*a^15*b^8 + 6208*a^16*b^7 - 3200*a^17*b^6 + 1728*a^18*b^5 - 1280*a^19*b^4 + 640*a^20*b^3 - 128*a^21*b^2 + (tan(e + f*x)*(7*a - 5*b)*(-a^7*b^5)^(1/2)*(256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*b^8 - 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*a^23*b^2))/(4*(a^9 - 2*a^8*b + a^7*b^2))))/(4*(a^9 - 2*a^8*b + a^7*b^2)))*(7*a - 5*b)*(-a^7*b^5)^(1/2)*1i)/(4*(a^9 - 2*a^8*b + a^7*b^2)))/(400*a^9*b^10 - 1520*a^10*b^9 + 1904*a^11*b^8 - 624*a^12*b^7 - 384*a^13*b^6 + 224*a^14*b^5 - ((tan(e + f*x)*(400*a^9*b^12 - 2320*a^10*b^11 + 5344*a^11*b^10 - 6112*a^12*b^9 + 3472*a^13*b^8 - 784*a^14*b^7 + 64*a^15*b^6 - 192*a^16*b^5 + 192*a^17*b^4 - 64*a^18*b^3) + ((7*a - 5*b)*(-a^7*b^5)^(1/2)*(2048*a^13*b^10 - 320*a^12*b^11 - 5440*a^14*b^9 + 7680*a^15*b^8 - 6208*a^16*b^7 + 3200*a^17*b^6 - 1728*a^18*b^5 + 1280*a^19*b^4 - 640*a^20*b^3 + 128*a^21*b^2 + (tan(e + f*x)*(7*a - 5*b)*(-a^7*b^5)^(1/2)*(256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*b^8 - 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*a^23*b^2))/(4*(a^9 - 2*a^8*b + a^7*b^2))))/(4*(a^9 - 2*a^8*b + a^7*b^2)))*(7*a - 5*b)*(-a^7*b^5)^(1/2))/(4*(a^9 - 2*a^8*b + a^7*b^2)) + ((tan(e + f*x)*(400*a^9*b^12 - 2320*a^10*b^11 + 5344*a^11*b^10 - 6112*a^12*b^9 + 3472*a^13*b^8 - 784*a^14*b^7 + 64*a^15*b^6 - 192*a^16*b^5 + 192*a^17*b^4 - 64*a^18*b^3)

```
 + ((7*a - 5*b)*(-a^7*b^5)^(1/2)*(320*a^12*b^11 - 2048*a^13*b^10 + 5440*a^1
4*b^9 - 7680*a^15*b^8 + 6208*a^16*b^7 - 3200*a^17*b^6 + 1728*a^18*b^5 - 128
0*a^19*b^4 + 640*a^20*b^3 - 128*a^21*b^2 + (tan(e + f*x)*(7*a - 5*b)*(-a^7*
b^5)^(1/2)*(256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*b^8 - 3584*a^18*b^7 +
 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*a^23*b^2))/(4*(a^9 - 2
*a^8*b + a^7*b^2))))/(4*(a^9 - 2*a^8*b + a^7*b^2)))*(7*a - 5*b)*(-a^7*b^5)^
(1/2))/(4*(a^9 - 2*a^8*b + a^7*b^2))))*(7*a - 5*b)*(-a^7*b^5)^(1/2)*1i)/(2*
f*(a^9 - 2*a^8*b + a^7*b^2))
```

3.236 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{x}{(a-b)^2}+\frac{(9a-7b)b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}(a-b)^2f}-\frac{(2a^3+2a^2b+2ab^2-7b^3)\cot(e+fx)}{2a^4(a-b)f}+\frac{(2a^2+2ab-7b^2}{6a^3(a-}$$

```
[Out] -x/(a-b)^2+1/2*(9*a-7*b)*b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(9/2)
/(a-b)^2/f-1/2*(2*a^3+2*a^2*b+2*a*b^2-7*b^3)*cot(f*x+e)/a^4/(a-b)/f+1/6*(2*
a^2+2*a*b-7*b^2)*cot(f*x+e)^3/a^3/(a-b)/f-1/10*(2*a-7*b)*cot(f*x+e)^5/a^2/(
a-b)/f-1/2*b*cot(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.23, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 483, 597, 536, 209, 211}

$$\frac{b^{7/2}(9a-7b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f(a-b)^2}-\frac{(2a-7b)\cot^5(e+fx)}{10a^2f(a-b)}+\frac{(2a^2+2ab-7b^2)\cot^3(e+fx)}{6a^3f(a-b)}-\frac{(2a^3+2a^2b+2ab^2-7b^3)\cot(e+fx)}{2a^4f(a-b)}-\frac{b\cot^5(e+fx)}{2af(a-b)\left(a+b\tan^2(e+fx)\right)}-\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] -(x/(a - b)^2) + ((9*a - 7*b)*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(2*a^(9/2)*(a - b)^2*f) - ((2*a^3 + 2*a^2*b + 2*a*b^2 - 7*b^3)*Cot[e + f
*x])/(2*a^4*(a - b)*f) + ((2*a^2 + 2*a*b - 7*b^2)*Cot[e + f*x]^3)/(6*a^3*(a
 - b)*f) - ((2*a - 7*b)*Cot[e + f*x]^5)/(10*a^2*(a - b)*f) - (b*Cot[e + f*x
]^5)/(2*a*(a - b)*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))} + \frac{\text{Subst}\left(\int \frac{2a-7b-7bx^2}{x^6(1+x^2)(a+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a-b)f}$$

$$= -\frac{(2a-7b)\cot^5(e+fx)}{10a^2(a-b)f} - \frac{b\cot^5(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))} - \frac{\text{Subst}\left(\int \frac{5(2a^2+\cdots}{x\cdots}\right.}{\cdots}$$

$$= \frac{(2a^2+2ab-7b^2)\cot^3(e+fx)}{6a^3(a-b)f} - \frac{(2a-7b)\cot^5(e+fx)}{10a^2(a-b)f} - \frac{b\cot^5(e+\cdots}{2a(a-b)f\,(a+b\cdots}$$

$$= -\frac{(2a^3+2a^2b+2ab^2-7b^3)\cot(e+fx)}{2a^4(a-b)f} + \frac{(2a^2+2ab-7b^2)\cot^3(e+fx)}{6a^3(a-b)f} - \cdots$$

$$= -\frac{(2a^3+2a^2b+2ab^2-7b^3)\cot(e+fx)}{2a^4(a-b)f} + \frac{(2a^2+2ab-7b^2)\cot^3(e+fx)}{6a^3(a-b)f} - \cdots$$

$$= -\frac{x}{(a-b)^2} + \frac{(9a-7b)b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}(a-b)^2f} - \frac{(2a^3+2a^2b+2ab^2-7b^3)\cdots}{2a^4(a-b)f}$$

Mathematica [A]
time = 4.71, size = 165, normalized size = 0.76

$$\frac{\frac{15(9a-7b)b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{9/2}(a-b)^2} - \frac{2\cot(e+fx)(23a^2+40ab+45b^2-a(11a+10b)\csc^2(e+fx)+3a^2\csc^4(e+fx))}{a^4} + \frac{15\left(-2(e+fx)+\frac{(a-b)b^4\sin(2(e+fx))}{a^4(a+b+(a-b)\cos(2(e+fx)))}\right)}{(a-b)^2}}{30f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

```
[Out] ((15*(9*a - 7*b)*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(9/2)*(
a - b)^2) - (2*Cot[e + f*x]*(23*a^2 + 40*a*b + 45*b^2 - a*(11*a + 10*b)*Csc
[e + f*x]^2 + 3*a^2*Csc[e + f*x]^4))/a^4 + (15*(-2*(e + f*x) + ((a - b)*b^4
*Sin[2*(e + f*x)])/(a^4*(a + b + (a - b)*Cos[2*(e + f*x)]))))/(a - b)^2)/(3
0*f)
```

Maple [A]
time = 0.47, size = 152, normalized size = 0.70

method	result

derivativedivides	$\frac{\frac{b^4\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(9a-7b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4(a-b)^2}-\frac{1}{5a^2\tan(fx+e)^5}-\frac{-a-2b}{3a^3\tan(fx+e)^3}-\frac{a^2+2ab+3b^2}{a^4\tan(fx+e)}-\frac{\arctan(\tan(fx+e}{(a-b)^2}}{f}$
default	$\frac{\frac{b^4\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\left(\tan^2(fx+e)\right)}+\frac{(9a-7b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4(a-b)^2}-\frac{1}{5a^2\tan(fx+e)^5}-\frac{-a-2b}{3a^3\tan(fx+e)^3}-\frac{a^2+2ab+3b^2}{a^4\tan(fx+e)}-\frac{\arctan(\tan(fx+e}{(a-b)^2}}{f}$
risch	$-\frac{x}{a^2-2ab+b^2}-\frac{i(240a^4b\,e^{10i(fx+e)}-738a^4e^{4i(fx+e)}b+168a^3b^2e^{4i(fx+e)}-880b^4e^{2i(fx+e)}a-12a^3b^2e^{2i(fx+e)}+132b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4/a^4/(a-b)^2*((1/2*a-1/2*b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(9*a-
7*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/5/a^2/tan(f*x+e)^5-1/3
*(-a-2*b)/a^3/tan(f*x+e)^3-(a^2+2*a*b+3*b^2)/a^4/tan(f*x+e)-1/(a-b)^2*arcta
n(tan(f*x+e)))
```

Maxima [A]
time = 0.51, size = 246, normalized size = 1.13

$$\frac{\frac{15\,(9\,ab^4-7\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^6-2\,a^5b+a^4b^2)\sqrt{ab}}-\frac{15\,(2\,a^3b+2\,a^2b^2+2\,ab^3-7\,b^4)\tan(fx+e)^6+10\,(3\,a^4+2\,a^3b+2\,a^2b^2-7\,ab^3)\tan(fx+e)^4+6\,a^4-6\,a^3b-2\,(5\,a^4+2\,a^3b-7\,a^2b^2)\tan(fx+e)^2}{(a^5b-a^4b^2)\tan(fx+e)^7+(a^6-a^5b)\tan(fx+e)^5}-\frac{30\,(fx+e)}{a^2-2\,ab+b^2}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(9*a*b^4 - 7*b^5)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^6 - 2*a^5*b
 + a^4*b^2)*sqrt(a*b)) - (15*(2*a^3*b + 2*a^2*b^2 + 2*a*b^3 - 7*b^4)*tan(f*
x + e)^6 + 10*(3*a^4 + 2*a^3*b + 2*a^2*b^2 - 7*a*b^3)*tan(f*x + e)^4 + 6*a^
4 - 6*a^3*b - 2*(5*a^4 + 2*a^3*b - 7*a^2*b^2)*tan(f*x + e)^2)/((a^5*b - a^4
*b^2)*tan(f*x + e)^7 + (a^6 - a^5*b)*tan(f*x + e)^5) - 30*(f*x + e)/(a^2 -
2*a*b + b^2))/f
```

Fricas [A]
time = 2.17, size = 698, normalized size = 3.20

$$\left[-\frac{120\,a^4bfx\tan(fx+e)^7+120\,a^5fx\tan(fx+e)^5+60\,(2\,a^4b-9\,ab^4+7\,b^5)\tan(fx+e)^6+24\,a^5-48\,a^4b+24\,a^3b^2+40\,(3\,a^5-a^4b-9\,a^2b^3+7\,ab^4)\tan(fx+e)^4-8\,(5\,a^5-3\,a^4b-9\,a^3b^2+7\,a^2b^3)\tan(fx+e)^2+15\,((9\,ab^4-7\,b^5)\tan(fx+e)^7+(9\,a^2b^3-7\,ab^4)\tan(fx+e)^5)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2-4\,(ab\tan(fx+e)^3-a^2\tan(fx+e))\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{120\,((a^6b-2\,a^5b^2+a^4b^3)f\tan(fx+e)^7+(a^7-2\,a^6b+a^5b^2)f\tan(fx+e)^5)},\ -\frac{60\,a^4bfx\tan(fx+e)^7+60\,a^5fx\tan(fx+e)^5+30\,(2\,a^4b-9\,ab^4+7\,b^5)\tan(fx+e)^6+12\,a^5-24\,a^4b+12\,a^3b^2+20\,(3\,a^5-a^4b-9\,a^2b^3+7\,ab^4)\tan(fx+e)^4-4\,(5\,a^5-3\,a^4b-9\,a^3b^2+7\,a^2b^3)\tan(fx+e)^2-15\,((9\,ab^4-7\,b^5)\tan(fx+e)^7+(9\,a^2b^3-7\,ab^4)\tan(fx+e)^5)\sqrt{\frac{b}{a}}\arctan\left(\frac{(b\tan(fx+e)^2-a)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)}{60\,((a^6b-2\,a^5b^2+a^4b^3)f\tan(fx+e)^7+(a^7-2\,a^6b+a^5b^2)f\tan(fx+e)^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(120*a^4*b*f*x*tan(f*x + e)^7 + 120*a^5*f*x*tan(f*x + e)^5 + 60*(2*
a^4*b - 9*a*b^4 + 7*b^5)*tan(f*x + e)^6 + 24*a^5 - 48*a^4*b + 24*a^3*b^2 +
40*(3*a^5 - a^4*b - 9*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^4 - 8*(5*a^5 - 3*a^4*
b - 9*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2 + 15*((9*a*b^4 - 7*b^5)*tan(f*x +
 e)^7 + (9*a^2*b^3 - 7*a*b^4)*tan(f*x + e)^5)*sqrt(-b/a)*log((b^2*tan(f*x +
 e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x +
e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a^6*b
 - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(
f*x + e)^5), -1/60*(60*a^4*b*f*x*tan(f*x + e)^7 + 60*a^5*f*x*tan(f*x + e)^5
 + 30*(2*a^4*b - 9*a*b^4 + 7*b^5)*tan(f*x + e)^6 + 12*a^5 - 24*a^4*b + 12*a
^3*b^2 + 20*(3*a^5 - a^4*b - 9*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^4 - 4*(5*a^5
 - 3*a^4*b - 9*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2 - 15*((9*a*b^4 - 7*b^5)*
tan(f*x + e)^7 + (9*a^2*b^3 - 7*a*b^4)*tan(f*x + e)^5)*sqrt(b/a)*arctan(1/2
*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^6*b - 2*a^5*b^2 +
a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.12, size = 225, normalized size = 1.03

$$\frac{\frac{15\,b^4\tan(fx+e)}{(a^5-a^4b)\left(b\tan(fx+e)^2+a\right)}+\frac{15\,(9\,ab^4-7\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^6-2\,a^5b+a^4b^2)\sqrt{ab}}-\frac{30\,(fx+e)}{a^2-2\,ab+b^2}-\frac{2\left(15\,a^2\tan(fx+e)^4+30\,ab\tan(fx+e)^4+45\,b^2\tan(fx+e)^4-5\,a^2\tan(fx+e)^2-10\,ab\tan(fx+e)^2+3\,a^2\right)}{a^4\tan(fx+e)^5}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*b^4*tan(f*x + e)/((a^5 - a^4*b)*(b*tan(f*x + e)^2 + a)) + 15*(9*a*
b^4 - 7*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/s
qrt(a*b)))/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(a*b)) - 30*(f*x + e)/(a^2 - 2*a*
b + b^2) - 2*(15*a^2*tan(f*x + e)^4 + 30*a*b*tan(f*x + e)^4 + 45*b^2*tan(f*
x + e)^4 - 5*a^2*tan(f*x + e)^2 - 10*a*b*tan(f*x + e)^2 + 3*a^2)/(a^4*tan(f
*x + e)^5))/f
```

Mupad [**B**]
time = 16.00, size = 2500, normalized size = 11.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^2,x)
```

```
[Out] - (1/(5*a) + (tan(e + f*x)^4*(5*a*b + 3*a^2 + 7*b^2))/(3*a^3) - (tan(e + f*
x)^2*(5*a + 7*b))/(15*a^2) + (tan(e + f*x)^6*(2*a*b^3 + 2*a^3*b - 7*b^4 + 2
*a^2*b^2))/(2*a^4*(a - b)))/(f*(a*tan(e + f*x)^5 + b*tan(e + f*x)^7)) - (2*
atan(((tan(e + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 1072
0*a^15*b^11 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 +
 192*a^22*b^4 - 64*a^23*b^3) + ((2816*a^17*b^11 - 448*a^16*b^12 - 7360*a^18
*b^10 + 10240*a^19*b^9 - 8000*a^20*b^8 + 3200*a^21*b^7 + 64*a^22*b^6 - 1280
*a^23*b^5 + 1280*a^24*b^4 - 640*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(25
6*a^20*b^10 - 1536*a^21*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5
 - 3584*a^26*b^4 + 1536*a^27*b^3 - 256*a^28*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2
))*1i)/(2*a^2 - 4*a*b + 2*b^2))/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(78
4*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11 + 5904*a^16
*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 + 192*a^22*b^4 - 64*a^23
*b^3) + ((448*a^16*b^12 - 2816*a^17*b^11 + 7360*a^18*b^10 - 10240*a^19*b^9
+ 8000*a^20*b^8 - 3200*a^21*b^7 - 64*a^22*b^6 + 1280*a^23*b^5 - 1280*a^24*b
^4 + 640*a^25*b^3 - 128*a^26*b^2 + (tan(e + f*x)*(256*a^20*b^10 - 1536*a^21
*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26*b^4 + 1536
*a^27*b^3 - 256*a^28*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b +
 2*b^2))/(2*a^2 - 4*a*b + 2*b^2))/(((tan(e + f*x)*(784*a^12*b^14 - 4368*a^1
3*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11 + 5904*a^16*b^10 - 1296*a^17*b^9
+ 64*a^20*b^6 - 192*a^21*b^5 + 192*a^22*b^4 - 64*a^23*b^3) + ((448*a^16*b^1
2 - 2816*a^17*b^11 + 7360*a^18*b^10 - 10240*a^19*b^9 + 8000*a^20*b^8 - 3200
*a^21*b^7 - 64*a^22*b^6 + 1280*a^23*b^5 - 1280*a^24*b^4 + 640*a^25*b^3 - 12
8*a^26*b^2 + (tan(e + f*x)*(256*a^20*b^10 - 1536*a^21*b^9 + 3584*a^22*b^8 -
 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26*b^4 + 1536*a^27*b^3 - 256*a^28*b
^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 -
4*a*b + 2*b^2) - ((tan(e + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14
*b^12 - 10720*a^15*b^11 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 19
2*a^21*b^5 + 192*a^22*b^4 - 64*a^23*b^3) + ((2816*a^17*b^11 - 448*a^16*b^12
 - 7360*a^18*b^10 + 10240*a^19*b^9 - 8000*a^20*b^8 + 3200*a^21*b^7 + 64*a^2
2*b^6 - 1280*a^23*b^5 + 1280*a^24*b^4 - 640*a^25*b^3 + 128*a^26*b^2 + (tan(
e + f*x)*(256*a^20*b^10 - 1536*a^21*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3
584*a^25*b^5 - 3584*a^26*b^4 + 1536*a^27*b^3 - 256*a^28*b^2)*1i)/(2*a^2 - 4
*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2) + 7
84*a^12*b^12 - 2800*a^13*b^11 + 3312*a^14*b^10 - 1296*a^15*b^9 + 224*a^16*b
^8 - 512*a^17*b^7 + 288*a^18*b^6)))/(f*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((
tan(e + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*
b^11 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 + 192*a^
22*b^4 - 64*a^23*b^3) + ((9*a - 7*b)*(-a^9*b^7)^(1/2)*(2816*a^17*b^11 - 448
*a^16*b^12 - 7360*a^18*b^10 + 10240*a^19*b^9 - 8000*a^20*b^8 + 3200*a^21*b^
7 + 64*a^22*b^6 - 1280*a^23*b^5 + 1280*a^24*b^4 - 640*a^25*b^3 + 128*a^26*b
```

^2 + (tan(e + f*x)*(9*a - 7*b)*(-a^9*b^7)^(1/2)*(256*a^20*b^10 - 1536*a^21*
b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26*b^4 + 1536*
a^27*b^3 - 256*a^28*b^2))/(4*(a^11 - 2*a^10*b + a^9*b^2))))/(4*(a^11 - 2*a^
10*b + a^9*b^2)))*(9*a - 7*b)*(-a^9*b^7)^(1/2)*1i)/(4*(a^11 - 2*a^10*b + a^
9*b^2)) + ((tan(e + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 -
10720*a^15*b^11 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*
b^5 + 192*a^22*b^4 - 64*a^23*b^3) + ((9*a - 7*b)*(-a^9*b^7)^(1/2)*(448*a^16
*b^12 - 2816*a^17*b^11 + 7360*a^18*b^10 - 10240*a^19*b^9 + 8000*a^20*b^8 -
3200*a^21*b^7 - 64*a^22*b^6 + 1280*a^23*b^5 - 1280*a^24*b^4 + 640*a^25*b^3
- 128*a^26*b^2 + (tan(e + f*x)*(9*a - 7*b)*(-a^9*b^7)^(1/2)*(256*a^20*b^10
- 1536*a^21*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26
*b^4 + 1536*a^27*b^3 - 256*a^28*b^2))/(4*(a^11 - 2*a^10*b + a^9*b^2))))/(4*
(a^11 - 2*a^10*b + a^9*b^2)))*(9*a - 7*b)*(-a^9*b^7)^(1/2)*1i)/(4*(a^11 - 2
*a^10*b + a^9*b^2)))/(784*a^12*b^12 - 2800*a^13*b^11 + 3312*a^14*b^10 - 129
6*a^15*b^9 + 224*a^16*b^8 - 512*a^17*b^7 + 288*a^18*b^6 - ((tan(e + f*x)*(7
84*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11 + 5904*a^1
6*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 + 192*a^22*b^4 - 64*a^2
3*b^3) + ((9*a - 7*b)*(-a^9*b^7)^(1/2)*(2816*a^17*b^11 - 448*a^16*b^12 - 73
60*a^18*b^10 + 10240*a^19*b^9 - 8000*a^20*b^8 + 3200*a^21*b^7 + 64*a^22*b^6
- 1280*a^23*b^5 + 1280*a^24*b^4 - 640*a^25*b^3 + 128*a^26*b^2 + (tan(e + f
x)(9*a - 7*b)*(-a^9*b^7)^(1/2)*(256*a^20*b^10 - 1536*a^21*b^9 + 3584*a^22
*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 3584*a^26*b^4 + 1536*a^27*b^3 - 256*
a^28*b^2))/(4*(a^11 - 2*a^10*b + a^9*b^2))))/(4*(a^11 - 2*a^10*b + a^9*b^2)
))*(9*a - 7*b)*(-a^9*b^7)^(1/2))/(4*(a^11 - 2*a^10*b + a^9*b^2)) + ((tan(e
+ f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11 +
5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 -...

3.237 $\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal. Leaf size=108

$$-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f}+\frac{a^2}{4(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^2}-\frac{a(a-2b)}{2(a-b)^2b^2f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^3/f+1/4*a^2/(a-b)/b^2/f/(a+b*t
an(f*x+e)^2)^2-1/2*a*(a-2*b)/(a-b)^2/b^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$\frac{a^2}{4b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{a(a-2b)}{2b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -1/2*Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/((a - b)^3*f) + a^2/(4*(a - b
)*b^2*f*(a + b*Tan[e + f*x]^2)^2) - (a*(a - 2*b))/(2*(a - b)^2*b^2*f*(a + b
*Tan[e + f*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^3(1+x)} - \frac{a^2}{(a-b)b(a+bx)^3} + \frac{a(a-2b)}{(a-b)^2b(a+bx)^2} + \frac{b}{(-a+b)^3(a+bx)}\right)\,dx, x, \tan^2\right.}{2f}$$
$$= -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f} + \frac{a^2}{4(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^2} -$$

Mathematica [A]
time = 0.77, size = 97, normalized size = 0.90

$$\frac{-4\log(\cos(e+fx)) - 2\log\left(a+b\tan^2(e+fx)\right) + \frac{a^2(a-b)^2}{b^2(a+b\tan^2(e+fx))^2} - \frac{2a(a-2b)(a-b)}{b^2(a+b\tan^2(e+fx))}}{4(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-4*Log[Cos[e + f*x]] - 2*Log[a + b*Tan[e + f*x]^2] + (a^2*(a - b)^2)/(b^2*
(a + b*Tan[e + f*x]^2)^2) - (2*a*(a - 2*b)*(a - b))/(b^2*(a + b*Tan[e + f*x
]^2)))/(4*(a - b)^3*f)
```

Maple [A]
time = 0.17, size = 117, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^3}+\frac{-\ln\left(a+b\left(\tan^2(fx+e)\right)\right)-\frac{a\left(a^2-3ab+2b^2\right)}{b^2\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{a^2\left(a^2-2ab+b^2\right)}{2b^2\left(a+b\left(\tan^2(fx+e)\right)\right)^2}}{2(a-b)^3}}{f}$
default	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^3}+\frac{-\ln\left(a+b\left(\tan^2(fx+e)\right)\right)-\frac{a\left(a^2-3ab+2b^2\right)}{b^2\left(a+b\left(\tan^2(fx+e)\right)\right)}+\frac{a^2\left(a^2-2ab+b^2\right)}{2b^2\left(a+b\left(\tan^2(fx+e)\right)\right)^2}}{2(a-b)^3}}{f}$
norman	$\frac{\frac{(-a+3b)a^2}{4b^2\left(a^2-2ab+b^2\right)f}+\frac{a(-a+2b)\left(\tan^2(fx+e)\right)}{2b\left(a^2-2ab+b^2\right)f}}{\left(a+b(\tan^2(fx+e))\right)^2}+\frac{\ln(1+\tan^2(fx+e))}{2f(a^3-3a^2b+3ab^2-b^3)}-\frac{\ln(a+b(\tan^2(fx+e)))}{2f(a^3-3a^2b+3ab^2-b^3)}$

risch	$\frac{ix}{a^3-3a^2b+3a\,b^2-b^3}+\frac{2ie}{f(a^3-3a^2b+3a\,b^2-b^3)}-\frac{4a\left(a\,e^{6i(fx+e)}-b\,e^{6i(fx+e)}+a\,e^{4i(fx+e)}+2b\,e^{4i(fx+e)}+a\,e^{2i(fx+e)}\right.}{\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a-b)^3*ln(1+tan(f*x+e)^2)+1/2/(a-b)^3*(-ln(a+b*tan(f*x+e)^2)-a*(a
^2-3*a*b+2*b^2)/b^2/(a+b*tan(f*x+e)^2)+1/2*a^2*(a^2-2*a*b+b^2)/b^2/(a+b*tan
(f*x+e)^2)^2))
```

Maxima [A]
time = 0.28, size = 193, normalized size = 1.79

$$\frac{\frac{4\,(a^2-ab)\sin(fx+e)^2-3\,a^2}{a^5-3\,a^4b+3\,a^3b^2-a^2b^3+(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)\sin(fx+e)^4-2\,(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)\sin(fx+e)^2}-\frac{2\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^3-3\,a^2b+3\,ab^2-b^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*(a^2 - a*b)*sin(f*x + e)^2 - 3*a^2)/(a^5 - 3*a^4*b + 3*a^3*b^2 - a^
2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin(f*x +
 e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x + e)^2) -
 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(106) = 212$.
time = 3.04, size = 214, normalized size = 1.98

$$\frac{(a^2-4\,ab)\tan\left(fx+e\right)^4-2\,(a^2+2\,ab)\tan\left(fx+e\right)^2-3\,a^2-2\,\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{4\left((a^3b^2-3\,a^2b^3+3\,ab^4-b^5)f\tan\left(fx+e\right)^4+2\,(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4)f\tan\left(fx+e\right)^2+(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - 4*a*b)*tan(f*x + e)^4 - 2*(a^2 + 2*a*b)*tan(f*x + e)^2 - 3*a^2
- 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(f*x + e)^2
 + a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f
*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3315 vs. $2(87) = 174$.
time = 73.27, size = 3315, normalized size = 30.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-3*tan(e + f*x)*
*4/(6*b**3*f*tan(e + f*x)**6 + 18*b**3*f*tan(e + f*x)**4 + 18*b**3*f*tan(e
+ f*x)**2 + 6*b**3*f) - 3*tan(e + f*x)**2/(6*b**3*f*tan(e + f*x)**6 + 18*b*
*3*f*tan(e + f*x)**4 + 18*b**3*f*tan(e + f*x)**2 + 6*b**3*f) - 1/(6*b**3*f*
tan(e + f*x)**6 + 18*b**3*f*tan(e + f*x)**4 + 18*b**3*f*tan(e + f*x)**2 + 6
*b**3*f), Eq(a, b)), ((log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*
f) - tan(e + f*x)**2/(2*f))/a**3, Eq(b, 0)), (x*tan(e)**5/(a + b*tan(e)**2)
**3, Eq(f, 0)), (-a**4/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*
a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**
2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e
+ f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e
+ f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*a**3*b*tan(e + f*x)**2/(4*a**5*b*
*2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e
 + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5
*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*
b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)
**4) + 4*a**3*b/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b*
*3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*
a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)*
*2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)*
*2 - 4*b**7*f*tan(e + f*x)**4) - 2*a**2*b**2*log(-sqrt(-a/b) + tan(e + f*x)
)/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*
b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f -
12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b*
*5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*
tan(e + f*x)**4) - 2*a**2*b**2*log(sqrt(-a/b) + tan(e + f*x))/(4*a**5*b**2*
f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e +
f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*
tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**
6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4
) + 2*a**2*b**2*log(tan(e + f*x)**2 + 1)/(4*a**5*b**2*f + 8*a**4*b**3*f*tan
(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**
4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*
a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 -
 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) + 6*a**2*b**2*tan(e
 + f*x)**2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f
+ 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*
b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 -
4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 -
4*b**7*f*tan(e + f*x)**4) - 3*a**2*b**2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(
e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4
```

```
*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a
**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 -
8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 4*a*b**3*log(-sqrt
(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e
 + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*
f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a*
*2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8
*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 4*a*b**3*log(sqrt(-
a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e +
 f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*
tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2
*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a
*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) + 4*a*b**3*log(tan(e +
f*x)**2 + 1)*tan(e + f*x)**2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2
 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e +
f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*
tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*
tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 4*a*b**3*tan(e + f*x)**2/(4*a
**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f
*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**
2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f +
 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e
+ f*x)**4) - 2*b**4*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5
*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*ta
n(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b
**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + ...
```

<u>**Giac**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(106) = 212.

time = 2.04, size = 469, normalized size = 4.34

$$-\frac{\frac{2\log\left(a+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{4\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{3a^2+\frac{20a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{32ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{50a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{128ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{96b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{20a^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{32ab(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{3a^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}}{(a^3-3a^2b+3ab^2-b^3)\left(a+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 4*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^2 + 20*a^2*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) - 32*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
50*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 128*a*b*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 + 96*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + 20*a^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 32*a*b*(cos(f*x + e)
 - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1
```

```
)^4)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1
)^2/(cos(f*x + e) + 1)^2)^2))/f
```

Mupad [B]
time = 12.52, size = 577, normalized size = 5.34

$$-\frac{a^3\,b\cos\left(e+f\,x\right)^4-\frac{a^4\cos\left(e+f\,x\right)^4}{4}-\frac{3\,a^2\,b^2\cos\left(e+f\,x\right)^4}{4}+b^4\sin\left(e+f\,x\right)^4\,\mathrm{atan}\left(\frac{a\sin\left(e+f\,x\right)^2-b\sin\left(e+f\,x\right)^2}{a\cos\left(e+f\,x\right)^2\,2{}\mathrm{i}+a\sin\left(e+f\,x\right)^2\,1{}\mathrm{i}+b\sin\left(e+f\,x\right)^2\,1{}\mathrm{i}}\right)\,1{}\mathrm{i}-a\,b^3\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2-\frac{a^3\,b\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2}{2}+a^2\,b^2\cos\left(e+f\,x\right)^4\,\mathrm{atan}\left(\frac{a\sin\left(e+f\,x\right)^2-b\sin\left(e+f\,x\right)^2}{a\cos\left(e+f\,x\right)^2\,2{}\mathrm{i}+a\sin\left(e+f\,x\right)^2\,1{}\mathrm{i}+b\sin\left(e+f\,x\right)^2\,1{}\mathrm{i}}\right)\,1{}\mathrm{i}+\frac{3\,a^2\,b^2\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2}{2}+a\,b^3\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2\,\mathrm{atan}\left(\frac{a\sin\left(e+f\,x\right)^2-b\sin\left(e+f\,x\right)^2}{a\cos\left(e+f\,x\right)^2\,2{}\mathrm{i}+a\sin\left(e+f\,x\right)^2\,1{}\mathrm{i}+b\sin\left(e+f\,x\right)^2\,1{}\mathrm{i}}\right)\,2{}\mathrm{i}}{f\,\left(-a^5\,b^2\cos\left(e+f\,x\right)^4+3\,a^4\,b^3\cos\left(e+f\,x\right)^4-2\,a^4\,b^3\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2-3\,a^3\,b^4\cos\left(e+f\,x\right)^4+6\,a^3\,b^4\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2-a^3\,b^4\sin\left(e+f\,x\right)^4+a^2\,b^5\cos\left(e+f\,x\right)^4-6\,a^2\,b^5\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2+3\,a^2\,b^5\sin\left(e+f\,x\right)^4+2\,a\,b^6\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2-3\,a\,b^6\sin\left(e+f\,x\right)^4+b^7\sin\left(e+f\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] -(a^3*b*cos(e + f*x)^4 - (a^4*cos(e + f*x)^4)/4 - (3*a^2*b^2*cos(e + f*x)^4
)/4 + b^4*sin(e + f*x)^4*atan((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(
e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i - a*b^3*cos(
e + f*x)^2*sin(e + f*x)^2 - (a^3*b*cos(e + f*x)^2*sin(e + f*x)^2)/2 + a^2*b
^2*cos(e + f*x)^4*atan((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x
)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i + (3*a^2*b^2*cos(e
+ f*x)^2*sin(e + f*x)^2)/2 + a*b^3*cos(e + f*x)^2*sin(e + f*x)^2*atan((a*si
n(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1i
 + b*sin(e + f*x)^2*1i))*2i)/(f*(b^7*sin(e + f*x)^4 - 3*a*b^6*sin(e + f*x)^
4 + a^2*b^5*cos(e + f*x)^4 - 3*a^3*b^4*cos(e + f*x)^4 + 3*a^4*b^3*cos(e + f
*x)^4 - a^5*b^2*cos(e + f*x)^4 + 3*a^2*b^5*sin(e + f*x)^4 - a^3*b^4*sin(e +
 f*x)^4 + 2*a*b^6*cos(e + f*x)^2*sin(e + f*x)^2 - 6*a^2*b^5*cos(e + f*x)^2*
sin(e + f*x)^2 + 6*a^3*b^4*cos(e + f*x)^2*sin(e + f*x)^2 - 2*a^4*b^3*cos(e
+ f*x)^2*sin(e + f*x)^2))
```

3.238 $\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^3} dx$

Optimal. Leaf size=97

$$\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f}-\frac{a}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2}-\frac{1}{2(a-b)^2f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] 1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^3/f-1/4*a/(a-b)/b/f/(a+b*tan(f*
x+e)^2)^2-1/2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 78}

$$-\frac{a}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/(2*(a - b)^3*f) - a/(4*(a - b)*b*f
*(a + b*Tan[e + f*x]^2)^2) - 1/(2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)^3(1+x)} + \frac{a}{(a-b)(a+bx)^3} + \frac{b}{(a-b)^2(a+bx)^2} + \frac{b}{(a-b)^3(a+bx)}\right)\,dx, x, \tan^2(\right.}{2f}$$

$$= \frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f} - \frac{a}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} - \frac{}{2(}$$

Mathematica [A]
time = 0.52, size = 87, normalized size = 0.90

$$\frac{4\log(\cos(e+fx)) + 2\log\left(a+b\tan^2(e+fx)\right) - \frac{a(a-b)^2}{b(a+b\tan^2(e+fx))^2} - \frac{2(a-b)}{a+b\tan^2(e+fx)}}{4(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (4*Log[Cos[e + f*x]] + 2*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)^2)/(b*(a +
b*Tan[e + f*x]^2)^2) - (2*(a - b))/(a + b*Tan[e + f*x]^2))/(4*(a - b)^3*f)
```

Maple [A]
time = 0.17, size = 101, normalized size = 1.04

method	result
derivativedivides	$\frac{-\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^3}+\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)-\frac{a-b}{a+b\left(\tan^2(fx+e)\right)}-\frac{a\left(a^2-2ab+b^2\right)}{2b\left(a+b\left(\tan^2(fx+e)\right)\right)^2}}{2(a-b)^3}}{f}$
default	$\frac{-\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^3}+\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)-\frac{a-b}{a+b\left(\tan^2(fx+e)\right)}-\frac{a\left(a^2-2ab+b^2\right)}{2b\left(a+b\left(\tan^2(fx+e)\right)\right)^2}}{2(a-b)^3}}{f}$
norman	$\frac{-\frac{b\left(\tan^2(fx+e)\right)}{2\left(a^2-2ab+b^2\right)f}+\frac{\left(-ab-b^2\right)a}{4b^2\left(a^2-2ab+b^2\right)f}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2} - \frac{\ln\left(1+\tan^2(fx+e)\right)}{2f\left(a^3-3a^2b+3ab^2-b^3\right)} + \frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{2f\left(a^3-3a^2b+3ab^2-b^3\right)}$

risch	$-\frac{ix}{a^3-3a^2b+3a\,b^2-b^3}-\frac{2ie}{f(a^3-3a^2b+3a\,b^2-b^3)}+\frac{2a^2\mathrm{e}^{6i(fx+e)}-2b^2\mathrm{e}^{6i(fx+e)}+4a^2\mathrm{e}^{4i(fx+e)}+4ab\,\mathrm{e}^{4i(fx+e)}+4b^2\mathrm{e}^{4}}{(a\,\mathrm{e}^{4i(fx+e)}-b\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{2i(fx+e)}+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2/(a-b)^3*ln(1+tan(f*x+e)^2)+1/2/(a-b)^3*(ln(a+b*tan(f*x+e)^2)-(a-b
)/(a+b*tan(f*x+e)^2)-1/2*a*(a^2-2*a*b+b^2)/b/(a+b*tan(f*x+e)^2)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(95) = 190$.
time = 0.28, size = 198, normalized size = 2.04

$$-\frac{\frac{2\,(a^2-b^2)\sin(fx+e)^2-2\,a^2-ab}{a^5-3\,a^4b+3\,a^3b^2-a^2b^3+(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)\sin(fx+e)^4-2\,(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)\sin(fx+e)^2}-\frac{2\,\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^3-3\,a^2b+3\,ab^2-b^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((2*(a^2 - b^2)*sin(f*x + e)^2 - 2*a^2 - a*b)/(a^5 - 3*a^4*b + 3*a^3*b
^2 - a^2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*si
n(f*x + e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x +
e)^2) - 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))
/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(95) = 190$.
time = 1.66, size = 220, normalized size = 2.27

$$\frac{(ab+2\,b^2)\tan(fx+e)^4+2\,(a^2+ab+b^2)\tan(fx+e)^2+2\,a^2+ab+2\,\left(b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{4\,\left((a^3b^2-3\,a^2b^3+3\,ab^4-b^5)f\tan(fx+e)^4+2\,(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4)f\tan(fx+e)^2+(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a*b + 2*b^2)*tan(f*x + e)^4 + 2*(a^2 + a*b + b^2)*tan(f*x + e)^2 + 2*
a^2 + a*b + 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(
f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^
5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x +
 e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2819 vs. $2(75) = 150$.
time = 74.58, size = 2819, normalized size = 29.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(tan(e +
 f*x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a**3, Eq(b, 0)), (-3*tan(e + f
*x)**2/(12*b**3*f*tan(e + f*x)**6 + 36*b**3*f*tan(e + f*x)**4 + 36*b**3*f*t
an(e + f*x)**2 + 12*b**3*f) - 1/(12*b**3*f*tan(e + f*x)**6 + 36*b**3*f*tan(
e + f*x)**4 + 36*b**3*f*tan(e + f*x)**2 + 12*b**3*f), Eq(a, b)), (x*tan(e)*
*3/(a + b*tan(e)**2)**3, Eq(f, 0)), (-a**3/(4*a**5*b*f + 8*a**4*b**2*f*tan(
e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3
*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a
**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 -
8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*a**2*b*log(-sqrt
(-a/b) + tan(e + f*x))/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**
4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 +
 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f
*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f
*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*a**2*b*log(sqrt(-a/b) + tan(e + f*x)
)/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**
3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*
a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*
f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan
(e + f*x)**4) - 2*a**2*b*log(tan(e + f*x)**2 + 1)/(4*a**5*b*f + 8*a**4*b**2
*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a*
*3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4
 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x
)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 4*a*b**2*lo
g(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**5*b*f + 8*a**4*b**2*f*t
an(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b
**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 2
4*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4
 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 4*a*b**2*log(sq
rt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e
+ f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f
*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**
2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*
a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) - 4*a*b**2*log(tan(e +
 f*x)**2 + 1)*tan(e + f*x)**2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 -
 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*
x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*ta
n(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*ta
n(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) - 2*a*b**2*tan(e + f*x)**2/(4*a**
5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(
e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**
```

```
4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a
*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x
)**4) + a*b**2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f
 + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3
*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 -
 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 -
 4*b**6*f*tan(e + f*x)**4) + 2*b**3*log(-sqrt(-a/b) + tan(e + f*x))*tan(e +
 f*x)**4/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a
**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*
f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**
2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**
6*f*tan(e + f*x)**4) + 2*b**3*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**
4/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**
3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*
a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*
f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan
(e + f*x)**4) - 2*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a**5*b*f
 + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f
*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*t
an(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5
*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4)
 + 2*b**3*tan(e + f*x)**2/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*
a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**
2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e
+ f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e
+ f*x)**2 - 4*b**6*f*tan(e + f*x)**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(95) = 190$.

time = 1.28, size = 506, normalized size = 5.22

$$\frac{\frac{2\log\left(a+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{4\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{3a^3+\frac{20a^3(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{32a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{34a^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{80a^2b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{48ab^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{16b^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{20a^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{32a^2b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{3a^3(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}}{(a^4-3a^3b+3a^2b^2-ab^3)\left(a+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a
^3 - 3*a^2*b + 3*a*b^2 - b^3) - 4*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^3 + 20*a^3*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - 32*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
 34*a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 80*a^2*b*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2 + 48*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + 16*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20*a^3*(cos(f*x
+ e) - 1)^3/(cos(f*x + e) + 1)^3 - 32*a^2*b*(cos(f*x + e) - 1)^3/(cos(f*x +
```

```
e) + 1)^3 + 3*a^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^4 - 3*a^3
*b + 3*a^2*b^2 - a*b^3)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*
b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2)^2))/f
```

Mupad [B]
time = 12.52, size = 532, normalized size = 5.48

$$\frac{\frac{a^3\cos(e+fx)^4}{4}-\frac{a\,b^2\cos(e+fx)^4}{4}+b^3\sin\left(e+f\,x\right)^4\mathrm{atan}\left(\frac{a\sin(e+fx)^2-b\sin(e+fx)^2}{a\cos(e+fx)^2\,2\mathrm{i}+a\sin(e+fx)^2\,1\mathrm{i}+b\sin(e+fx)^2\,1\mathrm{i}}\right)\,1\mathrm{i}-\frac{b^3\cos(e+fx)^2\sin(e+fx)^2}{2}+a^2\,b\cos\left(e+f\,x\right)^4\mathrm{atan}\left(\frac{a\sin(e+fx)^2-b\sin(e+fx)^2}{a\cos(e+fx)^2\,2\mathrm{i}+a\sin(e+fx)^2\,1\mathrm{i}+b\sin(e+fx)^2\,1\mathrm{i}}\right)\,1\mathrm{i}+\frac{a\,b^2\cos(e+fx)^2\sin(e+fx)^2}{2}+a\,b^2\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2\mathrm{atan}\left(\frac{a\sin(e+fx)^2-b\sin(e+fx)^2}{a\cos(e+fx)^2\,2\mathrm{i}+a\sin(e+fx)^2\,1\mathrm{i}+b\sin(e+fx)^2\,1\mathrm{i}}\right)\,2\mathrm{i}}{f\,\left(-a^5\,b\cos\left(e+f\,x\right)^4+3\,a^4\,b^2\cos\left(e+f\,x\right)^4-2\,a^4\,b^2\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2-3\,a^3\,b^3\cos\left(e+f\,x\right)^4+6\,a^3\,b^3\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2-a^3\,b^3\sin\left(e+f\,x\right)^4+a^2\,b^4\cos\left(e+f\,x\right)^4-6\,a^2\,b^4\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2+3\,a^2\,b^4\sin\left(e+f\,x\right)^4+2\,a\,b^5\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2-3\,a\,b^5\sin\left(e+f\,x\right)^4+b^6\sin\left(e+f\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((a^3*cos(e + f*x)^4)/4 - (a*b^2*cos(e + f*x)^4)/4 + b^3*sin(e + f*x)^4*ata
n((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*
x)^2*1i + b*sin(e + f*x)^2*1i))*1i - (b^3*cos(e + f*x)^2*sin(e + f*x)^2)/2
+ a^2*b*cos(e + f*x)^4*atan((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e
+ f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i + (a*b^2*cos(e
 + f*x)^2*sin(e + f*x)^2)/2 + a*b^2*cos(e + f*x)^2*sin(e + f*x)^2*atan((a*s
in(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1
i + b*sin(e + f*x)^2*1i))*2i)/(f*(b^6*sin(e + f*x)^4 - a^5*b*cos(e + f*x)^4
 - 3*a*b^5*sin(e + f*x)^4 + a^2*b^4*cos(e + f*x)^4 - 3*a^3*b^3*cos(e + f*x)
^4 + 3*a^4*b^2*cos(e + f*x)^4 + 3*a^2*b^4*sin(e + f*x)^4 - a^3*b^3*sin(e +
f*x)^4 + 2*a*b^5*cos(e + f*x)^2*sin(e + f*x)^2 - 6*a^2*b^4*cos(e + f*x)^2*s
in(e + f*x)^2 + 6*a^3*b^3*cos(e + f*x)^2*sin(e + f*x)^2 - 2*a^4*b^2*cos(e +
 f*x)^2*sin(e + f*x)^2))
```

3.239 $\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal. Leaf size=93

$$-\frac{\log \left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f}+\frac{1}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2}+\frac{1}{2(a-b)^2f\left(a+b\tan^2(e+fx)\right)}$$

```
[Out] -1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^3/f+1/4/(a-b)/f/(a+b*tan(f*x+e
)^2)^2+1/2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 455, 46}

$$\frac{1}{2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{1}{4f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\log \left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -1/2*Log[a*Cos[e + f*x]^2 + b*Sin[e + f*x]^2]/((a - b)^3*f) + 1/(4*(a - b)*
f*(a + b*Tan[e + f*x]^2)^2) + 1/(2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^3(1+x)} - \frac{b}{(a-b)(a+bx)^3} - \frac{b}{(a-b)^2(a+bx)^2} + \frac{b}{(-a+b)^3(a+bx)}\right)\,dx, x, \tan^2(\right.}{2f}$$

$$= -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f} + \frac{1}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{}{2(}$$

Mathematica [A]
time = 0.51, size = 82, normalized size = 0.88

$$\frac{-4\log(\cos(e+fx)) - 2\log\left(a+b\tan^2(e+fx)\right) + \frac{(a-b)^2}{\left(a+b\tan^2(e+fx)\right)^2} + \frac{2(a-b)}{a+b\tan^2(e+fx)}}{4(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
[Out] (-4*Log[Cos[e + f*x]] - 2*Log[a + b*Tan[e + f*x]^2] + (a - b)^2/(a + b*Tan[
e + f*x]^2)^2 + (2*(a - b))/(a + b*Tan[e + f*x]^2))/(4*(a - b)^3*f)
```

Maple [A]
time = 0.17, size = 108, normalized size = 1.16

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^3} - \frac{b\left(\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{b} - \frac{a-b}{b\left(a+b\left(\tan^2(fx+e)\right)\right)} - \frac{a^2-2ab+b^2}{2b\left(a+b\left(\tan^2(fx+e)\right)\right)^2}\right)}{2(a-b)^3}}{f}$
default	$\frac{\frac{\ln\left(1+\tan^2(fx+e)\right)}{2(a-b)^3} - \frac{b\left(\frac{\ln\left(a+b\left(\tan^2(fx+e)\right)\right)}{b} - \frac{a-b}{b\left(a+b\left(\tan^2(fx+e)\right)\right)} - \frac{a^2-2ab+b^2}{2b\left(a+b\left(\tan^2(fx+e)\right)\right)^2}\right)}{2(a-b)^3}}{f}$
norman	$\frac{\frac{3a\,b^2-b^3}{4b^2\left(a^2-2ab+b^2\right)f} + \frac{b\left(\tan^2(fx+e)\right)}{2\left(a^2-2ab+b^2\right)f}}{\left(a+b(\tan^2(fx+e))\right)^2} + \frac{\ln(1+\tan^2(fx+e))}{2f(a^3-3a^2b+3a\,b^2-b^3)} - \frac{\ln(a+b(\tan^2(fx+e)))}{2f(a^3-3a^2b+3a\,b^2-b^3)}$

risch	$\frac{ix}{a^3-3a^2b+3a\,b^2-b^3}+\frac{2ie}{f(a^3-3a^2b+3a\,b^2-b^3)}-\frac{4b(-a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}-a\,e^{2i(fx+e}}{(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/(a-b)^3*ln(1+tan(f*x+e)^2)-1/2*b/(a-b)^3*(1/b*ln(a+b*tan(f*x+e)^2)
-(a-b)/b/(a+b*tan(f*x+e)^2)-1/2*(a^2-2*a*b+b^2)/b/(a+b*tan(f*x+e)^2)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(91) = 182$.
time = 0.29, size = 196, normalized size = 2.11

$$\frac{\frac{4\,(ab-b^2)\sin(fx+e)^2-4\,ab+b^2}{a^5-3\,a^4b+3\,a^3b^2-a^2b^3+(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)\sin(fx+e)^4-2\,(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)\sin(fx+e)^2}-\frac{2\,\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^3-3\,a^2b+3\,ab^2-b^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*(a*b - b^2)*sin(f*x + e)^2 - 4*a*b + b^2)/(a^5 - 3*a^4*b + 3*a^3*b^
2 - a^2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin
(f*x + e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x + e
)^2) - 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/
f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(91) = 182$.
time = 1.69, size = 214, normalized size = 2.30

$$-\frac{3\,b^2\tan\left(fx+e\right)^4+2\,(2\,ab+b^2)\tan\left(fx+e\right)^2+4\,ab-b^2+2\,\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{4\,\left((a^3b^2-3\,a^2b^3+3\,ab^4-b^5)f\tan\left(fx+e\right)^4+2\,(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4)f\tan\left(fx+e\right)^2+(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*b^2*tan(f*x + e)^4 + 2*(2*a*b + b^2)*tan(f*x + e)^2 + 4*a*b - b^2 +
 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(f*x + e)^2
+ a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*
x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^
5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2846 vs. $2(73) = 146$.
time = 75.06, size = 2846, normalized size = 30.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)

[Out] Piecewise((zoo*x/tan(e)**5, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(6*b**3*f*tan(e + f*x)**6 + 18*b**3*f*tan(e + f*x)**4 + 18*b**3*f*tan(e + f*x)**2 + 6*b**3*f), Eq(a, b)), (x*tan(e)/(a + b*tan(e)**2)**3, Eq(f, 0)), (log(tan(e + f*x)**2 + 1)/(2*a**3*f), Eq(b, 0)), (-2*a**2*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 2*a**2*log(sqrt(-a/b) + tan(e + f*x))/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 2*a**2*log(tan(e + f*x)**2 + 1)/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 3*a**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 4*a*b*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 4*a*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 4*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 2*a*b*tan(e + f*x)**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 4*a*b/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b*

```
*2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12
*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3
*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*ta
n(e + f*x)**4) - 2*b**2*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**4/(4*
a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f
*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*t
an(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4
*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4)
 - 2*b**2*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**4/(4*a**5*f + 8*a**4
*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**
3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4
+ 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)
**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 2*b**2*log(t
an(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2
- 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)
**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(
e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(
e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) - 2*b**2*tan(e + f*x)**2/(4*a**5*f
+ 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4
- 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e +
f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(
e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + b**2
/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e
 + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3
*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*
b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 650 vs. 2(91) = 182.

time = 1.05, size = 650, normalized size = 6.99

$$-\frac{\frac{2\log\left(a+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{4\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{3a^4+\frac{12a^4(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{24a^2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8ab^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18a^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{16a^3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{48a^2b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{80ab^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{16b^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{12a^4(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{8a^3b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{24a^2b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{8ab^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{3a^4(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}}{(a^5-3a^4b+3a^3b^2-a^2b^3)\left(a+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 4*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a^4 + 12*a^4*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) - 8*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
 24*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b^3*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 18*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 -
 16*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 48*a^2*b^2*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + 80*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x +
```

```
e) + 1)^2 - 16*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^4*(cos(
f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 8*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*
x + e) + 1)^3 - 24*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*a*
b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^4*(cos(f*x + e) - 1)^4/
(cos(f*x + e) + 1)^4)/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*(a + 2*a*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2))/f
```

Mupad [B]
time = 12.46, size = 375, normalized size = 4.03

$$\frac{a^2\left(-3+\mathrm{atan}\left(\frac{a\tan(e+f\,x)^2\,1\mathrm{i}-b\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a+a\tan(e+f\,x)^2+b\tan(e+f\,x)^2}\right)\,4\mathrm{i}\right)+b^2\left(2\tan(e+f\,x)^2-1+\tan(e+f\,x)^4\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)^2\,1\mathrm{i}-b\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a+a\tan(e+f\,x)^2+b\tan(e+f\,x)^2}\right)\,4\mathrm{i}\right)+a\,b\left(4-2\tan(e+f\,x)^2+\tan(e+f\,x)^2\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)^2\,1\mathrm{i}-b\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a+a\tan(e+f\,x)^2+b\tan(e+f\,x)^2}\right)\,8\mathrm{i}\right)}{f\,\left(-4\,a^5-8\,a^4\,b\tan\left(e+f\,x\right)^2+12\,a^4\,b-4\,a^3\,b^2\tan\left(e+f\,x\right)^4+24\,a^3\,b^2\tan\left(e+f\,x\right)^2-12\,a^3\,b^2+12\,a^2\,b^3\tan\left(e+f\,x\right)^4-24\,a^2\,b^3\tan\left(e+f\,x\right)^2+4\,a^2\,b^3-12\,a\,b^4\tan\left(e+f\,x\right)^4+8\,a\,b^4\tan\left(e+f\,x\right)^2+4\,b^5\tan\left(e+f\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (a^2*(atan((a*tan(e + f*x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x
)^2 + b*tan(e + f*x)^2))*4i - 3) + b^2*(tan(e + f*x)^4*atan((a*tan(e + f*x)
^2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*4
i + 2*tan(e + f*x)^2 - 1) + a*b*(tan(e + f*x)^2*atan((a*tan(e + f*x)^2*1i -
 b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*8i - 2*t
an(e + f*x)^2 + 4))/(f*(12*a^4*b - 4*a^5 + 4*a^2*b^3 - 12*a^3*b^2 + 4*b^5*t
an(e + f*x)^4 + 8*a*b^4*tan(e + f*x)^2 - 8*a^4*b*tan(e + f*x)^2 - 12*a*b^4*
tan(e + f*x)^4 - 24*a^2*b^3*tan(e + f*x)^2 + 24*a^3*b^2*tan(e + f*x)^2 + 12
*a^2*b^3*tan(e + f*x)^4 - 4*a^3*b^2*tan(e + f*x)^4))
```

3.240 $\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal. Leaf size=148

$$\frac{\log(\cos(e+fx))}{(a-b)^3f}+\frac{\log(\tan(e+fx))}{a^3f}+\frac{b(3a^2-3ab+b^2)\log\left(a+b\tan^2(e+fx)\right)}{2a^3(a-b)^3f}-\frac{b}{4a(a-b)f\left(a+b\tan^2(e\right.}$$

```
[Out] ln(cos(f*x+e))/(a-b)^3/f+ln(tan(f*x+e))/a^3/f+1/2*b*(3*a^2-3*a*b+b^2)*ln(a+
b*tan(f*x+e)^2)/a^3/(a-b)^3/f-1/4*b/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/2*(2*a
-b)*b/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 457, 84}

$$\frac{\log(\tan(e+fx))}{a^3f}-\frac{b(2a-b)}{2a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{b(3a^2-3ab+b^2)\log\left(a+b\tan^2(e+fx)\right)}{2a^3f(a-b)^3}-\frac{b}{4af(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{\log(\cos(e+fx))}{f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] Log[Cos[e + f*x]]/((a - b)^3*f) + Log[Tan[e + f*x]]/(a^3*f) + (b*(3*a^2 - 3
*a*b + b^2)*Log[a + b*Tan[e + f*x]^2])/(2*a^3*(a - b)^3*f) - b/(4*a*(a - b)
*f*(a + b*Tan[e + f*x]^2)^2) - ((2*a - b)*b)/(2*a^2*(a - b)^2*f*(a + b*Tan[
e + f*x]^2))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{(a-b)^3(1+x)} + \frac{b^2}{a(a-b)(a+bx)^3} + \frac{(2a-b)b^2}{a^2(a-b)^2(a+bx)^2} + \frac{b^2(3a^2-3ab+b^2)}{a^3(a-b)^3(a+bx)}\right)\,dx\right.}{2f}$$
$$= \frac{\log(\cos(e+fx))}{(a-b)^3f} + \frac{\log(\tan(e+fx))}{a^3f} + \frac{b(3a^2-3ab+b^2)\log\left(a+b\tan^2(e+\right.}{2a^3(a-b)^3f}$$

Mathematica [A]

time = 1.21, size = 126, normalized size = 0.85

$$\frac{\frac{4\log(\cos(e+fx))}{(a-b)^3} + \frac{4\log(\tan(e+fx)) + \frac{b\left(2\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+fx)\right) - \frac{a(a-b)\left(a(5a-3b)+2(2a-b)b\tan^2(e+fx)\right)}{\left(a+b\tan^2(e+fx)\right)^2}\right)}{(a-b)^3}}{a^3}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((4*Log[Cos[e + f*x]])/(a - b)^3 + (4*Log[Tan[e + f*x]] + (b*(2*(3*a^2 - 3*
a*b + b^2)*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)*(a*(5*a - 3*b) + 2*(2*a -
 b)*b*Tan[e + f*x]^2))/(a + b*Tan[e + f*x]^2)^2))/(a - b)^3)/a^3)/(4*f)
```

Maple [A]

time = 0.39, size = 168, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{\ln(\cos(fx+e)+1)}{2a^3} + \frac{\ln(\cos(fx+e)-1)}{2a^3} + \frac{b\left(\frac{(3a-b)ab}{(a-b)^3\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)} + \frac{\left(3a^2-3ab+b^2\right)\ln\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(f\right.\right.}{(a-b)^3}\right.}{2a^3}}{f}$
default	$\frac{\frac{\ln(\cos(fx+e)+1)}{2a^3} + \frac{\ln(\cos(fx+e)-1)}{2a^3} + \frac{b\left(\frac{(3a-b)ab}{(a-b)^3\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)b+b\right)} + \frac{\left(3a^2-3ab+b^2\right)\ln\left(a\left(\cos^2(fx+e)\right)-\left(\cos^2(f\right.\right.}{(a-b)^3}\right.}{2a^3}}{f}$

norman	$\frac{\frac{(3ab-2b^2)b(\tan^2(fx+e))}{2a^2f(a^2-2ab+b^2)}+\frac{(5ab-3b^2)b^2(\tan^4(fx+e))}{4a^3f(a^2-2ab+b^2)}}{(a+b(\tan^2(fx+e)))^2}+\frac{\ln(\tan(fx+e))}{a^3f}-\frac{\ln(1+\tan^2(fx+e))}{2f(a^3-3a^2b+3ab^2-b^3)}+\frac{b(3a^2-3ab+b^2}{2fa^3(a^3}$
risch	$\frac{ix}{a^3-3a^2b+3ab^2-b^3}-\frac{2ix}{a^3}-\frac{2ie}{fa^3}-\frac{6ibx}{a(a^3-3a^2b+3ab^2-b^3)}-\frac{6ibe}{fa(a^3-3a^2b+3ab^2-b^3)}+\frac{6ib^2x}{a^2(a^3-3a^2b+3ab^2-b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2/a^3*ln(cos(f*x+e)+1)+1/2/a^3*ln(cos(f*x+e)-1)+1/2*b/a^3*((3*a-b)*a
*b/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+(3*a^2-3*a*b+b^2)/(a-b)^3*ln(a
*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/2*a^2/(a-b)^3*b^2/(a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)^2))
```

Maxima [A]
time = 0.30, size = 255, normalized size = 1.72

$$\frac{\frac{2\,(3\,a^2b-3\,ab^2+b^3)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^6-3\,a^5b+3\,a^4b^2-a^3b^3}+\frac{6\,a^2b^2-3\,ab^3-2\,(3\,a^2b^2-4\,ab^3+b^4)\sin(fx+e)^2}{a^7-3\,a^6b+3\,a^5b^2-a^4b^3+(a^7-5\,a^6b+10\,a^5b^2-10\,a^4b^3+5\,a^3b^4-a^2b^5)\sin(fx+e)^4-2\,(a^7-4\,a^6b+6\,a^5b^2-4\,a^4b^3+a^3b^4)\sin(fx+e)^2}+\frac{2\log\left(\sin(fx+e)^2\right)}{a^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*a^2*b - 3*a*b^2 + b^3)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^6 - 3*
a^5*b + 3*a^4*b^2 - a^3*b^3) + (6*a^2*b^2 - 3*a*b^3 - 2*(3*a^2*b^2 - 4*a*b^
3 + b^4)*sin(f*x + e)^2)/(a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3 + (a^7 - 5*a^
6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*sin(f*x + e)^4 - 2*(a^
7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*sin(f*x + e)^2) + 2*log(sin(
f*x + e)^2)/a^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(147) = 294.
time = 1.47, size = 434, normalized size = 2.93

$$\frac{6\,a^3b^2-3\,a^2b^3+(5\,a^2b^3-2\,ab^4)\tan(fx+e)^4+2\,(3\,a^3b^2+a^2b^3-ab^4)\tan(fx+e)^2+2\left(a^5-3\,a^4b+3\,a^3b^2-a^2b^3+(a^3b^2-3\,a^2b^3+3\,ab^4-b^5)\tan(fx+e)^4+2\,(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4)\tan(fx+e)^2\right)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)+2\left(3\,a^4b-3\,a^3b^2+a^2b^3+(3\,a^2b^3-3\,ab^4+b^5)\tan(fx+e)^4+2\,(3\,a^3b^2-3\,a^2b^3+ab^4)\tan(fx+e)^2\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{4\left((a^6b^2-3\,a^5b^3+3\,a^4b^4-a^3b^5)f\tan(fx+e)^4+2\,(a^7b-3\,a^6b^2+3\,a^5b^3-a^4b^4)f\tan(fx+e)^2+(a^8-3\,a^7b+3\,a^6b^2-a^5b^3)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^3*b^2 - 3*a^2*b^3 + (5*a^2*b^3 - 2*a*b^4)*tan(f*x + e)^4 + 2*(3*a^
3*b^2 + a^2*b^3 - a*b^4)*tan(f*x + e)^2 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^
2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x +
 e)^2 + 1)) + 2*(3*a^4*b - 3*a^3*b^2 + a^2*b^3 + (3*a^2*b^3 - 3*a*b^4 + b^5
)*tan(f*x + e)^4 + 2*(3*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*log((b
```

```
*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b
^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4
)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.29, size = 267, normalized size = 1.80

$$\frac{\frac{2\,(3\,a^2b-3\,ab^2+b^3)\,\log\left(\left|-a\sin(fx+e)^2+b\sin(fx+e)^2+a\right|\right)}{a^6-3\,a^5b+3\,a^4b^2-a^3b^3}-\frac{9\,a^3b\sin(fx+e)^4-18\,a^2b^2\sin(fx+e)^4+12\,ab^3\sin(fx+e)^4-3\,b^4\sin(fx+e)^4-18\,a^3b\sin(fx+e)^2+24\,a^2b^2\sin(fx+e)^2-8\,ab^3\sin(fx+e)^2+9\,a^3b-6\,a^2b^2}{(a^5-2\,a^4b+a^3b^2)\left(a\sin(fx+e)^2-b\sin(fx+e)^2-a\right)^2}+\frac{2\,\log\left(\sin(fx+e)^2\right)}{a^3}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a^2*b - 3*a*b^2 + b^3)*log(abs(-a*sin(f*x + e)^2 + b*sin(f*x + e)
^2 + a))/(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3) - (9*a^3*b*sin(f*x + e)^4 -
18*a^2*b^2*sin(f*x + e)^4 + 12*a*b^3*sin(f*x + e)^4 - 3*b^4*sin(f*x + e)^4
- 18*a^3*b*sin(f*x + e)^2 + 24*a^2*b^2*sin(f*x + e)^2 - 8*a*b^3*sin(f*x + e
)^2 + 9*a^3*b - 6*a^2*b^2)/((a^5 - 2*a^4*b + a^3*b^2)*(a*sin(f*x + e)^2 - b
*sin(f*x + e)^2 - a)^2) + 2*log(sin(f*x + e)^2)/a^3)/f
```

Mupad [B]
time = 12.59, size = 181, normalized size = 1.22

$$\frac{\ln\left(\tan(e+f\,x)\right)}{a^3\,f}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^3}-\frac{\frac{5\,a\,b-3\,b^2}{4\,a\,(a^2-2\,a\,b+b^2)}+\frac{b\tan(e+f\,x)^2\,(2\,a\,b-b^2)}{2\,a^2\,(a^2-2\,a\,b+b^2)}}{f\,\left(a^2+2\,a\,b\tan\left(e+f\,x\right)^2+b^2\tan\left(e+f\,x\right)^4\right)}+\frac{b\,\ln\left(b\tan(e+f\,x)^2+a\right)\,\left(3\,a^2-3\,a\,b+b^2\right)}{2\,a^3\,f\,(a-b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] log(tan(e + f*x))/(a^3*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^3) - ((5*a
*b - 3*b^2)/(4*a*(a^2 - 2*a*b + b^2)) + (b*tan(e + f*x)^2*(2*a*b - b^2))/(2
*a^2*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan(e + f*x
)^2)) + (b*log(a + b*tan(e + f*x)^2)*(3*a^2 - 3*a*b + b^2))/(2*a^3*f*(a - b
)^3)
```

3.241 $\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal. Leaf size=181

$$-\frac{\cot^2(e+fx)}{2a^3f}-\frac{\log(\cos(e+fx))}{(a-b)^3f}-\frac{(a+3b)\log(\tan(e+fx))}{a^4f}-\frac{b^2(6a^2-8ab+3b^2)\log\left(a+b\tan^2(e+fx)\right)}{2a^4(a-b)^3f}$$

```
[Out] -1/2*cot(f*x+e)^2/a^3/f-ln(cos(f*x+e))/(a-b)^3/f-(a+3*b)*ln(tan(f*x+e))/a^4
/f-1/2*b^2*(6*a^2-8*a*b+3*b^2)*ln(a+b*tan(f*x+e)^2)/a^4/(a-b)^3/f+1/4*b^2/a
^2/(a-b)/f/(a+b*tan(f*x+e)^2)^2+1/2*(3*a-2*b)*b^2/a^3/(a-b)^2/f/(a+b*tan(f*
x+e)^2)
```

Rubi [A]
time = 0.15, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{(a+3b)\log(\tan(e+fx))}{a^4f}+\frac{b^2(3a-2b)}{2a^3f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\cot^2(e+fx)}{2a^3f}+\frac{b^2}{4a^2f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{b^2(6a^2-8ab+3b^2)\log\left(a+b\tan^2(e+fx)\right)}{2a^4f(a-b)^3}-\frac{\log(\cos(e+fx))}{f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -1/2*Cot[e + f*x]^2/(a^3*f) - Log[Cos[e + f*x]]/((a - b)^3*f) - ((a + 3*b)*
Log[Tan[e + f*x]])/(a^4*f) - (b^2*(6*a^2 - 8*a*b + 3*b^2)*Log[a + b*Tan[e +
 f*x]^2])/(2*a^4*(a - b)^3*f) + b^2/(4*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2)
^2) + ((3*a - 2*b)*b^2)/(2*a^3*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^2} + \frac{-a-3b}{a^4x} + \frac{1}{(a-b)^3(1+x)} - \frac{b^3}{a^2(a-b)(a+bx)^3} - \frac{(3a-2b)b^3}{a^3(a-b)^2(a+bx)^2} - \frac{b^3(6a^2-8}{a^4(a-b)}\right.\right.}{2f}$$
$$= -\frac{\cot^2(e+fx)}{2a^3f} - \frac{\log(\cos(e+fx))}{(a-b)^3f} - \frac{(a+3b)\log(\tan(e+fx))}{a^4f} - \frac{b^2(6a^2-8a}{}$$

Mathematica [A]
time = 1.29, size = 144, normalized size = 0.80

$$-\frac{\frac{\cot^2(e+fx)}{a^3} - \frac{b^4}{2a^4(a-b)(b+a\cot^2(e+fx))^2} + \frac{(4a-3b)b^3}{a^4(a-b)^2(b+a\cot^2(e+fx))} + \frac{b^2(6a^2-8ab+3b^2)\log(b+a\cot^2(e+fx))}{a^4(a-b)^3} + \frac{2\log(\sin(e+fx))}{(a-b)^3}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -1/2*(Cot[e + f*x]^2/a^3 - b^4/(2*a^4*(a - b)*(b + a*Cot[e + f*x]^2)^2) + (
(4*a - 3*b)*b^3)/(a^4*(a - b)^2*(b + a*Cot[e + f*x]^2)) + (b^2*(6*a^2 - 8*a
*b + 3*b^2)*Log[b + a*Cot[e + f*x]^2])/(a^4*(a - b)^3) + (2*Log[Sin[e + f*x
]])/(a - b)^3)/f
```

Maple [A]
time = 0.42, size = 217, normalized size = 1.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/4/a^3/(cos(f*x+e)+1)+1/2*(-a-3*b)/a^4*ln(cos(f*x+e)+1)+1/4/a^3/(cos
(f*x+e)-1)+1/2*(-a-3*b)/a^4*ln(cos(f*x+e)-1)-1/2*b^2/a^4*(2*(2*a-b)*a*b/(a-
b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+(6*a^2-8*a*b+3*b^2)/(a-b)^3*ln(a*cos
```

```
(f*x+e)^2-cos(f*x+e)^2*b+b)-1/2*a^2/(a-b)^3*b^2/(a*cos(f*x+e)^2-cos(f*x+e)^
2*b+b)^2))
```

Maxima [A]
time = 0.31, size = 352, normalized size = 1.94

$$-\frac{\frac{2\left(6a^{2}b^{2}-8ab^{3}+3b^{4}\right)\log\left(-(a-b)\sin\left(fx+e\right)^{2}+a\right)}{a^{7}-3a^{6}b+3a^{5}b^{2}-a^{4}b^{3}}+\frac{2a^{5}-6a^{4}b+6a^{3}b^{2}-2a^{2}b^{3}+2\left(a^{5}-5a^{4}b+10a^{3}b^{2}-14a^{2}b^{3}+11ab^{4}-3b^{5}\right)\sin\left(fx+e\right)^{4}-\left(4a^{5}-16a^{4}b+24a^{3}b^{2}-24a^{2}b^{3}+9ab^{4}\right)\sin\left(fx+e\right)^{2}}{\left(a^{8}-5a^{7}b+10a^{6}b^{2}-10a^{5}b^{3}+5a^{4}b^{4}-a^{3}b^{5}\right)\sin\left(fx+e\right)^{6}-2\left(a^{8}-4a^{7}b+6a^{6}b^{2}-4a^{5}b^{3}+a^{4}b^{4}\right)\sin\left(fx+e\right)^{4}+\left(a^{8}-3a^{7}b+3a^{6}b^{2}-a^{5}b^{3}\right)\sin\left(fx+e\right)^{2}}+\frac{2(a+3b)\log\left(\sin\left(fx+e\right)^{2}\right)}{a^{4}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(6*a^2*b^2 - 8*a*b^3 + 3*b^4)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^7
 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) + (2*a^5 - 6*a^4*b + 6*a^3*b^2 - 2*a^2*b^
3 + 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 14*a^2*b^3 + 11*a*b^4 - 3*b^5)*sin(f*x
+ e)^4 - (4*a^5 - 16*a^4*b + 24*a^3*b^2 - 24*a^2*b^3 + 9*a*b^4)*sin(f*x + e
)^2)/((a^8 - 5*a^7*b + 10*a^6*b^2 - 10*a^5*b^3 + 5*a^4*b^4 - a^3*b^5)*sin(f
*x + e)^6 - 2*(a^8 - 4*a^7*b + 6*a^6*b^2 - 4*a^5*b^3 + a^4*b^4)*sin(f*x + e
)^4 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*sin(f*x + e)^2) + 2*(a + 3*b)*l
og(sin(f*x + e)^2)/a^4)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(179) = 358.
time = 2.00, size = 561, normalized size = 3.10

$$-\frac{\left(2a^{4}b^{2}-6a^{3}b^{3}+13a^{2}b^{4}-6ab^{5}\right)\tan\left(fx+e\right)^{6}+2a^{6}-6a^{5}b+6a^{4}b^{2}-2a^{3}b^{3}+2\left(2a^{5}b-5a^{4}b^{2}+7a^{3}b^{3}+2a^{2}b^{4}-3ab^{5}\right)\tan\left(fx+e\right)^{4}+\left(2a^{6}-2a^{5}b-6a^{4}b^{2}+18a^{3}b^{3}-9a^{2}b^{4}\right)\tan\left(fx+e\right)^{2}+2\left(\left(a^{4}b^{2}-6a^{2}b^{4}+8ab^{5}-3b^{6}\right)\tan\left(fx+e\right)^{6}+2\left(a^{5}b-6a^{3}b^{3}+8a^{2}b^{4}-3ab^{5}\right)\tan\left(fx+e\right)^{4}+\left(a^{6}-6a^{4}b^{2}+8a^{3}b^{3}-3a^{2}b^{4}\right)\tan\left(fx+e\right)^{2}\right)\log\left(\frac{\tan\left(fx+e\right)^{2}}{\tan\left(fx+e\right)^{2}+1}\right)+2\left(\left(6a^{2}b^{4}-8ab^{5}+3b^{6}\right)\tan\left(fx+e\right)^{6}+2\left(6a^{3}b^{3}-8a^{2}b^{4}+3ab^{5}\right)\tan\left(fx+e\right)^{4}+\left(6a^{4}b^{2}-8a^{3}b^{3}+3a^{2}b^{4}\right)\tan\left(fx+e\right)^{2}\right)\log\left(\frac{b\tan\left(fx+e\right)^{2}+a}{\tan\left(fx+e\right)^{2}+1}\right)}{4\left(\left(a^{7}b^{2}-3a^{6}b^{3}+3a^{5}b^{4}-a^{4}b^{5}\right)f\tan\left(fx+e\right)^{6}+2\left(a^{8}b-3a^{7}b^{2}+3a^{6}b^{3}-a^{5}b^{4}\right)f\tan\left(fx+e\right)^{4}+\left(a^{9}-3a^{8}b+3a^{7}b^{2}-a^{6}b^{3}\right)f\tan\left(fx+e\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*a^4*b^2 - 6*a^3*b^3 + 13*a^2*b^4 - 6*a*b^5)*tan(f*x + e)^6 + 2*a^6
 - 6*a^5*b + 6*a^4*b^2 - 2*a^3*b^3 + 2*(2*a^5*b - 5*a^4*b^2 + 7*a^3*b^3 + 2
*a^2*b^4 - 3*a*b^5)*tan(f*x + e)^4 + (2*a^6 - 2*a^5*b - 6*a^4*b^2 + 18*a^3*
b^3 - 9*a^2*b^4)*tan(f*x + e)^2 + 2*((a^4*b^2 - 6*a^2*b^4 + 8*a*b^5 - 3*b^6
)*tan(f*x + e)^6 + 2*(a^5*b - 6*a^3*b^3 + 8*a^2*b^4 - 3*a*b^5)*tan(f*x + e)
^4 + (a^6 - 6*a^4*b^2 + 8*a^3*b^3 - 3*a^2*b^4)*tan(f*x + e)^2)*log(tan(f*x
+ e)^2/(tan(f*x + e)^2 + 1)) + 2*((6*a^2*b^4 - 8*a*b^5 + 3*b^6)*tan(f*x + e
)^6 + 2*(6*a^3*b^3 - 8*a^2*b^4 + 3*a*b^5)*tan(f*x + e)^4 + (6*a^4*b^2 - 8*a
^3*b^3 + 3*a^2*b^4)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e
)^2 + 1)))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 +
2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8
*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 906 vs. 2(179) = 358.

time = 1.38, size = 906, normalized size = 5.01

$$-\frac{\frac{4\,(6\,a^2b^2-8\,ab^3+3\,b^4)\log\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^7-3\,a^6b+3\,a^5b^2-a^4b^3}-\frac{8\,\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{2\left(18\,a^4b^2-24\,a^3b^3+9\,a^2b^4+\frac{72\,a^4b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{208\,a^3b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{172\,a^2b^4(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{48\,ab^5(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{108\,a^4b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{368\,a^3b^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{502\,a^2b^4(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{288\,ab^5(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{64\,b^6(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{72\,a^4b^2(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{208\,a^3b^3(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{172\,a^2b^4(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{48\,ab^5(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{18\,a^4b^2(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}-\frac{24\,a^3b^3(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}+\frac{9\,a^2b^4(\cos(fx+e)-1)^4}{(\cos(fx+e)+1)^4}\right)}{(a^7-3\,a^6b+3\,a^5b^2-a^4b^3)\left(a+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)^2}+\frac{4\,(a+3\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4}-\frac{\left(a+\frac{4\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{12\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{a^4(\cos(fx+e)-1)}-\frac{\cos(fx+e)-1}{a^3(\cos(fx+e)+1)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*(6*a^2*b^2 - 8*a*b^3 + 3*b^4)*log(a + 2*a*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2)/(a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) - 8*log(
abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) - 2*(18*a^4*b^2 - 24*a^3*b^3 + 9*a^2*b^4 + 72*a^4*b^2*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) - 208*a^3*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
+ 172*a^2*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 48*a*b^5*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 108*a^4*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 - 368*a^3*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 502*a^2*b^
4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 288*a*b^5*(cos(f*x + e) - 1)^
2/(cos(f*x + e) + 1)^2 + 64*b^6*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 72*a^4*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 208*a^3*b^3*(cos(f*
x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 172*a^2*b^4*(cos(f*x + e) - 1)^3/(cos(
f*x + e) + 1)^3 - 48*a*b^5*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 18*a
^4*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 24*a^3*b^3*(cos(f*x + e)
 - 1)^4/(cos(f*x + e) + 1)^4 + 9*a^2*b^4*(cos(f*x + e) - 1)^4/(cos(f*x + e)
 + 1)^4)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*(a + 2*a*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) + 4*(a + 3*b)*log(abs(-cos(f*x + e)
+ 1)/abs(cos(f*x + e) + 1))/a^4 - (a + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) + 12*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/(a^4
*(cos(f*x + e) - 1)) - (cos(f*x + e) - 1)/(a^3*(cos(f*x + e) + 1)))/f
```

Mupad [B]

time = 13.46, size = 229, normalized size = 1.27

$$\frac{\ln\left({\mathrm{tan}\left(e+f\,x\right)}^2+1\right)}{2\,f\,{\left(a-b\right)}^3}-\frac{\frac{1}{2\,a}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^4\,\left(a^2\,b^2-5\,a\,b^3+3\,b^4\right)}{2\,a^3\,\left(a^2-2\,a\,b+b^2\right)}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(4\,a^2\,b-15\,a\,b^2+9\,b^3\right)}{4\,a^2\,\left(a^2-2\,a\,b+b^2\right)}}{f\,\left(a^2\,{\mathrm{tan}\left(e+f\,x\right)}^2+2\,a\,b\,{\mathrm{tan}\left(e+f\,x\right)}^4+b^2\,{\mathrm{tan}\left(e+f\,x\right)}^6\right)}-\frac{\ln\left(\mathrm{tan}\left(e+f\,x\right)\right)\,\left(a+3\,b\right)}{a^4\,f}-\frac{b^2\,\ln\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)\,\left(6\,a^2-8\,a\,b+3\,b^2\right)}{2\,a^4\,f\,{\left(a-b\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^3) - (1/(2*a) + (tan(e + f*x)^4*(3*b^4
 - 5*a*b^3 + a^2*b^2))/(2*a^3*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^2*(4*a^2
*b - 15*a*b^2 + 9*b^3))/(4*a^2*(a^2 - 2*a*b + b^2)))/(f*(a^2*tan(e + f*x)^2
 + b^2*tan(e + f*x)^6 + 2*a*b*tan(e + f*x)^4)) - (log(tan(e + f*x))*(a + 3*
b))/(a^4*f) - (b^2*log(a + b*tan(e + f*x)^2)*(6*a^2 - 8*a*b + 3*b^2))/(2*a^
4*f*(a - b)^3)
```

3.242 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=210

$$\frac{(a+3b)\cot^2(e+fx)}{2a^4f}-\frac{\cot^4(e+fx)}{4a^3f}+\frac{\log(\cos(e+fx))}{(a-b)^3f}+\frac{(a^2+3ab+6b^2)\log(\tan(e+fx))}{a^5f}+\frac{b^3(10a^2-15ab$$

```
[Out] 1/2*(a+3*b)*cot(f*x+e)^2/a^4/f-1/4*cot(f*x+e)^4/a^3/f+ln(cos(f*x+e))/(a-b)^
3/f+(a^2+3*a*b+6*b^2)*ln(tan(f*x+e))/a^5/f+1/2*b^3*(10*a^2-15*a*b+6*b^2)*ln
(a+b*tan(f*x+e)^2)/a^5/(a-b)^3/f-1/4*b^3/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1
/2*(4*a-3*b)*b^3/a^4/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.17, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 457, 90}

$$-\frac{b^3(4a-3b)}{2a^4f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{(a+3b)\cot^2(e+fx)}{2a^4f}-\frac{b^3}{4a^3f(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\cot^4(e+fx)}{4a^3f}+\frac{(a^2+3ab+6b^2)\log(\tan(e+fx))}{a^5f}+\frac{b^3(10a^2-15ab+6b^2)\log\left(a+b\tan^2(e+fx)\right)}{2a^5f(a-b)^3}+\frac{\log(\cos(e+fx))}{f(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((a + 3*b)*Cot[e + f*x]^2)/(2*a^4*f) - Cot[e + f*x]^4/(4*a^3*f) + Log[Cos[e
 + f*x]]/((a - b)^3*f) + ((a^2 + 3*a*b + 6*b^2)*Log[Tan[e + f*x]])/(a^5*f)
+ (b^3*(10*a^2 - 15*a*b + 6*b^2)*Log[a + b*Tan[e + f*x]^2])/(2*a^5*(a - b)^
3*f) - b^3/(4*a^3*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - ((4*a - 3*b)*b^3)/(
2*a^4*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
```

```
x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^3}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^3} + \frac{-a-3b}{a^4x^2} + \frac{a^2+3ab+6b^2}{a^5x} - \frac{1}{(a-b)^3(1+x)} + \frac{b^4}{a^3(a-b)(a+bx)^3} + \frac{(4a-3b)}{a^4(a-b)^2(a-}\right.\right.}{2f}$$

$$= \frac{(a+3b)\cot^2(e+fx)}{2a^4f} - \frac{\cot^4(e+fx)}{4a^3f} + \frac{\log(\cos(e+fx))}{(a-b)^3f} + \frac{(a^2+3ab+6b^2}{a}$$

Mathematica [A]
time = 1.76, size = 178, normalized size = 0.85

$$\frac{\frac{(a+3b)\cot^2(e+fx)}{a^4} - \frac{\cot^4(e+fx)}{2a^3} + \frac{2\log(\cos(e+fx))}{(a-b)^3} + \frac{4(a^2+3ab+6b^2)\log(\tan(e+fx)) + \frac{b^3\left(2\left(10a^2-15ab+6b^2\right)\log\left(a+b\tan^2(e+fx)\right) - \frac{a(a-b)\left(a(9a-7b)+2(4a-3b)b\tan^2(e+fx)\right)}{\left(a+b\tan^2(e+fx)\right)^2}\right)}{(a-b)^3}}{2a^5}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (((a + 3*b)*Cot[e + f*x]^2)/a^4 - Cot[e + f*x]^4/(2*a^3) + (2*Log[Cos[e + f
*x]])/(a - b)^3 + (4*(a^2 + 3*a*b + 6*b^2)*Log[Tan[e + f*x]] + (b^3*(2*(10*
a^2 - 15*a*b + 6*b^2)*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)*(a*(9*a - 7*b)
 + 2*(4*a - 3*b)*b*Tan[e + f*x]^2))/(a + b*Tan[e + f*x]^2)^2))/(a - b)^3)/(
2*a^5))/(2*f)
```

Maple [A]
time = 0.46, size = 272, normalized size = 1.30

method	result
derivativedivides	$-\frac{1}{16a^3(\cos(fx+e)+1)^2} - \frac{-7a-12b}{16a^4(\cos(fx+e)+1)} + \frac{\left(a^2+3ab+6b^2\right)\ln(\cos(fx+e)+1)}{2a^5} - \frac{1}{16a^3(\cos(fx+e)-1)^2} - \frac{7a+12b}{16a^4(\cos(fx+e)-1)} + \frac{(a}{}$

default	$\frac{-\frac{1}{16a^{3}(\cos(fx+e)+1)^{2}}-\frac{-7a-12b}{16a^{4}(\cos(fx+e)+1)}+\frac{\left(a^{2}+3ab+6b^{2}\right)\ln(\cos(fx+e)+1)}{2a^{5}}-\frac{1}{16a^{3}(\cos(fx+e)-1)^{2}}-\frac{7a+12b}{16a^{4}(\cos(fx+e)-1)}+\frac{(a^{2}\cdots}{}}{}$
norman	$\frac{-\frac{1}{4af}+\frac{(a+2b)\left(\tan^{2}(fx+e)\right)}{2a^{2}f}+\frac{\left(-4a^{3}b-3a^{2}b^{2}+27a\,b^{3}-18b^{4}\right)b^{2}\left(\tan^{8}(fx+e)\right)}{4a^{5}f\left(a^{2}-2ab+b^{2}\right)}+\frac{\left(-3a^{3}b-2a^{2}b^{2}+18a\,b^{3}-12b^{4}\right)b\left(\tan^{6}(fx+e)\right)}{2a^{4}f\left(a^{2}-2ab+b^{2}\right)}}{\tan(fx+e)^{4}(a+b(\tan^{2}(fx+e)))^{2}}+$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/16/a^3/(cos(f*x+e)+1)^2-1/16*(-7*a-12*b)/a^4/(cos(f*x+e)+1)+1/2*(a^
2+3*a*b+6*b^2)/a^5*ln(cos(f*x+e)+1)-1/16/a^3/(cos(f*x+e)-1)^2-1/16*(7*a+12*
b)/a^4/(cos(f*x+e)-1)+1/2*(a^2+3*a*b+6*b^2)/a^5*ln(cos(f*x+e)-1)+1/2*b^3/a^
5*((5*a-3*b)*a*b/(a-b)^3/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)+(10*a^2-15*a*b+6
*b^2)/(a-b)^3*ln(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)-1/2*a^2/(a-b)^3*b^2/(a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(207) = 414$.

time = 0.30, size = 424, normalized size = 2.02

$$\frac{\frac{2\left(10a^{2}b^{3}-15ab^{4}+6b^{5}\right)\log\left(-(a-b)\sin(fx+e)^{2}+a\right)}{a^{8}-3a^{7}b+3a^{6}b^{2}-a^{5}b^{3}}+\frac{2\left(2a^{6}-7a^{5}b+5a^{4}b^{2}+10a^{3}b^{3}-25a^{2}b^{4}+21ab^{5}-6b^{6}\right)\sin(fx+e)^{6}-a^{6}+3a^{5}b-3a^{4}b^{2}+a^{3}b^{3}-\left(9a^{6}-25a^{5}b+10a^{4}b^{2}+30a^{3}b^{3}-45a^{2}b^{4}+18ab^{5}\right)\sin(fx+e)^{4}+2\left(3a^{6}-7a^{5}b+3a^{4}b^{2}+3a^{3}b^{3}-2a^{2}b^{4}\right)\sin(fx+e)^{2}}{\left(a^{9}-5a^{8}b+10a^{7}b^{2}-10a^{6}b^{3}+5a^{5}b^{4}-a^{4}b^{5}\right)\sin(fx+e)^{8}-2\left(a^{9}-4a^{8}b+6a^{7}b^{2}-4a^{6}b^{3}+a^{5}b^{4}\right)\sin(fx+e)^{6}+\left(a^{9}-3a^{8}b+3a^{7}b^{2}-a^{6}b^{3}\right)\sin(fx+e)^{4}}+\frac{2\left(a^{2}+3ab+6b^{2}\right)\log\left(\sin(fx+e)^{2}\right)}{a^{5}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(10*a^2*b^3 - 15*a*b^4 + 6*b^5)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^
8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3) + (2*(2*a^6 - 7*a^5*b + 5*a^4*b^2 + 10*a
^3*b^3 - 25*a^2*b^4 + 21*a*b^5 - 6*b^6)*sin(f*x + e)^6 - a^6 + 3*a^5*b - 3*
a^4*b^2 + a^3*b^3 - (9*a^6 - 25*a^5*b + 10*a^4*b^2 + 30*a^3*b^3 - 45*a^2*b^
4 + 18*a*b^5)*sin(f*x + e)^4 + 2*(3*a^6 - 7*a^5*b + 3*a^4*b^2 + 3*a^3*b^3 -
 2*a^2*b^4)*sin(f*x + e)^2)/((a^9 - 5*a^8*b + 10*a^7*b^2 - 10*a^6*b^3 + 5*a
^5*b^4 - a^4*b^5)*sin(f*x + e)^8 - 2*(a^9 - 4*a^8*b + 6*a^7*b^2 - 4*a^6*b^3
 + a^5*b^4)*sin(f*x + e)^6 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*sin(f*x
+ e)^4) + 2*(a^2 + 3*a*b + 6*b^2)*log(sin(f*x + e)^2)/a^5)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(207) = 414$.

time = 4.44, size = 628, normalized size = 2.99

$$\frac{3\left(a^{5}b^{2}-a^{4}b^{3}-3a^{3}b^{4}+8a^{2}b^{5}-4ab^{6}\right)\tan(fx+e)^{8}-a^{7}+3a^{6}b-3a^{5}b^{2}+a^{4}b^{3}+2\left(3a^{6}b-2a^{5}b^{2}-9a^{4}b^{3}+14a^{3}b^{4}+3a^{2}b^{5}-6ab^{6}\right)\tan(fx+e)^{6}+\left(3a^{7}+a^{6}b-10a^{5}b^{2}-6a^{4}b^{3}+33a^{3}b^{4}-18a^{2}b^{5}\right)\tan(fx+e)^{4}+2\left(a^{7}-a^{6}b-3a^{5}b^{2}+5a^{4}b^{3}-2a^{3}b^{4}\right)\tan(fx+e)^{2}+2\left(\left(a^{5}b^{2}-10a^{2}b^{5}+15ab^{6}-6b^{7}\right)\tan(fx+e)^{8}+2\left(a^{6}b-10a^{3}b^{4}+15a^{2}b^{5}-6ab^{6}\right)\tan(fx+e)^{6}+\left(a^{7}-10a^{4}b^{3}+15a^{3}b^{4}-6a^{2}b^{5}\right)\tan(fx+e)^{4}\right)\log\left(\frac{\tan(fx+e)^{2}}{\tan(fx+e)^{2}+1}\right)+2\left(\left(10a^{2}b^{5}-15ab^{6}+6b^{7}\right)\tan(fx+e)^{8}+2\left(10a^{3}b^{4}-15a^{2}b^{5}+6ab^{6}\right)\tan(fx+e)^{6}+\left(10a^{4}b^{3}-15a^{3}b^{4}+6a^{2}b^{5}\right)\tan(fx+e)^{4}\right)\log\left(\frac{b\tan(fx+e)^{2}+a}{\tan(fx+e)^{2}+1}\right)}{4\left(\left(a^{8}b^{2}-3a^{7}b^{3}+3a^{6}b^{4}-a^{5}b^{5}\right)f\tan(fx+e)^{8}+2\left(a^{9}b-3a^{8}b^{2}+3a^{7}b^{3}-a^{6}b^{4}\right)f\tan(fx+e)^{6}+\left(a^{10}-3a^{9}b+3a^{8}b^{2}-a^{7}b^{3}\right)f\tan(fx+e)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(a^5*b^2 - a^4*b^3 - 3*a^3*b^4 + 8*a^2*b^5 - 4*a*b^6)*tan(f*x + e)^8
 - a^7 + 3*a^6*b - 3*a^5*b^2 + a^4*b^3 + 2*(3*a^6*b - 2*a^5*b^2 - 9*a^4*b^3
 + 14*a^3*b^4 + 3*a^2*b^5 - 6*a*b^6)*tan(f*x + e)^6 + (3*a^7 + a^6*b - 10*a
^5*b^2 - 6*a^4*b^3 + 33*a^3*b^4 - 18*a^2*b^5)*tan(f*x + e)^4 + 2*(a^7 - a^6
*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*tan(f*x + e)^2 + 2*((a^5*b^2 - 10*a
^2*b^5 + 15*a*b^6 - 6*b^7)*tan(f*x + e)^8 + 2*(a^6*b - 10*a^3*b^4 + 15*a^2*
b^5 - 6*a*b^6)*tan(f*x + e)^6 + (a^7 - 10*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5)
*tan(f*x + e)^4)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + 2*((10*a^2*b^5
- 15*a*b^6 + 6*b^7)*tan(f*x + e)^8 + 2*(10*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6)*
tan(f*x + e)^6 + (10*a^4*b^3 - 15*a^3*b^4 + 6*a^2*b^5)*tan(f*x + e)^4)*log(
(b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6
*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b
^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e
)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1497 vs. $2(207) = 414$.
time = 1.36, size = 1497, normalized size = 7.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(32*(10*a^2*b^3 - 15*a*b^4 + 6*b^5)*log(a + 2*a*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x +
e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3) - 64*
log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3 + 16*a^7*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 32*a^6*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 32*
a^4*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a^3*b^4*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 70*a^7*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 -
178*a^6*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 34*a^5*b^2*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + 586*a^4*b^3*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 - 752*a^3*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 272*
a^2*b^5*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 140*a^7*(cos(f*x + e) -
```

```
1)^3/(cos(f*x + e) + 1)^3 - 412*a^6*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
 1)^3 + 204*a^5*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 1356*a^4*b^
3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 3272*a^3*b^4*(cos(f*x + e) -
1)^3/(cos(f*x + e) + 1)^3 + 2496*a^2*b^5*(cos(f*x + e) - 1)^3/(cos(f*x + e)
 + 1)^3 - 640*a*b^6*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 145*a^7*(co
s(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 403*a^6*b*(cos(f*x + e) - 1)^4/(co
s(f*x + e) + 1)^4 + 211*a^5*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 +
 1487*a^4*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 3296*a^3*b^4*(cos
(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 2560*a^2*b^5*(cos(f*x + e) - 1)^4/(
cos(f*x + e) + 1)^4 - 256*a*b^6*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 -
 256*b^7*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 76*a^7*(cos(f*x + e) -
 1)^5/(cos(f*x + e) + 1)^5 - 140*a^6*b*(cos(f*x + e) - 1)^5/(cos(f*x + e) +
 1)^5 - 36*a^5*b^2*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 700*a^4*b^3*
(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 1624*a^3*b^4*(cos(f*x + e) - 1)
^5/(cos(f*x + e) + 1)^5 + 1152*a^2*b^5*(cos(f*x + e) - 1)^5/(cos(f*x + e) +
 1)^5 - 256*a*b^6*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 16*a^7*(cos(f
*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 160*a^4*b^3*(cos(f*x + e) - 1)^6/(cos
(f*x + e) + 1)^6 - 240*a^3*b^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 +
96*a^2*b^5*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((a^8 - 3*a^7*b + 3*a
^6*b^2 - a^5*b^3)*(a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 - 4*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)^2) + 32*(a^2 + 3*a*b +
6*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^5 - (12*a^3*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 24*a^2*b*(cos(f*x + e) - 1)/(cos(f*x +
e) + 1) + a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^6)/f
```

Mupad [B]
time = 13.51, size = 269, normalized size = 1.28

$$\frac{\frac{\tan(e+f\,x)^2\,(a+2\,b)}{2\,a^2}-\frac{1}{4\,a}+\frac{\tan(e+f\,x)^6\,(a^3\,b^2+a^2\,b^3-9\,a\,b^4+6\,b^5)}{2\,a^4\,(a^2-2\,a\,b+b^2)}+\frac{\tan(e+f\,x)^4\,(4\,a^3\,b+3\,a^2\,b^2-27\,a\,b^3+18\,b^4)}{4\,a^3\,(a^2-2\,a\,b+b^2)}}{f\,\left(a^2\tan\left(e+f\,x\right)^4+2\,a\,b\tan\left(e+f\,x\right)^6+b^2\tan\left(e+f\,x\right)^8\right)}-\frac{\ln\left(b\tan(e+f\,x)^2+a\right)\,\left(\frac{3\,b}{2\,a^4}+\frac{1}{2\,a^3}-\frac{1}{2\,(a-b)^3}+\frac{3\,b^2}{a^5}\right)}{f}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^3}+\frac{\ln\left(\tan(e+f\,x)\right)\,(a^2+3\,a\,b+6\,b^2)}{a^5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^2*(a + 2*b))/(2*a^2) - 1/(4*a) + (tan(e + f*x)^6*(6*b^5 - 9*
a*b^4 + a^2*b^3 + a^3*b^2))/(2*a^4*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^4*(
4*a^3*b - 27*a*b^3 + 18*b^4 + 3*a^2*b^2))/(4*a^3*(a^2 - 2*a*b + b^2)))/(f*(
a^2*tan(e + f*x)^4 + b^2*tan(e + f*x)^8 + 2*a*b*tan(e + f*x)^6)) - (log(a +
 b*tan(e + f*x)^2)*((3*b)/(2*a^4) + 1/(2*a^3) - 1/(2*(a - b)^3) + (3*b^2)/a
^5))/f - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^3) + (log(tan(e + f*x))*(3*a*
b + a^2 + 6*b^2))/(a^5*f)
```

3.243 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{x}{(a-b)^3}+\frac{\sqrt{a}\,(3a^2-10ab+15b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8(a-b)^3b^{5/2}f}-\frac{a\tan^3(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2}-\frac{a(3a}{8(a-b)^2b}$$

```
[Out] -x/(a-b)^3+1/8*(3*a^2-10*a*b+15*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*a^(
1/2)/(a-b)^3/b^(5/2)/f-1/4*a*tan(f*x+e)^3/(a-b)/b/f/(a+b*tan(f*x+e)^2)^2-1/
8*a*(3*a-7*b)*tan(f*x+e)/(a-b)^2/b^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]

time = 0.16, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 481, 592, 536, 209, 211}

$$\frac{\sqrt{a}\,(3a^2-10ab+15b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8b^{5/2}f(a-b)^3}-\frac{a(3a-7b)\tan(e+fx)}{8b^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan^3(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + (Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a]])/(8*(a - b)^3*b^(5/2)*f) - (a*Tan[e + f*x]^3)/(4*(a - b)*b*
f*(a + b*Tan[e + f*x]^2)^2) - (a*(3*a - 7*b)*Tan[e + f*x])/(8*(a - b)^2*b^2
*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{4(a-b)bf\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(3a-4b)x^2)}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+\right.}{4(a-b)bf}$$

$$= -\frac{a\tan^3(e+fx)}{4(a-b)bf\,(a+b\tan^2(e+fx))^2} - \frac{a(3a-7b)\tan(e+fx)}{8(a-b)^2b^2f\,(a+b\tan^2(e+fx))} - \text{Su}$$

$$= -\frac{a\tan^3(e+fx)}{4(a-b)bf\,(a+b\tan^2(e+fx))^2} - \frac{a(3a-7b)\tan(e+fx)}{8(a-b)^2b^2f\,(a+b\tan^2(e+fx))} - \text{Su}$$

$$= -\frac{x}{(a-b)^3} + \frac{\sqrt{a}\,(3a^2-10ab+15b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8(a-b)^3b^{5/2}f} - \frac{a\,\tan}{4(a-b)bf\,(a}$$

Mathematica [A]

time = 1.59, size = 142, normalized size = 0.93

$$\frac{-8(e+fx) + \frac{\sqrt{a}\,(3a^2-10ab+15b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}} - \frac{a(a-b)(3a^2-2ab-9b^2+3(a^2-4ab+3b^2)\cos(2(e+fx)))\sin(2(e+fx))}{b^2(a+b+(a-b)\cos(2(e+fx)))^2}}{8(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-8*(e + f*x) + (Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a]])/b^(5/2) - (a*(a - b)*(3*a^2 - 2*a*b - 9*b^2 + 3*(a^2 - 4*a*
b + 3*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b^2*(a + b + (a - b)*Cos[2*
(e + f*x)])^2))/(8*(a - b)^3*f)
```

Maple [A]

time = 0.35, size = 142, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3} + \frac{a\left(\frac{-\frac{(5a^2-14ab+9b^2)(\tan^3(fx+e))}{8b} - \frac{a(3a^2-10ab+7b^2)\tan(fx+e)}{8b^2}}{(a+b(\tan^2(fx+e)))^2} + \frac{(3a^2-10ab+15b^2)\arctan\left(\frac{b\tan(fx}{\sqrt{ab}}\right.}{8b^2\sqrt{ab}}\right.}{(a-b)^3}}{f}$

default	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{a\left(\frac{-\frac{(5a^2-14ab+9b^2)(\tan^3(fx+e))}{8b}-\frac{a(3a^2-10ab+7b^2)\tan(fx+e)}{8b^2}}{(a+b(\tan^2(fx+e)))^2}+\frac{(3a^2-10ab+15b^2)\arctan\left(\frac{b\tan(fx+e}{\sqrt{ab}}\right.}{8b^2\sqrt{ab}}\right.}{(a-b)^3}}{f}$
risch	$-\frac{x}{a^3-3a^2b+3a\,b^2-b^3}-\frac{i(3a^3e^{6i(fx+e)}-13a^2b\,e^{6i(fx+e)}+a\,b^2e^{6i(fx+e)}+9b^3e^{6i(fx+e)}+9a^3e^{4i(fx+e)}-21a^2b\,e^{4i(fx+e)}}{4(a\,e^{4i(fx+e)}-b\,e^{4i(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a-b)^3*arctan(tan(f*x+e))+a/(a-b)^3*((-1/8*(5*a^2-14*a*b+9*b^2)/b*
tan(f*x+e)^3-1/8*a*(3*a^2-10*a*b+7*b^2)/b^2*tan(f*x+e))/(a+b*tan(f*x+e)^2)^
2+1/8*(3*a^2-10*a*b+15*b^2)/b^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)
)))
```

Maxima [A]
time = 0.50, size = 235, normalized size = 1.54

$$\frac{\frac{(3a^3-10a^2b+15ab^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3b^2-3a^2b^3+3ab^4-b^5)\sqrt{ab}}-\frac{(5a^2b-9ab^2)\tan(fx+e)^3+(3a^3-7a^2b)\tan(fx+e)}{a^4b^2-2a^3b^3+a^2b^4+(a^2b^4-2ab^5+b^6)\tan(fx+e)^4+2(a^3b^3-2a^2b^4+ab^5)\tan(fx+e)^2}-\frac{8(fx+e)}{a^3-3a^2b+3ab^2-b^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^3 - 10*a^2*b + 15*a*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*b
^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sqrt(a*b)) - ((5*a^2*b - 9*a*b^2)*tan(f*x +
 e)^3 + (3*a^3 - 7*a^2*b)*tan(f*x + e))/(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 + (a
^2*b^4 - 2*a*b^5 + b^6)*tan(f*x + e)^4 + 2*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*ta
n(f*x + e)^2) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(144) = 288$.
time = 3.72, size = 767, normalized size = 5.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*b^4*f*x*tan(f*x + e)^4 + 64*a*b^3*f*x*tan(f*x + e)^2 + 32*a^2*b^
2*f*x + 4*(5*a^3*b - 14*a^2*b^2 + 9*a*b^3)*tan(f*x + e)^3 + ((3*a^2*b^2 - 1
0*a*b^3 + 15*b^4)*tan(f*x + e)^4 + 3*a^4 - 10*a^3*b + 15*a^2*b^2 + 2*(3*a^3
```

```
*b - 10*a^2*b^2 + 15*a*b^3)*tan(f*x + e)^2)*sqrt(-a/b)*log((b^2*tan(f*x + e
)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e)
)*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) + 4*(3*a^4
 - 10*a^3*b + 7*a^2*b^2)*tan(f*x + e))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^
7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x
 + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f), -1/16*(16*b^4*f*x
*tan(f*x + e)^4 + 32*a*b^3*f*x*tan(f*x + e)^2 + 16*a^2*b^2*f*x + 2*(5*a^3*b
 - 14*a^2*b^2 + 9*a*b^3)*tan(f*x + e)^3 - ((3*a^2*b^2 - 10*a*b^3 + 15*b^4)*
tan(f*x + e)^4 + 3*a^4 - 10*a^3*b + 15*a^2*b^2 + 2*(3*a^3*b - 10*a^2*b^2 +
15*a*b^3)*tan(f*x + e)^2)*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(
a/b)/(a*tan(f*x + e))) + 2*(3*a^4 - 10*a^3*b + 7*a^2*b^2)*tan(f*x + e))/((a
^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b
^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
 - a^2*b^5)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8974 vs. $2(133) = 266$.
time = 77.53, size = 8974, normalized size = 58.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e + f*x)**5/(
5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a**3, Eq(b, 0)), (x/b**3, Eq
(a, 0)), (15*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*ta
n(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*
x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*
tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*x)**2/(48*b**3*f*tan(e + f*
x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f
) + 15*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b*
*3*f*tan(e + f*x)**2 + 48*b**3*f) - 33*tan(e + f*x)**5/(48*b**3*f*tan(e + f
*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*
f) - 40*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x
)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) - 15*tan(e + f*x)/(48*b**3*f
*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2
+ 48*b**3*f), Eq(a, b)), (x*tan(e)**6/(a + b*tan(e)**2)**3, Eq(f, 0)), (3*a
**5*log(-sqrt(-a/b) + tan(e + f*x))/(16*a**5*b**3*f*sqrt(-a/b) + 32*a**4*b*
*4*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**4*f*sqrt(-a/b) + 16*a**3*b**5*
f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 +
48*a**3*b**5*f*sqrt(-a/b) - 48*a**2*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*
a**2*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**6*f*sqrt(-a/b) + 48*a*b
**7*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**2 -
 16*b**8*f*sqrt(-a/b)*tan(e + f*x)**4) - 3*a**5*log(sqrt(-a/b) + tan(e + f*
```

```
x))/(16*a**5*b**3*f*sqrt(-a/b) + 32*a**4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2
- 48*a**4*b**4*f*sqrt(-a/b) + 16*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 9
6*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**3*b**5*f*sqrt(-a/b) - 48*a
**2*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**2*b**6*f*sqrt(-a/b)*tan(e + f
*x)**2 - 16*a**2*b**6*f*sqrt(-a/b) + 48*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**4
 - 32*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*b**8*f*sqrt(-a/b)*tan(e + f*
x)**4) - 6*a**4*b*sqrt(-a/b)*tan(e + f*x)/(16*a**5*b**3*f*sqrt(-a/b) + 32*a
**4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**4*f*sqrt(-a/b) + 16*a**3
*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)
**2 + 48*a**3*b**5*f*sqrt(-a/b) - 48*a**2*b**6*f*sqrt(-a/b)*tan(e + f*x)**4
 + 96*a**2*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**6*f*sqrt(-a/b) +
48*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**7*f*sqrt(-a/b)*tan(e + f*x
)**2 - 16*b**8*f*sqrt(-a/b)*tan(e + f*x)**4) + 6*a**4*b*log(-sqrt(-a/b) + t
an(e + f*x))*tan(e + f*x)**2/(16*a**5*b**3*f*sqrt(-a/b) + 32*a**4*b**4*f*sq
rt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**4*f*sqrt(-a/b) + 16*a**3*b**5*f*sqrt(
-a/b)*tan(e + f*x)**4 - 96*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**3
*b**5*f*sqrt(-a/b) - 48*a**2*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**2*b*
*6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**6*f*sqrt(-a/b) + 48*a*b**7*f*s
qrt(-a/b)*tan(e + f*x)**4 - 32*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*b**
8*f*sqrt(-a/b)*tan(e + f*x)**4) - 10*a**4*b*log(-sqrt(-a/b) + tan(e + f*x))
/(16*a**5*b**3*f*sqrt(-a/b) + 32*a**4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 4
8*a**4*b**4*f*sqrt(-a/b) + 16*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a
**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**3*b**5*f*sqrt(-a/b) - 48*a**2
*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**2*b**6*f*sqrt(-a/b)*tan(e + f*x)
**2 - 16*a**2*b**6*f*sqrt(-a/b) + 48*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**4 -
32*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*b**8*f*sqrt(-a/b)*tan(e + f*x)*
*4) - 6*a**4*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(16*a**5*b**3
*f*sqrt(-a/b) + 32*a**4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**4*f*
sqrt(-a/b) + 16*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**3*b**5*f*sqr
t(-a/b)*tan(e + f*x)**2 + 48*a**3*b**5*f*sqrt(-a/b) - 48*a**2*b**6*f*sqrt(-
a/b)*tan(e + f*x)**4 + 96*a**2*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*
b**6*f*sqrt(-a/b) + 48*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**7*f*sq
rt(-a/b)*tan(e + f*x)**2 - 16*b**8*f*sqrt(-a/b)*tan(e + f*x)**4) + 10*a**4*
b*log(sqrt(-a/b) + tan(e + f*x))/(16*a**5*b**3*f*sqrt(-a/b) + 32*a**4*b**4*
f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**4*f*sqrt(-a/b) + 16*a**3*b**5*f*s
qrt(-a/b)*tan(e + f*x)**4 - 96*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*
a**3*b**5*f*sqrt(-a/b) - 48*a**2*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**
2*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**6*f*sqrt(-a/b) + 48*a*b**7
*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**2 - 16
*b**8*f*sqrt(-a/b)*tan(e + f*x)**4) - 10*a**3*b**2*sqrt(-a/b)*tan(e + f*x)*
*3/(16*a**5*b**3*f*sqrt(-a/b) + 32*a**4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 -
 48*a**4*b**4*f*sqrt(-a/b) + 16*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 96
*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**3*b**5*f*sqrt(-a/b) - 48*a*
*2*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**2*b**6*f*sqrt(-a/b)*tan(e + f*
x)**2 - 16*a**2*b**6*f*sqrt(-a/b) + 48*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**4
```

```
- 32*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*b...
```

Giac [A]
time = 2.50, size = 215, normalized size = 1.41

$$\frac{\frac{(3a^3-10a^2b+15ab^2)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^3b^2-3a^2b^3+3ab^4-b^5)\sqrt{ab}} - \frac{8(fx+e)}{a^3-3a^2b+3ab^2-b^3} - \frac{5a^2b\tan(fx+e)^3-9ab^2\tan(fx+e)^3+3a^3\tan(fx+e)-7a^2b\tan(fx+e)}{(a^2b^2-2ab^3+b^4)\left(b\tan(fx+e)^2+a\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^3 - 10*a^2*b + 15*a*b^2)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + a
rctan(b*tan(f*x + e)/sqrt(a*b)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sqr
t(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (5*a^2*b*tan(f*x +
e)^3 - 9*a*b^2*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) - 7*a^2*b*tan(f*x + e))/
((a^2*b^2 - 2*a*b^3 + b^4)*(b*tan(f*x + e)^2 + a)^2))/f
```

Mupad [B]
time = 15.52, size = 2500, normalized size = 16.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^3*(9*a*b - 5*a^2))/(8*(a^2*b - 2*a*b^2 + b^3)) + (a*tan(e +
f*x)*(7*a*b - 3*a^2))/(8*b*(a^2*b - 2*a*b^2 + b^3)))/(f*(a^2 + b^2*tan(e +
f*x)^4 + 2*a*b*tan(e + f*x)^2)) - (2*atan((((((224*a*b^10 - 1440*a^2*b^9 +
3936*a^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 9
6*a^8*b^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^
5*b^4 + a^6*b^3)) - (tan(e + f*x)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10 +
 1280*a^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5)*1
i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a
^3*b^4 + a^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)
*(9*a^6 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32
*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))/(6*a*b^2 - 6*a^2*b + 2
*a^3 - 2*b^3) - ((((224*a*b^10 - 1440*a^2*b^9 + 3936*a^3*b^8 - 5920*a^4*b^7
 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*b^3)/(64*(b^9 - 6*a*b
^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) + (tan(e
+ f*x)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10 + 1280*a^3*b^9 + 1280*a^4*b^
8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5)*1i)/(32*(6*a*b^2 - 6*a^2*b +
 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*1i)/(6*
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6
 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7 - 4*a*b^6 + 6*a^2*b^5
 - 4*a^3*b^4 + a^4*b^3)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((120*a*b^4
```

- 51*a^4*b + 9*a^5 - 185*a^2*b^3 + 139*a^3*b^2)/(32*(b^9 - 6*a*b^8 + 15*a^2
*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) + (((((224*a*b^10 -
1440*a^2*b^9 + 3936*a^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 +
800*a^7*b^4 - 96*a^8*b^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15
*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) - (tan(e + f*x)*(1280*a*b^11 - 256*b^12 -
2304*a^2*b^10 + 1280*a^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 -
256*a^7*b^5)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 +
6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)
+ (tan(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 19
0*a^4*b^2))/(32*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*1i)/(6*
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (((((224*a*b^10 - 1440*a^2*b^9 + 3936*a^
3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*b
^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 +
a^6*b^3)) + (tan(e + f*x)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10 + 1280*a
^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5)*1i)/(32*
(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4
+ a^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^6
- 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7 -
4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^
3 - 2*b^3))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)) - (atan((((-a*b^5)^(1
/2)*((tan(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 +
190*a^4*b^2))/(32*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) + ((-
a*b^5)^(1/2)*((224*a*b^10 - 1440*a^2*b^9 + 3936*a^3*b^8 - 5920*a^4*b^7 + 52
80*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*b^3)/(64*(b^9 - 6*a*b^8 +
15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) - (tan(e + f*x
)*(-a*b^5)^(1/2)*(3*a^2 - 10*a*b + 15*b^2)*(1280*a*b^11 - 256*b^12 - 2304*a
^2*b^10 + 1280*a^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a
^7*b^5))/(512*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5)*(b^7 - 4*a*b^6 + 6*a^2*
b^5 - 4*a^3*b^4 + a^4*b^3)))*(3*a^2 - 10*a*b + 15*b^2))/(16*(3*a*b^7 - b^8
- 3*a^2*b^6 + a^3*b^5)))*(3*a^2 - 10*a*b + 15*b^2)*1i)/(16*(3*a*b^7 - b^8 -
3*a^2*b^6 + a^3*b^5)) + ((-a*b^5)^(1/2)*((tan(e + f*x)*(9*a^6 - 60*a^5*b +
64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7 - 4*a*b^6 + 6*
a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) - ((-a*b^5)^(1/2)*((224*a*b^10 - 1440*a^2*b
^9 + 3936*a^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^
4 - 96*a^8*b^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 -
6*a^5*b^4 + a^6*b^3)) + (tan(e + f*x)*(-a*b^5)^(1/2)*(3*a^2 - 10*a*b + 15*
b^2)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10 + 1280*a^3*b^9 + 1280*a^4*b^8
- 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5))/(512*(3*a*b^7 - b^8 - 3*a^2*b
^6 + a^3*b^5)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))*(3*a^2 -
10*a*b + 15*b^2))/(16*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5)))*(3*a^2 - 10*a
*b + 15*b^2)*1i)/(16*(3*a*b^7 - b^8 - 3*a^2*b^6 + a^3*b^5)))/((120*a*b^4 -
51*a^4*b + 9*a^5 - 185*a^2*b^3 + 139*a^3*b^2)/(32*(b^9 - 6*a*b^8 + 15*a^2*b
^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b^3)) + ((-a*b^5)^(1/2)*((ta
n(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4
b^2))/(32(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) + ((-a*b^5)^(

```
1/2)*((224*a*b^10 - 1440*a^2*b^9 + 3936*a^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b
^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*b^3)/(...
```

3.244 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=145

$$\frac{x}{(a-b)^3}+\frac{(a^2-6ab-3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a-b)^3b^{3/2}f}-\frac{a\tan(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2}+\frac{(a-5b)\tan(e}{8(a-b)^2bf\left(a+b\tan\right.}$$

```
[Out] x/(a-b)^3+1/8*(a^2-6*a*b-3*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)^3/
b^(3/2)/f/a^(1/2)-1/4*a*tan(f*x+e)/(a-b)/b/f/(a+b*tan(f*x+e)^2)^2+1/8*(a-5*
b)*tan(f*x+e)/(a-b)^2/b/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 481, 541, 536, 209, 211}

$$\frac{(a^2-6ab-3b^2)\operatorname{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{3/2}f(a-b)^3}+\frac{(a-5b)\tan(e+fx)}{8bf(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{a\tan(e+fx)}{4bf(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] x/(a - b)^3 + ((a^2 - 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*Sqrt[a]*(a - b)^3*b^(3/2)*f) - (a*Tan[e + f*x])/(4*(a - b)*b*f*(a + b*
Tan[e + f*x]^2)^2) + ((a - 5*b)*Tan[e + f*x])/(8*(a - b)^2*b*f*(a + b*Tan[e
 + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
```

```
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{4(a-b)bf\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{a+(a-4b)x^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{4(a-b)bf\,(a+b\tan^2(e+fx))^2} + \frac{(a-5b)\tan(e+fx)}{8(a-b)^2bf\,(a+b\tan^2(e+fx))} + \text{Subs}$$

$$= -\frac{a\tan(e+fx)}{4(a-b)bf\,(a+b\tan^2(e+fx))^2} + \frac{(a-5b)\tan(e+fx)}{8(a-b)^2bf\,(a+b\tan^2(e+fx))} + \text{Subs}$$

$$= \frac{x}{(a-b)^3} + \frac{(a^2-6ab-3b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a-b)^3b^{3/2}f} - \frac{a\tan(e+fx)}{4(a-b)bf\,(a+b\tan^2(e$$

Mathematica [A]
time = 1.39, size = 136, normalized size = 0.94

$$\frac{8(e+fx) + \frac{(a^2-6ab-3b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} - \frac{(a-b)(a^2+2ab+5b^2+(a^2+4ab-5b^2)\cos(2(e+fx)))\sin(2(e+fx))}{b(a+b+(a-b)\cos(2(e+fx)))^2}}{8(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (8*(e + f*x) + ((a^2 - 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(Sqrt[a]*b^(3/2)) - ((a - b)*(a^2 + 2*a*b + 5*b^2 + (a^2 + 4*a*b - 5*b^2
)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b*(a + b + (a - b)*Cos[2*(e + f*x)])
^2))/(8*(a - b)^3*f)
```

Maple [A]
time = 0.34, size = 132, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{\frac{\left(\frac{1}{8}a^2-\frac{3}{4}ab+\frac{5}{8}b^2\right)\left(\tan^3(fx+e)\right)-\frac{a\left(a^2+2ab-3b^2\right)\tan(fx+e)}{8b}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(a^2-6ab-3b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{(a-b)^3}}{f}$

default	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{\frac{\left(\frac{1}{8}a^2-\frac{3}{4}ab+\frac{5}{8}b^2\right)\left(\tan^3(fx+e)\right)-\frac{a\left(a^2+2ab-3b^2\right)\tan(fx+e)}{8b}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(a^2-6ab-3b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{(a-b)^3}}{f}$
risch	$\frac{x}{a^3-3a^2b+3a\,b^2-b^3}-\frac{i\left(a^3\mathrm{e}^{6i(fx+e)}+9a^2b\,\mathrm{e}^{6i(fx+e)}-5a\,b^2\mathrm{e}^{6i(fx+e)}-5b^3\mathrm{e}^{6i(fx+e)}+3a^3\mathrm{e}^{4i(fx+e)}+17a^2b\,\mathrm{e}^{4i(fx+e)}+\right.}{4\left(a\,\mathrm{e}^{4i(fx+e)}-b\,\mathrm{e}^{4i(fx+e)}+2a\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^3*arctan(tan(f*x+e))+1/(a-b)^3*(((1/8*a^2-3/4*a*b+5/8*b^2)*tan
(f*x+e)^3-1/8*a*(a^2+2*a*b-3*b^2)/b*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(a
^2-6*a*b-3*b^2)/b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.52, size = 218, normalized size = 1.50

$$\frac{\frac{\left(a^2-6\,ab-3\,b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\left(a^3b-3\,a^2b^2+3\,ab^3-b^4\right)\sqrt{ab}}+\frac{\left(ab-5\,b^2\right)\tan(fx+e)^3-\left(a^2+3\,ab\right)\tan(fx+e)}{a^4b-2\,a^3b^2+a^2b^3+\left(a^2b^3-2\,ab^4+b^5\right)\tan(fx+e)^4+2\left(a^3b^2-2\,a^2b^3+ab^4\right)\tan(fx+e)^2}+\frac{8\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a^2 - 6*a*b - 3*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*b - 3*a^2
*b^2 + 3*a*b^3 - b^4)*sqrt(a*b)) + ((a*b - 5*b^2)*tan(f*x + e)^3 - (a^2 + 3
*a*b)*tan(f*x + e))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^2*b^3 - 2*a*b^4 + b^5
)*tan(f*x + e)^4 + 2*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*tan(f*x + e)^2) + 8*(f*x
 + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(136) = 272.
time = 4.03, size = 773, normalized size = 5.33

$$\left[\frac{32\,ab^4fx\tan(fx+e)^4+64\,a^2b^3fx\tan(fx+e)^2+32\,a^3b^2fx+4\left(a^3b^2-6\,a^2b^3+5\,ab^4\right)\tan(fx+e)^3-\left(\left(a^2b^2-6\,ab^3-3\,b^4\right)\tan(fx+e)^4+a^4-6\,a^3b-3\,a^2b^2+2\left(a^3b-6\,a^2b^2-3\,ab^3\right)\tan(fx+e)^2\right)\sqrt{-ab}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2-4\left(b\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{-ab}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)-4\left(a^4b+2\,a^3b^2-3\,a^2b^3\right)\tan(fx+e)}{32\left(\left(a^4b^4-3\,a^3b^5+3\,a^2b^6-ab^7\right)f\tan(fx+e)^4+2\left(a^5b^3-3\,a^4b^4+3\,a^3b^5-a^2b^6\right)f\tan(fx+e)^2+\left(a^6b^2-3\,a^5b^3+3\,a^4b^4-a^3b^5\right)f\right)},\ \frac{16\,ab^4fx\tan(fx+e)^4+32\,a^2b^3fx\tan(fx+e)^2+16\,a^3b^2fx+2\left(a^3b^2-6\,a^2b^3+5\,ab^4\right)\tan(fx+e)^3+\left(\left(a^2b^2-6\,ab^3-3\,b^4\right)\tan(fx+e)^4+a^4-6\,a^3b-3\,a^2b^2+2\left(a^3b-6\,a^2b^2-3\,ab^3\right)\tan(fx+e)^2\right)\sqrt{ab}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{ab}}{2\,ab\tan(fx+e)}\right)-2\left(a^4b+2\,a^3b^2-3\,a^2b^3\right)\tan(fx+e)}{16\left(\left(a^4b^4-3\,a^3b^5+3\,a^2b^6-ab^7\right)f\tan(fx+e)^4+2\left(a^5b^3-3\,a^4b^4+3\,a^3b^5-a^2b^6\right)f\tan(fx+e)^2+\left(a^6b^2-3\,a^5b^3+3\,a^4b^4-a^3b^5\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a*b^4*f*x*tan(f*x + e)^4 + 64*a^2*b^3*f*x*tan(f*x + e)^2 + 32*a^3
*b^2*f*x + 4*(a^3*b^2 - 6*a^2*b^3 + 5*a*b^4)*tan(f*x + e)^3 - ((a^2*b^2 - 6
*a*b^3 - 3*b^4)*tan(f*x + e)^4 + a^4 - 6*a^3*b - 3*a^2*b^2 + 2*(a^3*b - 6*a
^2*b^2 - 3*a*b^3)*tan(f*x + e)^2)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a*
```

```
b*tan(f*x + e)^2 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))/
(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^4*b + 2*a^3*b^2 -
 3*a^2*b^3)*tan(f*x + e))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(
f*x + e)^4 + 2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*tan(f*x + e)^2
 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f), 1/16*(16*a*b^4*f*x*tan(f
*x + e)^4 + 32*a^2*b^3*f*x*tan(f*x + e)^2 + 16*a^3*b^2*f*x + 2*(a^3*b^2 - 6
*a^2*b^3 + 5*a*b^4)*tan(f*x + e)^3 + ((a^2*b^2 - 6*a*b^3 - 3*b^4)*tan(f*x +
 e)^4 + a^4 - 6*a^3*b - 3*a^2*b^2 + 2*(a^3*b - 6*a^2*b^2 - 3*a*b^3)*tan(f*x
 + e)^2)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)/(a*b*tan(f*x
 + e))) - 2*(a^4*b + 2*a^3*b^2 - 3*a^2*b^3)*tan(f*x + e))/((a^4*b^4 - 3*a^3
*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^4 + 2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3
*b^5 - a^2*b^6)*f*tan(f*x + e)^2 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b
^5)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8957 vs. $2(126) = 252$.
time = 75.29, size = 8957, normalized size = 61.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((x + tan(e +
f*x)**3/(3*f) - tan(e + f*x)/f)/a**3, Eq(b, 0)), ((-x - 1/(f*tan(e + f*x)))
/b**3, Eq(a, 0)), (3*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)**6 + 144*b
**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 9*f*x*tan
(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*
b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 9*f*x*tan(e + f*x)**2/(48*b**3*f*tan(
e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*
b**3*f) + 3*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 1
44*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 3*tan(e + f*x)**5/(48*b**3*f*tan(e
 + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b
**3*f) - 8*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e +
f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) - 3*tan(e + f*x)/(48*b**3
*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**
2 + 48*b**3*f), Eq(a, b)), (x*tan(e)**4/(a + b*tan(e)**2)**3, Eq(f, 0)), (a
**4*log(-sqrt(-a/b) + tan(e + f*x))/(16*a**5*b**2*f*sqrt(-a/b) + 32*a**4*b*
*3*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**3*f*sqrt(-a/b) + 16*a**3*b**4*
f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 +
48*a**3*b**4*f*sqrt(-a/b) - 48*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*
a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b) + 48*a*b
**6*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 -
 16*b**7*f*sqrt(-a/b)*tan(e + f*x)**4) - a**4*log(sqrt(-a/b) + tan(e + f*x)
)/(16*a**5*b**2*f*sqrt(-a/b) + 32*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 -
```

48*a**4*b**3*f*sqrt(-a/b) + 16*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*
a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**3*b**4*f*sqrt(-a/b) - 48*a**
2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x
)**2 - 16*a**2*b**5*f*sqrt(-a/b) + 48*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 -
32*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*b**7*f*sqrt(-a/b)*tan(e + f*x)
4) - 2*a3*b*sqrt(-a/b)*tan(e + f*x)/(16*a**5*b**2*f*sqrt(-a/b) + 32*a**
4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**3*f*sqrt(-a/b) + 16*a**3*b
4*f*sqrt(-a/b)*tan(e + f*x)4 - 96*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**
2 + 48*a**3*b**4*f*sqrt(-a/b) - 48*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 +
96*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b) + 48
*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**6*f*sqrt(-a/b)*tan(e + f*x)*
*2 - 16*b**7*f*sqrt(-a/b)*tan(e + f*x)**4) + 2*a**3*b*log(-sqrt(-a/b) + tan
(e + f*x))*tan(e + f*x)**2/(16*a**5*b**2*f*sqrt(-a/b) + 32*a**4*b**3*f*sqrt
(-a/b)*tan(e + f*x)**2 - 48*a**4*b**3*f*sqrt(-a/b) + 16*a**3*b**4*f*sqrt(-a
/b)*tan(e + f*x)**4 - 96*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**3*b
4*f*sqrt(-a/b) - 48*a2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**2*b**5
*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b) + 48*a*b**6*f*sqr
t(-a/b)*tan(e + f*x)**4 - 32*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*b**7*
f*sqrt(-a/b)*tan(e + f*x)**4) - 6*a**3*b*log(-sqrt(-a/b) + tan(e + f*x))/(1
6*a**5*b**2*f*sqrt(-a/b) + 32*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a
4*b3*f*sqrt(-a/b) + 16*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**3
*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**3*b**4*f*sqrt(-a/b) - 48*a**2*b*
*5*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**2
- 16*a**2*b**5*f*sqrt(-a/b) + 48*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*
a*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*b**7*f*sqrt(-a/b)*tan(e + f*x)**4)
- 2*a**3*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(16*a**5*b**2*f*
sqrt(-a/b) + 32*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**3*f*sqr
t(-a/b) + 16*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**3*b**4*f*sqrt(-
a/b)*tan(e + f*x)**2 + 48*a**3*b**4*f*sqrt(-a/b) - 48*a**2*b**5*f*sqrt(-a/b
)*tan(e + f*x)**4 + 96*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**
5*f*sqrt(-a/b) + 48*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**6*f*sqrt(
-a/b)*tan(e + f*x)**2 - 16*b**7*f*sqrt(-a/b)*tan(e + f*x)**4) + 6*a**3*b*lo
g(sqrt(-a/b) + tan(e + f*x))/(16*a**5*b**2*f*sqrt(-a/b) + 32*a**4*b**3*f*sq
rt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**3*f*sqrt(-a/b) + 16*a**3*b**4*f*sqrt(
-a/b)*tan(e + f*x)**4 - 96*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**3
*b**4*f*sqrt(-a/b) - 48*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**2*b*
*5*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b) + 48*a*b**6*f*s
qrt(-a/b)*tan(e + f*x)**4 - 32*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*b**
7*f*sqrt(-a/b)*tan(e + f*x)**4) + 16*a**2*b**2*f*x*sqrt(-a/b)/(16*a**5*b**2
*f*sqrt(-a/b) + 32*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**3*f*
sqrt(-a/b) + 16*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**3*b**4*f*sqr
t(-a/b)*tan(e + f*x)**2 + 48*a**3*b**4*f*sqrt(-a/b) - 48*a**2*b**5*f*sqrt(-
a/b)*tan(e + f*x)**4 + 96*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*
b**5*f*sqrt(-a/b) + 48*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**6*f*sq
rt(-a/b)*tan(e + f*x)**2 - 16*b**7*f*sqrt(-a/b)...

Giac [A]
time = 1.46, size = 199, normalized size = 1.37

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(a^2-6\,ab-3\,b^2)}{(a^3b-3\,a^2b^2+3\,ab^3-b^4)\sqrt{ab}}+\frac{8\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{ab\tan(fx+e)^3-5\,b^2\tan(fx+e)^3-a^2\tan(fx+e)-3\,ab\tan(fx+e)}{(a^2b-2\,ab^2+b^3)\left(b\tan(fx+e)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(a^2 - 6*a*b - 3*b^2)/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sqrt(a*b)) +
8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (a*b*tan(f*x + e)^3 - 5*b^2*t
an(f*x + e)^3 - a^2*tan(f*x + e) - 3*a*b*tan(f*x + e))/((a^2*b - 2*a*b^2 +
b^3)*(b*tan(f*x + e)^2 + a)^2))/f
```

Mupad [B]
time = 15.49, size = 2500, normalized size = 17.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)^3*(a - 5*b))/(8*(a^2 - 2*a*b + b^2)) - (a*tan(e + f*x)*(a +
3*b))/(8*(a^2*b - 2*a*b^2 + b^3)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan
(e + f*x)^2)) - (2*atan((((((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^3*b
^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*b - 6*a*b
^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) - (tan(e + f*
x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4*b^6 - 23
04*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3
 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*1i)/(6*a*b^2 -
6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 + 73*b^
4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))/(6*a
*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1
440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*
b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) + (t
an(e + f*x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4
*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6*a*b^2 - 6*a^2*
b + 2*a^3 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*1i)/(6
*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^
4 + 73*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^
2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((((((544*a*b^8 - 96*b^9 - 1248*a
^2*b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)
/(64*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*
```

b^2)) - (tan(e + f*x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7
+ 1280*a^4*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6*a*b^
2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2
)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(36*a*b^3 - 12*
a^3*b + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3
- 4*a^3*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (27*a*b^2 - 11*a^2
*b + a^3 + 15*b^3)/(32*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 1
5*a^4*b^3 - 6*a^5*b^2)) + (((((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^3
*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*b - 6*a
*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) + (tan(e +
f*x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4*b^6 -
2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a
^3 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*1i)/(6*a*b^2
- 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 + 73*
b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*1i
)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*
b^3)) + (atan((((-a*b^3)^(1/2)*((tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 +
73*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2))
+ (((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^
5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 -
20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) - (tan(e + f*x)*(-a*b^3)^(1/2)*(6*a*b
 - a^2 + 3*b^2)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280
*a^4*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3))/(512*(a*b^6 - 3*a^2*
b^5 + 3*a^3*b^4 - a^4*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2))
)*(-a*b^3)^(1/2)*(6*a*b - a^2 + 3*b^2))/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4
- a^4*b^3)))*(6*a*b - a^2 + 3*b^2)*1i)/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 -
 a^4*b^3)) + ((-a*b^3)^(1/2)*((tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 + 73
*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)) -
(((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*
b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20
*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) + (tan(e + f*x)*(-a*b^3)^(1/2)*(6*a*b -
 a^2 + 3*b^2)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a
^4*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3))/(512*(a*b^6 - 3*a^2*b^
5 + 3*a^3*b^4 - a^4*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*
(-a*b^3)^(1/2)*(6*a*b - a^2 + 3*b^2))/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 -
a^4*b^3)))*(6*a*b - a^2 + 3*b^2)*1i)/(16*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a
^4*b^3)))/((27*a*b^2 - 11*a^2*b + a^3 + 15*b^3)/(32*(a^6*b - 6*a*b^6 + b^7
+ 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) - ((-a*b^3)^(1/2)*((ta
n(e + f*x)*(36*a*b^3 - 12*a^3*b + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(a^4*b -
4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)) + (((544*a*b^8 - 96*b^9 - 1248*a^2*
b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(6
4*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2
)) - (tan(e + f*x)*(-a*b^3)^(1/2)*(6*a*b - a^2 + 3*b^2)*(1280*a*b^9 - 256*b
^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4*b^6 - 2304*a^5*b^5 + 1280*a^6*
b^4 - 256*a^7*b^3))/(512*(a*b^6 - 3*a^2*b^5 + 3*a^3*b^4 - a^4*b^3)*(a^4*b -

```
4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-a*b...
```

3.245 $\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal. Leaf size=144

$$-\frac{x}{(a-b)^3}+\frac{(3a^2+6ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^3\sqrt{b}\,f}+\frac{\tan(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^2}+\frac{(3a+b)\tan(e+fx)}{8a(a-b)^2f\,(a+b\tan^2(e+fx))}$$

```
[Out] -x/(a-b)^3+1/8*(3*a^2+6*a*b-b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(3/2)
/(a-b)^3/f/b^(1/2)+1/4*tan(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^2+1/8*(3*a+b)*
tan(f*x+e)/a/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 482, 541, 536, 209, 211}

$$\frac{(3a^2+6ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}\,f(a-b)^3}+\frac{(3a+b)\tan(e+fx)}{8af(a-b)^2\,(a+b\tan^2(e+fx))}+\frac{\tan(e+fx)}{4f(a-b)\,(a+b\tan^2(e+fx))^2}-\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + ((3*a^2 + 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a]])/(8*a^(3/2)*(a - b)^3*Sqrt[b]*f) + Tan[e + f*x]/(4*(a - b)*f*(a + b*Tan
[e + f*x]^2)^2) + ((3*a + b)*Tan[e + f*x])/(8*a*(a - b)^2*f*(a + b*Tan[e +
f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{1-3x^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4(a-b)f}$$

$$= \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{(3a+b)\tan(e+fx)}{8a(a-b)^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}}{}$$

$$= \frac{\tan(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{(3a+b)\tan(e+fx)}{8a(a-b)^2f\left(a+b\tan^2(e+fx)\right)} - \frac{\text{Subst}}{}$$

$$= -\frac{x}{(a-b)^3} + \frac{(3a^2+6ab-b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^3\sqrt{b}\,f} + \frac{\tan(e+fx}{4(a-b)f\left(a+b\tan^2\right.}$$

Mathematica [A]
time = 1.40, size = 139, normalized size = 0.97

$$\frac{-8(e+fx) + \frac{(3a^2+6ab-b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} + \frac{(a-b)(5a^2+2ab+b^2+(5a^2-4ab-b^2)\cos(2(e+fx)))\sin(2(e+fx))}{a(a+b+(a-b)\cos(2(e+fx)))^2}}{8(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (-8*(e + f*x) + ((3*a^2 + 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
]])/(a^(3/2)*Sqrt[b]) + ((a - b)*(5*a^2 + 2*a*b + b^2 + (5*a^2 - 4*a*b - b^
2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a*(a + b + (a - b)*Cos[2*(e + f*x)]
)^2))/(8*(a - b)^3*f)
```

Maple [A]
time = 0.33, size = 137, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3} + \frac{\frac{\frac{b(3a^2-2ab-b^2)(\tan^3(fx+e))}{8a} + \left(\frac{5}{8}a^2-\frac{3}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2} + \frac{(3a^2+6ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{(a-b)^3}}{f}$

default	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{\frac{\frac{b\left(3a^2-2ab-b^2\right)\left(\tan^3(fx+e)\right)}{8a}+\left(\frac{5}{8}a^2-\frac{3}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(3a^2+6ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{(a-b)^3}}{f}$
risch	$-\frac{x}{a^3-3a^2b+3a\,b^2-b^3}+\frac{i\left(5a^3\mathrm{e}^{6i(fx+e)}+5a^2b\,\mathrm{e}^{6i(fx+e)}-9a\,b^2\mathrm{e}^{6i(fx+e)}-b^3\mathrm{e}^{6i(fx+e)}+15a^3\mathrm{e}^{4i(fx+e)}+13a^2b\,\mathrm{e}^{4i(fx+e)}\right.}{4\left(a\,\mathrm{e}^{4i(fx+e)}-b\,\mathrm{e}^{4i(fx+e)}+2a\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/(a-b)^3*arctan(tan(f*x+e))+1/(a-b)^3*((1/8*b*(3*a^2-2*a*b-b^2)/a*ta
n(f*x+e)^3+(5/8*a^2-3/4*a*b+1/8*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(
3*a^2+6*a*b-b^2)/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.50, size = 219, normalized size = 1.52

$$\frac{\frac{\left(3\,a^2+6\,ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\left(a^4-3\,a^3b+3\,a^2b^2-ab^3\right)\sqrt{ab}}+\frac{\left(3\,ab+b^2\right)\tan(fx+e)^3+\left(5\,a^2-ab\right)\tan(fx+e)}{a^5-2\,a^4b+a^3b^2+\left(a^3b^2-2\,a^2b^3+ab^4\right)\tan(fx+e)^4+2\left(a^4b-2\,a^3b^2+a^2b^3\right)\tan(fx+e)^2}-\frac{8\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 6*a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 3*a^3*b
 + 3*a^2*b^2 - a*b^3)*sqrt(a*b)) + ((3*a*b + b^2)*tan(f*x + e)^3 + (5*a^2 -
 a*b)*tan(f*x + e))/(a^5 - 2*a^4*b + a^3*b^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4
)*tan(f*x + e)^4 + 2*(a^4*b - 2*a^3*b^2 + a^2*b^3)*tan(f*x + e)^2) - 8*(f*x
 + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(135) = 270.
time = 3.71, size = 783, normalized size = 5.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^2*b^3*f*x*tan(f*x + e)^4 + 64*a^3*b^2*f*x*tan(f*x + e)^2 + 32*
a^4*b*f*x - 4*(3*a^3*b^2 - 2*a^2*b^3 - a*b^4)*tan(f*x + e)^3 + ((3*a^2*b^2
+ 6*a*b^3 - b^4)*tan(f*x + e)^4 + 3*a^4 + 6*a^3*b - a^2*b^2 + 2*(3*a^3*b +
6*a^2*b^2 - a*b^3)*tan(f*x + e)^2)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a
```

```
*b*tan(f*x + e)^2 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))
/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(5*a^4*b - 6*a^3*b^
2 + a^2*b^3)*tan(f*x + e))/((a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*t
an(f*x + e)^4 + 2*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e
)^2 + (a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f), -1/16*(16*a^2*b^3*f*x*t
an(f*x + e)^4 + 32*a^3*b^2*f*x*tan(f*x + e)^2 + 16*a^4*b*f*x - 2*(3*a^3*b^2
 - 2*a^2*b^3 - a*b^4)*tan(f*x + e)^3 - ((3*a^2*b^2 + 6*a*b^3 - b^4)*tan(f*x
 + e)^4 + 3*a^4 + 6*a^3*b - a^2*b^2 + 2*(3*a^3*b + 6*a^2*b^2 - a*b^3)*tan(f
*x + e)^2)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)/(a*b*tan(f
*x + e))) - 2*(5*a^4*b - 6*a^3*b^2 + a^2*b^3)*tan(f*x + e))/((a^5*b^3 - 3*a
^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*tan(f*x + e)^4 + 2*(a^6*b^2 - 3*a^5*b^3 + 3
*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^2 + (a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4
*b^4)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 9051 vs. $2(122) = 244$.
time = 76.24, size = 9051, normalized size = 62.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((x + 1/(f*tan
(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b**3, Eq(a, 0)), (3*f*x*tan(e + f*x)*
*6/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan
(e + f*x)**2 + 48*b**3*f) + 9*f*x*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**
6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) +
9*f*x*tan(e + f*x)**2/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)*
*4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 3*f*x/(48*b**3*f*tan(e + f*x
)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f)
 + 3*tan(e + f*x)**5/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**
4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 8*tan(e + f*x)**3/(48*b**3*f*
tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 +
 48*b**3*f) - 3*tan(e + f*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e
+ f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x*tan(e)**
2/(a + b*tan(e)**2)**3, Eq(f, 0)), ((-x + tan(e + f*x)/f)/a**3, Eq(b, 0)),
(3*a**4*log(-sqrt(-a/b) + tan(e + f*x))/(16*a**6*b*f*sqrt(-a/b) + 32*a**5*b
**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**5*b**2*f*sqrt(-a/b) + 16*a**4*b**3
*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 +
 48*a**4*b**3*f*sqrt(-a/b) - 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 + 96
*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**3*b**4*f*sqrt(-a/b) + 48*a*
*2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**2*b**5*f*sqrt(-a/b)*tan(e + f*
x)**2 - 16*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4) - 3*a**4*log(sqrt(-a/b) + t
an(e + f*x))/(16*a**6*b*f*sqrt(-a/b) + 32*a**5*b**2*f*sqrt(-a/b)*tan(e + f*
```

```
x)**2 - 48*a**5*b**2*f*sqrt(-a/b) + 16*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)*
*4 - 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**4*b**3*f*sqrt(-a/b)
- 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**3*b**4*f*sqrt(-a/b)*tan
(e + f*x)**2 - 16*a**3*b**4*f*sqrt(-a/b) + 48*a**2*b**5*f*sqrt(-a/b)*tan(e
+ f*x)**4 - 32*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a*b**6*f*sqrt(-a
/b)*tan(e + f*x)**4) - 16*a**3*b*f*x*sqrt(-a/b)/(16*a**6*b*f*sqrt(-a/b) + 3
2*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**5*b**2*f*sqrt(-a/b) + 16*a
**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f
*x)**2 + 48*a**4*b**3*f*sqrt(-a/b) - 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)
**4 + 96*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**3*b**4*f*sqrt(-a/b)
 + 48*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**2*b**5*f*sqrt(-a/b)*ta
n(e + f*x)**2 - 16*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4) + 10*a**3*b*sqrt(-a
/b)*tan(e + f*x)/(16*a**6*b*f*sqrt(-a/b) + 32*a**5*b**2*f*sqrt(-a/b)*tan(e
+ f*x)**2 - 48*a**5*b**2*f*sqrt(-a/b) + 16*a**4*b**3*f*sqrt(-a/b)*tan(e + f
*x)**4 - 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**4*b**3*f*sqrt(-a
/b) - 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**3*b**4*f*sqrt(-a/b)
*tan(e + f*x)**2 - 16*a**3*b**4*f*sqrt(-a/b) + 48*a**2*b**5*f*sqrt(-a/b)*ta
n(e + f*x)**4 - 32*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a*b**6*f*sqr
t(-a/b)*tan(e + f*x)**4) + 6*a**3*b*log(-sqrt(-a/b) + tan(e + f*x))*tan(e +
 f*x)**2/(16*a**6*b*f*sqrt(-a/b) + 32*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**
2 - 48*a**5*b**2*f*sqrt(-a/b) + 16*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 -
 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**4*b**3*f*sqrt(-a/b) - 48
*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**3*b**4*f*sqrt(-a/b)*tan(e +
 f*x)**2 - 16*a**3*b**4*f*sqrt(-a/b) + 48*a**2*b**5*f*sqrt(-a/b)*tan(e + f*
x)**4 - 32*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a*b**6*f*sqrt(-a/b)*
tan(e + f*x)**4) + 6*a**3*b*log(-sqrt(-a/b) + tan(e + f*x))/(16*a**6*b*f*sq
rt(-a/b) + 32*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**5*b**2*f*sqrt(
-a/b) + 16*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**4*b**3*f*sqrt(-a/
b)*tan(e + f*x)**2 + 48*a**4*b**3*f*sqrt(-a/b) - 48*a**3*b**4*f*sqrt(-a/b)*
tan(e + f*x)**4 + 96*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**3*b**4*
f*sqrt(-a/b) + 48*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**2*b**5*f*s
qrt(-a/b)*tan(e + f*x)**2 - 16*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4) - 6*a**
3*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(16*a**6*b*f*sqrt(-a/b)
+ 32*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**5*b**2*f*sqrt(-a/b) + 1
6*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**4*b**3*f*sqrt(-a/b)*tan(e
+ f*x)**2 + 48*a**4*b**3*f*sqrt(-a/b) - 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f
*x)**4 + 96*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**3*b**4*f*sqrt(-a
/b) + 48*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**2*b**5*f*sqrt(-a/b)
*tan(e + f*x)**2 - 16*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4) - 6*a**3*b*log(s
qrt(-a/b) + tan(e + f*x))/(16*a**6*b*f*sqrt(-a/b) + 32*a**5*b**2*f*sqrt(-a/
b)*tan(e + f*x)**2 - 48*a**5*b**2*f*sqrt(-a/b) + 16*a**4*b**3*f*sqrt(-a/b)*
tan(e + f*x)**4 - 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**4*b**3*
f*sqrt(-a/b) - 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**3*b**4*f*s
qrt(-a/b)*tan(e + f*x)**2 - 16*a**3*b**4*f*sqrt(-a/b) + 48*a**2*b**5*f*sqrt
(-a/b)*tan(e + f*x)**4 - 32*a**2*b**5*f*sqrt(-a...
```

Giac [A]
time = 1.12, size = 200, normalized size = 1.39

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)(3a^2+6ab-b^2)}{(a^4-3a^3b+3a^2b^2-ab^3)\sqrt{ab}}-\frac{8(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{3ab\tan(fx+e)^3+b^2\tan(fx+e)^3+5a^2\tan(fx+e)-ab\tan(fx+e)}{(a^3-2a^2b+ab^2)\left(b\tan(fx+e)^2+a\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)
))*(3*a^2 + 6*a*b - b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)) -
8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3*a*b*tan(f*x + e)^3 + b^2*t
an(f*x + e)^3 + 5*a^2*tan(f*x + e) - a*b*tan(f*x + e))/((a^3 - 2*a^2*b + a*
b^2)*(b*tan(f*x + e)^2 + a)^2))/f
```

Mupad [B]
time = 15.49, size = 2500, normalized size = 17.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] ((tan(e + f*x)*(5*a - b))/(8*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^3*(3*a*b
+ b^2))/(8*a*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan
(e + f*x)^2)) - (2*atan((((((32*a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 3040*a
^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(a^8
- 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) -
(tan(e + f*x)*(256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1
280*a^6*b^5 + 2304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2)*1i)/(32*(6*a*b^2 -
 6*a^2*b + 2*a^3 - 2*b^3)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)
))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^4*b - 12*a*
b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*
b^3 + 6*a^4*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((32*a*b^9 - 352
*a^2*b^8 + 1440*a^3*b^7 - 3040*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*
a^7*b^3 - 160*a^8*b^2)/(64*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^
4 - 20*a^5*b^3 + 15*a^6*b^2)) + (tan(e + f*x)*(256*a^2*b^9 - 1280*a^3*b^8 +
 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^5 + 2304*a^7*b^4 - 1280*a^8*b^3 +
 256*a^9*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^6 - 4*a^5*b +
a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)
+ (tan(e + f*x)*(9*a^4*b - 12*a*b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(
a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a
^3 - 2*b^3))/((((((32*a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 3040*a^4*b^6 + 3
680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(a^8 - 6*a^7*b
```

+ a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - (tan(e + f
x)(256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^
5 + 2304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b +
 2*a^3 - 2*b^3)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*1i)/(6*
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^4*b - 12*a*b^4 + b^5
+ 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^
4*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (3*a*b^3 + 9*a^3*b - b^4
 + 21*a^2*b^2)/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a
^5*b^3 + 15*a^6*b^2)) + (((((32*a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 3040*a
^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(a^8
- 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) +
(tan(e + f*x)*(256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1
280*a^6*b^5 + 2304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2)*1i)/(32*(6*a*b^2 -
 6*a^2*b + 2*a^3 - 2*b^3)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)
))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(9*a^4*b - 12*a*
b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*
b^3 + 6*a^4*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))))/(f*(6*a*b^2 -
 6*a^2*b + 2*a^3 - 2*b^3)) - (atan(((((tan(e + f*x)*(9*a^4*b - 12*a*b^4 + b
^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6
*a^4*b^2)) - ((-a^3*b)^(1/2)*((32*a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 3040
*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(a^
8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2))
- (tan(e + f*x)*(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b^2)*(256*a^2*b^9 - 1280*a^
3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^5 + 2304*a^7*b^4 - 1280*a^
8*b^3 + 256*a^9*b^2))/(512*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)*(a^6 -
 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(6*a*b + 3*a^2 - b^2))/(16*(a
^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)))*(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b
^2)*1i)/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)) + (((tan(e + f*x)*(9
*a^4*b - 12*a*b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^
2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)) + ((-a^3*b)^(1/2)*((32*a*b^9 - 352*a^2*b^8
+ 1440*a^3*b^7 - 3040*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 -
 160*a^8*b^2)/(64*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^
5*b^3 + 15*a^6*b^2)) + (tan(e + f*x)*(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b^2)*(
256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^5 + 2
304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2))/(512*(a^6*b - a^3*b^4 + 3*a^4*b^
3 - 3*a^5*b^2)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(6*a*b +
 3*a^2 - b^2))/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2)))*(-a^3*b)^(1/
2)*(6*a*b + 3*a^2 - b^2)*1i)/(16*(a^6*b - a^3*b^4 + 3*a^4*b^3 - 3*a^5*b^2))
)/((3*a*b^3 + 9*a^3*b - b^4 + 21*a^2*b^2)/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*
a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + (((tan(e + f*x)*(9*a^4*b
 - 12*a*b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4
- 4*a^3*b^3 + 6*a^4*b^2)) - ((-a^3*b)^(1/2)*((32*a*b^9 - 352*a^2*b^8 + 1440
*a^3*b^7 - 3040*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a
^8*b^2)/(64*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3
+ 15*a^6*b^2)) - (tan(e + f*x)*(-a^3*b)^(1/2)*(6*a*b + 3*a^2 - b^2)*(256*a^

```
2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*...
```

3.246 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=150

$$\frac{x}{(a-b)^3}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3f}-\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2}-\frac{(7a-3$$

```
[Out] x/(a-b)^3-1/8*(15*a^2-10*a*b+3*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))*b^(1
/2)/a^(5/2)/(a-b)^3/f-1/4*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(
7*a-3*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3742, 425, 541, 536, 209, 211}

$$-\frac{b(7a-3b)\tan(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}f(a-b)^3}-\frac{b\tan(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*f) - (b*Tan[e + f*x])/(4*a*(a - b)*f*(a
+ b*Tan[e + f*x]^2)^2) - ((7*a - 3*b)*b*Tan[e + f*x])/(8*a^2*(a - b)^2*f*(a
 + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+f\right.}{4a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Su}}{}$$

$$= -\frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)} + \frac{\text{Su}}{}$$

$$= \frac{x}{(a-b)^3} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3f} - \frac{b\tan(}{4a(a-b)f\,(a+}$$

Mathematica [A]

time = 1.32, size = 138, normalized size = 0.92

$$-\frac{-8\text{ArcTan}(\tan(e+fx))+\frac{\sqrt{b}\,(15a^2-10ab+3b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2(a-b)^2b\tan(e+fx)}{a(a+b\tan^2(e+fx))^2}+\frac{(7a-3b)(a-b)b\tan(e+fx)}{a^2(a+b\tan^2(e+fx))}}{8(a-b)^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3),x]
```

```
[Out] -1/8*(-8*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(
Sqrt[b]*Tan[e + f*x]])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[e + f*x])/(a*(
a + b*Tan[e + f*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[e + f*x])/(a^2*(a + b
*Tan[e + f*x]^2)))/((a - b)^3*f)
```

Maple [A]

time = 0.08, size = 142, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^3}-\frac{b\left(\frac{\frac{b\left(7a^2-10ab+3b^2\right)\left(\tan^3(fx+e)\right)}{8a^2}+\frac{\left(9a^2-14ab+5b^2\right)\tan(fx+e)}{8a}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2-10ab+3b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right.}{(a-b)^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^3}-\frac{b\left(\frac{\frac{b\left(7a^2-10ab+3b^2\right)\left(\tan^3(fx+e)\right)}{8a^2}+\frac{\left(9a^2-14ab+5b^2\right)\tan(fx+e)}{8a}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(15a^2-10ab+3b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right.}{(a-b)^3}}{f}$
risch	$\frac{x}{a^3-3a^2b+3ab^2-b^3}+\frac{i\left(9a^3e^{6i(fx+e)}+a^2b\,e^{6i(fx+e)}-13a\,b^2e^{6i(fx+e)}+3b^3e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+9a^2b\,e^{4i(fx+e)}+2\right.}{4\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^3*arctan(tan(f*x+e))-b/(a-b)^3*((1/8*b*(7*a^2-10*a*b+3*b^2)/a^
2*tan(f*x+e)^3+1/8*(9*a^2-14*a*b+5*b^2)/a*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+
1/8*(15*a^2-10*a*b+3*b^2)/a^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))
)
```

Maxima [A]

time = 0.50, size = 233, normalized size = 1.55

$$-\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}+\frac{(7\,ab^2-3\,b^3)\tan(fx+e)^3+(9\,a^2b-5\,ab^2)\tan(fx+e)}{a^6-2\,a^5b+a^4b^2+(a^4b^2-2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b-2\,a^4b^2+a^3b^3)\tan(fx+e)^2}-\frac{8\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5
- 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) + ((7*a*b^2 - 3*b^3)*tan(f*x +
e)^3 + (9*a^2*b - 5*a*b^2)*tan(f*x + e))/(a^6 - 2*a^5*b + a^4*b^2 + (a^4*b^
2 - 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^3)*t
an(f*x + e)^2) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(141) = 282$.
time = 4.66, size = 766, normalized size = 5.11

$$\left[\frac{32\,a^2b^2fx\tan\left(fx+e\right)^4+64\,a^3bfx\tan\left(fx+e\right)^2+32\,a^4fx-4\left(7\,a^2b^2-10\,ab^3+3\,b^4\right)\tan\left(fx+e\right)^3-\left(\left(15\,a^2b^2-10\,ab^3+3\,b^4\right)\tan\left(fx+e\right)^4+15\,a^4-10\,a^3b+3\,a^2b^2+2\left(15\,a^3b-10\,a^2b^2+3\,ab^3\right)\tan\left(fx+e\right)^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6ab\tan(fx+e)^2+a^2+4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2ab\tan(fx+e)^2+a^2}\right)-4\left(9\,a^3b-14\,a^2b^2+5\,ab^3\right)\tan\left(fx+e\right)}{32\left(\left(a^5b^2-3\,a^4b^3+3\,a^3b^4-a^2b^5\right)f\tan\left(fx+e\right)^4+2\left(a^6b-3\,a^5b^2+3\,a^4b^3-a^3b^4\right)f\tan\left(fx+e\right)^2+\left(a^7-3\,a^6b+3\,a^5b^2-a^4b^3\right)f\right)},\frac{16\,a^2b^2fx\tan\left(fx+e\right)^4+32\,a^3bfx\tan\left(fx+e\right)^2+16\,a^4fx-2\left(7\,a^2b^2-10\,ab^3+3\,b^4\right)\tan\left(fx+e\right)^3-\left(\left(15\,a^2b^2-10\,ab^3+3\,b^4\right)\tan\left(fx+e\right)^4+15\,a^4-10\,a^3b+3\,a^2b^2+2\left(15\,a^3b-10\,a^2b^2+3\,ab^3\right)\tan\left(fx+e\right)^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{b}{a}}}{2b\tan(fx+e)}\right)-2\left(9\,a^3b-14\,a^2b^2+5\,ab^3\right)\tan\left(fx+e\right)}{16\left(\left(a^5b^2-3\,a^4b^3+3\,a^3b^4-a^2b^5\right)f\tan\left(fx+e\right)^4+2\left(a^6b-3\,a^5b^2+3\,a^4b^3-a^3b^4\right)f\tan\left(fx+e\right)^2+\left(a^7-3\,a^6b+3\,a^5b^2-a^4b^3\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a^2*b^2*f*x*tan(f*x + e)^4 + 64*a^3*b*f*x*tan(f*x + e)^2 + 32*a^4
*f*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b
 - 10*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4
 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*s
qrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(9*a^3*b
- 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*ta
n(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/16*(16*a^2*b^2*f
*x*tan(f*x + e)^4 + 32*a^3*b*f*x*tan(f*x + e)^2 + 16*a^4*f*x - 2*(7*a^2*b^2
 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(
f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a^2*b^2 + 3*a
*b^3)*tan(f*x + e)^2)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)
/(b*tan(f*x + e))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8964 vs. $2(133) = 266$.
time = 75.98, size = 8964, normalized size = 59.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(e)**6, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**3, Eq(b,
 0)), ((-x - 1/(f*tan(e + f*x)) + 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e +
f*x)**5))/b**3, Eq(a, 0)), (15*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)*
*6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) +
 45*f*x*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x
)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*x)**2/(48
*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f
*x)**2 + 48*b**3*f) + 15*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e
+ f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 15*tan(e + f*x)**5/(4
8*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e +
f*x)**2 + 48*b**3*f) + 40*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*
b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 33*tan(e
 + f*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*
f*tan(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x/(a + b*tan(e)**2)**3, Eq(f, 0
)), (16*a**4*f*x*sqrt(-a/b)/(16*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*
tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e
+ f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt
(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a
/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)
*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5
*f*sqrt(-a/b)*tan(e + f*x)**4) - 15*a**4*log(-sqrt(-a/b) + tan(e + f*x))/(1
6*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*
sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqr
t(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-
a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*
b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**
4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4)
 + 15*a**4*log(sqrt(-a/b) + tan(e + f*x))/(16*a**7*f*sqrt(-a/b) + 32*a**6*b
*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqr
t(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a*
*5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*
b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**
4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2
- 16*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4) + 32*a**3*b*f*x*sqrt(-a/b)*tan
(e + f*x)**2/(16*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2
 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*
a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**
4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x
)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**
4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b)*t
an(e + f*x)**4) - 18*a**3*b*sqrt(-a/b)*tan(e + f*x)/(16*a**7*f*sqrt(-a/b) +
 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*
b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)*
*2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4
+ 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 4
```

```
8*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e
+ f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4) - 30*a**3*b*log(-sqr
t(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(16*a**7*f*sqrt(-a/b) + 32*a**6*b*f
*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(
-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**5
*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*b*
*3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**4*
f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 -
16*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4) + 10*a**3*b*log(-sqrt(-a/b) + ta
n(e + f*x))/(16*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2
- 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a
**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**4
*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)
**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4
 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*sqrt(-a/b)*ta
n(e + f*x)**4) + 30*a**3*b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(
16*a**7*f*sqrt(-a/b) + 32*a**6*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f
*sqrt(-a/b) + 16*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sq
rt(-a/b)*tan(e + f*x)**2 + 48*a**5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(
-a/b)*tan(e + f*x)**4 + 96*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4
*b**3*f*sqrt(-a/b) + 48*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b*
*4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*...
```

Giac [A]
time = 0.65, size = 213, normalized size = 1.42

$$-\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}-\frac{8\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{7\,ab^2\tan(fx+e)^3-3\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)-5\,ab^2\tan(fx+e)}{(a^4-2\,a^3b+a^2b^2)\left(b\tan(fx+e)^2+a\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (7*a*b^2*tan(f*x +
 e)^3 - 3*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e))
/((a^4 - 2*a^3*b + a^2*b^2)*(b*tan(f*x + e)^2 + a)^2))/f
```

Mupad [B]
time = 15.35, size = 2500, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^3,x)
```

[Out] (atan((((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) - (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) - ((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) - (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2))))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b +

3*b^2)*1i)/(8*f*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) - ((tan(e + f*x)^3*(7*a*b^2 - 3*b^3))/(8*a^2*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)*(9*a*b - 5*b^2))/(8*a*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan(e + f*x)^2)) - (2*atan((((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) - (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(e + f*x)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a^3*b^3)/(32...

3.247 $\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^3} dx$

Optimal. Leaf size=189

$$-\frac{x}{(a-b)^3}+\frac{b^{3/2}(35a^2-42ab+15b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^3f}-\frac{(8a^2-27ab+15b^2)\cot(e+fx)}{8a^3(a-b)^2f}-\frac{b}{4a(a-b)f}$$

```
[Out] -x/(a-b)^3+1/8*b^(3/2)*(35*a^2-42*a*b+15*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(
1/2))/a^(7/2)/(a-b)^3/f-1/8*(8*a^2-27*a*b+15*b^2)*cot(f*x+e)/a^3/(a-b)^2/f-
1/4*b*cot(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(9*a-5*b)*b*cot(f*x+e)/
a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.20, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3751, 483, 593, 597, 536, 209, 211}

$$-\frac{b(9a-5b)\cot(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{b^{3/2}(35a^2-42ab+15b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f(a-b)^3}-\frac{(8a^2-27ab+15b^2)\cot(e+fx)}{8a^3f(a-b)^2}-\frac{b\cot(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + (b^(3/2)*(35*a^2 - 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e
+ f*x])/Sqrt[a]])/(8*a^(7/2)*(a - b)^3*f) - ((8*a^2 - 27*a*b + 15*b^2)*Cot[
e + f*x])/(8*a^3*(a - b)^2*f) - (b*Cot[e + f*x])/(4*a*(a - b)*f*(a + b*Tan[
e + f*x]^2)^2) - ((9*a - 5*b)*b*Cot[e + f*x])/(8*a^2*(a - b)^2*f*(a + b*Tan
[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{4a-5b-5bx^2}{x^2(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+f\right.}{4a(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{(9a-5b)b\cot(e+fx)}{8a^2(a-b)^2f\,(a+b\tan^2(e+fx))} + \text{Sul}$$

$$= -\frac{(8a^2-27ab+15b^2)\cot(e+fx)}{8a^3(a-b)^2f} - \frac{b\cot(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{}{8a^2(}$$

$$= -\frac{(8a^2-27ab+15b^2)\cot(e+fx)}{8a^3(a-b)^2f} - \frac{b\cot(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{}{8a^2(}$$

$$= -\frac{x}{(a-b)^3} + \frac{b^{3/2}(35a^2-42ab+15b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^3f} - \frac{(8a^2-27ab+}{8a^3(}$$

Mathematica [A]
time = 1.55, size = 174, normalized size = 0.92

$$\frac{\frac{8(e+fx)}{(-a+b)^3} + \frac{b^{3/2}(35a^2-42ab+15b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)^3} - \frac{8\cot(e+fx)}{a^3} - \frac{4b^3\sin(2(e+fx))}{a^2(a-b)^2(a+b+(a-b)\cos(2(e+fx)))^2} + \frac{(13a-7b)b^2\sin(2(e+fx))}{a^3(a-b)^2(a+b+(a-b)\cos(2(e+fx)))}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((8*(e + f*x))/(-a + b)^3 + (b^(3/2)*(35*a^2 - 42*a*b + 15*b^2)*ArcTan[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(a - b)^3) - (8*Cot[e + f*x])/a^3 - (
4*b^3*Sin[2*(e + f*x)])/(a^2*(a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2
) + ((13*a - 7*b)*b^2*Sin[2*(e + f*x)])/(a^3*(a - b)^2*(a + b + (a - b)*Cos
[2*(e + f*x)])))/(8*f)
```

Maple [A]
time = 0.52, size = 153, normalized size = 0.81

method	result

derivativedivides	$\frac{\frac{b^{2}\left(\frac{\left(\frac{11}{8}a^{2}b-\frac{9}{4}a\,b^{2}+\frac{7}{8}b^{3}\right)\left(\tan^{3}(fx+e)\right)+\frac{a\left(13a^{2}-22ab+9b^{2}\right)\tan(fx+e)}{8}}{\left(a+b\left(\tan^{2}(fx+e)\right)\right)^{2}}+\frac{\left(35a^{2}-42ab+15b^{2}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}(a-b)^{3}}-\frac{}{a^{3}\tan(}}{f}$
default	$\frac{\frac{b^{2}\left(\frac{\left(\frac{11}{8}a^{2}b-\frac{9}{4}a\,b^{2}+\frac{7}{8}b^{3}\right)\left(\tan^{3}(fx+e)\right)+\frac{a\left(13a^{2}-22ab+9b^{2}\right)\tan(fx+e)}{8}}{\left(a+b\left(\tan^{2}(fx+e)\right)\right)^{2}}+\frac{\left(35a^{2}-42ab+15b^{2}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}(a-b)^{3}}-\frac{}{a^{3}\tan(}}{f}$
risch	$-\frac{x}{a^{3}-3a^{2}b+3a\,b^{2}-b^{3}}-\frac{i\left(-112a^{4}e^{4i(fx+e)}b+96a^{3}b^{2}e^{4i(fx+e)}-158b^{4}e^{2i(fx+e)}a+90a^{3}b^{2}e^{2i(fx+e)}+72b^{3}e^{2i(fx+e)}a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2/a^3/(a-b)^3*(((11/8*a^2*b-9/4*a*b^2+7/8*b^3)*tan(f*x+e)^3+1/8*a*(1
3*a^2-22*a*b+9*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(35*a^2-42*a*b+15*
b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/a^3/tan(f*x+e)-1/(a-b)
^3*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.51, size = 282, normalized size = 1.49

$$\frac{\frac{\left(35\,a^{2}b^{2}-42\,ab^{3}+15\,b^{4}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\left(a^{6}-3\,a^{5}b+3\,a^{4}b^{2}-a^{3}b^{3}\right)\sqrt{ab}}-\frac{\left(8\,a^{2}b^{2}-27\,ab^{3}+15\,b^{4}\right)\tan(fx+e)^{4}+8\,a^{4}-16\,a^{3}b+8\,a^{2}b^{2}+\left(16\,a^{3}b-45\,a^{2}b^{2}+25\,ab^{3}\right)\tan(fx+e)^{2}}{\left(a^{5}b^{2}-2\,a^{4}b^{3}+a^{3}b^{4}\right)\tan(fx+e)^{5}+2\left(a^{6}b-2\,a^{5}b^{2}+a^{4}b^{3}\right)\tan(fx+e)^{3}+\left(a^{7}-2\,a^{6}b+a^{5}b^{2}\right)\tan(fx+e)}-\frac{8\,(fx+e)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((35*a^2*b^2 - 42*a*b^3 + 15*b^4)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^
6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*sqrt(a*b)) - ((8*a^2*b^2 - 27*a*b^3 + 15
*b^4)*tan(f*x + e)^4 + 8*a^4 - 16*a^3*b + 8*a^2*b^2 + (16*a^3*b - 45*a^2*b^
2 + 25*a*b^3)*tan(f*x + e)^2)/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*tan(f*x + e)
^5 + 2*(a^6*b - 2*a^5*b^2 + a^4*b^3)*tan(f*x + e)^3 + (a^7 - 2*a^6*b + a^5*
b^2)*tan(f*x + e)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(179) = 358$.
time = 2.80, size = 911, normalized size = 4.82

$$\left[-\frac{32\,a^{3}b^{2}fx\tan\left(fx+e\right)^{5}+64\,a^{4}bfx\tan\left(fx+e\right)^{3}+32\,a^{5}fx\tan\left(fx+e\right)+32\,a^{5}-96\,a^{4}b+96\,a^{3}b^{2}-32\,a^{2}b^{3}+4\left(8\,a^{3}b^{2}-35\,a^{2}b^{3}+42\,ab^{4}-15\,b^{5}\right)\tan\left(fx+e\right)^{4}+4\left(16\,a^{4}b-61\,a^{3}b^{2}+70\,a^{2}b^{3}-25\,ab^{4}\right)\tan\left(fx+e\right)^{2}+\left(\left(35\,a^{2}b^{3}-42\,ab^{4}+15\,b^{5}\right)\tan\left(fx+e\right)^{5}+2\left(35\,a^{3}b^{2}-42\,a^{2}b^{3}+15\,ab^{4}\right)\tan\left(fx+e\right)^{3}+\left(35\,a^{4}b-42\,a^{3}b^{2}+15\,a^{2}b^{3}\right)\tan\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^{2}\tan(fx+e)^{4}-6\,ab\tan(fx+e)^{2}+a^{2}-4\left(ab\tan(fx+e)^{3}-a^{2}\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^{2}\tan(fx+e)^{4}+2\,ab\tan(fx+e)^{2}+a^{2}}\right)}{32\left(\left(a^{6}b^{2}-3\,a^{5}b^{3}+3\,a^{4}b^{4}-a^{3}b^{5}\right)f\tan\left(fx+e\right)^{5}+2\left(a^{7}b-3\,a^{6}b^{2}+3\,a^{5}b^{3}-a^{4}b^{4}\right)f\tan\left(fx+e\right)^{3}+\left(a^{8}-3\,a^{7}b+3\,a^{6}b^{2}-a^{5}b^{3}\right)f\tan\left(fx+e\right)\right)},-\frac{16\,a^{3}b^{2}fx\tan\left(fx+e\right)^{5}+32\,a^{4}bfx\tan\left(fx+e\right)^{3}+16\,a^{5}fx\tan\left(fx+e\right)+16\,a^{5}-48\,a^{4}b+48\,a^{3}b^{2}-16\,a^{2}b^{3}+2\left(8\,a^{3}b^{2}-35\,a^{2}b^{3}+42\,ab^{4}-15\,b^{5}\right)\tan\left(fx+e\right)^{4}+2\left(16\,a^{4}b-61\,a^{3}b^{2}+70\,a^{2}b^{3}-25\,ab^{4}\right)\tan\left(fx+e\right)^{2}-\left(\left(35\,a^{2}b^{3}-42\,ab^{4}+15\,b^{5}\right)\tan\left(fx+e\right)^{5}+2\left(35\,a^{3}b^{2}-42\,a^{2}b^{3}+15\,ab^{4}\right)\tan\left(fx+e\right)^{3}+\left(35\,a^{4}b-42\,a^{3}b^{2}+15\,a^{2}b^{3}\right)\tan\left(fx+e\right)\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^{2}-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)}{16\left(\left(a^{6}b^{2}-3\,a^{5}b^{3}+3\,a^{4}b^{4}-a^{3}b^{5}\right)f\tan\left(fx+e\right)^{5}+2\left(a^{7}b-3\,a^{6}b^{2}+3\,a^{5}b^{3}-a^{4}b^{4}\right)f\tan\left(fx+e\right)^{3}+\left(a^{8}-3\,a^{7}b+3\,a^{6}b^{2}-a^{5}b^{3}\right)f\tan\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^3*b^2*f*x*tan(f*x + e)^5 + 64*a^4*b*f*x*tan(f*x + e)^3 + 32*a^
5*f*x*tan(f*x + e) + 32*a^5 - 96*a^4*b + 96*a^3*b^2 - 32*a^2*b^3 + 4*(8*a^3
*b^2 - 35*a^2*b^3 + 42*a*b^4 - 15*b^5)*tan(f*x + e)^4 + 4*(16*a^4*b - 61*a^
3*b^2 + 70*a^2*b^3 - 25*a*b^4)*tan(f*x + e)^2 + ((35*a^2*b^3 - 42*a*b^4 + 1
5*b^5)*tan(f*x + e)^5 + 2*(35*a^3*b^2 - 42*a^2*b^3 + 15*a*b^4)*tan(f*x + e)
^3 + (35*a^4*b - 42*a^3*b^2 + 15*a^2*b^3)*tan(f*x + e))*sqrt(-b/a)*log((b^2
*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*
tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)
))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5 + 2*(a^7*b
 - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a^7*b + 3*a
^6*b^2 - a^5*b^3)*f*tan(f*x + e)), -1/16*(16*a^3*b^2*f*x*tan(f*x + e)^5 + 3
2*a^4*b*f*x*tan(f*x + e)^3 + 16*a^5*f*x*tan(f*x + e) + 16*a^5 - 48*a^4*b +
48*a^3*b^2 - 16*a^2*b^3 + 2*(8*a^3*b^2 - 35*a^2*b^3 + 42*a*b^4 - 15*b^5)*ta
n(f*x + e)^4 + 2*(16*a^4*b - 61*a^3*b^2 + 70*a^2*b^3 - 25*a*b^4)*tan(f*x +
e)^2 - ((35*a^2*b^3 - 42*a*b^4 + 15*b^5)*tan(f*x + e)^5 + 2*(35*a^3*b^2 - 4
2*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^3 + (35*a^4*b - 42*a^3*b^2 + 15*a^2*b^3)
*tan(f*x + e))*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan
(f*x + e))))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5
+ 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a
^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.23, size = 231, normalized size = 1.22

$$\frac{\frac{(35\,a^2b^2-42\,ab^3+15\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^6-3\,a^5b+3\,a^4b^2-a^3b^3)\sqrt{ab}}-\frac{8\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{11\,ab^3\tan(fx+e)^3-7\,b^4\tan(fx+e)^3+13\,a^2b^2\tan(fx+e)-9\,ab^3\tan(fx+e)}{(a^5-2\,a^4b+a^3b^2)\left(b\tan(fx+e)^2+a\right)^2}-\frac{8}{a^3\tan(fx+e)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 - 42*a*b^3 + 15*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
+ arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*
```

```
sqrt(a*b)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (11*a*b^3*tan(f*
x + e)^3 - 7*b^4*tan(f*x + e)^3 + 13*a^2*b^2*tan(f*x + e) - 9*a*b^3*tan(f*x
 + e))/((a^5 - 2*a^4*b + a^3*b^2)*(b*tan(f*x + e)^2 + a)^2) - 8/(a^3*tan(f*
x + e)))/f
```

Mupad [B]
time = 15.05, size = 915, normalized size = 4.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (atan((b^5*tan(e + f*x)*(-a^7*b^3)^(3/2)*225i - a*b^4*tan(e + f*x)*(-a^7*b^
3)^(3/2)*1260i + a^4*b*tan(e + f*x)*(-a^7*b^3)^(3/2)*1225i + a^14*b*tan(e +
 f*x)*(-a^7*b^3)^(1/2)*64i + a^2*b^3*tan(e + f*x)*(-a^7*b^3)^(3/2)*2814i -
a^3*b^2*tan(e + f*x)*(-a^7*b^3)^(3/2)*2940i)/(225*a^11*b^9 - 1260*a^12*b^8
+ 2814*a^13*b^7 - 2940*a^14*b^6 + 1225*a^15*b^5 - 64*a^18*b^2))*(-a^7*b^3)^
(1/2)*(35*a^2 - 42*a*b + 15*b^2)*1i)/(8*f*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8
*b^2)) - (1/a + (tan(e + f*x)^4*(15*b^4 - 27*a*b^3 + 8*a^2*b^2))/(8*a^3*(a^
2 - 2*a*b + b^2)) + (tan(e + f*x)^2*(16*a^2*b - 45*a*b^2 + 25*b^3))/(8*a^2*
(a^2 - 2*a*b + b^2)))/(f*(a^2*tan(e + f*x) + b^2*tan(e + f*x)^5 + 2*a*b*tan
(e + f*x)^3)) - (2*atan((2*tan(e + f*x)*((262144*a^15*b^15 - 2883584*a^16*b
^14 + 14155776*a^17*b^13 - 40370176*a^18*b^12 + 72089600*a^19*b^11 - 778567
68*a^20*b^10 + 34603008*a^21*b^9 + 34603008*a^22*b^8 - 77856768*a^23*b^7 +
72089600*a^24*b^6 - 40370176*a^25*b^5 + 14155776*a^26*b^4 - 2883584*a^27*b^
3 + 262144*a^28*b^2)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)^2 - 230400*a^9*b^1
5 + 2672640*a^10*b^14 - 14078976*a^11*b^13 + 44261376*a^12*b^12 - 91801600*
a^13*b^11 + 131051520*a^14*b^10 - 130287616*a^15*b^9 + 89219072*a^16*b^8 -
40743936*a^17*b^7 + 11847680*a^18*b^6 - 2237440*a^19*b^5 + 393216*a^20*b^4
- 65536*a^21*b^3))/((6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*((2*(245760*a^12*b^
15 - 2899968*a^13*b^14 + 15613952*a^14*b^13 - 50577408*a^15*b^12 + 10928128
0*a^16*b^11 - 164659200*a^17*b^10 + 174882816*a^18*b^9 - 127893504*a^19*b^8
 + 58638336*a^20*b^7 - 10567680*a^21*b^6 - 5160960*a^22*b^5 + 4145152*a^23*
b^4 - 1179648*a^24*b^3 + 131072*a^25*b^2))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b
^3)^2 + 230400*a^9*b^12 - 1981440*a^10*b^11 + 7443456*a^11*b^10 - 15879168*
a^12*b^9 + 20933632*a^13*b^8 - 17363968*a^14*b^7 + 8788992*a^15*b^6 - 24586
24*a^16*b^5 + 286720*a^17*b^4))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))
```

3.248 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=240

$$\frac{x}{(a-b)^3}-\frac{b^{5/2}(63a^2-90ab+35b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^3f}+\frac{(8a^3+8a^2b-55ab^2+35b^3)\cot(e+fx)}{8a^4(a-b)^2f}-\frac{(8a^2}{}$$

[Out] x/(a-b)^3-1/8*b^(5/2)*(63*a^2-90*a*b+35*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(9/2)/(a-b)^3/f+1/8*(8*a^3+8*a^2*b-55*a*b^2+35*b^3)*cot(f*x+e)/a^4/(a-b)^2/f-1/24*(8*a^2-55*a*b+35*b^2)*cot(f*x+e)^3/a^3/(a-b)^2/f-1/4*b*cot(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(11*a-7*b)*b*cot(f*x+e)^3/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)

Rubi [A]

time = 0.25, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3751, 483, 593, 597, 536, 209, 211}

$$-\frac{b(11a-7b)\cot^3(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}-\frac{b^{5/2}(63a^2-90ab+35b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f(a-b)^3}-\frac{(8a^2-55ab+35b^2)\cot^3(e+fx)}{24a^3f(a-b)^2}+\frac{(8a^3+8a^2b-55ab^2+35b^3)\cot(e+fx)}{8a^4f(a-b)^2}-\frac{b\cot^3(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)^2}+\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]

[Out] x/(a - b)^3 - (b^(5/2)*(63*a^2 - 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(8*a^(9/2)*(a - b)^3*f) + ((8*a^3 + 8*a^2*b - 55*a*b^2 + 35*b^3)*Cot[e + f*x])/(8*a^4*(a - b)^2*f) - ((8*a^2 - 55*a*b + 35*b^2)*Cot[e + f*x]^3)/(24*a^3*(a - b)^2*f) - (b*Cot[e + f*x]^3)/(4*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - ((11*a - 7*b)*b*Cot[e + f*x]^3)/(8*a^2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
```

```
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{4a-7b-7bx^2}{x^4(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+f\right.}{4a(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{(11a-7b)b\cot^3(e+fx)}{8a^2(a-b)^2f\,(a+b\tan^2(e+fx))} + \text{Sul}$$

$$= -\frac{(8a^2-55ab+35b^2)\cot^3(e+fx)}{24a^3(a-b)^2f} - \frac{b\cot^3(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{}{8a^2}$$

$$= \frac{(8a^3+8a^2b-55ab^2+35b^3)\cot(e+fx)}{8a^4(a-b)^2f} - \frac{(8a^2-55ab+35b^2)\cot^3(e+fx)}{24a^3(a-b)^2f}$$

$$= \frac{(8a^3+8a^2b-55ab^2+35b^3)\cot(e+fx)}{8a^4(a-b)^2f} - \frac{(8a^2-55ab+35b^2)\cot^3(e+fx)}{24a^3(a-b)^2f}$$

$$= \frac{x}{(a-b)^3} - \frac{b^{5/2}(63a^2-90ab+35b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^3f} + \frac{(8a^3+8a^2b-5}{8a}$$

Mathematica [A]
time = 3.32, size = 184, normalized size = 0.77

$$\frac{-\frac{3b^{5/2}(63a^2-90ab+35b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{9/2}(a-b)^3} - \frac{8\cot(e+fx)(-4a-9b+a\csc^2(e+fx))}{a^4} + \frac{3\left(8(e+fx)-\frac{(a-b)b^3(17a^2+2ab-11b^2+(17a^2-28ab+11b^2)\cos(2(e+fx)))\sin(2(e+fx))}{a^4(a+b+(a-b)\cos(2(e+fx)))^2}\right)}{(a-b)^3}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] ((-3*b^(5/2)*(63*a^2 - 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a]])/(a^(9/2)*(a - b)^3) - (8*Cot[e + f*x]*(-4*a - 9*b + a*Csc[e + f*x]^2))
/a^4 + (3*(8*(e + f*x) - ((a - b)*b^3*(17*a^2 + 2*a*b - 11*b^2 + (17*a^2 -
28*a*b + 11*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a^4*(a + b + (a - b)*
Cos[2*(e + f*x)])^2)))/(a - b)^3)/(24*f)
```

Maple [A]
time = 0.54, size = 173, normalized size = 0.72

method	result

derivativedivides	$-\frac{\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b-\frac{13}{4}a\,b^2+\frac{11}{8}b^3\right)\left(\tan^3(fx+e)\right)+\frac{a\left(17a^2-30ab+13b^2\right)\tan(fx+e)}{8}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(63a^2-90ab+35b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4(a-b)^3}-\frac{}{3}\cdots}{f}$
default	$-\frac{\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b-\frac{13}{4}a\,b^2+\frac{11}{8}b^3\right)\left(\tan^3(fx+e)\right)+\frac{a\left(17a^2-30ab+13b^2\right)\tan(fx+e)}{8}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(63a^2-90ab+35b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4(a-b)^3}-\frac{}{3}\cdots}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b^3/a^4/(a-b)^3*(((15/8*a^2*b-13/4*a*b^2+11/8*b^3)*tan(f*x+e)^3+1/8*a
*(17*a^2-30*a*b+13*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(63*a^2-90*a*b
+35*b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/3/a^3/tan(f*x+e)^3
-(-a-3*b)/a^4/tan(f*x+e)+1/(a-b)^3*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.50, size = 340, normalized size = 1.42

$$-\frac{\frac{3\left(63\,a^2b^3-90\,ab^4+35\,b^5\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\left(a^7-3\,a^6b+3\,a^5b^2-a^4b^3\right)\sqrt{ab}}-\frac{3\left(8\,a^3b^2+8\,a^2b^3-55\,ab^4+35\,b^5\right)\tan(fx+e)^6-8\,a^5+16\,a^4b-8\,a^3b^2+\left(48\,a^4b+40\,a^3b^2-275\,a^2b^3+175\,ab^4\right)\tan(fx+e)^4+8\left(3\,a^5+a^4b-11\,a^3b^2+7\,a^2b^3\right)\tan(fx+e)^2}{\left(a^6b^2-2\,a^5b^3+a^4b^4\right)\tan(fx+e)^7+2\left(a^7b-2\,a^6b^2+a^5b^3\right)\tan(fx+e)^5+\left(a^8-2\,a^7b+a^6b^2\right)\tan(fx+e)^3}-\frac{24\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(3*(63*a^2*b^3 - 90*a*b^4 + 35*b^5)*arctan(b*tan(f*x + e)/sqrt(a*b))/
((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*sqrt(a*b)) - (3*(8*a^3*b^2 + 8*a^2*b
^3 - 55*a*b^4 + 35*b^5)*tan(f*x + e)^6 - 8*a^5 + 16*a^4*b - 8*a^3*b^2 + (48
*a^4*b + 40*a^3*b^2 - 275*a^2*b^3 + 175*a*b^4)*tan(f*x + e)^4 + 8*(3*a^5 +
a^4*b - 11*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2)/((a^6*b^2 - 2*a^5*b^3 + a^4
*b^4)*tan(f*x + e)^7 + 2*(a^7*b - 2*a^6*b^2 + a^5*b^3)*tan(f*x + e)^5 + (a^
8 - 2*a^7*b + a^6*b^2)*tan(f*x + e)^3) - 24*(f*x + e)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(229) = 458$.
time = 2.96, size = 1038, normalized size = 4.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(96*a^4*b^2*f*x*tan(f*x + e)^7 + 192*a^5*b*f*x*tan(f*x + e)^5 + 96*a^
6*f*x*tan(f*x + e)^3 + 12*(8*a^4*b^2 - 63*a^2*b^4 + 90*a*b^5 - 35*b^6)*tan(
f*x + e)^6 - 32*a^6 + 96*a^5*b - 96*a^4*b^2 + 32*a^3*b^3 + 4*(48*a^5*b - 8*
a^4*b^2 - 315*a^3*b^3 + 450*a^2*b^4 - 175*a*b^5)*tan(f*x + e)^4 + 32*(3*a^6
 - 2*a^5*b - 12*a^4*b^2 + 18*a^3*b^3 - 7*a^2*b^4)*tan(f*x + e)^2 - 3*((63*a
^2*b^4 - 90*a*b^5 + 35*b^6)*tan(f*x + e)^7 + 2*(63*a^3*b^3 - 90*a^2*b^4 + 3
5*a*b^5)*tan(f*x + e)^5 + (63*a^4*b^2 - 90*a^3*b^3 + 35*a^2*b^4)*tan(f*x +
e)^3)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(
a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*
a*b*tan(f*x + e)^2 + a^2)))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*
tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)
^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^3), 1/48*(48*a^4*
b^2*f*x*tan(f*x + e)^7 + 96*a^5*b*f*x*tan(f*x + e)^5 + 48*a^6*f*x*tan(f*x +
 e)^3 + 6*(8*a^4*b^2 - 63*a^2*b^4 + 90*a*b^5 - 35*b^6)*tan(f*x + e)^6 - 16*
a^6 + 48*a^5*b - 48*a^4*b^2 + 16*a^3*b^3 + 2*(48*a^5*b - 8*a^4*b^2 - 315*a^
3*b^3 + 450*a^2*b^4 - 175*a*b^5)*tan(f*x + e)^4 + 16*(3*a^6 - 2*a^5*b - 12*
a^4*b^2 + 18*a^3*b^3 - 7*a^2*b^4)*tan(f*x + e)^2 - 3*((63*a^2*b^4 - 90*a*b^
5 + 35*b^6)*tan(f*x + e)^7 + 2*(63*a^3*b^3 - 90*a^2*b^4 + 35*a*b^5)*tan(f*x
 + e)^5 + (63*a^4*b^2 - 90*a^3*b^3 + 35*a^2*b^4)*tan(f*x + e)^3)*sqrt(b/a)*
arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^7*b^2 -
3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 +
3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^
3)*f*tan(f*x + e)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.26, size = 261, normalized size = 1.09

$$-\frac{\frac{3\,(63\,a^2b^3-90\,ab^4+35\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^7-3\,a^6b+3\,a^5b^2-a^4b^3)\sqrt{ab}}-\frac{24\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{3\left(15\,ab^4\tan(fx+e)^3-11\,b^5\tan(fx+e)^3+17\,a^2b^3\tan(fx+e)-13\,ab^4\tan(fx+e)\right)}{(a^6-2\,a^5b+a^4b^2)\left(b\tan(fx+e)^2+a\right)^2}-\frac{8\left(3\,a\tan(fx+e)^2+9\,b\tan(fx+e)^2-a\right)}{a^4\tan(fx+e)^3}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3 - 90*a*b^4 + 35*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b
```

```
^3)*sqrt(a*b)) - 24*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 3*(15*a*b^4
*tan(f*x + e)^3 - 11*b^5*tan(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) - 13*a*b^
4*tan(f*x + e))/((a^6 - 2*a^5*b + a^4*b^2)*(b*tan(f*x + e)^2 + a)^2) - 8*(3
*a*tan(f*x + e)^2 + 9*b*tan(f*x + e)^2 - a)/(a^4*tan(f*x + e)^3))/f
```

Mupad [B]
time = 15.36, size = 986, normalized size = 4.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^3,x)

[Out] (2*atan((2*tan(e + f*x)*((262144*a^20*b^15 - 2883584*a^21*b^14 + 14155776*a
^22*b^13 - 40370176*a^23*b^12 + 72089600*a^24*b^11 - 77856768*a^25*b^10 + 3
4603008*a^26*b^9 + 34603008*a^27*b^8 - 77856768*a^28*b^7 + 72089600*a^29*b^
6 - 40370176*a^30*b^5 + 14155776*a^31*b^4 - 2883584*a^32*b^3 + 262144*a^33*
b^2)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)^2 - 1254400*a^12*b^17 + 13977600*a
^13*b^16 - 70333440*a^14*b^15 + 210329600*a^15*b^14 - 413730816*a^16*b^13 +
 559067136*a^17*b^12 - 525322240*a^18*b^11 + 338780160*a^19*b^10 - 14351257
6*a^20*b^9 + 36390912*a^21*b^8 - 5047296*a^22*b^7 + 1310720*a^23*b^6 - 9830
40*a^24*b^5 + 393216*a^25*b^4 - 65536*a^26*b^3))/((6*a*b^2 - 6*a^2*b + 2*a^
3 - 2*b^3)*((2*(573440*a^16*b^16 - 6635520*a^17*b^15 + 34947072*a^18*b^14 -
 110542848*a^19*b^13 + 233275392*a^20*b^12 - 344883200*a^21*b^11 + 36519936
0*a^22*b^10 - 279281664*a^23*b^9 + 155959296*a^24*b^8 - 67518464*a^25*b^7 +
 27279360*a^26*b^6 - 12042240*a^27*b^5 + 4718592*a^28*b^4 - 1179648*a^29*b^
3 + 131072*a^30*b^2))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)^2 + 1254400*a^12*
b^14 - 10214400*a^13*b^13 + 35927040*a^14*b^12 - 70650880*a^15*b^11 + 83495
936*a^16*b^10 - 58242048*a^17*b^9 + 20216832*a^18*b^8 - 17408*a^19*b^7 - 22
85568*a^20*b^6 + 516096*a^21*b^5))))/(f*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)
) + ((tan(e + f*x)^2*(3*a + 7*b))/(3*a^2) - 1/(3*a) + (tan(e + f*x)^6*(35*b
^5 - 55*a*b^4 + 8*a^2*b^3 + 8*a^3*b^2))/(8*a^4*(a^2 - 2*a*b + b^2)) + (tan(
e + f*x)^4*(48*a^3*b - 275*a*b^3 + 175*b^4 + 40*a^2*b^2))/(24*a^3*(a^2 - 2*
a*b + b^2)))/(f*(a^2*tan(e + f*x)^3 + b^2*tan(e + f*x)^7 + 2*a*b*tan(e + f*
x)^5)) - (atan((b^5*tan(e + f*x)*(-a^9*b^5)^(3/2)*1225i - a*b^4*tan(e + f*x
)*(-a^9*b^5)^(3/2)*6300i + a^4*b*tan(e + f*x)*(-a^9*b^5)^(3/2)*3969i + a^18
*b*tan(e + f*x)*(-a^9*b^5)^(1/2)*64i + a^2*b^3*tan(e + f*x)*(-a^9*b^5)^(3/2
)*12510i - a^3*b^2*tan(e + f*x)*(-a^9*b^5)^(3/2)*11340i)/(1225*a^14*b^12 -
6300*a^15*b^11 + 12510*a^16*b^10 - 11340*a^17*b^9 + 3969*a^18*b^8 - 64*a^23
*b^3))*(-a^9*b^5)^(1/2)*(63*a^2 - 90*a*b + 35*b^2)*1i)/(8*f*(3*a^11*b - a^1
2 + a^9*b^3 - 3*a^10*b^2))
```

3.249 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=297

$$-\frac{x}{(a-b)^3}+\frac{b^{7/2}(99a^2-154ab+63b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}(a-b)^3f}-\frac{(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4)\cot(e+$$

```
[Out] -x/(a-b)^3+1/8*b^(7/2)*(99*a^2-154*a*b+63*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^
(1/2))/a^(11/2)/(a-b)^3/f-1/8*(8*a^4+8*a^3*b+8*a^2*b^2-91*a*b^3+63*b^4)*cot
(f*x+e)/a^5/(a-b)^2/f+1/24*(8*a^3+8*a^2*b-91*a*b^2+63*b^3)*cot(f*x+e)^3/a^4
/(a-b)^2/f-1/40*(8*a^2-91*a*b+63*b^2)*cot(f*x+e)^5/a^3/(a-b)^2/f-1/4*b*cot(
f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(13*a-9*b)*b*cot(f*x+e)^5/a^2/(
a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Rubi [A]
time = 0.32, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3751, 483, 593, 597, 536, 209, 211}

$$-\frac{b(13a-9b)\cot^5(e+fx)}{8a^2f(a-b)^2\left(a+b\tan^2(e+fx)\right)}+\frac{b^{7/2}(99a^2-154ab+63b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f(a-b)^3}-\frac{(8a^2-91ab+63b^2)\cot^5(e+fx)}{40a^3f(a-b)^2}+\frac{(8a^3+8a^2b-91ab^2+63b^3)\cot^3(e+fx)}{24a^4f(a-b)^2}-\frac{(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4)\cot(e+fx)}{8a^5f(a-b)^2}-\frac{b\cot^5(e+fx)}{4af(a-b)\left(a+b\tan^2(e+fx)\right)^2}-\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] -(x/(a - b)^3) + (b^(7/2)*(99*a^2 - 154*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e
 + f*x])/Sqrt[a]])/(8*a^(11/2)*(a - b)^3*f) - ((8*a^4 + 8*a^3*b + 8*a^2*b^2
 - 91*a*b^3 + 63*b^4)*Cot[e + f*x])/(8*a^5*(a - b)^2*f) + ((8*a^3 + 8*a^2*b
 - 91*a*b^2 + 63*b^3)*Cot[e + f*x]^3)/(24*a^4*(a - b)^2*f) - ((8*a^2 - 91*a
*b + 63*b^2)*Cot[e + f*x]^5)/(40*a^3*(a - b)^2*f) - (b*Cot[e + f*x]^5)/(4*a
*(a - b)*f*(a + b*Tan[e + f*x]^2)^2) - ((13*a - 9*b)*b*Cot[e + f*x]^5)/(8*a
^2*(a - b)^2*f*(a + b*Tan[e + f*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} + \frac{\text{Subst}\left(\int \frac{4a-9b-9bx^2}{x^6(1+x^2)(a+bx^2)^2}\,dx, x, \tan(e+f\right.}{4a(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{(13a-9b)b\cot^5(e+fx)}{8a^2(a-b)^2f\,(a+b\tan^2(e+fx))} + \text{Sul}$$

$$= -\frac{(8a^2-91ab+63b^2)\cot^5(e+fx)}{40a^3(a-b)^2f} - \frac{b\cot^5(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{}{8a^2}$$

$$= \frac{(8a^3+8a^2b-91ab^2+63b^3)\cot^3(e+fx)}{24a^4(a-b)^2f} - \frac{(8a^2-91ab+63b^2)\cot^5(e+fx)}{40a^3(a-b)^2f}$$

$$= -\frac{(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4)\cot(e+fx)}{8a^5(a-b)^2f} + \frac{(8a^3+8a^2b-91ab^2+6}{24a^4(a-b}$$

$$= -\frac{(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4)\cot(e+fx)}{8a^5(a-b)^2f} + \frac{(8a^3+8a^2b-91ab^2+6}{24a^4(a-b}$$

$$= -\frac{x}{(a-b)^3} + \frac{b^{7/2}(99a^2-154ab+63b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}(a-b)^3f} - \frac{(8a^4+8a^3b}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 949 vs. 2(297) = 594.
time = 6.25, size = 949, normalized size = 3.20

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

```
[Out] (b^(7/2)*(99*a^2 - 154*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(8*a^(11/2)*(a - b)^3*f) + (Csc[e + f*x]^5*(-3184*a^7*Cos[e + f*x] + 7440
*a^6*b*Cos[e + f*x] - 12000*a^5*b^2*Cos[e + f*x] + 10240*a^4*b^3*Cos[e + f*
x] + 6450*a^3*b^4*Cos[e + f*x] + 714*a^2*b^5*Cos[e + f*x] - 22890*a*b^6*Cos
[e + f*x] + 13230*b^7*Cos[e + f*x] - 1536*a^7*Cos[3*(e + f*x)] + 7648*a^6*b
*Cos[3*(e + f*x)] - 2912*a^5*b^2*Cos[3*(e + f*x)] - 1152*a^4*b^3*Cos[3*(e +
 f*x)] - 14872*a^3*b^4*Cos[3*(e + f*x)] - 12796*a^2*b^5*Cos[3*(e + f*x)] +
52080*a*b^6*Cos[3*(e + f*x)] - 26460*b^7*Cos[3*(e + f*x)] - 704*a^7*Cos[5*(
```

e + f*x)] + 2656*a^6*b*Cos[5*(e + f*x)] - 4128*a^5*b^2*Cos[5*(e + f*x)] - 3712*a^4*b^3*Cos[5*(e + f*x)] + 5504*a^3*b^4*Cos[5*(e + f*x)] + 27684*a^2*b^5*Cos[5*(e + f*x)] - 46200*a*b^6*Cos[5*(e + f*x)] + 18900*b^7*Cos[5*(e + f*x)] - 536*a^7*Cos[7*(e + f*x)] + 248*a^6*b*Cos[7*(e + f*x)] + 768*a^5*b^2*Cos[7*(e + f*x)] + 128*a^4*b^3*Cos[7*(e + f*x)] + 6553*a^3*b^4*Cos[7*(e + f*x)] - 21441*a^2*b^5*Cos[7*(e + f*x)] + 20895*a*b^6*Cos[7*(e + f*x)] - 6615*b^7*Cos[7*(e + f*x)] - 184*a^7*Cos[9*(e + f*x)] + 440*a^6*b*Cos[9*(e + f*x)] - 160*a^5*b^2*Cos[9*(e + f*x)] + 640*a^4*b^3*Cos[9*(e + f*x)] - 3635*a^3*b^4*Cos[9*(e + f*x)] + 5839*a^2*b^5*Cos[9*(e + f*x)] - 3885*a*b^6*Cos[9*(e + f*x)] + 945*b^7*Cos[9*(e + f*x)] - 720*a^7*(e + f*x)*Sin[e + f*x] - 3360*a^6*b*(e + f*x)*Sin[e + f*x] - 15120*a^5*b^2*(e + f*x)*Sin[e + f*x] - 480*a^7*(e + f*x)*Sin[3*(e + f*x)] + 10080*a^5*b^2*(e + f*x)*Sin[3*(e + f*x)] + 480*a^7*(e + f*x)*Sin[5*(e + f*x)] + 1920*a^6*b*(e + f*x)*Sin[5*(e + f*x)] - 4320*a^5*b^2*(e + f*x)*Sin[5*(e + f*x)] + 120*a^7*(e + f*x)*Sin[7*(e + f*x)] - 1200*a^6*b*(e + f*x)*Sin[7*(e + f*x)] + 1080*a^5*b^2*(e + f*x)*Sin[7*(e + f*x)] - 120*a^7*(e + f*x)*Sin[9*(e + f*x)] + 240*a^6*b*(e + f*x)*Sin[9*(e + f*x)] - 120*a^5*b^2*(e + f*x)*Sin[9*(e + f*x)]))/(7680*a^5*(a - b)^3*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])^2)

Maple [A]

time = 0.60, size = 199, normalized size = 0.67

method	result
derivativedivides	$\frac{\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b-\frac{17}{4}a\,b^2+\frac{15}{8}b^3\right)\left(\tan^3(fx+e)\right)+\frac{a\left(21a^2-38ab+17b^2\right)\tan(fx+e)}{8}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(99a^2-154ab+63b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^5(a-b)^3}-\frac{}{5a}}{f}$
default	$\frac{\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b-\frac{17}{4}a\,b^2+\frac{15}{8}b^3\right)\left(\tan^3(fx+e)\right)+\frac{a\left(21a^2-38ab+17b^2\right)\tan(fx+e)}{8}}{\left(a+b\left(\tan^2(fx+e)\right)\right)^2}+\frac{\left(99a^2-154ab+63b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^5(a-b)^3}-\frac{}{5a}}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4/a^5/(a-b)^3*(((19/8*a^2*b-17/4*a*b^2+15/8*b^3)*tan(f*x+e)^3+1/8*a*
(21*a^2-38*a*b+17*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(99*a^2-154*a*b
+63*b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/5/a^3/tan(f*x+e)^5
-1/3*(-a-3*b)/a^4/tan(f*x+e)^3-(a^2+3*a*b+6*b^2)/a^5/tan(f*x+e)-1/(a-b)^3*a
rctan(tan(f*x+e)))
```

Maxima [A]

time = 0.50, size = 405, normalized size = 1.36

$$\frac{\frac{15\,(99\,a^2b^4-154\,ab^5+63\,b^6)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^8-3\,a^7b+3\,a^6b^2-a^5b^3)\sqrt{ab}}-\frac{15\,(8\,a^4b^2+8\,a^3b^3+8\,a^2b^4-91\,ab^5+63\,b^6)\tan(fx+e)^8+5\,(48\,a^5b+40\,a^4b^2+40\,a^3b^3-455\,a^2b^4+315\,ab^5)\tan(fx+e)^6+24\,a^6-48\,a^5b+24\,a^4b^2+8\,(15\,a^6+5\,a^5b+8\,a^4b^2-91\,a^3b^3+63\,a^2b^4)\tan(fx+e)^4-8\,(5\,a^6-a^5b-13\,a^4b^2+9\,a^3b^3)\tan(fx+e)^2}{(a^7b^2-2\,a^6b^3+a^5b^4)\tan(fx+e)^9+2\,(a^8b-2\,a^7b^2+a^6b^3)\tan(fx+e)^7+(a^9-2\,a^8b+a^7b^2)\tan(fx+e)^5}-\frac{120\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(15*(99*a^2*b^4 - 154*a*b^5 + 63*b^6)*arctan(b*tan(f*x + e)/sqrt(a*b)
)/((a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*sqrt(a*b)) - (15*(8*a^4*b^2 + 8*a^
3*b^3 + 8*a^2*b^4 - 91*a*b^5 + 63*b^6)*tan(f*x + e)^8 + 5*(48*a^5*b + 40*a^
4*b^2 + 40*a^3*b^3 - 455*a^2*b^4 + 315*a*b^5)*tan(f*x + e)^6 + 24*a^6 - 48*
a^5*b + 24*a^4*b^2 + 8*(15*a^6 + 5*a^5*b + 8*a^4*b^2 - 91*a^3*b^3 + 63*a^2*
b^4)*tan(f*x + e)^4 - 8*(5*a^6 - a^5*b - 13*a^4*b^2 + 9*a^3*b^3)*tan(f*x +
e)^2)/((a^7*b^2 - 2*a^6*b^3 + a^5*b^4)*tan(f*x + e)^9 + 2*(a^8*b - 2*a^7*b^
2 + a^6*b^3)*tan(f*x + e)^7 + (a^9 - 2*a^8*b + a^7*b^2)*tan(f*x + e)^5) - 1
20*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Fricas [A]
time = 4.39, size = 1148, normalized size = 3.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(480*a^5*b^2*f*x*tan(f*x + e)^9 + 960*a^6*b*f*x*tan(f*x + e)^7 + 48
0*a^7*f*x*tan(f*x + e)^5 + 60*(8*a^5*b^2 - 99*a^2*b^5 + 154*a*b^6 - 63*b^7)
*tan(f*x + e)^8 + 96*a^7 - 288*a^6*b + 288*a^5*b^2 - 96*a^4*b^3 + 20*(48*a^
6*b - 8*a^5*b^2 - 495*a^3*b^4 + 770*a^2*b^5 - 315*a*b^6)*tan(f*x + e)^6 + 3
2*(15*a^7 - 10*a^6*b + 3*a^5*b^2 - 99*a^4*b^3 + 154*a^3*b^4 - 63*a^2*b^5)*t
an(f*x + e)^4 - 32*(5*a^7 - 6*a^6*b - 12*a^5*b^2 + 22*a^4*b^3 - 9*a^3*b^4)*
tan(f*x + e)^2 + 15*((99*a^2*b^5 - 154*a*b^6 + 63*b^7)*tan(f*x + e)^9 + 2*(
99*a^3*b^4 - 154*a^2*b^5 + 63*a*b^6)*tan(f*x + e)^7 + (99*a^4*b^3 - 154*a^3
*b^4 + 63*a^2*b^5)*tan(f*x + e)^5)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a
*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b
/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a^8*b^2 - 3*a^7*
b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*
b^3 - a^6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*
tan(f*x + e)^5), -1/240*(240*a^5*b^2*f*x*tan(f*x + e)^9 + 480*a^6*b*f*x*tan
(f*x + e)^7 + 240*a^7*f*x*tan(f*x + e)^5 + 30*(8*a^5*b^2 - 99*a^2*b^5 + 154
*a*b^6 - 63*b^7)*tan(f*x + e)^8 + 48*a^7 - 144*a^6*b + 144*a^5*b^2 - 48*a^4
*b^3 + 10*(48*a^6*b - 8*a^5*b^2 - 495*a^3*b^4 + 770*a^2*b^5 - 315*a*b^6)*ta
n(f*x + e)^6 + 16*(15*a^7 - 10*a^6*b + 3*a^5*b^2 - 99*a^4*b^3 + 154*a^3*b^4
 - 63*a^2*b^5)*tan(f*x + e)^4 - 16*(5*a^7 - 6*a^6*b - 12*a^5*b^2 + 22*a^4*b
^3 - 9*a^3*b^4)*tan(f*x + e)^2 - 15*((99*a^2*b^5 - 154*a*b^6 + 63*b^7)*tan(
```

```
f*x + e)^9 + 2*(99*a^3*b^4 - 154*a^2*b^5 + 63*a*b^6)*tan(f*x + e)^7 + (99*a
^4*b^3 - 154*a^3*b^4 + 63*a^2*b^5)*tan(f*x + e)^5)*sqrt(b/a)*arctan(1/2*(b*
tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^8*b^2 - 3*a^7*b^3 + 3*
a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^
6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x
+ e)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.46, size = 307, normalized size = 1.03

$$\frac{\frac{15\,(99\,a^2b^4-154\,ab^5+63\,b^6)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)\right)}{(a^8-3\,a^7b+3\,a^6b^2-a^5b^3)\sqrt{ab}}-\frac{120\,(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{15\left(19\,ab^5\tan(fx+e)^3-15\,b^6\tan(fx+e)^3+21\,a^2b^4\tan(fx+e)-17\,ab^5\tan(fx+e)\right)}{(a^7-2\,a^6b+a^5b^2)\left(b\tan(fx+e)^2+a\right)^2}-\frac{8\left(15\,a^2\tan(fx+e)^4+45\,ab\tan(fx+e)^4+90\,b^2\tan(fx+e)^4-5\,a^2\tan(fx+e)^2-15\,ab\tan(fx+e)^2+3\,a^2\right)}{a^5\tan(fx+e)^5}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(99*a^2*b^4 - 154*a*b^5 + 63*b^6)*(pi*floor((f*x + e)/pi + 1/2)*s
gn(b) + arctan(b*tan(f*x + e)/sqrt(a*b)))/((a^8 - 3*a^7*b + 3*a^6*b^2 - a^5
*b^3)*sqrt(a*b)) - 120*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 15*(19*a
*b^5*tan(f*x + e)^3 - 15*b^6*tan(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) - 17*
a*b^5*tan(f*x + e))/((a^7 - 2*a^6*b + a^5*b^2)*(b*tan(f*x + e)^2 + a)^2) -
8*(15*a^2*tan(f*x + e)^4 + 45*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 -
5*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 + 3*a^2)/(a^5*tan(f*x + e)^5))
/f
```

Mupad [**B**]
time = 16.13, size = 2500, normalized size = 8.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^3,x)
```

```
[Out] (atan((b^5*tan(e + f*x)*(-a^11*b^7)^(3/2)*3969i - a*b^4*tan(e + f*x)*(-a^11
*b^7)^(3/2)*19404i + a^4*b*tan(e + f*x)*(-a^11*b^7)^(3/2)*9801i + a^22*b*ta
n(e + f*x)*(-a^11*b^7)^(1/2)*64i + a^2*b^3*tan(e + f*x)*(-a^11*b^7)^(3/2)*3
```

6190i - a^3*b^2*tan(e + f*x)*(-a^11*b^7)^(3/2)*30492i)/(3969*a^17*b^15 - 19
404*a^18*b^14 + 36190*a^19*b^13 - 30492*a^20*b^12 + 9801*a^21*b^11 - 64*a^2
8*b^4))*(-a^11*b^7)^(1/2)*(99*a^2 - 154*a*b + 63*b^2)*1i)/(8*f*(3*a^13*b -
a^14 + a^11*b^3 - 3*a^12*b^2)) - (1/(5*a) + (tan(e + f*x)^4*(35*a*b + 15*a^
2 + 63*b^2))/(15*a^3) - (tan(e + f*x)^2*(5*a + 9*b))/(15*a^2) + (tan(e + f*
x)^6*(48*a^4*b - 455*a*b^4 + 315*b^5 + 40*a^2*b^3 + 40*a^3*b^2))/(24*a^4*(a
^2 - 2*a*b + b^2)) + (tan(e + f*x)^8*(63*b^6 - 91*a*b^5 + 8*a^2*b^4 + 8*a^3
*b^3 + 8*a^4*b^2))/(8*a^5*(a^2 - 2*a*b + b^2)))/(f*(a^2*tan(e + f*x)^5 + b^
2*tan(e + f*x)^9 + 2*a*b*tan(e + f*x)^7)) - (2*atan(((((1032192*a^20*b^17 -
11812864*a^21*b^16 + 61489152*a^22*b^15 - 192135168*a^23*b^14 + 400392192*
a^24*b^13 - 584220672*a^25*b^12 + 608862208*a^26*b^11 - 452296704*a^27*b^10
+ 231653376*a^28*b^9 - 71122944*a^29*b^8 + 606208*a^30*b^7 + 14893056*a^31
*b^6 - 11010048*a^32*b^5 + 4718592*a^33*b^4 - 1179648*a^34*b^3 + 131072*a^3
5*b^2 + (tan(e + f*x)*(262144*a^25*b^15 - 2883584*a^26*b^14 + 14155776*a^27
*b^13 - 40370176*a^28*b^12 + 72089600*a^29*b^11 - 77856768*a^30*b^10 + 3460
3008*a^31*b^9 + 34603008*a^32*b^8 - 77856768*a^33*b^7 + 72089600*a^34*b^6 -
40370176*a^35*b^5 + 14155776*a^36*b^4 - 2883584*a^37*b^3 + 262144*a^38*b^2
)*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 -
2*b^3) + tan(e + f*x)*(4064256*a^15*b^19 - 44255232*a^16*b^18 + 217240576*
a^17*b^17 - 632905728*a^18*b^16 + 1211615232*a^19*b^15 - 1592176640*a^20*b^
14 + 1454180352*a^21*b^13 - 911302656*a^22*b^12 + 374944768*a^23*b^11 - 914
41152*a^24*b^10 + 10101760*a^25*b^9 - 393216*a^26*b^8 + 983040*a^27*b^7 - 1
310720*a^28*b^6 + 983040*a^29*b^5 - 393216*a^30*b^4 + 65536*a^31*b^3))/(6*a
*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (((1032192*a^20*b^17 - 11812864*a^21*b^16
+ 61489152*a^22*b^15 - 192135168*a^23*b^14 + 400392192*a^24*b^13 - 5842206
72*a^25*b^12 + 608862208*a^26*b^11 - 452296704*a^27*b^10 + 231653376*a^28*b
^9 - 71122944*a^29*b^8 + 606208*a^30*b^7 + 14893056*a^31*b^6 - 11010048*a^3
2*b^5 + 4718592*a^33*b^4 - 1179648*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*
x)*(262144*a^25*b^15 - 2883584*a^26*b^14 + 14155776*a^27*b^13 - 40370176*a^
28*b^12 + 72089600*a^29*b^11 - 77856768*a^30*b^10 + 34603008*a^31*b^9 + 346
03008*a^32*b^8 - 77856768*a^33*b^7 + 72089600*a^34*b^6 - 40370176*a^35*b^5
+ 14155776*a^36*b^4 - 2883584*a^37*b^3 + 262144*a^38*b^2)*1i)/(6*a*b^2 - 6*
a^2*b + 2*a^3 - 2*b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - tan(e + f
x)(4064256*a^15*b^19 - 44255232*a^16*b^18 + 217240576*a^17*b^17 - 6329057
28*a^18*b^16 + 1211615232*a^19*b^15 - 1592176640*a^20*b^14 + 1454180352*a^2
1*b^13 - 911302656*a^22*b^12 + 374944768*a^23*b^11 - 91441152*a^24*b^10 + 1
0101760*a^25*b^9 - 393216*a^26*b^8 + 983040*a^27*b^7 - 1310720*a^28*b^6 + 9
83040*a^29*b^5 - 393216*a^30*b^4 + 65536*a^31*b^3))/(6*a*b^2 - 6*a^2*b + 2*
a^3 - 2*b^3))/(((((1032192*a^20*b^17 - 11812864*a^21*b^16 + 61489152*a^22*b
^15 - 192135168*a^23*b^14 + 400392192*a^24*b^13 - 584220672*a^25*b^12 + 608
862208*a^26*b^11 - 452296704*a^27*b^10 + 231653376*a^28*b^9 - 71122944*a^29
*b^8 + 606208*a^30*b^7 + 14893056*a^31*b^6 - 11010048*a^32*b^5 + 4718592*a^
33*b^4 - 1179648*a^34*b^3 + 131072*a^35*b^2 + (tan(e + f*x)*(262144*a^25*b^
15 - 2883584*a^26*b^14 + 14155776*a^27*b^13 - 40370176*a^28*b^12 + 72089600
*a^29*b^11 - 77856768*a^30*b^10 + 34603008*a^31*b^9 + 34603008*a^32*b^8 - 7

7856768*a^33*b^7 + 72089600*a^34*b^6 - 40370176*a^35*b^5 + 14155776*a^36*b^4 - 2883584*a^37*b^3 + 262144*a^38*b^2)*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + tan(e + f*x)*(4064256*a^15*b^19 - 44255232*a^16*b^18 + 217240576*a^17*b^17 - 632905728*a^18*b^16 + 1211615232*a^19*b^15 - 1592176640*a^20*b^14 + 1454180352*a^21*b^13 - 911302656*a^22*b^12 + 374944768*a^23*b^11 - 91441152*a^24*b^10 + 10101760*a^25*b^9 - 393216*a^26*b^8 + 983040*a^27*b^7 - 1310720*a^28*b^6 + 983040*a^29*b^5 - 393216*a^30*b^4 + 65536*a^31*b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + ((((1032192*a^20*b^17 - 11812864*a^21*b^16 + 61489152*a^22*b^15 - 192135168*a^23*b^14 + 400392192*a^24*b^13 - 584220672*a^25*b^12 + 608862208*a^26*b^11 - 452296704*a^27*b^10 + 231653376*a^28*b^9 - 71122944*a^29*b^8 + 606208*a^30*b^7 + 14893056*a^31*b^6 - 11010048*a^32*b^5 + 4718592*a^33*b^4 - 1179648*a^34*b^3 + 131072*a^35*b^2 - (tan(e + f*x)*(262144*a^25*b^15 - 2883584*a^26*b^14 + 14155776*a^27*b^13 - 40370176*a^28*b^12 + 72089600*a^29*b^11 - 77856768*a^30*b^10 + 34603008*a^31*b^9 + 34603008*a^32*b^8 - 77856768*a^33*b^7 + 72089600*a^34*b^6 - 40370176*a^35*b^5 + 14155776*a^36*b^4 - 2883584*a^37*b^3 + 262144*a^38*b^2)*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - tan(e + f*x)*(4064256*a^15*b^19 - 44255232*a^16*b^18 + 217240576*a^17*b^17 - 632905728*a...

3.250 $\int (a+b\tan^2(c+dx))^4\, dx$

Optimal. Leaf size=115

$$(a-b)^4x+\frac{(2a-b)b(2a^2-2ab+b^2)\tan(c+dx)}{d}+\frac{b^2(6a^2-4ab+b^2)\tan^3(c+dx)}{3d}+\frac{(4a-b)b^3\tan^5(c+dx)}{5d}+$$

```
[Out] (a-b)^4*x+(2*a-b)*b*(2*a^2-2*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(6*a^2-4*a*b+b^2
)*tan(d*x+c)^3/d+1/5*(4*a-b)*b^3*tan(d*x+c)^5/d+1/7*b^4*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 398, 209}

$$\frac{b^2(6a^2-4ab+b^2)\tan^3(c+dx)}{3d}+\frac{b(2a-b)\left(2a^2-2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^3(4a-b)\tan^5(c+dx)}{5d}+x(a-b)^4+\frac{b^4\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^4,x]
```

```
[Out] (a - b)^4*x + ((2*a - b)*b*(2*a^2 - 2*a*b + b^2)*Tan[c + d*x])/d + (b^2*(6*
a^2 - 4*a*b + b^2)*Tan[c + d*x]^3)/(3*d) + ((4*a - b)*b^3*Tan[c + d*x]^5)/(
5*d) + (b^4*Tan[c + d*x]^7)/(7*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int (a+b\tan^2(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b(2a^2-2ab+b^2)+b^2(6a^2-4ab+b^2)\,x^2+(4a-b)b^3x^4\right.\right.}{d}$$
$$= \frac{(2a-b)b(2a^2-2ab+b^2)\tan(c+dx)}{d}+\frac{b^2(6a^2-4ab+b^2)\tan^3(c+dx)}{3d}+$$
$$= (a-b)^4x+\frac{(2a-b)b(2a^2-2ab+b^2)\tan(c+dx)}{d}+\frac{b^2(6a^2-4ab+b^2)\tan^3}{3d}$$

Mathematica [A]
time = 1.25, size = 137, normalized size = 1.19

$$\frac{\tan(c+dx)\left(\frac{105(a-b)^4\tanh^{-1}\left(\sqrt{-\tan^2(c+dx)}\right)}{\sqrt{-\tan^2(c+dx)}}+b(105(4a^3-6a^2b+4ab^2-b^3)+35b(6a^2-4ab+b^2)\tan^2(c+dx)+21(4a-b)b^2\tan^4(c+dx)+15b^3\tan^6(c+dx))\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^4,x]
[Out] (Tan[c + d*x]*((105*(a - b)^4*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c +
 d*x]^2] + b*(105*(4*a^3 - 6*a^2*b + 4*a*b^2 - b^3) + 35*b*(6*a^2 - 4*a*b +
 b^2)*Tan[c + d*x]^2 + 21*(4*a - b)*b^2*Tan[c + d*x]^4 + 15*b^3*Tan[c + d*x
]^6)))/(105*d)
```

Maple [A]
time = 0.07, size = 173, normalized size = 1.50

method	result
norman	$(a^4-4a^3b+6a^2b^2-4a\,b^3+b^4)\,x+\frac{b(4a^3-6a^2b+4a\,b^2-b^3)\tan(dx+c)}{d}+\frac{b^4(\tan^7(dx+c))}{7d}+\frac{b^2(6a^2-}{}$
derivativedivides	$\frac{\frac{b^4(\tan^7(dx+c))}{7}+\frac{4a\,b^3(\tan^5(dx+c))}{5}-\frac{b^4(\tan^5(dx+c))}{5}+2a^2b^2(\tan^3(dx+c))-\frac{4a\,b^3(\tan^3(dx+c))}{3}+\frac{b^4(\tan^3(dx+c))}{3}+4a^3b\,t}{d}$
default	$\frac{\frac{b^4(\tan^7(dx+c))}{7}+\frac{4a\,b^3(\tan^5(dx+c))}{5}-\frac{b^4(\tan^5(dx+c))}{5}+2a^2b^2(\tan^3(dx+c))-\frac{4a\,b^3(\tan^3(dx+c))}{3}+\frac{b^4(\tan^3(dx+c))}{3}+4a^3b\,t}{d}$
risch	$a^4x-4a^3bx+6a^2b^2x-4a\,b^3x+b^4x-\frac{8ib(44b^3-161a\,b^2+210a^2b-105a^3-1575a^3e^{4i(dx+c)}+609b^3e^{4i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/7*b^4*tan(d*x+c)^7+4/5*a*b^3*tan(d*x+c)^5-1/5*b^4*tan(d*x+c)^5+2*a^2
*b^2*tan(d*x+c)^3-4/3*a*b^3*tan(d*x+c)^3+1/3*b^4*tan(d*x+c)^3+4*a^3*b*tan(d
```

```
*x+c)-6*a^2*b^2*tan(d*x+c)+4*a*b^3*tan(d*x+c)-b^4*tan(d*x+c)+(a^4-4*a^3*b+6
*a^2*b^2-4*a*b^3+b^4)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 162, normalized size = 1.41

$$a^4x - \frac{4\,(dx + c - \tan(dx + c))a^3b}{d} + \frac{2\left(\tan(dx + c)^3 + 3\,dx + 3\,c - 3\,\tan(dx + c)\right)a^2b^2}{d} + \frac{4\left(3\,\tan(dx + c)^5 - 5\,\tan(dx + c)^3 - 15\,dx - 15\,c + 15\,\tan(dx + c)\right)ab^3}{15\,d} + \frac{(15\,\tan(dx + c)^7 - 21\,\tan(dx + c)^5 + 35\,\tan(dx + c)^3 + 105\,dx + 105\,c - 105\,\tan(dx + c))b^4}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x - 4*(d*x + c - tan(d*x + c))*a^3*b/d + 2*(tan(d*x + c)^3 + 3*d*x + 3*
c - 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 -
 15*d*x - 15*c + 15*tan(d*x + c))*a*b^3/d + 1/105*(15*tan(d*x + c)^7 - 21*t
an(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c - 105*tan(d*x + c))*b^4
/d
```

Fricas [A]
time = 4.24, size = 134, normalized size = 1.17

$$\frac{15\,b^4\tan(dx + c)^7 + 21\,(4\,ab^3 - b^4)\tan(dx + c)^5 + 35\,(6\,a^2b^2 - 4\,ab^3 + b^4)\tan(dx + c)^3 + 105\,(a^4 - 4\,a^3b + 6\,a^2b^2 - 4\,ab^3 + b^4)dx + 105\,(4\,a^3b - 6\,a^2b^2 + 4\,ab^3 - b^4)\tan(dx + c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 21*(4*a*b^3 - b^4)*tan(d*x + c)^5 + 35*(6*a^
2*b^2 - 4*a*b^3 + b^4)*tan(d*x + c)^3 + 105*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x + 105*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*tan(d*x + c))/
d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(100) = 200$.
time = 0.23, size = 209, normalized size = 1.82

$$\begin{cases} a^4x - 4a^3bx + \frac{4a^3b\tan(c+dx)}{d} + 6a^2b^2x + \frac{2a^2b^2\tan^3(c+dx)}{d} - \frac{6a^2b^2\tan(c+dx)}{d} - 4ab^3x + \frac{4ab^3\tan^5(c+dx)}{5d} - \frac{4ab^3\tan^3(c+dx)}{3d} + \frac{4ab^3\tan(c+dx)}{d} + b^4x + \frac{b^4\tan^7(c+dx)}{7d} - \frac{b^4\tan^5(c+dx)}{5d} + \frac{b^4\tan^3(c+dx)}{3d} - \frac{b^4\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a + b\tan^2(c))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**4,x)
```

```
[Out] Piecewise((a**4*x - 4*a**3*b*x + 4*a**3*b*tan(c + d*x)/d + 6*a**2*b**2*x +
2*a**2*b**2*tan(c + d*x)**3/d - 6*a**2*b**2*tan(c + d*x)/d - 4*a*b**3*x + 4
*a*b**3*tan(c + d*x)**5/(5*d) - 4*a*b**3*tan(c + d*x)**3/(3*d) + 4*a*b**3*t
an(c + d*x)/d + b**4*x + b**4*tan(c + d*x)**7/(7*d) - b**4*tan(c + d*x)**5/
(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c + d*x)/d, Ne(d, 0)), (x*(a
+ b*tan(c)**2)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2209 vs. $2(109) = 218$.
time = 2.27, size = 2209, normalized size = 19.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^4*d*x*tan(d*x)^7*tan(c)^7 - 420*a^3*b*d*x*tan(d*x)^7*tan(c)^7
+ 630*a^2*b^2*d*x*tan(d*x)^7*tan(c)^7 - 420*a*b^3*d*x*tan(d*x)^7*tan(c)^7 +
 105*b^4*d*x*tan(d*x)^7*tan(c)^7 - 735*a^4*d*x*tan(d*x)^6*tan(c)^6 + 2940*a
^3*b*d*x*tan(d*x)^6*tan(c)^6 - 4410*a^2*b^2*d*x*tan(d*x)^6*tan(c)^6 + 2940*
a*b^3*d*x*tan(d*x)^6*tan(c)^6 - 735*b^4*d*x*tan(d*x)^6*tan(c)^6 - 420*a^3*b
*tan(d*x)^7*tan(c)^6 + 630*a^2*b^2*tan(d*x)^7*tan(c)^6 - 420*a*b^3*tan(d*x)
^7*tan(c)^6 + 105*b^4*tan(d*x)^7*tan(c)^6 - 420*a^3*b*tan(d*x)^6*tan(c)^7 +
 630*a^2*b^2*tan(d*x)^6*tan(c)^7 - 420*a*b^3*tan(d*x)^6*tan(c)^7 + 105*b^4*
tan(d*x)^6*tan(c)^7 + 2205*a^4*d*x*tan(d*x)^5*tan(c)^5 - 8820*a^3*b*d*x*tan
(d*x)^5*tan(c)^5 + 13230*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 - 8820*a*b^3*d*x*t
an(d*x)^5*tan(c)^5 + 2205*b^4*d*x*tan(d*x)^5*tan(c)^5 - 210*a^2*b^2*tan(d*x
)^7*tan(c)^4 + 140*a*b^3*tan(d*x)^7*tan(c)^4 - 35*b^4*tan(d*x)^7*tan(c)^4 +
 2520*a^3*b*tan(d*x)^6*tan(c)^5 - 4410*a^2*b^2*tan(d*x)^6*tan(c)^5 + 2940*a
*b^3*tan(d*x)^6*tan(c)^5 - 735*b^4*tan(d*x)^6*tan(c)^5 + 2520*a^3*b*tan(d*x
)^5*tan(c)^6 - 4410*a^2*b^2*tan(d*x)^5*tan(c)^6 + 2940*a*b^3*tan(d*x)^5*tan
(c)^6 - 735*b^4*tan(d*x)^5*tan(c)^6 - 210*a^2*b^2*tan(d*x)^4*tan(c)^7 + 140
*a*b^3*tan(d*x)^4*tan(c)^7 - 35*b^4*tan(d*x)^4*tan(c)^7 - 3675*a^4*d*x*tan(
d*x)^4*tan(c)^4 + 14700*a^3*b*d*x*tan(d*x)^4*tan(c)^4 - 22050*a^2*b^2*d*x*t
an(d*x)^4*tan(c)^4 + 14700*a*b^3*d*x*tan(d*x)^4*tan(c)^4 - 3675*b^4*d*x*tan
(d*x)^4*tan(c)^4 - 84*a*b^3*tan(d*x)^7*tan(c)^2 + 21*b^4*tan(d*x)^7*tan(c)^
2 + 840*a^2*b^2*tan(d*x)^6*tan(c)^3 - 980*a*b^3*tan(d*x)^6*tan(c)^3 + 245*b
^4*tan(d*x)^6*tan(c)^3 - 6300*a^3*b*tan(d*x)^5*tan(c)^4 + 11970*a^2*b^2*tan
(d*x)^5*tan(c)^4 - 8820*a*b^3*tan(d*x)^5*tan(c)^4 + 2205*b^4*tan(d*x)^5*tan
(c)^4 - 6300*a^3*b*tan(d*x)^4*tan(c)^5 + 11970*a^2*b^2*tan(d*x)^4*tan(c)^5
- 8820*a*b^3*tan(d*x)^4*tan(c)^5 + 2205*b^4*tan(d*x)^4*tan(c)^5 + 840*a^2*b
^2*tan(d*x)^3*tan(c)^6 - 980*a*b^3*tan(d*x)^3*tan(c)^6 + 245*b^4*tan(d*x)^3
*tan(c)^6 - 84*a*b^3*tan(d*x)^2*tan(c)^7 + 21*b^4*tan(d*x)^2*tan(c)^7 + 367
5*a^4*d*x*tan(d*x)^3*tan(c)^3 - 14700*a^3*b*d*x*tan(d*x)^3*tan(c)^3 + 22050
*a^2*b^2*d*x*tan(d*x)^3*tan(c)^3 - 14700*a*b^3*d*x*tan(d*x)^3*tan(c)^3 + 36
75*b^4*d*x*tan(d*x)^3*tan(c)^3 - 15*b^4*tan(d*x)^7 + 168*a*b^3*tan(d*x)^6*t
an(c) - 147*b^4*tan(d*x)^6*tan(c) - 1260*a^2*b^2*tan(d*x)^5*tan(c)^2 + 1680
*a*b^3*tan(d*x)^5*tan(c)^2 - 735*b^4*tan(d*x)^5*tan(c)^2 + 8400*a^3*b*tan(d
*x)^4*tan(c)^3 - 16380*a^2*b^2*tan(d*x)^4*tan(c)^3 + 12600*a*b^3*tan(d*x)^4
*tan(c)^3 - 3675*b^4*tan(d*x)^4*tan(c)^3 + 8400*a^3*b*tan(d*x)^3*tan(c)^4 -
 16380*a^2*b^2*tan(d*x)^3*tan(c)^4 + 12600*a*b^3*tan(d*x)^3*tan(c)^4 - 3675
*b^4*tan(d*x)^3*tan(c)^4 - 1260*a^2*b^2*tan(d*x)^2*tan(c)^5 + 1680*a*b^3*ta
```

```
n(d*x)^2*tan(c)^5 - 735*b^4*tan(d*x)^2*tan(c)^5 + 168*a*b^3*tan(d*x)*tan(c)
^6 - 147*b^4*tan(d*x)*tan(c)^6 - 15*b^4*tan(c)^7 - 2205*a^4*d*x*tan(d*x)^2*
tan(c)^2 + 8820*a^3*b*d*x*tan(d*x)^2*tan(c)^2 - 13230*a^2*b^2*d*x*tan(d*x)^
2*tan(c)^2 + 8820*a*b^3*d*x*tan(d*x)^2*tan(c)^2 - 2205*b^4*d*x*tan(d*x)^2*t
an(c)^2 - 84*a*b^3*tan(d*x)^5 + 21*b^4*tan(d*x)^5 + 840*a^2*b^2*tan(d*x)^4*
tan(c) - 980*a*b^3*tan(d*x)^4*tan(c) + 245*b^4*tan(d*x)^4*tan(c) - 6300*a^3
*b*tan(d*x)^3*tan(c)^2 + 11970*a^2*b^2*tan(d*x)^3*tan(c)^2 - 8820*a*b^3*tan
(d*x)^3*tan(c)^2 + 2205*b^4*tan(d*x)^3*tan(c)^2 - 6300*a^3*b*tan(d*x)^2*tan
(c)^3 + 11970*a^2*b^2*tan(d*x)^2*tan(c)^3 - 8820*a*b^3*tan(d*x)^2*tan(c)^3
+ 2205*b^4*tan(d*x)^2*tan(c)^3 + 840*a^2*b^2*tan(d*x)*tan(c)^4 - 980*a*b^3*
tan(d*x)*tan(c)^4 + 245*b^4*tan(d*x)*tan(c)^4 - 84*a*b^3*tan(c)^5 + 21*b^4*
tan(c)^5 + 735*a^4*d*x*tan(d*x)*tan(c) - 2940*a^3*b*d*x*tan(d*x)*tan(c) + 4
410*a^2*b^2*d*x*tan(d*x)*tan(c) - 2940*a*b^3*d*x*tan(d*x)*tan(c) + 735*b^4*
d*x*tan(d*x)*tan(c) - 210*a^2*b^2*tan(d*x)^3 + 140*a*b^3*tan(d*x)^3 - 35*b^
4*tan(d*x)^3 + 2520*a^3*b*tan(d*x)^2*tan(c) - 4410*a^2*b^2*tan(d*x)^2*tan(c
) + 2940*a*b^3*tan(d*x)^2*tan(c) - 735*b^4*tan(d*x)^2*tan(c) + 2520*a^3*b*t
an(d*x)*tan(c)^2 - 4410*a^2*b^2*tan(d*x)*tan(c)^2 + 2940*a*b^3*tan(d*x)*tan
(c)^2 - 735*b^4*tan(d*x)*tan(c)^2 - 210*a^2*b^2*tan(c)^3 + 140*a*b^3*tan(c)
^3 - 35*b^4*tan(c)^3 - 105*a^4*d*x + 420*a^3*b*d*x - 630*a^2*b^2*d*x + 420*
a*b^3*d*x - 105*b^4*d*x - 420*a^3*b*tan(d*x) + 630*a^2*b^2*tan(d*x) - 420*a
*b^3*tan(d*x) + 105*b^4*tan(d*x) - 420*a^3*b*tan(c) + 630*a^2*b^2*tan(c) -
420*a*b^3*tan(c) + 105*b^4*tan(c))/(d*tan(d*x)^7*tan(c)^7 - 7*d*tan(d*x)^6*
tan(c)^6 + 21*d*tan(d*x)^5*tan(c)^5 - 35*d*tan(d*x)^4*tan(c)^4 + 35*d*tan(d
*x)^3*tan(c)^3 - 21*d*tan(d*x)^2*tan(c)^2 + 7*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 11.38, size = 164, normalized size = 1.43

$$\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)^4}{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}\right)\,(a-b)^4}{d}+\frac{b^4\tan(c+d\,x)^7}{7\,d}+\frac{\tan(c+d\,x)^3\,\left(2\,a^2\,b^2-\frac{4\,a\,b^3}{3}+\frac{b^4}{3}\right)}{d}+\frac{\tan(c+d\,x)^5\,\left(\frac{4\,a\,b^3}{5}-\frac{b^4}{5}\right)}{d}+\frac{\tan(c+d\,x)\,\left(4\,a^3\,b-6\,a^2\,b^2+4\,a\,b^3-b^4\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^4,x)
```

```
[Out] (atan((tan(c + d*x)*(a - b)^4)/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))
*(a - b)^4)/d + (b^4*tan(c + d*x)^7)/(7*d) + (tan(c + d*x)^3*(b^4/3 - (4*a*
b^3)/3 + 2*a^2*b^2))/d + (tan(c + d*x)^5*((4*a*b^3)/5 - b^4/5))/d + (tan(c
+ d*x)*(4*a*b^3 + 4*a^3*b - b^4 - 6*a^2*b^2))/d
```

3.251 $\int (a+b\tan^2(c+dx))^3 \, dx$

Optimal. Leaf size=77

$$(a-b)^3x+\frac{b(3a^2-3ab+b^2)\tan(c+dx)}{d}+\frac{(3a-b)b^2\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

```
[Out] (a-b)^3*x+b*(3*a^2-3*a*b+b^2)*tan(d*x+c)/d+1/3*(3*a-b)*b^2*tan(d*x+c)^3/d+1
/5*b^3*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 398, 209}

$$\frac{b(3a^2-3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a-b)\tan^3(c+dx)}{3d}+x(a-b)^3+\frac{b^3\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^3,x]
```

```
[Out] (a - b)^3*x + (b*(3*a^2 - 3*a*b + b^2)*Tan[c + d*x])/d + ((3*a - b)*b^2*Tan
[c + d*x]^3)/(3*d) + (b^3*Tan[c + d*x]^5)/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b(3a^2-3ab+b^2)+(3a-b)b^2x^2+b^3x^4+\frac{(a-b)^3}{1+x^2}\right)dx, x, \tan(c+dx\right.}{d}$$
$$= \frac{b(3a^2-3ab+b^2)\tan(c+dx)}{d}+\frac{(3a-b)b^2\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}+$$
$$= (a-b)^3x+\frac{b(3a^2-3ab+b^2)\tan(c+dx)}{d}+\frac{(3a-b)b^2\tan^3(c+dx)}{3d}+\frac{b^3\tan^5}{}$$

Mathematica [A]
time = 0.62, size = 102, normalized size = 1.32

$$\frac{\tan(c+dx)\left(\frac{15(a-b)^3\tanh^{-1}\left(\sqrt{-\tan^2(c+dx)}\right)}{\sqrt{-\tan^2(c+dx)}}+b(45a^2-15ab(3-\tan^2(c+dx))+b^2(15-5\tan^2(c+dx)+3\tan^4(c+dx)))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^3,x]
```

```
[Out] (Tan[c + d*x]*((15*(a - b)^3*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c +
d*x]^2] + b*(45*a^2 - 15*a*b*(3 - Tan[c + d*x]^2) + b^2*(15 - 5*Tan[c + d*x
]^2 + 3*Tan[c + d*x]^4))))/(15*d)
```

Maple [A]
time = 0.05, size = 108, normalized size = 1.40

method	result
norman	$(a^3-3a^2b+3a\,b^2-b^3)\,x+\frac{b(3a^2-3ab+b^2)\tan(dx+c)}{d}+\frac{b^3(\tan^5(dx+c))}{5d}+\frac{(3a-b)b^2(\tan^3(dx+c))}{3d}$
derivativedivides	$\frac{\frac{b^3\left(\tan^5(dx+c)\right)}{5}+a\,b^2(\tan^3(dx+c))-\frac{b^3\left(\tan^3(dx+c)\right)}{3}+3a^2b\tan(dx+c)-3a\,b^2\tan(dx+c)+b^3\tan(dx+c)+(a^3-3a^2b+3a\,b^2}{d}$
default	$\frac{\frac{b^3\left(\tan^5(dx+c)\right)}{5}+a\,b^2(\tan^3(dx+c))-\frac{b^3\left(\tan^3(dx+c)\right)}{3}+3a^2b\tan(dx+c)-3a\,b^2\tan(dx+c)+b^3\tan(dx+c)+(a^3-3a^2b+3a\,b^2}{d}$
risch	$a^3x-3a^2bx+3a\,b^2x-b^3x+\frac{2ib(45a^2e^{8i(dx+c)}-90ab\,e^{8i(dx+c)}+45b^2e^{8i(dx+c)}+180a^2e^{6i(dx+c)}-270ab\,e^{6i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b^3*tan(d*x+c)^5+a*b^2*tan(d*x+c)^3-1/3*b^3*tan(d*x+c)^3+3*a^2*b*t
an(d*x+c)-3*a*b^2*tan(d*x+c)+b^3*tan(d*x+c)+(a^3-3*a^2*b+3*a*b^2-b^3)*arcta
n(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 104, normalized size = 1.35

$$a^3x - \frac{3\,(dx+c-\tan(dx+c))a^2b}{d} + \frac{(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c))ab^2}{d} + \frac{(3\,\tan(dx+c)^5-5\,\tan(dx+c)^3-15\,dx-15\,c+15\,\tan(dx+c))b^3}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 3*(d*x + c - tan(d*x + c))*a^2*b/d + (tan(d*x + c)^3 + 3*d*x + 3*c
- 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*
d*x - 15*c + 15*tan(d*x + c))*b^3/d
```

Fricas [A]
time = 3.60, size = 90, normalized size = 1.17

$$\frac{3\,b^3\tan(dx+c)^5+5\,(3\,ab^2-b^3)\tan(dx+c)^3+15\,(a^3-3\,a^2b+3\,ab^2-b^3)dx+15\,(3\,a^2b-3\,ab^2+b^3)\tan(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^3*tan(d*x + c)^5 + 5*(3*a*b^2 - b^3)*tan(d*x + c)^3 + 15*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d*x + 15*(3*a^2*b - 3*a*b^2 + b^3)*tan(d*x + c))/d
```

Sympy [A]
time = 0.15, size = 126, normalized size = 1.64

$$\begin{cases} a^3x - 3a^2bx + \frac{3a^2b\tan(c+dx)}{d} + 3ab^2x + \frac{ab^2\tan^3(c+dx)}{d} - \frac{3ab^2\tan(c+dx)}{d} - b^3x + \frac{b^3\tan^5(c+dx)}{5d} - \frac{b^3\tan^3(c+dx)}{3d} + \frac{b^3\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\tan^2(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*x + 3*a**2*b*tan(c + d*x)/d + 3*a*b**2*x + a*b
**2*tan(c + d*x)**3/d - 3*a*b**2*tan(c + d*x)/d - b**3*x + b**3*tan(c + d*x
)**5/(5*d) - b**3*tan(c + d*x)**3/(3*d) + b**3*tan(c + d*x)/d, Ne(d, 0)), (
x*(a + b*tan(c)**2)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(73) = 146.
time = 0.92, size = 1027, normalized size = 13.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="giac")
```

[Out] 1/15*(15*a^3*d*x*tan(d*x)^5*tan(c)^5 - 45*a^2*b*d*x*tan(d*x)^5*tan(c)^5 + 4
5*a*b^2*d*x*tan(d*x)^5*tan(c)^5 - 15*b^3*d*x*tan(d*x)^5*tan(c)^5 - 75*a^3*d
*x*tan(d*x)^4*tan(c)^4 + 225*a^2*b*d*x*tan(d*x)^4*tan(c)^4 - 225*a*b^2*d*x*
tan(d*x)^4*tan(c)^4 + 75*b^3*d*x*tan(d*x)^4*tan(c)^4 - 45*a^2*b*tan(d*x)^5*
tan(c)^4 + 45*a*b^2*tan(d*x)^5*tan(c)^4 - 15*b^3*tan(d*x)^5*tan(c)^4 - 45*a
^2*b*tan(d*x)^4*tan(c)^5 + 45*a*b^2*tan(d*x)^4*tan(c)^5 - 15*b^3*tan(d*x)^4
*tan(c)^5 + 150*a^3*d*x*tan(d*x)^3*tan(c)^3 - 450*a^2*b*d*x*tan(d*x)^3*tan(
c)^3 + 450*a*b^2*d*x*tan(d*x)^3*tan(c)^3 - 150*b^3*d*x*tan(d*x)^3*tan(c)^3
- 15*a*b^2*tan(d*x)^5*tan(c)^2 + 5*b^3*tan(d*x)^5*tan(c)^2 + 180*a^2*b*tan(
d*x)^4*tan(c)^3 - 225*a*b^2*tan(d*x)^4*tan(c)^3 + 75*b^3*tan(d*x)^4*tan(c)^
3 + 180*a^2*b*tan(d*x)^3*tan(c)^4 - 225*a*b^2*tan(d*x)^3*tan(c)^4 + 75*b^3*
tan(d*x)^3*tan(c)^4 - 15*a*b^2*tan(d*x)^2*tan(c)^5 + 5*b^3*tan(d*x)^2*tan(c
)^5 - 150*a^3*d*x*tan(d*x)^2*tan(c)^2 + 450*a^2*b*d*x*tan(d*x)^2*tan(c)^2 -
 450*a*b^2*d*x*tan(d*x)^2*tan(c)^2 + 150*b^3*d*x*tan(d*x)^2*tan(c)^2 - 3*b^
3*tan(d*x)^5 + 30*a*b^2*tan(d*x)^4*tan(c) - 25*b^3*tan(d*x)^4*tan(c) - 270*
a^2*b*tan(d*x)^3*tan(c)^2 + 360*a*b^2*tan(d*x)^3*tan(c)^2 - 150*b^3*tan(d*x
)^3*tan(c)^2 - 270*a^2*b*tan(d*x)^2*tan(c)^3 + 360*a*b^2*tan(d*x)^2*tan(c)^
3 - 150*b^3*tan(d*x)^2*tan(c)^3 + 30*a*b^2*tan(d*x)*tan(c)^4 - 25*b^3*tan(d
*x)*tan(c)^4 - 3*b^3*tan(c)^5 + 75*a^3*d*x*tan(d*x)*tan(c) - 225*a^2*b*d*x*
tan(d*x)*tan(c) + 225*a*b^2*d*x*tan(d*x)*tan(c) - 75*b^3*d*x*tan(d*x)*tan(c
) - 15*a*b^2*tan(d*x)^3 + 5*b^3*tan(d*x)^3 + 180*a^2*b*tan(d*x)^2*tan(c) -
225*a*b^2*tan(d*x)^2*tan(c) + 75*b^3*tan(d*x)^2*tan(c) + 180*a^2*b*tan(d*x)
*tan(c)^2 - 225*a*b^2*tan(d*x)*tan(c)^2 + 75*b^3*tan(d*x)*tan(c)^2 - 15*a*b
^2*tan(c)^3 + 5*b^3*tan(c)^3 - 15*a^3*d*x + 45*a^2*b*d*x - 45*a*b^2*d*x + 1
5*b^3*d*x - 45*a^2*b*tan(d*x) + 45*a*b^2*tan(d*x) - 15*b^3*tan(d*x) - 45*a^
2*b*tan(c) + 45*a*b^2*tan(c) - 15*b^3*tan(c))/(d*tan(d*x)^5*tan(c)^5 - 5*d*
tan(d*x)^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 +
 5*d*tan(d*x)*tan(c) - d)

Mupad [B]
time = 11.47, size = 115, normalized size = 1.49

$$\frac{b^3\tan(c+d\,x)^5}{5\,d}+\frac{\tan(c+d\,x)\,\left(3\,a^2\,b-3\,a\,b^2+b^3\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)^3}{a^3-3\,a^2\,b+3\,a\,b^2-b^3}\right)\,(a-b)^3}{d}+\frac{\tan(c+d\,x)^3\,\left(a\,b^2-\frac{b^3}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(c + d*x)^2)^3,x)

[Out] (b^3*tan(c + d*x)^5)/(5*d) + (tan(c + d*x)*(3*a^2*b - 3*a*b^2 + b^3))/d + (
atan((tan(c + d*x)*(a - b)^3)/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(a - b)^3)/d
 + (tan(c + d*x)^3*(a*b^2 - b^3/3))/d

3.252 $\int (a+b\tan^2(c+dx))^2 \, dx$

Optimal. Leaf size=46

$$(a-b)^2x+\frac{(2a-b)b\tan(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

```
[Out] (a-b)^2*x+(2*a-b)*b*tan(d*x+c)/d+1/3*b^2*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 398, 209}

$$\frac{b(2a-b)\tan(c+dx)}{d}+x(a-b)^2+\frac{b^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a - b)^2*x + ((2*a - b)*b*Tan[c + d*x])/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(2a-b)b\tan(c+dx)}{d} + \frac{b^2\tan^3(c+dx)}{3d} + \frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+\right.}{d}$$
$$= (a-b)^2x + \frac{(2a-b)b\tan(c+dx)}{d} + \frac{b^2\tan^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.38, size = 73, normalized size = 1.59

$$\frac{\tan(c+dx)\left(\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\tan^2(c+dx)}\right)}{\sqrt{-\tan^2(c+dx)}} + b(6a-b(3-\tan^2(c+dx)))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Tan[c + d*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c + d
*x]^2] + b*(6*a - b*(3 - Tan[c + d*x]^2))))/(3*d)
```

Maple [A]
time = 0.03, size = 59, normalized size = 1.28

method	result	size
norman	$(a^2-2ab+b^2)\,x+\frac{(2a-b)b\tan(dx+c)}{d}+\frac{b^2\left(\tan^3(dx+c)\right)}{3d}$	49
derivativedivides	$\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}+2ab\tan(dx+c)-b^2\tan(dx+c)+(a^2-2ab+b^2)\arctan(\tan(dx+c))}{d}$	59
default	$\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}+2ab\tan(dx+c)-b^2\tan(dx+c)+(a^2-2ab+b^2)\arctan(\tan(dx+c))}{d}$	59
risch	$x\,a^2-2xab+x\,b^2-\frac{4ib\left(-3a\,e^{4i(dx+c)}+3b\,e^{4i(dx+c)}-6a\,e^{2i(dx+c)}+3b\,e^{2i(dx+c)}-3a+2b\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^2*tan(d*x+c)^3+2*a*b*tan(d*x+c)-b^2*tan(d*x+c)+(a^2-2*a*b+b^2)*a
rctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 58, normalized size = 1.26

$$a^2x - \frac{2\,(dx + c - \tan\left(dx + c\right))ab}{d} + \frac{\left(\tan\left(dx + c\right)^3 + 3\,dx + 3\,c - 3\,\tan\left(dx + c\right)\right)b^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*(d*x + c - tan(d*x + c))*a*b/d + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*
c - 3*tan(d*x + c))*b^2/d
```

Fricas [A]
time = 1.75, size = 51, normalized size = 1.11

$$\frac{b^2\tan\left(dx + c\right)^3 + 3\,(a^2 - 2\,ab + b^2)dx + 3\,(2\,ab - b^2)\tan\left(dx + c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*d*x + 3*(2*a*b - b^2)*tan(d
*x + c))/d
```

Sympy [A]
time = 0.10, size = 68, normalized size = 1.48

$$\begin{cases} a^2x - 2abx + \frac{2ab\tan\left(c+dx\right)}{d} + b^2x + \frac{b^2\tan^3\left(c+dx\right)}{3d} - \frac{b^2\tan\left(c+dx\right)}{d} & \text{for } d \neq 0 \\ x(a + b\tan^2\left(c\right))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(c + d*x)/d + b**2*x + b**2*tan(c +
d*x)**3/(3*d) - b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**2)**2, Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(44) = 88.
time = 0.65, size = 359, normalized size = 7.80

$$\frac{3\,a^2dx\tan\left(dx\right)^3\tan\left(c\right)^3 - 6\,abdx\tan\left(dx\right)^3\tan\left(c\right)^3 + 3\,b^2dx\tan\left(dx\right)^3\tan\left(c\right)^3 - 9\,a^2dx\tan\left(dx\right)^2\tan\left(c\right)^2 + 18\,abdx\tan\left(dx\right)^2\tan\left(c\right)^2 - 9\,b^2dx\tan\left(dx\right)^2\tan\left(c\right)^2 - 6\,ab\tan\left(dx\right)^3\tan\left(c\right)^2 + 3\,b^2\tan\left(dx\right)^3\tan\left(c\right)^2 - 6\,ab\tan\left(dx\right)^2\tan\left(c\right)^3 + 3\,b^2\tan\left(dx\right)^2\tan\left(c\right)^3 + 9\,a^2dx\tan\left(dx\right)\tan\left(c\right) - 18\,abdx\tan\left(dx\right)\tan\left(c\right) + 9\,b^2dx\tan\left(dx\right)\tan\left(c\right) - b^2\tan\left(dx\right)^3 + 12\,ab\tan\left(dx\right)^2\tan\left(c\right) - 9\,b^2\tan\left(dx\right)^2\tan\left(c\right) + 12\,ab\tan\left(dx\right)\tan\left(c\right)^2 - 9\,b^2\tan\left(dx\right)\tan\left(c\right)^2 - b^2\tan\left(c\right)^3 - 3\,a^2dx + 6\,abdx - 3\,b^2dx - 6\,ab\tan\left(dx\right) + 3\,b^2\tan\left(dx\right) - 6\,ab\tan\left(c\right) + 3\,b^2\tan\left(c\right)}{3\,\left(d\tan\left(dx\right)^3\tan\left(c\right)^3 - 3\,d\tan\left(dx\right)^2\tan\left(c\right)^2 + 3\,d\tan\left(dx\right)\tan\left(c\right) - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*d*x*tan(d*x)^3*tan(c)^3 - 6*a*b*d*x*tan(d*x)^3*tan(c)^3 + 3*b^2*
d*x*tan(d*x)^3*tan(c)^3 - 9*a^2*d*x*tan(d*x)^2*tan(c)^2 + 18*a*b*d*x*tan(d*
x)^2*tan(c)^2 - 9*b^2*d*x*tan(d*x)^2*tan(c)^2 - 6*a*b*tan(d*x)^3*tan(c)^2 +
 3*b^2*tan(d*x)^3*tan(c)^2 - 6*a*b*tan(d*x)^2*tan(c)^3 + 3*b^2*tan(d*x)^2*t
an(c)^3 + 9*a^2*d*x*tan(d*x)*tan(c) - 18*a*b*d*x*tan(d*x)*tan(c) + 9*b^2*d*
x*tan(d*x)*tan(c) - b^2*tan(d*x)^3 + 12*a*b*tan(d*x)^2*tan(c) - 9*b^2*tan(d
*x)^2*tan(c) + 12*a*b*tan(d*x)*tan(c)^2 - 9*b^2*tan(d*x)*tan(c)^2 - b^2*tan
(c)^3 - 3*a^2*d*x + 6*a*b*d*x - 3*b^2*d*x - 6*a*b*tan(d*x) + 3*b^2*tan(d*x)
 - 6*a*b*tan(c) + 3*b^2*tan(c))/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan
(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 11.42, size = 76, normalized size = 1.65

$$\frac{\tan(c+d\,x)\,\left(2\,a\,b-b^2\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,{(a-b)}^2}{a^2-2\,a\,b+b^2}\right)\,{(a-b)}^2}{d}+\frac{b^2\,{\tan(c+d\,x)}^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] (tan(c + d*x)*(2*a*b - b^2))/d + (atan((tan(c + d*x)*(a - b)^2)/(a^2 - 2*a*
b + b^2))*(a - b)^2)/d + (b^2*tan(c + d*x)^3)/(3*d)
```

3.253 $\int \left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=19

$$ax-bx+\frac{b\tan(c+dx)}{d}$$

```
[Out] a*x-b*x+b*tan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3554, 8}

$$ax+\frac{b\tan(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[c + d*x]^2,x]
```

```
[Out] a*x - b*x + (b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\tan^2(c+dx)\right)\,dx &= ax+b\int \tan^2(c+dx)\,dx\\ &= ax+\frac{b\tan(c+dx)}{d}-b\int 1\,dx\\ &= ax-bx+\frac{b\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.47

$$ax-\frac{b\text{ArcTan}(\tan(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[c + d*x]^2,x]
[Out] a*x - (b*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.53

method	result	size
norman	$(a-b)\,x+\frac{b\tan(dx+c)}{d}$	20
derivativedivides	$\frac{b\tan(dx+c)+(a-b)\arctan(\tan(dx+c))}{d}$	27
default	$ax+\frac{b\tan(dx+c)}{d}-\frac{b\arctan(\tan(dx+c))}{d}$	29
risch	$ax-bx+\frac{2ib}{d(e^{2i(dx+c)}+1)}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] a*x+b*tan(d*x+c)/d-b/d*arctan(tan(d*x+c))
```

Maxima [A]
time = 0.50, size = 23, normalized size = 1.21

$$ax-\frac{(dx+c-\tan(dx+c))b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^2,x, algorithm="maxima")
[Out] a*x - (d*x + c - tan(d*x + c))*b/d
```

Fricas [A]
time = 3.34, size = 21, normalized size = 1.11

$$\frac{(a-b)dx+b\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^2,x, algorithm="fricas")
[Out] ((a - b)*d*x + b*tan(d*x + c))/d
```

Sympy [A]
time = 0.05, size = 20, normalized size = 1.05

$$ax+b\left(\begin{cases}-x+\frac{\tan(c+dx)}{d} & \text{for } d\neq 0\\ x\tan^2(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((-x + tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(19) = 38.
time = 0.53, size = 231, normalized size = 12.16

$$ax+\frac{\left(\pi-4\,dx\tan\left(dx\right)\tan\left(c\right)-\pi\mathrm{sgn}\left(2\,\tan\left(dx\right)^{2}\tan\left(c\right)+2\,\tan\left(dx\right)\tan\left(c\right)^{2}-2\,\tan\left(dx\right)-2\,\tan\left(c\right)\right)\tan\left(dx\right)\tan\left(c\right)-\pi\tan\left(dx\right)\tan\left(c\right)+2\,\arctan\left(\frac{\tan(dx)\tan(c)-1}{\tan(dx)+\tan(c)}\right)\tan\left(dx\right)\tan\left(c\right)+2\,\arctan\left(\frac{\tan(dx)+\tan(c)}{\tan(dx)\tan(c)-1}\right)\tan\left(dx\right)\tan\left(c\right)+4\,dx+\pi\mathrm{sgn}\left(2\,\tan\left(dx\right)^{2}\tan\left(c\right)+2\,\tan\left(dx\right)\tan\left(c\right)^{2}-2\,\tan\left(dx\right)-2\,\tan\left(c\right)\right)-2\,\arctan\left(\frac{\tan(dx)\tan(c)-1}{\tan(dx)+\tan(c)}\right)-2\,\arctan\left(\frac{\tan(dx)+\tan(c)}{\tan(dx)\tan(c)-1}\right)-4\,\tan\left(dx\right)-4\,\tan\left(c\right)\right)b}{4\left(d\tan\left(dx\right)\tan\left(c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x + 1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c)
 + 2*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*
arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x +
 pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))
- 2*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x)
+ tan(c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))*b/(d*tan(d*x)*tan
(c) - d)
```

Mupad [B]
time = 11.44, size = 21, normalized size = 1.11

$$\frac{b\tan(c+d\,x)+d\,x\,(a-b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(c + d*x)^2,x)
```

```
[Out] (b*tan(c + d*x) + d*x*(a - b))/d
```

3.254 $\int \frac{1}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)d}$$

```
[Out] x/(a-b)-arctan(b^(1/2)*tan(d*x+c)/a^(1/2))*b^(1/2)/(a-b)/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3741, 3756, 211}

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a -
b)*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3741

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx = \frac{x}{a-b} - \frac{b\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx}{a-b}$$
$$= \frac{x}{a-b} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{(a-b)d}$$
$$= \frac{x}{a-b} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)d}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.98

$$\frac{\text{ArcTan}(\tan(c+dx)) - \frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{ad-bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(-1),x]
```

```
[Out] (ArcTan[Tan[c + d*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sq
rt[a])/(a*d - b*d)
```

Maple [A]
time = 0.16, size = 50, normalized size = 1.00

method	result	size
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}+\frac{\arctan(\tan(dx+c))}{a-b}}{d}$	50
default	$\frac{-\frac{b\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}+\frac{\arctan(\tan(dx+c))}{a-b}}{d}$	50
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(e^{2i(dx+c)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)d}-\frac{\sqrt{-ab}\,\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)d}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(-1/(a-b)*b/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/(a-b)*arctan (tan(d*x+c)))

Maxima [A]
time = 0.56, size = 48, normalized size = 0.96

$$-\frac{\frac{b\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}-\frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="maxima")

[Out] -(b*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (d*x + c)/(a - b))/d

Fricas [A]
time = 3.66, size = 182, normalized size = 3.64

$$\left[\frac{4\,dx-\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(dx+c)^4-6\,ab\tan(dx+c)^2+a^2+4\left(ab\tan(dx+c)^3-a^2\tan(dx+c)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(dx+c)^4+2\,ab\tan(dx+c)^2+a^2}\right)}{4\,(a-b)d},\ \frac{2\,dx-\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(dx+c)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(dx+c)}\right)}{2\,(a-b)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="fricas")

[Out] [1/4*(4*d*x - sqrt(-b/a)*log((b^2*tan(d*x + c)^4 - 6*a*b*tan(d*x + c)^2 + a ^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*sqrt(-b/a))/(b^2*tan(d*x + c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)))/((a - b)*d), 1/2*(2*d*x - sqrt(b/a)*arc tan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)/(b*tan(d*x + c))))/((a - b)*d)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(37) = 74$.
time = 1.25, size = 240, normalized size = 4.80

$$\begin{cases} \frac{\tilde{\infty}x}{\tan^2(c)} & \text{for}\ a=0\wedge b=0\wedge d=0 \\ \frac{x}{a} & \text{for}\ b=0 \\ \frac{-x-\frac{1}{d\tan(c+dx)}}{b} & \text{for}\ a=0 \\ \frac{dx\tan^2(c+dx)}{2bd\tan^2(c+dx)+2bd}+\frac{dx}{2bd\tan^2(c+dx)+2bd}+\frac{\tan(c+dx)}{2bd\tan^2(c+dx)+2bd} & \text{for}\ a=b \\ \frac{x}{a+b\tan^2(c)} & \text{for}\ d=0 \\ \frac{2dx\sqrt{-\frac{a}{b}}}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}}-\frac{\log\left(-\sqrt{-\frac{a}{b}}+\tan(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}}+\frac{\log\left(\sqrt{-\frac{a}{b}}+\tan(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0)
), ((-x - 1/(d*tan(c + d*x)))/b, Eq(a, 0)), (d*x*tan(c + d*x)**2/(2*b*d*tan
(c + d*x)**2 + 2*b*d) + d*x/(2*b*d*tan(c + d*x)**2 + 2*b*d) + tan(c + d*x)/
(2*b*d*tan(c + d*x)**2 + 2*b*d), Eq(a, b)), (x/(a + b*tan(c)**2), Eq(d, 0))
, (2*d*x*sqrt(-a/b)/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)) - log(-sqrt(-a/b)
 + tan(c + d*x))/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)) + log(sqrt(-a/b) + t
an(c + d*x))/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)), True))
```

Giac [A]
time = 0.59, size = 65, normalized size = 1.30

$$-\frac{\frac{\left(\pi \left\lfloor \frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,(a-b)}-\frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))*
b/(sqrt(a*b)*(a - b)) - (d*x + c)/(a - b))/d
```

Mupad [B]
time = 11.90, size = 948, normalized size = 18.96

$$-\frac{\mathrm{atan}\left(\frac{\frac{-4b^3\tan(c+dx)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(c+dx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1\mathrm{i}}{2a-2b}\right)1\mathrm{i}}{2a-2b}}{2a-2b}+\frac{-4b^3\tan(c+dx)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(c+dx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1\mathrm{i}}{2a-2b}\right)1\mathrm{i}}{2a-2b}}{2a-2b}}{\frac{\left(-4b^3\tan(c+dx)+\frac{\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(c+dx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1\mathrm{i}}{2a-2b}\right)1\mathrm{i}}{2a-2b}\right)1\mathrm{i}}{2a-2b}-\frac{\left(-4b^3\tan(c+dx)+\frac{\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(c+dx)\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)1\mathrm{i}}{2a-2b}\right)1\mathrm{i}}{2a-2b}\right)1\mathrm{i}}{2a-2b}}\right)}{d\,(a-b)}+\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-ab}\left(2b^3\tan(c+dx)-\frac{\sqrt{-ab}\left(2b^4-4ab^3+2a^2b^2+\frac{\tan(c+dx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4(ab-a^2)}\right)}{2(ab-a^2)}\right)1\mathrm{i}}{ab-a^2}+\frac{\sqrt{-ab}\left(2b^3\tan(c+dx)-\frac{\sqrt{-ab}\left(4ab^3-2b^4-2a^2b^2+\frac{\tan(c+dx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4(ab-a^2)}\right)}{2(ab-a^2)}\right)1\mathrm{i}}{ab-a^2}}{\frac{\sqrt{-ab}\left(2b^3\tan(c+dx)-\frac{\sqrt{-ab}\left(2b^4-4ab^3+2a^2b^2+\frac{\tan(c+dx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4(ab-a^2)}\right)}{2(ab-a^2)}\right)}{ab-a^2}-\frac{\sqrt{-ab}\left(2b^3\tan(c+dx)-\frac{\sqrt{-ab}\left(4ab^3-2b^4-2a^2b^2+\frac{\tan(c+dx)\sqrt{-ab}\left(-8a^3b^2+8a^2b^3+8ab^4-8b^5\right)}{4(ab-a^2)}\right)}{2(ab-a^2)}\right)}{ab-a^2}}\right)\sqrt{-ab}\,1\mathrm{i}}{a\,d\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*(2*b^3*tan(c + d*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 +
 2*a^2*b^2 + (tan(c + d*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^
3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2) + ((-a*b)^(1/2)*
(2*b^3*tan(c + d*x) - ((-a*b)^(1/2)*(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan(c +
 d*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2
))))/(2*(a*b - a^2)))*1i)/(a*b - a^2))/(((-a*b)^(1/2)*(2*b^3*tan(c + d*x) -
 ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 + 2*a^2*b^2 + (tan(c + d*x)*(-a*b)^(1/2)*(8
*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))
)/(a*b - a^2) - ((-a*b)^(1/2)*(2*b^3*tan(c + d*x) - ((-a*b)^(1/2)*(4*a*b^3
- 2*b^4 - 2*a^2*b^2 + (tan(c + d*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b
^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2))))/(a*b - a^2))*(-a*b)^(
```

```
1/2)*1i)/(a*d*(a - b)) - atan(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(c + d*
x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*
b) - 4*b^3*tan(c + d*x))/(2*a - 2*b) + (((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (ta
n(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(
2*a - 2*b) - 4*b^3*tan(c + d*x))/(2*a - 2*b))/(((((4*b^4 - 8*a*b^3 + 4*a^2*
b^2 + (tan(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*
b))*1i)/(2*a - 2*b) - 4*b^3*tan(c + d*x))*1i)/(2*a - 2*b) - ((((8*a*b^3 - 4
*b^4 - 4*a^2*b^2 + (tan(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*
1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(c + d*x))*1i)/(2*a - 2*b)))/(d
*(a - b))
```

3.255 $\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{x}{(a-b)^2}-\frac{(3a-b)\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d}-\frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)}$$

```
[Out] x/(a-b)^2-1/2*(3*a-b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))*b^(1/2)/a^(3/2)/(a
-b)^2/d-1/2*b*tan(d*x+c)/a/(a-b)/d/(a+b*tan(d*x+c)^2)
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.357$, Rules used = {3742, 425, 536, 209, 211}

$$-\frac{\sqrt{b}\,(3a-b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}-\frac{b\tan(c+dx)}{2ad(a-b)\left(a+b\tan^2(c+dx)\right)}+\frac{x}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] x/(a - b)^2 - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2
*a^(3/2)*(a - b)^2*d) - (b*Tan[c + d*x])/(2*a*(a - b)*d*(a + b*Tan[c + d*x]
^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= -\frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{2a(a-b)d}\\ &= -\frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a-b)^2d}-\frac{((3a-\cdots}{\cdots}\\ &= \frac{x}{(a-b)^2}-\frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d}-\frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx\cdots\right.}\end{aligned}$$

Mathematica [A]
time = 0.70, size = 88, normalized size = 0.91

$$\frac{2\text{ArcTan}(\tan(c+dx))+\frac{\sqrt{b}\,(-3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{b(-a+b)\tan(c+dx)}{a(a+b\tan^2(c+dx))}}{2(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(-2),x]
```

```
[Out] (2*ArcTan[Tan[c + d*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])
/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[c + d*x])/(a*(a + b*Tan[c + d*x]^2)))/
(2*(a - b)^2*d)
```

Maple [A]

time = 0.23, size = 93, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{(a-b)\tan(dx+c)}{2a\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(dx+c))}{(a-b)^2}}{d}$
default	$\frac{-\frac{b\left(\frac{(a-b)\tan(dx+c)}{2a\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(dx+c))}{(a-b)^2}}{d}$
risch	$\frac{x}{a^2-2ab+b^2}+\frac{ib\left(a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}+a-b\right)}{da(-a+b)^2\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2i(dx+c)}+\right.}{4a(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a-b)^2*b*(1/2/a*(a-b)*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*(3*a-b)/a/
(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))+1/(a-b)^2*arctan(tan(d*x+c)))
```

Maxima [A]

time = 0.51, size = 114, normalized size = 1.18

$$-\frac{\frac{b\tan(dx+c)}{a^3-a^2b+(a^2b-ab^2)\tan(dx+c)^2}+\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2-2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*tan(d*x + c)/(a^3 - a^2*b + (a^2*b - a*b^2)*tan(d*x + c)^2) + (3*a*
b - b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b
)) - 2*(d*x + c)/(a^2 - 2*a*b + b^2))/d
```

Fricas [A]

time = 2.31, size = 390, normalized size = 4.02

$$\left[\frac{8\,abdx\tan(dx+c)^2+8\,a^2dx-\left((3\,ab-b^2)\tan(dx+c)^2+3\,a^2-ab\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{b^2\tan(dx+c)^4-6\,ab\tan(dx+c)^2+a^2+4\left(ab\tan(dx+c)^3-a^2\tan(dx+c)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(dx+c)^4+2\,ab\tan(dx+c)^2+a^2}\right)-4\,(ab-b^2)\tan(dx+c)}{8\left((a^3b-2\,a^2b^2+ab^3)d\tan(dx+c)^2+(a^4-2\,a^3b+a^2b^2)d\right)},\ \frac{4\,abdx\tan(dx+c)^2+4\,a^2dx-\left((3\,ab-b^2)\tan(dx+c)^2+3\,a^2-ab\right)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{\left(b\tan(dx+c)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(dx+c)}\right)-2\,(ab-b^2)\tan(dx+c)}{4\left((a^3b-2\,a^2b^2+ab^3)d\tan(dx+c)^2+(a^4-2\,a^3b+a^2b^2)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/8*(8*a*b*d*x*tan(d*x + c)^2 + 8*a^2*d*x - ((3*a*b - b^2)*tan(d*x + c)^2 + 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(d*x + c)^4 - 6*a*b*tan(d*x + c)^2 + a^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*sqrt(-b/a))/(b^2*tan(d*x + c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)) - 4*(a*b - b^2)*tan(d*x + c))/((a^3*b - 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^2 + (a^4 - 2*a^3*b + a^2*b^2)*d), 1/4*(4*a*b*d*x*tan(d*x + c)^2 + 4*a^2*d*x - ((3*a*b - b^2)*tan(d*x + c)^2 + 3*a^2 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)/(b*tan(d*x + c))) - 2*(a*b - b^2)*tan(d*x + c))/((a^3*b - 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^2 + (a^4 - 2*a^3*b + a^2*b^2)*d)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. $2(78) = 156$.
time = 10.58, size = 2125, normalized size = 21.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c)**2)**2,x)

[Out] Piecewise((zoo*x/tan(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**2, Eq(b, 0)), ((x + 1/(d*tan(c + d*x)) - 1/(3*d*tan(c + d*x)**3))/b**2, Eq(a, 0)), (3*d*x*tan(c + d*x)**4/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d*x)**2 + 8*b**2*d) + 6*d*x*tan(c + d*x)**2/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d*x)**2 + 8*b**2*d) + 3*d*x/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d*x)**2 + 8*b**2*d) + 3*tan(c + d*x)**3/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d*x)**2 + 8*b**2*d) + 5*tan(c + d*x)/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d*x)**2 + 8*b**2*d), Eq(a, b)), (x/(a + b*tan(c)**2)**2, Eq(d, 0)), (4*a**2*d*x*sqrt(-a/b)/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) - 3*a**2*log(-sqrt(-a/b) + tan(c + d*x))/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) + 3*a**2*log(sqrt(-a/b) + tan(c + d*x))/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) + 4*a*b*d*x*sqrt(-a/b)*tan(c + d*x)**2/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) - 2*a*b*sqrt(-a/b)*tan(c + d*x)/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sq

```
rt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) - 3*a*b*log(-sqrt(-a/b) +
 tan(c + d*x))*tan(c + d*x)**2/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)
*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c +
 d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2
) + a*b*log(-sqrt(-a/b) + tan(c + d*x))/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*s
qrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b
)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c
+ d*x)**2) + 3*a*b*log(sqrt(-a/b) + tan(c + d*x))*tan(c + d*x)**2/(4*a**4*d
*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b)
 - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*
a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) - a*b*log(sqrt(-a/b) + tan(c + d*x))/(
4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sq
rt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a
/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) + 2*b**2*sqrt(-a/b)*tan(c + d*
x)/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*
d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqr
t(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) + b**2*log(-sqrt(-a/b) + t
an(c + d*x))*tan(c + d*x)**2/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*t
an(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d
*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2)
- b**2*log(sqrt(-a/b) + tan(c + d*x))*tan(c + d*x)**2/(4*a**4*d*sqrt(-a/b)
+ 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b*
*2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqr
t(-a/b)*tan(c + d*x)**2), True))
```

Giac [A]
time = 0.58, size = 122, normalized size = 1.26

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(3\,ab-b^2)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2-2\,ab+b^2}+\frac{b\tan(dx+c)}{\left(b\tan(dx+c)^2+a\right)(a^2-ab)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b
)))*(3*a*b - b^2)/((a^3 - 2*a^2*b + a*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 -
2*a*b + b^2) + b*tan(d*x + c)/((b*tan(d*x + c)^2 + a)*(a^2 - a*b)))/d
```

Mupad [B]
time = 13.15, size = 2489, normalized size = 25.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*tan(c + d*x)^2)^2,x)

[Out] (2*atan((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(c + d*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(c + d*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) - (tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((3*a*b^3)/2 - b^4/2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2))))/(d*(2*a^2 - 4*a*b + 2*b^2)) - (atan((((-a^3*b)^(1/2)*((tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) - (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2*a^4*b + a^3*b^2)) + ((-a^3*b)^(1/2)*((tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(c + d*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b)*1i)/(4*(a^5 - 2*a^4*b + a^3*b^2)))/(((3*a*b^3)/2 - b^4/2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + ((-a^3*b)^(1/2)*((tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^2*b^2)) - (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b + a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)

```
) - ((-a^3*b)^(1/2)*((tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 -
2*a^3*b + a^2*b^2)) + (((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 1
8*a^5*b^3 - 4*a^6*b^2)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(c + d*x
)*(-a^3*b)^(1/2)*(3*a - b)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b
^4 - 48*a^6*b^3 + 16*a^7*b^2))/(8*(a^4 - 2*a^3*b + a^2*b^2)*(a^5 - 2*a^4*b
+ a^3*b^2)))*(-a^3*b)^(1/2)*(3*a - b))/(4*(a^5 - 2*a^4*b + a^3*b^2)))*(3*a
- b))/(4*(a^5 - 2*a^4*b + a^3*b^2))))*(-a^3*b)^(1/2)*(3*a - b)*1i)/(2*d*(a^
5 - 2*a^4*b + a^3*b^2)) - (b*tan(c + d*x))/(2*a*d*(a + b*tan(c + d*x)^2)*(a
 - b))
```

3.256 $\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=150

$$\frac{x}{(a-b)^3}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d}-\frac{b\tan(c+dx)}{4a(a-b)d\left(a+b\tan^2(c+dx)\right)^2}-\frac{(7a-3b}{8a^2(a-b)^2d\,(}$$

```
[Out] x/(a-b)^3-1/8*(15*a^2-10*a*b+3*b^2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))*b^(1
/2)/a^(5/2)/(a-b)^3/d-1/4*b*tan(d*x+c)/a/(a-b)/d/(a+b*tan(d*x+c)^2)^2-1/8*(
7*a-3*b)*b*tan(d*x+c)/a^2/(a-b)^2/d/(a+b*tan(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3742, 425, 541, 536, 209, 211}

$$-\frac{b(7a-3b)\tan(c+dx)}{8a^2d(a-b)^2\left(a+b\tan^2(c+dx)\right)}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}-\frac{b\tan(c+dx)}{4ad(a-b)\left(a+b\tan^2(c+dx)\right)^2}+\frac{x}{(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] x/(a - b)^3 - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tan[c + d*
x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Tan[c + d*x])/(4*a*(a - b)*d*(a
+ b*Tan[c + d*x]^2)^2) - ((7*a - 3*b)*b*Tan[c + d*x])/(8*a^2*(a - b)^2*d*(a
 + b*Tan[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{(a+b\tan^2(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{4a(a-b)d\,(a+b\tan^2(c+dx))^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{4a(a-b)d}$$

$$= -\frac{b\tan(c+dx)}{4a(a-b)d\,(a+b\tan^2(c+dx))^2} - \frac{(7a-3b)b\tan(c+dx)}{8a^2(a-b)^2d\,(a+b\tan^2(c+dx))} + \text{Subs}$$

$$= -\frac{b\tan(c+dx)}{4a(a-b)d\,(a+b\tan^2(c+dx))^2} - \frac{(7a-3b)b\tan(c+dx)}{8a^2(a-b)^2d\,(a+b\tan^2(c+dx))} + \text{Subs}$$

$$= \frac{x}{(a-b)^3} - \frac{\sqrt{b}\,(15a^2-10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} - \frac{b\tan(c}{4a(a-b)d\,(a+b}$$

Mathematica [A]
time = 1.32, size = 138, normalized size = 0.92

$$-\frac{-8\text{ArcTan}(\tan(c+dx)) + \frac{\sqrt{b}\,(15a^2-10ab+3b^2)\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2(a-b)^2b\tan(c+dx)}{a(a+b\tan^2(c+dx))^2} + \frac{(7a-3b)(a-b)b\tan(c+dx)}{a^2(a+b\tan^2(c+dx))}}{8(a-b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(-3),x]
```

```
[Out] -1/8*(-8*ArcTan[Tan[c + d*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(
Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[c + d*x])/(a*(
a + b*Tan[c + d*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[c + d*x])/(a^2*(a + b
*Tan[c + d*x]^2)))/((a - b)^3*d)
```

Maple [A]
time = 0.36, size = 142, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)(\tan^3(dx+c))}{8a^2}+\frac{(9a^2-14ab+5b^2)\tan(dx+c)}{8a}}{(a+b(\tan^2(dx+c)))^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\arctan(\tan(dx+}{(a-b)^3}}{d}$

default	$\frac{-\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)(\tan^3(dx+c))}{8a^2}+\frac{(9a^2-14ab+5b^2)\tan(dx+c)}{8a}}{\left(a+b(\tan^2(dx+c))\right)^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\arctan(\tan(d}{(a-b)^3}}{d}$
risch	$\frac{x}{a^3-3a^2b+3a\,b^2-b^3}+\frac{ib(9a^3e^{6i(dx+c)}+a^2b\,e^{6i(dx+c)}-13a\,b^2e^{6i(dx+c)}+3b^3e^{6i(dx+c)}+27a^3e^{4i(dx+c)}+9a^2b\,e^{4i(dx+c)}}{4(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b/(a-b)^3*((1/8*b*(7*a^2-10*a*b+3*b^2)/a^2*tan(d*x+c)^3+1/8*(9*a^2-14
*a*b+5*b^2)/a*tan(d*x+c))/(a+b*tan(d*x+c)^2)^2+1/8*(15*a^2-10*a*b+3*b^2)/a^
2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))+1/(a-b)^3*arctan(tan(d*x+c)
))
```

Maxima [A]
time = 0.52, size = 227, normalized size = 1.51

$$-\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}+\frac{(7\,ab^2-3\,b^3)\tan(dx+c)^3+(9\,a^2b-5\,ab^2)\tan(dx+c)}{a^6-2\,a^5b+a^4b^2+(a^4b^2-2\,a^3b^3+a^2b^4)\tan(dx+c)^4+2\,(a^5b-2\,a^4b^2+a^3b^3)\tan(dx+c)^2}-\frac{8\,(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^5
- 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) + ((7*a*b^2 - 3*b^3)*tan(d*x +
c)^3 + (9*a^2*b - 5*a*b^2)*tan(d*x + c))/(a^6 - 2*a^5*b + a^4*b^2 + (a^4*b^
2 - 2*a^3*b^3 + a^2*b^4)*tan(d*x + c)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^3)*t
an(d*x + c)^2) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(136) = 272.
time = 2.89, size = 742, normalized size = 4.95

$$\left[\frac{32\,a^2b^2dx\tan(dx+c)^4+64\,a^3bdx\tan(dx+c)^2+32\,a^4dx-4\,(7\,a^2b^2-10\,ab^3+3\,b^4)\tan(dx+c)^3-\left((15\,a^2b^2-10\,ab^3+3\,b^4)\tan(dx+c)^4+15\,a^4-10\,a^3b+3\,a^2b^2+2\,(15\,a^3b-10\,a^2b^2+3\,ab^3)\tan(dx+c)^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(dx+c)^4-6\,ab\tan(dx+c)^2+a^2+4\,\left(ab\tan(dx+c)^3-a^2\tan(dx+c)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(dx+c)^4+2\,ab\tan(dx+c)^2+a^2}\right)-4\,(9\,a^3b-14\,a^2b^2+5\,ab^3)\tan(dx+c)}{32\,\left((a^5b^2-3\,a^4b^3+3\,a^3b^4-a^2b^5)d\tan(dx+c)^4+2\,(a^6b-3\,a^5b^2+3\,a^4b^3-a^3b^4)d\tan(dx+c)^2+(a^7-3\,a^6b+3\,a^5b^2-a^4b^3)d\right)},\ \frac{16\,a^2b^2dx\tan(dx+c)^4+32\,a^3bdx\tan(dx+c)^2+16\,a^4dx-2\,(7\,a^2b^2-10\,ab^3+3\,b^4)\tan(dx+c)^3-\left((15\,a^2b^2-10\,ab^3+3\,b^4)\tan(dx+c)^4+15\,a^4-10\,a^3b+3\,a^2b^2+2\,(15\,a^3b-10\,a^2b^2+3\,ab^3)\tan(dx+c)^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(dx+c)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(dx+c)}\right)-2\,(9\,a^3b-14\,a^2b^2+5\,ab^3)\tan(dx+c)}{16\,\left((a^5b^2-3\,a^4b^3+3\,a^3b^4-a^2b^5)d\tan(dx+c)^4+2\,(a^6b-3\,a^5b^2+3\,a^4b^3-a^3b^4)d\tan(dx+c)^2+(a^7-3\,a^6b+3\,a^5b^2-a^4b^3)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*a^2*b^2*d*x*tan(d*x + c)^4 + 64*a^3*b*d*x*tan(d*x + c)^2 + 32*a^4
*d*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(d*x + c)^3 - ((15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*tan(d*x + c)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b
```

```
 - 10*a^2*b^2 + 3*a*b^3)*tan(d*x + c)^2)*sqrt(-b/a)*log((b^2*tan(d*x + c)^4
 - 6*a*b*tan(d*x + c)^2 + a^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*s
qrt(-b/a))/(b^2*tan(d*x + c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)) - 4*(9*a^3*b
- 14*a^2*b^2 + 5*a*b^3)*tan(d*x + c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*d*tan(d*x + c)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*ta
n(d*x + c)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d), 1/16*(16*a^2*b^2*d
*x*tan(d*x + c)^4 + 32*a^3*b*d*x*tan(d*x + c)^2 + 16*a^4*d*x - 2*(7*a^2*b^2
 - 10*a*b^3 + 3*b^4)*tan(d*x + c)^3 - ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(
d*x + c)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a^2*b^2 + 3*a
*b^3)*tan(d*x + c)^2)*sqrt(b/a)*arctan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)
/(b*tan(d*x + c))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(d*x + c))/((a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*tan(d*x + c)^4 + 2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*d*tan(d*x + c)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8964 vs. $2(133) = 266$.
time = 51.35, size = 8964, normalized size = 59.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x/tan(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**3, Eq(b,
 0)), ((-x - 1/(d*tan(c + d*x)) + 1/(3*d*tan(c + d*x)**3) - 1/(5*d*tan(c +
d*x)**5))/b**3, Eq(a, 0)), (15*d*x*tan(c + d*x)**6/(48*b**3*d*tan(c + d*x)*
*6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d) +
 45*d*x*tan(c + d*x)**4/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x
)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d) + 45*d*x*tan(c + d*x)**2/(48
*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*tan(c + d
*x)**2 + 48*b**3*d) + 15*d*x/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c
+ d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d) + 15*tan(c + d*x)**5/(4
8*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*tan(c +
d*x)**2 + 48*b**3*d) + 40*tan(c + d*x)**3/(48*b**3*d*tan(c + d*x)**6 + 144*
b**3*d*tan(c + d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d) + 33*tan(c
 + d*x)/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*
d*tan(c + d*x)**2 + 48*b**3*d), Eq(a, b)), (x/(a + b*tan(c)**2)**3, Eq(d, 0
)), (16*a**4*d*x*sqrt(-a/b)/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*
tan(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*tan(c
+ d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 48*a**5*b**2*d*sqrt
(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a
/b)*tan(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)
*tan(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**2*b**5
*d*sqrt(-a/b)*tan(c + d*x)**4) - 15*a**4*log(-sqrt(-a/b) + tan(c + d*x))/(1
```

6*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*tan(c + d*x)**4) + 15*a**4*log(sqrt(-a/b) + tan(c + d*x))/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*tan(c + d*x)**4) + 32*a**3*b*d*x*sqrt(-a/b)*tan(c + d*x)**2/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*tan(c + d*x)**4) - 18*a**3*b*sqrt(-a/b)*tan(c + d*x)/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*tan(c + d*x)**4) - 30*a**3*b*log(-sqrt(-a/b) + tan(c + d*x))*tan(c + d*x)**2/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*tan(c + d*x)**4) + 10*a**3*b*log(-sqrt(-a/b) + tan(c + d*x))/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**2*b**5*d*sqrt(-a/b)*tan(c + d*x)**4) + 30*a**3*b*log(sqrt(-a/b) + tan(c + d*x))*tan(c + d*x)**2/(16*a**7*d*sqrt(-a/b) + 32*a**6*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 48*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**4 + 96*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**3*b**4*d*sqrt(-a/b)*tan(c + d*x)**4 - 32*a**3*b*

```
*4*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**2*b**5*...
```

Giac [A]
time = 0.66, size = 205, normalized size = 1.37

$$-\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{7\,ab^2\tan(dx+c)^3-3\,b^3\tan(dx+c)^3+9\,a^2b\tan(dx+c)-5\,ab^2\tan(dx+c)}{(a^4-2\,a^3b+a^2b^2)\left(b\tan(dx+c)^2+a\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) +
arctan(b*tan(d*x + c)/sqrt(a*b)))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sq
rt(a*b)) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (7*a*b^2*tan(d*x +
 c)^3 - 3*b^3*tan(d*x + c)^3 + 9*a^2*b*tan(d*x + c) - 5*a*b^2*tan(d*x + c))
/((a^4 - 2*a^3*b + a^2*b^2)*(b*tan(d*x + c)^2 + a)^2))/d
```

Mupad [B]
time = 13.98, size = 2500, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^2)^3,x)
```

```
[Out] (atan((((-a^5*b)^(1/2)*((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300
*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b
^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^
6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^1
0 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2))
- (tan(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 128
0*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 128
0*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8
 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10
*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b
 + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b)^(1/2)*
((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*a^4*b^3)
)/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^2*b^10 -
 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 +
5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 -
 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(c + d*x)*(-a^5*b
)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^
7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b
^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 -
4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/(16*(3*a
```

^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^
7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 + 105*a
^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3
 + 15*a^8*b^2)) - ((-a^5*b)^(1/2)*((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^
2*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^
3 + 6*a^6*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^
7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^
2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15
*a^8*b^2)) - (tan(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^
4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^
9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^
6*b^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(
15*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a
^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^5*b
)^(1/2)*((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 + 289*
a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((96*a^
2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^
7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a
^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(c + d*x)
*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*b^8 + 230
4*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^10*b^3 + 25
6*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4*a^7*b + a^
4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2))/
(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^2 - 10*a*b + 3*b^2))/(16*
(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2))))*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b +
3*b^2)*1i)/(8*d*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) - (2*atan((((((96*a^
2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^
7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a
^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) - (tan(c + d*x)
*(256*a^4*b^9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 +
 2304*a^9*b^4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b +
2*a^3 - 2*b^3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a
*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2
*b^5 - 300*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3
 + 6*a^6*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((96*a^2*b^10 - 800
*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280
*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^9*b + a^4*b^6 - 6*a
^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(c + d*x)*(256*a^4*b^
9 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^
4 - 1280*a^10*b^3 + 256*a^11*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^
3)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(6*a*b^2 - 6*a^2
*b + 2*a^3 - 2*b^3) + (tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a
^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2
)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((51*a*b^5 - 9*b^6 - 115*a^2*b^4 +
 105*a^3*b^3)/(32*(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a

```
^7*b^3 + 15*a^8*b^2)) + (((((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 681
6*a^5*b^7 + 9760*a^6*b^6 - 9056*a^7*b^5 + 5280*...
```

3.257 $\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=54

$$\frac{3}{8}\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}-\frac{3}{8}\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{4}\sqrt{a\sec^2(x)}\,\tan^3(x)$$

```
[Out] 3/8*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)-3/8*(a*sec(x)^2)^(1/2)*tan(x)
+1/4*(a*sec(x)^2)^(1/2)*tan(x)^3
```

Rubi [A]
time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3738, 4210, 2691, 3855}

$$\frac{1}{4}\tan^3(x)\sqrt{a\sec^2(x)}-\frac{3}{8}\tan(x)\sqrt{a\sec^2(x)}+\frac{3}{8}\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2])/8 - (3*Sqrt[a*Sec[x]^2]*Tan[x])
/8 + (Sqrt[a*Sec[x]^2]*Tan[x]^3)/4
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
```

```
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \sqrt{a\sec^2(x)}\tan^4(x)\,dx\\ &= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\tan^4(x)\,dx\\ &= \frac{1}{4}\sqrt{a\sec^2(x)}\tan^3(x)-\frac{1}{4}\left(3\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\tan^2(x)\,dx\\ &= -\frac{3}{8}\sqrt{a\sec^2(x)}\tan(x)+\frac{1}{4}\sqrt{a\sec^2(x)}\tan^3(x)+\frac{1}{8}\left(3\cos(x)\sqrt{a\sec^2(x)}\right)\\ &= \frac{3}{8}\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}-\frac{3}{8}\sqrt{a\sec^2(x)}\tan(x)+\frac{1}{4}\sqrt{a\sec^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 0.59

$$\frac{1}{8}\sqrt{a\sec^2(x)}\left(3\tanh^{-1}(\sin(x))\cos(x)-3\tan(x)+2\tan^3(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4*Sqrt[a + a*Tan[x]^2],x]
[Out] (Sqrt[a*Sec[x]^2]*(3*ArcTanh[Sin[x]]*Cos[x] - 3*Tan[x] + 2*Tan[x]^3))/8
```

Maple [A]
time = 0.12, size = 56, normalized size = 1.04

method	result
derivativedivides	$\frac{\tan(x)\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{4a}-\frac{5\sqrt{a+a\left(\tan^2(x)\right)}\,\tan(x)}{8}+\frac{3\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\left(\tan^2(x)\right)}\right.}{8}$
default	$\frac{\tan(x)\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{4a}-\frac{5\sqrt{a+a\left(\tan^2(x)\right)}\,\tan(x)}{8}+\frac{3\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\left(\tan^2(x)\right)}\right.}{8}$
risch	$\frac{i\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(5\,e^{6ix}-3\,e^{4ix}+3\,e^{2ix}-5)}{4(e^{2ix}+1)^3}-\frac{3\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x)}{4}+\frac{3\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*tan(x)*(a+a*tan(x)^2)^(3/2)/a-5/8*(a+a*tan(x)^2)^(1/2)*tan(x)+3/8*a^(1/
2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(42) = 84$.
time = 0.85, size = 860, normalized size = 15.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(4*(5*sin(7*x) - 3*sin(5*x) + 3*sin(3*x) - 5*sin(x))*cos(8*x) - 40*(2
*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) - 16*(3*sin(5*x) - 3*sin(3*x)
 + 5*sin(x))*cos(6*x) + 24*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) + 24*(3*sin(3
*x) - 5*sin(x))*cos(4*x) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*
cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6
*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2
*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x)
 + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2
*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) +
1) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2
+ 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(2*x)
 + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(4*x)
 + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin(6*x
) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 +
8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(5*cos(7*x) - 3
*cos(5*x) + 3*cos(3*x) - 5*cos(x))*sin(8*x) + 20*(4*cos(6*x) + 6*cos(4*x) +
 4*cos(2*x) + 1)*sin(7*x) + 16*(3*cos(5*x) - 3*cos(3*x) + 5*cos(x))*sin(6*x
) - 12*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) - 24*(3*cos(3*x) - 5*cos(x))*
sin(4*x) + 12*(4*cos(2*x) + 1)*sin(3*x) - 48*cos(3*x)*sin(2*x) + 80*cos(x)*
sin(2*x) - 80*cos(2*x)*sin(x) - 20*sin(x))*sqrt(a)/(2*(4*cos(6*x) + 6*cos(4
*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) +
1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2
+ 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8
*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)
^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)
```

Fricas [A]
time = 4.79, size = 56, normalized size = 1.04

$$\frac{1}{8}\sqrt{a\tan\left(x\right)^{2}+a}\,\left(2\,\tan\left(x\right)^{3}-3\,\tan\left(x\right)\right)+\frac{3}{16}\,\sqrt{a}\,\log\left(2\,a\tan\left(x\right)^{2}+2\,\sqrt{a\tan\left(x\right)^{2}+a}\,\sqrt{a}\,\tan\left(x\right)+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(a*tan(x)^2 + a)*(2*tan(x)^3 - 3*tan(x)) + 3/16*sqrt(a)*log(2*a*tan
(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\tan^{2}\left(x\right)+1\right)}\,\tan^{4}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x)**4,x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**4, x)
```

Giac [A]
time = 0.44, size = 48, normalized size = 0.89

$$\frac{1}{8}\sqrt{a\tan\left(x\right)^{2}+a}\,\left(2\,\tan\left(x\right)^{2}-3\right)\tan\left(x\right)-\frac{3}{8}\sqrt{a}\,\log\left(\left|-\sqrt{a}\,\tan\left(x\right)+\sqrt{a\tan\left(x\right)^{2}+a}\,\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 - 3)*tan(x) - 3/8*sqrt(a)*log(abs(-sqr
t(a)*tan(x) + sqrt(a*tan(x)^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)^{4}\,\sqrt{a\tan\left(x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^4*(a + a*tan(x)^2)^(1/2), x)
```

3.258 $\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=30

$$-\sqrt{a\sec^2(x)}+\frac{\left(a\sec^2(x)\right)^{3/2}}{3a}$$

```
[Out] 1/3*(a*sec(x)^2)^(3/2)/a-(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3738, 4209, 45}

$$\frac{\left(a\sec^2(x)\right)^{3/2}}{3a}-\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -Sqrt[a*Sec[x]^2] + (a*Sec[x]^2)^(3/2)/(3*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx = \int \sqrt{a\sec^2(x)}\,\tan^3(x)\,dx$$
$$= \frac{1}{2}a\text{Subst}\left(\int \frac{-1+x}{\sqrt{ax}}\,dx, x, \sec^2(x)\right)$$
$$= \frac{1}{2}a\text{Subst}\left(\int \left(-\frac{1}{\sqrt{ax}}+\frac{\sqrt{ax}}{a}\right)\,dx, x, \sec^2(x)\right)$$
$$= -\sqrt{a\sec^2(x)}+\frac{\left(a\sec^2(x)\right)^{3/2}}{3a}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.67

$$\frac{1}{3}\sqrt{a\sec^2(x)}\,\left(-3+\sec^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (Sqrt[a*Sec[x]^2]*(-3 + Sec[x]^2))/3
```

Maple [A]
time = 0.06, size = 29, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{3a}-\sqrt{a+a\left(\tan^2\left(x\right)\right)}$	29
default	$\frac{\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{3a}-\sqrt{a+a\left(\tan^2\left(x\right)\right)}$	29
risch	$-\frac{2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(3\,e^{4ix}+2\,e^{2ix}+3\right)}{3\left(e^{2ix}+1\right)^2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*(a+a*tan(x)^2)^(3/2)-(a+a*tan(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(24) = 48$.
time = 0.54, size = 276, normalized size = 9.20

$$-\frac{2\left(\left(3\cos\left(5x\right)+2\cos\left(3x\right)+3\cos\left(x\right)\right)\cos\left(6x\right)+3\left(3\cos\left(4x\right)+3\cos\left(2x\right)+1\right)\cos\left(5x\right)+3\left(2\cos\left(3x\right)+3\cos\left(x\right)\right)\cos\left(4x\right)+2\left(3\cos\left(2x\right)+1\right)\cos\left(3x\right)+9\cos\left(2x\right)\cos\left(x\right)+\left(3\sin\left(5x\right)+2\sin\left(3x\right)+3\sin\left(x\right)\right)\sin\left(6x\right)+9\left(\sin\left(4x\right)+\sin\left(2x\right)\right)\sin\left(5x\right)+3\left(2\sin\left(3x\right)+3\sin\left(x\right)\right)\sin\left(4x\right)+6\sin\left(3x\right)\sin\left(2x\right)+9\sin\left(2x\right)\sin\left(x\right)+3\cos\left(x\right)\right)\sqrt{a}}{3\left(2\left(3\cos\left(4x\right)+3\cos\left(2x\right)+1\right)\cos\left(6x\right)+\cos\left(6x\right)^2+6\left(3\cos\left(2x\right)+1\right)\cos\left(4x\right)+9\cos\left(4x\right)^2+9\cos\left(2x\right)^2+6\left(\sin\left(4x\right)+\sin\left(2x\right)\right)\sin\left(6x\right)+\sin\left(6x\right)^2+9\sin\left(4x\right)^2+18\sin\left(4x\right)\sin\left(2x\right)+9\sin\left(2x\right)^2+6\cos\left(2x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -2/3*((3*cos(5*x) + 2*cos(3*x) + 3*cos(x))*cos(6*x) + 3*(3*cos(4*x) + 3*cos
(2*x) + 1)*cos(5*x) + 3*(2*cos(3*x) + 3*cos(x))*cos(4*x) + 2*(3*cos(2*x) +
1)*cos(3*x) + 9*cos(2*x)*cos(x) + (3*sin(5*x) + 2*sin(3*x) + 3*sin(x))*sin(
6*x) + 9*(sin(4*x) + sin(2*x))*sin(5*x) + 3*(2*sin(3*x) + 3*sin(x))*sin(4*x
) + 6*sin(3*x)*sin(2*x) + 9*sin(2*x)*sin(x) + 3*cos(x))*sqrt(a)/(2*(3*cos(4
*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(4*x) +
 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(6*x)^
2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)
```

Fricas [A]
time = 1.77, size = 18, normalized size = 0.60

$$\frac{1}{3}\sqrt{a\tan\left(x\right)^{2}+a}\,\left(\tan\left(x\right)^{2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*tan(x)^2 + a)*(tan(x)^2 - 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\tan^{2}\left(x\right)+1\right)}\,\tan^{3}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x)**3,x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**3, x)
```

Giac [A]
time = 0.42, size = 29, normalized size = 0.97

$$\frac{\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}-3\,\sqrt{a\tan\left(x\right)^{2}+a}\,a}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*((a*tan(x)^2 + a)^(3/2) - 3*sqrt(a*tan(x)^2 + a)*a)/a
```

Mupad [B]
time = 11.64, size = 19, normalized size = 0.63

$$-\frac{\sqrt{2}\,\sqrt{a}\,\left(6\cos\left(x\right)^2-2\right)}{3\,{\left(2\cos\left(x\right)^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a + a*tan(x)^2)^(1/2),x)
[Out] -(2^(1/2)*a^(1/2)*(6*cos(x)^2 - 2))/(3*(2*cos(x)^2)^(3/2))
```

3.259 $\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}+\frac{1}{2}\sqrt{a\sec^2(x)}\,\tan(x)$$

```
[Out] -1/2*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)+1/2*(a*sec(x)^2)^(1/2)*tan(x
)
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3738, 4210, 2691, 3855}

$$\frac{1}{2}\tan(x)\sqrt{a\sec^2(x)}-\frac{1}{2}\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -1/2*(ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2]) + (Sqrt[a*Sec[x]^2]*Tan[x])/
2
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
```

```
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \sqrt{a\sec^2(x)}\,\tan^2(x)\,dx\\ &= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\tan^2(x)\,dx\\ &= \frac{1}{2}\sqrt{a\sec^2(x)}\,\tan(x)-\frac{1}{2}\left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}+\frac{1}{2}\sqrt{a\sec^2(x)}\,\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{1}{2}\sqrt{a\sec^2(x)}\,\left(-\tanh^{-1}(\sin(x))\cos(x)+\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2*Sqrt[a + a*Tan[x]^2],x]
[Out] (Sqrt[a*Sec[x]^2]*(-(ArcTanh[Sin[x]]*Cos[x]) + Tan[x]))/2
```

Maple [A]
time = 0.06, size = 39, normalized size = 1.08

method	result
derivativedivides	$\frac{\sqrt{a+a\left(\tan^2\left(x\right)\right)}\,\tan\left(x\right)}{2}-\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan\left(x\right)+\sqrt{a+a\left(\tan^2\left(x\right)\right)}\right)}{2}$
default	$\frac{\sqrt{a+a\left(\tan^2\left(x\right)\right)}\,\tan\left(x\right)}{2}-\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan\left(x\right)+\sqrt{a+a\left(\tan^2\left(x\right)\right)}\right)}{2}$
risch	$-\frac{i\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}-1\right)}{e^{2ix}+1}+\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}-i\right)\cos\left(x\right)-\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}+i\right)\cos\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a+a*tan(x)^2)^(1/2)*tan(x)-1/2*a^(1/2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2
)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(28) = 56$.
time = 0.56, size = 295, normalized size = 8.19

$$\frac{\left(4\left(\sin\left(3x\right)-\sin\left(x\right)\right)\cos\left(4x\right)-\left(2\left(2\cos\left(2x\right)+1\right)\cos\left(4x\right)+\cos\left(4x\right)^{2}+4\cos\left(2x\right)^{2}+\sin\left(4x\right)^{2}+4\sin\left(4x\right)\sin\left(2x\right)+4\sin\left(2x\right)^{2}+4\cos\left(2x\right)+1\right)\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\sin\left(x\right)+1\right)+\left(2\left(2\cos\left(2x\right)+1\right)\cos\left(4x\right)+\cos\left(4x\right)^{2}+4\cos\left(2x\right)^{2}+\sin\left(4x\right)^{2}+4\sin\left(4x\right)\sin\left(2x\right)+4\sin\left(2x\right)^{2}+4\cos\left(2x\right)+1\right)\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\sin\left(x\right)+1\right)-4\left(\cos\left(3x\right)-\cos\left(x\right)\right)\sin\left(4x\right)+4\left(2\cos\left(2x\right)+1\right)\sin\left(3x\right)-8\cos\left(3x\right)\sin\left(2x\right)+8\cos\left(x\right)\sin\left(2x\right)-8\cos\left(2x\right)\sin\left(x\right)-4\sin\left(x\right)\right)\sqrt{a}}{4\left(2\left(2\cos\left(2x\right)+1\right)\cos\left(4x\right)+\cos\left(4x\right)^{2}+4\cos\left(2x\right)^{2}+\sin\left(4x\right)^{2}+4\sin\left(4x\right)\sin\left(2x\right)+4\sin\left(2x\right)^{2}+4\cos\left(2x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*x) - sin(x))*cos(4*x) - (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*c
os(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (2*(2*cos(2*x) + 1)*
cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4
*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(
cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x)*sin(
2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))*sqrt(a)/(2*(2*cos(
2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*si
n(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Fricas [A]
time = 2.80, size = 47, normalized size = 1.31

$$\frac{1}{4}\sqrt{a}\log\left(2a\tan\left(x\right)^{2}-2\sqrt{a\tan\left(x\right)^{2}+a}\sqrt{a}\tan\left(x\right)+a\right)+\frac{1}{2}\sqrt{a\tan\left(x\right)^{2}+a}\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(a)*log(2*a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a) +
 1/2*sqrt(a*tan(x)^2 + a)*tan(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\tan^{2}\left(x\right)+1\right)}\tan^{2}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x)**2,x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**2, x)
```

Giac [A]
time = 0.47, size = 40, normalized size = 1.11

$$\frac{1}{2}\sqrt{a}\log\left(\left|-\sqrt{a}\tan\left(x\right)+\sqrt{a\tan\left(x\right)^{2}+a}\right|\right)+\frac{1}{2}\sqrt{a\tan\left(x\right)^{2}+a}\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a))) + 1/2*sqrt(a*t
an(x)^2 + a)*tan(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \tan(x)^2\,\sqrt{a\,\tan\left(x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^2*(a + a*tan(x)^2)^(1/2), x)
```

3.260 $\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=10

$$\sqrt{a\sec^2(x)}$$

[Out] (a*sec(x)^2)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3738, 4209, 32}

$$\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]*Sqrt[a + a*Tan[x]^2],x]

[Out] Sqrt[a*Sec[x]^2]

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \sqrt{a\sec^2(x)}\,\tan(x)\,dx\\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{\sqrt{ax}}\,dx, x, \sec^2(x)\right)\\
&= \sqrt{a\sec^2(x)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.04, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\sqrt{a+a\left(\tan^2\left(x\right)\right)}$	11
default	$\sqrt{a+a\left(\tan^2\left(x\right)\right)}$	11
risch	$2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^2}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(x)^2)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] (a+a*tan(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*tan(x)^2 + a)*tan(x), x)
```

Fricas [A]
time = 4.00, size = 10, normalized size = 1.00

$$\sqrt{a\tan\left(x\right)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] sqrt(a*tan(x)^2 + a)
```

Sympy [**A**]
time = 0.32, size = 10, normalized size = 1.00

$$\sqrt{a\tan^{2}\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)**2)**(1/2)*tan(x),x)
```

```
[Out] sqrt(a*tan(x)**2 + a)
```

Giac [**A**]
time = 0.43, size = 10, normalized size = 1.00

$$\sqrt{a\tan\left(x\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] sqrt(a*tan(x)^2 + a)
```

Mupad [**B**]
time = 11.63, size = 10, normalized size = 1.00

$$\frac{\sqrt{a}}{\sqrt{\cos\left(x\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] a^(1/2)/(cos(x)^2)^(1/2)
```

3.261 $\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=24

$$-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

```
[Out] -arctanh((a*sec(x)^2)^(1/2)/a^(1/2))*a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3738, 4209, 65, 213}

$$-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
```

```
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \cot(x)\sqrt{a\sec^2(x)}\,dx\\ &= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx,x,\sec^2(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a\sec^2(x)}\right)\\ &= -\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.25

$$\cos(x)\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + a*Tan[x]^2],x]
[Out] Cos[x]*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.21, size = 23, normalized size = 0.96

method	result	size
default	$\cos(x)\sqrt{\frac{a}{\cos(x)^2}}\ \ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)$	23
risch	$-2\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\ \ln(e^{ix}+1)\cos(x)+2\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\ \ln(e^{ix}-1)\cos(x)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] cos(x)*(a/cos(x)^2)^(1/2)*ln(-(-1+cos(x))/sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.
time = 0.56, size = 38, normalized size = 1.58

$$-\frac{1}{2}\sqrt{a}\left(\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*(log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(
x)^2 - 2*cos(x) + 1))
```

Fricas [**A**]
time = 3.21, size = 63, normalized size = 2.62

$$\left[\frac{1}{2}\sqrt{a}\log\left(\frac{a\tan(x)^2 - 2\sqrt{a\tan(x)^2+a}\sqrt{a}+2a}{\tan(x)^2}\right),\sqrt{-a}\arctan\left(\frac{\sqrt{a\tan(x)^2+a}\sqrt{-a}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)
^2), sqrt(-a)*arctan(sqrt(a*tan(x)^2 + a)*sqrt(-a)/a)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\tan^2(x)+1\right)}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x), x)
```

Giac [**A**]
time = 0.43, size = 24, normalized size = 1.00

$$\frac{a\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [**B**]
time = 0.18, size = 12, normalized size = 0.50

$$-\sqrt{a}\,\mathrm{atanh}\left(\sqrt{\frac{1}{\cos(x)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + a*tan(x)^2)^(1/2),x)
[Out] -a^(1/2)*atanh((1/cos(x)^2)^(1/2))
```

3.262 $\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=14

$$-\cot(x)\sqrt{a\sec^2(x)}$$

```
[Out] -cot(x)*(a*sec(x)^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3738, 4210, 2686, 8}

$$-\cot(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[a*Sec[x]^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx = \int \cot^2(x)\sqrt{a\sec^2(x)}\,dx$$
$$= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \cot(x)\csc(x)\,dx$$
$$= -\left(\left(\cos(x)\sqrt{a\sec^2(x)}\right)\text{Subst}(\int 1\,dx, x, \csc(x))\right)$$
$$= -\cot(x)\sqrt{a\sec^2(x)}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\cot(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[a*Sec[x]^2])
```

Maple [A]
time = 0.13, size = 17, normalized size = 1.21

method	result	size
default	$-\frac{\cos(x)\sqrt{\frac{a}{\cos(x)^2}}}{\sin(x)}$	17
risch	$-\frac{2i\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}{e^{2ix}-1}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)/sin(x)*(a/cos(x)^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 17, normalized size = 1.21

$$-\frac{\sqrt{\tan(x)^2+1}\,\sqrt{a}}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

[Out] -sqrt(tan(x)^2 + 1)*sqrt(a)/tan(x)

Fricas [A]
time = 2.55, size = 16, normalized size = 1.14

$$-\frac{\sqrt{a\tan(x)^2 + a}}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")

[Out] -sqrt(a*tan(x)^2 + a)/tan(x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a(\tan^2(x) + 1)}\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)**2*(a+a*tan(x)**2)**(1/2),x)

[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.
time = 0.48, size = 32, normalized size = 2.29

$$\frac{2a^{\frac{3}{2}}}{\left(\sqrt{a}\tan(x) - \sqrt{a\tan(x)^2 + a}\right)^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")

[Out] 2*a^(3/2)/((sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a)

Mupad [B]
time = 11.98, size = 25, normalized size = 1.79

$$\frac{2\sqrt{a}\cos(x)\sin(x)}{\sqrt{\cos(x)^2}\left(2\cos(x)^2 - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^2*(a + a*tan(x)^2)^(1/2),x)

[Out] (2*a^(1/2)*cos(x)*sin(x))/((cos(x)^2)^(1/2)*(2*cos(x)^2 - 2))

3.263 $\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=45

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)-\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}$$

```
[Out] 1/2*arctanh((a*sec(x)^2)^(1/2)/a^(1/2))*a^(1/2)-1/2*cot(x)^2*(a*sec(x)^2)^(
1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3738, 4209, 44, 65, 213}

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)-\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]])/2 - (Cot[x]^2*Sqrt[a*Sec[x]^2])
/2
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx &= \int \cot^3(x)\sqrt{a\sec^2(x)}\,dx \\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{(-1+x)^2\sqrt{ax}}\,dx,x,\sec^2(x)\right) \\
&= -\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)} - \frac{1}{4}a\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx,x,\sec^2(x)\right) \\
&= -\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a\sec^2(x)}\right) \\
&= \frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right) - \frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 38, normalized size = 0.84

$$-\frac{1}{2}\cos(x)\left(\cot(x)\csc(x)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Sqrt[a + a*Tan[x]^2],x]
[Out] -1/2*(Cos[x]*(Cot[x]*Csc[x] - Log[Cos[x/2]] + Log[Sin[x/2]])*Sqrt[a*Sec[x]^
2])
```

Maple [A]
time = 0.16, size = 51, normalized size = 1.13

method	result	size
default	$\frac{\left(\left(\cos^2(x)\right)\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)-\cos(x)-\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)\right)\cos(x)\sqrt{\frac{a}{\cos(x)^2}}}{2\sin(x)^2}$	51
risch	$\frac{\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,(\mathrm{e}^{2ix}+1)^2}{(\mathrm{e}^{2ix}-1)^2}-\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,\ln\left(\mathrm{e}^{ix}-1\right)\cos\left(x\right)+\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,\ln\left(\mathrm{e}^{ix}+1\right)\cos\left(x\right)$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(cos(x)^2*ln(-(-1+cos(x))/sin(x))-cos(x)-ln(-(-1+cos(x))/sin(x)))*cos(x
)*(a/cos(x)^2)^(1/2)/sin(x)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(33) = 66$.
time = 0.56, size = 303, normalized size = 6.73

$$-\frac{\left(4\left(\cos\left(3x\right)+\cos\left(x\right)\right)\cos\left(4x\right)-4\left(2\cos\left(2x\right)-1\right)\cos\left(3x\right)-8\cos\left(2x\right)\cos\left(x\right)-\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\sin\left(2x\right)-4\sin\left(2x\right)^2+4\cos\left(2x\right)-1\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\cos\left(x\right)+1\right)+\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\sin\left(2x\right)-4\sin\left(2x\right)^2+4\cos\left(2x\right)-1\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\cos\left(x\right)+1\right)+4\left(\sin\left(3x\right)+\sin\left(x\right)\right)\sin\left(4x\right)-8\sin\left(3x\right)\sin\left(2x\right)-8\sin\left(2x\right)\sin\left(x\right)+4\cos\left(x\right)\right)\sqrt{a}}{4\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\sin\left(2x\right)-4\sin\left(2x\right)^2+4\cos\left(2x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos(
2*x)*cos(x) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - si
n(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 -
4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x)
 - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))*sin(4
*x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))*sqrt(a)/(2*(2*cos
(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*s
in(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

Fricas [A]
time = 2.40, size = 58, normalized size = 1.29

$$\frac{\sqrt{a}\,\log\left(\frac{a\tan(x)^2+2\sqrt{a\tan\left(x\right)^2+a}\,\sqrt{a}+2a}{\tan(x)^2}\right)\tan\left(x\right)^2-2\sqrt{a\tan\left(x\right)^2+a}}{4\tan\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a)*log((a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)
^2)*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a))/tan(x)^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\tan^{2}\left(x\right)+1\right)}\,\cot^{3}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**3, x)
```

Giac [A]
time = 0.43, size = 42, normalized size = 0.93

$$-\frac{a\arctan\left(\frac{\sqrt{a\tan\left(x\right)^{2}+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}-\frac{\sqrt{a\tan\left(x\right)^{2}+a}}{2\tan\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a) - 1/2*sqrt(a*tan(x)^2
 + a)/tan(x)^2
```

Mupad [B]
time = 12.04, size = 37, normalized size = 0.82

$$\frac{\sqrt{a}\ \mathrm{atanh}\left(\frac{\sqrt{a\tan\left(x\right)^{2}+a}}{\sqrt{a}}\right)}{2}-\frac{\sqrt{a\tan\left(x\right)^{2}+a}}{2\tan\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] (a^(1/2)*atanh((a + a*tan(x)^2)^(1/2)/a^(1/2)))/2 - (a + a*tan(x)^2)^(1/2)/
(2*tan(x)^2)
```

3.264 $\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal. Leaf size=34

$$\cot(x)\sqrt{a\sec^2(x)}-\frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}$$

```
[Out] cot(x)*(a*sec(x)^2)^(1/2)-1/3*cot(x)*csc(x)^2*(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {3738, 4210, 2686}

$$\cot(x)\sqrt{a\sec^2(x)}-\frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] Cot[x]*Sqrt[a*Sec[x]^2] - (Cot[x]*Csc[x]^2*Sqrt[a*Sec[x]^2])/3
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx = \int \cot^4(x)\sqrt{a\sec^2(x)}\,dx$$
$$= \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int \cot^3(x)\csc(x)\,dx$$
$$= -\left(\left(\cos(x)\sqrt{a\sec^2(x)}\right)\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \csc(x)\right)\right)$$
$$= \cot(x)\sqrt{a\sec^2(x)} - \frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.65

$$-\frac{1}{3}\cot(x)\left(-3+\csc^2(x)\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -1/3*(Cot[x]*(-3 + Csc[x]^2)*Sqrt[a*Sec[x]^2])
```

Maple [A]
time = 0.14, size = 25, normalized size = 0.74

method	result	size
default	$-\frac{(3(\cos^2(x))-2)\cos(x)\sqrt{\frac{a}{\cos(x)^2}}}{3\sin(x)^3}$	25
risch	$\frac{2i\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)(3\,e^{4ix}-2\,e^{2ix}+3)}{3(e^{2ix}-1)^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*cos(x)^2-2)*cos(x)*(a/cos(x)^2)^(1/2)/sin(x)^3
```

Maxima [A]
time = 0.53, size = 29, normalized size = 0.85

$$\frac{\left(2\sqrt{a}\tan(x)^2-\sqrt{a}\right)\sqrt{\tan(x)^2+1}}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*sqrt(a)*tan(x)^2 - sqrt(a))*sqrt(tan(x)^2 + 1)/tan(x)^3
```

Fricas [A]
time = 2.31, size = 24, normalized size = 0.71

$$\frac{\sqrt{a\tan(x)^2+a}\,\left(2\,\tan(x)^2-1\right)}{3\,\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 - 1)/tan(x)^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\tan^2(x)+1\right)}\,\cot^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4*(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.
time = 0.43, size = 59, normalized size = 1.74

$$\frac{4\left(3\left(\sqrt{a}\,\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a\right)a^{\frac{5}{2}}}{3\left(\left(\sqrt{a}\,\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a)*a^(5/2)/((sqrt(a)*tan
(x) - sqrt(a*tan(x)^2 + a))^2 - a)^3
```

Mupad [B]
time = 11.85, size = 40, normalized size = 1.18

$$\frac{\sqrt{2}\,\sqrt{a}\,\left(2\,\sin(2\,x)-6\,\sin(2\,x)\,\left(2\cos(x)^2-1\right)\right)}{24\,\sqrt{2\cos(x)^2}\,\left(\cos(x)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4*(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*a^(1/2)*(2*sin(2*x) - 6*sin(2*x)*(2*cos(x)^2 - 1)))/(24*(2*cos(x)^
2)^(1/2)*(cos(x)^2 - 1)^2)
```

3.265 $\int \sqrt{a+a\tan^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$

```
[Out] arctanh(a^(1/2)*tan(d*x+c)/(a*sec(d*x+c)^2)^(1/2))*a^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3738, 4207, 223, 212}

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Tan[c + d*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a+a\tan^2(c+dx)}\,dx=\int \sqrt{a\sec^2(c+dx)}\,dx$$
$$=\frac{a\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{a\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$
$$=\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.86

$$\frac{\tanh^{-1}(\sin(c+dx))\cos(c+dx)\sqrt{a\sec^2(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Tan[c + d*x]^2],x]
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]*Sqrt[a*Sec[c + d*x]^2])/d
```

Maple [A]
time = 0.15, size = 34, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\left(\tan^2\left(dx+c\right)\right)}\right)}{d}$	34
default	$\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\left(\tan^2\left(dx+c\right)\right)}\right)}{d}$	34
risch	$\frac{2\ln\left(e^{idx}+ie^{-ic}\right)\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\,\cos(dx+c)}{d}-\frac{2\ln\left(e^{idx}-ie^{-ic}\right)\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\,\cos(dx+c)}{d}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/d*a^(1/2)*ln(a^(1/2)*tan(d*x+c)+(a+a*tan(d*x+c)^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.

time = 0.55, size = 65, normalized size = 1.81

$$\frac{\sqrt{a}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\sin\left(dx+c\right)+1\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 4.40, size = 90, normalized size = 2.50

$$\left[\frac{\sqrt{a}\log\left(2\,a\tan\left(dx+c\right)^2+2\,\sqrt{a\tan\left(dx+c\right)^2+a}\,\sqrt{a}\,\tan\left(dx+c\right)+a\right)}{2\,d},-\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{a\tan\left(dx+c\right)^2+a}\,\sqrt{-a}}{a\tan(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(a)*
tan(d*x + c) + a)/d, -sqrt(-a)*arctan(sqrt(a*tan(d*x + c)^2 + a)*sqrt(-a)/(
a*tan(d*x + c)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\tan^2\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*tan(c + d*x)**2 + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(30) = 60$.
time = 0.62, size = 66, normalized size = 1.83

$$-\frac{\left(\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)\right)\sqrt{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(tan(1/2*d*x + 1/2*c)^4 - 1) - log(
abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(tan(1/2*d*x + 1/2*c)^4 - 1))*sqrt(a)/d
```

Mupad [B]
time = 11.86, size = 41, normalized size = 1.14

$$\begin{cases} 0 & \text{if } a = 0 \\ \frac{\sqrt{a}\, \ln\left(\sqrt{a}\, \tan(c+d\,x)+\sqrt{a\tan\left(c+d\,x\right)^2+a}\right)}{d} & \text{if } a \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^(1/2),x)
```

```
[Out] piecewise(a == 0, 0, a ~= 0, (a^(1/2)*log(a^(1/2)*tan(c + d*x) + (a + a*tan
(c + d*x)^2)^(1/2)))/d)
```

3.266 $\int \tan^3(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}+\frac{\left(a\sec^2(x)\right)^{5/2}}{5a}$$

```
[Out] -1/3*(a*sec(x)^2)^(3/2)+1/5*(a*sec(x)^2)^(5/2)/a
```

Rubi [A]
time = 0.07, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {3738, 4209, 45}

$$\frac{\left(a\sec^2(x)\right)^{5/2}}{5a}-\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -1/3*(a*Sec[x]^2)^(3/2) + (a*Sec[x]^2)^(5/2)/(5*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \tan^3(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx &= \int \left(a\sec^2(x)\right)^{3/2}\tan^3(x)\,dx\\ &= \frac{1}{2}a\text{Subst}\left(\int (-1+x)\sqrt{ax}\,dx, x, \sec^2(x)\right)\\ &= \frac{1}{2}a\text{Subst}\left(\int \left(-\sqrt{ax}+\frac{(ax)^{3/2}}{a}\right)dx, x, \sec^2(x)\right)\\ &= -\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}+\frac{\left(a\sec^2(x)\right)^{5/2}}{5a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.69

$$\frac{1}{15}\left(a\sec^2(x)\right)^{3/2}\left(-5+3\sec^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] ((a*Sec[x]^2)^(3/2)*(-5 + 3*Sec[x]^2))/15
```

Maple [A]
time = 0.06, size = 29, normalized size = 0.91

method	result	size
derivativedivides	$\frac{(a+a(\tan^2(x)))^{\frac{5}{2}}}{5a}-\frac{(a+a(\tan^2(x)))^{\frac{3}{2}}}{3}$	29
default	$\frac{(a+a(\tan^2(x)))^{\frac{5}{2}}}{5a}-\frac{(a+a(\tan^2(x)))^{\frac{3}{2}}}{3}$	29
risch	$-\frac{8a\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(5\,e^{6ix}-2\,e^{4ix}+5\,e^{2ix})}{15(e^{2ix}+1)^4}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/a*(a+a*tan(x)^2)^(5/2)-1/3*(a+a*tan(x)^2)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(24) = 48.
time = 0.58, size = 559, normalized size = 17.47

$$-\frac{8\left(50a\cos(4x)\cos(3x)+50a\sin(4x)\sin(3x)+25a\sin(3x)\sin(2x)+(5a\cos(7x)-2a\cos(5x)+5a\cos(3x))\cos(10x)+5(5a\cos(7x)-2a\cos(5x)+5a\cos(3x))\cos(8x)+5(10a\cos(6x)+10a\cos(4x)+5a\cos(2x)+a)\cos(7x)-10(2a\cos(5x)-5a\cos(3x))\cos(6x)-2(10a\cos(4x)+5a\cos(2x)+a)\cos(5x)+5(5a\cos(2x)+a)\cos(3x)+(5a\sin(7x)-2a\sin(5x)+5a\sin(3x))\sin(10x)+5(5a\sin(7x)-2a\sin(5x)+5a\sin(3x))\sin(8x)+25(2a\sin(6x)+2a\sin(4x)+a\sin(2x))\sin(7x)-10(2a\sin(5x)-5a\sin(3x))\sin(6x)-10(2a\sin(4x)+a\sin(2x))\sin(5x)\right)\sqrt{a}}{15\left(2(5\cos(8x)+10\cos(6x)+10\cos(4x)+5\cos(2x)+1)\cos(10x)+\cos(10x)^2+10(10\cos(6x)+10\cos(4x)+5\cos(2x)+1)\cos(8x)+25\cos(8x)^2+20(10\cos(4x)+5\cos(2x)+1)\cos(6x)+100\cos(6x)^2+20(5\cos(2x)+1)\cos(4x)+100\cos(4x)^2+25\cos(2x)^2+10(\sin(8x)+2\sin(6x)+2\sin(4x)+\sin(2x))\sin(10x)+\sin(10x)^2+50(2\sin(6x)+2\sin(4x)+\sin(2x))\sin(8x)+25\sin(8x)^2+100(2\sin(4x)+\sin(2x))\sin(6x)+100\sin(6x)^2+100\sin(4x)^2+100\sin(4x)\sin(2x)+25\sin(2x)^2+10\cos(2x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/15*(50*a*cos(4*x)*cos(3*x) + 50*a*sin(4*x)*sin(3*x) + 25*a*sin(3*x)*sin(
2*x) + (5*a*cos(7*x) - 2*a*cos(5*x) + 5*a*cos(3*x))*cos(10*x) + 5*(5*a*cos(
7*x) - 2*a*cos(5*x) + 5*a*cos(3*x))*cos(8*x) + 5*(10*a*cos(6*x) + 10*a*cos(
4*x) + 5*a*cos(2*x) + a)*cos(7*x) - 10*(2*a*cos(5*x) - 5*a*cos(3*x))*cos(6*
x) - 2*(10*a*cos(4*x) + 5*a*cos(2*x) + a)*cos(5*x) + 5*(5*a*cos(2*x) + a)*c
os(3*x) + (5*a*sin(7*x) - 2*a*sin(5*x) + 5*a*sin(3*x))*sin(10*x) + 5*(5*a*s
in(7*x) - 2*a*sin(5*x) + 5*a*sin(3*x))*sin(8*x) + 25*(2*a*sin(6*x) + 2*a*si
n(4*x) + a*sin(2*x))*sin(7*x) - 10*(2*a*sin(5*x) - 5*a*sin(3*x))*sin(6*x) -
 10*(2*a*sin(4*x) + a*sin(2*x))*sin(5*x))*sqrt(a)/(2*(5*cos(8*x) + 10*cos(6
*x) + 10*cos(4*x) + 5*cos(2*x) + 1)*cos(10*x) + cos(10*x)^2 + 10*(10*cos(6*
x) + 10*cos(4*x) + 5*cos(2*x) + 1)*cos(8*x) + 25*cos(8*x)^2 + 20*(10*cos(4*
x) + 5*cos(2*x) + 1)*cos(6*x) + 100*cos(6*x)^2 + 20*(5*cos(2*x) + 1)*cos(4*
x) + 100*cos(4*x)^2 + 25*cos(2*x)^2 + 10*(sin(8*x) + 2*sin(6*x) + 2*sin(4*x
) + sin(2*x))*sin(10*x) + sin(10*x)^2 + 50*(2*sin(6*x) + 2*sin(4*x) + sin(2
*x))*sin(8*x) + 25*sin(8*x)^2 + 100*(2*sin(4*x) + sin(2*x))*sin(6*x) + 100*
sin(6*x)^2 + 100*sin(4*x)^2 + 100*sin(4*x)*sin(2*x) + 25*sin(2*x)^2 + 10*co
s(2*x) + 1)
```

Fricas [A]
time = 2.42, size = 29, normalized size = 0.91

$$\frac{1}{15}\left(3\,a\tan\left(x\right)^{4}+a\tan\left(x\right)^{2}-2\,a\right)\sqrt{a\tan\left(x\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*tan(x)^4 + a*tan(x)^2 - 2*a)*sqrt(a*tan(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\tan^{2}\left(x\right)+1\right)\right)^{\frac{3}{2}}\tan^{3}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*tan(x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(24) = 48.
time = 0.42, size = 72, normalized size = 2.25

$$\frac{1}{3}\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}-\sqrt{a\tan\left(x\right)^{2}+a}\,a+\frac{3\left(a\tan\left(x\right)^{2}+a\right)^{\frac{5}{2}}-10\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}a+15\sqrt{a\tan\left(x\right)^{2}+a}\,a^{2}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(a*tan(x)^2 + a)^(3/2) - sqrt(a*tan(x)^2 + a)*a + 1/15*(3*(a*tan(x)^2 +
 a)^(5/2) - 10*(a*tan(x)^2 + a)^(3/2)*a + 15*sqrt(a*tan(x)^2 + a)*a^2)/a
```

Mupad [B]
time = 12.01, size = 19, normalized size = 0.59

$$-\frac{2\sqrt{2}\,a^{3/2}\left(10\cos\left(x\right)^2-6\right)}{15\left(2\cos\left(x\right)^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] -(2*2^(1/2)*a^(3/2)*(10*cos(x)^2 - 6))/(15*(2*cos(x)^2)^(5/2))
```

3.267 $\int \tan^2(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=59

$$-\frac{1}{8}a\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}-\frac{1}{8}a\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{4}a\sec^2(x)\sqrt{a\sec^2(x)}\,\tan(x)$$

```
[Out] -1/8*a*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)-1/8*a*(a*sec(x)^2)^(1/2)*t
an(x)+1/4*a*sec(x)^2*(a*sec(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3738, 4210, 2691, 3853, 3855}

$$\frac{1}{4}a\tan(x)\sec^2(x)\sqrt{a\sec^2(x)}-\frac{1}{8}a\tan(x)\sqrt{a\sec^2(x)}-\frac{1}{8}a\cos(x)\sqrt{a\sec^2(x)}\,\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -1/8*(a*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2]) - (a*Sqrt[a*Sec[x]^2]*Tan[
x])/8 + (a*Sec[x]^2*Sqrt[a*Sec[x]^2]*Tan[x])/4
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \tan^2(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx &= \int \left(a\sec^2(x)\right)^{3/2}\tan^2(x)\,dx\\ &= \left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec^3(x)\tan^2(x)\,dx\\ &= \frac{1}{4}a\sec^2(x)\sqrt{a\sec^2(x)}\,\tan(x)-\frac{1}{4}\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int \sec^3(x)\,dx\\ &= -\frac{1}{8}a\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{4}a\sec^2(x)\sqrt{a\sec^2(x)}\,\tan(x)-\frac{1}{8}\Big(a\cos(x)\sqrt{}\\ &= -\frac{1}{8}a\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}-\frac{1}{8}a\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{4}a\sec\end{aligned}$$

Mathematica [A]
time = 0.05, size = 34, normalized size = 0.58

$$\frac{1}{8}\left(a\sec^2(x)\right)^{3/2}\left(-\tanh^{-1}(\sin(x))\cos^3(x)-\cos(x)\sin(x)+2\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] ((a*Sec[x]^2)^(3/2)*(-(ArcTanh[Sin[x]]*Cos[x]^3) - Cos[x]*Sin[x] + 2*Tan[x]
))/8
```

Maple [A]
time = 0.06, size = 57, normalized size = 0.97

method	result

derivativedivides	$\frac{\tan(x)\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{a\left(\frac{\sqrt{a+a\left(\tan^2(x)\right)}\,\tan(x)}{2}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\left(\tan^2(x)\right)}\right)}{2}\right.}{4}$
default	$\frac{\tan(x)\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{4}-\frac{a\left(\frac{\sqrt{a+a\left(\tan^2(x)\right)}\,\tan(x)}{2}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\left(\tan^2(x)\right)}\right)}{2}\right.}{4}$
risch	$\frac{ia\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(e^{6ix}-7\,e^{4ix}+7\,e^{2ix}-1\right)}{4\left(e^{2ix}+1\right)^3}-\frac{a\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}+i\right)\cos(x)}{4}+\frac{a\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}-i\right)\cos(x)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*tan(x)*(a+a*tan(x)^2)^(3/2)-1/4*a*(1/2*(a+a*tan(x)^2)^(1/2)*tan(x)+1/2*
a^(1/2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(47) = 94$.
time = 0.86, size = 934, normalized size = 15.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(112*a*cos(3*x)*sin(2*x) - 16*a*cos(x)*sin(2*x) + 16*a*cos(2*x)*sin(x)
 - 4*(a*sin(7*x) - 7*a*sin(5*x) + 7*a*sin(3*x) - a*sin(x))*cos(8*x) + 8*(2*
a*sin(6*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*cos(7*x) + 16*(7*a*sin(5*x) - 7*a
*sin(3*x) + a*sin(x))*cos(6*x) - 56*(3*a*sin(4*x) + 2*a*sin(2*x))*cos(5*x)
- 24*(7*a*sin(3*x) - a*sin(x))*cos(4*x) - (a*cos(8*x)^2 + 16*a*cos(6*x)^2 +
 36*a*cos(4*x)^2 + 16*a*cos(2*x)^2 + a*sin(8*x)^2 + 16*a*sin(6*x)^2 + 36*a*
sin(4*x)^2 + 48*a*sin(4*x)*sin(2*x) + 16*a*sin(2*x)^2 + 2*(4*a*cos(6*x) + 6
*a*cos(4*x) + 4*a*cos(2*x) + a)*cos(8*x) + 8*(6*a*cos(4*x) + 4*a*cos(2*x) +
 a)*cos(6*x) + 12*(4*a*cos(2*x) + a)*cos(4*x) + 8*a*cos(2*x) + 4*(2*a*sin(6
*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*sin(8*x) + 16*(3*a*sin(4*x) + 2*a*sin(2*
x))*sin(6*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*cos(8*x)^2 +
 16*a*cos(6*x)^2 + 36*a*cos(4*x)^2 + 16*a*cos(2*x)^2 + a*sin(8*x)^2 + 16*a*
sin(6*x)^2 + 36*a*sin(4*x)^2 + 48*a*sin(4*x)*sin(2*x) + 16*a*sin(2*x)^2 + 2
*(4*a*cos(6*x) + 6*a*cos(4*x) + 4*a*cos(2*x) + a)*cos(8*x) + 8*(6*a*cos(4*x
) + 4*a*cos(2*x) + a)*cos(6*x) + 12*(4*a*cos(2*x) + a)*cos(4*x) + 8*a*cos(2
*x) + 4*(2*a*sin(6*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*sin(8*x) + 16*(3*a*sin
(4*x) + 2*a*sin(2*x))*sin(6*x) + a)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
 + 4*(a*cos(7*x) - 7*a*cos(5*x) + 7*a*cos(3*x) - a*cos(x))*sin(8*x) - 4*(4*
```

```
a*cos(6*x) + 6*a*cos(4*x) + 4*a*cos(2*x) + a)*sin(7*x) - 16*(7*a*cos(5*x) -
 7*a*cos(3*x) + a*cos(x))*sin(6*x) + 28*(6*a*cos(4*x) + 4*a*cos(2*x) + a)*s
in(5*x) + 24*(7*a*cos(3*x) - a*cos(x))*sin(4*x) - 28*(4*a*cos(2*x) + a)*sin
(3*x) + 4*a*sin(x))*sqrt(a)/(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*c
os(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*
x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*
sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x)
+ 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*
x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)
```

Fricas [A]
time = 2.02, size = 57, normalized size = 0.97

$$\frac{1}{16}\,a^{\frac{3}{2}}\log\left(2\,a\tan\left(x\right)^{2}-2\,\sqrt{a\tan\left(x\right)^{2}+a}\,\sqrt{a}\,\tan\left(x\right)+a\right)+\frac{1}{8}\left(2\,a\tan\left(x\right)^{3}+a\tan\left(x\right)\right)\sqrt{a\tan\left(x\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*a^(3/2)*log(2*a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
+ 1/8*(2*a*tan(x)^3 + a*tan(x))*sqrt(a*tan(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\tan^{2}\left(x\right)+1\right)\right)^{\frac{3}{2}}\tan^{2}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*tan(x)**2, x)
```

Giac [A]
time = 0.45, size = 49, normalized size = 0.83

$$\frac{1}{8}\left(\sqrt{a\tan\left(x\right)^{2}+a}\,\left(2\,\tan\left(x\right)^{2}+1\right)\tan\left(x\right)+\sqrt{a}\,\log\left(\left|-\sqrt{a}\,\tan\left(x\right)+\sqrt{a\tan\left(x\right)^{2}+a}\right|\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 + 1)*tan(x) + sqrt(a)*log(abs(-sqrt(a
)*tan(x) + sqrt(a*tan(x)^2 + a))))*a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\tan(x)^2\,\left(a\tan(x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] int(tan(x)^2*(a + a*tan(x)^2)^(3/2), x)
```

3.268 $\int \tan(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

```
[Out] 1/3*(a*sec(x)^2)^(3/2)
```

Rubi [A]

time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3738, 4209, 32}

$$\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (a*Sec[x]^2)^(3/2)/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \tan(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx &= \int \left(a\sec^2(x)\right)^{3/2}\tan(x)\,dx\\ &= \frac{1}{2}a\text{Subst}\left(\int \sqrt{ax}\,dx, x, \sec^2(x)\right)\\ &= \frac{1}{3}\left(a\sec^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(a + a*Tan[x]^2)^(3/2),x]
[Out] (a*Sec[x]^2)^(3/2)/3
```

Maple [A]
time = 0.04, size = 13, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{3}$	13
default	$\frac{\left(a+a\left(\tan^2(x)\right)\right)^{\frac{3}{2}}}{3}$	13
risch	$\frac{8a\,\mathrm{e}^{2ix}\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^2}}}{3\left(\mathrm{e}^{2ix}+1\right)^2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(a+a*tan(x)^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((a*tan(x)^2 + a)^(3/2)*tan(x), x)
```

Fricas [A]
time = 2.14, size = 12, normalized size = 0.86

$$\frac{1}{3}\left(a\tan\left(x\right)^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*tan(x)^2 + a)^(3/2)
```

Sympy [A]
time = 1.08, size = 12, normalized size = 0.86

$$\frac{(a\tan^2(x) + a)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] (a*tan(x)**2 + a)**(3/2)/3
```

Giac [A]
time = 0.44, size = 12, normalized size = 0.86

$$\frac{1}{3}\left(a\tan(x)^2 + a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(a*tan(x)^2 + a)^(3/2)
```

Mupad [B]
time = 0.18, size = 11, normalized size = 0.79

$$\frac{a^{3/2}}{3\left(\cos(x)^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] a^(3/2)/(3*(cos(x)^2)^(3/2))
```

3.269 $\int \cot(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=37

$$-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)+a\sqrt{a\sec^2(x)}$$

```
[Out] -a^(3/2)*arctanh((a*sec(x)^2)^(1/2)/a^(1/2))+a*(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3738, 4209, 52, 65, 213}

$$a\sqrt{a\sec^2(x)}-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(a^(3/2)*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]) + a*Sqrt[a*Sec[x]^2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx &= \int \cot(x)\left(a\sec^2(x)\right)^{3/2}\,dx\\ &= \frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{ax}}{-1+x}\,dx, x, \sec^2(x)\right)\\ &= a\sqrt{a\sec^2(x)}+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx, x, \sec^2(x)\right)\\ &= a\sqrt{a\sec^2(x)}+a\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sec^2(x)}\right)\\ &= -a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)+a\sqrt{a\sec^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.92

$$a\left(1+\cos(x)\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] a*(1 + Cos[x]*(-Log[Cos[x/2]] + Log[Sin[x/2]]))*Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.86

method	result	size

default	$\left(\cos \left(x \right) \ln \left(-\frac{-1+\cos \left(x \right)}{\sin \left(x \right)}\right)+\cos \left(x \right)+1\right) \left(\cos^{2}\left(x \right)\right) \left(\frac{a}{\cos \left(x \right)^{2}}\right)^{\frac{3}{2}}$	32
risch	$2a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}-2a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\,\ln\left(\mathrm{e}^{ix}+1\right)\cos\left(x\right)+2a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\,\ln\left(\mathrm{e}^{ix}-1\right)\cos\left(x\right)$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (cos(x)*ln(-(-1+cos(x))/sin(x))+cos(x)+1)*cos(x)^2*(a/cos(x)^2)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(29) = 58$.
time = 0.55, size = 134, normalized size = 3.62

$$\frac{\left(4\,a\cos\left(2\,x\right)\cos\left(x\right)+4\,a\sin\left(2\,x\right)\sin\left(x\right)+4\,a\cos\left(x\right)-\left(a\cos\left(2\,x\right)^{2}+a\sin\left(2\,x\right)^{2}+2\,a\cos\left(2\,x\right)+a\right)\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)+\left(a\cos\left(2\,x\right)^{2}+a\sin\left(2\,x\right)^{2}+2\,a\cos\left(2\,x\right)+a\right)\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\cos\left(x\right)+1\right)\right)\sqrt{a}}{2\,\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*cos(2*x)*cos(x) + 4*a*sin(2*x)*sin(x) + 4*a*cos(x) - (a*cos(2*x)^2
 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
 + (a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1))*sqrt(a)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A]
time = 2.53, size = 49, normalized size = 1.32

$$\frac{1}{2}\,a^{\frac{3}{2}}\log\left(\frac{a\tan\left(x\right)^{2}-2\,\sqrt{a\tan\left(x\right)^{2}+a}\,\sqrt{a}+2\,a}{\tan\left(x\right)^{2}}\right)+\sqrt{a\tan\left(x\right)^{2}+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*a^(3/2)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)^
2) + sqrt(a*tan(x)^2 + a)*a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\tan^{2}\left(x\right)+1\right)\right)^{\frac{3}{2}}\cot\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*cot(x), x)
```

Giac [A]
time = 0.41, size = 42, normalized size = 1.14

$$a^2\left(\frac{\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{a\tan(x)^2+a}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] a^2*(arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(a*tan(x)^2 + a)/
a)
```

Mupad [B]
time = 11.67, size = 33, normalized size = 0.89

$$a\sqrt{a\tan(x)^2+a}-a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] a*(a + a*tan(x)^2)^(1/2) - a^(3/2)*atanh((a + a*tan(x)^2)^(1/2)/a^(1/2))
```

3.270 $\int \cot^2(x) \left(a+a\tan^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=33

$$a \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} - a\cot(x)\sqrt{a\sec^2(x)}$$

```
[Out] a*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)-a*cot(x)*(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3738, 4210, 2701, 327, 213}

$$a\cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x)) - a\cot(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] a*ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2] - a*Cot[x]*Sqrt[a*Sec[x]^2]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx &= \int \cot^2(x)\left(a\sec^2(x)\right)^{3/2}\,dx\\ &= \left(a\cos(x)\sqrt{a\sec^2(x)}\right)\int \csc^2(x)\sec(x)\,dx\\ &= -\left(\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(x)\right)\right)\\ &= -a\cot(x)\sqrt{a\sec^2(x)}-\left(a\cos(x)\sqrt{a\sec^2(x)}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\right.\\ &= a\tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}-a\cot(x)\sqrt{a\sec^2(x)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.02, size = 27, normalized size = 0.82

$$-a\cot(x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(x)\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*(a + a*Tan[x]^2)^(3/2),x]
[Out] -(a*Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[x]^2]*Sqrt[a*Sec[x]^2])
```

Maple [A]

time = 0.17, size = 54, normalized size = 1.64

method	result	si
default	$\frac{\left(\ln\left(\frac{1-\cos(x)+\sin(x)}{\sin(x)}\right)\sin(x)-\ln\left(-\frac{\cos(x)-1+\sin(x)}{\sin(x)}\right)\sin(x)-1\right)\left(\cos^3(x)\right)\left(\frac{a}{\cos(x)^2}\right)^{\frac{3}{2}}}{\sin(x)}$	54

risch	$-\frac{2ia(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}{e^{2ix}-1}-2a\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\ \ln(e^{ix}-i)\cos(x)+2a\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\ \ln(e^{ix}+i)\cos(x)$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((1-cos(x)+sin(x))/sin(x))*sin(x)-ln(-(cos(x)-1+sin(x))/sin(x))*sin(x)-1
)*cos(x)^3*(a/cos(x)^2)^(3/2)/sin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(29) = 58$.
time = 0.54, size = 134, normalized size = 4.06

$$-\frac{\left(4\,a\cos(x)\sin(2\,x)-4\,a\cos(2\,x)\sin(x)-\left(a\cos(2\,x)^2+a\sin(2\,x)^2-2\,a\cos(2\,x)+a\right)\log\left(\cos(x)^2+\sin(x)^2+2\,\sin(x)+1\right)+\left(a\cos(2\,x)^2+a\sin(2\,x)^2-2\,a\cos(2\,x)+a\right)\log\left(\cos(x)^2+\sin(x)^2-2\,\sin(x)+1\right)+4\,a\sin(x)\right)\sqrt{a}}{2\left(\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*cos(x)*sin(2*x) - 4*a*cos(2*x)*sin(x) - (a*cos(2*x)^2 + a*sin(2*x
)^2 - 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*cos(2*
x)^2 + a*sin(2*x)^2 - 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1) + 4*a*sin(x))*sqrt(a)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)
```

Fricas [A]
time = 3.39, size = 53, normalized size = 1.61

$$\frac{a^{\frac{3}{2}}\log\left(2\,a\tan(x)^2+2\,\sqrt{a\tan(x)^2+a}\,\sqrt{a}\,\tan(x)+a\right)\tan(x)-2\,\sqrt{a\tan(x)^2+a}\,a}{2\,\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)*
tan(x) - 2*sqrt(a*tan(x)^2 + a)*a)/tan(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\tan^2(x)+1\right)\right)^{\frac{3}{2}}\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral((a*(tan(x)**2 + 1))**(3/2)*cot(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.
time = 0.45, size = 62, normalized size = 1.88

$$-\frac{1}{2}\left(\sqrt{a}\,\log\left(\left(\sqrt{a}\,\tan\left(x\right)-\sqrt{a\tan\left(x\right)^{2}+a}\right)^{2}\right)-\frac{4\,a^{\frac{3}{2}}}{\left(\sqrt{a}\,\tan\left(x\right)-\sqrt{a\tan\left(x\right)^{2}+a}\right)^{2}-a}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(a)*log((sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2) - 4*a^(3/2)/((
sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a))*a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cot(x)^2\,\left(a\tan(x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] int(cot(x)^2*(a + a*tan(x)^2)^(3/2), x)
```

3.271 $\int \left(a+a\tan^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=68

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d}+\frac{a\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{2d}$$

```
[Out] 1/2*a^(3/2)*arctanh(a^(1/2)*tan(d*x+c)/(a*sec(d*x+c)^2)^(1/2))/d+1/2*a*(a*s
ec(d*x+c)^2)^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3738, 4207, 201, 223, 212}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d}+\frac{a\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/(2*d) + (a
*Sqrt[a*Sec[c + d*x]^2]*Tan[c + d*x])/(2*d)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+a\tan^2(c+dx)\right)^{3/2}\,dx &= \int \left(a\sec^2(c+dx)\right)^{3/2}\,dx\\
&= \frac{a\text{Subst}\left(\int \sqrt{a+ax^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{a\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{2d}+\frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx,x,\tan(c+dx)\right)}{2d}\\
&= \frac{a\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{2d}+\frac{a^2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d}\\
&= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d}+\frac{a\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.63

$$
\frac{a\sqrt{a\sec^2(c+dx)}\,\left(\tanh^{-1}(\sin(c+dx))\cos(c+dx)+\tan(c+dx)\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Sec[c + d*x]^2]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + Tan[c + d*x
]))/(2*d)
```

Maple [A]
time = 0.10, size = 60, normalized size = 0.88

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)\sqrt{a+a\left(\tan^2(dx+c)\right)}}{2}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\left(\tan^2(dx+c)\right)}\right)}{2}\right)}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)\sqrt{a+a\left(\tan^2(dx+c)\right)}}{2}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\left(\tan^2(dx+c)\right)}\right)}{2}\right)}{d}$
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}-1\right)}{\left(e^{2i(dx+c)}+1\right)d}-\frac{\ln\left(e^{idx}-ie^{-ic}\right)\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\,a\cos(dx+c)}{d}+\frac{\ln\left(e^{idx}+ie^{-ic}\right)\sqrt{\frac{}{(e}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(1/2*tan(d*x+c)*(a+a*tan(d*x+c)^2)^(1/2)+1/2*a^(1/2)*ln(a^(1/2)*tan(d
*x+c)+(a+a*tan(d*x+c)^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(56) = 112$.
time = 0.53, size = 556, normalized size = 8.18

$$-\frac{\left(8a\cos(3dx+3c)\sin(2dx+2c)-8a\cos(dx+c)\sin(2dx+2c)+8a\cos(2dx+2c)\sin(dx+c)-4\left(a\sin(3dx+3c)-a\sin(dx+c)\right)\cos(4dx+4c)-\left(a\cos(4dx+4c)^2+4a\cos(2dx+2c)^2+a\sin(4dx+4c)^2+4a\sin(4dx+4c)\sin(2dx+2c)+4a\sin(2dx+2c)^2+2\left(2a\cos(2dx+2c)+a\right)\cos(4dx+4c)+4a\cos(2dx+2c)+a\right)\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)+\left(a\cos(4dx+4c)^2+4a\cos(2dx+2c)^2+a\sin(4dx+4c)^2+4a\sin(4dx+4c)\sin(2dx+2c)+4a\sin(2dx+2c)^2+2\left(2a\cos(2dx+2c)+a\right)\cos(4dx+4c)+4a\cos(2dx+2c)+a\right)\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)+4\left(a\cos(3dx+3c)-a\cos(dx+c)\right)\sin(4dx+4c)-4\left(2a\cos(2dx+2c)+a\right)\sin(3dx+3c)+4a\sin(dx+c)\right)\sqrt{a}}{4\left(2\left(2\cos(2dx+2c)+1\right)\cos(4dx+4c)+\cos(4dx+4c)^2+4\cos(2dx+2c)^2+\sin(4dx+4c)^2+4\sin(4dx+4c)\sin(2dx+2c)+4\sin(2dx+2c)^2+4\cos(2dx+2c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 8*a*cos(d*x + c)*sin(2*d*x +
2*c) + 8*a*cos(2*d*x + 2*c)*sin(d*x + c) - 4*(a*sin(3*d*x + 3*c) - a*sin(d*
x + c))*cos(4*d*x + 4*c) - (a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 +
 a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d
*x + 2*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x
 + 2*c) + a)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + (a
*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*
x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*log(cos(d*x + c)
^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*(a*cos(3*d*x + 3*c) - a*cos(d
*x + c))*sin(4*d*x + 4*c) - 4*(2*a*cos(2*d*x + 2*c) + a)*sin(3*d*x + 3*c) +
 4*a*sin(d*x + c))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*
d)
```

Fricas [A]
time = 3.82, size = 72, normalized size = 1.06

$$\frac{a^{\frac{3}{2}}\log\left(2\,a\tan\left(dx+c\right)^2+2\,\sqrt{a\tan\left(dx+c\right)^2+a}\,\sqrt{a}\,\tan\left(dx+c\right)+a\right)+2\,\sqrt{a\tan\left(dx+c\right)^2+a}\,a\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(a^(3/2)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(a)*
tan(d*x + c) + a) + 2*sqrt(a*tan(d*x + c)^2 + a)*a*tan(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\tan^2\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. $2(56) = 112$.
time = 1.90, size = 2125, normalized size = 31.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)
^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*
c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c) - a^(3/2)*s
gn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*
x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*
c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1))*log(abs(-tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) + 1))/(tan(1/2*c) - 1) - (a^(3/2)*sgn(tan(1/2*d*x)
^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/
2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/
2*d*x)*tan(1/2*c) + 1)*tan(1/2*c) + a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4
 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/
2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*
c) + 1))*log(abs(-tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) + 1))
```

/(tan(1/2*c) + 1) - 2*(a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^3*tan(1/2*c)^8 + 6*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^3*tan(1/2*c)^6 + 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^2*tan(1/2*c)^7 + a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)*tan(1/2*c)^8 - 18*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^2*tan(1/2*c)^5 - 10*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)*tan(1/2*c)^6 - 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^7 - 6*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^3*tan(1/2*c)^2 - 18*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^2*tan(1/2*c)^3 + 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^5 - a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^3 + 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^2*tan(1/2*c) + 10*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^3 - a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x) - 2*

```
a^(3/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - t
an(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 -
 tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c))/((tan(1/2*d*x)^2
*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
 1)^2*(tan(1/2*c)^4 - 2*tan(1/2*c)^2 + 1)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\tan(c+d\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)^2)^(3/2), x)
```

3.272 $\int (a+a\tan^2(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d}+\frac{3a^2\sqrt{a\sec^2(c+dx)}\tan(c+dx)}{8d}+\frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d}$$

```
[Out] 3/8*a^(5/2)*arctanh(a^(1/2)*tan(d*x+c)/(a*sec(d*x+c)^2)^(1/2))/d+1/4*a*(a*s
ec(d*x+c)^2)^(3/2)*tan(d*x+c)/d+3/8*a^2*(a*sec(d*x+c)^2)^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3738, 4207, 201, 223, 212}

$$\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d}+\frac{3a^2\tan(c+dx)\sqrt{a\sec^2(c+dx)}}{8d}+\frac{a\tan(c+dx)\left(a\sec^2(c+dx)\right)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a*Sec[c + d*x]^2]])/(8*d) +
(3*a^2*Sqrt[a*Sec[c + d*x]^2]*Tan[c + d*x])/(8*d) + (a*(a*Sec[c + d*x]^2)^(
3/2)*Tan[c + d*x])/(4*d)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \left(a+a\tan^2(c+dx)\right)^{5/2}\,dx = \int \left(a\sec^2(c+dx)\right)^{5/2}\,dx$$
$$= \frac{a\text{Subst}\left(\int \left(a+ax^2\right)^{3/2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d} + \frac{(3a^2)\,\text{Subst}\left(\int \sqrt{a+ax^2}\,dx, x, \tan(c+\right.}{4d}$$
$$= \frac{3a^2\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{8d} + \frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d} + \frac{(3a^3}{}$$
$$= \frac{3a^2\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{8d} + \frac{a\left(a\sec^2(c+dx)\right)^{3/2}\tan(c+dx)}{4d} + \frac{(3a^3}{}$$
$$= \frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d} + \frac{3a^2\sqrt{a\sec^2(c+dx)}\,\tan(c+dx)}{8d} + \frac{a(}{}$$

Mathematica [A]
time = 0.15, size = 65, normalized size = 0.66

$$\frac{a^2\cos(c+dx)\sqrt{a\sec^2(c+dx)}\,\left(3\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\left(3+2\sec^2(c+dx)\right)\tan(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] (a^2*Cos[c + d*x]*Sqrt[a*Sec[c + d*x]^2]*(3*ArcTanh[Sin[c + d*x]] + Sec[c +
 d*x]*(3 + 2*Sec[c + d*x]^2)*Tan[c + d*x]))/(8*d)
```

Maple [A]
time = 0.11, size = 86, normalized size = 0.88

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\tan(dx+c)\sqrt{a+a\left(\tan^2(dx+c)\right)}}{2}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+}\right.}{2}\right.}{4}\right.}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\tan(dx+c)\sqrt{a+a\left(\tan^2(dx+c)\right)}}{2}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+}\right.}{2}\right.}{4}\right.}{d}$
risch	$-\frac{ia^2\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(3\,e^{6i(dx+c)}+11\,e^{4i(dx+c)}-11\,e^{2i(dx+c)}-3\right)}{4\left(e^{2i(dx+c)}+1\right)^3 d}+\frac{3\ln\left(e^{idx}+ie^{-ic}\right)\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\,a^2\cos}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(1/4*tan(d*x+c)*(a+a*tan(d*x+c)^2)^(3/2)+3/4*a*(1/2*tan(d*x+c)*(a+a*t
an(d*x+c)^2)^(1/2)+1/2*a^(1/2)*ln(a^(1/2)*tan(d*x+c)+(a+a*tan(d*x+c)^2)^(1/
2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(82) = 164$.
time = 0.85, size = 1769, normalized size = 18.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(176*a^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 48*a^2*cos(d*x + c)*sin(2
*d*x + 2*c) - 48*a^2*cos(2*d*x + 2*c)*sin(d*x + c) - 12*a^2*sin(d*x + c) +
4*(3*a^2*sin(7*d*x + 7*c) + 11*a^2*sin(5*d*x + 5*c) - 11*a^2*sin(3*d*x + 3*
c) - 3*a^2*sin(d*x + c))*cos(8*d*x + 8*c) - 24*(2*a^2*sin(6*d*x + 6*c) + 3*
a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 16*(11*a^
2*sin(5*d*x + 5*c) - 11*a^2*sin(3*d*x + 3*c) - 3*a^2*sin(d*x + c))*cos(6*d*
x + 6*c) - 88*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*cos(5*d*x +
 5*c) - 24*(11*a^2*sin(3*d*x + 3*c) + 3*a^2*sin(d*x + c))*cos(4*d*x + 4*c)
+ 3*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x
+ 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(
6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*
```

```
d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(
4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) +
a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c)
+ a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c
) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x +
2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) +
 1) - 3*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*
d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*
sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 +
 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c
) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2
*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x +
 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x +
 2*c))*sin(6*d*x + 6*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x +
c) + 1) - 4*(3*a^2*cos(7*d*x + 7*c) + 11*a^2*cos(5*d*x + 5*c) - 11*a^2*cos(
3*d*x + 3*c) - 3*a^2*cos(d*x + c))*sin(8*d*x + 8*c) + 12*(4*a^2*cos(6*d*x +
 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*sin(7*d*x +
7*c) - 16*(11*a^2*cos(5*d*x + 5*c) - 11*a^2*cos(3*d*x + 3*c) - 3*a^2*cos(d*
x + c))*sin(6*d*x + 6*c) + 44*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2
*c) + a^2)*sin(5*d*x + 5*c) + 24*(11*a^2*cos(3*d*x + 3*c) + 3*a^2*cos(d*x +
 c))*sin(4*d*x + 4*c) - 44*(4*a^2*cos(2*d*x + 2*c) + a^2)*sin(3*d*x + 3*c))
*sqrt(a)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c)
+ 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c
)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x +
2*c) + 1)*d)
```

Fricas [A]
time = 2.67, size = 91, normalized size = 0.93

$$\frac{3\,a^{\frac{5}{2}}\log\left(2\,a\tan\left(dx+c\right)^{2}+2\,\sqrt{a\tan\left(dx+c\right)^{2}+a}\,\sqrt{a}\,\tan\left(dx+c\right)+a\right)+2\,\left(2\,a^{2}\tan\left(dx+c\right)^{3}+5\,a^{2}\tan\left(dx+c\right)\right)\sqrt{a\tan\left(dx+c\right)^{2}+a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*a^(5/2)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(
a)*tan(d*x + c) + a) + 2*(2*a^2*tan(d*x + c)^3 + 5*a^2*tan(d*x + c))*sqrt(a
*tan(d*x + c)^2 + a))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\tan^2{(c+dx)}+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7084 vs. $2(82) = 164$.
time = 3.63, size = 7084, normalized size = 72.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*
c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/
2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c) - a^(5/2)
*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*
d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/
2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1))*log(abs(-tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) + tan(1/2*c) + 1))/(tan(1/2*c) - 1) - 3*(a^(5/2)*sgn(tan(1/2*
d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*ta
n(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*ta
n(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c) + a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*
c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*ta
n(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(
1/2*c) + 1))*log(abs(-tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) +
 1))/(tan(1/2*c) + 1) - 2*(5*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*ta
n(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) -
4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)
*tan(1/2*d*x)^7*tan(1/2*c)^16 + 34*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4
- 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2
*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c
) + 1)*tan(1/2*d*x)^7*tan(1/2*c)^14 - 6*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*
c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*ta
n(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(
1/2*c) + 1)*tan(1/2*d*x)^6*tan(1/2*c)^15 + 3*a^(5/2)*sgn(tan(1/2*d*x)^4*tan
(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)
^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)
```

```
*tan(1/2*c) + 1)*tan(1/2*d*x)^5*tan(1/2*c)^16 - 58*a^(5/2)*sgn(tan(1/2*d*x)
^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/
2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/
2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^7*tan(1/2*c)^12 - 354*a^(5/2)*sgn(tan(1
/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4
*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4
*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^6*tan(1/2*c)^13 - 162*a^(5/2)*sg
n(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x
)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c
)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^5*tan(1/2*c)^14 - 30*a^(5
/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1
/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan
(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^4*tan(1/2*c)^15 + 3
*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 -
tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3
 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^3*tan(1/2*c)^1
6 - 6*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)
^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*
c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^7*tan(1/2
*c)^10 + 330*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan
(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*t
an(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^6*
tan(1/2*c)^11 + 1514*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*
x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/
2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2
*d*x)^5*tan(1/2*c)^12 + 1062*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*ta
n(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) -
4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)
*tan(1/2*d*x)^4*tan(1/2*c)^13 + 318*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4
 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/
2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*
c) + 1)*tan(1/2*d*x)^3*tan(1/2*c)^14 + 46*a^(5/2)*sgn(tan(1/2*d*x)^4*tan(1/
2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*
tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*ta
n(1/2*c) + 1)*tan(1/2*d*x)^2*tan(1/2*c)^15 + 5*a^(5/2)*sgn(tan(1/2*d*x)^4*t
an(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*
x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*
x)*tan(1/2*c) + 1)*tan(1/2*d*x)*tan(1/2*c)^16 + 30*a^(5/2)*sgn(tan(1/2*d*x)
^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/
2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/
2*d*x)*tan(1/2*c) + 1)*tan(1/2*d*x)^6*tan(1/2*c)^9 - 506*a^(5/2)*sgn(tan(1/
2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\tan(c+d\,x)^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tan(c + d*x)^2)^(5/2),x)
[Out] int((a + a*tan(c + d*x)^2)^(5/2), x)
```

3.273 $\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{\sqrt{a\sec^2(x)}}+\frac{\sqrt{a\sec^2(x)}}{a}$$

```
[Out] 1/(a*sec(x)^2)^(1/2)+(a*sec(x)^2)^(1/2)/a
```

Rubi [A]

time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3738, 4209, 45}

$$\frac{\sqrt{a\sec^2(x)}}{a}+\frac{1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] 1/Sqrt[a*Sec[x]^2] + Sqrt[a*Sec[x]^2]/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\tan^3(x)}{\sqrt{a\sec^2(x)}}\,dx \\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{-1+x}{(ax)^{3/2}}\,dx, x, \sec^2(x)\right) \\
&= \frac{1}{2}a\text{Subst}\left(\int \left(-\frac{1}{(ax)^{3/2}}+\frac{1}{a\sqrt{ax}}\right)\,dx, x, \sec^2(x)\right) \\
&= \frac{1}{\sqrt{a\sec^2(x)}}+\frac{\sqrt{a\sec^2(x)}}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 0.68

$$\frac{1+\sec^2(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/Sqrt[a + a*Tan[x]^2],x]
[Out] (1 + Sec[x]^2)/Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.06, size = 26, normalized size = 1.04

method	result	size
derivativedivides	$\frac{\sqrt{a+a\left(\tan^2\left(x\right)\right)}}{a}+\frac{1}{\sqrt{a+a\left(\tan^2\left(x\right)\right)}}$	26
default	$\frac{\sqrt{a+a\left(\tan^2\left(x\right)\right)}}{a}+\frac{1}{\sqrt{a+a\left(\tan^2\left(x\right)\right)}}$	26
risch	$\frac{e^{2ix}}{2\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}+\frac{1}{2(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{2\,e^{2ix}}{\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)^2}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/a*(a+a*tan(x)^2)^(1/2)+1/(a+a*tan(x)^2)^(1/2)
```

Maxima [A]
time = 0.31, size = 37, normalized size = 1.48

$$\frac{\left(\sin\left(x\right)^2-2\right)\sqrt{\sin\left(x\right)+1}\,\sqrt{-\sin\left(x\right)+1}}{\sqrt{a}\,\sin\left(x\right)^2-\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] (sin(x)^2 - 2)*sqrt(sin(x) + 1)*sqrt(-sin(x) + 1)/(sqrt(a)*sin(x)^2 - sqrt(
a))
```

Fricas [A]
time = 2.42, size = 17, normalized size = 0.68

$$\frac{\tan(x)^2 + 2}{\sqrt{a\tan(x)^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] (tan(x)^2 + 2)/sqrt(a*tan(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(x)}{\sqrt{a(\tan^2(x) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+a*tan(x)**2)**(1/2),x)
[Out] Integral(tan(x)**3/sqrt(a*(tan(x)**2 + 1)), x)
```

Giac [A]
time = 0.44, size = 27, normalized size = 1.08

$$\frac{\sqrt{a\tan(x)^2 + a} + \frac{a}{\sqrt{a\tan(x)^2 + a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] (sqrt(a*tan(x)^2 + a) + a/sqrt(a*tan(x)^2 + a))/a
```

Mupad [B]
time = 0.29, size = 22, normalized size = 0.88

$$\frac{\sqrt{2}\,(\cos(2x) + 3)}{2\sqrt{a}\,\sqrt{\cos(2x) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + a*tan(x)^2)^(1/2),x)
[Out] (2^(1/2)*(cos(2*x) + 3))/(2*a^(1/2)*(cos(2*x) + 1)^(1/2))
```

3.274 $\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\sin(x))\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

```
[Out] arctanh(sin(x))*sec(x)/(a*sec(x)^2)^(1/2)-tan(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3738, 4210, 2672, 327, 212}

$$\frac{\sec(x)\tanh^{-1}(\sin(x))}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] (ArcTanh[Sin[x]]*Sec[x])/Sqrt[a*Sec[x]^2] - Tan[x]/Sqrt[a*Sec[x]^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\tan^2(x)}{\sqrt{a\sec^2(x)}}\,dx \\
&= \frac{\sec(x)\int \sin(x)\tan(x)\,dx}{\sqrt{a\sec^2(x)}} \\
&= \frac{\sec(x)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(x)\right)}{\sqrt{a\sec^2(x)}} \\
&= -\frac{\tan(x)}{\sqrt{a\sec^2(x)}}+\frac{\sec(x)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{\sqrt{a\sec^2(x)}} \\
&= \frac{\tanh^{-1}(\sin(x))\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 1.58

$$-\frac{\sec(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sin(x)\right)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/Sqrt[a + a*Tan[x]^2],x]
[Out] -((Sec[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x]))/S
qrt[a*Sec[x]^2])
```

Maple [A]
time = 0.06, size = 38, normalized size = 1.23

method	result
derivativedivides	$\frac{\ln\left(\sqrt{a}\ \tan(x)+\sqrt{a+a\left(\tan^{2}\left(x\right)\right)}\right)}{\sqrt{a}}-\frac{\tan(x)}{\sqrt{a+a\left(\tan^{2}\left(x\right)\right)}}$
default	$\frac{\ln\left(\sqrt{a}\ \tan(x)+\sqrt{a+a\left(\tan^{2}\left(x\right)\right)}\right)}{\sqrt{a}}-\frac{\tan(x)}{\sqrt{a+a\left(\tan^{2}\left(x\right)\right)}}$
risch	$\frac{i\mathrm{e}^{2ix}}{2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}\ (\mathrm{e}^{2ix}+1)}-\frac{i}{2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}\ (\mathrm{e}^{2ix}+1)}-\frac{\mathrm{e}^{ix}\ln(\mathrm{e}^{ix}-i)}{\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}\ (\mathrm{e}^{2ix}+1)}+\frac{\mathrm{e}^{ix}\ln(\mathrm{e}^{ix}+i)}{\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}\ (\mathrm{e}^{2ix}+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))/a^(1/2)-tan(x)/(a+a*tan(x)^2)^(1/2)
```

Maxima [A]
time = 0.55, size = 42, normalized size = 1.35

$$\frac{\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\sin\left(x\right)+1\right)-\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\sin\left(x\right)+1\right)-2\sin\left(x\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1) - 2*sin(x))/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(27) = 54$.
time = 3.91, size = 64, normalized size = 2.06

$$\frac{\left(\tan\left(x\right)^{2}+1\right)\sqrt{a}\ \log\left(2\,a\tan\left(x\right)^{2}+2\,\sqrt{a\tan\left(x\right)^{2}+a}\,\sqrt{a}\ \tan\left(x\right)+a\right)-2\,\sqrt{a\tan\left(x\right)^{2}+a}\ \tan\left(x\right)}{2\left(a\tan\left(x\right)^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*((tan(x)^2 + 1)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(
a)*tan(x) + a) - 2*sqrt(a*tan(x)^2 + a)*tan(x))/(a*tan(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{2}\left(x\right)}{\sqrt{a\left(\tan^{2}\left(x\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)**2/sqrt(a*(tan(x)**2 + 1)), x)
```

Giac [A]
time = 0.47, size = 40, normalized size = 1.29

$$-\frac{\log\left(\left|-\sqrt{a}\tan\left(x\right)+\sqrt{a\tan\left(x\right)^{2}+a}\right|\right)}{\sqrt{a}}-\frac{\tan\left(x\right)}{\sqrt{a\tan\left(x\right)^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a)))/sqrt(a) - tan(x)/sqrt(a*t
an(x)^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(x)^2}{\sqrt{a\tan\left(x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)^2/(a + a*tan(x)^2)^(1/2), x)
```

3.275 $\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

```
[Out] -1/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3738, 4209, 32}

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -(1/Sqrt[a*Sec[x]^2])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{a\sec^2(x)}}\,dx$$
$$= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \sec^2(x)\right)$$
$$= -\frac{1}{\sqrt{a\sec^2(x)}}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + a*Tan[x]^2],x]
[Out] -(1/Sqrt[a*Sec[x]^2])
```

Maple [A]
time = 0.04, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{1}{\sqrt{a+a\left(\tan^2(x)\right)}}$	13
default	$-\frac{1}{\sqrt{a+a\left(\tan^2(x)\right)}}$	13
risch	$-\frac{e^{2ix}}{2\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}-\frac{1}{2(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(a+a*tan(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(x)/sqrt(a*tan(x)^2 + a), x)
```

Fricas [A]
time = 2.62, size = 12, normalized size = 1.00

$$-\frac{1}{\sqrt{a\tan(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/sqrt(a*tan(x)^2 + a)
```

Sympy [A]
time = 0.33, size = 14, normalized size = 1.17

$$-\frac{1}{\sqrt{a\tan^2(x)+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)**2)**(1/2),x)
[Out] -1/sqrt(a*tan(x)**2 + a)
```

Giac [A]
time = 0.40, size = 12, normalized size = 1.00

$$-\frac{1}{\sqrt{a\tan(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/sqrt(a*tan(x)^2 + a)
```

Mupad [B]
time = 11.80, size = 11, normalized size = 0.92

$$-\frac{\sqrt{\cos(x)^2}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + a*tan(x)^2)^(1/2),x)
[Out] -(cos(x)^2)^(1/2)/a^(1/2)
```

3.276 $\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{1}{\sqrt{a\sec^2(x)}}$$

[Out] -arctanh((a*sec(x)^2)^(1/2)/a^(1/2))/a^(1/2)+1/(a*sec(x)^2)^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3738, 4209, 53, 65, 213}

$$\frac{1}{\sqrt{a\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/Sqrt[a + a*Tan[x]^2],x]

[Out] -(ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]/Sqrt[a]) + 1/Sqrt[a*Sec[x]^2]

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\cot(x)}{\sqrt{a\sec^2(x)}}\,dx\\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{3/2}}\,dx, x, \sec^2(x)\right)\\
&= \frac{1}{\sqrt{a\sec^2(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx, x, \sec^2(x)\right)\\
&= \frac{1}{\sqrt{a\sec^2(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx, x, \sqrt{a\sec^2(x)}\right)}{a}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{1}{\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.91

$$\frac{\left(\cos(x)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sec(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + a*Tan[x]^2],x]
[Out] ((Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]])*Sec[x])/Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.14, size = 29, normalized size = 0.83

method	result	size
default	$\frac{\cos(x)+\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)+1}{\sqrt{\frac{a}{\cos(x)^2}}\,\cos(x)}$	29
risch	$\frac{e^{2ix}}{2\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}+\frac{1}{2(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{e^{ix}\ln(e^{ix}-1)}{\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}-\frac{e^{ix}\ln(e^{ix}+1)}{\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}$	148

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (cos(x)+ln(-(-1+cos(x))/sin(x))+1)/(a/cos(x)^2)^(1/2)/cos(x)
```

Maxima [A]
time = 0.55, size = 42, normalized size = 1.20

$$\frac{2\cos(x)-\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*(2*cos(x) - log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1))/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.
time = 2.64, size = 66, normalized size = 1.89

$$\frac{\left(\tan(x)^2+1\right)\sqrt{a}\,\log\left(\frac{a\tan(x)^2-2\sqrt{a\tan(x)^2+a}\,\sqrt{a}+2a}{\tan(x)^2}\right)+2\sqrt{a\tan(x)^2+a}}{2\left(a\tan(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*((tan(x)^2 + 1)*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a
) + 2*a)/tan(x)^2) + 2*sqrt(a*tan(x)^2 + a))/(a*tan(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot(x)}{\sqrt{a\left(\tan^2(x)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a*(tan(x)**2 + 1)), x)
```

Giac [A]
time = 0.43, size = 34, normalized size = 0.97

$$\frac{\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{1}{\sqrt{a\tan(x)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a) + 1/sqrt(a*tan(x)^2 + a)
```

Mupad [B]
time = 0.14, size = 31, normalized size = 0.89

$$\frac{1}{\sqrt{a\tan(x)^2+a}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] 1/(a + a*tan(x)^2)^(1/2) - atanh((a + a*tan(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.277 $\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

```
[Out] -csc(x)*sec(x)/(a*sec(x)^2)^(1/2)-tan(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3738, 4210, 2670, 14}

$$-\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

```
[Out] -((Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2]) - Tan[x]/Sqrt[a*Sec[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx &= \int \frac{\cot^2(x)}{\sqrt{a\sec^2(x)}}\,dx\\ &= \frac{\sec(x)\int \cos(x)\cot^2(x)\,dx}{\sqrt{a\sec^2(x)}}\\ &= -\frac{\sec(x)\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -\sin(x)\right)}{\sqrt{a\sec^2(x)}}\\ &= -\frac{\sec(x)\mathrm{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, -\sin(x)\right)}{\sqrt{a\sec^2(x)}}\\ &= -\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.71

$$\frac{-\csc(x)\sec(x)-\tan(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/Sqrt[a + a*Tan[x]^2],x]
[Out] (-(Csc[x]*Sec[x]) - Tan[x])/Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.14, size = 24, normalized size = 0.77

method	result	size
default	$\frac{\cos^2(x)-2}{\sin(x)\cos(x)\sqrt{\frac{a}{\cos(x)^2}}}$	24
risch	$\frac{i e^{2ix}}{2\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}-\frac{i}{2\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}-\frac{2i e^{2ix}}{\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)(e^{2ix}-1)}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] (cos(x)^2-2)/sin(x)/cos(x)/(a/cos(x)^2)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(27) = 54$.
time = 0.55, size = 128, normalized size = 4.13

$$\frac{\left(\left(\sin\left(3\,x\right)-\sin\left(x\right)\right)\cos\left(4\,x\right)-\left(\cos\left(3\,x\right)-\cos\left(x\right)\right)\sin\left(4\,x\right)-\left(6\,\cos\left(2\,x\right)-1\right)\sin\left(3\,x\right)+6\,\cos\left(3\,x\right)\sin\left(2\,x\right)-6\,\cos\left(x\right)\sin\left(2\,x\right)+6\,\cos\left(2\,x\right)\sin\left(x\right)-\sin\left(x\right)\right)\sqrt{a}}{2\,\left(a\cos\left(3\,x\right)^{2}-2\,a\cos\left(3\,x\right)\cos\left(x\right)+a\cos\left(x\right)^{2}+a\sin\left(3\,x\right)^{2}-2\,a\sin\left(3\,x\right)\sin\left(x\right)+a\sin\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x) - (6*cos(2
*x) - 1)*sin(3*x) + 6*cos(3*x)*sin(2*x) - 6*cos(x)*sin(2*x) + 6*cos(2*x)*si
n(x) - sin(x))*sqrt(a)/(a*cos(3*x)^2 - 2*a*cos(3*x)*cos(x) + a*cos(x)^2 + a
*sin(3*x)^2 - 2*a*sin(3*x)*sin(x) + a*sin(x)^2)
```

Fricas [A]
time = 3.23, size = 33, normalized size = 1.06

$$-\frac{\sqrt{a\tan\left(x\right)^{2}+a}\,\left(2\,\tan\left(x\right)^{2}+1\right)}{a\tan\left(x\right)^{3}+a\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 + 1)/(a*tan(x)^3 + a*tan(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{2}\left(x\right)}{\sqrt{a\left(\tan^{2}\left(x\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+a*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)**2/sqrt(a*(tan(x)**2 + 1)), x)
```

Giac [A]
time = 0.45, size = 47, normalized size = 1.52

$$-\frac{\tan\left(x\right)}{\sqrt{a\tan\left(x\right)^{2}+a}}+\frac{2\,\sqrt{a}}{\left(\sqrt{a}\,\tan\left(x\right)-\sqrt{a\tan\left(x\right)^{2}+a}\right)^{2}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -tan(x)/sqrt(a*tan(x)^2 + a) + 2*sqrt(a)/((sqrt(a)*tan(x) - sqrt(a*tan(x)^2
 + a))^2 - a)
```

Mupad [B]
time = 11.82, size = 40, normalized size = 1.29

$$\frac{\sqrt{2}\,\left(6\,\sin\left(2\,x\right)-2\,\sin\left(2\,x\right)\,\left(2\cos\left(x\right)^2-1\right)\right)}{8\,\sqrt{a}\,\sqrt{2\cos\left(x\right)^2}\,\left(\cos\left(x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + a*tan(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*(6*sin(2*x) - 2*sin(2*x)*(2*cos(x)^2 - 1)))/(8*a^(1/2)*(2*cos(x)^2
)^(1/2)*(cos(x)^2 - 1))
```

3.278 $\int \frac{\tan^3(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}-\frac{1}{a\sqrt{a\sec^2(x)}}$$

```
[Out] 1/3/(a*sec(x)^2)^(3/2)-1/a/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3738, 4209, 45}

$$\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}-\frac{1}{a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] 1/(3*(a*Sec[x]^2)^(3/2)) - 1/(a*Sqrt[a*Sec[x]^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx = \int \frac{\tan^3(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx$$
$$= \frac{1}{2}a\text{Subst}\left(\int \frac{-1+x}{(ax)^{5/2}}\,dx, x, \sec^2(x)\right)$$
$$= \frac{1}{2}a\text{Subst}\left(\int \left(-\frac{1}{(ax)^{5/2}}+\frac{1}{a(ax)^{3/2}}\right)\,dx, x, \sec^2(x)\right)$$
$$= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}} - \frac{1}{a\sqrt{a\sec^2(x)}}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.77

$$\frac{-5+\cos(2x)}{6a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (-5 + Cos[2*x])/(6*a*Sqrt[a*Sec[x]^2])
```

Maple [A]
time = 0.05, size = 29, normalized size = 0.97

method	result
derivativedivides	$-\frac{1}{a\sqrt{a+a\left(\tan^2\left(x\right)\right)}}+\frac{1}{3\left(a+a\left(\tan^2\left(x\right)\right)\right)^{\frac{3}{2}}}$
default	$-\frac{1}{a\sqrt{a+a\left(\tan^2\left(x\right)\right)}}+\frac{1}{3\left(a+a\left(\tan^2\left(x\right)\right)\right)^{\frac{3}{2}}}$
risch	$\frac{e^{4ix}}{24a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}}-\frac{3\,e^{2ix}}{8a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}}-\frac{3}{8\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(e^{2ix}+1\right)a}+\frac{e^{-2ix}}{24a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2i}}{\left(e^{2ix}+\right.}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/(a+a*tan(x)^2)^(1/2)+1/3/(a+a*tan(x)^2)^(3/2)
```

Maxima [A]
time = 0.30, size = 38, normalized size = 1.27

$$\frac{\left(\sin\left(x\right)^2+2\right)\left(\sin\left(x\right)+1\right)^{\frac{3}{2}}\left(-\sin\left(x\right)+1\right)^{\frac{3}{2}}}{3\left(a^{\frac{3}{2}}\sin\left(x\right)^2-a^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sin(x)^2 + 2)*(sin(x) + 1)^(3/2)*(-sin(x) + 1)^(3/2)/(a^(3/2)*sin(x)^2
 - a^(3/2))
```

Fricas [A]
time = 1.97, size = 43, normalized size = 1.43

$$-\frac{\sqrt{a\tan\left(x\right)^{2}+a}\,\left(3\,\tan\left(x\right)^{2}+2\right)}{3\,\left(a^{2}\tan\left(x\right)^{4}+2\,a^{2}\tan\left(x\right)^{2}+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(a*tan(x)^2 + a)*(3*tan(x)^2 + 2)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 +
 a^2)
```

Sympy [A]
time = 1.96, size = 36, normalized size = 1.20

$$\begin{cases} \frac{\frac{a}{3\left(a\tan^{2}\left(x\right)+a\right)^{\frac{3}{2}}}-\frac{1}{\sqrt{a\tan^{2}\left(x\right)+a}}}{a} & \text{for}\: a \neq 0 \\ \tilde{\infty}\tan^{4}\left(x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Piecewise(((a/(3*(a*tan(x)**2 + a)**(3/2)) - 1/sqrt(a*tan(x)**2 + a))/a, Ne
(a, 0)), (zoo*tan(x)**4, True))
```

Giac [A]
time = 0.41, size = 26, normalized size = 0.87

$$-\frac{3\,a\tan\left(x\right)^{2}+2\,a}{3\,\left(a\tan\left(x\right)^{2}+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*tan(x)^2 + 2*a)/((a*tan(x)^2 + a)^(3/2)*a)
```

Mupad [B]
time = 11.75, size = 29, normalized size = 0.97

$$-\frac{\left(\mathrm{tan}(x)^{2}+\frac{2}{3}\right)\,\sqrt{a\tan\left(x\right)^{2}+a}}{a^{2}\,\left(\tan\left(x\right)^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + a*tan(x)^2)^(3/2),x)
[Out] -((tan(x)^2 + 2/3)*(a + a*tan(x)^2)^(1/2))/(a^2*(tan(x)^2 + 1)^2)
```

3.279 $\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

```
[Out] 1/3*sin(x)^2*tan(x)/a/(a*sec(x)^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3738, 4210, 2644, 30}

$$\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \int \frac{\tan^2(x)}{(a\sec^2(x))^{3/2}}\,dx$$
$$= \frac{\sec(x)\int \cos(x)\sin^2(x)\,dx}{a\sqrt{a\sec^2(x)}}$$
$$= \frac{\sec(x)\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right)}{a\sqrt{a\sec^2(x)}}$$
$$= \frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.78

$$\frac{\tan^3(x)}{3\,(a\sec^2(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/(a + a*Tan[x]^2)^(3/2),x]
[Out] Tan[x]^3/(3*(a*Sec[x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.
time = 0.05, size = 56, normalized size = 2.43

method	result
derivativedivides	$\frac{\tan(x)}{a\sqrt{a+a(\tan^2(x))}} - a\left(\frac{\tan(x)}{3a(a+a(\tan^2(x)))^{\frac{3}{2}}} + \frac{2\tan(x)}{3a^2\sqrt{a+a(\tan^2(x))}}\right)$
default	$\frac{\tan(x)}{a\sqrt{a+a(\tan^2(x))}} - a\left(\frac{\tan(x)}{3a(a+a(\tan^2(x)))^{\frac{3}{2}}} + \frac{2\tan(x)}{3a^2\sqrt{a+a(\tan^2(x))}}\right)$
risch	$\frac{ie^{4ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}} - \frac{ie^{2ix}}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}} + \frac{i}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}} - \frac{ie^{-2ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*tan(x)/(a+a*tan(x)^2)^(1/2)-a*(1/3/a*tan(x)/(a+a*tan(x)^2)^(3/2)+2/3/a^
2*tan(x)/(a+a*tan(x)^2)^(1/2))
```

Maxima [A]
time = 0.55, size = 14, normalized size = 0.61

$$-\frac{\sin(3x) - 3\sin(x)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
[Out] -1/12*(sin(3*x) - 3*sin(x))/a^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.
time = 3.95, size = 39, normalized size = 1.70

$$\frac{\sqrt{a\tan(x)^2 + a}\,\tan(x)^3}{3\left(a^2\tan(x)^4 + 2a^2\tan(x)^2 + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*sqrt(a*tan(x)^2 + a)*tan(x)^3/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{\left(a\left(\tan^2(x) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+a*tan(x)**2)**(3/2),x)
[Out] Integral(tan(x)**2/(a*(tan(x)**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.44, size = 16, normalized size = 0.70

$$\frac{\tan(x)^3}{3\left(a\tan(x)^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3/(a*tan(x)^2 + a)^(3/2)
```

Mupad [B]
time = 11.65, size = 28, normalized size = 1.22

$$\frac{\tan(x)^3}{\left(3\,a\tan\left(x\right)^2+3\,a\right)\,\sqrt{a\tan\left(x\right)^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + a*tan(x)^2)^(3/2),x)
[Out] tan(x)^3/((3*a + 3*a*tan(x)^2)*(a + a*tan(x)^2)^(1/2))
```

3.280 $\int \frac{\tan(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

[Out] -1/3/(a*sec(x)^2)^(3/2)

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3738, 4209, 32}

$$-\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/(a + a*Tan[x]^2)^(3/2),x]

[Out] -1/3*1/(a*Sec[x]^2)^(3/2)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx &= \int \frac{\tan(x)}{(a\sec^2(x))^{3/2}}\,dx \\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{(ax)^{5/2}}\,dx, x, \sec^2(x)\right) \\
&= -\frac{1}{3\,(a\sec^2(x))^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{3\,(a\sec^2(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -1/3*1/(a*Sec[x]^2)^(3/2)
```

Maple [A]
time = 0.04, size = 13, normalized size = 0.93

method	result
derivativedivides	$-\frac{1}{3(a+a(\tan^2(x)))^{\frac{3}{2}}}$
default	$-\frac{1}{3(a+a(\tan^2(x)))^{\frac{3}{2}}}$
risch	$-\frac{e^{4ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{e^{2ix}}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{1}{8\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)a}-\frac{e^{-2ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e}{(e^{2i}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a+a*tan(x)^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(a*tan(x)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(10) = 20$.
time = 1.46, size = 35, normalized size = 2.50

$$-\frac{\sqrt{a \tan(x)^2 + a}}{3\left(a^2 \tan(x)^4 + 2\,a^2 \tan(x)^2 + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*sqrt(a*tan(x)^2 + a)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

Sympy [A]
time = 1.55, size = 15, normalized size = 1.07

$$-\frac{1}{3\left(a \tan^2(x) + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)**2)**(3/2),x)
[Out] -1/(3*(a*tan(x)**2 + a)**(3/2))
```

Giac [A]
time = 0.41, size = 12, normalized size = 0.86

$$-\frac{1}{3\left(a \tan(x)^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/3/(a*tan(x)^2 + a)^(3/2)
```

Mupad [B]
time = 0.15, size = 23, normalized size = 1.64

$$-\frac{\sqrt{a \tan(x)^2 + a}}{3\,a^2 \left(\tan(x)^2 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + a*tan(x)^2)^(3/2),x)
[Out] -(a + a*tan(x)^2)^(1/2)/(3*a^2*(tan(x)^2 + 1)^2)
```

3.281 $\int \frac{\cot(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}$$

```
[Out] -arctanh((a*sec(x)^2)^(1/2)/a^(1/2))/a^(3/2)+1/3/(a*sec(x)^2)^(3/2)+1/a/(a*
sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3738, 4209, 53, 65, 213}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}+\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]/a^(3/2)) + 1/(3*(a*Sec[x]^2)^(3/2)) + 1
/(a*Sqrt[a*Sec[x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4209

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Dist[b/(2*f), Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x]
, x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx &= \int \frac{\cot(x)}{\left(a\sec^2(x)\right)^{3/2}}\,dx \\
&= \frac{1}{2}a\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{5/2}}\,dx,x,\sec^2(x)\right) \\
&= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)(ax)^{3/2}}\,dx,x,\sec^2(x)\right) \\
&= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{ax}}\,dx,x,\sec^2(x)\right)}{2a} \\
&= \frac{1}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}+\frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^2}{a}}\,dx,x,\sqrt{a\sec^2(x)}\right)}{a^2} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{1}{a\sqrt{a\sec^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 0.89

$$\frac{15+\cos(3x)\sec(x)+12\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sec(x)}{12a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] (15 + Cos[3*x]*Sec[x] + 12*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sec[x])/(12*a*S
qrt[a*Sec[x]^2])
```

Maple [A]
time = 0.15, size = 38, normalized size = 0.72

method	result
default	$\frac{\cos^3(x)+3\cos(x)+3\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)+4}{3\cos(x)^3\left(\frac{a}{\cos(x)^2}\right)^{\frac{3}{2}}}$
risch	$\frac{e^{4ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{5\,e^{2ix}}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{5}{8\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)a}+\frac{e^{-2ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+$ — $a(e^{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(cos(x)^3+3*cos(x)+3*ln(-(-1+cos(x))/sin(x))+4)/cos(x)^3/(a/cos(x)^2)^(
3/2)
```

Maxima [A]
time = 0.55, size = 48, normalized size = 0.91

$$\frac{\cos(3x)+15\cos(x)-6\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+6\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)}{12\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(cos(3*x) + 15*cos(x) - 6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 6*
log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))/a^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(41) = 82.
time = 3.39, size = 94, normalized size = 1.77

$$\frac{3\left(\tan(x)^4+2\tan(x)^2+1\right)\sqrt{a}\,\log\left(\frac{a\tan(x)^2-2\sqrt{a\tan(x)^2+a}\,\sqrt{a}+2a}{\tan(x)^2}\right)+2\sqrt{a\tan(x)^2+a}\,\left(3\tan(x)^2+4\right)}{6\left(a^2\tan(x)^4+2a^2\tan(x)^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(tan(x)^4 + 2*tan(x)^2 + 1)*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x
)^2 + a)*sqrt(a) + 2*a)/tan(x)^2) + 2*sqrt(a*tan(x)^2 + a)*(3*tan(x)^2 + 4)
)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(a(\tan^2(x)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral(cot(x)/(a*(tan(x)**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.43, size = 53, normalized size = 1.00

$$\frac{\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3\,a\tan(x)^2+4\,a}{3\left(a\tan(x)^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/3*(3*a*tan(x)^2 + 4*
a)/((a*tan(x)^2 + a)^(3/2)*a)
```

Mupad [B]
time = 11.69, size = 46, normalized size = 0.87

$$\frac{\frac{a\tan(x)^2+a}{a}+\frac{1}{3}}{\left(a\tan(x)^2+a\right)^{3/2}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] ((a + a*tan(x)^2)/a + 1/3)/(a + a*tan(x)^2)^(3/2) - atanh((a + a*tan(x)^2)^
(1/2)/a^(1/2))/a^(3/2)
```

3.282 $\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}}-\frac{2\tan(x)}{a\sqrt{a\sec^2(x)}}+\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

```
[Out] -csc(x)*sec(x)/a/(a*sec(x)^2)^(1/2)-2*tan(x)/a/(a*sec(x)^2)^(1/2)+1/3*sin(x
)^2*tan(x)/a/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3738, 4210, 2670, 276}

$$-\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}}-\frac{2\tan(x)}{a\sqrt{a\sec^2(x)}}+\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

```
[Out] -((Csc[x]*Sec[x])/(a*Sqrt[a*Sec[x]^2])) - (2*Tan[x])/(a*Sqrt[a*Sec[x]^2]) +
 (Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
```

```
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx &= \int \frac{\cot^2(x)}{(a\sec^2(x))^{3/2}}\,dx\\ &= \frac{\sec(x)\int \cos^3(x)\cot^2(x)\,dx}{a\sqrt{a\sec^2(x)}}\\ &= -\frac{\sec(x)\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, -\sin(x)\right)}{a\sqrt{a\sec^2(x)}}\\ &= -\frac{\sec(x)\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, -\sin(x)\right)}{a\sqrt{a\sec^2(x)}}\\ &= -\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}}-\frac{2\tan(x)}{a\sqrt{a\sec^2(x)}}+\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 0.52

$$\frac{\sec^3(x)\left(-3\csc(x)-6\sin(x)+\sin^3(x)\right)}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/(a + a*Tan[x]^2)^(3/2),x]
[Out] (Sec[x]^3*(-3*Csc[x] - 6*Sin[x] + Sin[x]^3))/(3*(a*Sec[x]^2)^(3/2))
```

Maple [A]
time = 0.14, size = 31, normalized size = 0.52

method	result
default	$\frac{\cos^4(x)+4(\cos^2(x))-8}{3\sin(x)\cos(x)^3\left(\frac{a}{\cos(x)^2}\right)^{\frac{3}{2}}}$
risch	$\frac{i e^{4ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{7i e^{2ix}}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{7i}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{i e^{-2ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{}{a(e^{2ix}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(cos(x)^4+4*cos(x)^2-8)/sin(x)/cos(x)^3/(a/cos(x)^2)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(52) = 104$.
time = 0.56, size = 225, normalized size = 3.75

$$\frac{\left(\left(\sin\left(5\,x\right)-\sin\left(3\,x\right)\right)\cos\left(8\,x\right)+20\left(\sin\left(5\,x\right)-\sin\left(3\,x\right)\right)\cos\left(6\,x\right)+10\left(9\,\sin\left(4\,x\right)-2\,\sin\left(2\,x\right)\right)\cos\left(5\,x\right)-\left(\cos\left(5\,x\right)-\cos\left(3\,x\right)\right)\sin\left(8\,x\right)-20\left(\cos\left(5\,x\right)-\cos\left(3\,x\right)\right)\sin\left(6\,x\right)-\left(90\,\cos\left(4\,x\right)-20\,\cos\left(2\,x\right)-1\right)\sin\left(5\,x\right)-90\,\cos\left(3\,x\right)\sin\left(4\,x\right)-\left(20\,\cos\left(2\,x\right)+1\right)\sin\left(3\,x\right)+90\,\cos\left(4\,x\right)\sin\left(3\,x\right)+20\,\cos\left(3\,x\right)\sin\left(2\,x\right)\right)\sqrt{a}}{24\left(a^{2}\cos\left(5\,x\right)^{2}-2\,a^{2}\cos\left(5\,x\right)\cos\left(3\,x\right)+a^{2}\cos\left(3\,x\right)^{2}+a^{2}\sin\left(5\,x\right)^{2}-2\,a^{2}\sin\left(5\,x\right)\sin\left(3\,x\right)+a^{2}\sin\left(3\,x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*((sin(5*x) - sin(3*x))*cos(8*x) + 20*(sin(5*x) - sin(3*x))*cos(6*x) +
10*(9*sin(4*x) - 2*sin(2*x))*cos(5*x) - (cos(5*x) - cos(3*x))*sin(8*x) - 20
*(cos(5*x) - cos(3*x))*sin(6*x) - (90*cos(4*x) - 20*cos(2*x) - 1)*sin(5*x)
- 90*cos(3*x)*sin(4*x) - (20*cos(2*x) + 1)*sin(3*x) + 90*cos(4*x)*sin(3*x)
+ 20*cos(3*x)*sin(2*x))*sqrt(a)/(a^2*cos(5*x)^2 - 2*a^2*cos(5*x)*cos(3*x) +
 a^2*cos(3*x)^2 + a^2*sin(5*x)^2 - 2*a^2*sin(5*x)*sin(3*x) + a^2*sin(3*x)^2
)
```

Fricas [A]
time = 1.47, size = 52, normalized size = 0.87

$$-\frac{\left(8\,\tan\left(x\right)^{4}+12\,\tan\left(x\right)^{2}+3\right)\sqrt{a\tan\left(x\right)^{2}+a}}{3\left(a^{2}\tan\left(x\right)^{5}+2\,a^{2}\tan\left(x\right)^{3}+a^{2}\tan\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*tan(x)^4 + 12*tan(x)^2 + 3)*sqrt(a*tan(x)^2 + a)/(a^2*tan(x)^5 + 2*
a^2*tan(x)^3 + a^2*tan(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{2}\left(x\right)}{\left(a\left(\tan^{2}\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+a*tan(x)**2)**(3/2),x)
```

```
[Out] Integral(cot(x)**2/(a*(tan(x)**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.47, size = 55, normalized size = 0.92

$$-\frac{\left(5 \tan \left(x\right)^{2}+6\right) \tan \left(x\right)}{3 \left(a \tan \left(x\right)^{2}+a\right)^{\frac{3}{2}}}+\frac{2}{\left(\left(\sqrt{a} \tan \left(x\right)-\sqrt{a \tan \left(x\right)^{2}+a}\right)^{2}-a\right) \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(5*tan(x)^2 + 6)*tan(x)/(a*tan(x)^2 + a)^(3/2) + 2/(((sqrt(a)*tan(x) -
 sqrt(a*tan(x)^2 + a))^2 - a)*sqrt(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{cot}(x)^2}{\left(a \tan \left(x\right)^2+a\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + a*tan(x)^2)^(3/2),x)
```

```
[Out] int(cot(x)^2/(a + a*tan(x)^2)^(3/2), x)
```

3.283 $\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

```
[Out] tan(d*x+c)/d/(a*sec(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3738, 4207, 197}

$$\frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Tan[c + d*x]^2],x]
```

```
[Out] Tan[c + d*x]/(d*Sqrt[a*Sec[c + d*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec^2(c+dx)}}\,dx$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{(a+ax^2)^{3/2}}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 1.00

$$\frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Tan[c + d*x]^2],x]
[Out] Tan[c + d*x]/(d*Sqrt[a*Sec[c + d*x]^2])
```

Maple [A]
time = 0.09, size = 25, normalized size = 1.04

method	result	size
derivativedivides	$\frac{\tan(dx+c)}{d\sqrt{a+a\left(\tan^2(dx+c)\right)}}$	25
default	$\frac{\tan(dx+c)}{d\sqrt{a+a\left(\tan^2(dx+c)\right)}}$	25
risch	$-\frac{i e^{2i(dx+c)}}{2d\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}}+\frac{i}{2\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/d*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2)
```

Maxima [A]
time = 0.54, size = 13, normalized size = 0.54

$$\frac{\sin(dx+c)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] sin(d*x + c)/(sqrt(a)*d)
```

Fricas [A]
time = 3.21, size = 38, normalized size = 1.58

$$\frac{\sqrt{a\tan\left(dx+c\right)^2+a}\,\tan\left(dx+c\right)}{ad\tan\left(dx+c\right)^2+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*tan(d*x + c)^2 + a)*tan(d*x + c)/(a*d*tan(d*x + c)^2 + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\tan^2\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(1/2),x)
[Out] Integral(1/sqrt(a*tan(c + d*x)**2 + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.
time = 0.67, size = 47, normalized size = 1.96

$$-\frac{2}{\sqrt{a}\,d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] -2/(sqrt(a)*d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))*sgn(tan(1/2*d
*x + 1/2*c)^4 - 1))
```

Mupad [B]
time = 12.07, size = 55, normalized size = 2.29

$$\frac{\sin\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}{4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+3}}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^(1/2),x)
[Out] (sin(2*c + 2*d*x)*((a*(cos(2*c + 2*d*x) + 1))/(4*cos(2*c + 2*d*x) + cos(4*c
 + 4*d*x) + 3))^(1/2))/(a*d)
```

3.284 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}$$

```
[Out] 1/3*tan(d*x+c)/d/(a*sec(d*x+c)^2)^(3/2)+2/3*tan(d*x+c)/a/d/(a*sec(d*x+c)^2)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3738, 4207, 198, 197}

$$\frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}+\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] Tan[c + d*x]/(3*d*(a*Sec[c + d*x]^2)^(3/2)) + (2*Tan[c + d*x])/(3*a*d*Sqrt[
a*Sec[c + d*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
```

```
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx &= \int \frac{1}{\left(a\sec^2(c+dx)\right)^{3/2}}\,dx\\
&= \frac{a\text{Subst}\left(\int \frac{1}{(a+ax^2)^{5/2}}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(a+ax^2)^{3/2}}\,dx, x, \tan(c+dx)\right)}{3d}\\
&= \frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.69

$$
-\frac{\left(-3+\sin^2(c+dx)\right)\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-3/2),x]
[Out] -1/3*((-3 + Sin[c + d*x]^2)*Tan[c + d*x])/(a*d*Sqrt[a*Sec[c + d*x]^2])
```

Maple [A]
time = 0.08, size = 57, normalized size = 0.98

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)}{3a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(dx+c)}{3a^2\sqrt{a+a\left(\tan^2(dx+c)\right)}}\right)}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)}{3a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(dx+c)}{3a^2\sqrt{a+a\left(\tan^2(dx+c)\right)}}\right)}{d}$
risch	$-\frac{i e^{4i(dx+c)}}{24d\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)a}-\frac{3i e^{2i(dx+c)}}{8d\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)a}+\frac{3i}{8a\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\,e^{2i}}{(e^{2i(dx}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

[Out] 1/d*a*(1/3/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(3/2)+2/3/a^2*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2))

Maxima [A]
time = 0.54, size = 26, normalized size = 0.45

$$\frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(dx+c\right)}{12\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(d*x + c))/(a^(3/2)*d)
```

Fricas [A]
time = 3.68, size = 70, normalized size = 1.21

$$\frac{\sqrt{a\tan\left(dx+c\right)^{2}+a}\,\left(2\,\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)}{3\,\left(a^{2}d\tan\left(dx+c\right)^{4}+2\,a^{2}d\tan\left(dx+c\right)^{2}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*tan(d*x + c)^2 + a)*(2*tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d*tan(d*x + c)^4 + 2*a^2*d*tan(d*x + c)^2 + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\tan^{2}\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 691 vs. 2(50) = 100.
time = 1.91, size = 691, normalized size = 11.91

$$-\frac{2\left(3\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{5}\tan\left(\frac{1}{2}c\right)^{6}-9\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{5}\tan\left(\frac{1}{2}c\right)^{4}-18\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{4}\tan\left(\frac{1}{2}c\right)^{5}+2\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)^{6}+9\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{5}\tan\left(\frac{1}{2}c\right)^{2}+36\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{4}\tan\left(\frac{1}{2}c\right)^{3}+42\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)^{4}+3\sqrt{a}\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^{6}-3\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{5}-18\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{4}\tan\left(\frac{1}{2}c\right)-42\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)^{2}-48\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{2}\tan\left(\frac{1}{2}c\right)^{3}-9\sqrt{a}\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^{4}-6\sqrt{a}\tan\left(\frac{1}{2}c\right)^{5}-2\sqrt{a}\tan\left(\frac{1}{2}dx\right)^{3}+9\sqrt{a}\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^{2}-4\sqrt{a}\tan\left(\frac{1}{2}c\right)^{3}-3\sqrt{a}\tan\left(\frac{1}{2}dx\right)-6\sqrt{a}\tan\left(\frac{1}{2}c\right)\right)}{3\left(a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}dx\right)^{4}-4\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)-4\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)+1\right)\tan\left(\frac{1}{2}c\right)^{6}+3a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}dx\right)^{4}-4\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)-4\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)+1\right)\tan\left(\frac{1}{2}c\right)^{4}+3a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}dx\right)^{4}-4\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)-4\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)+1\right)\tan\left(\frac{1}{2}c\right)^{2}+a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}dx\right)^{4}-4\tan\left(\frac{1}{2}dx\right)^{3}\tan\left(\frac{1}{2}c\right)-4\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}c\right)^{4}-4\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)+1\right)\right)\left(\tan\left(\frac{1}{2}dx\right)^{2}+1\right)^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(a)*tan(1/2*d*x)^5*tan(1/2*c)^6 - 9*sqrt(a)*tan(1/2*d*x)^5*tan(
1/2*c)^4 - 18*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c)^5 + 2*sqrt(a)*tan(1/2*d*x)^
3*tan(1/2*c)^6 + 9*sqrt(a)*tan(1/2*d*x)^5*tan(1/2*c)^2 + 36*sqrt(a)*tan(1/2
*d*x)^4*tan(1/2*c)^3 + 42*sqrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^4 + 3*sqrt(a)*t
an(1/2*d*x)*tan(1/2*c)^6 - 3*sqrt(a)*tan(1/2*d*x)^5 - 18*sqrt(a)*tan(1/2*d*
x)^4*tan(1/2*c) - 42*sqrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^2 - 48*sqrt(a)*tan(1
/2*d*x)^2*tan(1/2*c)^3 - 9*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^4 - 6*sqrt(a)*ta
n(1/2*c)^5 - 2*sqrt(a)*tan(1/2*d*x)^3 + 9*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^2
 - 4*sqrt(a)*tan(1/2*c)^3 - 3*sqrt(a)*tan(1/2*d*x) - 6*sqrt(a)*tan(1/2*c))/
((a^2*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan
(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - t
an(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^6 + 3*a^2*sgn(tan(1
/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4
*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4
*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^4 + 3*a^2*sgn(tan(1/2*d*x)^4*tan(1
/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3
*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*t
an(1/2*c) + 1)*tan(1/2*c)^2 + a^2*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1
/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*t
an(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1))*(
tan(1/2*d*x)^2 + 1)^3*d)
```

Mupad [B]
time = 11.71, size = 35, normalized size = 0.60

$$\frac{\frac{2\tan(c+d\,x)^3}{3}+\tan(c+d\,x)}{d\,{\left(a\,{\tan\left(c+d\,x\right)}^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^(3/2),x)
```

```
[Out] (tan(c + d*x) + (2*tan(c + d*x)^3)/3)/(d*(a + a*tan(c + d*x)^2)^(3/2))
```

3.285 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}$$

```
[Out] 1/5*tan(d*x+c)/d/(a*sec(d*x+c)^2)^(5/2)+4/15*tan(d*x+c)/a/d/(a*sec(d*x+c)^2
)^(3/2)+8/15*tan(d*x+c)/a^2/d/(a*sec(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3738, 4207, 198, 197}

$$\frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}+\frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] Tan[c + d*x]/(5*d*(a*Sec[c + d*x]^2)^(5/2)) + (4*Tan[c + d*x])/(15*a*d*(a*S
ec[c + d*x]^2)^(3/2)) + (8*Tan[c + d*x])/(15*a^2*d*Sqrt[a*Sec[c + d*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
```

```
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sec^2(c+dx)\right)^{5/2}}\,dx$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{(a+ax^2)^{7/2}}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{4\text{Subst}\left(\int \frac{1}{(a+ax^2)^{5/2}}\,dx, x, \tan(c+dx)\right)}{5d}$$
$$= \frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{8\text{Subst}\left(\int \frac{1}{(a+ax^2)^{3/2}}\,dx,\right.}{15ad}$$
$$= \frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}} + \frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}} + \frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}$$

Mathematica [A]

time = 0.13, size = 59, normalized size = 0.67

$$\frac{\cos(c+dx)\sqrt{a\sec^2(c+dx)}\,(150\sin(c+dx)+25\sin(3(c+dx))+3\sin(5(c+dx)))}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-5/2),x]
[Out] (Cos[c + d*x]*Sqrt[a*Sec[c + d*x]^2]*(150*Sin[c + d*x] + 25*Sin[3*(c + d*x)
] + 3*Sin[5*(c + d*x)]))/(240*a^3*d)
```

Maple [A]

time = 0.10, size = 88, normalized size = 1.00

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)}{5a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}+\frac{\frac{4\tan(dx+c)}{15a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\left(\tan^2(dx+c)\right)}}}{a}\right)}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)}{5a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}+\frac{\frac{4\tan(dx+c)}{15a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\left(\tan^2(dx+c)\right)}}}{a}\right)}{d}$

risch	$-\frac{i e^{6 i (dx+c)}}{160 d \sqrt{\frac{a e^{2 i (dx+c)}}{\left(e^{2 i (dx+c)}+1\right)^{2}}}\, \left(e^{2 i (dx+c)}+1\right) a^{2}}-\frac{5 i e^{2 i (dx+c)}}{16 d \sqrt{\frac{a e^{2 i (dx+c)}}{\left(e^{2 i (dx+c)}+1\right)^{2}}}\, \left(e^{2 i (dx+c)}+1\right) a^{2}}+\frac{5 i}{16 a^{2} \left(e^{2 i (dx+c)}+1\right) \sqrt{\frac{}{(e}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(1/5/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(5/2)+4/5/a*(1/3/a*tan(d*x+c)/(a
+a*tan(d*x+c)^2)^(3/2)+2/3/a^2*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2)))
```

Maxima [A]
time = 0.55, size = 39, normalized size = 0.44

$$\frac{3 \sin (5 dx + 5 c) + 25 \sin (3 dx + 3 c) + 150 \sin (dx + c)}{240 a^{\frac{5}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*sin(5*d*x + 5*c) + 25*sin(3*d*x + 3*c) + 150*sin(d*x + c))/(a^(5/2
)*d)
```

Fricas [A]
time = 1.54, size = 94, normalized size = 1.07

$$\frac{\left(8 \tan (dx+c)^{5} + 20 \tan (dx+c)^{3} + 15 \tan (dx+c)\right) \sqrt{a \tan (dx+c)^{2} + a}}{15 \left(a^{3} d \tan (dx+c)^{6} + 3 a^{3} d \tan (dx+c)^{4} + 3 a^{3} d \tan (dx+c)^{2} + a^{3} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*tan(d*x + c)^5 + 20*tan(d*x + c)^3 + 15*tan(d*x + c))*sqrt(a*tan(d*
x + c)^2 + a)/(a^3*d*tan(d*x + c)^6 + 3*a^3*d*tan(d*x + c)^4 + 3*a^3*d*tan(
d*x + c)^2 + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \tan^{2}(c + dx) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a*tan(c + d*x)**2 + a)**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1429 vs. $2(76) = 152$.
time = 1.87, size = 1429, normalized size = 16.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(a)*tan(1/2*d*x)^9*tan(1/2*c)^10 - 75*sqrt(a)*tan(1/2*d*x)^9*
tan(1/2*c)^8 - 150*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c)^9 + 20*sqrt(a)*tan(1/2
*d*x)^7*tan(1/2*c)^10 + 150*sqrt(a)*tan(1/2*d*x)^9*tan(1/2*c)^6 + 600*sqrt(
a)*tan(1/2*d*x)^8*tan(1/2*c)^7 + 700*sqrt(a)*tan(1/2*d*x)^7*tan(1/2*c)^8 +
58*sqrt(a)*tan(1/2*d*x)^5*tan(1/2*c)^10 - 150*sqrt(a)*tan(1/2*d*x)^9*tan(1/
2*c)^4 - 900*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c)^5 - 2200*sqrt(a)*tan(1/2*d*x
)^7*tan(1/2*c)^6 - 2400*sqrt(a)*tan(1/2*d*x)^6*tan(1/2*c)^7 - 610*sqrt(a)*t
an(1/2*d*x)^5*tan(1/2*c)^8 - 300*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c)^9 + 20*s
qrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^10 + 75*sqrt(a)*tan(1/2*d*x)^9*tan(1/2*c)^
2 + 600*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c)^3 + 2200*sqrt(a)*tan(1/2*d*x)^7*t
an(1/2*c)^4 + 4800*sqrt(a)*tan(1/2*d*x)^6*tan(1/2*c)^5 + 5380*sqrt(a)*tan(1
/2*d*x)^5*tan(1/2*c)^6 + 2000*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c)^7 + 700*sqr
t(a)*tan(1/2*d*x)^3*tan(1/2*c)^8 + 15*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^10 -
15*sqrt(a)*tan(1/2*d*x)^9 - 150*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c) - 700*sqr
t(a)*tan(1/2*d*x)^7*tan(1/2*c)^2 - 2400*sqrt(a)*tan(1/2*d*x)^6*tan(1/2*c)^3
 - 5380*sqrt(a)*tan(1/2*d*x)^5*tan(1/2*c)^4 - 5960*sqrt(a)*tan(1/2*d*x)^4*t
an(1/2*c)^5 - 2200*sqrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^6 - 800*sqrt(a)*tan(1/
2*d*x)^2*tan(1/2*c)^7 - 75*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^8 - 30*sqrt(a)*t
an(1/2*c)^9 - 20*sqrt(a)*tan(1/2*d*x)^7 + 610*sqrt(a)*tan(1/2*d*x)^5*tan(1/
2*c)^2 + 2000*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c)^3 + 2200*sqrt(a)*tan(1/2*d*
x)^3*tan(1/2*c)^4 + 320*sqrt(a)*tan(1/2*d*x)^2*tan(1/2*c)^5 + 150*sqrt(a)*t
an(1/2*d*x)*tan(1/2*c)^6 - 40*sqrt(a)*tan(1/2*c)^7 - 58*sqrt(a)*tan(1/2*d*x
)^5 - 300*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c) - 700*sqrt(a)*tan(1/2*d*x)^3*ta
n(1/2*c)^2 - 800*sqrt(a)*tan(1/2*d*x)^2*tan(1/2*c)^3 - 150*sqrt(a)*tan(1/2*
d*x)*tan(1/2*c)^4 - 116*sqrt(a)*tan(1/2*c)^5 - 20*sqrt(a)*tan(1/2*d*x)^3 +
75*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^2 - 40*sqrt(a)*tan(1/2*c)^3 - 15*sqrt(a)
*tan(1/2*d*x) - 30*sqrt(a)*tan(1/2*c))/((a^3*sgn(tan(1/2*d*x)^4*tan(1/2*c)^
4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1
/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2
*c) + 1)*tan(1/2*c)^10 + 5*a^3*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*
d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(
1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1
/2*c)^8 + 10*a^3*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2
*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1
/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^6 + 10*a
^3*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/
```

```
2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(
1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^4 + 5*a^3*sgn(tan(1/2*
d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*ta
n(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*ta
n(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^2 + a^3*sgn(tan(1/2*d*x)^4*tan(1/2*c)
^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(
1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/
2*c) + 1))*(tan(1/2*d*x)^2 + 1)^5*d)
```

Mupad [B]
time = 0.20, size = 47, normalized size = 0.53

$$\frac{\tan(c+d\,x)\,\left(8\tan(c+d\,x)^4+20\tan(c+d\,x)^2+15\right)}{15\,d\,{\left(a\tan\left(c+d\,x\right)^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^(5/2),x)
```

```
[Out] (tan(c + d*x)*(20*tan(c + d*x)^2 + 8*tan(c + d*x)^4 + 15))/(15*d*(a + a*tan
(c + d*x)^2)^(5/2))
```

3.286 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}}+\frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{16\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}$$

```
[Out] 1/7*tan(d*x+c)/d/(a*sec(d*x+c)^2)^(7/2)+6/35*tan(d*x+c)/a/d/(a*sec(d*x+c)^2
)^(5/2)+8/35*tan(d*x+c)/a^2/d/(a*sec(d*x+c)^2)^(3/2)+16/35*tan(d*x+c)/a^3/d
/(a*sec(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3738, 4207, 198, 197}

$$\frac{16\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}+\frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Tan[c + d*x]^2)^(-7/2),x]
```

```
[Out] Tan[c + d*x]/(7*d*(a*Sec[c + d*x]^2)^(7/2)) + (6*Tan[c + d*x])/(35*a*d*(a*S
ec[c + d*x]^2)^(5/2)) + (8*Tan[c + d*x])/(35*a^2*d*(a*Sec[c + d*x]^2)^(3/2)
) + (16*Tan[c + d*x])/(35*a^3*d*Sqrt[a*Sec[c + d*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a+a\tan^2(c+dx))^{7/2}}\,dx = \int \frac{1}{(a\sec^2(c+dx))^{7/2}}\,dx$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{(a+ax^2)^{9/2}}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{7d\,(a\sec^2(c+dx))^{7/2}} + \frac{6\text{Subst}\left(\int \frac{1}{(a+ax^2)^{7/2}}\,dx, x, \tan(c+dx)\right)}{7d}$$
$$= \frac{\tan(c+dx)}{7d\,(a\sec^2(c+dx))^{7/2}} + \frac{6\tan(c+dx)}{35ad\,(a\sec^2(c+dx))^{5/2}} + \frac{24\text{Subst}\left(\int \frac{1}{(a+ax^2)^{5/2}}\,dx,\right.}{35ad}$$
$$= \frac{\tan(c+dx)}{7d\,(a\sec^2(c+dx))^{7/2}} + \frac{6\tan(c+dx)}{35ad\,(a\sec^2(c+dx))^{5/2}} + \frac{8\tan(c+dx)}{35a^2d\,(a\sec^2(c+dx))^{3/2}}$$
$$= \frac{\tan(c+dx)}{7d\,(a\sec^2(c+dx))^{7/2}} + \frac{6\tan(c+dx)}{35ad\,(a\sec^2(c+dx))^{5/2}} + \frac{8\tan(c+dx)}{35a^2d\,(a\sec^2(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.16, size = 69, normalized size = 0.58

$$\frac{\cos(c+dx)\sqrt{a\sec^2(c+dx)}\,(1225\sin(c+dx)+245\sin(3(c+dx))+49\sin(5(c+dx))+5\sin(7(c+dx)))}{2240a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Tan[c + d*x]^2)^(-7/2),x]
[Out] (Cos[c + d*x]*Sqrt[a*Sec[c + d*x]^2]*(1225*Sin[c + d*x] + 245*Sin[3*(c + d*
x)] + 49*Sin[5*(c + d*x)] + 5*Sin[7*(c + d*x)]))/(2240*a^4*d)
```

Maple [A]
time = 0.10, size = 119, normalized size = 1.01

method	result

derivativedivides	$\frac{a\left(\frac{\tan(dx+c)}{7a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{7}{2}}}+\frac{\frac{6\tan(dx+c)}{35a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4\tan(dx+c)}{15a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\left(\tan^2\left(dx+c\right)\right)}}\right)}{7a}}{a}\right)}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)}{7a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{7}{2}}}+\frac{\frac{6\tan(dx+c)}{35a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4\tan(dx+c)}{15a\left(a+a\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\left(\tan^2\left(dx+c\right)\right)}}\right)}{7a}}{a}\right)}{d}$
risch	$-\frac{i\mathrm{e}^{8i(dx+c)}}{896d\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}}\left(\mathrm{e}^{2i(dx+c)}+1\right)a^3}-\frac{35i\mathrm{e}^{2i(dx+c)}}{128d\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}}\left(\mathrm{e}^{2i(dx+c)}+1\right)a^3}+\frac{35}{128a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*tan(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(1/7/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(7/2)+6/7/a*(1/5/a*tan(d*x+c)/(a
+a*tan(d*x+c)^2)^(5/2)+4/5/a*(1/3/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(3/2)+2/3
/a^2*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2))))
```

Maxima [A]
time = 0.55, size = 50, normalized size = 0.42

$$\frac{5\sin\left(7\,dx+7\,c\right)+49\sin\left(5\,dx+5\,c\right)+245\sin\left(3\,dx+3\,c\right)+1225\sin\left(dx+c\right)}{2240\,a^{\frac{7}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2240*(5*sin(7*d*x + 7*c) + 49*sin(5*d*x + 5*c) + 245*sin(3*d*x + 3*c) + 1
225*sin(d*x + c))/(a^(7/2)*d)
```

Fricas [A]
time = 3.19, size = 118, normalized size = 1.00

$$\frac{\left(16\tan\left(dx+c\right)^7+56\tan\left(dx+c\right)^5+70\tan\left(dx+c\right)^3+35\tan\left(dx+c\right)\right)\sqrt{a\tan\left(dx+c\right)^2+a}}{35\left(a^4d\tan\left(dx+c\right)^8+4\,a^4d\tan\left(dx+c\right)^6+6\,a^4d\tan\left(dx+c\right)^4+4\,a^4d\tan\left(dx+c\right)^2+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="fricas")

[Out] 1/35*(16*tan(d*x + c)^7 + 56*tan(d*x + c)^5 + 70*tan(d*x + c)^3 + 35*tan(d*x + c))*sqrt(a*tan(d*x + c)^2 + a)/(a^4*d*tan(d*x + c)^8 + 4*a^4*d*tan(d*x + c)^6 + 6*a^4*d*tan(d*x + c)^4 + 4*a^4*d*tan(d*x + c)^2 + a^4*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\tan^2\left(c+dx\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*tan(d*x+c)**2)**(7/2),x)

[Out] Integral((a*tan(c + d*x)**2 + a)**(-7/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2455 vs. 2(102) = 204.
time = 2.87, size = 2455, normalized size = 20.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="giac")

[Out] -2/35*(35*sqrt(a)*tan(1/2*d*x)^13*tan(1/2*c)^14 - 245*sqrt(a)*tan(1/2*d*x)^13*tan(1/2*c)^12 - 490*sqrt(a)*tan(1/2*d*x)^12*tan(1/2*c)^13 + 70*sqrt(a)*tan(1/2*d*x)^11*tan(1/2*c)^14 + 735*sqrt(a)*tan(1/2*d*x)^13*tan(1/2*c)^10 + 2940*sqrt(a)*tan(1/2*d*x)^12*tan(1/2*c)^11 + 3430*sqrt(a)*tan(1/2*d*x)^11*tan(1/2*c)^12 + 301*sqrt(a)*tan(1/2*d*x)^9*tan(1/2*c)^14 - 1225*sqrt(a)*tan(1/2*d*x)^13*tan(1/2*c)^8 - 7350*sqrt(a)*tan(1/2*d*x)^12*tan(1/2*c)^9 - 18130*sqrt(a)*tan(1/2*d*x)^11*tan(1/2*c)^10 - 19600*sqrt(a)*tan(1/2*d*x)^10*tan(1/2*c)^11 - 5243*sqrt(a)*tan(1/2*d*x)^9*tan(1/2*c)^12 - 2450*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c)^13 + 212*sqrt(a)*tan(1/2*d*x)^7*tan(1/2*c)^14 + 1225*sqrt(a)*tan(1/2*d*x)^13*tan(1/2*c)^6 + 9800*sqrt(a)*tan(1/2*d*x)^12*tan(1/2*c)^7 + 36750*sqrt(a)*tan(1/2*d*x)^11*tan(1/2*c)^8 + 78400*sqrt(a)*tan(1/2*d*x)^10*tan(1/2*c)^9 + 84721*sqrt(a)*tan(1/2*d*x)^9*tan(1/2*c)^10 + 34300*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c)^11 + 11284*sqrt(a)*tan(1/2*d*x)^7*tan(1/2*c)^12 + 301*sqrt(a)*tan(1/2*d*x)^5*tan(1/2*c)^14 - 735*sqrt(a)*tan(1/2*d*x)^13*tan(1/2*c)^4 - 7350*sqrt(a)*tan(1/2*d*x)^12*tan(1/2*c)^5 - 36750*sqrt(a)*tan(1/2*d*x)^11*tan(1/2*c)^6 - 117600*sqrt(a)*tan(1/2*d*x)^10*tan(1/2*c)^7 - 230055*sqrt(a)*tan(1/2*d*x)^9*tan(1/2*c)^8 - 240590*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c)^9 - 113148*sqrt(a)*tan(1/2*d*x)^7*tan(1/2*c)^10 - 39200*sqrt(a)*tan(1/2*d*x)^6*tan(1/2*c)^11 - 5243*sqrt(a)*tan(1/2*d*x)^5*tan(1/2*c)^12 - 1470*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c)^13 + 70*sqrt(a)*tan(1/2*d*x)^3*ta

n(1/2*c)^14 + 245*sqrt(a)*tan(1/2*d*x)^13*tan(1/2*c)^2 + 2940*sqrt(a)*tan(1
/2*d*x)^12*tan(1/2*c)^3 + 18130*sqrt(a)*tan(1/2*d*x)^11*tan(1/2*c)^4 + 7840
0*sqrt(a)*tan(1/2*d*x)^10*tan(1/2*c)^5 + 230055*sqrt(a)*tan(1/2*d*x)^9*tan(
1/2*c)^6 + 417480*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c)^7 + 424900*sqrt(a)*tan(
1/2*d*x)^7*tan(1/2*c)^8 + 219520*sqrt(a)*tan(1/2*d*x)^6*tan(1/2*c)^9 + 8472
1*sqrt(a)*tan(1/2*d*x)^5*tan(1/2*c)^10 + 16660*sqrt(a)*tan(1/2*d*x)^4*tan(1
/2*c)^11 + 3430*sqrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^12 + 35*sqrt(a)*tan(1/2*d
*x)*tan(1/2*c)^14 - 35*sqrt(a)*tan(1/2*d*x)^13 - 490*sqrt(a)*tan(1/2*d*x)^1
2*tan(1/2*c) - 3430*sqrt(a)*tan(1/2*d*x)^11*tan(1/2*c)^2 - 19600*sqrt(a)*ta
n(1/2*d*x)^10*tan(1/2*c)^3 - 84721*sqrt(a)*tan(1/2*d*x)^9*tan(1/2*c)^4 - 24
0590*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c)^5 - 424900*sqrt(a)*tan(1/2*d*x)^7*ta
n(1/2*c)^6 - 432320*sqrt(a)*tan(1/2*d*x)^6*tan(1/2*c)^7 - 230055*sqrt(a)*ta
n(1/2*d*x)^5*tan(1/2*c)^8 - 91042*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c)^9 - 181
30*sqrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^10 - 3920*sqrt(a)*tan(1/2*d*x)^2*tan(1
/2*c)^11 - 245*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^12 - 70*sqrt(a)*tan(1/2*c)^1
3 - 70*sqrt(a)*tan(1/2*d*x)^11 + 5243*sqrt(a)*tan(1/2*d*x)^9*tan(1/2*c)^2 +
 34300*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c)^3 + 113148*sqrt(a)*tan(1/2*d*x)^7*
tan(1/2*c)^4 + 219520*sqrt(a)*tan(1/2*d*x)^6*tan(1/2*c)^5 + 230055*sqrt(a)*
tan(1/2*d*x)^5*tan(1/2*c)^6 + 108696*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c)^7 +
36750*sqrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^8 + 3136*sqrt(a)*tan(1/2*d*x)^2*tan
(1/2*c)^9 + 735*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^10 - 140*sqrt(a)*tan(1/2*c)
^11 - 301*sqrt(a)*tan(1/2*d*x)^9 - 2450*sqrt(a)*tan(1/2*d*x)^8*tan(1/2*c) -
 11284*sqrt(a)*tan(1/2*d*x)^7*tan(1/2*c)^2 - 39200*sqrt(a)*tan(1/2*d*x)^6*t
an(1/2*c)^3 - 84721*sqrt(a)*tan(1/2*d*x)^5*tan(1/2*c)^4 - 91042*sqrt(a)*tan
(1/2*d*x)^4*tan(1/2*c)^5 - 36750*sqrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^6 - 1276
8*sqrt(a)*tan(1/2*d*x)^2*tan(1/2*c)^7 - 1225*sqrt(a)*tan(1/2*d*x)*tan(1/2*c
)^8 - 602*sqrt(a)*tan(1/2*c)^9 - 212*sqrt(a)*tan(1/2*d*x)^7 + 5243*sqrt(a)*
tan(1/2*d*x)^5*tan(1/2*c)^2 + 16660*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c)^3 + 1
8130*sqrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^4 + 3136*sqrt(a)*tan(1/2*d*x)^2*tan(
1/2*c)^5 + 1225*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^6 - 424*sqrt(a)*tan(1/2*c)^
7 - 301*sqrt(a)*tan(1/2*d*x)^5 - 1470*sqrt(a)*tan(1/2*d*x)^4*tan(1/2*c) - 3
430*sqrt(a)*tan(1/2*d*x)^3*tan(1/2*c)^2 - 3920*sqrt(a)*tan(1/2*d*x)^2*tan(1
/2*c)^3 - 735*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^4 - 602*sqrt(a)*tan(1/2*c)^5
 - 70*sqrt(a)*tan(1/2*d*x)^3 + 245*sqrt(a)*tan(1/2*d*x)*tan(1/2*c)^2 - 140*s
qrt(a)*tan(1/2*c)^3 - 35*sqrt(a)*tan(1/2*d*x) - 70*sqrt(a)*tan(1/2*c))/((a^
4*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2
*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1
/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^14 + 7*a^4*sgn(tan(1/2*
d*x)^4*tan(1/2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*ta
n(1/2*d*x)^3*tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*ta
n(1/2*d*x)*tan(1/2*c) + 1)*tan(1/2*c)^12 + 21*a^4*sgn(tan(1/2*d*x)^4*tan(1/
2*c)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*
tan(1/2*c) - 4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*ta
n(1/2*c) + 1)*tan(1/2*c)^10 + 35*a^4*sgn(tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*ta
n(1/2*d*x)^3*tan(1/2*c)^3 - tan(1/2*d*x)^4 - 4*tan(1/2*d*x)^3*tan(1/2*c) -

```
4*tan(1/2*d*x)*tan(1/2*c)^3 - tan(1/2*c)^4 - 4*tan(1/2*d*x)*tan(1/2*c) + 1)
*tan(1/2*c)^8 + 35*a^4*sgn(tan(1/2*d*x)^4*tan(1...
```

Mupad [B]
time = 11.78, size = 161, normalized size = 1.36

$$\frac{16\tan(c+d\,x)\,\sqrt{a\tan\left(c+d\,x\right)^2+a}}{35\,a^4\,d\,\left(\tan\left(c+d\,x\right)^2+1\right)}+\frac{8\tan(c+d\,x)\,\sqrt{a\tan\left(c+d\,x\right)^2+a}}{35\,a^4\,d\,\left(\tan\left(c+d\,x\right)^2+1\right)^2}+\frac{6\tan(c+d\,x)\,\sqrt{a\tan\left(c+d\,x\right)^2+a}}{35\,a^4\,d\,\left(\tan\left(c+d\,x\right)^2+1\right)^3}+\frac{\tan(c+d\,x)\,\sqrt{a\tan\left(c+d\,x\right)^2+a}}{7\,a^4\,d\,\left(\tan\left(c+d\,x\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*tan(c + d*x)^2)^(7/2),x)
```

```
[Out] (16*tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(35*a^4*d*(tan(c + d*x)^2 +
1)) + (8*tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(35*a^4*d*(tan(c + d*x)
^2 + 1)^2) + (6*tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(35*a^4*d*(tan(c
 + d*x)^2 + 1)^3) + (tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(7*a^4*d*(t
an(c + d*x)^2 + 1)^4)
```

3.287 $\int (1+\tan^2(x))^{3/2}\, dx$

Optimal. Leaf size=22

$$\frac{1}{2}\sinh^{-1}(\tan(x))+\frac{1}{2}\sqrt{\sec^2(x)}\,\tan(x)$$

```
[Out] 1/2*arcsinh(tan(x))+1/2*(sec(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3738, 4207, 201, 221}

$$\frac{1}{2}\tan(x)\sqrt{\sec^2(x)}+\frac{1}{2}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^2)^(3/2),x]
```

```
[Out] ArcSinh[Tan[x]]/2 + (Sqrt[Sec[x]^2]*Tan[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(1+\tan^2(x)\right)^{3/2}\,dx &= \int \sec^2(x)^{3/2}\,dx\\
&= \text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\sqrt{\sec^2(x)}\,\tan(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\sinh^{-1}(\tan(x))+\frac{1}{2}\sqrt{\sec^2(x)}\,\tan(x)
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(22) = 44.
time = 0.05, size = 52, normalized size = 2.36

$$\frac{1}{2}\cos(x)\sqrt{\sec^2(x)}\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sec(x)\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^2)^(3/2),x]
[Out] (Cos[x]*Sqrt[Sec[x]^2]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]
] + Sec[x]*Tan[x]))/2
```

Maple [A]
time = 0.06, size = 19, normalized size = 0.86

method	result
derivativedivides	$\frac{\tan(x)\sqrt{1+\tan^2(x)}}{2}+\frac{\operatorname{arcsinh}(\tan(x))}{2}$
default	$\frac{\tan(x)\sqrt{1+\tan^2(x)}}{2}+\frac{\operatorname{arcsinh}(\tan(x))}{2}$
risch	$-\frac{i\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}-1)}{e^{2ix}+1}-\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x)+\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*tan(x)*(1+tan(x)^2)^(1/2)+1/2*arcsinh(tan(x))
```

Maxima [A]
time = 0.51, size = 18, normalized size = 0.82

$$\frac{1}{2}\sqrt{\tan(x)^2+1}\,\tan(x)+\frac{1}{2}\operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*arcsinh(tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(16) = 32$.
time = 2.13, size = 72, normalized size = 3.27

$$\frac{1}{2}\sqrt{\tan(x)^2+1}\,\tan(x)+\frac{1}{4}\log\left(\frac{\tan(x)^2+\sqrt{\tan(x)^2+1}\,\tan(x)+1}{\tan(x)^2+1}\right)-\frac{1}{4}\log\left(\frac{\tan(x)^2-\sqrt{\tan(x)^2+1}\,\tan(x)+1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/4*log((tan(x)^2 + sqrt(tan(x)^2 + 1)*tan(
x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - sqrt(tan(x)^2 + 1)*tan(x) + 1
)/(tan(x)^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\tan^2(x)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)**2)**(3/2),x)
```

```
[Out] Integral((tan(x)**2 + 1)**(3/2), x)
```

Giac [A]
time = 0.42, size = 29, normalized size = 1.32

$$\frac{1}{2}\sqrt{\tan(x)^2+1}\,\tan(x)-\frac{1}{2}\log\left(\sqrt{\tan(x)^2+1}-\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) - 1/2*log(sqrt(tan(x)^2 + 1) - tan(x))
```

Mupad [B]
time = 0.11, size = 18, normalized size = 0.82

$$\frac{\operatorname{asinh}(\tan(x))}{2}+\frac{\tan(x)\,\sqrt{\tan(x)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^2 + 1)^(3/2),x)
```

```
[Out] asinh(tan(x))/2 + (tan(x)*(tan(x)^2 + 1)^(1/2))/2
```

3.288 $\int \sqrt{1+\tan^2(x)}\,dx$

Optimal. Leaf size=3

$$\sinh^{-1}(\tan(x))$$

```
[Out] arcsinh(tan(x))
```

Rubi [A]

time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3738, 4207, 221}

$$\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Tan[x]^2],x]
```

```
[Out] ArcSinh[Tan[x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\tan^2(x)}\,dx &= \int \sqrt{\sec^2(x)}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\tan(x)\right)\\
&= \sinh^{-1}(\tan(x))
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(3) = 6.
time = 0.01, size = 44, normalized size = 14.67

$$\cos(x)\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)\sqrt{\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Tan[x]^2],x]
```

```
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[Sec[x]^2
]
```

Maple [A]
time = 0.04, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\operatorname{arcsinh}(\tan(x))$	4
default	$\operatorname{arcsinh}(\tan(x))$	4
risch	$2\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)-2\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(tan(x))
```

Maxima [A]
time = 0.51, size = 3, normalized size = 1.00

$$\operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(3) = 6.
time = 1.88, size = 60, normalized size = 20.00

$$\frac{1}{2}\log\left(\frac{\tan(x)^2+\sqrt{\tan(x)^2+1}\,\tan(x)+1}{\tan(x)^2+1}\right)-\frac{1}{2}\log\left(\frac{\tan(x)^2-\sqrt{\tan(x)^2+1}\,\tan(x)+1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((tan(x)^2 + sqrt(tan(x)^2 + 1)*tan(x) + 1)/(tan(x)^2 + 1)) - 1/2*lo
g((tan(x)^2 - sqrt(tan(x)^2 + 1)*tan(x) + 1)/(tan(x)^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan^2(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)**2)**(1/2),x)
[Out] Integral(sqrt(tan(x)**2 + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(3) = 6$.
time = 0.46, size = 16, normalized size = 5.33

$$-\log\left(\sqrt{\tan(x)^2 + 1} - \tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(sqrt(tan(x)^2 + 1) - tan(x))
```

Mupad [B]
time = 0.06, size = 3, normalized size = 1.00

$$\mathrm{asinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^2 + 1)^(1/2),x)
[Out] asinh(tan(x))
```

3.289 $\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx$

Optimal. Leaf size=11

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

```
[Out] tan(x)/(sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3738, 4207, 197}

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Tan[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx = \int \frac{1}{\sqrt{\sec^2(x)}}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Tan[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

Maple [A]
time = 0.03, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\tan(x)}{\sqrt{1+\tan^2(x)}}$	12
default	$\frac{\tan(x)}{\sqrt{1+\tan^2(x)}}$	12
risch	$-\frac{i\mathrm{e}^{2ix}}{2\sqrt{\frac{\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,(\mathrm{e}^{2ix}+1)}+\frac{i}{2\sqrt{\frac{\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,(\mathrm{e}^{2ix}+1)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+tan(x)^2)^(1/2)*tan(x)
```

Maxima [A]
time = 0.30, size = 11, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

Fricas [A]
time = 2.72, size = 11, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

Sympy [A]
time = 0.16, size = 12, normalized size = 1.09

$$\frac{\tan(x)}{\sqrt{\tan^2(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**2)**(1/2),x)
```

```
[Out] tan(x)/sqrt(tan(x)**2 + 1)
```

Giac [A]
time = 0.41, size = 11, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.82

$$\tan(x)\,\sqrt{\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^2 + 1)^(1/2),x)
```

```
[Out] tan(x)*(cos(x)^2)^(1/2)
```

3.290 $\int \left(-1-\tan^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)-\frac{1}{2}\sqrt{-\sec^2(x)}\,\tan(x)$$

```
[Out] 1/2*arctan(tan(x)/(-sec(x)^2)^(1/2))-1/2*(-sec(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3738, 4207, 201, 223, 209}

$$\frac{1}{2}\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)-\frac{1}{2}\tan(x)\sqrt{-\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Tan[x]^2)^(3/2),x]
```

```
[Out] ArcTan[Tan[x]/Sqrt[-Sec[x]^2]]/2 - (Sqrt[-Sec[x]^2]*Tan[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(-1-\tan^2(x)\right)^{3/2} dx &= \int \left(-\sec^2(x)\right)^{3/2} dx\\ &= -\text{Subst}\left(\int \sqrt{-1-x^2}\, dx, x, \tan(x)\right)\\ &= -\frac{1}{2}\sqrt{-\sec^2(x)}\tan(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\, dx, x, \tan(x)\right)\\ &= -\frac{1}{2}\sqrt{-\sec^2(x)}\tan(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\, dx, x, \frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)-\frac{1}{2}\sqrt{-\sec^2(x)}\tan(x)\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(35) = 70.
time = 0.05, size = 72, normalized size = 2.06

$$\frac{1}{4}\cos(x)\sqrt{-\sec^2(x)}\left(2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{1}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}+\frac{1}{-1+\sin(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Tan[x]^2)^(3/2),x]
[Out] (Cos[x]*Sqrt[-Sec[x]^2]*(2*Log[Cos[x/2] - Sin[x/2]] - 2*Log[Cos[x/2] + Sin[
x/2]] + (Cos[x/2] + Sin[x/2])^(-2) + (-1 + Sin[x])^(-1)))/4
```

Maple [**A**]
time = 0.07, size = 32, normalized size = 0.91

method	result
derivativedivides	$-\frac{\tan(x)\sqrt{-1-\left(\tan^2(x)\right)}}{2}+\frac{\arctan\left(\frac{\tan(x)}{\sqrt{-1-\left(\tan^2(x)\right)}}\right)}{2}$

default	$-\frac{\tan(x)\sqrt{-1-(\tan^2(x))}}{2}+\frac{\arctan\left(\frac{\tan(x)}{\sqrt{-1-(\tan^2(x))}}\right)}{2}$
risch	$\frac{i\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}-1)}{e^{2ix}+1}-\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)+\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*tan(x)*(-1-tan(x)^2)^(1/2)+1/2*arctan(tan(x)/(-1-tan(x)^2)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.57, size = 20, normalized size = 0.57

$$-\frac{1}{2}\sqrt{-\tan(x)^2-1}\,\tan(x)-\frac{1}{2}i\,\operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-tan(x)^2 - 1)*tan(x) - 1/2*I*arcsinh(tan(x))
```

Fricas [C] Result contains complex when optimal does not.
time = 2.05, size = 73, normalized size = 2.09

$$\frac{\left(-i\,e^{(4ix)}-2i\,e^{(2ix)}-i\right)\log\left(e^{(ix)}+i\right)+\left(i\,e^{(4ix)}+2i\,e^{(2ix)}+i\right)\log\left(e^{(ix)}-i\right)-2\,e^{(3ix)}+2\,e^{(ix)}}{2\left(e^{(4ix)}+2\,e^{(2ix)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*((-I*e^(4*I*x) - 2*I*e^(2*I*x) - I)*log(e^(I*x) + I) + (I*e^(4*I*x) + 2
*I*e^(2*I*x) + I)*log(e^(I*x) - I) - 2*e^(3*I*x) + 2*e^(I*x))/(e^(4*I*x) +
2*e^(2*I*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\tan^2(x)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)**2)**(3/2),x)
[Out] Integral((-tan(x)**2 - 1)**(3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 29, normalized size = 0.83

$$-\frac{1}{2}i\sqrt{\tan(x)^2+1}\,\tan(x)+\frac{1}{2}i\log\left(\sqrt{\tan(x)^2+1}-\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*I*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*I*log(sqrt(tan(x)^2 + 1) - tan(x))
```

Mupad [B]
time = 11.59, size = 31, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{\tan(x)}{\sqrt{-\tan(x)^2-1}}\right)}{2}-\frac{\tan(x)\,\sqrt{-\tan(x)^2-1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- tan(x)^2 - 1)^(3/2),x)
```

```
[Out] atan(tan(x)/(- tan(x)^2 - 1)^(1/2))/2 - (tan(x)*(- tan(x)^2 - 1)^(1/2))/2
```

3.291 $\int \sqrt{-1-\tan^2(x)}\,dx$

Optimal. Leaf size=16

$$-\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$

```
[Out] -arctan(tan(x)/(-sec(x)^2)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3738, 4207, 223, 209}

$$-\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Tan[x]^2],x]
```

```
[Out] -ArcTan[Tan[x]/Sqrt[-Sec[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{-1-\tan^2(x)}\,dx = \int \sqrt{-\sec^2(x)}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \tan(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$
$$= -\tan^{-1}\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.
time = 0.01, size = 46, normalized size = 2.88

$$\cos(x)\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)\sqrt{-\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Tan[x]^2],x]
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[-Sec[x]^
2]
```

Maple [A]
time = 0.06, size = 17, normalized size = 1.06

method	result	size
derivativedivides	$-\arctan\left(\frac{\tan(x)}{\sqrt{-1-(\tan^2(x))}}\right)$	17
default	$-\arctan\left(\frac{\tan(x)}{\sqrt{-1-(\tan^2(x))}}\right)$	17
risch	$-2\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x)+2\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctan(tan(x)/(-1-tan(x)^2)^(1/2))
```

Maxima [A]
time = 0.62, size = 17, normalized size = 1.06

$$\arctan(\cos(x), \sin(x)+1)+\arctan(\cos(x), -\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] arctan2(cos(x), sin(x) + 1) + arctan2(cos(x), -sin(x) + 1)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.92, size = 19, normalized size = 1.19

$$i \log \left(e^{(i x)} + i\right) - i \log \left(e^{(i x)} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] I*log(e^(I*x) + I) - I*log(e^(I*x) - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\tan^{2}{\left(x \right)} - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)**2)**(1/2),x)
[Out] Integral(sqrt(-tan(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 16, normalized size = 1.00

$$-i \log \left(\sqrt{\tan \left(x\right)^{2} + 1} - \tan \left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tan(x)^2)^(1/2),x, algorithm="giac")
[Out] -I*log(sqrt(tan(x)^2 + 1) - tan(x))
```

Mupad [B]
time = 0.11, size = 16, normalized size = 1.00

$$-\mathrm{atan}\left(\frac{\mathrm{tan}\left(x\right)}{\sqrt{-{\mathrm{tan}\left(x\right)}^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- tan(x)^2 - 1)^(1/2),x)
[Out] -atan(tan(x)/(- tan(x)^2 - 1)^(1/2))
```

3.292 $\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

```
[Out] tan(x)/(-sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3738, 4207, 197}

$$\frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Tan[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[-Sec[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx = \int \frac{1}{\sqrt{-\sec^2(x)}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{(-1-x^2)^{3/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Tan[x]^2],x]
[Out] Tan[x]/Sqrt[-Sec[x]^2]
```

Maple [A]
time = 0.06, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\tan(x)}{\sqrt{-1-\left(\tan^2(x)\right)}}$	14
default	$\frac{\tan(x)}{\sqrt{-1-\left(\tan^2(x)\right)}}$	14
risch	$-\frac{i e^{2ix}}{2\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}+\frac{i}{2\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] tan(x)/(-1-tan(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-tan(x)^2 - 1), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.11, size = 12, normalized size = 0.92

$$-\frac{1}{2}\left(e^{(2ix)}-1\right)e^{(-ix)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(e^(2*I*x) - 1)*e^(-I*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\tan^{2}(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-tan(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 12, normalized size = 0.92

$$-\frac{i\tan(x)}{\sqrt{\tan(x)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -I*tan(x)/sqrt(tan(x)^2 + 1)
```

Mupad [B]
time = 11.98, size = 13, normalized size = 1.00

$$-\frac{\sqrt{2}\,\sin(2x)\,1\mathrm{i}}{2\sqrt{2\cos(x)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- tan(x)^2 - 1)^(1/2),x)
```

```
[Out] -(2^(1/2)*sin(2*x)*1i)/(2*(2*cos(x)^2)^(1/2))
```

3.293 $\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{(a+$$

[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f+(a+b*tan(f*x+e)^2)^(1/2)/f-1/3*(a+b)*(a+b*tan(f*x+e)^2)^(3/2)/b^2/f+1/5*(a+b*tan(f*x+e)^2)^(5/2)/b^2/f

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 457, 90, 52, 65, 214}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]

[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a + b*Tan[e + f*x]^2]/f - ((a + b)*(a + b*Tan[e + f*x]^2)^(3/2))/(3*b^2*f) + (a + b*Tan[e + f*x]^2)^(5/2)/(5*b^2*f)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-b)\sqrt{a+bx}}{b} + \frac{\sqrt{a+bx}}{1+x} + \frac{(a+bx)^{3/2}}{b}\right)\,dx, x, \tan^2(\right.}{2f}$$

$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f} + \frac{\text{Su}}{}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{(a+b}{}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{(a+b}{}$$

$$= -\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\tan^2(e+}}{f}$$

Mathematica [A]
time = 0.93, size = 109, normalized size = 0.93

$$\frac{-15\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) + \frac{\sqrt{a+b\tan^2(e+fx)}\left(-2a^2-5ab+15b^2+(a-5b)b\tan^2(e+fx)+3b^2\tan^4(e+fx)\right)}{b^2}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-15*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (Sqrt[a
+ b*Tan[e + f*x]^2]*(-2*a^2 - 5*a*b + 15*b^2 + (a - 5*b)*b*Tan[e + f*x]^2 +
 3*b^2*Tan[e + f*x]^4))/b^2)/(15*f)
```

Maple [A]
time = 0.08, size = 157, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{(\tan^2(fx+e))(a+b(\tan^2(fx+e)))^{\frac{3}{2}}}{5b} - \frac{2a(a+b(\tan^2(fx+e)))^{\frac{3}{2}}}{15b^2} - \frac{(a+b(\tan^2(fx+e)))^{\frac{3}{2}}}{3b} + b\left(\frac{\sqrt{a+b(\tan^2(fx+e)}}{b}\right.}{f}$
default	$\frac{\frac{(\tan^2(fx+e))(a+b(\tan^2(fx+e)))^{\frac{3}{2}}}{5b} - \frac{2a(a+b(\tan^2(fx+e)))^{\frac{3}{2}}}{15b^2} - \frac{(a+b(\tan^2(fx+e)))^{\frac{3}{2}}}{3b} + b\left(\frac{\sqrt{a+b(\tan^2(fx+e)}}{b}\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/5*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2)/b-2/15*a/b^2*(a+b*tan(f*x+e)
^2)^(3/2)-1/3*(a+b*tan(f*x+e)^2)^(3/2)/b+b*(1/b*(a+b*tan(f*x+e)^2)^(1/2)-1/
(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))+a/(-a+b)^(1/2)*
arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

Fricas [A]
time = 3.27, size = 334, normalized size = 2.85

$$\left[\frac{15\sqrt{a-b}\,b^2\log\left(-\frac{b^2\tan(fx+e)^4+2(4ab-3b^2)\tan(fx+e)^2-4(b\tan(fx+e)^2+2a-b)\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)+4(3b^2\tan(fx+e)^4+(ab-5b^2)\tan(fx+e)^2-2a^2-5ab+15b^2)\sqrt{b\tan(fx+e)^2+a}}{60b^2f}, \frac{15\sqrt{-a+b}\,b^2\arctan\left(\frac{2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a+b}}{b\tan(fx+e)^2+2a-b}\right)+2(3b^2\tan(fx+e)^4+(ab-5b^2)\tan(fx+e)^2-2a^2-5ab+15b^2)\sqrt{b\tan(fx+e)^2+a}}{30b^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*sqrt(a - b)*b^2*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(
f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4
*(3*b^2*tan(f*x + e)^4 + (a*b - 5*b^2)*tan(f*x + e)^2 - 2*a^2 - 5*a*b + 15*
b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/30*(15*sqrt(-a + b)*b^2*arctan(
```

```
2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2
*(3*b^2*tan(f*x + e)^4 + (a*b - 5*b^2)*tan(f*x + e)^2 - 2*a^2 - 5*a*b + 15*
b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2{(e + fx)}}\tan^5{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**5, x)
```

Giac [A]
time = 0.49, size = 143, normalized size = 1.22

$$\frac{(a-b)\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{3\left(b\tan(fx+e)^2+a\right)^{\frac{5}{2}}b^8f^4-5\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}ab^8f^4-5\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}b^9f^4+15\sqrt{b\tan(fx+e)^2+a}\,b^{10}f^4}{15\,b^{10}f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] (a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) +
1/15*(3*(b*tan(f*x + e)^2 + a)^(5/2)*b^8*f^4 - 5*(b*tan(f*x + e)^2 + a)^(3/
2)*a*b^8*f^4 - 5*(b*tan(f*x + e)^2 + a)^(3/2)*b^9*f^4 + 15*sqrt(b*tan(f*x +
 e)^2 + a)*b^10*f^4)/(b^10*f^5)
```

Mupad [B]
time = 20.97, size = 157, normalized size = 1.34

$$\frac{\left(b\tan(e+f\,x)^2+a\right)^{5/2}}{5\,b^2\,f}-\left(\frac{2\,a}{3\,b^2\,f}-\frac{a-b}{3\,b^2\,f}\right)\left(b\tan(e+f\,x)^2+a\right)^{3/2}-\sqrt{b\tan(e+f\,x)^2+a}\,\left(\left(\frac{2\,a}{b^2\,f}-\frac{a-b}{b^2\,f}\right)(a-b)-\frac{a^2}{b^2\,f}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}\,1\mathrm{i}}{\sqrt{a-b}}\right)\sqrt{a-b}\,1\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (atan(((a + b*tan(e + f*x)^2)^(1/2)*1i)/(a - b)^(1/2))*(a - b)^(1/2)*1i)/f
- ((2*a)/(3*b^2*f) - (a - b)/(3*b^2*f))*(a + b*tan(e + f*x)^2)^(3/2) - (a +
 b*tan(e + f*x)^2)^(1/2)*(((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a - b) - a^2/(
b^2*f)) + (a + b*tan(e + f*x)^2)^(5/2)/(5*b^2*f)
```

3.294 $\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a+b\tan^2(e+fx))^{3/2}}{3bf}$$

```
[Out] arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f-(a+b*tan(f*x+e)
^2)^(1/2)/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/b/f
```

Rubi [A]

time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3751, 457, 81, 52, 65, 214}

$$\frac{(a+b\tan^2(e+fx))^{3/2}}{3bf}-\frac{\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - Sqrt[a +
b*Tan[e + f*x]^2]/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*b*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b\tan^2(e+fx))^{3/2}}{3bf} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+f$$

$$= -\frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{(a+b\tan^2(e+fx))^{3/2}}{3bf} - \frac{(a-b)\text{Su}$$

$$= -\frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{(a+b\tan^2(e+fx))^{3/2}}{3bf} - \frac{(a-b)\text{Su}$$

$$= \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{\sqrt{a+b\tan^2(e+}}{f}$$

Mathematica [A]
time = 0.24, size = 82, normalized size = 0.93

$$\frac{3\sqrt{a-b}\,b\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) + \sqrt{a+b\tan^2(e+fx)}\,(a-3b+b\tan^2(e+fx))}{3bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (3*Sqrt[a - b]*b*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a +
 b*Tan[e + f*x]^2]*(a - 3*b + b*Tan[e + f*x]^2))/(3*b*f)
```

Maple [A]
time = 0.06, size = 112, normalized size = 1.27

method	result

derivativedivides	$\frac{\frac{\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{3b}-b\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)-\frac{a\arctan\left(\frac{\sqrt{\ldots}}{\ldots}\right)}{\ldots}}{f}$
default	$\frac{\frac{\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{3b}-b\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)-\frac{a\arctan\left(\frac{\sqrt{\ldots}}{\ldots}\right)}{\ldots}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*(a+b*tan(f*x+e)^2)^(3/2)/b-b*(1/b*(a+b*tan(f*x+e)^2)^(1/2)-1/(-a+b
)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))-a/(-a+b)^(1/2)*arcta
n((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

Fricas [A]
time = 2.32, size = 266, normalized size = 3.02

$$\left[\frac{3\sqrt{a-b}\,b\log\left(-\frac{b^2\tan(fx+e)^4+2(4ab-3b^2)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)+4\left(b\tan(fx+e)^2+a-3b\right)\sqrt{b\tan(fx+e)^2+a}}{12bf},\ -\frac{3\sqrt{-a+b}\,b\arctan\left(\frac{2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a+b}}{b\tan(fx+e)^2+2a-b}\right)-2\left(b\tan(fx+e)^2+a-3b\right)\sqrt{b\tan(fx+e)^2+a}}{6bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(a - b)*b*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x
 + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
- b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*(b
*tan(f*x + e)^2 + a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/6*(3*sqrt(
-a + b)*b*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^
2 + 2*a - b)) - 2*(b*tan(f*x + e)^2 + a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/
(b*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2(e + fx)}\,\tan^3(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**3, x)
```

Giac [**A**]
time = 0.48, size = 96, normalized size = 1.09

$$-\frac{(a-b)\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f} + \frac{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}b^2f^2 - 3\sqrt{b\tan(fx+e)^2+a}\,b^3f^2}{3\,b^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -(a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) +
 1/3*((b*tan(f*x + e)^2 + a)^(3/2)*b^2*f^2 - 3*sqrt(b*tan(f*x + e)^2 + a)*b
^3*f^2)/(b^3*f^3)
```

Mupad [**B**]
time = 14.38, size = 76, normalized size = 0.86

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}}{\sqrt{a-b}}\right)\sqrt{a-b}}{f} - \frac{\sqrt{b\tan(e+f\,x)^2+a}}{f} + \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{3\,b\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b)^(1/2))*(a - b)^(1/2))/f - (a +
b*tan(e + f*x)^2)^(1/2)/f + (a + b*tan(e + f*x)^2)^(3/2)/(3*b*f)
```

3.295 $\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f+(a+b*tan(f*x+e
)^2)^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3751, 455, 52, 65, 214}

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a
 + b*Tan[e + f*x]^2]/f
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \text{ta}\right.}{2f}$$

$$= \frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\text{t}}\right.}{bf}$$

$$= -\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\tan^2(e+}}{f}$$

Mathematica [A]

time = 0.03, size = 59, normalized size = 0.95

$$\frac{-\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]

[Out] (-(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]) + Sqrt[a + b*Tan[e + f*x]^2])/f

Maple [A]
time = 0.05, size = 91, normalized size = 1.47

method	result
derivativedivides	$\frac{b\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)+\frac{a\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+\right.\right.}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}}{f}$
default	$\frac{b\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)+\frac{a\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+\right.\right.}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)

[Out] 1/f*(b*(1/b*(a+b*tan(f*x+e)^2)^(1/2)-1/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))+a/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")

[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e), x)

Fricas [A]
time = 3.27, size = 224, normalized size = 3.61

$$\left[\frac{\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2-4\,\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)+4\,\sqrt{b\tan\left(fx+e\right)^2+a}}{4\,f},\ \frac{\sqrt{-a+b}\arctan\left(\frac{2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{b\tan(fx+e)^2+2\,a-b}\right)+2\,\sqrt{b\tan\left(fx+e\right)^2+a}}{2\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)
^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)
+ 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(b*
tan(f*x + e)^2 + a))/f, 1/2*(sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*sqrt(b*tan(f*x + e)^2 + a
))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\tan^2\left(e+fx\right)}\,\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x), x)
```

Giac [A]
time = 0.45, size = 60, normalized size = 0.97

$$\frac{(a-b)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] (a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) +
sqrt(b*tan(f*x + e)^2 + a)/f
```

Mupad [B]
time = 12.51, size = 54, normalized size = 0.87

$$\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{f}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sqrt{a-b}}\right)\,\sqrt{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(1/2)/f - (atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b
)^(1/2))*(a - b)^(1/2))/f
```

3.296 $\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+arctanh((a+b*tan(f*x+e
)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 457, 85, 65, 214}

$$\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/f) + (Sqrt[a - b]*A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{(a-b)\text{Subst}\left(\int \frac{\quad}{(1+\quad}\right.}{}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} - \frac{(a-b)\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{f}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.97

$$\frac{-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]) + Sqrt[a - b]*ArcTa
nh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(62) = 124$.
time = 0.71, size = 615, normalized size = 8.31

method	result
default	$-\frac{\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{\cos(fx+e)^2}}\,\sqrt{4}\,\cos(fx+e)(\cos(fx+e)-1)\left(\ln\left(-\frac{2(\cos(fx+e)-1)\left(\cos(fx+e)\sqrt{a}\,\sqrt{\frac{a(\cos^2(fx+e)\cdots}{(\cos(\cdots}}\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*4^(1/2)*cos(f
*x+e)*(cos(f*x+e)-1)*(ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x
+e)^2/a^(1/2))*a^(1/2)*(a-b)^(1/2)-a^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(
cos(f*x+e)-1))*(a-b)^(1/2)+2*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*a-2*l
n(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*b)/sin(f*x+e)^2/((a*cos(f*x+e)^2-co
s(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)/(a-b)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 3.13, size = 398, normalized size = 5.38

$$\left[\frac{\sqrt{a-b}\log\left(\frac{b\tan\left(fx+e\right)^{2}+2\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^{2}+1}\right)+\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^{2}-2\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{a}+2a}{\tan\left(fx+e\right)^{2}}\right)}{2f},\ \frac{2\sqrt{-a+b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{-a+b}}{a-b}\right)+\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^{2}-2\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{a}+2a}{\tan\left(fx+e\right)^{2}}\right)}{2f},\ \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{-a}}{a}\right)+\sqrt{a-b}\log\left(\frac{b\tan\left(fx+e\right)^{2}+2\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^{2}+1}\right)}{2f},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{-a}}{a}\right)+\sqrt{-a+b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}\sqrt{-a+b}}{a-b}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + sqrt(a)*log((b*tan(f*x + e)^2 -
2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/f, 1/2*(2*sqrt
(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + sqrt(a)
*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*
x + e)^2))/f, 1/2*(2*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)
 + sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
- b) + 2*a - b)/(tan(f*x + e)^2 + 1)))/f, (sqrt(-a)*arctan(sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(-a)/a) + sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sq
rt(-a + b)/(a - b)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\tan^{2}\left(e+fx\right)}\,\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(64) = 128.
time = 1.01, size = 207, normalized size = 2.80

$$-\frac{\left(\frac{2a\arctan\left(-\frac{\sqrt{a-b}\sin\left(fx+e\right)^{2}-\sqrt{a\sin\left(fx+e\right)^{4}-b\sin\left(fx+e\right)^{4}-2a\sin\left(fx+e\right)^{2}+b\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{a-b}\log\left(\left|-2\left(\sqrt{a-b}\sin\left(fx+e\right)^{2}-\sqrt{a\sin\left(fx+e\right)^{4}-b\sin\left(fx+e\right)^{4}-2a\sin\left(fx+e\right)^{2}+b\sin\left(fx+e\right)^{2}+a}\right)\left(a-b\right)+\left(2a-b\right)\sqrt{a-b}\right|\right)\right)\mathrm{sgn}\left(\sin\left(fx+e\right)^{2}-1\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*arctan(-(sqrt(a - b)*sin(f*x + e)^2 - sqrt(a*sin(f*x + e)^4 - b*s
in(f*x + e)^4 - 2*a*sin(f*x + e)^2 + b*sin(f*x + e)^2 + a))/sqrt(-a))/sqrt(
-a) + sqrt(a - b)*log(abs(-2*(sqrt(a - b)*sin(f*x + e)^2 - sqrt(a*sin(f*x +
```

```
e)^4 - b*sin(f*x + e)^4 - 2*a*sin(f*x + e)^2 + b*sin(f*x + e)^2 + a))*(a -
b) + (2*a - b)*sqrt(a - b))))*sgn(sin(f*x + e)^2 - 1)/f
```

Mupad [B]
time = 0.29, size = 83, normalized size = 1.12

$$-\frac{\sqrt{a}\ \mathrm{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sqrt{a}}\right)}{f}-\frac{\mathrm{atanh}\left(\frac{a\,b^3\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\ \sqrt{a-b}}{a\,b^4-a^2\,b^3}\right)\ \sqrt{a-b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] - (a^(1/2)*atanh((a + b*tan(e + f*x)^2)^(1/2)/a^(1/2)))/f - (atanh((a*b^3*(
a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(a*b^4 - a^2*b^3))*(a - b)^(1/2)
)/f
```

3.297 $\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=115

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^2(e+fx)\sqrt{a+}}{2f}$$

```
[Out] 1/2*(2*a-b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-arctanh((a+
b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f-1/2*cot(f*x+e)^2*(a+b*tan(
f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3751, 457, 101, 162, 65, 214}

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2/Sqrt[a]])/(2*Sqrt[a]*f) - (Sq
rt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - (Cot[e + f*x
]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^3(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2a+b)-\frac{bx}{2}}{x(1+x)\sqrt{a+bx}}\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a}}\right.}{2}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\, d\right.}{}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.25, size = 115, normalized size = 1.00

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right) - \sqrt{a}\left(2\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) + \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)}{2\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] - Sqrt[a]*(2*Sqrt[a
- b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Cot[e + f*x]^2*Sqrt[
a + b*Tan[e + f*x]^2]))/(2*Sqrt[a]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2134 vs. $2(97) = 194$.
time = 0.47, size = 2135, normalized size = 18.57

method	result	size
default	Expression too large to display	2135

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8/f*(cos(f*x+e)-1)*(-4*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*4^(1/2)*cos(f*x+e)^2*a^(5/2)-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*4^(1/2)*cos(f*x+e)^2*a^(3/2)*(a-b)^(1/2)+4*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*4^(1/2)*cos(f*x+e)^2*a^(3/2)*b+8*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)^2*a^(1/2)*(a-b)^(1/2)+4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*4^(1/2)*cos(f*x+e)^2*a^(1/2)*(a-b)^(1/2)*b+2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*4^(1/2)*cos(f*x+e)*a^(3/2)*(a-b)^(1/2)-2*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*4^(1/2)*cos(f*x+e)^2*(a-b)^(1/2)*a^2+ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*4^(1/2)*cos(f*x+e)^2*(a-b)^(1/2)*a*b+2*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*4^(1/2)*cos(f*x+e)^2*(a-b)^(1/2)*a^2-ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*4^(1/2)*cos(f*x+e)^2*(a-b)^(1/2)*a*b+4*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*4^(1/2)*a^(5/2)+16*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)*a^(1/2)*(a-b)^(1/2)-4*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*4^(1/2)*a^(3/2)*b+8*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)*a^(1/2)*(a-b)^(1/2)-4*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*4^(1/2)*a^(1/2)*(a-b)^(1/2)*b+2*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*4^(1/2)*(a-b)^(1/2)*a^2-ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*4^

```
(1/2)*(a-b)^(1/2)*a*b-2*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f
*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*4^(1
/2)*(a-b)^(1/2)*a^2+ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e
)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*4^(1/2)*
(a-b)^(1/2)*a*b)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2
)^(1/2)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+
e)^4/a^(3/2)/(a-b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

Fricas [A]
time = 2.30, size = 624, normalized size = 5.43

$$\left[\frac{2\sqrt{a-b}\,a\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2-(2a-b)\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a}+2a}{\tan\left(fx+e\right)^2}\right)\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,a}{4af\tan\left(fx+e\right)^2},\ -\frac{4a\sqrt{-a+b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{a-b}\right)\tan\left(fx+e\right)^2+(2a-b)\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a}+2a}{\tan\left(fx+e\right)^2}\right)\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,a}{4af\tan\left(fx+e\right)^2},\ -\frac{\sqrt{-a}(2a-b)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a}}{a}\right)\tan\left(fx+e\right)^2-\sqrt{a-b}\,a\log\left(\frac{b\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^2+\sqrt{b\tan\left(fx+e\right)^2+a}\,a}{2af\tan\left(fx+e\right)^2},\ -\frac{\sqrt{-a}(2a-b)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a}}{a}\right)\tan\left(fx+e\right)^2+2a\sqrt{-a+b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{a-b}\right)\tan\left(fx+e\right)^2+\sqrt{b\tan\left(fx+e\right)^2+a}\,a}{2af\tan\left(fx+e\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/4*(2*sqrt(a - b)*a*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 - (2*a - b)*sqr
t(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/ta
n(f*x + e)^2)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(a*f*tan(f*x
 + e)^2), -1/4*(4*a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
 + b)/(a - b))*tan(f*x + e)^2 + (2*a - b)*sqrt(a)*log((b*tan(f*x + e)^2 - 2
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 +
 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(a*f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a
- b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 - sqrt(a
- b)*a*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2
*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)^2 + a)*a
)/(a*f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a - b)*arctan(sqrt(b*tan(f*x + e)
^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 + 2*a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)^2
+ a)*a)/(a*f*tan(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\tan^2\left(e+fx\right)}\,\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 11.87, size = 238, normalized size = 2.07

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,b^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{2\,\left(\frac{a\,b^4}{2}-\frac{3\,b^5}{4}+\frac{b^6}{4\,a}\right)}-\frac{3\,b^5\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{4\,\sqrt{a}\,\left(\frac{a\,b^4}{2}-\frac{3\,b^5}{4}+\frac{b^6}{4\,a}\right)}+\frac{b^6\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{4\,a^{3/2}\,\left(\frac{a\,b^4}{2}-\frac{3\,b^5}{4}+\frac{b^6}{4\,a}\right)}\right)\,\left(2\,a-b\right)}{2\,\sqrt{a}\,f}-\frac{\operatorname{atanh}\left(\frac{b^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\sqrt{a-b}}{2\,\left(\frac{a\,b^4}{2}-\frac{b^5}{2}\right)}\right)\,\sqrt{a-b}}{f}-\frac{b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{2\,\left(f\,\left(b\tan\left(e+f\,x\right)^2+a\right)-a\,f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (atanh((a^(1/2)*b^4*(a + b*tan(e + f*x)^2)^(1/2))/(2*((a*b^4)/2 - (3*b^5)/4
 + b^6/(4*a))) - (3*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*a^(1/2)*((a*b^4)/2
 - (3*b^5)/4 + b^6/(4*a))) + (b^6*(a + b*tan(e + f*x)^2)^(1/2))/(4*a^(3/2)*
((a*b^4)/2 - (3*b^5)/4 + b^6/(4*a))))*(2*a - b))/(2*a^(1/2)*f) - (atanh((b^
4*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(2*((a*b^4)/2 - b^5/2)))*(a -
 b)^(1/2))/f - (b*(a + b*tan(e + f*x)^2)^(1/2))/(2*(f*(a + b*tan(e + f*x)^2
) - a*f))
```

3.298 $\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=163

$$-\frac{(8a^2-4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{(4a-b)\,}{}$$

```
[Out] -1/8*(8*a^2-4*a*b-b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+
arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f+1/8*(4*a-b)*cot
(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/4*cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^
(1/2)/f
```

Rubi [A]
time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3751, 457, 101, 156, 162, 65, 214}

$$-\frac{(8a^2-4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+\frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/8*((8*a^2 - 4*a*b - b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(a
^(3/2)*f) + (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f
 + ((4*a - b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*a*f) - (Cot[e +
 f*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
```

```
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^5(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a+b)-\frac{3bx}{2}}{x^2(1+x)\sqrt{a+bx}}\right.}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+}}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+}}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+}}{4f}$$

$$= -\frac{(8a^2-4ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{\sqrt{a-b}\ \text{t}}{}$$

Mathematica [A]
time = 0.88, size = 138, normalized size = 0.85

$$\frac{(-8a^2+4ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)-\cot^2(e+fx)\left(-4a+b+2a\cot^2(e+fx)\right)\sqrt{a+b\tan^2(e+fx)}\right)}{8a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 4*a*b + b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sqrt[
a]*(8*a*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] - Cot[e
 + f*x]^2*(-4*a + b + 2*a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2]))/(8*a
^(3/2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5675 vs. $2(141) = 282$.
time = 0.35, size = 5676, normalized size = 34.82

method	result	size
default	Expression too large to display	5676

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

Fricas [A]
time = 2.76, size = 765, normalized size = 4.69

$$\left[\frac{8\sqrt{a-b}\,a^2\log\left(\frac{b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4-\left(8a^2-4ab-b^2\right)\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a}+2a}{\tan\left(fx+e\right)^2}\right)\tan\left(fx+e\right)^4+2\left(\left(4a^2-ab\right)\tan\left(fx+e\right)^2-2a^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{16a^2f\tan\left(fx+e\right)^4}, \frac{16a^2\sqrt{-a+b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{a-b}\right)\tan\left(fx+e\right)^4-\left(8a^2-4ab-b^2\right)\sqrt{a}\log\left(\frac{b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a}+2a}{\tan\left(fx+e\right)^2}\right)\tan\left(fx+e\right)^4+2\left(\left(4a^2-ab\right)\tan\left(fx+e\right)^2-2a^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{16a^2f\tan\left(fx+e\right)^4}, \frac{4\sqrt{a-b}\,a^2\log\left(\frac{b\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2a-b}{\tan\left(fx+e\right)^2+1}\right)\tan\left(fx+e\right)^4+\left(8a^2-4ab-b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a}}{a}\right)\tan\left(fx+e\right)^4+\left(\left(4a^2-ab\right)\tan\left(fx+e\right)^2-2a^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{8a^2f\tan\left(fx+e\right)^4}, \frac{8a^2\sqrt{-a+b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{a-b}\right)\tan\left(fx+e\right)^4+\left(8a^2-4ab-b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a}}{a}\right)\tan\left(fx+e\right)^4+\left(\left(4a^2-ab\right)\tan\left(fx+e\right)^2-2a^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{8a^2f\tan\left(fx+e\right)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 - (8*a^2 - 4
*a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sq
rt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 + 2*((4*a^2 - a*b)*tan(f*x + e)
^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(f*x + e)^4), 1/16*(16*a^
2*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan
(f*x + e)^4 - (8*a^2 - 4*a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 + 2*((4
*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(
f*x + e)^4), 1/8*(4*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 +
 (8*a^2 - 4*a*b - b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/
a)*tan(f*x + e)^4 + ((4*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x +
 e)^2 + a))/(a^2*f*tan(f*x + e)^4), 1/8*(8*a^2*sqrt(-a + b)*arctan(-sqrt(b*
tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^2 - 4*a*b -
 b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4
 + ((4*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*
f*tan(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2(e + fx)}\,\cot^5(e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 12.02, size = 542, normalized size = 3.33

$$\frac{\mathrm{atanh}\left(\frac{3b^7\sqrt{b\tan(e+fx)^2+a}}{64\sqrt{a^3}\left(\frac{ab^5}{4}-\frac{11b^6}{32}+\frac{3b^7}{64a}+\frac{11b^8}{256a^2}+\frac{b^9}{256a^3}\right)}-\frac{11b^6\sqrt{b\tan(e+fx)^2+a}}{32\sqrt{a^3}\left(\frac{b^5}{4}-\frac{11b^6}{32a}+\frac{3b^7}{64a^2}+\frac{11b^8}{256a^3}+\frac{b^9}{256a^4}\right)}+\frac{11b^8\sqrt{b\tan(e+fx)^2+a}}{256\sqrt{a^3}\left(\frac{3b^7}{64}-\frac{11ab^6}{32}+\frac{a^2b^5}{4}+\frac{11b^8}{256a}+\frac{b^9}{256a^2}\right)}+\frac{b^9\sqrt{b\tan(e+fx)^2+a}}{256\sqrt{a^3}\left(\frac{3ab^7}{64}+\frac{11b^8}{256}-\frac{11a^2b^6}{32}+\frac{a^3b^5}{4}+\frac{b^9}{256a}\right)}+\frac{ab^5\sqrt{b\tan(e+fx)^2+a}}{4\sqrt{a^3}\left(\frac{b^5}{4}-\frac{11b^6}{32a}+\frac{3b^7}{64a^2}+\frac{11b^8}{256a^3}+\frac{b^9}{256a^4}\right)}\right)\left(-8a^2+4ab+b^2\right)}{8f\sqrt{a^3}}-\frac{\mathrm{atanh}\left(\frac{b^5\sqrt{b\tan(e+fx)^2+a}\,\sqrt{a-b}}{4\left(\frac{7b^6}{32}-\frac{ab^5}{4}+\frac{b^7}{32a}\right)}+\frac{b^6\sqrt{b\tan(e+fx)^2+a}\,\sqrt{a-b}}{32\left(\frac{7ab^6}{32}+\frac{b^7}{32}-\frac{a^2b^5}{4}\right)}\right)\sqrt{a-b}}{f}-\frac{\sqrt{b\tan(e+fx)^2+a}\left(\frac{ab}{2}+\frac{b^2}{8}\right)-\frac{b\left(b\tan(e+fx)^2+a\right)^{3/2}(4a-b)}{8a}}{f\left(b\tan(e+fx)^2+a\right)^2+a^2f-2af\left(b\tan(e+fx)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (atanh((3*b^7*(a + b*tan(e + f*x)^2)^(1/2))/(64*(a^3)^(1/2)*((a*b^5)/4 - (1
1*b^6)/32 + (3*b^7)/(64*a) + (11*b^8)/(256*a^2) + b^9/(256*a^3))) - (11*b^6
*(a + b*tan(e + f*x)^2)^(1/2))/(32*(a^3)^(1/2)*(b^5/4 - (11*b^6)/(32*a) + (
3*b^7)/(64*a^2) + (11*b^8)/(256*a^3) + b^9/(256*a^4))) + (11*b^8*(a + b*tan
(e + f*x)^2)^(1/2))/(256*(a^3)^(1/2)*((3*b^7)/64 - (11*a*b^6)/32 + (a^2*b^5
)/4 + (11*b^8)/(256*a) + b^9/(256*a^2))) + (b^9*(a + b*tan(e + f*x)^2)^(1/2
))/(256*(a^3)^(1/2)*((3*a*b^7)/64 + (11*b^8)/256 - (11*a^2*b^6)/32 + (a^3*b
^5)/4 + b^9/(256*a))) + (a*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*(a^3)^(1/2)
*(b^5/4 - (11*b^6)/(32*a) + (3*b^7)/(64*a^2) + (11*b^8)/(256*a^3) + b^9/(25
6*a^4))))*(4*a*b - 8*a^2 + b^2))/(8*f*(a^3)^(1/2)) - (atanh((b^5*(a + b*tan
(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(4*((7*b^6)/32 - (a*b^5)/4 + b^7/(32*a)))
 + (b^6*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(32*((7*a*b^6)/32 + b^7
/32 - (a^2*b^5)/4)))*(a - b)^(1/2))/f - ((a + b*tan(e + f*x)^2)^(1/2)*((a*b
)/2 + b^2/8) - (b*(a + b*tan(e + f*x)^2)^(3/2)*(4*a - b))/(8*a))/(f*(a + b*
tan(e + f*x)^2)^2 + a^2*f - 2*a*f*(a + b*tan(e + f*x)^2))
```

3.299 $\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=222

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(a^3+2a^2b+8ab^2-16b^3)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{5/2}f}$$

```
[Out] 1/16*(a^3+2*a^2*b+8*a*b^2-16*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e
)^2)^(1/2))/b^(5/2)/f-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2
))*(a-b)^(1/2)/f-1/16*(a-2*b)*(a+4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b
^2/f+1/24*(a-6*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/6*(a+b*tan(f*
x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.23, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 489, 596, 537, 223, 212, 385, 209}

$$\frac{(a^3+2a^2b+8ab^2-16b^3)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{5/2}f}-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f}+\frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f}+\frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) + ((a^3 + 2*a^2*b + 8*a*b^2 - 16*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) - ((a - 2*b)*(a + 4*b)*Tan[e +
f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(16*b^2*f) + ((a - 6*b)*Tan[e + f*x]^3*Sqr
t[a + b*Tan[e + f*x]^2])/(24*b*f) + (Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]
^2])/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{x^4(5a+(-a+6b)x^2)}{(1+x^2)\sqrt{a+bx^2}} \cdots\right.}{6f}$$

$$= \frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a \cdots}}{\cdots}$$

$$= -\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b \cdots}{\cdots}$$

$$= -\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b \cdots}{\cdots}$$

$$= -\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b \cdots}{\cdots}$$

$$= -\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+2a^2b+8ab \cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 5.95, size = 420, normalized size = 1.89

$$\frac{3\sqrt{2}a(a^2+2ab-8b^2)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\tan(e+fx)+48\sqrt{2}ab^2\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\tan(e+fx)-(30a^3+62a^2b-224ab^2-104b^3+(45a^3+91a^2b-332ab^2+84b^3)\cos(2(e+fx))+2(9a^3+17a^2b-80ab^2-12b^3)\cos(4(e+fx))+3a^3\cos(6(e+fx))+5a^2b\cos(6(e+fx))-52ab^2\cos(6(e+fx))+44b^3\cos(6(e+fx)))\csc^4(2(e+fx))\sin^2(e+fx)\tan^3(e+fx)}{48\sqrt{2}b^2f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (3*Sqrt[2]*a*(a^2 + 2*a*b - 8*b^2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])
*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)
])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Tan[e + f*x] + 48*Sqrt[2]*a*b^2*Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b))
, ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2
]], 1]*Tan[e + f*x] - (30*a^3 + 62*a^2*b - 224*a*b^2 - 104*b^3 + (45*a^3 +
91*a^2*b - 332*a*b^2 + 84*b^3)*Cos[2*(e + f*x)] + 2*(9*a^3 + 17*a^2*b - 80*
```

```
a*b^2 - 12*b^3)*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)] + 5*a^2*b*Cos[6*(
e + f*x)] - 52*a*b^2*Cos[6*(e + f*x)] + 44*b^3*Cos[6*(e + f*x)])*Csc[2*(e +
 f*x)]^4*Sin[e + f*x]^2*Tan[e + f*x]^3)/(48*Sqrt[2]*b^2*f*Sqrt[(a + b + (a
- b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(196) = 392$.

time = 0.06, size = 429, normalized size = 1.93

method	result
derivativedivides	$\frac{\frac{\left(\tan^3(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{6b} - \frac{a\left(\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{4b} - \frac{a\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}\,\tan(fx+e)}{2} + \cdots\right.}{\cdots}\right.}{2b}}{\cdots}$
default	$\frac{\frac{\left(\tan^3(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{6b} - \frac{a\left(\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{4b} - \frac{a\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}\,\tan(fx+e)}{2} + \cdots\right.}{\cdots}\right.}{2b}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/6*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/b-1/2*a/b*(1/4*tan(f*x+e)*(a
+b*tan(f*x+e)^2)^(3/2)/b-1/4*a/b*(1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)+1
/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))))-1/4*tan(f*x+
e)*(a+b*tan(f*x+e)^2)^(3/2)/b+1/4*a/b*(1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x
+e)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2)))+1/2*(a+b
*tan(f*x+e)^2)^(1/2)*tan(f*x+e)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*ta
n(f*x+e)^2)^(1/2))-b*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/
2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(
f*x+e)^2)^(1/2)*tan(f*x+e)))-a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)
/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Fricas [**A**]
time = 5.77, size = 864, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/96*(48*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 3*(a^3
 + 2*a^2*b + 8*a*b^2 - 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(8*b^3*tan(f*x + e)^5 + 2*(
a*b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/96*(96*sqrt(a - b)*b^3*arctan(-sqrt(
b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(a^3 + 2*a^2*b + 8*a*
b^2 - 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(b)*tan(f*x + e) + a) - 2*(8*b^3*tan(f*x + e)^5 + 2*(a*b^2 - 6*b^3)*ta
n(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e
)^2 + a))/(b^3*f), 1/48*(24*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2
 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e
)^2 + 1)) - 3*(a^3 + 2*a^2*b + 8*a*b^2 - 16*b^3)*sqrt(-b)*arctan(sqrt(b*tan
(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (8*b^3*tan(f*x + e)^5 + 2*(a*
b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqr
t(b*tan(f*x + e)^2 + a))/(b^3*f), -1/48*(48*sqrt(a - b)*b^3*arctan(-sqrt(b*
tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(a^3 + 2*a^2*b + 8*a*b^
2 - 16*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x
+ e))) - (8*b^3*tan(f*x + e)^5 + 2*(a*b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*
b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^{2}\left(e + fx\right)}\,\tan^{6}\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e+f\,x)^6\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.300 $\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(a^2+4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{(a-4b)\,\mathrm{t}}{}$$

```
[Out] -1/8*(a^2+4*a*b-8*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))
/b^(3/2)/f+arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1
/2)/f+1/8*(a-4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*(a+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A]

time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 489, 596, 537, 223, 212, 385, 209}

$$-\frac{(a^2+4ab-8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f - ((a^2 + 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(8*b^(3/2)*f) + ((a - 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x
]^2])/(8*b*f) + (Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{x^2(3a+(-a+4b)x^2)}{(1+x^2)\sqrt{a+bx^2}}\right.}{4f}$$

$$= \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a}}{4}$$

$$= \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a}}{4}$$

$$= \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a}}{4}$$

$$= \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^2+4ab-8b^2)\,\text{ta}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 6.17, size = 767, normalized size = 4.54

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/4*(-((b*(a^2 - 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2
*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*
Cos[2*(e + f*x)]))) - (4*b*(-4*a*b + 4*b^2)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt
[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot
[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[
((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*Ell
ipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/S
```

```
qrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a
 - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Co
s[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[
((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e +
 f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(
e + f*x)]]]))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]])/(b*f) + (Sqrt[(a + b
+ a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sec[e
+ f*x]*(a*Sin[e + f*x] - 6*b*Sin[e + f*x]))/(8*b) + (Sec[e + f*x]^2*Tan[e +
 f*x])/4))/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(147) = 294$.

time = 0.06, size = 307, normalized size = 1.82

method	result
derivativedivides	$\dfrac{\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{4b} - \frac{a\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}\,\tan(fx+e)}{2} + \frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2 \right.}\right.}{2\sqrt{b}}\right.}{4b}}{}$
default	$\dfrac{\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{4b} - \frac{a\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}\,\tan(fx+e)}{2} + \frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2 \right.}\right.}{2\sqrt{b}}\right.}{4b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/b-1/4*a/b*(1/2*(a+b*tan(f*x+e)
^2)^(1/2)*tan(f*x+e)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)
^(1/2)))-1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)-1/2*a/b^(1/2)*ln(b^(1/2)*t
an(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+b*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e
)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b)
)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))+a*(b^4*(a-b))^(1/2)/b^2/(a-b)
*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Fricas [A]
time = 5.02, size = 705, normalized size = 4.17

$$\left[\frac{8\sqrt{-a+b}\,b^{2}\log\left(-\frac{(a-2b)\tan(fx+e)^{2}+2\sqrt{b\tan(fx+e)^{2}+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)-(a^{2}+4ab-8b^{2})\sqrt{b}\log\left(2b\tan(fx+e)^{2}+2\sqrt{b\tan(fx+e)^{2}+a}\,\sqrt{b}\tan(fx+e)+a\right)+2\left(2b^{2}\tan(fx+e)^{3}+(ab-4b^{2})\tan(fx+e)\right)\sqrt{b\tan(fx+e)^{2}+a}}{16b^{2}f}, \frac{16\sqrt{a-b}\,b^{2}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^{2}+a}}{\sqrt{a-b}\tan(fx+e)}\right)-(a^{2}+4ab-8b^{2})\sqrt{b}\log\left(2b\tan(fx+e)^{2}+2\sqrt{b\tan(fx+e)^{2}+a}\,\sqrt{b}\tan(fx+e)+a\right)+2\left(2b^{2}\tan(fx+e)^{3}+(ab-4b^{2})\tan(fx+e)\right)\sqrt{b\tan(fx+e)^{2}+a}}{16b^{2}f}, \frac{4\sqrt{-a+b}\,b^{2}\log\left(-\frac{(a-2b)\tan(fx+e)^{2}+2\sqrt{b\tan(fx+e)^{2}+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)+(a^{2}+4ab-8b^{2})\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(fx+e)^{2}+a}\,\sqrt{-b}}{b\tan(fx+e)}\right)+\left(2b^{2}\tan(fx+e)^{3}+(ab-4b^{2})\tan(fx+e)\right)\sqrt{b\tan(fx+e)^{2}+a}}{8b^{2}f}, \frac{8\sqrt{a-b}\,b^{2}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^{2}+a}}{\sqrt{a-b}\tan(fx+e)}\right)+(a^{2}+4ab-8b^{2})\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(fx+e)^{2}+a}\,\sqrt{-b}}{b\tan(fx+e)}\right)+\left(2b^{2}\tan(fx+e)^{3}+(ab-4b^{2})\tan(fx+e)\right)\sqrt{b\tan(fx+e)^{2}+a}}{8b^{2}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (a^2 +
4*a*b - 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(b)*tan(f*x + e) + a) + 2*(2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f
*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/16*(16*sqrt(a - b)*b^2*arct
an(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (a^2 + 4*a*b -
 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
b)*tan(f*x + e) + a) + 2*(2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f*x + e)
)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/8*(4*sqrt(-a + b)*b^2*log(-((a - 2
*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e)
 - a)/(tan(f*x + e)^2 + 1)) + (a^2 + 4*a*b - 8*b^2)*sqrt(-b)*arctan(sqrt(b*
tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (2*b^2*tan(f*x + e)^3 + (a
*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/8*(8*sqrt(
a - b)*b^2*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) +
 (a^2 + 4*a*b - 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/
(b*tan(f*x + e))) + (2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f*x + e))*sqr
t(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^{2}{\left(e + fx \right)}}\,\tan^{4}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.301 $\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}\,f}+\frac{\tan(e+fx)\sqrt{a}}{}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+1/2*
(a-2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*
(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]

time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3751, 489, 537, 223, 212, 385, 209}

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) + ((a - 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/(2*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a+(-a+2b)x^2}{(1+x^2)\sqrt{a+bx^2}}\right.}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}}\right.}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx\right.}{}$$

$$= -\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-2b)\tanh^{-1}\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 4.01, size = 251, normalized size = 2.04

$$\frac{\left(-\sqrt{2}a\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+2\sqrt{2}a\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)\right)\tan(e+fx)}{2\sqrt{2}f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-(Sqrt[2]*a*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*E
llipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]
/Sqrt[2]], 1]) + 2*Sqrt[2]*a*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + (a + b + (a - b)*Cos[2*(e + f
*x)])*Sec[e + f*x]^2)*Tan[e + f*x])/(2*Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[
2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(105) = 210$.
time = 0.06, size = 223, normalized size = 1.81

method	result
derivativedivides	$\frac{\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}\tan(fx+e)}{2}+\frac{a\ln\left(\sqrt{b}\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{2\sqrt{b}}-b\left(\frac{\ln\left(\sqrt{b}\tan(\right.}{}\right.}{}$
default	$\frac{\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}\tan(fx+e)}{2}+\frac{a\ln\left(\sqrt{b}\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{2\sqrt{b}}-b\left(\frac{\ln\left(\sqrt{b}\tan(\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f
*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-b*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)
^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1
/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))-a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arc
tan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Fricas [A]
time = 3.66, size = 569, normalized size = 4.63

$$\left[-\frac{(a-2b)\sqrt{b}\log\left(2b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{b}\tan(fx+e)+a\right)-2\sqrt{-a+b}\,b\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)-2\sqrt{b\tan(fx+e)^2+a}\,b\tan(fx+e)}{4bf}, -\frac{4\sqrt{a-b}\,b\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{a-b}\tan(fx+e)}\right)+(a-2b)\sqrt{b}\log\left(2b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{b}\tan(fx+e)+a\right)-2\sqrt{b\tan(fx+e)^2+a}\,b\tan(fx+e)}{4bf}, -\frac{(a-2b)\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}\sqrt{-b}}{b\tan(fx+e)}\right)-\sqrt{-a+b}\,b\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)-\sqrt{b\tan(fx+e)^2+a}\,b\tan(fx+e)}{2bf}, -\frac{2\sqrt{a-b}\,b\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{a-b}\tan(fx+e)}\right)+(a-2b)\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}\sqrt{-b}}{b\tan(fx+e)}\right)-\sqrt{b\tan(fx+e)^2+a}\,b\tan(fx+e)}{2bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(b)*tan(f*x + e) + a) - 2*sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x +
e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x
 + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f), -1/4*(4
*sqrt(a - b)*b*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e)
```

```
)) + (a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(b)*tan(f*x + e) + a) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/
(b*f), -1/2*((a - 2*b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/
(b*tan(f*x + e))) - sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))
- sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f), -1/2*(2*sqrt(a - b)*b*a
rctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (a - 2*b)*s
qrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt
(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2\left(e + fx\right)}\tan^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(e+f\,x\right)}^2\,\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.302 $\int \sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+arcta
nh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3742, 399, 223, 212, 385, 209}

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\right.}{}$$
$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{1-bx}\right.}{}$$
$$= \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+}\right.}{f}$$

Mathematica [A]
time = 0.24, size = 108, normalized size = 1.27

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{b}\,\tan^2(e+fx)-\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{b}\,\log\left(-\sqrt{b}\,\tan(e+fx)+\sqrt{a+b\tan^2(e+fx)}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[e + f*x]^2 - Tan[e + f*x]*Sqrt
[a + b*Tan[e + f*x]^2])/Sqrt[a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Tan[e + f*x])
+ Sqrt[a + b*Tan[e + f*x]^2]])/f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(73) = 146$.
time = 0.06, size = 167, normalized size = 1.96

method	result
derivativedivides	$\frac{b\left(\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan\right.}}\right.}{b^2(a-b)}\right.}{f}$
default	$\frac{b\left(\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan\right.}}\right.}{b^2(a-b)}\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-(b^4*(a-b))
^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e)))+a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(
1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.81, size = 432, normalized size = 5.08

$$\left[\frac{\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{b}\tan\left(fx+e\right)+a\right)+\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(fx+e)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)}{2\,f}, \frac{2\,\sqrt{a-b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{a-b}\tan(fx+e)}\right)+\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{b}\tan\left(fx+e\right)+a\right)}{2\,f}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-b}}{b\tan(fx+e)}\right)-\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(fx+e)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)}{2\,f}, \frac{\sqrt{a-b}\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{a-b}\tan(fx+e)}\right)-\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-b}}{b\tan(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)
*tan(f*x + e) + a) + sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)))/
f, 1/2*(2*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f
*x + e))) + sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(b)*tan(f*x + e) + a))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
 e)^2 + 1)))/f, (sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b
)*tan(f*x + e))) - sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*t
an(f*x + e))))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^{2}\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(1/2), x)
```

3.303 $\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f-cot(
f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3751, 486, 12, 385, 209}

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{-a+b}{(1+x^2)\sqrt{a+bx^2}}\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{}}\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{1-(-a+b)}\right.}{}$$

$$= -\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.18, size = 64, normalized size = 0.85

$$-\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{(a-b)\tan^2(e+fx)}{a+b\tan^2(e+fx)}\right)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]

[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -(((a - b)*Tan[e + f*x]^2)/(a + b*Tan[e + f*x]^2))]*Sqrt[a + b*Tan[e + f*x]^2])/f)

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.76, size = 2233, normalized size = 29.77

method	result	size
default	Expression too large to display	2233

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)*(EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*cos(f*x+e)*sin(f*x+e)*a-EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*cos(f*x+e)*sin(f*x+e)*b-2*a*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)+2*b*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(

```
f*x+e)*cos(f*x+e)+2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x
+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/
(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2
)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*sin(f*x+e)-2^(1/2)*((I*cos(f*x+e)*b^(1/2
)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e
)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-c
os(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1
)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^
(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*b*sin(f*x+e)-2
*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e
)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)
^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)
^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/
sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-
a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*x+e)+2*2
^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*
a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(
1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(
1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/si
n(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+
2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b*sin(f*x+e)+((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a-((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
*b)/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/sin(f*x+e)/((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Fricas [A]
time = 3.76, size = 273, normalized size = 3.64

$$\left[\frac{\sqrt{-a+b}\log\left(-\frac{(a^2-8ab+8b^2)\tan(fx+e)^4-2(3a^2-4ab)\tan(fx+e)^2+a^2-4\left((a-2b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)\tan(fx+e)-4\sqrt{b\tan(fx+e)^2+a}}{4f\tan(fx+e)}, -\frac{\sqrt{a-b}\arctan\left(-\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}\tan(fx+e)}{(a-2b)\tan(fx+e)^2-a}\right)\tan(fx+e)+2\sqrt{b\tan(fx+e)^2+a}}{2f\tan(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^
2 + 1))*tan(f*x + e) - 4*sqrt(b*tan(f*x + e)^2 + a))/(f*tan(f*x + e)), -1/2
*(sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)
/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e) + 2*sqrt(b*tan(f*x + e)^2 + a
))/(f*tan(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^2{(e + fx)}}\cot^2{(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}{\left(e+f\,x\right)}^2\,\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.304 $\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}-\frac{\cot^3(e+fx)\sqrt{a-}}{3}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+1/3*(
3*a-b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/3*cot(f*x+e)^3*(a+b*tan(f*
x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 486, 597, 12, 385, 209}

$$\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/f + ((3*a - b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f) - (Cot[e +
 f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{-3a+b-2bx^2}{x^2(1+x^2)\sqrt{a+b}} \cdots\right.}{3\cdots}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+\cdots}}{3\cdots}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+\cdots}}{3\cdots}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+\cdots}}{3\cdots}$$

$$= \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-b)\cot(e+f\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.36, size = 241, normalized size = 2.06

$$-\frac{\cos^2(e+fx)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(\frac{\sec^2(e+fx)\left(\text{ArcSin}\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\right)\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}+\sqrt{\cos^2(e+fx)+\frac{b\sin^2(e+fx)}{a}}\right)(a-2b\tan^2(e+fx))}{\sqrt{\cos^2(e+fx)+\frac{b\sin^2(e+fx)}{a}}\,(a+b\tan^2(e+fx))}-\frac{4(a-b)\,{}_2F_1\left(2,2;\frac{3}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\sin^2(e+fx)(a+b\tan^2(e+fx))}{a^2}\right)}{3f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/3*(Cos[e + f*x]^2*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2]*(1 + (b*Tan[
e + f*x]^2)/a)*((Sec[e + f*x]^2*(ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*S
qrt[((a - b)*Sin[e + f*x]^2)/a] + Sqrt[Cos[e + f*x]^2 + (b*Sin[e + f*x]^2)/
a])*(a - 2*b*Tan[e + f*x]^2))/(Sqrt[Cos[e + f*x]^2 + (b*Sin[e + f*x]^2)/a]*
(a + b*Tan[e + f*x]^2)) - (4*(a - b)*Hypergeometric2F1[2, 2, 3/2, ((a - b)*
Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2))/f
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 4518, normalized size = 38.62

method	result	size
default	Expression too large to display	4518

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3/f*(3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^2-3*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b-6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^2+6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a*b+3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2-3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b-6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b

```
^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*Ell
ipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e)
,-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^
(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^
2+6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*
x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a
-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)
/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/
2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/
2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*co
s(f*x+e)^2*a*b-3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)
^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(
cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)
*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2+3*sin(f*x+e)*cos(
f*x+e)*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)
*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)
+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(
1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a
*b+8*b^2)/a^2)^(1/2))*a*b+6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*
(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(
f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(
a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,
(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2-6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)
^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)
^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Fricas [A]
time = 3.76, size = 329, normalized size = 2.81

$$\left[\frac{3\,a\sqrt{-a+b}\,\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2+4\,\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^3+4\,\left((3\,a-b)\tan\left(fx+e\right)^2-a\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{12\,af\tan\left(fx+e\right)^3},\ \frac{3\,\sqrt{a-b}\,a\arctan\left(-\frac{2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}\,\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan\left(fx+e\right)^3+2\,\left((3\,a-b)\tan\left(fx+e\right)^2-a\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{6\,af\tan\left(fx+e\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a
^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x
+ e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x
+ e)^2 + 1))*tan(f*x + e)^3 + 4*((3*a - b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f
*x + e)^2 + a))/(a*f*tan(f*x + e)^3), 1/6*(3*sqrt(a - b)*a*arctan(-2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a
))*tan(f*x + e)^3 + 2*((3*a - b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f*x + e)^2
+ a))/(a*f*tan(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^2{(e+fx)}}\,\cot^4{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}(e+f\,x)^4\,\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.305 $\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal. Leaf size=167

$$-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}+\frac{(5a-$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f-1/15
*(15*a^2-5*a*b-2*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f+1/15*(5*a-b
)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e
)^2)^(1/2)/f
```

Rubi [A]

time = 0.16, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 486, 597, 12, 385, 209}

$$-\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f) - ((15*a^2 - 5*a*b - 2*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/
(15*a^2*f) + ((5*a - b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a*f)
 - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{-5a+b-4bx^2}{x^4(1+x^2)\sqrt{a+b}}\right.}{5}$$

$$= \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} - \frac{\cot^5(e+fx)\sqrt{a}}{5}$$

$$= -\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} + \frac{(5a-}{}$$

$$= -\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} + \frac{(5a-}{}$$

$$= -\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} + \frac{(5a-}{}$$

$$= -\frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2-5ab-2b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 8.81, size = 321, normalized size = 1.92

$$-\frac{\cos^4(e+fx)\cot^5(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(-8(a-b)\,{}_2F_1\left(2,2;\frac{3}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\tan^2(e+fx)\left(2a-3b\tan^2(e+fx)\right)\left(a+b\tan^2(e+fx)\right)^2+8(a-b)\,{}_3F_2\left(2,2,2;1,\frac{3}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^3+\frac{a^2\sec^4(e+fx)\left(\text{ArcSin}\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\right)\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}+\sqrt{\cos^2(e+fx)+\frac{b\sin^2(e+fx)}{a}}\right)\left(3a^2-4ab\tan^2(e+fx)+8b^2\tan^4(e+fx)\right)}{\sqrt{\cos^2(e+fx)+\frac{b\sin^2(e+fx)}{a}}}\right)}{15a^3f\sqrt{a+b\tan^2(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/15*(Cos[e + f*x]^4*Cot[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*(-8*(a - b)
*Hypergeometric2F1[2, 2, 3/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2*(2
*a - 3*b*Tan[e + f*x]^2)*(a + b*Tan[e + f*x]^2)^2 + 8*(a - b)*Hypergeometri
cPFQ[{2, 2, 2}, {1, 3/2}, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2*(a + b
*Tan[e + f*x]^2)^3 + (a^2*Sec[e + f*x]^4*(ArcSin[Sqrt[((a - b)*Sin[e + f*x]
^2)/a]]*Sqrt[((a - b)*Sin[e + f*x]^2)/a] + Sqrt[Cos[e + f*x]^2 + (b*Sin[e +
```

```
 f*x]^2)/a])*(3*a^2 - 4*a*b*Tan[e + f*x]^2 + 8*b^2*Tan[e + f*x]^4))/Sqrt[Co
s[e + f*x]^2 + (b*Sin[e + f*x]^2)/a]))/(a^3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.41, size = 6894, normalized size = 41.28

method	result	size
default	Expression too large to display	6894

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Fricas [A]
time = 5.25, size = 395, normalized size = 2.37

$$\left[\frac{15\,a^{2}\sqrt{-a+b}\,\log\left(-\frac{(a^{2}-8\,ab+8\,b^{2})\tan(fx+e)^{4}-2\,(3\,a^{2}-4\,ab)\tan(fx+e)^{2}+a^{2}-4\,((a-2\,b)\tan(fx+e)^{3}-a\tan(fx+e))\sqrt{b\tan(fx+e)^{2}+a}\,\sqrt{-a+b}}{\tan(fx+e)^{4}+2\,\tan(fx+e)^{2}+1}\right)\tan(fx+e)^{5}-4\,\left((15\,a^{2}-5\,ab-2\,b^{2})\tan(fx+e)^{4}-(5\,a^{2}-ab)\tan(fx+e)^{2}+3\,a^{2}\right)\sqrt{b\tan(fx+e)^{2}+a}}{60\,a^{2}f\tan(fx+e)^{5}},\ -\frac{15\,\sqrt{a-b}\,a^{2}\arctan\left(-\frac{2\,\sqrt{b\tan(fx+e)^{2}+a}\,\sqrt{a-b}\,\tan(fx+e)}{(a-2\,b)\tan(fx+e)^{2}-a}\right)\tan(fx+e)^{5}+2\,\left((15\,a^{2}-5\,ab-2\,b^{2})\tan(fx+e)^{4}-(5\,a^{2}-ab)\tan(fx+e)^{2}+3\,a^{2}\right)\sqrt{b\tan(fx+e)^{2}+a}}{30\,a^{2}f\tan(fx+e)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*a^2*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(
3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f
*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f
*x + e)^2 + 1))*tan(f*x + e)^5 - 4*((15*a^2 - 5*a*b - 2*b^2)*tan(f*x + e)^4
 - (5*a^2 - a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f
*tan(f*x + e)^5), -1/30*(15*sqrt(a - b)*a^2*arctan(-2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)
^5 + 2*((15*a^2 - 5*a*b - 2*b^2)*tan(f*x + e)^4 - (5*a^2 - a*b)*tan(f*x + e
)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(f*x + e)^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^{2}\left(e+fx\right)}\,\cot^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^6\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.306 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=145

$$-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+$$

[Out] -(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a-b)*(a+b*tan(f*x+e)^2)^(1/2)/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/f-1/5*(a+b)*(a+b*tan(f*x+e)^2)^(5/2)/b^2/f+1/7*(a+b*tan(f*x+e)^2)^(7/2)/b^2/f

Rubi [A]
time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 457, 90, 52, 65, 214}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]

[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + ((a - b)*Sqrt[a + b*Tan[e + f*x]^2])/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*f) - ((a + b)*(a + b*Tan[e + f*x]^2)^(5/2))/(5*b^2*f) + (a + b*Tan[e + f*x]^2)^(7/2)/(7*b^2*f)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-b)(a+bx)^{3/2}}{b} + \frac{(a+bx)^{3/2}}{1+x} + \frac{(a+bx)^{5/2}}{b}\right)\,dx, x, \tan^2(e+\right.}{2f}$$

$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f} + \frac{\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2f} + $$

$$= \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} - \frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f} + $$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} - $$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} - $$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(a-b)\sqrt{a+}}{}$$

Mathematica [A]
time = 0.92, size = 139, normalized size = 0.96

$$\frac{\frac{2}{3}\left(a+b\tan^2(e+fx)\right)^{3/2} - \frac{2(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2} + \frac{2\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2} + 2(a-b)\left(-\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) + \sqrt{a+b\tan^2(e+fx)}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
[Out] ((2*(a + b*Tan[e + f*x]^2)^(3/2))/3 - (2*(a + b)*(a + b*Tan[e + f*x]^2)^(5/
2))/(5*b^2) + (2*(a + b*Tan[e + f*x]^2)^(7/2))/(7*b^2) + 2*(a - b)*(-(Sqrt[
a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]) + Sqrt[a + b*Tan[e
+ f*x]^2]))/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(125) = 250.
time = 0.06, size = 263, normalized size = 1.81

method	result
derivativedivides	$\frac{\frac{\left(\tan^2(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{35b^2}-\frac{\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{5b}+b^2\left(\frac{\left(\tan^2(fx+e)\right)\sqrt{a+b\left(\tan\right.}}{3b}\right.}{}$
default	$\frac{\frac{\left(\tan^2(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{35b^2}-\frac{\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{5b}+b^2\left(\frac{\left(\tan^2(fx+e)\right)\sqrt{a+b\left(\tan\right.}}{3b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/7*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(5/2)/b-2/35*a/b^2*(a+b*tan(f*x+e)
^2)^(5/2)-1/5*(a+b*tan(f*x+e)^2)^(5/2)/b+b^2*(1/3*tan(f*x+e)^2/b*(a+b*tan(f
*x+e)^2)^(1/2)-2/3*a/b^2*(a+b*tan(f*x+e)^2)^(1/2)-1/b*(a+b*tan(f*x+e)^2)^(1
/2)+1/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))+2*a*b*(1/
b*(a+b*tan(f*x+e)^2)^(1/2)-1/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(
-a+b)^(1/2)))+a^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

Fricas [A]
time = 4.51, size = 430, normalized size = 2.97

$$\left[-\frac{105\,(ab^2-b^3)\sqrt{a-b}\,\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)-4\,\left(15\,b^3\tan(fx+e)^6+3\,(8\,ab^2-7\,b^3)\tan(fx+e)^4-6\,a^3-21\,a^2b+140\,ab^2-105\,b^3+(3\,a^2b-42\,ab^2+35\,b^3)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{420\,b^2f},\ \frac{105\,(ab^2-b^3)\sqrt{-a+b}\,\arctan\left(\frac{2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a+b}}{b\tan(fx+e)^2+2\,a-b}\right)+2\,\left(15\,b^3\tan(fx+e)^6+3\,(8\,ab^2-7\,b^3)\tan(fx+e)^4-6\,a^3-21\,a^2b+140\,ab^2-105\,b^3+(3\,a^2b-42\,ab^2+35\,b^3)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{210\,b^2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/420*(105*(a*b^2 - b^3)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b
 - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e
)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e
```

```
)^2 + 1)) - 4*(15*b^3*tan(f*x + e)^6 + 3*(8*a*b^2 - 7*b^3)*tan(f*x + e)^4 -
 6*a^3 - 21*a^2*b + 140*a*b^2 - 105*b^3 + (3*a^2*b - 42*a*b^2 + 35*b^3)*tan
(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/210*(105*(a*b^2 - b^3)*
sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x +
e)^2 + 2*a - b)) + 2*(15*b^3*tan(f*x + e)^6 + 3*(8*a*b^2 - 7*b^3)*tan(f*x +
 e)^4 - 6*a^3 - 21*a^2*b + 140*a*b^2 - 105*b^3 + (3*a^2*b - 42*a*b^2 + 35*b
^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}} \tan^5\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**5, x)
```

Giac [A]
time = 0.63, size = 196, normalized size = 1.35

$$\frac{\left(a^2 - 2\,ab + b^2\right)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2 + a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f} + \frac{15\left(b\tan\left(fx+e\right)^2 + a\right)^{\frac{7}{2}} b^{12} f^6 - 21\left(b\tan\left(fx+e\right)^2 + a\right)^{\frac{5}{2}} a b^{12} f^6 - 21\left(b\tan\left(fx+e\right)^2 + a\right)^{\frac{5}{2}} b^{13} f^6 + 35\left(b\tan\left(fx+e\right)^2 + a\right)^{\frac{3}{2}} b^{14} f^6 + 105\sqrt{b\tan\left(fx+e\right)^2 + a}\, a b^{14} f^6 - 105\sqrt{b\tan\left(fx+e\right)^2 + a}\, b^{15} f^6}{105\,b^{14} f^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (a^2 - 2*a*b + b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-
a + b)*f) + 1/105*(15*(b*tan(f*x + e)^2 + a)^(7/2)*b^12*f^6 - 21*(b*tan(f*x
 + e)^2 + a)^(5/2)*a*b^12*f^6 - 21*(b*tan(f*x + e)^2 + a)^(5/2)*b^13*f^6 +
35*(b*tan(f*x + e)^2 + a)^(3/2)*b^14*f^6 + 105*sqrt(b*tan(f*x + e)^2 + a)*a
*b^14*f^6 - 105*sqrt(b*tan(f*x + e)^2 + a)*b^15*f^6)/(b^14*f^7)
```

Mupad [B]
time = 40.84, size = 233, normalized size = 1.61

$$\frac{\left(b\tan(e+f\,x)^2 + a\right)^{7/2}}{7\,b^2\,f} - \left(\frac{2\,a}{5\,b^2\,f} - \frac{a-b}{5\,b^2\,f}\right)\left(b\tan(e+f\,x)^2 + a\right)^{5/2} - \sqrt{b\tan\left(e+f\,x\right)^2 + a}\,\left(a-b\right)\left(\left(\frac{2\,a}{b^2\,f} - \frac{a-b}{b^2\,f}\right)\left(a-b\right) - \frac{a^2}{b^2\,f}\right) - \left(b\tan(e+f\,x)^2 + a\right)^{3/2}\left(\frac{\left(\frac{2\,a}{b^2\,f} - \frac{a-b}{b^2\,f}\right)\left(a-b\right)}{3} - \frac{a^2}{3\,b^2\,f}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^2 + a}\,(a-b)^{3/2}\,1\mathrm{i}}{a^2 - 2\,a\,b + b^2}\right)\left(a-b\right)^{3/2}\,1\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(7/2)/(7*b^2*f) - ((2*a)/(5*b^2*f) - (a - b)/(5*b^2*
f))*(a + b*tan(e + f*x)^2)^(5/2) - (a + b*tan(e + f*x)^2)^(1/2)*(a - b)*(((
2*a)/(b^2*f) - (a - b)/(b^2*f))*(a - b) - a^2/(b^2*f)) - (a + b*tan(e + f*x
)^2)^(3/2)*((((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a - b))/3 - a^2/(3*b^2*f))
+ (atan(((a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(3/2)*1i)/(a^2 - 2*a*b + b^2)
)*(a - b)^(3/2)*1i)/f
```

3.307 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}-\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+$$

[Out] (a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f-(a-b)*(a+b*tan(f*x+e)^2)^(1/2)/f-1/3*(a+b*tan(f*x+e)^2)^(3/2)/f+1/5*(a+b*tan(f*x+e)^2)^(5/2)/b/f

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = {3751, 457, 81, 52, 65, 214}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf}-\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]

[Out] ((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - ((a - b)*Sqrt[a + b*Tan[e + f*x]^2])/f - (a + b*Tan[e + f*x]^2)^(3/2)/(3*f) + (a + b*Tan[e + f*x]^2)^(5/2)/(5*b*f)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x^3(a+bx^2)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b\tan^2(e+fx))^{5/2}}{5bf} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}dx, x, \tan^2(e+f\right.}{2f}$$

$$= -\frac{(a+b\tan^2(e+fx))^{3/2}}{3f} + \frac{(a+b\tan^2(e+fx))^{5/2}}{5bf} - \frac{(a-b}{}$$

$$= -\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b\tan^2(e+fx))^{3/2}}{3f} + $$

$$= -\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b\tan^2(e+fx))^{3/2}}{3f} + $$

$$= \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{(a-b)\sqrt{a+}}{}$$

Mathematica [A]
time = 0.59, size = 112, normalized size = 0.97

$$\frac{15(a-b)^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a+b\tan^2(e+fx)}\,(3a^2-20ab+15b^2+(6a-5b)b\tan^2(e+fx)+3b^2\tan^4(e+fx))}{15bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (15*(a - b)^(3/2)*b*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[
a + b*Tan[e + f*x]^2]*(3*a^2 - 20*a*b + 15*b^2 + (6*a - 5*b)*b*Tan[e + f*x]
^2 + 3*b^2*Tan[e + f*x]^4))/(15*b*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(100) = 200.
time = 0.06, size = 218, normalized size = 1.88

method	result
derivativedivides	$\frac{\frac{\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{5b}-b^2\left(\frac{\left(\tan^2(fx+e)\right)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{3b}-\frac{2a\sqrt{a+b\left(\tan^2(fx+e)\right)}}{3b^2}-\sqrt{a-}\right.}{}$
default	$\frac{\frac{\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{5b}-b^2\left(\frac{\left(\tan^2(fx+e)\right)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{3b}-\frac{2a\sqrt{a+b\left(\tan^2(fx+e)\right)}}{3b^2}-\sqrt{a-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/5*(a+b*tan(f*x+e)^2)^(5/2)/b-b^2*(1/3*tan(f*x+e)^2/b*(a+b*tan(f*x+e)
^2)^(1/2)-2/3*a/b^2*(a+b*tan(f*x+e)^2)^(1/2)-1/b*(a+b*tan(f*x+e)^2)^(1/2)+1
/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))-2*a*b*(1/b*(a+
b*tan(f*x+e)^2)^(1/2)-1/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)
^(1/2)))-a^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

Fricas [A]
time = 4.36, size = 348, normalized size = 3.00

$$\left[-\frac{15\,(ab-b^2)\sqrt{a-b}\,\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2-4\,\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)-4\,\left(3\,b^2\tan\left(fx+e\right)^4+\left(6\,ab-5\,b^2\right)\tan\left(fx+e\right)^2+3\,a^2-20\,ab+15\,b^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{60\,bf},\ -\frac{15\,(ab-b^2)\sqrt{-a+b}\,\arctan\left(\frac{2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{b\tan(fx+e)^2+2\,a-b}\right)-2\,\left(3\,b^2\tan\left(fx+e\right)^4+\left(6\,ab-5\,b^2\right)\tan\left(fx+e\right)^2+3\,a^2-20\,ab+15\,b^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{30\,bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b - b^2)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*
b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2
+ a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2
+ 1)) - 4*(3*b^2*tan(f*x + e)^4 + (6*a*b - 5*b^2)*tan(f*x + e)^2 + 3*a^2 -
```

```
20*a*b + 15*b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/30*(15*(a*b - b^2)*s
qrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e
)^2 + 2*a - b)) - 2*(3*b^2*tan(f*x + e)^4 + (6*a*b - 5*b^2)*tan(f*x + e)^2
+ 3*a^2 - 20*a*b + 15*b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**3, x)
```

Giac [A]
time = 0.57, size = 150, normalized size = 1.29

$$-\frac{\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{3\,\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}b^4f^4-5\,\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^5f^4-15\,\sqrt{b\tan\left(fx+e\right)^2+a}\,ab^5f^4+15\,\sqrt{b\tan\left(fx+e\right)^2+a}\,b^6f^4}{15\,b^5f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(a^2 - 2*a*b + b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(
-a + b)*f) + 1/15*(3*(b*tan(f*x + e)^2 + a)^(5/2)*b^4*f^4 - 5*(b*tan(f*x +
e)^2 + a)^(3/2)*b^5*f^4 - 15*sqrt(b*tan(f*x + e)^2 + a)*a*b^5*f^4 + 15*sqrt
(b*tan(f*x + e)^2 + a)*b^6*f^4)/(b^5*f^5)
```

Mupad [B]
time = 22.57, size = 156, normalized size = 1.34

$$\frac{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{5/2}}{5\,b\,f}-\left(\frac{a}{3\,b\,f}-\frac{a-b}{3\,b\,f}\right)\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2}-\left(\frac{a}{b\,f}-\frac{a-b}{b\,f}\right)\,\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}\,\left(a-b\right)-\frac{\mathrm{atan}\left(\frac{\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}\,{\left(a-b\right)}^{3/2}\,1{}\mathrm{i}}{a^2-2\,a\,b+b^2}\right)\,{\left(a-b\right)}^{3/2}\,1{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(5/2)/(5*b*f) - (a/(3*b*f) - (a - b)/(3*b*f))*(a + b
*tan(e + f*x)^2)^(3/2) - (a/(b*f) - (a - b)/(b*f))*(a + b*tan(e + f*x)^2)^(
1/2)*(a - b) - (atan(((a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(3/2)*1i)/(a^2 -
 2*a*b + b^2))*(a - b)^(3/2)*1i)/f
```

3.308 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=90

$$-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a-b)*(a+b*tan
(f*x+e)^2)^(1/2)/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3751, 455, 52, 65, 214}

$$\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
- b)*Sqrt[a + b*Tan[e + f*x]^2])/f + (a + b*Tan[e + f*x]^2)^(3/2)/(3*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}dx, x, \tan\right.}{2f}$$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-}{}$$

$$= \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-}{}$$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(a-b)\sqrt{a+}}{}$$

Mathematica [A]

time = 0.28, size = 80, normalized size = 0.89

$$\frac{-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a+b\tan^2(e+fx)}\,(4a-3b+b\tan^2(e+fx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a
 + b*Tan[e + f*x]^2]*(4*a - 3*b + b*Tan[e + f*x]^2))/(3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(78) = 156$.
time = 0.05, size = 197, normalized size = 2.19

method	result
derivativedivides	$\frac{b^2\left(\frac{(\tan^2(fx+e))\sqrt{a+b(\tan^2(fx+e))}}{3b}-\frac{2a\sqrt{a+b(\tan^2(fx+e))}}{3b^2}-\frac{\sqrt{a+b(\tan^2(fx+e}}{b}\right.}{}$
default	$\frac{b^2\left(\frac{(\tan^2(fx+e))\sqrt{a+b(\tan^2(fx+e))}}{3b}-\frac{2a\sqrt{a+b(\tan^2(fx+e))}}{3b^2}-\frac{\sqrt{a+b(\tan^2(fx+e}}{b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/3*tan(f*x+e)^2/b*(a+b*tan(f*x+e)^2)^(1/2)-2/3*a/b^2*(a+b*tan(f*
x+e)^2)^(1/2)-1/b*(a+b*tan(f*x+e)^2)^(1/2)+1/(-a+b)^(1/2)*arctan((a+b*tan(f
*x+e)^2)^(1/2)/(-a+b)^(1/2)))+2*a*b*(1/b*(a+b*tan(f*x+e)^2)^(1/2)-1/(-a+b)^
(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))+a^2/(-a+b)^(1/2)*arcta
n((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

Fricas [A]
time = 1.94, size = 267, normalized size = 2.97

$$\left[-\frac{3\,(a-b)^{\frac{3}{2}}\log\left(-\frac{b^{2}\tan(fx+e)^{4}+2\,(4\,ab-3\,b^{2})\tan(fx+e)^{2}+4\,\left(b\tan(fx+e)^{2}+2\,a-b\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{a-b}+8\,a^{2}-8\,ab+b^{2}}{\tan(fx+e)^{4}+2\,\tan(fx+e)^{2}+1}\right)-4\,\left(b\tan\left(fx+e\right)^{2}+4\,a-3\,b\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}}{12\,f},\ \frac{3\,(a-b)\sqrt{-a+b}\,\arctan\left(\frac{2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}}{b\tan(fx+e)^{2}+2\,a-b}\right)+2\,\left(b\tan\left(fx+e\right)^{2}+4\,a-3\,b\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}}{6\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a - b)^(3/2)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*
x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
 - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(
b*tan(f*x + e)^2 + 4*a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/f, 1/6*(3*(a - b)
*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x +
 e)^2 + 2*a - b)) + 2*(b*tan(f*x + e)^2 + 4*a - 3*b)*sqrt(b*tan(f*x + e)^2
+ a))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^{2}\left(e + fx\right)\right)^{\frac{3}{2}} \tan\left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

Giac [A]
time = 0.52, size = 114, normalized size = 1.27

$$\frac{(a^{2}-2\,ab+b^{2})\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}f^{2}+3\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,af^{2}-3\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,bf^{2}}{3\,f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (a^2 - 2*a*b + b^2)*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-
a + b)*f) + 1/3*((b*tan(f*x + e)^2 + a)^(3/2)*f^2 + 3*sqrt(b*tan(f*x + e)^2
 + a)*a*f^2 - 3*sqrt(b*tan(f*x + e)^2 + a)*b*f^2)/f^3
```

Mupad [B]
time = 14.99, size = 91, normalized size = 1.01

$$\frac{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}}{3\,f}+\frac{\sqrt{b\,{\tan\left(e+f\,x\right)}^2+a}\,\left(a-b\right)}{f}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,{\tan\left(e+f\,x\right)}^2+a}\,{\left(a-b\right)}^{3/2}}{a^2-2\,a\,b+b^2}\right)\,{\left(a-b\right)}^{3/2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(3/2)/(3*f) + ((a + b*tan(e + f*x)^2)^(1/2)*(a - b))
/f - (atanh(((a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(3/2))/(a^2 - 2*a*b + b^2
))*(a - b)^(3/2))/f
```

3.309 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{b\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] -a^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f+(a-b)^(3/2)*arctanh((a
+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+b*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]

time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 457, 86, 162, 65, 214}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/f) + ((a - b)^(3/2)
*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f + (b*Sqrt[a + b*Tan[e +
 f*x]^2])/f
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{b\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{a^2+(2a-b)bx}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(\right.}{2f}$$

$$= \frac{b\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e\right.}{2f}$$

$$= \frac{b\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{a^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan}\right.}{bf}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.17, size = 90, normalized size = 0.95

$$\frac{-a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+b\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]) + (a - b)^(3/2)*Arc
Tanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + b*Sqrt[a + b*Tan[e + f*x]^2]
)/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1764 vs. $2(81) = 162$.
time = 0.30, size = 1765, normalized size = 18.58

method	result	size
default	Expression too large to display	1765

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(cos(f*x+e)-1)^3*(2*cos(f*x+e)*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f
*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x
+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+
e))*a^(9/2)-4*cos(f*x+e)*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*a^(7/2)*b
+2*cos(f*x+e)*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/
(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*a^(5/2)*b^2+2*cos(f*
x+e)*a^(5/2)*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)*b+2*b*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a
^(5/2)*(a-b)^(1/2)+3*cos(f*x+e)*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((
a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos
(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+
b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)*a^3*b-6*cos(f*x+e)*ln(-4*(cos(f*x+e)-1
)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x
+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)*a^2*b^2+3*cos(
f*x+e)*ln(-4*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)
^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*
(a-b)^(1/2)*a*b^3+cos(f*x+e)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*
x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/
sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)*a^4-3*cos(f*x+e)*ln(-2*(cos(f*x+e)-1)*(co
s(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)
-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)*a^3*b+6*cos(f*x+e)*
ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(
1/2)*a^2*b^2-3*cos(f*x+e)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin
(f*x+e)^2/a^(1/2))*(a-b)^(1/2)*a*b^3-cos(f*x+e)*ln(-4*(cos(f*x+e)*a^(1/2)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)
+b)/(cos(f*x+e)-1))*(a-b)^(1/2)*a^4)*cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+
e)^2*b+b)/cos(f*x+e)^2)^(3/2)*4^(1/2)/sin(f*x+e)^6/((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/a^(5/2)/(a-b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

Fricas [A]
time = 5.92, size = 619, normalized size = 6.52

$$\left[-\frac{(a-b)^{\frac{3}{2}}\log\left(-\frac{b^2\tan(fx+e)^4+2(4ab-3b^2)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)-2a^{\frac{3}{2}}\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{a}+2a}{\tan(fx+e)^2}\right)-4\sqrt{b\tan(fx+e)^2+a}\,b}{4f}, \frac{4\sqrt{-a}\,a\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}\sqrt{-a}}{a}\right)-(a-b)^{\frac{3}{2}}\log\left(-\frac{b^2\tan(fx+e)^4+2(4ab-3b^2)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)+4\sqrt{b\tan(fx+e)^2+a}\,b}{4f}, \frac{(-a+b)^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{b\tan(fx+e)^2+2a-b}\right)+a^{\frac{3}{2}}\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{a}+2a}{\tan(fx+e)^2}\right)+2\sqrt{b\tan(fx+e)^2+a}\,b}{2f}, \frac{2\sqrt{-a}\,a\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}\sqrt{-a}}{a}\right)+(-a+b)^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{b\tan(fx+e)^2+2a-b}\right)+2\sqrt{b\tan(fx+e)^2+a}\,b}{2f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - b)^(3/2)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x +
 e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 2*a^(3
/2)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan
(f*x + e)^2) - 4*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/4*(4*sqrt(-a)*a*arctan(
sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) - (a - b)^(3/2)*log(-(b^2*tan(f*x +
e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sq
rt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4
 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/2*((-a + b
)^(3/2)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2
+ 2*a - b)) + a^(3/2)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/2*(2*
sqrt(-a)*a*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + (-a + b)^(3/2)*a
rctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b
)) + 2*sqrt(b*tan(f*x + e)^2 + a)*b)/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**B**]
time = 12.03, size = 546, normalized size = 5.75

$$\frac{b\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}{f}+\frac{\mathrm{atanh}\left(\frac{6\,a^3\,b^3\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7}-\frac{6\,a^2\,b^4\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7}+\frac{2\,a\,b^5\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7}\right)\,\sqrt{{\left(a-b\right)}^3}}{f}-\frac{\mathrm{atanh}\left(\frac{2\,b^6\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}-\frac{8\,a\,b^5\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}+\frac{12\,a^2\,b^4\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}-\frac{6\,a^3\,b^3\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}\right)\,\sqrt{a^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (b*(a + b*tan(e + f*x)^2)^(1/2))/f + (atanh((6*a^3*b^3*(a + b*tan(e + f*x)^
2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*
a^3*b^5 - 18*a^4*b^4 + 6*a^5*b^3) - (6*a^2*b^4*(a + b*tan(e + f*x)^2)^(1/2)
*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5
- 18*a^4*b^4 + 6*a^5*b^3) + (2*a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2
- 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18*a^4*b
^4 + 6*a^5*b^3))*((a - b)^3)^(1/2))/f - (atanh((2*b^6*(a + b*tan(e + f*x)^2
)^(1/2)*(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) - (8*
a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12
*a^4*b^4 - 6*a^5*b^3) + (12*a^2*b^4*(a + b*tan(e + f*x)^2)^(1/2)*(a^3)^(1/2
))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) - (6*a^3*b^3*(a + b*tan
(e + f*x)^2)^(1/2)*(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5
*b^3))*(a^3)^(1/2))/f
```

3.310 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^2(e+$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+1/2*(2*a-3*b)*
arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f-1/2*a*cot(f*x+e)^2*(a+b
*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 457, 100, 162, 65, 214}

$$\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*f) - (
(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f - (a*Cot[e
 + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^3(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= -\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(2a-3b)+\frac{1}{2}}{x(1+x)\sqrt{a}}\right.}{}$$
$$= -\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{(a(2a-3b))\text{Subst}\left(\right.}{}$$
$$= -\frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f} - \frac{(a(2a-3b))\text{Subst}\left(\right.}{}$$
$$= \frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{(a-b)^{3/2}}{}$$

Mathematica [A]
time = 0.23, size = 109, normalized size = 0.94

$$\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right) - 2(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) - a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] - 2*(a - b
)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] - a*Cot[e + f*x]^2*
Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. 2(98) = 196.
time = 0.41, size = 2011, normalized size = 17.34

method	result	size
default	Expression too large to display	2011

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)

```
[Out] 1/8/f*4^(1/2)*(cos(f*x+e)-1)^2*(2*cos(f*x+e)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+
e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(
1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^(3/2)*(a-b)^(1/2)-2*cos(f*x+e)*ln(-
4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+co
s(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*a^(3/2)*(a-b)^(1/2)-3*cos(f*x+e)
*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*a^(1/2
)*(a-b)^(1/2)*b-2*a^(3/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(
f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e
)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin
(f*x+e)^2/a^(1/2))*(a-b)^(1/2)+3*cos(f*x+e)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*c
os(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/
(cos(f*x+e)-1))*a^(1/2)*(a-b)^(1/2)*b+2*a^(3/2)*ln(-4*(cos(f*x+e)*a^(1/2)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)
+b)/(cos(f*x+e)-1))*(a-b)^(1/2)+3*a^(1/2)*b*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e
)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)-3*a^(1/2)*ln(-4*(cos(f*x+e
)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*co
s(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)-1))*b*(a-b)^(1/2)-2*cos(f*x+e)*((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*(a-b)^(1/2)*a+4*cos(f*x+e)*ln(4*(
a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^
(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*a^2-8*cos(f*x+e)*ln(4*(a-b)^(1/2)*cos(f*
x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/
2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*
a-4*b*cos(f*x+e))*a*b+4*cos(f*x+e)*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+
e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)
^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))
*b^2-4*a^2*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(
cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(co
s(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))+8*ln(4*(a-b)^(1/2)*cos(
f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(
1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e
```

```
)*a-4*b*cos(f*x+e))*a*b-4*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(
f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*
x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*b^2)*cos
(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)^6/(a-b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

Fricas [A]
time = 1.81, size = 616, normalized size = 5.31

$$\left[-\frac{2 \, {\left(a - b\right)}^{\frac{3}{2}} \log\left(\frac{b \tan\left(f x + e\right)^{2} + 2 \, \sqrt{b \tan\left(f x + e\right)^{2} + a} \sqrt{a - b} + 2 \, a - b}{\tan\left(f x + e\right)^{2} + 1}\right) \tan\left(f x + e\right)^{2} + {\left(2 \, a - 3 \, b\right)} \sqrt{a} \log\left(\frac{b \tan\left(f x + e\right)^{2} - 2 \, \sqrt{b \tan\left(f x + e\right)^{2} + a} \sqrt{a} + 2 \, a}{\tan\left(f x + e\right)^{2}}\right) \tan\left(f x + e\right)^{2} + 2 \, \sqrt{b \tan\left(f x + e\right)^{2} + a} a}{4 \, f \tan\left(f x + e\right)^{2}}, -\frac{4 \, {\left(a - b\right)} \sqrt{-a + b} \arctan\left(-\frac{\sqrt{b \tan\left(f x + e\right)^{2} + a} \sqrt{-a + b}}{a - b}\right) \tan\left(f x + e\right)^{2} + {\left(2 \, a - 3 \, b\right)} \sqrt{a} \log\left(\frac{b \tan\left(f x + e\right)^{2} - 2 \, \sqrt{b \tan\left(f x + e\right)^{2} + a} \sqrt{a} + 2 \, a}{\tan\left(f x + e\right)^{2}}\right) \tan\left(f x + e\right)^{2} + 2 \, \sqrt{b \tan\left(f x + e\right)^{2} + a} a}{4 \, f \tan\left(f x + e\right)^{2}}, -\frac{\sqrt{-a} {\left(2 \, a - 3 \, b\right)} \arctan\left(\frac{\sqrt{b \tan\left(f x + e\right)^{2} + a} \sqrt{-a}}{a}\right) \tan\left(f x + e\right)^{2} + {\left(a - b\right)}^{\frac{3}{2}} \log\left(\frac{b \tan\left(f x + e\right)^{2} + 2 \, \sqrt{b \tan\left(f x + e\right)^{2} + a} \sqrt{a - b} + 2 \, a - b}{\tan\left(f x + e\right)^{2} + 1}\right) \tan\left(f x + e\right)^{2} + \sqrt{b \tan\left(f x + e\right)^{2} + a} a}{2 \, f \tan\left(f x + e\right)^{2}}, -\frac{\sqrt{-a} {\left(2 \, a - 3 \, b\right)} \arctan\left(\frac{\sqrt{b \tan\left(f x + e\right)^{2} + a} \sqrt{-a}}{a}\right) \tan\left(f x + e\right)^{2} + 2 \, {\left(a - b\right)} \sqrt{-a + b} \arctan\left(-\frac{\sqrt{b \tan\left(f x + e\right)^{2} + a} \sqrt{-a + b}}{a - b}\right) \tan\left(f x + e\right)^{2} + \sqrt{b \tan\left(f x + e\right)^{2} + a} a}{2 \, f \tan\left(f x + e\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a - b)^(3/2)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (2*a - 3*b)*
sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)
/tan(f*x + e)^2)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*
x + e)^2), -1/4*(4*(a - b)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*
sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + (2*a - 3*b)*sqrt(a)*log((b*tan(f*x +
 e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x
 + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)^2), -1/2*(sqrt(-a
)*(2*a - 3*b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2
+ (a - b)^(3/2)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
 - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)
^2 + a)*a)/(f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a - 3*b)*arctan(sqrt(b*tan
(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 + 2*(a - b)*sqrt(-a + b)*arctan
(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + sqrt(b*
tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}} \cot^{3}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**B**]
time = 12.03, size = 447, normalized size = 3.85

$$\frac{\operatorname{atanh}\left(\frac{3\,a^2\,b^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{2\left(-\frac{3\,a^4\,b^4}{2}+5\,a^3\,b^5-\frac{11\,a^2\,b^6}{2}+2\,a\,b^7\right)}-\frac{2\,a\,b^5\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{-\frac{3\,a^4\,b^4}{2}+5\,a^3\,b^5-\frac{11\,a^2\,b^6}{2}+2\,a\,b^7}\right)\,\sqrt{{\left(a-b\right)}^3}}{f}+\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{3\,\sqrt{a}\,b^7\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{-\frac{3\,a^4\,b^4}{2}+\frac{23\,a^3\,b^5}{4}-\frac{29\,a^2\,b^6}{4}+3\,a\,b^7}-\frac{29\,a^{3/2}\,b^6\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{4\left(-\frac{3\,a^4\,b^4}{2}+\frac{23\,a^3\,b^5}{4}-\frac{29\,a^2\,b^6}{4}+3\,a\,b^7\right)}+\frac{23\,a^{5/2}\,b^5\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{4\left(-\frac{3\,a^4\,b^4}{2}+\frac{23\,a^3\,b^5}{4}-\frac{29\,a^2\,b^6}{4}+3\,a\,b^7\right)}-\frac{3\,a^{7/2}\,b^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{2\left(-\frac{3\,a^4\,b^4}{2}+\frac{23\,a^3\,b^5}{4}-\frac{29\,a^2\,b^6}{4}+3\,a\,b^7\right)}\right)\,\left(2\,a-3\,b\right)}{2\,f}-\frac{a\,b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{2\,\left(f\,\left(b\tan\left(e+f\,x\right)^2+a\right)-a\,f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (atanh((3*a^2*b^4*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b
^3)^(1/2))/(2*(2*a*b^7 - (11*a^2*b^6)/2 + 5*a^3*b^5 - (3*a^4*b^4)/2)) - (2*
a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(
2*a*b^7 - (11*a^2*b^6)/2 + 5*a^3*b^5 - (3*a^4*b^4)/2))*((a - b)^3)^(1/2))/f
 + (a^(1/2)*atanh((3*a^(1/2)*b^7*(a + b*tan(e + f*x)^2)^(1/2))/(3*a*b^7 - (
29*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4*b^4)/2) - (29*a^(3/2)*b^6*(a + b*ta
n(e + f*x)^2)^(1/2))/(4*(3*a*b^7 - (29*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4
*b^4)/2)) + (23*a^(5/2)*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*(3*a*b^7 - (29
*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4*b^4)/2)) - (3*a^(7/2)*b^4*(a + b*tan(
e + f*x)^2)^(1/2))/(2*(3*a*b^7 - (29*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4*b
^4)/2)))*(2*a - 3*b))/(2*f) - (a*b*(a + b*tan(e + f*x)^2)^(1/2))/(2*(f*(a +
 b*tan(e + f*x)^2) - a*f))
```

3.311 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=161

$$-\frac{(8a^2-12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{(4a$$

[Out] (a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f-1/8*(8*a^2-12*a*b+3*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+1/8*(4*a-5*b)*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)/f-1/4*a*cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2)/f

Rubi [A]
time = 0.15, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3751, 457, 100, 156, 162, 65, 214}

$$-\frac{(8a^2-12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}-\frac{a\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}+\frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]

[Out] -1/8*((8*a^2 - 12*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(Sqrt[a]*f) + ((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f + ((4*a - 5*b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*f) - (a*Cot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^5(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{a\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4a-5b)+}{x^2(1+x)\sqrt{}}\right.}{}$$

$$= \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx}{}$$

$$= \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx}{}$$

$$= \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx}{}$$

$$= -\frac{(8a^2-12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}\,f} + \frac{(a}{}$$

Mathematica [A]
time = 0.94, size = 140, normalized size = 0.87

$$\frac{(-8a^2+12ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\cot^2(e+fx)\left(4a-5b-2a\cot^2(e+fx)\right)\sqrt{a+b\tan^2(e+fx)}\right)}{8\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-8*a^2 + 12*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sq
rt[a]*(8*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Co
t[e + f*x]^2*(4*a - 5*b - 2*a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2]))/
(8*Sqrt[a]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5223 vs. $2(139) = 278$.
time = 0.30, size = 5224, normalized size = 32.45

method	result	size
default	Expression too large to display	5224

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

Fricas [A]
time = 3.37, size = 784, normalized size = 4.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^2 - a*b)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
+ e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 - (
8*a^2 - 12*a*b + 3*b^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*((4*a^2 - 5*a*b
)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4),
 1/16*(16*(a^2 - a*b)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^2 - 12*a*b + 3*b^2)*sqrt(a)*log((b*t
an(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)
*tan(f*x + e)^4 + 2*((4*a^2 - 5*a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x
 + e)^2 + a))/(a*f*tan(f*x + e)^4), 1/8*((8*a^2 - 12*a*b + 3*b^2)*sqrt(-a)*
arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4 - 4*(a^2 - a*b
)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
 b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + ((4*a^2 - 5*a*b)*tan(
f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4), 1/8*(
(8*a^2 - 12*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
)/a)*tan(f*x + e)^4 + 8*(a^2 - a*b)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e
)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + ((4*a^2 - 5*a*b)*tan(f*x +
e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 12.25, size = 578, normalized size = 3.59

$$\frac{\sqrt{b\tan\left(e+fx\right)^2+a}\,\left(\frac{3ab^2}{8}-\frac{a^2b}{2}\right)+\frac{b\left(b\tan(e+fx)^2+a\right)^{3/2}(4a-5b)}{8}}{f\left(b\tan\left(e+fx\right)^2+a\right)^2+a^2f-2af\,\left(b\tan\left(e+fx\right)^2+a\right)}-\frac{\mathrm{atanh}\left(\frac{9b^6\sqrt{b\tan\left(e+fx\right)^2+a}\,\sqrt{a^3-3a^2b+3ab^2-b^3}}{32\left(\frac{a^3b^5}{4}-\frac{25a^2b^6}{32}+\frac{13ab^7}{16}-\frac{9b^8}{32}\right)}-\frac{ab^5\sqrt{b\tan\left(e+fx\right)^2+a}\,\sqrt{a^3-3a^2b+3ab^2-b^3}}{4\left(\frac{a^3b^5}{4}-\frac{25a^2b^6}{32}+\frac{13ab^7}{16}-\frac{9b^8}{32}\right)}\right)\sqrt{(a-b)^3}}{f}-\frac{\mathrm{atanh}\left(\frac{75\sqrt{a}\,b^7\sqrt{b\tan\left(e+fx\right)^2+a}}{64\left(\frac{75ab^7}{64}-\frac{159b^8}{256}-\frac{29a^2b^6}{32}+\frac{a^3b^5}{4}+\frac{27b^9}{256a}\right)}-\frac{159b^8\sqrt{b\tan\left(e+fx\right)^2+a}}{256\sqrt{a}\,\left(\frac{75ab^7}{64}-\frac{159b^8}{256}-\frac{29a^2b^6}{32}+\frac{a^3b^5}{4}+\frac{27b^9}{256a}\right)}-\frac{29a^{3/2}b^6\sqrt{b\tan\left(e+fx\right)^2+a}}{32\left(\frac{75ab^7}{64}-\frac{159b^8}{256}-\frac{29a^2b^6}{32}+\frac{a^3b^5}{4}+\frac{27b^9}{256a}\right)}+\frac{a^{5/2}b^5\sqrt{b\tan\left(e+fx\right)^2+a}}{4\left(\frac{75ab^7}{64}-\frac{159b^8}{256}-\frac{29a^2b^6}{32}+\frac{a^3b^5}{4}+\frac{27b^9}{256a}\right)}+\frac{27b^9\sqrt{b\tan\left(e+fx\right)^2+a}}{256a^{3/2}\left(\frac{75ab^7}{64}-\frac{159b^8}{256}-\frac{29a^2b^6}{32}+\frac{a^3b^5}{4}+\frac{27b^9}{256a}\right)}\right)\left(8a^2-12ab+3b^2\right)}{8\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] ((a + b*tan(e + f*x)^2)^(1/2)*((3*a*b^2)/8 - (a^2*b)/2) + (b*(a + b*tan(e +
 f*x)^2)^(3/2)*(4*a - 5*b))/8)/(f*(a + b*tan(e + f*x)^2)^2 + a^2*f - 2*a*f*
(a + b*tan(e + f*x)^2)) - (atanh((9*b^6*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b
^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(32*((13*a*b^7)/16 - (9*b^8)/32 - (25*a^2*
b^6)/32 + (a^3*b^5)/4)) - (a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2 - 3*
a^2*b + a^3 - b^3)^(1/2))/(4*((13*a*b^7)/16 - (9*b^8)/32 - (25*a^2*b^6)/32
+ (a^3*b^5)/4)))*((a - b)^3)^(1/2))/f - (atanh((75*a^(1/2)*b^7*(a + b*tan(e
 + f*x)^2)^(1/2))/(64*((75*a*b^7)/64 - (159*b^8)/256 - (29*a^2*b^6)/32 + (a
^3*b^5)/4 + (27*b^9)/(256*a))) - (159*b^8*(a + b*tan(e + f*x)^2)^(1/2))/(25
6*a^(1/2)*((75*a*b^7)/64 - (159*b^8)/256 - (29*a^2*b^6)/32 + (a^3*b^5)/4 +
(27*b^9)/(256*a))) - (29*a^(3/2)*b^6*(a + b*tan(e + f*x)^2)^(1/2))/(32*((75
*a*b^7)/64 - (159*b^8)/256 - (29*a^2*b^6)/32 + (a^3*b^5)/4 + (27*b^9)/(256*
a))) + (a^(5/2)*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*((75*a*b^7)/64 - (159*
b^8)/256 - (29*a^2*b^6)/32 + (a^3*b^5)/4 + (27*b^9)/(256*a))) + (27*b^9*(a
+ b*tan(e + f*x)^2)^(1/2))/(256*a^(3/2)*((75*a*b^7)/64 - (159*b^8)/256 - (2
9*a^2*b^6)/32 + (a^3*b^5)/4 + (27*b^9)/(256*a))))*(8*a^2 - 12*a*b + 3*b^2))
/(8*a^(1/2)*f)
```

3.312 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=294

$$-\frac{(a-b)^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a^4+8a^3b+48a^2b^2-192ab^3+128b^4)\tanh^{-1}\left(\frac{\sqrt{b}\ \text{ta}}{\sqrt{a+b\,\text{ta}}}\right.}{128b^{5/2}f}$$

```
[Out] -(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/12
8*(3*a^4+8*a^3*b+48*a^2*b^2-192*a*b^3+128*b^4)*arctanh(b^(1/2)*tan(f*x+e)/(
a+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/128*(3*a^3+8*a^2*b-80*a*b^2+64*b^3)*(a
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-56*a*b+48*b^2)*(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/48*(9*a-8*b)*(a+b*tan(f*x+e)^2)^(1/2)*
tan(f*x+e)^5/f+1/8*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.30, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 488, 596, 537, 223, 212, 385, 209}

$$\frac{(3a^2-56ab+48b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{192bf}-\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f}+\frac{(3a^4+8a^3b+48a^2b^2-192ab^3+128b^4)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{128b^{5/2}f}-\frac{(a-b)^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{(9a-8b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{48f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + ((3*a^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*ArcTanh[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) - ((3*a^3
+ 8*a^2*b - 80*a*b^2 + 64*b^3)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(12
8*b^2*f) + ((3*a^2 - 56*a*b + 48*b^2)*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x
]^2])/(192*b*f) + ((9*a - 8*b)*Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(
48*f) + (b*Tan[e + f*x]^7*Sqrt[a + b*Tan[e + f*x]^2])/(8*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{\text{Subst}\left(\int \frac{x^6(a(8a-7b)+(9a}{(1+x^2)\sqrt{a+}}\right.}{}$$

$$= \frac{(9a-8b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)}{}$$

$$= \frac{(3a^2-56ab+48b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{192bf} + \frac{(9a}{}$$

$$= -\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx}}{128b^2f}$$

$$= -\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx}}{128b^2f}$$

$$= -\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx}}{128b^2f}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^4+8a^3b+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 6.36, size = 908, normalized size = 3.09

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((b*(3*a^4 + 8*a^3*b - 16*a^2*b^2 - 64*a*b^3 + 64*b^4)*Sqrt[(a + b + (a -
 b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]
*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b
)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[
e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)]))) - (4*b*(-64*a^2*b^2 + 1
28*a*b^3 - 64*b^4)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(
e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((
a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a +
b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^
4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
- (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[
2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[
1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]))))/Sqrt[a + b
+ (a - b)*Cos[2*(e + f*x)]])/(64*b^2*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)]
 - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sec[e + f*x]^5*(9*a*Sin[e
+ f*x] - 26*b*Sin[e + f*x]))/48 + (Sec[e + f*x]^3*(3*a^2*Sin[e + f*x] - 128
*a*b*Sin[e + f*x] + 184*b^2*Sin[e + f*x]))/(192*b) + (Sec[e + f*x]*(-9*a^3*
Sin[e + f*x] - 30*a^2*b*Sin[e + f*x] + 424*a*b^2*Sin[e + f*x] - 400*b^3*Sin
[e + f*x]))/(384*b^2) + (b*Sec[e + f*x]^6*Tan[e + f*x])/8))/f
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(264) = 528$.
time = 0.06, size = 667, normalized size = 2.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/8*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(5/2)/b-3/8*a/b*(1/6*tan(f*x+e)*(a
+b*tan(f*x+e)^2)^(5/2)/b-1/6*a/b*(1/4*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3
/4*a*(1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(
f*x+e)+(a+b*tan(f*x+e)^2)^(1/2)))))-1/6*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(5/2)
/b+1/6*a/b*(1/4*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/4*a*(1/2*(a+b*tan(f*x
+e)^2)^(1/2)*tan(f*x+e)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)
^2)^(1/2))))+1/4*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/4*a*(1/2*(a+b*tan(f*
x+e)^2)^(1/2)*tan(f*x+e)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e
)^2)^(1/2)))-b^2*(1/2*tan(f*x+e)/b*(a+b*tan(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*l
n(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-ln(b^(1/2)*tan(f*x+e)+(a+b*t
an(f*x+e)^2)^(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b
^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))-2*a*b*(ln(b^(1/2)*tan
(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arcta
n(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))-a^2*(b^
4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)
```

```
^2)^(1/2)*tan(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

Fricas [A]
time = 7.25, size = 1101, normalized size = 3.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/768*(3*(3*a^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(b)*log(
2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)
 - 384*(a*b^3 - b^4)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) +
 2*(48*b^4*tan(f*x + e)^7 + 8*(9*a*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b
^2 - 56*a*b^3 + 48*b^4)*tan(f*x + e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3
+ 64*b^4)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/384*(3*(3*a
^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(-b)*arctan(sqrt(b*tan
(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + 192*(a*b^3 - b^4)*sqrt(-a + b
)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b
)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (48*b^4*tan(f*x + e)^7 + 8*(9*a
*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*tan(f*x +
e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*sqrt(b*tan
(f*x + e)^2 + a))/(b^3*f), -1/768*(768*(a*b^3 - b^4)*sqrt(a - b)*arctan(-sq
rt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - 3*(3*a^4 + 8*a^3*b +
 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(48*b^4*tan(f*x + e)^7
+ 8*(9*a*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*ta
n(f*x + e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*sq
rt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/384*(384*(a*b^3 - b^4)*sqrt(a - b)*ar
ctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(3*a^4 + 8
*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(-b)*arctan(sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (48*b^4*tan(f*x + e)^7 + 8*(9*a*b^3
 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*tan(f*x + e)^3
 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*sqrt(b*tan(f*x
 + e)^2 + a))/(b^3*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{tan}\left(e+f\,x\right)}^6\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.313 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=224

$$\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(a^3+6a^2b-24ab^2+16b^3)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}-$$

```
[Out] (a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-1/16*
(a^3+6*a^2*b-24*a*b^2+16*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)
^(1/2))/b^(3/2)/f+1/16*(a^2-10*a*b+8*b^2)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+
e)/b/f+1/24*(7*a-6*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f+1/6*b*(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.24, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 488, 596, 537, 223, 212, 385, 209}

$$\frac{(a^2-10ab+8b^2)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf}-\frac{(a^3+6a^2b-24ab^2+16b^3)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f}+\frac{(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f - ((a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b*Tan[e + f*x]^2]])/(16*b^(3/2)*f) + ((a^2 - 10*a*b + 8*b^2)*Tan[e
 + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(16*b*f) + ((7*a - 6*b)*Tan[e + f*x]^3*
Sqrt[a + b*Tan[e + f*x]^2])/(24*f) + (b*Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f
*x]^2])/(6*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{x^4(a(6a-5b)+(7a}{(1+x^2)\sqrt{a+}}\right.}{}$$

$$= \frac{(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)}{}$$

$$= \frac{(a^2-10ab+8b^2)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} + \frac{(7a-6}{}$$

$$= \frac{(a^2-10ab+8b^2)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} + \frac{(7a-6}{}$$

$$= \frac{(a^2-10ab+8b^2)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} + \frac{(7a-6}{}$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^3+6a^2b-24a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 5.16, size = 442, normalized size = 1.97

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (((30*a^3 - 266*a^2*b + 200*a*b^2 + 104*b^3 + (45*a^3 - 433*a^2*b + 296*a*b
^2 - 84*b^3)*Cos[2*(e + f*x)] + 2*(9*a^3 - 107*a^2*b + 92*a*b^2 + 12*b^3)*C
os[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)] - 47*a^2*b*Cos[6*(e + f*x)] + 88*a
*b^2*Cos[6*(e + f*x)] - 44*b^3*Cos[6*(e + f*x)])*Csc[2*(e + f*x)]^4 - 3*Sqr
t[2]*a*(a^2 - 10*a*b + 8*b^2)*Cot[e + f*x]^2*Csc[e + f*x]^2*Sqrt[((a + b +
```

```
(a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b
+ (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 48*Sqrt[2]*a*
b*(-a + b)*Cot[e + f*x]^2*Csc[e + f*x]^2*Sqrt[((a + b + (a - b)*Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e + f*x]^2*Tan
[e + f*x]^3)/(48*Sqrt[2]*b*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e
+ f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(198) = 396$.

time = 0.06, size = 519, normalized size = 2.32

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}\,\tan(fx+e)}{2}+\frac{a}{\cdots}\right.}{\cdots}\right.}{6b}}{\cdots}$
default	$\frac{\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}\,\tan(fx+e)}{2}+\frac{a}{\cdots}\right.}{\cdots}\right.}{6b}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/6*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(5/2)/b-1/6*a/b*(1/4*tan(f*x+e)*(a+b
*tan(f*x+e)^2)^(3/2)+3/4*a*(1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)+1/2*a/b
^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))))-1/4*tan(f*x+e)*(a+
b*tan(f*x+e)^2)^(3/2)-3/4*a*(1/2*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)+1/2*a/
b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2)))+b^2*(1/2*tan(f*x+e
)/b*(a+b*tan(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f
*x+e)^2)^(1/2))-ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)+(b^
4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)
^2)^(1/2)*tan(f*x+e)))+2*a*b*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2
))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(
a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))+a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan
(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Fricas [**A**]
time = 7.47, size = 899, normalized size = 4.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/96*(3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2
 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 48*(a*b^2 - b^3
)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*(8*b^3*tan(f*x +
 e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^2*b - 10*a*b^2 + 8*b^3)*t
an(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/48*(3*(a^3 + 6*a^2*b -
24*a*b^2 + 16*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*t
an(f*x + e))) - 24*(a*b^2 - b^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) + (8*b^3*tan(f*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3
*(a^2*b - 10*a*b^2 + 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*
f), 1/96*(96*(a*b^2 - b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(
sqrt(a - b)*tan(f*x + e))) + 3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(b)*
log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e)
+ a) + 2*(8*b^3*tan(f*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^
2*b - 10*a*b^2 + 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f),
1/48*(48*(a*b^2 - b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt
(a - b)*tan(f*x + e))) + 3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(-b)*arc
tan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (8*b^3*tan(f*x
+ e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^2*b - 10*a*b^2 + 8*b^3)*
tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}}\tan^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e+f\,x)^4\,{\left(b\tan(e+f\,x)^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.314 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$-\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a^2-12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{b}\,f}+\frac{(5a-$$

```
[Out] -(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/8*
(3*a^2-12*a*b+8*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f
/b^(1/2)+1/8*(5*a-4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*b*(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.17, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 488, 596, 537, 223, 212, 385, 209}

$$\frac{(3a^2-12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{b}\,f}-\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}+\frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + ((3*a^2 - 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b*Tan[e + f*x]^2]])/(8*Sqrt[b]*f) + ((5*a - 4*b)*Tan[e + f*x]*Sqrt[a + b*T
an[e + f*x]^2])/(8*f) + (b*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2(a(4a-3b)+(5a}{(1+x^2)\sqrt{a+}}\right.}{}$$

$$= \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)}{}$$

$$= \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)}{}$$

$$= \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)}{}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^2-12ab+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 6.22, size = 771, normalized size = 4.48

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((b*(a^2 + 4*a*b - 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[
2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)
])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)
*Cos[2*(e + f*x)])) + (4*b*(4*a^2 - 8*a*b + 4*b^2)*Sqrt[1 + Cos[2*(e + f*x)
]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)
]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*
```

```
x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^
2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a +
 b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*
(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSi
n[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*
Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*
Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(4*f) + (Sqrt[
(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(
(Sec[e + f*x]*(5*a*Sin[e + f*x] - 6*b*Sin[e + f*x]))/8 + (b*Sec[e + f*x]^2*
Tan[e + f*x])/4))/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(150) = 300$.
time = 0.06, size = 409, normalized size = 2.38

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\tan(fx+e)}{2}+\frac{a\ln\left(\sqrt{b}\tan(fx+e)+\sqrt{a+b\left(\mathrm{ta}\right.}\right.}{2\sqrt{b}}\right.}{4}}{}$
default	$\frac{\frac{\tan(fx+e)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\tan(fx+e)}{2}+\frac{a\ln\left(\sqrt{b}\tan(fx+e)+\sqrt{a+b\left(\mathrm{ta}\right.}\right.}{2\sqrt{b}}\right.}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/4*a*(1/2*(a+b*tan(f*x+e)^2)^
(1/2)*tan(f*x+e)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/
2)))-b^2*(1/2*tan(f*x+e)/b*(a+b*tan(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2
)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e
)^2)^(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b)
)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))-2*a*b*(ln(b^(1/2)*tan(f*x+e)+
(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a
-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))-a^2*(b^4*(a-b))
^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Fricas [A]
time = 4.55, size = 742, normalized size = 4.31

$$\left[\frac{(3a^2-12ab+8b^2)\sqrt{b}\log\left(2b\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\sqrt{b}\tan(fx+e)+a\right)-8(ab-b^2)\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)+2\left(2b^2\tan(fx+e)^3+(5ab-4b^2)\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}}{16bf}, -\frac{(3a^2-12ab+8b^2)\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}\sqrt{-b}}{b\tan(fx+e)}\right)+4(ab-b^2)\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)-\left(2b^2\tan(fx+e)^3+(5ab-4b^2)\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}}{8bf}, -\frac{16(ab-b^2)\sqrt{a-b}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{a-b}\tan(fx+e)}\right)-(3a^2-12ab+8b^2)\sqrt{b}\log\left(2b\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\sqrt{b}\tan(fx+e)+a\right)-2\left(2b^2\tan(fx+e)^3+(5ab-4b^2)\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}}{16bf}, -\frac{8(ab-b^2)\sqrt{a-b}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{a-b}\tan(fx+e)}\right)+(3a^2-12ab+8b^2)\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}\sqrt{-b}}{b\tan(fx+e)}\right)-\left(2b^2\tan(fx+e)^3+(5ab-4b^2)\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}}{8bf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/16*((3*a^2 - 12*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*t
an(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 8*(a*b - b^2)*sqrt(-a + b)*l
og(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*t
an(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*(2*b^2*tan(f*x + e)^3 + (5*a*b -
 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/8*((3*a^2 - 12*
a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x
 + e))) + 4*(a*b - b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)
) - (2*b^2*tan(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x +
e)^2 + a))/(b*f), -1/16*(16*(a*b - b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x
+ e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a^2 - 12*a*b + 8*b^2)*sqrt(b)*
log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e)
+ a) - 2*(2*b^2*tan(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f
*x + e)^2 + a))/(b*f), -1/8*(8*(a*b - b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f
*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (3*a^2 - 12*a*b + 8*b^2)*sqrt(
-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (2*b^2*t
an(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(
b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}} \tan^2\left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^2\,{\left(b\tan(e+f\,x)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.315 $\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)}{}$$

```
[Out] (a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/2*(
3*a-2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2
*b*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3742, 427, 537, 223, 212, 385, 209}

$$\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{\text{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{(1+x^2)\sqrt{a+bx^2}}\,dx,x,\tan(e\right.}{2f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^2\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx,\right.}{f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}+\frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{}{\sqrt{}}\right.}{f}$$

$$=\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-2b)\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a+}}\right.}{2f}$$

Mathematica [A]
time = 0.43, size = 142, normalized size = 1.14

$$\frac{-2(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{b}\,\tan^2(e+fx)-\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{b}\,(-3a+2b)\log\left(-\sqrt{b}\,\tan(e+fx)+\sqrt{a+b\tan^2(e+fx)}\right)+b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - b)^(3/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[e + f*x]^2 - Tan[e + f*x]*S
qrt[a + b*Tan[e + f*x]^2])/Sqrt[a - b]] + Sqrt[b]*(-3*a + 2*b)*Log[-(Sqrt[b
]*Tan[e + f*x]) + Sqrt[a + b*Tan[e + f*x]^2]] + b*Tan[e + f*x]*Sqrt[a + b*T
an[e + f*x]^2])/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(107) = 214$.
time = 0.00, size = 327, normalized size = 2.62

method	result

derivativedivides	$\frac{b^2\left(\frac{\tan(fx+e)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\mathrm{ta}\right.}{}\right.}{}$
default	$\frac{b^2\left(\frac{\tan(fx+e)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\mathrm{ta}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*(1/2*tan(f*x+e)/b*(a+b*tan(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2
)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e
)^2)^(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b)
)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))+2*a*b*(ln(b^(1/2)*tan(f*x+e)+
(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a
-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))+a^2*(b^4*(a-b))
^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A]
time = 4.55, size = 567, normalized size = 4.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(a - b)*sqrt(-a + b)*log(-((a - 2*b)*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/
```

```
(tan(f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, -1/
2*((3*a - 2*b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f
*x + e))) - (-a + b)^(3/2)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - sqrt(b
*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/4*(4*(a - b)^(3/2)*arctan(-sqrt(b
*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*sqrt(b)*log(
2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)
 + 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/2*(2*(a - b)^(3/2)*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a - 2*b)*s
qrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt
(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^{2}\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^(3/2), x)
```

3.316 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=114

$$-\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

```
[Out] -(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+b^(3
/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-a*cot(f*x+e)*(a+
b*tan(f*x+e)^2)^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3751, 485, 537, 223, 212, 385, 209}

$$-\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^
2]])/f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/f - (a*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{-a(a-2b)+b}{(1+x^2)\sqrt{a+}}\right.}{}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)^2\text{Subst}\left(\int \frac{}{(1+}\right.}{}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a-b)^2\text{Subst}\left(\int \frac{}{1-(}\right.}{}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{b^{3/2}\tanh^{-1}\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 4.40, size = 256, normalized size = 2.25

$$-\frac{a\left((a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)+\sqrt{2}(a-2b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+\sqrt{2}(-a+b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\tan(e+fx)}{\sqrt{2}f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 + Sqrt[2]*(a - 2*b)
*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcS
in[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]
 + Sqrt[2]*(-a + b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2
)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)
])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[(a + b +
(a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.50, size = 3333, normalized size = 29.24

method	result	size

default	Expression too large to display	3333

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(
f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*
(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+
1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1
/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*
b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x+e)*a^2-2*sin(f*x+e)*cos(f*x+e)*2^(1
/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2
)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*
x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a
^2)^(1/2))*a*b-2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)
^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(
cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1
/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*
cos(f*x+e)*sin(f*x+e)*a^2+4*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*
(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(
f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(
a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,
(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2))*cos(f*x+e)*sin(f*x+e)*a*b-2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)
^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)
^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+
e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(
1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)*cos(f*x+e)+2*2^(1/2)*((I*cos(f*x+e)*b^
(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f
*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/
2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x
+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),1/(2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)*cos(f*x+e)+2^(1/2)*((I*cos(f
*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b
)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(
```

```
a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((
cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^
(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^
2*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(co
s(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a
+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*b*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+
e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-
cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)
-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)+4*2^(1/2)*((I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+
e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-
cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)
-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*b*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+
e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-
cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)
-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2))*b^2*sin(f*x+e)+2*2^(1/2)*((I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+
e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Fricas [A]
time = 4.44, size = 756, normalized size = 6.63

$$\left[\frac{2\,b^{\frac{3}{2}}\log\left(2\,b\tan\left(fx+e\right)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{b}\tan\left(fx+e\right)+a\right)\tan\left(fx+e\right)-\left(a-b\right)\sqrt{-a+b}\log\left(-\frac{\left(a^{2}-8\,ab+8\,b^{2}\right)\tan\left(fx+e\right)^{4}-2\,\left(3\,a^{2}-4\,ab\right)\tan\left(fx+e\right)^{2}+a^{2}+4\,\left(\left(a-2\,b\right)\tan\left(fx+e\right)^{3}-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)-4\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a}{4\,f\tan\left(fx+e\right)},\ -\frac{4\,\sqrt{-b}\,b\arctan\left(\frac{\sqrt{-b}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\right)\tan\left(fx+e\right)+\left(a-b\right)\sqrt{-a+b}\log\left(-\frac{\left(a^{2}-8\,ab+8\,b^{2}\right)\tan\left(fx+e\right)^{4}-2\,\left(3\,a^{2}-4\,ab\right)\tan\left(fx+e\right)^{2}+a^{2}+4\,\left(\left(a-2\,b\right)\tan\left(fx+e\right)^{3}-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+1}\right)\tan\left(fx+e\right)+4\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a}{4\,f\tan\left(fx+e\right)},\ -\frac{\left(a-b\right)^{\frac{3}{2}}\arctan\left(-\frac{2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{a-b}\tan\left(fx+e\right)}{\left(a-2\,b\right)\tan\left(fx+e\right)^{2}-a}\right)\tan\left(fx+e\right)-b^{\frac{3}{2}}\log\left(2\,b\tan\left(fx+e\right)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{b}\tan\left(fx+e\right)+a\right)\tan\left(fx+e\right)+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a}{2\,f\tan\left(fx+e\right)},\ -\frac{\left(a-b\right)^{\frac{3}{2}}\arctan\left(-\frac{2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{a-b}\tan\left(fx+e\right)}{\left(a-2\,b\right)\tan\left(fx+e\right)^{2}-a}\right)\tan\left(fx+e\right)+2\,\sqrt{-b}\,b\arctan\left(\frac{\sqrt{-b}\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a}}\right)\tan\left(fx+e\right)+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,a}{2\,f\tan\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b^(3/2)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
b)*tan(f*x + e) + a)*tan(f*x + e) - (a - b)*sqrt(-a + b)*log(-((a^2 - 8*a*b
 + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a -
 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
 b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))*tan(f*x + e) - 4*sqrt(b*tan(f
*x + e)^2 + a)*a)/(f*tan(f*x + e)), -1/4*(4*sqrt(-b)*b*arctan(sqrt(-b)*tan(
f*x + e)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e) + (a - b)*sqrt(-a + b)*lo
g(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2
 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))*tan(f*x + e)
+ 4*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)), -1/2*((a - b)^(3/2)*arc
tan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f
*x + e)^2 - a))*tan(f*x + e) - b^(3/2)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)*tan(f*x + e) + 2*sqrt(b*tan(f*x
 + e)^2 + a)*a)/(f*tan(f*x + e)), -1/2*((a - b)^(3/2)*arctan(-2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*ta
n(f*x + e) + 2*sqrt(-b)*b*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^
2 + a))*tan(f*x + e) + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^{\frac{3}{2}}\cot^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}\left(e+f\,x\right)^2\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.317 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=115

$$\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}-\frac{a\cot^3(e+fx)\sqrt{}}{}$$

[Out] (a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/3*(3*a-4*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f-1/3*a*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/f

Rubi [A]

time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 485, 597, 12, 385, 209}

$$\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}+\frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]

[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f + ((3*a - 4*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*f) - (a*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*f)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{-a(3a-4b)-(2a}{x^2(1+x^2)\sqrt{a}}\right.}{}$$

$$= \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{}}{}$$

$$= \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{}}{}$$

$$= \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{}}{}$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-4b)\cot(e+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.24, size = 78, normalized size = 0.68

$$-\frac{\cot(e+fx)\left(b+a\cot^2(e+fx)\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{(a-b)\tan^2(e+fx)}{a+b\tan^2(e+fx)}\right)\sqrt{a+b\tan^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*(Cot[e + f*x]*(b + a*Cot[e + f*x]^2)*Hypergeometric2F1[-3/2, 1, -1/2,
-(((a - b)*Tan[e + f*x]^2)/(a + b*Tan[e + f*x]^2))]*Sqrt[a + b*Tan[e + f*x]
^2])/f
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.37, size = 6591, normalized size = 57.31

method	result	size

default	Expression too large to display	6591

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Fricas [A]
time = 3.13, size = 326, normalized size = 2.83

$$\left[-\frac{3\,(a-b)\sqrt{-a+b}\,\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2-4\,\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^3-4\,\left((3\,a-4\,b)\tan\left(fx+e\right)^2-a\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{12\,f\tan\left(fx+e\right)^3},\ \frac{3\,(a-b)^{\frac{3}{2}}\arctan\left(-\frac{2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}\,\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan\left(fx+e\right)^3+2\,\left((3\,a-4\,b)\tan\left(fx+e\right)^2-a\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{6\,f\tan\left(fx+e\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a - b)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 -
 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*t
an(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*t
an(f*x + e)^2 + 1))*tan(f*x + e)^3 - 4*((3*a - 4*b)*tan(f*x + e)^2 - a)*sqr
t(b*tan(f*x + e)^2 + a))/(f*tan(f*x + e)^3), 1/6*(3*(a - b)^(3/2)*arctan(-2
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e
)^2 - a))*tan(f*x + e)^3 + 2*((3*a - 4*b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f*
x + e)^2 + a))/(f*tan(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^4\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.318 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$-\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}+\frac{(5a\cdots}{\cdots}$$

[Out] -(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-1/15*(15*a^2-20*a*b+3*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f+1/15*(5*a-6*b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/f-1/5*a*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2)/f

Rubi [A]
time = 0.17, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3751, 485, 597, 12, 385, 209}

$$-\frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}-\frac{(a-b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f}+\frac{(5a-6b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]

[Out] -(((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f) - ((15*a^2 - 20*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(15*a*f) + ((5*a - 6*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*f) - (a*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*f)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{-a(5a-6b)-(}{x^4(1+x^2)\sqrt{a}}\right.}{}$$

$$= \frac{(5a-6b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15f} - \frac{a\cot^5(e+fx}{}$$

$$= -\frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} + \frac{(5}{}$$

$$= -\frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} + \frac{(5}{}$$

$$= -\frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} + \frac{(5}{}$$

$$= -\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2-20ab}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.67, size = 140, normalized size = 0.85

$$-\frac{\cos(e+fx)\left(b+a\cot^2(e+fx)\right)^2\left(a(-2b+3a\cot^2(e+fx))\,{}_2F_1\left(1,1;-\frac{1}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)+2(a-b)(a+b+(a-b)\cos(2(e+fx)))\,{}_2F_1\left(2,2;\frac{1}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\right)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*(Cos[e + f*x]*(b + a*Cot[e + f*x]^2)^2*(a*(-2*b + 3*a*Cot[e + f*x]^2)
*Hypergeometric2F1[1, 1, -1/2, ((a - b)*Sin[e + f*x]^2)/a] + 2*(a - b)*(a +
 b + (a - b)*Cos[2*(e + f*x)])*Hypergeometric2F1[2, 2, 1/2, ((a - b)*Sin[e
+ f*x]^2)/a])*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a^3*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.43, size = 10026, normalized size = 60.76

method	result	size
default	Expression too large to display	10026

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Fricas [A]
time = 3.97, size = 405, normalized size = 2.45

$$\left[-\frac{15\,(a^2-ab)\sqrt{-a+b}\,\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2+4\,\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a+b}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)\tan(fx+e)^5+4\,\left((15\,a^2-20\,ab+3\,b^2)\tan(fx+e)^4-(5\,a^2-6\,ab)\tan(fx+e)^2+3\,a^2\right)\sqrt{b\tan(fx+e)^2+a}}{60\,af\tan(fx+e)^5},\ -\frac{15\,(a^2-ab)\sqrt{a-b}\,\arctan\left(-\frac{2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a-b}\,\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan(fx+e)^5+2\,\left((15\,a^2-20\,ab+3\,b^2)\tan(fx+e)^4-(5\,a^2-6\,ab)\tan(fx+e)^2+3\,a^2\right)\sqrt{b\tan(fx+e)^2+a}}{30\,af\tan(fx+e)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a^2 - a*b)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e
)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3
- a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4
+ 2*tan(f*x + e)^2 + 1))*tan(f*x + e)^5 + 4*((15*a^2 - 20*a*b + 3*b^2)*tan(
f*x + e)^4 - (5*a^2 - 6*a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2
+ a))/(a*f*tan(f*x + e)^5), -1/30*(15*(a^2 - a*b)*sqrt(a - b)*arctan(-2*sqr
t(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2
- a))*tan(f*x + e)^5 + 2*((15*a^2 - 20*a*b + 3*b^2)*tan(f*x + e)^4 - (5*a^2
 - 6*a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x
+ e)^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^6\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.319 $\int (a+b\tan^2(c+dx))^{5/2}\,dx$

Optimal. Leaf size=170

$$\frac{(a-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{d}+\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{8d}+\frac{(7\cdots}{}$$

[Out] (a-b)^(5/2)*arctan((a-b)^(1/2)*tan(d*x+c)/(a+b*tan(d*x+c)^2)^(1/2))/d+1/8*(15*a^2-20*a*b+8*b^2)*arctanh(b^(1/2)*tan(d*x+c)/(a+b*tan(d*x+c)^2)^(1/2))*b^(1/2)/d+1/8*(7*a-4*b)*b*(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*b*tan(d*x+c)*(a+b*tan(d*x+c)^2)^(3/2)/d

Rubi [A]
time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3742, 427, 542, 537, 223, 212, 385, 209}

$$\frac{\sqrt{b}\,(15a^2-20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{8d}+\frac{(a-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{d}+\frac{b\tan(c+dx)\,(a+b\tan^2(c+dx))^{3/2}}{4d}+\frac{b(7a-4b)\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[c + d*x]^2)^(5/2),x]

[Out] ((a - b)^(5/2)*ArcTan[(Sqrt[a - b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^2]])/d + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^2]])/(8*d) + ((7*a - 4*b)*b*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]^2])/(8*d) + (b*Tan[c + d*x]*(a + b*Tan[c + d*x]^2)^(3/2))/(4*d)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{5/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/2}}{4d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\,(a(4a-b)+(7a-4b)bx^2}{1+x^2}\right.}{4d}$$

$$= \frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b\tan^2(c+\right.}{4d}$$

$$= \frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b\tan^2(c+\right.}{4d}$$

$$= \frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b\tan^2(c+\right.}{4d}$$

$$= \frac{(a-b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{d} + \frac{\sqrt{b}\,(15a^2-20ab+8b^2)\tanh^{-}}{8d}$$

Mathematica [A]
time = 0.70, size = 169, normalized size = 0.99

$$\frac{-8(a-b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{b}\,\tan^2(c+dx)-\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{\sqrt{a-b}}\right)-\sqrt{b}\,(15a^2-20ab+8b^2)\log\left(-\sqrt{b}\,\tan(c+dx)+\sqrt{a+b\tan^2(c+dx)}\right)+b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}\,(9a-4b+2b\tan^2(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] (-8*(a - b)^(5/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[c + d*x]^2 - Tan[c + d*x]*S
qrt[a + b*Tan[c + d*x]^2])/Sqrt[a - b]] - Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)
*Log[-(Sqrt[b]*Tan[c + d*x]) + Sqrt[a + b*Tan[c + d*x]^2]] + b*Tan[c + d*x]
*Sqrt[a + b*Tan[c + d*x]^2]*(9*a - 4*b + 2*b*Tan[c + d*x]^2))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(148) = 296.
time = 0.13, size = 578, normalized size = 3.40

method	result

derivativedivides	$b^3\left(\frac{\left(\tan^3(dx+c)\right)\sqrt{a+b\left(\tan^2(dx+c)\right)}}{4b}-\frac{3a\left(\frac{\tan(dx+c)\sqrt{a+b\left(\tan^2(dx+c)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(dx+\right.}{}\right.}{4b}\right.$
default	$b^3\left(\frac{\left(\tan^3(dx+c)\right)\sqrt{a+b\left(\tan^2(dx+c)\right)}}{4b}-\frac{3a\left(\frac{\tan(dx+c)\sqrt{a+b\left(\tan^2(dx+c)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(dx+\right.}{}\right.}{4b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/4*tan(d*x+c)^3/b*(a+b*tan(d*x+c)^2)^(1/2)-3/4*a/b*(1/2*tan(d*x+
c)/b*(a+b*tan(d*x+c)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*tan(d*x+c)+(a+b*tan(
d*x+c)^2)^(1/2)))-1/2*tan(d*x+c)/b*(a+b*tan(d*x+c)^2)^(1/2)+1/2*a/b^(3/2)*l
n(b^(1/2)*tan(d*x+c)+(a+b*tan(d*x+c)^2)^(1/2))+ln(b^(1/2)*tan(d*x+c)+(a+b*t
an(d*x+c)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b
^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)))+3*a*b^2*(1/2*tan(d*x+
c)/b*(a+b*tan(d*x+c)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*tan(d*x+c)+(a+b*tan(
d*x+c)^2)^(1/2))-ln(b^(1/2)*tan(d*x+c)+(a+b*tan(d*x+c)^2)^(1/2))/b^(1/2)+(b
^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c
)^2)^(1/2)*tan(d*x+c)))+3*a^2*b*(ln(b^(1/2)*tan(d*x+c)+(a+b*tan(d*x+c)^2)^(
1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2
)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)))+a^3*(b^4*(a-b))^(1/2)/b^2/(a-b)*arc
tan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^2 + a)^(5/2), x)
```

Fricas [A]
time = 5.10, size = 703, normalized size = 4.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*((15*a^2 - 20*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(d*x + c)^2 + 2*sqrt(b*
tan(d*x + c)^2 + a)*sqrt(b)*tan(d*x + c) + a) + 8*(a^2 - 2*a*b + b^2)*sqrt(
-a + b)*log(-((a - 2*b)*tan(d*x + c)^2 + 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(
-a + b)*tan(d*x + c) - a)/(tan(d*x + c)^2 + 1)) + 2*(2*b^2*tan(d*x + c)^3 +
 (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*tan(d*x + c)^2 + a))/d, 1/16*(16*(a^2
 - 2*a*b + b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(d*x + c)^2 + a)/(sqrt(a - b)
*tan(d*x + c))) + (15*a^2 - 20*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(d*x + c)^2
+ 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(b)*tan(d*x + c) + a) + 2*(2*b^2*tan(d*x
 + c)^3 + (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*tan(d*x + c)^2 + a))/d, -1/8
*((15*a^2 - 20*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(d*x + c)^2 + a)*sqrt
(-b)/(b*tan(d*x + c))) - 4*(a^2 - 2*a*b + b^2)*sqrt(-a + b)*log(-((a - 2*b)
*tan(d*x + c)^2 + 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(-a + b)*tan(d*x + c) -
a)/(tan(d*x + c)^2 + 1)) - (2*b^2*tan(d*x + c)^3 + (9*a*b - 4*b^2)*tan(d*x
+ c))*sqrt(b*tan(d*x + c)^2 + a))/d, 1/8*(8*(a^2 - 2*a*b + b^2)*sqrt(a - b)
*arctan(-sqrt(b*tan(d*x + c)^2 + a)/(sqrt(a - b)*tan(d*x + c))) - (15*a^2 -
 20*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(d*x + c)^2 + a)*sqrt(-b)/(b*tan
(d*x + c))) + (2*b^2*tan(d*x + c)^3 + (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*
tan(d*x + c)^2 + a))/d]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(c + dx\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^2+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^(5/2),x)
[Out] int((a + b*tan(c + d*x)^2)^(5/2), x)
```

3.320 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}-\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)-(a+b)*(a+b*tan
(f*x+e)^2)^(1/2)/b^2/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/b^2/f
```

Rubi [A]
time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3751, 457, 90, 65, 214}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)) - ((a +
b)*Sqrt[a + b*Tan[e + f*x]^2])/(b^2*f) + (a + b*Tan[e + f*x]^2)^(3/2)/(3*b^
2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)\sqrt{a+bx^2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a+bx}}\,dx,x,\tan^2(e+fx)\right)}{2f}$$

$$=\frac{\text{Subst}\left(\int \left(\frac{-a-b}{b\sqrt{a+bx}}+\frac{1}{(1+x)\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)\,dx,x,\tan^2(e+fx)\right)}{2f}$$

$$=-\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\text{Subst}\left(\int \frac{}{(1+x)\cdots}\right.}{}$$

$$=-\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\cdots}\right.}{}$$

$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}-\frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f}+\frac{(a+\cdots}{}$$

Mathematica [A]
time = 1.63, size = 87, normalized size = 0.92

$$-\frac{\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{2(2a+3b-b\tan^2(e+fx))\sqrt{a+b\tan^2(e+fx)}}{3b^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/2*((2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b] + (2*
(2*a + 3*b - b*Tan[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2])/(3*b^2))/f
```

Maple [A]
time = 0.06, size = 103, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{(\tan^2(fx+e))\sqrt{a+b(\tan^2(fx+e))}}{3b}-\frac{2a\sqrt{a+b(\tan^2(fx+e))}}{3b^2}-\frac{\sqrt{a+b(\tan^2(fx+e))}}{b}}{f}$
default	$\frac{\frac{(\tan^2(fx+e))\sqrt{a+b(\tan^2(fx+e))}}{3b}-\frac{2a\sqrt{a+b(\tan^2(fx+e))}}{3b^2}-\frac{\sqrt{a+b(\tan^2(fx+e))}}{b}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*tan(f*x+e)^2/b*(a+b*tan(f*x+e)^2)^(1/2)-2/3*a/b^2*(a+b*tan(f*x+e)^
2)^(1/2)-1/b*(a+b*tan(f*x+e)^2)^(1/2)+1/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)
^2)^(1/2)/(-a+b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^5/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 3.32, size = 326, normalized size = 3.43

$$\left[\frac{3\,\sqrt{a-b}\,b^2\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2-4\,\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)+4\,\left((ab-b^2)\tan\left(fx+e\right)^2-2\,a^2-ab+3\,b^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{12\,(ab^2-b^3)f}, \frac{3\,\sqrt{-a+b}\,b^2\arctan\left(\frac{2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{b\tan(fx+e)^2+2\,a-b}\right)+2\,\left((ab-b^2)\tan\left(fx+e\right)^2-2\,a^2-ab+3\,b^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{6\,(ab^2-b^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(a - b)*b^2*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f
*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*
((a*b - b^2)*tan(f*x + e)^2 - 2*a^2 - a*b + 3*b^2)*sqrt(b*tan(f*x + e)^2 +
a))/((a*b^2 - b^3)*f), 1/6*(3*sqrt(-a + b)*b^2*arctan(2*sqrt(b*tan(f*x + e)
^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*((a*b - b^2)*tan(f*x
 + e)^2 - 2*a^2 - a*b + 3*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a*b^2 - b^3)*f
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [A]
time = 0.50, size = 114, normalized size = 1.20

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}+\frac{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^4f^2-3\,\sqrt{b\tan\left(fx+e\right)^2+a}\,ab^4f^2-3\,\sqrt{b\tan\left(fx+e\right)^2+a}\,b^5f^2}{3\,b^6f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) + 1/3*((b*
tan(f*x + e)^2 + a)^(3/2)*b^4*f^2 - 3*sqrt(b*tan(f*x + e)^2 + a)*a*b^4*f^2
- 3*sqrt(b*tan(f*x + e)^2 + a)*b^5*f^2)/(b^6*f^3)
```

Mupad [B]
time = 12.88, size = 97, normalized size = 1.02

$$\frac{{\left(b\tan(e+f\,x)^2+a\right)}^{3/2}}{3\,b^2\,f}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sqrt{a-b}}\right)}{f\,\sqrt{a-b}}-\left(\frac{2\,a}{b^2\,f}-\frac{a-b}{b^2\,f}\right)\,\sqrt{b\tan\left(e+f\,x\right)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(3/2)/(3*b^2*f) - atanh((a + b*tan(e + f*x)^2)^(1/2)
/(a - b)^(1/2))/(f*(a - b)^(1/2)) - ((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a +
b*tan(e + f*x)^2)^(1/2)
```

3.321 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{bf}$$

```
[Out] arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)+(a+b*tan(f*x+e)
^2)^(1/2)/b/f
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3751, 457, 81, 65, 214}

$$\frac{\sqrt{a+b\tan^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f) + Sqrt[a +
b*Tan[e + f*x]^2]/(b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\tan^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a+b\tan^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f} + \frac{\sqrt{a+b\tan^2(e+fx)}}{bf}
\end{aligned}$$

Mathematica [A]

time = 0.20, size = 62, normalized size = 0.97

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b}}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]

[Out] (ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/Sqrt[a - b] + Sqrt[a + b*Tan[e + f*x]^2]/b)/f

Maple [A]
time = 0.06, size = 56, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}}{f}$	56
default	$\frac{\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}}{f}$	56

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/f*(1/b*(a+b*tan(f*x+e)^2)^(1/2)-1/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tan(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)

Fricas [A]
time = 3.39, size = 258, normalized size = 4.03

$$\left[\frac{\sqrt{a-b}\,b\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)+4\sqrt{b\tan\left(fx+e\right)^2+a}\,(a-b)}{4\,(ab-b^2)f},\ -\frac{\sqrt{-a+b}\,b\arctan\left(\frac{2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{b\tan(fx+e)^2+2\,a-b}\right)-2\sqrt{b\tan\left(fx+e\right)^2+a}\,(a-b)}{2\,(ab-b^2)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*b*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x +
e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b
) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(
b*tan(f*x + e)^2 + a)*(a - b))/((a*b - b^2)*f), -1/2*(sqrt(-a + b)*b*arctan
(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) -
2*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a*b - b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [A]
time = 0.46, size = 62, normalized size = 0.97

$$-\frac{\frac{b\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}-\frac{\sqrt{b\tan(fx+e)^2+a}}{f}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f) - sqrt
(b*tan(f*x + e)^2 + a)/f)/b
```

Mupad [B]
time = 12.33, size = 56, normalized size = 0.88

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}}{\sqrt{a-b}}\right)}{f\,\sqrt{a-b}}+\frac{\sqrt{b\tan(e+f\,x)^2+a}}{b\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b)^(1/2))/(f*(a - b)^(1/2)) + (a +
b*tan(e + f*x)^2)^(1/2)/(b*f)
```

3.322 $\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3751, 455, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]

[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Maple [A]
time = 0.09, size = 35, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$	35
default	$\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/f/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 4.17, size = 193, normalized size = 4.71

$$\left[\frac{\log\left(-\frac{b^2\tan\left(fx+e\right)^4+2\left(4ab-3b^2\right)\tan\left(fx+e\right)^2-4\left(b\tan\left(fx+e\right)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}\right)}{4\sqrt{a-b}\,f}, \frac{\sqrt{-a+b}\arctan\left(\frac{2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}}{b\tan\left(fx+e\right)^2+2a-b}\right)}{2\left(a-b\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/4*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan
(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a
*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/(sqrt(a - b)*f), 1/2*sqr
t(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^
2 + 2*a - b))/((a - b)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [A]
time = 0.45, size = 35, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/(sqrt(-a + b)*f)
```

Mupad [B]
time = 12.34, size = 35, normalized size = 0.85

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^{2}+a}}{\sqrt{a-b}}\right)}{f\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] -atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b)^(1/2))/(f*(a - b)^(1/2))
```

3.323 $\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+arctanh((a+b*tan(f*x+e
)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 457, 88, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \text{ta}\right.}{2f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+fx)}\right)}{bf} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{bf} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}
\end{aligned}
$$

Mathematica [**A**]

time = 0.06, size = 72, normalized size = 0.97

$$\frac{-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a +
b*Tan[e + f*x]^2]/Sqrt[a - b]]/Sqrt[a - b])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(62) = 124$.
time = 0.39, size = 496, normalized size = 6.70

method	result
default	$-\frac{\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{(\cos(fx+e)+1)^2}}\left(\ln\left(-\frac{2(\cos(fx+e)-1)\left(\cos(fx+e)\sqrt{a}\sqrt{\frac{a(\cos^2(fx+e))-(\cos^2(fx+e))b+b}{(\cos(fx+e)+1)^2}}-\cos(fx+e)\right.}{\sin(fx+e)^2\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*(ln(-2*(c
os(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)+2*l
n(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1
)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^
2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*a^(1/2)-ln(-4*(cos(f*x+e)*a^(1/2)*(
(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-
cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)
+b)/(cos(f*x+e)-1))*(a-b)^(1/2))*sin(f*x+e)^2/((a*cos(f*x+e)^2-cos(f*x+e)^2
*b+b)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(cos(f*x+e)-1)/a^(1/2)/(a-b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(cot(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)

Fricas [A]
time = 3.94, size = 462, normalized size = 6.24

$$\left[\frac{\sqrt{a-b}\,a\log\left(\frac{b\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a-b}+2a-b}{\tan(fx+e)^2+1}\right)+(a-b)\sqrt{a}\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a}+2a}{\tan(fx+e)^2}\right)}{2\,(a^2-ab)f},\ \frac{2a\sqrt{-a+b}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a+b}}{a-b}\right)+(a-b)\sqrt{a}\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a}+2a}{\tan(fx+e)^2}\right)}{2\,(a^2-ab)f},\ \frac{2\sqrt{-a}\,(a-b)\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a}}{a}\right)+\sqrt{a-b}\,a\log\left(\frac{b\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{a-b}+2a-b}{\tan(fx+e)^2+1}\right)}{2\,(a^2-ab)f},\ \frac{\sqrt{-a}\,(a-b)\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a}}{a}\right)+a\sqrt{-a+b}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a+b}}{a-b}\right)}{(a^2-ab)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*(sqrt(a - b)*a*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + (a - b)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/((a^2 - a*b)*f), 1/2*(2*a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + (a - b)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/((a^2 - a*b)*f), 1/2*(2*sqrt(-a)*(a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + sqrt(a - b)*a*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)))/((a^2 - a*b)*f), (sqrt(-a)*(a - b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + a*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)))/((a^2 - a*b)*f)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)

[Out] Integral(cot(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(64) = 128.
time = 1.60, size = 208, normalized size = 2.81

$$-\frac{\frac{2\arctan\left(-\frac{\sqrt{a-b}\,\sin(fx+e)^2-\sqrt{a\sin(fx+e)^4-b\sin(fx+e)^4-2a\sin(fx+e)^2+b\sin(fx+e)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\log\left(\left|-2\left(\sqrt{a-b}\,\sin(fx+e)^2-\sqrt{a\sin(fx+e)^4-b\sin(fx+e)^4-2a\sin(fx+e)^2+b\sin(fx+e)^2+a}\right)(a-b)+(2a-b)\sqrt{a-b}\right|\right)}{\sqrt{a-b}}}{2f\,\mathrm{sgn}\left(\sin(fx+e)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*(2*arctan(-(sqrt(a - b)*sin(f*x + e)^2 - sqrt(a*sin(f*x + e)^4 - b*sin(f*x + e)^4 - 2*a*sin(f*x + e)^2 + b*sin(f*x + e)^2 + a))/sqrt(-a))/sqrt(-a

```
) + log(abs(-2*(sqrt(a - b)*sin(f*x + e)^2 - sqrt(a*sin(f*x + e)^4 - b*sin(
f*x + e)^4 - 2*a*sin(f*x + e)^2 + b*sin(f*x + e)^2 + a))*(a - b) + (2*a - b
)*sqrt(a - b)))/sqrt(a - b))/(f*sgn(sin(f*x + e)^2 - 1))
```

Mupad [B]
time = 12.03, size = 232, normalized size = 3.14

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sqrt{a}}\right)}{\sqrt{a}\,f}-\frac{\operatorname{atanh}\left(\frac{4\,a\,b^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\left(\frac{2\,b^4\,f^3}{a\,f^3-b\,f^3}-\frac{2\,a\,b^3\,f^3}{a\,f^3-b\,f^3}\right)\,\sqrt{a-b}}-\frac{2\,b^3\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\left(\frac{2\,b^4\,f^3}{a\,f^3-b\,f^3}-\frac{2\,a\,b^3\,f^3}{a\,f^3-b\,f^3}\right)\,\sqrt{a-b}}+\frac{2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\left(a\,f^3-b\,f^3\right)}{b\,f^3\,\sqrt{a-b}}\right)}{f\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] - atanh((a + b*tan(e + f*x)^2)^(1/2)/a^(1/2))/(a^(1/2)*f) - atanh((4*a*b^2*
(a + b*tan(e + f*x)^2)^(1/2))/(((2*b^4*f^3)/(a*f^3 - b*f^3) - (2*a*b^3*f^3)
/(a*f^3 - b*f^3))*(a - b)^(1/2)) - (2*b^3*(a + b*tan(e + f*x)^2)^(1/2))/(((
2*b^4*f^3)/(a*f^3 - b*f^3) - (2*a*b^3*f^3)/(a*f^3 - b*f^3))*(a - b)^(1/2))
+ (2*(a + b*tan(e + f*x)^2)^(1/2)*(a*f^3 - b*f^3))/(b*f^3*(a - b)^(1/2)))/(
f*(a - b)^(1/2))
```

3.324 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=116

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}$$

```
[Out] 1/2*(2*a+b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-arctanh((a+
b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)-1/2*cot(f*x+e)^2*(a+b*tan(
f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3751, 457, 105, 162, 65, 214}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a + b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - Arc
Tanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f) - (Cot[e + f*x
]^2*Sqrt[a + b*Tan[e + f*x]^2])/(2*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2a+b)+\frac{bx}{2}}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e\right.}{2af}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e\right.}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e\right.}}{bf}$$

$$= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right.}{\sqrt{a-b}\,f}$$

Mathematica [A]
time = 0.52, size = 135, normalized size = 1.16

$$\frac{(2a^2-ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(-2a\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+(-a+b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)}{2a^{3/2}(a-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((2*a^2 - a*b - b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sqrt[a]*
(-2*a*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (-a + b
)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2]))/(2*a^(3/2)*(a - b)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3600 vs. 2(98) = 196.
time = 0.44, size = 3601, normalized size = 31.04

method	result	size

default	Expression too large to display	3601

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4/f*(4*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*a^(5/2)-2*cos(f*x+e)^3*a^(5/2)*(a-b)^(1/2)+4*cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*a^(5/2)+2*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)*a^2+cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)*a*b-2*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*(a-b)^(1/2)*a^2-cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*(a-b)^(1/2)*a*b+2*cos(f*x+e)^3*a^(3/2)*(a-b)^(1/2)*b-4*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*a^(5/2)+2*cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)*a^2+cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(1/2)*a*b-2*cos(f*x+e)^2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(

```
1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*(a-b)^(1/2)*a^2-cos(f*x+e)^2*((a*
cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(cos(f*x+e)*a^
(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+((a*cos(f*
x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f*x+e)*a-b*cos
(f*x+e)+b)/(cos(f*x+e)-1))*(a-b)^(1/2)*a*b-4*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(4*(a-b)^(1/2)*cos(f*x+e)*((a*cos(f*x+e)^2-c
os(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)*a-4*b*cos(f*x+e))*a^(5/
2)-2*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*
ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f*x
+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-b)^(
1/2)*a^2-cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*
b+b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos
(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)/sin(f*x+e)^2/a^(1/2))*(a-
b)^(1/2)*a*b+2*cos(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)
^2)^(1/2)*ln(-4*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos
(f*x+e)+1)^2)^(1/2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*(a-b)^(1/2)*a^2+co
s(f*x+e)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*ln(-4*(
cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/
2)+((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+cos(f
*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)-1))*(a-b)^(1/2)*a*b-2*cos(f*x+e)*a^(3/2
)*(a-b)^(1/2)*b-2*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2
)*ln(-2*(cos(f*x+e)-1)*(cos(f*x+e)*a^(1/2)*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+
b)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)+((a*cos(f*x+e)^2-cos(f
*x+e)^2*b+b)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)+b)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 3.30, size = 729, normalized size = 6.28

$$\left[\frac{2\sqrt{a-b}\,a^2\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2a-b}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+\left(2a^2-ab-b^2\right)\sqrt{a}\,\log\left(\frac{b\tan(fx+e)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a}+2a}{\tan(fx+e)^2}\right)\tan\left(fx+e\right)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\left(a^2-ab\right)}{4\left(a^3-a^2b\right)f\tan\left(fx+e\right)^2},\ -\frac{4a^2\sqrt{-a+b}\,\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{a-b}\right)\tan\left(fx+e\right)^2-\left(2a^2-ab-b^2\right)\sqrt{a}\,\log\left(\frac{b\tan(fx+e)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a}+2a}{\tan(fx+e)^2}\right)\tan\left(fx+e\right)^2+2\sqrt{b\tan\left(fx+e\right)^2+a}\left(a^2-ab\right)}{4\left(a^3-a^2b\right)f\tan\left(fx+e\right)^2},\ \frac{\sqrt{a-b}\,a^2\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}+2a-b}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2-\left(2a^2-ab-b^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a}}{a}\right)\tan\left(fx+e\right)^2-\sqrt{b\tan\left(fx+e\right)^2+a}\left(a^2-ab\right)}{2\left(a^3-a^2b\right)f\tan\left(fx+e\right)^2},\ -\frac{2a^2\sqrt{-a+b}\,\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{a-b}\right)\tan\left(fx+e\right)^2+\left(2a^2-ab-b^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a}}{a}\right)\tan\left(fx+e\right)^2+\sqrt{b\tan\left(fx+e\right)^2+a}\left(a^2-ab\right)}{2\left(a^3-a^2b\right)f\tan\left(fx+e\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (2*a^2 - a*
b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*(a^
2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), -1/4*(4*a^2*sqrt(-a + b)*arctan
(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 - (2*a^2
- a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*(a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), 1/2*(sqrt(a - b)*a^2*log((b
*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(
f*x + e)^2 + 1))*tan(f*x + e)^2 - (2*a^2 - a*b - b^2)*sqrt(-a)*arctan(sqrt(
b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^2 - sqrt(b*tan(f*x + e)^2 +
a)*(a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), -1/2*(2*a^2*sqrt(-a + b)*
arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^2 + (
2*a^2 - a*b - b^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*t
an(f*x + e)^2 + sqrt(b*tan(f*x + e)^2 + a)*(a^2 - a*b))/((a^3 - a^2*b)*f*ta
n(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 0.44, size = 830, normalized size = 7.16

$$\frac{\operatorname{atanh}\left(\frac{b^6\sqrt{b\tan(e+fx)^2+a}}{4\sqrt{a^3}\left(\frac{3ab^4}{2}+\frac{5b^5}{4}+\frac{b^6}{4a}\right)}+\frac{3b^4\sqrt{b\tan(e+fx)^2+a}}{2\sqrt{a^3}\left(\frac{3b^4}{2a}+\frac{5b^5}{4a^2}+\frac{b^6}{4a^3}\right)}+\frac{5b^5\sqrt{b\tan(e+fx)^2+a}}{4\sqrt{a^3}\left(\frac{3b^4}{2}+\frac{5b^5}{4a}+\frac{b^6}{4a^2}\right)}\right)(2a+b)}{2f\sqrt{a^3}}-\frac{b\sqrt{b\tan(e+fx)^2+a}}{2a\left(f\left(b\tan(e+fx)^2+a\right)-af\right)}+\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{\frac{2ab^4f^2+2a^2b^3f^2}{2a^2f^3}-\frac{\sqrt{b\tan(e+fx)^2+a}\,(16a^2b^3f^2-32a^3b^2f^2)}{8a^2f^3\sqrt{a-b}}}{2f\sqrt{a-b}}-\frac{\sqrt{b\tan(e+fx)^2+a}\,(4ab^3+b^4+8a^2b^2)}{4a^2f^2}\right)1i}{f\sqrt{a-b}}-\frac{\left(\frac{\frac{2ab^4f^2+2a^2b^3f^2}{2a^2f^3}+\frac{\sqrt{b\tan(e+fx)^2+a}\,(16a^2b^3f^2-32a^3b^2f^2)}{8a^2f^3\sqrt{a-b}}}{2f\sqrt{a-b}}+\frac{\sqrt{b\tan(e+fx)^2+a}\,(4ab^3+b^4+8a^2b^2)}{4a^2f^2}\right)1i}{f\sqrt{a-b}}}{\frac{\frac{\frac{2ab^4f^2+2a^2b^3f^2}{2a^2f^3}-\frac{\sqrt{b\tan(e+fx)^2+a}\,(16a^2b^3f^2-32a^3b^2f^2)}{8a^2f^3\sqrt{a-b}}}{2f\sqrt{a-b}}-\frac{\sqrt{b\tan(e+fx)^2+a}\,(4ab^3+b^4+8a^2b^2)}{4a^2f^2}}{f\sqrt{a-b}}+\frac{\frac{\frac{2ab^4f^2+2a^2b^3f^2}{2a^2f^3}+\frac{\sqrt{b\tan(e+fx)^2+a}\,(16a^2b^3f^2-32a^3b^2f^2)}{8a^2f^3\sqrt{a-b}}}{2f\sqrt{a-b}}+\frac{\sqrt{b\tan(e+fx)^2+a}\,(4ab^3+b^4+8a^2b^2)}{4a^2f^2}}{f\sqrt{a-b}}-\frac{\frac{b^4}{2}+ab^3}{a^2f^3}}\right)1i}{f\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] (atan((((((2*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*a^2*f^3) - ((a + b*tan(e + f*x)^
2)^(1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*f^2))/(8*a^2*f^3*(a - b)^(1/2)))/(2*f
*(a - b)^(1/2)) - ((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b^3 + b^4 + 8*a^2*b^2)
)/(4*a^2*f^2))*1i)/(f*(a - b)^(1/2)) - ((((2*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*
a^2*f^3) + ((a + b*tan(e + f*x)^2)^(1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*f^2))
/(8*a^2*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(
1/2)*(4*a*b^3 + b^4 + 8*a^2*b^2))/(4*a^2*f^2))*1i)/(f*(a - b)^(1/2)))/((((2
*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*a^2*f^3) - ((a + b*tan(e + f*x)^2)^(1/2)*(16
*a^2*b^3*f^2 - 32*a^3*b^2*f^2))/(8*a^2*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/
2)) - ((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b^3 + b^4 + 8*a^2*b^2))/(4*a^2*f^2
))/(f*(a - b)^(1/2)) + (((2*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*a^2*f^3) + ((a +
b*tan(e + f*x)^2)^(1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*f^2))/(8*a^2*f^3*(a -
b)^(1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b^3 + b
^4 + 8*a^2*b^2))/(4*a^2*f^2))/(f*(a - b)^(1/2)) - (a*b^3 + b^4/2)/(a^2*f^3)
))*1i)/(f*(a - b)^(1/2)) - (b*(a + b*tan(e + f*x)^2)^(1/2))/(2*a*(f*(a + b*
tan(e + f*x)^2) - a*f)) + (atanh((b^6*(a + b*tan(e + f*x)^2)^(1/2))/(4*(a^3
)^(1/2)*((3*a*b^4)/2 + (5*b^5)/4 + b^6/(4*a))) + (3*b^4*(a + b*tan(e + f*x)
^2)^(1/2))/(2*(a^3)^(1/2)*((3*b^4)/(2*a) + (5*b^5)/(4*a^2) + b^6/(4*a^3)))
+ (5*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*(a^3)^(1/2)*((3*b^4)/2 + (5*b^5)/
(4*a) + b^6/(4*a^2))))*(2*a + b))/(2*f*(a^3)^(1/2))
```

3.325 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=166

$$-\frac{(8a^2+4ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}+\frac{(4a+3b)\cot^2(}{}$$

```
[Out] -1/8*(8*a^2+4*a*b+3*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/
f+arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)+1/8*(4*a+3*b)
*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/4*cot(f*x+e)^4*(a+b*tan(f*x+
e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3751, 457, 105, 156, 162, 65, 214}

$$\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f}-\frac{(8a^2+4ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}-\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/8*((8*a^2 + 4*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2/Sqrt[a]])/
(a^(5/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*
f) + ((4*a + 3*b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/(8*a^2*f) - (C
ot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2])/(4*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+3b)+\frac{3bx}{2}}{x^2(1+x)\sqrt{a+bx}}\,dx, x, \tan^2\right.}{4af}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e}{4af}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e}{4af}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e}{4af}$$

$$= -\frac{(8a^2+4ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{t}}}{\sqrt{}}\right.}{\sqrt{a-}}$$

Mathematica [A]
time = 1.28, size = 162, normalized size = 0.98

$$\frac{(-8a^3+4a^2b+ab^2+3b^3)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a^2\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+(-a+b)\cot^2(e+fx)\left(-4a-3b+2a\cot^2(e+fx)\right)\sqrt{a+b\tan^2(e+fx)}\right)}{8a^{5/2}(a-b)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ((-8*a^3 + 4*a^2*b + a*b^2 + 3*b^3)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt
[a]] + Sqrt[a]*(8*a^2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a
 - b]] + (-a + b)*Cot[e + f*x]^2*(-4*a - 3*b + 2*a*Cot[e + f*x]^2)*Sqrt[a +
 b*Tan[e + f*x]^2]))/(8*a^(5/2)*(a - b)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7640 vs. 2(144) = 288.
time = 0.36, size = 7641, normalized size = 46.03

method	result	size
default	Expression too large to display	7641

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^5/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 4.26, size = 893, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a - b)*a^3*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (8*a^3 - 4
*a^2*b - a*b^2 - 3*b^3)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*(2*a^3 - 2*a^2*
b - (4*a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(
(a^4 - a^3*b)*f*tan(f*x + e)^4), 1/16*(16*a^3*sqrt(-a + b)*arctan(-sqrt(b*t
an(f*x + e)^2 + a)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^3 - 4*a^2*b
- a*b^2 - 3*b^3)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*(2*a^3 - 2*a^2*b - (4*
a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 -
a^3*b)*f*tan(f*x + e)^4), 1/8*(4*sqrt(a - b)*a^3*log((b*tan(f*x + e)^2 + 2*
sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan
(f*x + e)^4 + (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*sqrt(-a)*arctan(sqrt(b*tan(
f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4 - (2*a^3 - 2*a^2*b - (4*a^3 - a^
2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f
*tan(f*x + e)^4), 1/8*(8*a^3*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a
)*sqrt(-a + b)/(a - b))*tan(f*x + e)^4 + (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*
sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a)*tan(f*x + e)^4 - (2*
```

```
a^3 - 2*a^2*b - (4*a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x +
e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 12.14, size = 1215, normalized size = 7.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] - (((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b + 5*b^2))/(8*a) - (b*(a + b*tan(e +
 f*x)^2)^(3/2)*(4*a + 3*b))/(8*a^2))/(f*(a + b*tan(e + f*x)^2)^2 + a^2*f -
2*a*f*(a + b*tan(e + f*x)^2)) - (atan((((((((3*a^2*b^5*f^2)/2 + (a^3*b^4*f^2
)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^3) - ((a + b*tan(e + f*x)^2)^(1/2)*(256*a^4*b
^3*f^2 - 512*a^5*b^2*f^2))/(128*a^4*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2))
 - ((a + b*tan(e + f*x)^2)^(1/2)*(24*a*b^5 + 9*b^6 + 64*a^2*b^4 + 64*a^3*b^
3 + 128*a^4*b^2))/(64*a^4*f^2))*1i)/(f*(a - b)^(1/2)) - (((((3*a^2*b^5*f^2)
/2 + (a^3*b^4*f^2)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^3) + ((a + b*tan(e + f*x)^2)
^(1/2)*(256*a^4*b^3*f^2 - 512*a^5*b^2*f^2))/(128*a^4*f^3*(a - b)^(1/2)))/(2
```

```
*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(1/2)*(24*a*b^5 + 9*b^6 + 64*a^
2*b^4 + 64*a^3*b^3 + 128*a^4*b^2))/(64*a^4*f^2))*1i)/(f*(a - b)^(1/2)))/(((
((3*a^2*b^5*f^2)/2 + (a^3*b^4*f^2)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^3) - ((a + b
*tan(e + f*x)^2)^(1/2)*(256*a^4*b^3*f^2 - 512*a^5*b^2*f^2))/(128*a^4*f^3*(a
 - b)^(1/2)))/(2*f*(a - b)^(1/2)) - ((a + b*tan(e + f*x)^2)^(1/2)*(24*a*b^5
 + 9*b^6 + 64*a^2*b^4 + 64*a^3*b^3 + 128*a^4*b^2))/(64*a^4*f^2))/(f*(a - b)
^(1/2)) + ((((3*a^2*b^5*f^2)/2 + (a^3*b^4*f^2)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^
3) + ((a + b*tan(e + f*x)^2)^(1/2)*(256*a^4*b^3*f^2 - 512*a^5*b^2*f^2))/(12
8*a^4*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(1/
2)*(24*a*b^5 + 9*b^6 + 64*a^2*b^4 + 64*a^3*b^3 + 128*a^4*b^2))/(64*a^4*f^2)
)/(f*(a - b)^(1/2)) - ((3*a*b^5)/4 + (9*b^6)/32 + (5*a^2*b^4)/4 + a^3*b^3)/
(a^4*f^3)))*1i)/(f*(a - b)^(1/2)) - (atanh((35*b^6*(a + b*tan(e + f*x)^2)^(
1/2))/(32*(a^5)^(1/2)*((5*b^5)/(4*a) + (35*b^6)/(32*a^2) + (63*b^7)/(64*a^3
) + (81*b^8)/(256*a^4) + (27*b^9)/(256*a^5))) + (5*b^5*(a + b*tan(e + f*x)^
2)^(1/2))/(4*(a^5)^(1/2)*((5*b^5)/(4*a^2) + (35*b^6)/(32*a^3) + (63*b^7)/(6
4*a^4) + (81*b^8)/(256*a^5) + (27*b^9)/(256*a^6))) + (63*b^7*(a + b*tan(e +
 f*x)^2)^(1/2))/(64*(a^5)^(1/2)*((5*b^5)/4 + (35*b^6)/(32*a) + (63*b^7)/(64
*a^2) + (81*b^8)/(256*a^3) + (27*b^9)/(256*a^4))) + (81*b^8*(a + b*tan(e +
f*x)^2)^(1/2))/(256*(a^5)^(1/2)*((5*a*b^5)/4 + (35*b^6)/32 + (63*b^7)/(64*a
) + (81*b^8)/(256*a^2) + (27*b^9)/(256*a^3))) + (27*b^9*(a + b*tan(e + f*x)
^2)^(1/2))/(256*(a^5)^(1/2)*((35*a*b^6)/32 + (63*b^7)/64 + (5*a^2*b^5)/4 +
(81*b^8)/(256*a) + (27*b^9)/(256*a^2))))*(4*a*b + 8*a^2 + 3*b^2))/(8*f*(a^5
)^(1/2))
```

3.326 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=177

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}+\frac{(3a^2+4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}-\frac{(3a+4b)\tan(e}{}$$

```
[Out] 1/8*(3*a^2+4*a*b+8*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)
)/b^(5/2)/f-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)
^(1/2)-1/8*(3*a+4*b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f
```

Rubi [A]
time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 490, 596, 537, 223, 212, 385, 209}

$$\frac{(3a^2+4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) + ((3*a^2 + 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*
Tan[e + f*x]^2]])/(8*b^(5/2)*f) - ((3*a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[
e + f*x]^2])/(8*b^2*f) + (Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*b*f
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf} - \frac{\text{Subst}\left(\int \frac{x^2(3a+(3a+4b)x^2)}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e\right.}{4bf}$$

$$= -\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e}}{4bf}$$

$$= -\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e}}{4bf}$$

$$= -\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e}}{4bf}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} + \frac{(3a^2+4ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\text{ta}}{\sqrt{a+b\,\text{ta}}}\right.}{8b^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 6.23, size = 768, normalized size = 4.34

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (-((b*(3*a^2 + 4*a*b + 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 +
Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a
- b)*Cos[2*(e + f*x)]))) + (16*b^3*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b +
 (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]
^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[Ar
```

```
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos
[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/
(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]
])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(4*b^2*f) + (Sqrt[(a + b + a*C
os[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((-3*Sec[e +
f*x]*(a*Sin[e + f*x] + 2*b*Sin[e + f*x]))/(8*b^2) + (Sec[e + f*x]^2*Tan[e +
 f*x])/(4*b)))/f
```

Maple [A]
time = 0.07, size = 248, normalized size = 1.40

method	result
derivativedivides	$\frac{\frac{\left(\tan^3(fx+e)\right)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{4b}-\frac{3a\left(\frac{\tan(fx+e)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\ \tan(fx+e)+\sqrt{}\right.}{}\right.}{4b}}{}$
default	$\frac{\frac{\left(\tan^3(fx+e)\right)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{4b}-\frac{3a\left(\frac{\tan(fx+e)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\ \tan(fx+e)+\sqrt{}\right.}{}\right.}{4b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*tan(f*x+e)^3/b*(a+b*tan(f*x+e)^2)^(1/2)-3/4*a/b*(1/2*tan(f*x+e)/b*
(a+b*tan(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e
)^2)^(1/2)))-1/2*tan(f*x+e)/b*(a+b*tan(f*x+e)^2)^(1/2)+1/2*a/b^(3/2)*ln(b^(
1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*
x+e)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a
-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 5.96, size = 851, normalized size = 4.81

$$\left[-\frac{8\sqrt{-a+b}\,b^{3}\log\left(-\frac{(a-2b)\tan(fx+e)^{2}+2\sqrt{b\tan(fx+e)^{2}+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)-(3a^{3}+a^{2}b+4ab^{2}-8b^{3})\sqrt{b}\log\left(2b\tan(fx+e)^{2}+2\sqrt{b\tan(fx+e)^{2}+a}\sqrt{b}\tan(fx+e)+a\right)-2\left(2(ab^{2}-b^{3})\tan(fx+e)^{3}-(3a^{2}b+ab^{2}-4b^{3})\tan(fx+e)\right)\sqrt{b\tan(fx+e)^{2}+a}}{16(ab^{3}-b^{4})f}, -\frac{4\sqrt{-a+b}\,b^{3}\log\left(-\frac{(a-2b)\tan(fx+e)^{2}+2\sqrt{b\tan(fx+e)^{2}+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)+(3a^{3}+a^{2}b+4ab^{2}-8b^{3})\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(fx+e)^{2}+a}\sqrt{-b}}{b\tan(fx+e)}\right)-\left(2(ab^{2}-b^{3})\tan(fx+e)^{3}-(3a^{2}b+ab^{2}-4b^{3})\tan(fx+e)\right)\sqrt{b\tan(fx+e)^{2}+a}}{8(ab^{3}-b^{4})f}, -\frac{16\sqrt{a-b}\,b^{3}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^{2}+a}}{\sqrt{a-b}\tan(fx+e)}\right)-(3a^{3}+a^{2}b+4ab^{2}-8b^{3})\sqrt{b}\log\left(2b\tan(fx+e)^{2}+2\sqrt{b\tan(fx+e)^{2}+a}\sqrt{b}\tan(fx+e)+a\right)-2\left(2(ab^{2}-b^{3})\tan(fx+e)^{3}-(3a^{2}b+ab^{2}-4b^{3})\tan(fx+e)\right)\sqrt{b\tan(fx+e)^{2}+a}}{16(ab^{3}-b^{4})f}, -\frac{8\sqrt{a-b}\,b^{3}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^{2}+a}}{\sqrt{a-b}\tan(fx+e)}\right)+(3a^{3}+a^{2}b+4ab^{2}-8b^{3})\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(fx+e)^{2}+a}\sqrt{-b}}{b\tan(fx+e)}\right)-\left(2(ab^{2}-b^{3})\tan(fx+e)^{3}-(3a^{2}b+ab^{2}-4b^{3})\tan(fx+e)\right)\sqrt{b\tan(fx+e)^{2}+a}}{8(ab^{3}-b^{4})f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (3*a^3
 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f
*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(2*(a*b^2 - b^3)*tan(f*x + e)^
3 - (3*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a
*b^3 - b^4)*f), -1/8*(4*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 + 2
*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2
+ 1)) + (3*a^3 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x +
e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3
*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 -
 b^4)*f), -1/16*(16*sqrt(a - b)*b^3*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqr
t(a - b)*tan(f*x + e))) - (3*a^3 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(b)*log(2*b
*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) -
2*(2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))
*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 - b^4)*f), -1/8*(8*sqrt(a - b)*b^3*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (3*a^3 + a^2*
b + 4*a*b^2 - 8*b^3)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b
*tan(f*x + e))) - (2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3*a^2*b + a*b^2 - 4*b^
3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 - b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{6}\left(e+fx\right)}{\sqrt{a+b\tan^{2}\left(e+fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^6}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.327 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=125

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf}$$

```
[Out] -1/2*(a+2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f
+arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)+1/2*
(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

Rubi [A]
time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3751, 490, 537, 223, 212, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{3/2}f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f) - ((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])
/(2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf} - \frac{\operatorname{Subst}\left(\int \frac{a+(a+2b)x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e\right.}{2bf}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf} + \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e\right.}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+}{\sqrt{a+b\tan^2}}\right.}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} - \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right.}{2b^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 4.84, size = 271, normalized size = 2.17

$$-\frac{\left(\sqrt{2}\,a(-a+b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,F\left(\operatorname{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)-2\sqrt{2}\,ab\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\Pi\left(-\frac{b}{a-b};\operatorname{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+(a-b)(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)\right)\tan(e+fx)}{2\sqrt{2}\,b(-a+b)f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/2*((Sqrt[2]*a*(-a + b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
+ f*x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2]*a*b*Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (
a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + (a - b)*(a + b +
 (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2)*Tan[e + f*x])/(Sqrt[2]*b*(-a + b
)*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Maple [A]
time = 0.07, size = 157, normalized size = 1.26

method	result

derivativedivides	$\frac{\frac{\tan(fx+e)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\tan(fx+\right.}{}}{f}$
default	$\frac{\frac{\tan(fx+e)\sqrt{a+b\left(\tan^2(fx+e)\right)}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\tan(fx+\right.}{}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*tan(f*x+e)/b*(a+b*tan(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*tan
(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^
(1/2))/b^(1/2)+(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/
2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 5.57, size = 677, normalized size = 5.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (a^2 +
a*b - 2*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(b)*tan(f*x + e) + a) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*
x + e))/((a*b^2 - b^3)*f), -1/2*(sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x +
 e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*
x + e)^2 + 1)) - (a^2 + a*b - 2*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-b)/(b*tan(f*x + e))) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*ta
n(f*x + e))/((a*b^2 - b^3)*f), 1/4*(4*sqrt(a - b)*b^2*arctan(-sqrt(b*tan(f*
```

```
x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (a^2 + a*b - 2*b^2)*sqrt(b)*log
(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a
) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a*b^2 - b^3)*f
), 1/2*(2*sqrt(a - b)*b^2*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*t
an(f*x + e))) + (a^2 + a*b - 2*b^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan
(f*x + e))/((a*b^2 - b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^4}{\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.328 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=86

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)+arct
anh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 494, 223, 212, 385, 209}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[b]
*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x,\right.}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x,\right.}{f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 0.53, size = 149, normalized size = 1.73

$$\frac{a\csc^2(e+fx)\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\sin(2(e+fx))}{2(a-b)bf\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] (a*Csc[e + f*x]^2*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Co
s[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a + b + (a - b)*Cos[2
*(e + f*x)])*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(2*(a - b)*b*f*Sqrt[((a + b
+ (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b])
```

Maple [A]
time = 0.06, size = 100, normalized size = 1.16

method	result
derivativedivides	$\frac{\frac{\ln\left(\sqrt{b}\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b)}\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\sqrt{a+b\left(\tan^2(\cdots\right.}}\right)}{b^2(a-b)}}{f}$
default	$\frac{\frac{\ln\left(\sqrt{b}\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b)}\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\sqrt{a+b\left(\tan^2(\cdots\right.}}\right)}{b^2(a-b)}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1
/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*t
an(f*x+e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 4.84, size = 501, normalized size = 5.83

$$\left[\frac{(a-b)\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{b}\,\tan\left(fx+e\right)+a\right)-\sqrt{-a+b}\,b\log\left(-\frac{(a-2\,b)\tan(fx+e)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)}{2\,(ab-b^{2})f},-\frac{2\,(a-b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-b}}{b\tan(fx+e)}\right)+\sqrt{-a+b}\,b\log\left(-\frac{(a-2\,b)\tan(fx+e)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)}{2\,(ab-b^{2})f},-\frac{2\,\sqrt{a-b}\,b\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{a-b}\,\tan(fx+e)}\right)-(a-b)\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^{2}+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{b}\,\tan\left(fx+e\right)+a\right)}{2\,(ab-b^{2})f},-\frac{\sqrt{a-b}\,b\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{a-b}\,\tan(fx+e)}\right)+(a-b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-b}}{b\tan(fx+e)}\right)}{(ab-b^{2})f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a - b)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(b)*tan(f*x + e) + a) - sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x + e)^2
+ 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)
^2 + 1)))/((a*b - b^2)*f), -1/2*(2*(a - b)*sqrt(-b)*arctan(sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + sqrt(-a + b)*b*log(-((a - 2*b)*tan(
f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(t
an(f*x + e)^2 + 1)))/((a*b - b^2)*f), -1/2*(2*sqrt(a - b)*b*arctan(-sqrt(b*
tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (a - b)*sqrt(b)*log(2*b*t
an(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a))/((a
*b - b^2)*f), -(sqrt(a - b)*b*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a -
b)*tan(f*x + e))) + (a - b)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt
(-b)/(b*tan(f*x + e))))/((a*b - b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}{\left(e + f x \right)}}{\sqrt{a + b \tan^{2}{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{tan}\left(e+f\,x\right)}^2}{\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.329 $\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3742, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[e + f*x]^2],x]
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f)
```

Maple [A]
time = 0.06, size = 67, normalized size = 1.46

method	result	size
derivativedivides	$\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b(\tan^2(fx+e))}}\right)}{f\,b^2(a-b)}$	67
default	$\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b(\tan^2(fx+e))}}\right)}{f\,b^2(a-b)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan
(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.84, size = 131, normalized size = 2.85

$$\left[-\frac{\sqrt{-a+b}\,\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)}{2(a-b)f},\ \frac{\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{a-b}\,\tan(fx+e)}\right)}{\sqrt{a-b}\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))/((a - b)*f), arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e)))/(sqrt(a - b)*f)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [**B**]
time = 12.69, size = 40, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\tan(e+f\,x)\,\sqrt{a-b}}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\right)}{f\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] atan((tan(e + f*x)*(a - b)^(1/2))/(a + b*tan(e + f*x)^2)^(1/2))/(f*(a - b)^
(1/2))
```

3.330 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)-cot(
f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3751, 491, 12, 385, 209}

$$-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{a}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e\right.}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+}{\sqrt{a+b\tan}}\right.}{f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 8.17, size = 212, normalized size = 2.72

$$-\frac{\cos^2(e+fx)\cot(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(\frac{4\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\sin^2(e+fx)\left(a^2-b^2\tan^2(e+fx)+ab\left(-1+\tan^2(e+fx)\right)\right)}{3a^2}+\frac{\operatorname{ArcSin}\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\right)\left(a+2b\tan^2(e+fx)\right)}{a\sqrt{\frac{\cos^2(e+fx)\sin^2(e+fx)\left(a^2-b^2\tan^2(e+fx)+ab\left(-1+\tan^2(e+fx)\right)\right)}{a^2}}}\right)}{f\sqrt{a+b\tan^2(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -((Cos[e + f*x]^2*Cot[e + f*x]*(1 + (b*Tan[e + f*x]^2)/a)*((4*Hypergeometri
c2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a^2 - b^2*Tan[e
 + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/(3*a^2) + (ArcSin[Sqrt[((a - b)*Sin
[e + f*x]^2)/a]]*(a + 2*b*Tan[e + f*x]^2))/(a*Sqrt[(Cos[e + f*x]^2*Sin[e +
f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2])))/(f*S
qrt[a + b*Tan[e + f*x]^2]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.41, size = 1195, normalized size = 15.32

method	result	size
default	Expression too large to display	1195

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/si
n(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^
2)/a^2)^(1/2))*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)
*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(co
s(f*x+e)+1)/a)^(1/2)*cos(f*x+e)*sin(f*x+e)*a-2*a*2^(1/2)*((I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*
x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2
)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)+2^(1/2)*((I*cos(f*x+e
)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(c
os(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)
^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2
)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a*sin(
f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+c
os(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/
```

```
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+
e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b
)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a*sin(f*
x+e)+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a-((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b
)/a)^(1/2)*b)/cos(f*x+e)/sin(f*x+e)/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(
f*x+e)^2)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [A]
time = 3.22, size = 305, normalized size = 3.91

$$\left[-\frac{a\sqrt{-a+b}\log\left(-\frac{(a^2-8ab+8b^2)\tan(fx+e)^4-2(3a^2-4ab)\tan(fx+e)^2+a^2+4\left((a-2b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)\tan(fx+e)+4\sqrt{b\tan(fx+e)^2+a}(a-b)}{4(a^2-ab)f\tan(fx+e)}, -\frac{\sqrt{a-b}\,a\arctan\left(-\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}\tan(fx+e)}{(a-2b)\tan(fx+e)^2-a}\right)\tan(fx+e)+2\sqrt{b\tan(fx+e)^2+a}(a-b)}{2(a^2-ab)f\tan(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/4*(a*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2
 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x +
e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x +
e)^2 + 1))*tan(f*x + e) + 4*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2 - a*b
)*f*tan(f*x + e)), -1/2*(sqrt(a - b)*a*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e) + 2*
sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2 - a*b)*f*tan(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^2}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.331 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=120

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)+1/3*(
3*a+2*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)^3*(a+b*ta
n(f*x+e)^2)^(1/2)/a/f
```

Rubi [A]
time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 491, 597, 12, 385, 209}

$$\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]*
f) + ((3*a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*f) - (Cot
[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} + \frac{\text{Subst}\left(\int \frac{-3a-2b-2bx^2}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan\right.}{3af}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+}}{3af}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+}}{3af}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+}}{3af}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx}}{3a^2f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.09, size = 263, normalized size = 2.19

$$-\frac{\cos^2(e+fx)\cot^3(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(-12b(-a+b)(-a-b+(-a+b)\cos(2(e+fx)))\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\tan^4(e+fx)-8(a-b)\,{}_3F_2\left(2,2,2;1,\frac{5}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2+\frac{6a\text{ArcSin}\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\right)\left(a^2-4ab\tan^2(e+fx)-8b^2\tan^4(e+fx)\right)}{\sqrt{\frac{(a-b)\sin^2(2(e+fx))\left(a+b\tan^2(e+fx)\right)}{a^2}}}\right)}{9a^3f\sqrt{a+b\tan^2(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/9*(Cos[e + f*x]^2*Cot[e + f*x]^3*(1 + (b*Tan[e + f*x]^2)/a)*(-12*b*(-a +
 b)*(-a - b + (-a + b)*Cos[2*(e + f*x)])*Hypergeometric2F1[2, 2, 5/2, ((a -
 b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^4 - 8*(a - b)*HypergeometricPFQ[{2, 2,
2}, {1, 5/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x
]^2)^2 + (6*a*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a^2 - 4*a*b*Tan[e +
 f*x]^2 - 8*b^2*Tan[e + f*x]^4))/Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b*Ta
n[e + f*x]^2))/a^2]))/(a^3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.51, size = 2433, normalized size = 20.28

method	result	size
default	Expression too large to display	2433

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+
cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1
/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x
+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-
8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^2-6*2^(1/2)*((I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(
cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((co
s(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^2+3*2^(1/2)
*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*co
s(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I
*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*E
llipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e
),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)
^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2-6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^
(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e
)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*cos(f*x+e)^2*a^2-3*2^(1/2)*((I*cos(f*x+e)*b
^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(
f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1
/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x
+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(
a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e
)*sin(f*x+e)*a^2+6*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2
)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^
(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
```

```
)*cos(f*x+e)*sin(f*x+e)*a^2-3*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-
2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*co
s(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2
)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2*sin(f*x+e)+6*2^(1/2)*((I*c
os(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+
e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*Ellipti
cPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2
)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)+4*((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^4*a^2-2*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)*cos(f*x+e)^4*a*b-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
*cos(f*x+e)^4*b^2-3*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*
a^2+5*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*a*b+4*((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2*b^2-3*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)*a*b-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^2)/co
s(f*x+e)/sin(f*x+e)^3/((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(1/2
)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/a^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Fricas [**A**]
time = 3.32, size = 377, normalized size = 3.14

$$\left[-\frac{3\,a^2\sqrt{-a+b}\,\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2-4\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^3-4\left((3\,a^2-ab-2\,b^2)\tan\left(fx+e\right)^2-a^2+ab\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{12\,(a^3-a^2b)f\tan\left(fx+e\right)^3},\ \frac{3\sqrt{a-b}\,a^2\arctan\left(-\frac{2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}\,\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan\left(fx+e\right)^3+2\left((3\,a^2-ab-2\,b^2)\tan\left(fx+e\right)^2-a^2+ab\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{6\,(a^3-a^2b)f\tan\left(fx+e\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/12*(3*a^2*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(
3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f
*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f
*x + e)^2 + 1))*tan(f*x + e)^3 - 4*((3*a^2 - a*b - 2*b^2)*tan(f*x + e)^2 -
a^2 + a*b)*sqrt(b*tan(f*x + e)^2 + a))/((a^3 - a^2*b)*f*tan(f*x + e)^3), 1/
6*(3*sqrt(a - b)*a^2*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f
```

```
*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)^3 + 2*((3*a^2 - a*b -
2*b^2)*tan(f*x + e)^2 - a^2 + a*b)*sqrt(b*tan(f*x + e)^2 + a))/((a^3 - a^2*
b)*f*tan(f*x + e)^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cot(e+f\,x)}^4}{\sqrt{b\,{\tan\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.332 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal. Leaf size=170

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f}-\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}+\frac{(5a+4b)\cot^3}{}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)-1/15
*(15*a^2+10*a*b+8*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/f+1/15*(5*a+
4*b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/5*cot(f*x+e)^5*(a+b*tan(
f*x+e)^2)^(1/2)/a/f
```

Rubi [A]

time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 491, 597, 12, 385, 209}

$$\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}-\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f}-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b
]*f)) - ((15*a^2 + 10*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])
/(15*a^3*f) + ((5*a + 4*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a
^2*f) - (Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(5*a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{-5a-4b-4bx^2}{x^4(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan\right.}{5af}$$

$$= \frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+}}{5af}$$

$$= -\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+}{}$$

$$= -\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+}{}$$

$$= -\frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}\,f} - \frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b}}{15a^3f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 12.24, size = 1374, normalized size = 8.08

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

```
[Out] -1/45*(Cos[e + f*x]^4*Cot[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*(9*a^4*ArcS
in[Sqrt[((a - b)*Sin[e + f*x]^2)/a]] + 9*a^4*ArcSin[Sqrt[((a - b)*Sin[e + f
*x]^2)/a]]*Tan[e + f*x]^2 - 18*a^3*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a
]]*Tan[e + f*x]^2 - 18*a^3*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e
 + f*x]^4 + 72*a^2*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x
]^4 + 72*a^2*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^6 +
```

```
144*a*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^6 + 144*a*b
^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^8 - 4*a^4*Hypergeo
metric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2*Sqrt[(Cos[e
 + f*x]^2*Sin[e + f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]
^2)))/a^2] + 4*a^3*b*Hypergeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/
a]*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*Sin[e + f*x]^2*(a^2 - b^2*Tan[e + f*
x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2] - 12*a^3*b*Hypergeometric2F1[2, 2,
5/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x]^2*Sin[e
+ f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2] + 12*
a^2*b^2*Hypergeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*
x]^4*Sqrt[(Cos[e + f*x]^2*Sin[e + f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-
1 + Tan[e + f*x]^2)))/a^2] + 168*a^2*b^2*Hypergeometric2F1[2, 2, 5/2, ((a -
 b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^6*Sqrt[(Cos[e + f*x]^2*Sin[e + f*x]^2*(
a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2] - 168*a*b^3*Hyp
ergeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^6*Sqrt[(
Cos[e + f*x]^2*Sin[e + f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e +
 f*x]^2)))/a^2] + 176*a*b^3*Hypergeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f
*x]^2)/a]*Tan[e + f*x]^8*Sqrt[(Cos[e + f*x]^2*Sin[e + f*x]^2*(a^2 - b^2*Tan
[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2] - 176*b^4*Hypergeometric2F1[
2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^8*Sqrt[(Cos[e + f*x]^2*
Sin[e + f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2]
 + 16*(a - b)*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 5/2}, ((a - b)*Sin[e +
 f*x]^2)/a]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^3*Sqrt[(Cos[e + f*x]^2*Si
n[e + f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2] -
 24*HypergeometricPFQ[{2, 2, 2}, {1, 5/2}, ((a - b)*Sin[e + f*x]^2)/a]*Tan[
e + f*x]^2*(a + b*Tan[e + f*x]^2)^2*Sqrt[(Cos[e + f*x]^2*Sin[e + f*x]^2*(a^
2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2]*(a^2 + 4*b^2*Tan[
e + f*x]^2 - a*(b + 4*b*Tan[e + f*x]^2))))/(a^4*f*Sqrt[a + b*Tan[e + f*x]^2
]*Sqrt[(Cos[e + f*x]^2*Sin[e + f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 +
 Tan[e + f*x]^2)))/a^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.42, size = 3741, normalized size = 22.01

method	result	size
default	Expression too large to display	3741

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/f*(-30*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/
2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b
^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(
f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*
```

(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(
f*x+e)*a^3-30*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^
(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e
)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*
I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3-30*cos(f*x+e)*sin(
f*x+e)*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)
*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)
+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^
(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^3+15*cos
(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/
2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3+15*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)*a^2*b+10*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^2+24*cos(f*x
+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3-24*cos(f*x+e)^2*((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3+23*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^
(1/2)+a-2*b)/a)^(1/2)*a^3-35*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)*a^3+15*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3-
9*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b-6*cos(f*x+e)
^6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^2+34*cos(f*x+e)^4*((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b+22*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)*a*b^2-40*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)/a)^(1/2)*a^2*b-26*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
*a*b^2+8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3-8*cos(f*x+e)^6*((2*I
b^(1/2)(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3-30*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*
(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+
1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos
(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)
*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a
-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^5*sin(f*x+e)*a^3+15*2^(1/2)*((I*cos(f
*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b
)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(
a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((
cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^
(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*co
s(f*x+e)^5*sin(f*x+e)*a^3-30*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)
b^(1/2)(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(co
s(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^

```
(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(
f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b
^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^3+15*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*
((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos
(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*El
lipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e)
,((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^
(1/2))*a^3+60*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^
(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^
(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e
)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I
*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2))*a^3-30*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*
b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f...
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 4.79, size = 457, normalized size = 2.69

$$\left[-\frac{15\,a^3\sqrt{-a+b}\,\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2+4\,\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^5+4\left(\left(15\,a^3-5\,a^2b-2\,ab^2-8\,b^3\right)\tan\left(fx+e\right)^4+3\,a^3-3\,a^2b-\left(5\,a^3-a^2b-4\,ab^2\right)\tan\left(fx+e\right)^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{60\,(a^4-a^3b)f\tan\left(fx+e\right)^5},\ -\frac{15\,\sqrt{a-b}\,a^3\arctan\left(-\frac{2\,\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}\,\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan\left(fx+e\right)^5+2\left(\left(15\,a^3-5\,a^2b-2\,ab^2-8\,b^3\right)\tan\left(fx+e\right)^4+3\,a^3-3\,a^2b-\left(5\,a^3-a^2b-4\,ab^2\right)\tan\left(fx+e\right)^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{30\,(a^4-a^3b)f\tan\left(fx+e\right)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*a^3*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*
(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(
f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(
f*x + e)^2 + 1))*tan(f*x + e)^5 + 4*((15*a^3 - 5*a^2*b - 2*a*b^2 - 8*b^3)*t
an(f*x + e)^4 + 3*a^3 - 3*a^2*b - (5*a^3 - a^2*b - 4*a*b^2)*tan(f*x + e)^2)
*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^5), -1/30*(15*sq
rt(a - b)*a^3*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)
/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)^5 + 2*((15*a^3 - 5*a^2*b - 2*
a*b^2 - 8*b^3)*tan(f*x + e)^4 + 3*a^3 - 3*a^2*b - (5*a^3 - a^2*b - 4*a*b^2)
```

```
*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^
5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^6}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2), x)
```

3.333 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}+\frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+a^2/(a-b)/b^2/
f/(a+b*tan(f*x+e)^2)^(1/2)+(a+b*tan(f*x+e)^2)^(1/2)/b^2/f
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3751, 457, 89, 65, 214}

$$\frac{a^2}{b^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + a^2/
((a - b)*b^2*f*Sqrt[a + b*Tan[e + f*x]^2]) + Sqrt[a + b*Tan[e + f*x]^2]/(b^
2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{(a-b)b(a+bx)^{3/2}} + \frac{1}{b\sqrt{a+bx}} + \frac{1}{(a-b)(1+x)\sqrt{a+bx}}\right) dx, x, \tan^2(\right.}{2f}$$

$$= \frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f} + \frac{\text{Subst}\left(\int \frac{}{(1+x}\right.}{}$$

$$= \frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.26, size = 84, normalized size = 0.86

$$\frac{b^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right) + (a-b)\left(2a+b+b\tan^2(e+fx)\right)}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (a -
 b)*(2*a + b + b*Tan[e + f*x]^2))/((a - b)*b^2*f*Sqrt[a + b*Tan[e + f*x]^2]
)
```

Maple [A]
time = 0.07, size = 130, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{\tan^2(fx+e)}{b\sqrt{a+b(\tan^2(fx+e))}}+\frac{2a}{b^2\sqrt{a+b(\tan^2(fx+e))}}+\frac{1}{b\sqrt{a+b(\tan^2(fx+e))}}+\frac{\arctan\left(\frac{\sqrt{\ldots}}{\ldots}\right.}{\ldots}}{f}$
default	$\frac{\frac{\tan^2(fx+e)}{b\sqrt{a+b(\tan^2(fx+e))}}+\frac{2a}{b^2\sqrt{a+b(\tan^2(fx+e))}}+\frac{1}{b\sqrt{a+b(\tan^2(fx+e))}}+\frac{\arctan\left(\frac{\sqrt{\ldots}}{\ldots}\right.}{\ldots}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(tan(f*x+e)^2/b/(a+b*tan(f*x+e)^2)^(1/2)+2*a/b^2/(a+b*tan(f*x+e)^2)^(1/
2)+1/b/(a+b*tan(f*x+e)^2)^(1/2)+1/(a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)
^2)^(1/2)/(-a+b)^(1/2))+1/(a-b)/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^5/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(91) = 182$.
time = 3.25, size = 448, normalized size = 4.57

$$\left[-\frac{\left(b^3\tan(fx+e)^2+ab^2\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2(4ab-3b^2)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)-4\left(2a^3-3a^2b+ab^2+\left(a^2b-2ab^2+b^3\right)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{4\left(\left(a^2b^3-2ab^4+b^5\right)f\tan(fx+e)^2+\left(a^3b^2-2a^2b^3+ab^4\right)f\right)}, \frac{\left(b^3\tan(fx+e)^2+ab^2\right)\sqrt{-a+b}\arctan\left(\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{b\tan(fx+e)^2+2a-b}\right)+2\left(2a^3-3a^2b+ab^2+\left(a^2b-2ab^2+b^3\right)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{2\left(\left(a^2b^3-2ab^4+b^5\right)f\tan(fx+e)^2+\left(a^3b^2-2a^2b^3+ab^4\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 +
2*(4*a*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*ta
n(f*x + e)^2 + 1)) - 4*(2*a^3 - 3*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*t
an(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan
(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), 1/2*((b^3*tan(f*x + e)^2 +
a*b^2)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan
```

```
(f*x + e)^2 + 2*a - b)) + 2*(2*a^3 - 3*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b
^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^3 - 2*a*b^4 + b^5)*
f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [A]
time = 0.56, size = 99, normalized size = 1.01

$$\frac{a^2}{(ab^2f-b^3f)\sqrt{b\tan(fx+e)^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{-a+b}}\right)}{(af-bf)\sqrt{-a+b}}+\frac{\sqrt{b\tan(fx+e)^2+a}}{b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] a^2/((a*b^2*f - b^3*f)*sqrt(b*tan(f*x + e)^2 + a)) + arctan(sqrt(b*tan(f*x
+ e)^2 + a)/sqrt(-a + b))/((a*f - b*f)*sqrt(-a + b)) + sqrt(b*tan(f*x + e)^
2 + a)/(b^2*f)
```

Mupad [B]
time = 13.73, size = 112, normalized size = 1.14

$$\frac{\sqrt{b\tan(e+fx)^2+a}}{b^2f}+\frac{a^2}{b^2f\sqrt{b\tan(e+fx)^2+a}\,(a-b)}+\frac{\mathrm{atan}\left(\frac{a\sqrt{b\tan(e+fx)^2+a}\,1\mathrm{i}-b\sqrt{b\tan(e+fx)^2+a}\,1\mathrm{i}}{(a-b)^{3/2}}\right)1\mathrm{i}}{f\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (a + b*tan(e + f*x)^2)^(1/2)/(b^2*f) + (atan((a*(a + b*tan(e + f*x)^2)^(1/2
)*1i - b*(a + b*tan(e + f*x)^2)^(1/2)*1i)/(a - b)^(3/2))*1i)/(f*(a - b)^(3/
2)) + a^2/(b^2*f*(a + b*tan(e + f*x)^2)^(1/2)*(a - b))
```

3.334 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}-\frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f-a/(a-b)/b/f/(a+
b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3751, 457, 79, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{a}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - a/((a -
 b)*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= -\frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+\right.}{2(a-b)f} \\
&= -\frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e}\right.}{(a-b)bf} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 75, normalized size = 1.03

$$\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\frac{a(-a+b)}{b\sqrt{a+b\tan^2(e+fx)}}}{(a-b)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (a*(-a + b))
/(b*Sqrt[a + b*Tan[e + f*x]^2]))/((a - b)^2*f)
```

Maple [A]
time = 0.06, size = 87, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{1}{b\sqrt{a+b\left(\tan^2(fx+e)\right)}}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}-\frac{1}{(a-b)\sqrt{a+b\left(\tan^2(fx+e)\right)}}}{f}$
default	$\frac{-\frac{1}{b\sqrt{a+b\left(\tan^2(fx+e)\right)}}-\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}-\frac{1}{(a-b)\sqrt{a+b\left(\tan^2(fx+e)\right)}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/b/(a+b*tan(f*x+e)^2)^(1/2)-1/(a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f*x
+e)^2)^(1/2)/(-a+b)^(1/2))-1/(a-b)/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(67) = 134$.
time = 2.98, size = 372, normalized size = 5.10

$$\left[-\frac{\left(b^2\tan\left(fx+e\right)^2+ab\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\left(4ab-3b^2\right)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)+4\sqrt{b\tan\left(fx+e\right)^2+a}\left(a^2-ab\right)}{4\left(\left(a^2b^2-2ab^3+b^4\right)f\tan\left(fx+e\right)^2+\left(a^3b-2a^2b^2+ab^3\right)f\right)}, -\frac{\left(b^2\tan\left(fx+e\right)^2+ab\right)\sqrt{-a+b}\arctan\left(\frac{2\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{-a+b}}{b\tan(fx+e)^2+2a-b}\right)+2\sqrt{b\tan\left(fx+e\right)^2+a}\left(a^2-ab\right)}{2\left(\left(a^2b^2-2ab^3+b^4\right)f\tan\left(fx+e\right)^2+\left(a^3b-2a^2b^2+ab^3\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*tan(f*x + e)^2 + a*b)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*
(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(
f*x + e)^2 + 1)) + 4*sqrt(b*tan(f*x + e)^2 + a)*(a^2 - a*b))/((a^2*b^2 - 2*
a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f), -1/2*((b^2*
tan(f*x + e)^2 + a*b)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a^2
- a*b))/((a^2*b^2 - 2*a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 +
a*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [A]
time = 0.51, size = 76, normalized size = 1.04

$$-\frac{\frac{b\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{(af-bf)\sqrt{-a+b}}+\frac{a}{\sqrt{b\tan\left(fx+e\right)^2+a}\,(af-bf)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a*f - b*f)*sqrt(-a +
b)) + a/(sqrt(b*tan(f*x + e)^2 + a)*(a*f - b*f)))/b
```

Mupad [B]
time = 13.10, size = 90, normalized size = 1.23

$$-\frac{a}{b\,f\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\left(a-b\right)}-\frac{\mathrm{atan}\left(\frac{a\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}-b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}}{{\left(a-b\right)}^{3/2}}\right)\,1\mathrm{i}}{f\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] - (atan((a*(a + b*tan(e + f*x)^2)^(1/2)*1i - b*(a + b*tan(e + f*x)^2)^(1/2)
*1i)/(a - b)^(3/2))*1i)/(f*(a - b)^(3/2)) - a/(b*f*(a + b*tan(e + f*x)^2)^(
1/2)*(a - b))
```

3.335 $\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}+\frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1/(a-b)/f/(a+b
*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3751, 455, 53, 65, 214}

$$\frac{1}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + 1/((
a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\Big(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+fx}{2(a-b)f}$$

$$= \frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\Big(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+f}}{(a-b)bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 56, normalized size = 0.81

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right)}{(-a+b)f\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)]/((-a + b)
*f*Sqrt[a + b*Tan[e + f*x]^2]))
```

Maple [A]
time = 0.05, size = 66, normalized size = 0.96

method	result	size
derivativedivides	$\frac{\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}+\frac{1}{(a-b)\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}}{f}$	66
default	$\frac{\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}+\frac{1}{(a-b)\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}}{f}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/(
a-b)/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(63) = 126.

time = 3.30, size = 346, normalized size = 5.01

$$\left[-\frac{\left(b\tan\left(fx+e\right)^{2}+a\right)\sqrt{a-b}\,\log\left(-\frac{b^{2}\tan(fx+e)^{4}+2\,(4\,ab-3\,b^{2})\tan(fx+e)^{2}+4\left(b\tan(fx+e)^{2}+2\,a-b\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{a-b}+8\,a^{2}-8\,ab+b^{2}}{\tan(fx+e)^{4}+2\,\tan(fx+e)^{2}+1}\right)-4\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,(a-b)}{4\left((a^{2}b-2\,ab^{2}+b^{3})f\tan\left(fx+e\right)^{2}+(a^{3}-2\,a^{2}b+ab^{2})f\right)},\ \frac{\left(b\tan\left(fx+e\right)^{2}+a\right)\sqrt{-a+b}\,\arctan\left(\frac{2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}}{b\tan(fx+e)^{2}+2\,a-b}\right)+2\,\sqrt{b\tan\left(fx+e\right)^{2}+a}\,(a-b)}{2\left((a^{2}b-2\,ab^{2}+b^{3})f\tan\left(fx+e\right)^{2}+(a^{3}-2\,a^{2}b+ab^{2})f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*tan(f*x + e)^2 + a)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a
*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x
+ e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x
+ e)^2 + 1)) - 4*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2*b - 2*a*b^2 + b^
3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f), 1/2*((b*tan(f*x + e)^2 +
a)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x
 + e)^2 + 2*a - b)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2*b - 2*a*b
^2 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f)]
```

Sympy [A]
time = 11.08, size = 56, normalized size = 0.81

$$\frac{1}{f\left(a-b\right)\sqrt{a+b\tan^{2}\left(e+fx\right)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tan^{2}\left(e+fx\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] 1/(f*(a - b)*sqrt(a + b*tan(e + f*x)**2)) + atan(sqrt(a + b*tan(e + f*x)**2
)/sqrt(-a + b))/(f*sqrt(-a + b)*(a - b))
```

Giac [A]
time = 0.49, size = 69, normalized size = 1.00

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{(af-bf)\sqrt{-a+b}}+\frac{1}{\sqrt{b\tan\left(fx+e\right)^{2}+a}\,(af-bf)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a*f - b*f)*sqrt(-a + b))
+ 1/(sqrt(b*tan(f*x + e)^2 + a)*(a*f - b*f))
```

Mupad [B]
time = 13.07, size = 85, normalized size = 1.23

$$\frac{1}{f\sqrt{b\tan(e+fx)^2+a}\ (a-b)}+\frac{\operatorname{atan}\left(\frac{a\sqrt{b\tan(e+fx)^2+a}\ 1\mathrm{i}-b\sqrt{b\tan(e+fx)^2+a}\ 1\mathrm{i}}{(a-b)^{3/2}}\right)\ 1\mathrm{i}}{f\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] 1/(f*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)) + (atan((a*(a + b*tan(e + f*x)^2
)^(1/2)*1i - b*(a + b*tan(e + f*x)^2)^(1/2)*1i)/(a - b)^(3/2))*1i)/(f*(a -
b)^(3/2))
```

3.336 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}-\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+arctanh((a+b*tan(f*x+e
)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f-b/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 457, 87, 162, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - b/(a*(a - b)*f*Sqrt[a
 + b*Tan[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-b-bx}{x(1+x)\sqrt{a+bx}}\,dx, x, \tan^2(e+\right.}{2a(a-b)f}$$

$$= -\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^2(e+fx\right.}{2af}$$

$$= -\frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^2(e+}\right.}{abf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 91, normalized size = 0.86

$$\frac{-a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\tan^2(e+fx)}{a}\right)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)]) + (a
- b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[e + f*x]^2)/a])/(a*(a - b)*
f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32887 vs. 2(92) = 184.
time = 0.97, size = 32888, normalized size = 310.26

method	result	size
default	Expression too large to display	32888

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(95) = 190$.
time = 3.46, size = 952, normalized size = 8.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*b*tan(f*x + e)^2 + a^3)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*s
qrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - (a
^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(a)*log(
(b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e
)^2) + 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 +
a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f), 1/2*(2*(a^2*b*tan
(f*x + e)^2 + a^3)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
+ b)/(a - b)) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x +
e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a)
+ 2*a)/tan(f*x + e)^2) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^
4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f),
 1/2*(2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sq
rt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) - (a^2*b*tan(f*x + e)^
2 + a^3)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - 2*(a^2*b - a*b^2)*sqrt(b*tan(
f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2
*a^4*b + a^3*b^2)*f), ((a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan
```

```
(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + (a^2*
b*tan(f*x + e)^2 + a^3)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqr
t(-a + b)/(a - b)) - (a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b -
2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e + fx)}{(a + b\tan^2(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 12.64, size = 1922, normalized size = 18.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] b/(f*(a + b*tan(e + f*x)^2)^(1/2)*(a*b - a^2)) - atanh((2*a^2*b^8*f^2*(a +
b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*a^
3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)) - (12*a^3*b^7
*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f
^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)) + (
30*a^4*b^6*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12
*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3
*f^2)) - (38*a^5*b^5*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^
```

8*f^2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)) + (24*a^6*b^4*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)) - (6*a^7*b^3*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)))/(f*(a^3)^(1/2)) + (atan((((((a + b*tan(e + f*x)^2)^(1/2)*(2*a^3*b^7*f^3 - 10*a^4*b^6*f^3 + 22*a^5*b^5*f^3 - 26*a^6*b^4*f^3 + 16*a^7*b^3*f^3 - 4*a^8*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(12*a^5*b^7*f^4 - 2*a^4*b^8*f^4 - 28*a^6*b^6*f^4 + 32*a^7*b^5*f^4 - 18*a^8*b^4*f^4 + 4*a^9*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8*f^5 - 56*a^6*b^7*f^5 + 160*a^7*b^6*f^5 - 240*a^8*b^5*f^5 + 200*a^9*b^4*f^5 - 88*a^10*b^3*f^5 + 16*a^11*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) + ((((a + b*tan(e + f*x)^2)^(1/2)*(2*a^3*b^7*f^3 - 10*a^4*b^6*f^3 + 22*a^5*b^5*f^3 - 26*a^6*b^4*f^3 + 16*a^7*b^3*f^3 - 4*a^8*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(2*a^4*b^8*f^4 - 12*a^5*b^7*f^4 + 28*a^6*b^6*f^4 - 32*a^7*b^5*f^4 + 18*a^8*b^4*f^4 - 4*a^9*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8*f^5 - 56*a^6*b^7*f^5 + 160*a^7*b^6*f^5 - 240*a^8*b^5*f^5 + 200*a^9*b^4*f^5 - 88*a^10*b^3*f^5 + 16*a^11*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3))/(2*a^3*b^6*f^2 - 6*a^4*b^5*f^2 + 6*a^5*b^4*f^2 - 2*a^6*b^3*f^2 - ((((a + b*tan(e + f*x)^2)^(1/2)*(2*a^3*b^7*f^3 - 10*a^4*b^6*f^3 + 22*a^5*b^5*f^3 - 26*a^6*b^4*f^3 + 16*a^7*b^3*f^3 - 4*a^8*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(12*a^5*b^7*f^4 - 2*a^4*b^8*f^4 - 28*a^6*b^6*f^4 + 32*a^7*b^5*f^4 - 18*a^8*b^4*f^4 + 4*a^9*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8*f^5 - 56*a^6*b^7*f^5 + 160*a^7*b^6*f^5 - 240*a^8*b^5*f^5 + 200*a^9*b^4*f^5 - 88*a^10*b^3*f^5 + 16*a^11*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2))/(f*(a - b)^3) + ((((a + b*tan(e + f*x)^2)^(1/2)*(2*a^3*b^7*f^3 - 10*a^4*b^6*f^3 + 22*a^5*b^5*f^3 - 26*a^6*b^4*f^3 + 16*a^7*b^3*f^3 - 4*a^8*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(2*a^4*b^8*f^4 - 12*a^5*b^7*f^4 + 28*a^6*b^6*f^4 - 32*a^7*b^5*f^4 + 18*a^8*b^4*f^4 - 4*a^9*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(8*a^5*b^8*f^5 - 56*a^6*b^7*f^5 + 160*a^7*b^6*f^5 - 240*a^8*b^5*f^5 + 200*a^9*b^4*f^5 - 88*a^10*b^3*f^5 + 16*a^11*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2))/(f*(a - b)^3)))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3)

3.337 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=157

$$\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}-\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e}$$

```
[Out] 1/2*(2*a+3*b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-arctanh((
a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f-1/2*(a-3*b)*b/a^2/(a-b)/
f/(a+b*tan(f*x+e)^2)^(1/2)-1/2*cot(f*x+e)^2/a/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3751, 457, 105, 157, 162, 65, 214}

$$\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{b(a-3b)}{2a^2f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) - A
rcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f) - ((a - 3*
b)*b)/(2*a^2*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - Cot[e + f*x]^2/(2*a*f*
Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2a+3b)+\frac{3bx}{2}}{x(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2af}$$

$$= -\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+f}}{\sqrt{a-b}}\right.}{(a-b)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.30, size = 115, normalized size = 0.73

$$\frac{-2a^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(a\cot^2(e+fx)+(2a+3b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\tan^2(e+fx)}{a}\right)\right)}{2a^2(-a+b)f\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*a^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (
a - b)*(a*Cot[e + f*x]^2 + (2*a + 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 +
(b*Tan[e + f*x]^2)/a]))/(2*a^2*(-a + b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 54352 vs. $2(135) = 270$.
time = 1.41, size = 54353, normalized size = 346.20

method	result	size
default	Expression too large to display	54353

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(140) = 280$.
time = 4.75, size = 1300, normalized size = 8.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^3*b*tan(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(a - b)*log((b*tan
(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x
+ e)^2 + 1)) - ((2*a^3*b - a^2*b^2 - 4*a*b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a
^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x +
e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*(a^4
 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(b
*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^
6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/4*(4*(a^3*b*tan(f*x + e)^4 + a
^4*tan(f*x + e)^2)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
+ b)/(a - b)) - ((2*a^3*b - a^2*b^2 - 4*a*b^3 + 3*b^4)*tan(f*x + e)^4 + (2*
a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x +
 e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*(a^
4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(
b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a
```

```
^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/2*(((2*a^3*b - a^2*b^2 - 4*a*
b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x
 + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + (a^3*b*ta
n(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sq
rt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + (a^
4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(
b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a
^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/2*(((2*a^3*b - a^2*b^2 - 4*a*
b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x
 + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 2*(a^3*b*
tan(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x +
e)^2 + a)*sqrt(-a + b)/(a - b)) + (a^4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2
*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4
*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)
^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 12.54, size = 2483, normalized size = 15.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] (b^2/(a*b - a^2) + (b*(a + b*tan(e + f*x)^2)*(a - 3*b))/(2*a*(a*b - a^2)))/
(f*(a + b*tan(e + f*x)^2)^(3/2) - a*f*(a + b*tan(e + f*x)^2)^(1/2)) - (atan
((((((a + b*tan(e + f*x)^2)^(1/2)*(144*a^6*b^9*f^3 - 528*a^7*b^8*f^3 + 544*
a^8*b^7*f^3 + 160*a^9*b^6*f^3 - 496*a^10*b^5*f^3 - 16*a^11*b^4*f^3 + 320*a^
12*b^3*f^3 - 128*a^13*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(512*a^9*b^8*f^4 - 9
6*a^8*b^9*f^4 - 1056*a^10*b^7*f^4 + 1024*a^11*b^6*f^4 - 416*a^12*b^5*f^4 +
32*a^14*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(256*a^10
*b^8*f^5 - 1792*a^11*b^7*f^5 + 5120*a^12*b^6*f^5 - 7680*a^13*b^5*f^5 + 6400
*a^14*b^4*f^5 - 2816*a^15*b^3*f^5 + 512*a^16*b^2*f^5))/(4*f*(a - b)^3)))/(2
*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) + ((((a + b*tan(e + f*x)
^2)^(1/2)*(144*a^6*b^9*f^3 - 528*a^7*b^8*f^3 + 544*a^8*b^7*f^3 + 160*a^9*b^
6*f^3 - 496*a^10*b^5*f^3 - 16*a^11*b^4*f^3 + 320*a^12*b^3*f^3 - 128*a^13*b^
2*f^3))/2 + (((a - b)^3)^(1/2)*(96*a^8*b^9*f^4 - 512*a^9*b^8*f^4 + 1056*a^1
0*b^7*f^4 - 1024*a^11*b^6*f^4 + 416*a^12*b^5*f^4 - 32*a^14*b^3*f^4 + ((a +
b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(256*a^10*b^8*f^5 - 1792*a^11*b^7
*f^5 + 5120*a^12*b^6*f^5 - 7680*a^13*b^5*f^5 + 6400*a^14*b^4*f^5 - 2816*a^1
5*b^3*f^5 + 512*a^16*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^
3)^(1/2)*1i)/(f*(a - b)^3))/(144*a^6*b^8*f^2 - 384*a^7*b^7*f^2 + 256*a^8*b^
6*f^2 + 96*a^9*b^5*f^2 - 144*a^10*b^4*f^2 + 32*a^11*b^3*f^2 - ((((a + b*tan
(e + f*x)^2)^(1/2)*(144*a^6*b^9*f^3 - 528*a^7*b^8*f^3 + 544*a^8*b^7*f^3 + 1
60*a^9*b^6*f^3 - 496*a^10*b^5*f^3 - 16*a^11*b^4*f^3 + 320*a^12*b^3*f^3 - 12
8*a^13*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(512*a^9*b^8*f^4 - 96*a^8*b^9*f^4 -
 1056*a^10*b^7*f^4 + 1024*a^11*b^6*f^4 - 416*a^12*b^5*f^4 + 32*a^14*b^3*f^4
 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(256*a^10*b^8*f^5 - 1792
*a^11*b^7*f^5 + 5120*a^12*b^6*f^5 - 7680*a^13*b^5*f^5 + 6400*a^14*b^4*f^5 -
 2816*a^15*b^3*f^5 + 512*a^16*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*
((a - b)^3)^(1/2))/(f*(a - b)^3) + ((((a + b*tan(e + f*x)^2)^(1/2)*(144*a^6
*b^9*f^3 - 528*a^7*b^8*f^3 + 544*a^8*b^7*f^3 + 160*a^9*b^6*f^3 - 496*a^10*b
^5*f^3 - 16*a^11*b^4*f^3 + 320*a^12*b^3*f^3 - 128*a^13*b^2*f^3))/2 + (((a -
 b)^3)^(1/2)*(96*a^8*b^9*f^4 - 512*a^9*b^8*f^4 + 1056*a^10*b^7*f^4 - 1024*a
^11*b^6*f^4 + 416*a^12*b^5*f^4 - 32*a^14*b^3*f^4 + ((a + b*tan(e + f*x)^2)^
(1/2)*((a - b)^3)^(1/2)*(256*a^10*b^8*f^5 - 1792*a^11*b^7*f^5 + 5120*a^12*b
^6*f^5 - 7680*a^13*b^5*f^5 + 6400*a^14*b^4*f^5 - 2816*a^15*b^3*f^5 + 512*a^
16*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2))/(f*(a -
b)^3)))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) + (atanh((216*a^5*b^11*f^2*(a +
 b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2
 + 936*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 +
 440*a^9*b^5*f^2 - 240*a^10*b^4*f^2)) - (864*a^6*b^10*f^2*(a + b*tan(e + f*
x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 936*a^5*b^
9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*
f^2 - 240*a^10*b^4*f^2)) + (936*a^7*b^9*f^2*(a + b*tan(e + f*x)^2)^(1/2))/(
(a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 936*a^5*b^9*f^2 + 496*a^
6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*f^2 - 240*a^10
```

```
*b^4*f^2)) + (496*a^8*b^8*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2
16*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 936*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 14
64*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*f^2 - 240*a^10*b^4*f^2)) - (
1464*a^9*b^7*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f
^2 - 864*a^4*b^10*f^2 + 936*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^
2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*f^2 - 240*a^10*b^4*f^2)) + (480*a^10*b^6*
f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*
b^10*f^2 + 936*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b
^6*f^2 + 440*a^9*b^5*f^2 - 240*a^10*b^4*f^2)) + (440*a^11*b^5*f^2*(a + b*ta
n(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 93
6*a^5*b^9*f^2 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*
a^9*b^5*f^2 - 240*a^10*b^4*f^2)) - (240*a^12*b^4*f^2*(a + b*tan(e + f*x)^2)
^(1/2))/((a^5)^(1/2)*(216*a^3*b^11*f^2 - 864*a^4*b^10*f^2 + 936*a^5*b^9*f^2
 + 496*a^6*b^8*f^2 - 1464*a^7*b^7*f^2 + 480*a^8*b^6*f^2 + 440*a^9*b^5*f^2 -
 240*a^10*b^4*f^2)))*(2*a + 3*b))/(2*f*(a^5)^(1/2))
```

3.338 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{(8a^2+12ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}+\frac{b(4a^2}{8a^3(a-b)f}$$

```
[Out] -1/8*(8*a^2+12*a*b+15*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2
)/f+arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1/8*b*(4*a^
2+3*a*b-15*b^2)/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/8*(4*a+5*b)*cot(f*x+
e)^2/a^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^4/a/f/(a+b*tan(f*x+e)^2)
^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 457, 105, 156, 157, 162, 65, 214}

$$\frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}}-\frac{(8a^2+12ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{b(4a^2+3ab-15b^2)}{8a^3f(a-b)\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}-\frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((8*a^2 + 12*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]
)/(a^(7/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3
/2)*f) + (b*(4*a^2 + 3*a*b - 15*b^2))/(8*a^3*(a - b)*f*Sqrt[a + b*Tan[e + f
*x]^2]) + ((4*a + 5*b)*Cot[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Tan[e + f*x]^2])
 - Cot[e + f*x]^4/(4*a*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+5b)+\frac{5bx}{2}}{x^2(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx)\right)}{4af}$$

$$= \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(8a^2+\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{b(4a^2+3ab-15b^2)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{c}\cdots}{4af\sqrt{a\cdots}}$$

$$= \frac{b(4a^2+3ab-15b^2)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{c}\cdots}{4af\sqrt{a\cdots}}$$

$$= \frac{b(4a^2+3ab-15b^2)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{c}\cdots}{4af\sqrt{a\cdots}}$$

$$= -\frac{(8a^2+12ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+\cdots}}{\cdots}\right.}{(a-\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.80, size = 142, normalized size = 0.66

$$\frac{8a^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(a\cot^2(e+fx)\left(-4a-5b+2a\cot^2(e+fx)\right)-\left(8a^2+12ab+15b^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\tan^2(e+fx)}{a}\right)\right)}{8a^3(-a+b)f\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (8*a^3*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (a
 - b)*(a*Cot[e + f*x]^2*(-4*a - 5*b + 2*a*Cot[e + f*x]^2) - (8*a^2 + 12*a*b
 + 15*b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[e + f*x]^2)/a]))/(8*a
^3*(-a + b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79933 vs. 2(189) = 378.
time = 2.10, size = 79934, normalized size = 371.79

method	result	size
default	Expression too large to display	79934

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 4.44, size = 1574, normalized size = 7.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(a - b)*log((b*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x
```

```
 + e)^2 + 1)) - ((8*a^4*b - 4*a^3*b^2 - a^2*b^3 - 18*a*b^4 + 15*b^5)*tan(f*
x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)
^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) +
2*a)/tan(f*x + e)^2) + 2*(2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2
- 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*
a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 +
a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4), 1/
16*(16*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(-a + b)*arctan(-sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + ((8*a^4*b - 4*a^3*b^2 - a^2
*b^3 - 18*a*b^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*
a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(
b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(2*a^5 - 4*a^4*b +
 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4 - (
4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e
)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2*a^6*b
+ a^5*b^2)*f*tan(f*x + e)^4), 1/8*(((8*a^4*b - 4*a^3*b^2 - a^2*b^3 - 18*a*b
^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*
a*b^4)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/
a) - 4*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(a - b)*log((b*tan(f
*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x +
e)^2 + 1)) - (2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3
 + 15*a*b^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan
(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*t
an(f*x + e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4), 1/8*(((8*a^4*b
 - 4*a^3*b^2 - a^2*b^3 - 18*a*b^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4
*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(
b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 8*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x +
 e)^4)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)
) - (2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b
^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan(f*x + e)
^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x +
e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4)]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 13.13, size = 2118, normalized size = 9.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] (atan((((((a + b*tan(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*b
^10*f^3 + 854016*a^11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3 -
 680960*a^14*b^6*f^3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 327680*a
^17*b^3*f^3 - 131072*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(638976*a^13*b^9
*f^4 - 122880*a^12*b^10*f^4 - 1318912*a^14*b^8*f^4 + 1376256*a^15*b^7*f^4 -
 794624*a^16*b^6*f^4 + 311296*a^17*b^5*f^4 - 122880*a^18*b^4*f^4 + 32768*a^
19*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144*a^15*b
^8*f^5 - 1835008*a^16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18*b^5*f^5
 + 6553600*a^19*b^4*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f^5))/(4*f
*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) + ((((a
+ b*tan(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*b^10*f^3 + 854
016*a^11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3 - 680960*a^14*
b^6*f^3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 327680*a^17*b^3*f^3 -
 131072*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(122880*a^12*b^10*f^4 - 63897
6*a^13*b^9*f^4 + 1318912*a^14*b^8*f^4 - 1376256*a^15*b^7*f^4 + 794624*a^16*
b^6*f^4 - 311296*a^17*b^5*f^4 + 122880*a^18*b^4*f^4 - 32768*a^19*b^3*f^4 +
((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144*a^15*b^8*f^5 - 1835
008*a^16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18*b^5*f^5 + 6553600*a^
19*b^4*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f^5))/(4*f*(a - b)^3)))
/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3))/(230400*a^9*b^10*f^2
 - 552960*a^10*b^9*f^2 + 301056*a^11*b^8*f^2 + 36864*a^12*b^7*f^2 + 123904*
a^13*b^6*f^2 - 147456*a^14*b^5*f^2 - 24576*a^15*b^4*f^2 + 32768*a^16*b^3*f^
2 - ((((a + b*tan(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*b^10
*f^3 + 854016*a^11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3 - 68
0960*a^14*b^6*f^3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 327680*a^17
```

*b^3*f^3 - 131072*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(638976*a^13*b^9*f^4 - 122880*a^12*b^10*f^4 - 1318912*a^14*b^8*f^4 + 1376256*a^15*b^7*f^4 - 794624*a^16*b^6*f^4 + 311296*a^17*b^5*f^4 - 122880*a^18*b^4*f^4 + 32768*a^19*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144*a^15*b^8*f^5 - 1835008*a^16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18*b^5*f^5 + 6553600*a^19*b^4*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2))/(f*(a - b)^3) + ((((a + b*tan(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*b^10*f^3 + 854016*a^11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3 - 680960*a^14*b^6*f^3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 327680*a^17*b^3*f^3 - 131072*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(122880*a^12*b^10*f^4 - 638976*a^13*b^9*f^4 + 1318912*a^14*b^8*f^4 - 1376256*a^15*b^7*f^4 + 794624*a^16*b^6*f^4 - 311296*a^17*b^5*f^4 + 122880*a^18*b^4*f^4 - 32768*a^19*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144*a^15*b^8*f^5 - 1835008*a^16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18*b^5*f^5 + 6553600*a^19*b^4*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2))/(f*(a - b)^3)))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) - (b^3/(a*(a - b)) - (b*(a + b*tan(e + f*x)^2)*(5*a*b + 4*a^2 - 25*b^2))/(8*(a^2*b - a^3)) + (b*(a + b*tan(e + f*x)^2)^2*(3*a*b + 4*a^2 - 15*b^2))/(8*(a^3*b - a^4)))/(f*(a + b*tan(e + f*x)^2)^(5/2) + a^2*f*(a + b*tan(e + f*x)^2)^(1/2) - 2*a*f*(a + b*tan(e + f*x)^2)^(3/2)) - (atan((a^15*b^12*(a + b*tan(e + f*x)^2)^(1/2)*3375i - a^16*b^11*(a + b*tan(e + f*x)^2)^(1/2)*12150i + a^17*b^10*(a + b*tan(e + f*x)^2)^(1/2)*13905i - a^18*b^9*(a + b*tan(e + f*x)^2)^(1/2)*6912i + a^19*b^8*(a + b*tan(e + f*x)^2)^(1/2)*10953i - a^20*b^7*(a + b*tan(e + f*x)^2)^(1/2)*16542i + a^21*b^6*(a + b*tan(e + f*x)^2)^(1/2)*7343i - a^22*b^5*(a + b*tan(e + f*x)^2)^(1/2)*1932i + a^23*b^4*(a + b*tan(e + f*x)^2)^(1/2)*4200i - a^24*b^3*(a + b*tan(e + f*x)^2)^(1/2)*2240i)/(a^7*(a^7)^(1/2)*(a^7*(6912*a*b^9 - 13905*b^10 - 10953*a^2*b^8 + 16542*a^3*b^7 - 7343*a^4*b^6 + 1932*a^5*b^5 - 4200*a^6*b^4 + 2240*a^7*b^3) - 3375*a^5*b^12 + 12150*a^6*b^11)))*(12*a*b + 8*a^2 + 15*b^2)*1i)/(8*f*(a^7)^(1/2))

3.339 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{5/2}f}-\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-1/2*
(3*a+2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+1/
2*(3*a-b)*(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a-b)/b^2/f-a*tan(f*x+e)^3/(a
-b)/b/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 481, 596, 537, 223, 212, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{5/2}f}+\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b^2f(a-b)}-\frac{a\tan^3(e+fx)}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - ((3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*
x]^2]])/(2*b^(5/2)*f) - (a*Tan[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Tan[e +
f*x]^2]) + ((3*a - b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a - b)*b
^2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(3a-b)x^2)}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{(a-b)bf}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+f}}{2(a-b)b^2f}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+f}}{2(a-b)b^2f}$$

$$= -\frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+f}}{2(a-b)b^2f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+f}}\right.}{2b^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 5.91, size = 327, normalized size = 1.80

$$\frac{\left(2(a-b)\left(3a^2-b^2+(3a^2-2ab+b^2)\cos(2(e+fx))\right)\csc(2(e+fx))-\sqrt{2}a(3a^2-4ab+b^2)\cot(e+fx)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+2\sqrt{2}ab^2\cot(e+fx)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sec^2(e+fx)\sin(2(e+fx))\tan(e+fx)}{4\sqrt{2}(a-b)^2b^2f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*(a - b)*(3*a^2 - b^2 + (3*a^2 - 2*a*b + b^2)*Cos[2*(e + f*x)])*Csc[2*(e
 + f*x)] - Sqrt[2]*a*(3*a^2 - 4*a*b + b^2)*Cot[e + f*x]*Sqrt[((a + b + (a -
 b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 2*Sqrt[2]*a*b^2*C
ot[e + f*x]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Ell
```

```
ipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)]*Tan[e + f*x])/(
4*Sqrt[2]*(a - b)^2*b^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f
*x]^2])
```

Maple [A]
time = 0.06, size = 270, normalized size = 1.48

method	result
derivativedivides	$\frac{\frac{\tan^3(fx+e)}{2b\sqrt{a+b\left(\tan^2(fx+e)\right)}}-\frac{3a\left(-\frac{\tan(fx+e)}{b\sqrt{a+b\left(\tan^2(fx+e)\right)}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2\right.}\right.}{b^{\frac{3}{2}}}\right.}{2b}}{}$
default	$\frac{\frac{\tan^3(fx+e)}{2b\sqrt{a+b\left(\tan^2(fx+e)\right)}}-\frac{3a\left(-\frac{\tan(fx+e)}{b\sqrt{a+b\left(\tan^2(fx+e)\right)}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2\right.}\right.}{b^{\frac{3}{2}}}\right.}{2b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*tan(f*x+e)^3/b/(a+b*tan(f*x+e)^2)^(1/2)-3/2*a/b*(-tan(f*x+e)/b/(a+
b*tan(f*x+e)^2)^(1/2)+1/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1
/2)))+tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(1/2)-1/b^(3/2)*ln(b^(1/2)*tan(f*x+e)
+(a+b*tan(f*x+e)^2)^(1/2))+tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)-1/(a-b)^2*
(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)
^(1/2)*tan(f*x+e))+1/(a-b)*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.73, size = 1253, normalized size = 6.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b - 4*a^2*b^2 - a*b^3 +
 2*b^4)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt(
-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((a^2*b^2 - 2*a*b^3 + b
^4)*tan(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan
(f*x + e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 -
2*a^2*b^4 + a*b^5)*f), 1/2*((3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b
 - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + (b^4*tan(f*x + e)^2 + a*b^3)*sqr
t(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqr
t(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + ((a^2*b^2 - 2*a*b^3 + b
^4)*tan(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan
(f*x + e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 -
2*a^2*b^4 + a*b^5)*f), -1/4*(4*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt(a - b)*arc
tan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (3*a^4 - 4*a^
3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e
)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*
tan(f*x + e) + a) - 2*((a^2*b^2 - 2*a*b^3 + b^4)*tan(f*x + e)^3 + (3*a^3*b
 - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^4 -
2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 - 2*a^2*b^4 + a*b^5)*f), -1/2*(2
*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a
)/(sqrt(a - b)*tan(f*x + e))) - (3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a
^3*b - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - ((a^2*b^2 - 2*a*b^3 + b^4)*t
an(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x
+ e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3*b^3 - 2*a^2
*b^4 + a*b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^6}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.340 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f}-\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f+arcta
nh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-a*tan(f*x+e)/(a-b
)/b/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3751, 481, 537, 223, 212, 385, 209}

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f}-\frac{a\tan(e+fx)}{bf(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(b^(3/2)*
f) - (a*Tan[e + f*x])/((a - b)*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+(a-b)x^2}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+ \right.}{(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+ \right.}{(a-b)f}$$

$$= -\frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+f}{\sqrt{a+b\tan^2(}}\right.}{(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{}{(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 1.95, size = 250, normalized size = 2.03

$$\frac{a\left(-a+b+\frac{(a-b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)}{\sqrt{2}}-\frac{b\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)}{\sqrt{2}}\right)\sec^2(e+fx)\sin(2(e+fx))}{\sqrt{2}\,(a-b)^2bf\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(-a + b + ((a - b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]/Sqrt[2]], 1])/Sqrt[2] - (b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)
*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])/Sqrt[2])*Sec[e + f*x]^2
*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)^2*b*f*Sqrt[(a + b + (a - b)*Cos[2*(e +
f*x)])*Sec[e + f*x]^2])
```

Maple [A]
time = 0.06, size = 182, normalized size = 1.48

method	result
derivativedivides	$-\frac{\tan(fx+e)}{b\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\right)}{b^{\frac{3}{2}}}-\frac{\tan(fx+e)}{a\sqrt{a+b\left(\tan^2\left(f\right.\right.}}$
default	$-\frac{\tan(fx+e)}{b\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2\left(fx+e\right)\right)}\right)}{b^{\frac{3}{2}}}-\frac{\tan(fx+e)}{a\sqrt{a+b\left(\tan^2\left(f\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(1/2)+1/b^(3/2)*ln(b^(1/2)*tan(f*x+e)
+(a+b*tan(f*x+e)^2)^(1/2))-tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)+1/(a-b)^2*
(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)
^(1/2)*tan(f*x+e))-1/(a-b)*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(115) = 230$.
time = 5.48, size = 1016, normalized size = 8.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt
(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x +
 e) + a) + (b^3*tan(f*x + e)^2 + a*b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*
x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan
(f*x + e)^2 + 1)) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a)*tan(f*x +
e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*
b^4)*f), -1/2*(2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x +
```

```
 e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))
) - (b^3*tan(f*x + e)^2 + a*b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) + 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e))/((a
^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)
, 1/2*(2*(b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x +
e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2
*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan
(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f
*x + e)^2 + a)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 +
(a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), ((b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)
*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - (a^3 - 2*
a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) - (a^2*b - a*b^2)*sqrt(b
*tan(f*x + e)^2 + a)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e
)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
[Out] Integral(tan(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^4}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.341 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}+\frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f+tan(
f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3751, 482, 385, 209}

$$\frac{\tan(e+fx)}{f(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) + Tan[e + f*x]/((a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+f$$

$$= \frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} + \frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

Mathematica [A]
time = 2.22, size = 154, normalized size = 1.90

$$\frac{\tan(e+fx)\left(\tanh^{-1}\left(\frac{\sqrt{\frac{(-a+b)\tan^2(e+fx)}{a}}}{\sqrt{1+\frac{b\tan^2(e+fx)}{a}}}\right)(b+a\cot^2(e+fx))\sqrt{\frac{(-a+b)\tan^2(e+fx)}{a}}+(a-b)\sqrt{1+\frac{b\tan^2(e+fx)}{a}}\right)}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}\sqrt{1+\frac{b\tan^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] (Tan[e + f*x]*(ArcTanh[Sqrt[((-a + b)*Tan[e + f*x]^2)/a]/Sqrt[1 + (b*Tan[e
+ f*x]^2)/a]]*(b + a*Cot[e + f*x]^2)*Sqrt[((-a + b)*Tan[e + f*x]^2)/a] + (a
 - b)*Sqrt[1 + (b*Tan[e + f*x]^2)/a]))/((a - b)^2*f*Sqrt[a + b*Tan[e + f*x]
^2]*Sqrt[1 + (b*Tan[e + f*x]^2)/a])
```

Maple [A]
time = 0.06, size = 126, normalized size = 1.56

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{a\sqrt{a+b\left(\tan^2(fx+e)\right)}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{(a-b)^2b^2}+\cdots}{f}$
default	$\frac{\frac{\tan(fx+e)}{a\sqrt{a+b\left(\tan^2(fx+e)\right)}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{(a-b)^2b^2}+\cdots}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)-1/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*
arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/(
a-b)*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 1.74, size = 299, normalized size = 3.69

$$\left[\frac{\left(b\tan\left(fx+e\right)^2+a\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)+2\sqrt{b\tan\left(fx+e\right)^2+a}\,(a-b)\tan\left(fx+e\right)}{2\left(\left(a^2b-2ab^2+b^3\right)f\tan\left(fx+e\right)^2+\left(a^3-2a^2b+ab^2\right)f\right)},-\frac{\left(b\tan\left(fx+e\right)^2+a\right)\sqrt{a-b}\,\arctan\left(-\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{a-b}\,\tan(fx+e)}\right)-\sqrt{b\tan\left(fx+e\right)^2+a}\,(a-b)\tan\left(fx+e\right)}{\left(a^2b-2ab^2+b^3\right)f\tan\left(fx+e\right)^2+\left(a^3-2a^2b+ab^2\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*tan(f*x + e)^2 + a)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 -
2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2
 + 1)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a - b)*tan(f*x + e))/((a^2*b - 2*a*b
^2 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f), -((b*tan(f*x + e)^
2 + a)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x
+ e))) - sqrt(b*tan(f*x + e)^2 + a)*(a - b)*tan(f*x + e))/((a^2*b - 2*a*b^2
 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e + fx)}{(a + b\tan^2(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e + f\,x)^2}{\left(b\tan\left(e + f\,x\right)^2 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.342 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*tan
(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3742, 390, 385, 209}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\tan(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Tan[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 :> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e+\right.}{(a-b)f} \\
&= -\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\tan(e+}{\sqrt{a+b\tan^2}}\right.}{(a-b)f} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.22, size = 214, normalized size = 2.52

$$\frac{4\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(a(a-b)\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\tan^2(e+fx)+\frac{15(3a+2b\tan^2(e+fx))\left(-2\text{ArcSin}\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\right)(a\cos^2(e+fx)+b\sin^2(e+fx))+a\sqrt{\frac{(a-b)\sin^2(2(e+fx))\,(a+b\tan^2(e+fx))}{a^2}}\right)}{\left(\frac{(a-b)\sin^2(2(e+fx))(a+b\tan^2(e+fx))}{a^2}\right)^{3/2}}\right)}{15a^4f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

```
[Out] (4*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]*(a*(a - b)*Hyperg
eometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2 + (15*(3*
a + 2*b*Tan[e + f*x]^2)*(-2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a*Cos
[e + f*x]^2 + b*Sin[e + f*x]^2) + a*Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b
*Tan[e + f*x]^2))/a^2]))/(((a - b)*Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2
))/a^2)^(3/2)))/(15*a^4*f)
```

Maple [A]
time = 0.00, size = 102, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{(a-b)^2b^2}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\left(\tan^2(fx+e)\right)}}}{f}$
default	$\frac{\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{(a-b)^2b^2}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\left(\tan^2(fx+e)\right)}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+
b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/(a-b)*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)
^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 3.25, size = 324, normalized size = 3.81

$$\left[\frac{\left(ab\tan(fx+e)^2+a^2\right)\sqrt{-a+b}\,\log\left(-\frac{(a-2b)\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\,\sqrt{-a+b}\,\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)-2\sqrt{b\tan(fx+e)^2+a}\,(ab-b^2)\tan(fx+e)}{2\left((a^3b-2a^2b^2+ab^3)f\tan(fx+e)^2+(a^4-2a^3b+a^2b^2)f\right)},\ \frac{\left(ab\tan(fx+e)^2+a^2\right)\sqrt{a-b}\,\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{a-b}\,\tan(fx+e)}\right)-\sqrt{b\tan(fx+e)^2+a}\,(ab-b^2)\tan(fx+e)}{(a^3b-2a^2b^2+ab^3)f\tan(fx+e)^2+(a^4-2a^3b+a^2b^2)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^
2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a^3*b
 - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), ((a*
b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqr
t(a - b)*tan(f*x + e))) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x +
e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^
2)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan^{2}\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.343 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2(a-b)f}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*co
t(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-(a-2*b)*cot(f*x+e)*(a+b*tan(f*x
+e)^2)^(1/2)/a^2/(a-b)/f
```

Rubi [A]
time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 483, 597, 12, 385, 209}

$$-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2f(a-b)}-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\cot(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - (b*Cot[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - ((a -
 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a^2*(a - b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-2b-2bx^2}{x^2(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(\right.}{a(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+}}{a^2(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+}}{a^2(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+}}{a^2(a-b)f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.01, size = 882, normalized size = 6.89

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Cos[e + f*x]^2*Cot[e + f*x]*((3*a*Csc[e + f*x]^2)/(a - b) + (12*b*Sec[e
+ f*x]^2)/(a - b) + (16*(a - b)*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[e
 + f*x]^2)/a]*Sin[e + f*x]^2)/(15*a) + (8*(a - b)*HypergeometricPFQ[{2, 2,
2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(15*a) + (8*b^2*S
ec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)) + (8*(a - b)*b*Hypergeometric2F1[
2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*a^
2) + (16*(a - b)*b*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e +
```

```
f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(15*a^2) + (8*(a - b)*b^2*Hyperge
ometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*
x]^4)/(5*a^3) + (8*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a -
 b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(15*a^3) - (3*ArcSin[
Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/((((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt
[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) - (12*b*ArcSin[Sqrt[((a - b)*S
in[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sq
rt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) - (8*b^2*ArcSin[Sqrt[((a - b
)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*(((a - b)*Sin[e + f*x]^2)/a)^(3/
2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) + (3*ArcSin[Sqrt[((a -
b)*Sin[e + f*x]^2)/a]])/Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*
Tan[e + f*x]^2))/a^2] + (12*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[
e + f*x]^2)/(a*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f
*x]^2))/a^2]) + (8*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x
]^4)/(a^2*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2
))/a^2])))/(a*f*Sqrt[a + b*Tan[e + f*x]^2]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.44, size = 1305, normalized size = 10.20

method	result	size
default	Expression too large to display	1305

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a
-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)
/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f
*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2
)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)*sin(f*x
+e)*a^2-2*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+
cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1
/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x
+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-
b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*x
+e)*sin(f*x+e)*a^2+2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-
b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)
/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)
+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/
2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^2*sin(f*x+e)-2*2^(1/2)*((I*cos(f*x+e)*b
^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(
```

```
f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1
/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*
x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2
)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(
1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^2*sin(f*x+e)+((2*I*b^(1/2)*(a-b)^(1/2)+
a-2*b)/a)^(1/2)*cos(f*x+e)^2*a^2-2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2
)*cos(f*x+e)^2*a*b+2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*cos(f*x+e)^2
*b^2+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b-2*((2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)/a)^(1/2)*b^2)*cos(f*x+e)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/co
s(f*x+e)^2)^(3/2)/sin(f*x+e)/a^2/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/
(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A]
time = 3.21, size = 493, normalized size = 3.85

$$\left[\frac{\left(a^2 b\tan\left(fx+e\right)^3+a^3\tan\left(fx+e\right)\right)\sqrt{-a+b}\,\log\left(-\frac{\left(a^2-8\,ab+8\,b^2\right)\tan(fx+e)^4-2\left(3\,a^2-4\,ab\right)\tan(fx+e)^2+a^2-4\left(\left(a-2\,b\right)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{-a+b}}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)-4\left(a^3-2\,a^2b+ab^2+\left(a^2b-3\,ab^2+2\,b^3\right)\tan\left(fx+e\right)^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{4\left(\left(a^4b-2\,a^3b^2+a^2b^3\right)f\tan\left(fx+e\right)^3+\left(a^5-2\,a^4b+a^3b^2\right)f\tan\left(fx+e\right)\right)},\ -\frac{\left(a^2 b\tan\left(fx+e\right)^3+a^3\tan\left(fx+e\right)\right)\sqrt{a-b}\,\arctan\left(-\frac{2\sqrt{b\tan\left(fx+e\right)^2+a}\,\sqrt{a-b}\,\tan(fx+e)}{\left(a-2\,b\right)\tan(fx+e)^2-a}\right)+2\left(a^3-2\,a^2b+ab^2+\left(a^2b-3\,ab^2+2\,b^3\right)\tan\left(fx+e\right)^2\right)\sqrt{b\tan\left(fx+e\right)^2+a}}{2\left(\left(a^4b-2\,a^3b^2+a^2b^3\right)f\tan\left(fx+e\right)^3+\left(a^5-2\,a^4b+a^3b^2\right)f\tan\left(fx+e\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/4*((a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))*sqrt(-a + b)*log(-((a^2 - 8
*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*(
(a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(a^3 - 2*a^2*b + a*b^
2 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(
(a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*
f*tan(f*x + e)), -1/2*((a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))*sqrt(a - b
)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*
tan(f*x + e)^2 - a)) + 2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 3*a*b^2 + 2*b^3)
*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 + a^2*b^3)
*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*f*tan(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^2}{{\left(b\tan\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.344 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=184

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}+\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b}}{3a^3(a-b)f}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*cot
(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*(3*a-4*b)*(a+2*b)*cot(f*x+
e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)/f-1/3*(a-4*b)*cot(f*x+e)^3*(a+b*tan(f
*x+e)^2)^(1/2)/a^2/(a-b)/f
```

Rubi [A]
time = 0.17, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$,
Rules used = {3751, 483, 597, 12, 385, 209}

$$\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)}-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f(a-b)}+\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\cot^3(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3/2
)*f) - (b*Cot[e + f*x]^3)/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) + ((3*a
- 4*b)*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^3*(a - b)*f)
 - ((a - 4*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^2*(a - b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-4b-4bx^2}{x^4(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(\right.}{a(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+}}{3a^2(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b}}{3a^3(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b}}{3a^3(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b}}{3a^3(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 11.82, size = 1398, normalized size = 7.60

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/45*(Cos[e + f*x]^2*Cot[e + f*x]^3*((45*a*Csc[e + f*x]^2)/(a - b) - (270*
b*Sec[e + f*x]^2)/(a - b) + (4*(a - b)*Hypergeometric2F1[2, 2, 7/2, ((a - b
)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/a - (24*(a - b)*HypergeometricPFQ[{2,
```

```
2, 2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/a - (16*(a - b
)*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*
Sin[e + f*x]^2)/a - (1080*b^2*Sec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)) -
(132*(a - b)*b*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin
[e + f*x]^2*Tan[e + f*x]^2)/a^2 - (144*(a - b)*b*HypergeometricPFQ[{2, 2, 2
}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/a^2
 - (48*(a - b)*b*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Sin[
e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/a^2 - (720*b^3*Sec[e + f*x]^2
*Tan[e + f*x]^4)/(a^2*(a - b)) - (312*(a - b)*b^2*Hypergeometric2F1[2, 2, 7
/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 - (216*(
a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/
a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 - (48*(a - b)*b^2*HypergeometricPFQ[{
2, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e
+ f*x]^4)/a^3 - (176*(a - b)*b^3*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[
e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/a^4 - (96*(a - b)*b^3*Hyperge
ometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*
Tan[e + f*x]^6)/a^4 - (16*(a - b)*b^3*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1
, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/a^4 - (4
5*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/((((a - b)*Sin[e + f*x]^2)/a)^(
3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) + (270*b*ArcSin[Sqrt[
((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*(((a - b)*Sin[e + f*x]^2)/a
)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) + (1080*b^2*ArcSin
[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*(((a - b)*Sin[e + f
*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) + (720*b^3
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^6)/(a^3*(((a - b)*Si
n[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) + (
45*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/Sqrt[((a - b)*Cos[e + f*x]^2*S
in[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (270*b*ArcSin[Sqrt[((a - b)*Si
n[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f
*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (1080*b^2*ArcSin[Sqrt[((a - b)*Sin[e
+ f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x
]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (720*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f
*x]^2)/a]]*Tan[e + f*x]^6)/(a^3*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2
*(a + b*Tan[e + f*x]^2))/a^2])))/(a*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.37, size = 2577, normalized size = 14.01

method	result	size
default	Expression too large to display	2577

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(6*cos(f*x+e)^3*sin(f*x+e)*2^(1/2
)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*c
```

os(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-
I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*
EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x
+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/
a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^3-3*cos(f*x+e)^3*sin(
f*x+e)*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos
(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)
*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)
+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(
1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a
*b+8*b^2)/a^2)^(1/2))*a^3+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*
b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos
(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(
1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f
x+e)-1)((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^3-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*((
I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f
*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*Elli
pticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),(
(8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1
/2))*a^3-6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)
-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)
(-2(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b
*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*
b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2))*a^3+3*cos(f*x+e)*sin(f*x+e)*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*
(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+
1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos
(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1
/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3-6*2^(1/2)*((
I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f
*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*Elli
pticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),
-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(
1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*a^3+3*2^(1/2)*((
I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f
*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^
(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*Elli
pticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),(
(8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1

```
/2))*sin(f*x+e)*a^3-4*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2
)*a^3+3*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b+6*cos(
f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^2-8*cos(f*x+e)^4*((2
*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3+3*cos(f*x+e)^2*((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2)*a^3-5*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b
)/a)^(1/2)*a^2*b-8*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a
*b^2+16*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3+3*((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b+2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b
)/a)^(1/2)*a*b^2-8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3)*cos(f*x+e
)^3*((a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)^3/a^3
/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [A]
time = 4.62, size = 603, normalized size = 3.28

$$\left[\frac{3\left(a^{3} b \tan\left(f x+e\right)^{5}+a^{4} \tan\left(f x+e\right)^{3}\right) \sqrt{-a+b} \log\left(-\frac{\left(a^{2}-8 a b+8 b^{2}\right) \tan\left(f x+e\right)^{4}-2\left(3 a^{2}-4 a b\right) \tan\left(f x+e\right)^{2}+a^{2}+4\left(\left(a-2 b\right) \tan\left(f x+e\right)^{3}-a \tan\left(f x+e\right)\right) \sqrt{b \tan\left(f x+e\right)^{2}+a} \sqrt{-a+b}}{\tan\left(f x+e\right)^{4}+2 \tan\left(f x+e\right)^{2}+1}\right)+4\left(\left(3 a^{3} b-a^{2} b^{2}-10 a b^{3}+8 b^{4}\right) \tan\left(f x+e\right)^{4}-a^{4}+2 a^{3} b-a^{2} b^{2}+\left(3 a^{4}-2 a^{3} b-5 a^{2} b^{2}+4 a b^{3}\right) \tan\left(f x+e\right)^{2}\right) \sqrt{b \tan\left(f x+e\right)^{2}+a}}{12\left(\left(a^{5} b-2 a^{4} b^{2}+a^{3} b^{3}\right) f \tan\left(f x+e\right)^{5}+\left(a^{6}-2 a^{5} b+a^{4} b^{2}\right) f \tan\left(f x+e\right)^{3}\right)}, \frac{3\left(a^{3} b \tan\left(f x+e\right)^{5}+a^{4} \tan\left(f x+e\right)^{3}\right) \sqrt{a-b} \arctan\left(-\frac{2 \sqrt{b \tan\left(f x+e\right)^{2}+a} \sqrt{a-b} \tan\left(f x+e\right)}{\left(a-2 b\right) \tan\left(f x+e\right)^{2}-a}\right)+2\left(\left(3 a^{3} b-a^{2} b^{2}-10 a b^{3}+8 b^{4}\right) \tan\left(f x+e\right)^{4}-a^{4}+2 a^{3} b-a^{2} b^{2}+\left(3 a^{4}-2 a^{3} b-5 a^{2} b^{2}+4 a b^{3}\right) \tan\left(f x+e\right)^{2}\right) \sqrt{b \tan\left(f x+e\right)^{2}+a}}{6\left(\left(a^{5} b-2 a^{4} b^{2}+a^{3} b^{3}\right) f \tan\left(f x+e\right)^{5}+\left(a^{6}-2 a^{5} b+a^{4} b^{2}\right) f \tan\left(f x+e\right)^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] [1/12*(3*(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3)*sqrt(-a + b)*log(-((a^
2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2
+ 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*((3*a^3*b - a^2*
b^2 - 10*a*b^3 + 8*b^4)*tan(f*x + e)^4 - a^4 + 2*a^3*b - a^2*b^2 + (3*a^4 -
 2*a^3*b - 5*a^2*b^2 + 4*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))
/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2
)*f*tan(f*x + e)^3), 1/6*(3*(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3)*sqr
t(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a
- 2*b)*tan(f*x + e)^2 - a)) + 2*((3*a^3*b - a^2*b^2 - 10*a*b^3 + 8*b^4)*tan
(f*x + e)^4 - a^4 + 2*a^3*b - a^2*b^2 + (3*a^4 - 2*a^3*b - 5*a^2*b^2 + 4*a*
b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*
b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^4}{{\left(b\tan\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2), x)
```

3.345 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}-\frac{(15a^3+10a^2b+8ab^2-48b^3)\cot(e-}{15a^4(a-b)}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*co
t(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^3+10*a^2*b+8*a*b^2
-48*b^3)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^4/(a-b)/f+1/15*(5*a^2+4*a*b-
24*b^2)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)/f-1/5*(a-6*b)*cot(f
*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2)/a^2/(a-b)/f
```

Rubi [A]

time = 0.23, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 483, 597, 12, 385, 209}

$$-\frac{(a-6b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^2f(a-b)}+\frac{(5a^2+4ab-24b^2)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f(a-b)}-\frac{(15a^3+10a^2b+8ab^2-48b^3)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^4f(a-b)}-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\cot^5(e+fx)}{af(a-b)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(3
/2)*f)) - (b*Cot[e + f*x]^5)/(a*(a - b)*f*Sqrt[a + b*Tan[e + f*x]^2]) - ((1
5*a^3 + 10*a^2*b + 8*a*b^2 - 48*b^3)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2
])/(15*a^4*(a - b)*f) + ((5*a^2 + 4*a*b - 24*b^2)*Cot[e + f*x]^3*Sqrt[a + b
*Tan[e + f*x]^2])/(15*a^3*(a - b)*f) - ((a - 6*b)*Cot[e + f*x]^5*Sqrt[a + b
*Tan[e + f*x]^2])/(5*a^2*(a - b)*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-6b-6bx^2}{x^6(1+x^2)\sqrt{a+bx^2}}\,dx, x, \tan(e\right.}{a(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-6b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+}}{5a^2(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(5a^2+4ab-24b^2)\cot^3(e+fx)\sqrt{a+b}}{15a^3(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15a^3+10a^2b+8ab^2-48b^3)\cot(e+f}{15a^4(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15a^3+10a^2b+8ab^2-48b^3)\cot(e+f}{15a^4(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15a^3+10a^2b+8ab^2-48b^3)\cot(e+f}{15a^4(a-b)f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 15.45, size = 1994, normalized size = 7.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

[Out] -1/5*(Cos[e + f*x]^2*Cot[e + f*x]^5*((3*a*Csc[e + f*x]^2)/(a - b) - (8*b*Se
c[e + f*x]^2)/(a - b) - (16*(a - b)*HypergeometricPFQ[{2, 2, 2}, {1, 7/2},
((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(9*a) + (32*(a - b)*Hypergeomet
ricPFQ[{2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e +
 f*x]^2)/(45*a) + (48*b^2*Sec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)) - (16*
(a - b)*b*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e +
f*x]^2*Tan[e + f*x]^2)/(9*a^2) + (32*(a - b)*b*HypergeometricPFQ[{2, 2, 2},
 {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(9*a^
2) + (64*(a - b)*b*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Si
n[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(9*a^2) + (128*(a - b)*b*Hy
pergeometricPFQ[{2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a
]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(45*a^2) + (192*b^3*Sec[e + f*x]^2*Tan[e +
 f*x]^4)/(a^2*(a - b)) + (80*(a - b)*b^2*Hypergeometric2F1[2, 2, 7/2, ((a -
 b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(3*a^3) + (112*(a - b
)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Si
n[e + f*x]^2*Tan[e + f*x]^4)/(3*a^3) + (64*(a - b)*b^2*HypergeometricPFQ[{2
, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e +
 f*x]^4)/(3*a^3) + (64*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2, 2, 2}, {1, 1
, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(15*a
^3) + (128*b^4*Sec[e + f*x]^2*Tan[e + f*x]^6)/(a^3*(a - b)) + (64*(a - b)*b
^3*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*
Tan[e + f*x]^6)/a^4 + (512*(a - b)*b^3*HypergeometricPFQ[{2, 2, 2}, {1, 7/2
}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/(9*a^4) + (64
*(a - b)*b^3*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Sin[e +
f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/(3*a^4) + (128*(a - b)*b^3*Hyperg
eometricPFQ[{2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Si
n[e + f*x]^2*Tan[e + f*x]^6)/(45*a^4) + (320*(a - b)*b^4*Hypergeometric2F1[
2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^8)/(9*a^
5) + (224*(a - b)*b^4*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e
 + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^8)/(9*a^5) + (64*(a - b)*b^4*Hype
rgeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e
+ f*x]^2*Tan[e + f*x]^8)/(9*a^5) + (32*(a - b)*b^4*HypergeometricPFQ[{2, 2,
 2, 2, 2}, {1, 1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e
 + f*x]^8)/(45*a^5) - (3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/((((a -
b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]
) + (8*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*(((a -
 b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a
]) - (48*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*
(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]
^2))/a]) - (192*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^6
)/(a^3*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e
 + f*x]^2))/a]) - (128*b^4*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e +
 f*x]^8)/(a^4*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a +
b*Tan[e + f*x]^2))/a]) + (3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/Sqrt[
((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (8*b*

```
ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*Sqrt[((a - b)*C
os[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) + (48*b^2*ArcSin
[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*Sqrt[((a - b)*Cos[e
 + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) + (192*b^3*ArcSin[Sq
rt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^6)/(a^3*Sqrt[((a - b)*Cos[e +
f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) + (128*b^4*ArcSin[Sqrt[
((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^8)/(a^4*Sqrt[((a - b)*Cos[e + f*x
]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])))/(a*f*Sqrt[a + b*Tan[e +
f*x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.43, size = 3925, normalized size = 15.58

method	result	size
default	Expression too large to display	3925

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/f/(a*cos(f*x+e)^2-cos(f*x+e)^2*b+b)^2*(-10*((2*I*b^(1/2)*(a-b)^(1/2)+a
-2*b)/a)^(1/2)*a^2*b^2-8*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^3-48
*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^4-144*cos(f*x+e)^
2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^4+144*cos(f*x+e)^4*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^4-23*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2)*a^4+35*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^
(1/2)*a^4-15*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^4+48*
((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^4+30*2^(1/2)*((I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*
x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2
)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/
2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*sin(f*x+e)*a^4-15*2^(1/2)*((I*cos(f*x+e)*b^
(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f
*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/
2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+
e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a
-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*sin(f*x+e)
*a^4-34*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3*b+40*cos
(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3*b+12*cos(f*x+e)^6*(
(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3*b+12*cos(f*x+e)^6*((2*I*b^(1/2
)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b^2+32*cos(f*x+e)^6*((2*I*b^(1/2)*(a-b)^(
1/2)+a-2*b)/a)^(1/2)*a*b^3-28*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)*a^2*b^2-72*cos(f*x+e)^4*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)
*a*b^3+26*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b^2+48
```

```
*cos(f*x+e)^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^3-15*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3*b+30*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-
b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/
a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*
x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1
/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2))*cos(f*x+e)^5*sin(f*x+e)*a^4-15*2^(1/2)*((I*cos(f*x+
e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(
cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos
(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/
2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f
*x+e)^5*sin(f*x+e)*a^4+30*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1
/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I
*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*
x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-
b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-
(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*a^4-15*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b
)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a
)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x
+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*
I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-
4*I*b^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^4*sin(f*x
+e)*a^4-60*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)
+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*
x+e)+1)/a)^(1/2)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)
/a)^(1/2)/sin(f*x+e),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a
-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*cos(f*
x+e)^3*sin(f*x+e)*a^4+30*2^(1/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/
2)*(a-b)^(1/2)+cos(f*x+e)*a-b*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*
cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x
+e)-b)/(cos(f*x+e)+1)/a)^(1/2)*EllipticF((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)
^(1/2)+a-2*b)/a)^(1/2)/sin(f*x+e),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-
b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*cos(f*x+e)^3*sin(f*x+e)*a^4-60*2^(1
/2)*((I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+cos(f*x+e)*a-b
*cos(f*x+e)+b)/(cos(f*x+e)+1)/a)^(1/2)*(-2*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2
)-I*b^(1/2)*(a-b)^(1/2)-cos(f*x+e)*a+b*cos(f*x+e)-b)/(cos(f*x+e)+1)/a)^(1/2
)*EllipticPi((cos(f*x+e)-1)*((2*I*b^(1/2)*(a-b)...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.24, size = 713, normalized size = 2.83

$$\left[\frac{15\left(a^{4} b \tan\left(fx+e\right)^{7}+a^{5} \tan\left(fx+e\right)^{5}\right)\sqrt{-a+b}\,\log\left(-\frac{\left(a^{2}-8ab+8b^{2}\right)\tan\left(fx+e\right)^{4}-2\left(3a^{2}-4ab\right)\tan\left(fx+e\right)^{2}+a^{2}-4\left(\left(a-2b\right)\tan\left(fx+e\right)^{3}-a\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{-a+b}}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}\right)-4\left(\left(15a^{4}b-5a^{3}b^{2}-2a^{2}b^{3}-56ab^{4}+48b^{5}\right)\tan\left(fx+e\right)^{6}+3a^{5}-6a^{4}b+3a^{3}b^{2}+\left(15a^{5}-10a^{4}b-a^{3}b^{2}-28a^{2}b^{3}+24ab^{4}\right)\tan\left(fx+e\right)^{4}-\left(5a^{5}-4a^{4}b-7a^{3}b^{2}+6a^{2}b^{3}\right)\tan\left(fx+e\right)^{2}\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}}{60\left(\left(a^{6}b-2a^{5}b^{2}+a^{4}b^{3}\right)f\tan\left(fx+e\right)^{7}+\left(a^{7}-2a^{6}b+a^{5}b^{2}\right)f\tan\left(fx+e\right)^{5}\right)},\ -\frac{15\left(a^{4}b\tan\left(fx+e\right)^{7}+a^{5}\tan\left(fx+e\right)^{5}\right)\sqrt{a-b}\,\arctan\left(-\frac{2\sqrt{b\tan\left(fx+e\right)^{2}+a}\,\sqrt{a-b}\,\tan\left(fx+e\right)}{\left(a-2b\right)\tan\left(fx+e\right)^{2}-a}\right)+2\left(\left(15a^{4}b-5a^{3}b^{2}-2a^{2}b^{3}-56ab^{4}+48b^{5}\right)\tan\left(fx+e\right)^{6}+3a^{5}-6a^{4}b+3a^{3}b^{2}+\left(15a^{5}-10a^{4}b-a^{3}b^{2}-28a^{2}b^{3}+24ab^{4}\right)\tan\left(fx+e\right)^{4}-\left(5a^{5}-4a^{4}b-7a^{3}b^{2}+6a^{2}b^{3}\right)\tan\left(fx+e\right)^{2}\right)\sqrt{b\tan\left(fx+e\right)^{2}+a}}{30\left(\left(a^{6}b-2a^{5}b^{2}+a^{4}b^{3}\right)f\tan\left(fx+e\right)^{7}+\left(a^{7}-2a^{6}b+a^{5}b^{2}\right)f\tan\left(fx+e\right)^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^4*b*tan(f*x + e)^7 + a^5*tan(f*x + e)^5)*sqrt(-a + b)*log(-((a
^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2
 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*((15*a^4*b - 5*
a^3*b^2 - 2*a^2*b^3 - 56*a*b^4 + 48*b^5)*tan(f*x + e)^6 + 3*a^5 - 6*a^4*b +
 3*a^3*b^2 + (15*a^5 - 10*a^4*b - a^3*b^2 - 28*a^2*b^3 + 24*a*b^4)*tan(f*x
+ e)^4 - (5*a^5 - 4*a^4*b - 7*a^3*b^2 + 6*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*t
an(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7
- 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^5), -1/30*(15*(a^4*b*tan(f*x + e)^7 + a
^5*tan(f*x + e)^5)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
- b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((15*a^4*b - 5*a^3*b^
2 - 2*a^2*b^3 - 56*a*b^4 + 48*b^5)*tan(f*x + e)^6 + 3*a^5 - 6*a^4*b + 3*a^3
*b^2 + (15*a^5 - 10*a^4*b - a^3*b^2 - 28*a^2*b^3 + 24*a*b^4)*tan(f*x + e)^4
 - (5*a^5 - 4*a^4*b - 7*a^3*b^2 + 6*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x
 + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^
6*b + a^5*b^2)*f*tan(f*x + e)^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{6}\left(e+fx\right)}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.346 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}+\frac{a^2}{3(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{a(a-2b)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-a*(a-2*b)/(a-b
)^2/b^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*a^2/(a-b)/b^2/f/(a+b*tan(f*x+e)^2)^(
3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3751, 457, 89, 65, 214}

$$\frac{a^2}{3b^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{a(a-2b)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + a^2/
(3*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (a*(a - 2*b))/((a - b)^2*b
^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{(a-b)b(a+bx)^{5/2}} + \frac{a(a-2b)}{(a-b)^2b(a+bx)^{3/2}} + \frac{1}{(a-b)^2(1+x)\sqrt{a+bx}}\right) dx, x, \text{ta}\right.}{2f}$$

$$= \frac{a^2}{3(a-b)b^2f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{a(a-2b)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} + \text{S}$$

$$= \frac{a^2}{3(a-b)b^2f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{a(a-2b)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} + \text{S}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{a^2}{3(a-b)b^2f\,(a+b\tan^2(e+fx))^{3/2}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.32, size = 91, normalized size = 0.79

$$\frac{b^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right) - (a-b)\left(2a-b+3b\tan^2(e+fx)\right)}{3(a-b)b^2f\,(a+b\tan^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (b^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)] - (a
- b)*(2*a - b + 3*b*Tan[e + f*x]^2))/(3*(a - b)*b^2*f*(a + b*Tan[e + f*x]^2
)^(3/2))
```

Maple [A]
time = 0.07, size = 155, normalized size = 1.35

method	result

derivativedivides	$\frac{-\frac{\tan^2(fx+e)}{b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{2a}{3b^2\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{1}{3b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{1}{3(a-b)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a}}{}\right.}{(}}{f}$
default	$\frac{-\frac{\tan^2(fx+e)}{b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{2a}{3b^2\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{1}{3b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{1}{3(a-b)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a}}{}\right.}{(}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-tan(f*x+e)^2/b/(a+b*tan(f*x+e)^2)^(3/2)-2/3*a/b^2/(a+b*tan(f*x+e)^2)^
(3/2)+1/3/b/(a+b*tan(f*x+e)^2)^(3/2)+1/3/(a-b)/(a+b*tan(f*x+e)^2)^(3/2)+1/(
a-b)^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/(a-b)^2
/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(106) = 212$.
time = 4.61, size = 628, normalized size = 5.46

$$\left[\frac{3\left(b^4\tan(fx+e)^4+2ab^3\tan(fx+e)^2+a^2b^2\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2(4ab-3b^2)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)-4\left(2a^4-7a^3b+5a^2b^2+3(a^3b-3a^2b^2+2ab^3)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{12\left((a^3b^4-3a^2b^5+3ab^6-b^7)f\tan(fx+e)^4+2(a^4b^3-3a^3b^4+3a^2b^5-ab^6)f\tan(fx+e)^2+(a^5b^2-3a^4b^3+3a^3b^4-a^2b^5)f\right)}, \frac{3\left(b^4\tan(fx+e)^4+2ab^3\tan(fx+e)^2+a^2b^2\right)\sqrt{-a+b}\arctan\left(\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{b\tan(fx+e)^2+2a-b}\right)-2\left(2a^4-7a^3b+5a^2b^2+3(a^3b-3a^2b^2+2ab^3)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{6\left((a^3b^4-3a^2b^5+3ab^6-b^7)f\tan(fx+e)^4+2(a^4b^3-3a^3b^4+3a^2b^5-ab^6)f\tan(fx+e)^2+(a^5b^2-3a^4b^3+3a^3b^4-a^2b^5)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^4*tan(f*x + e)^4 + 2*a*b^3*tan(f*x + e)^2 + a^2*b^2)*sqrt(a - b
)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*
x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b
+ b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(2*a^4 - 7*a^3*b + 5*a^
```

```
2*b^2 + 3*(a^3*b - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)
^2 + a))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b
^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3
 + 3*a^3*b^4 - a^2*b^5)*f), 1/6*(3*(b^4*tan(f*x + e)^4 + 2*a*b^3*tan(f*x +
e)^2 + a^2*b^2)*sqrt(-a + b)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)/(b*tan(f*x + e)^2 + 2*a - b)) - 2*(2*a^4 - 7*a^3*b + 5*a^2*b^2 + 3*(a^3*
b - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*
b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b^4
+ 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 -
a^2*b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [A]
time = 0.61, size = 137, normalized size = 1.19

$$\frac{\arctan\left(\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{-a+b}}\right)}{(a^2f-2\,abf+b^2f)\sqrt{-a+b}}-\frac{3\left(b\tan(fx+e)^2+a\right)a^2-a^3-6\left(b\tan(fx+e)^2+a\right)ab+a^2b}{3\,(a^2b^2f-2\,ab^3f+b^4f)\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a^2*f - 2*a*b*f + b^2*f)*
sqrt(-a + b)) - 1/3*(3*(b*tan(f*x + e)^2 + a)*a^2 - a^3 - 6*(b*tan(f*x + e)
^2 + a)*a*b + a^2*b)/((a^2*b^2*f - 2*a*b^3*f + b^4*f)*(b*tan(f*x + e)^2 + a
)^(3/2))
```

Mupad [B]
time = 16.03, size = 148, normalized size = 1.29

$$\frac{\frac{a^2}{3\,(a-b)}+\frac{\left(b\tan(e+f\,x)^2+a\right)\,(2\,a\,b-a^2)}{(a-b)^2}}{b^2\,f\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}}+\frac{\mathrm{atan}\left(\frac{a^2\,\sqrt{b\tan(e+f\,x)^2+a}\,1{}\mathrm{i}+b^2\,\sqrt{b\tan(e+f\,x)^2+a}\,1{}\mathrm{i}-a\,b\,\sqrt{b\tan(e+f\,x)^2+a}\,2{}\mathrm{i}}{(a-b)^{5/2}}\right)\,1{}\mathrm{i}}{f\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] (atan((a^2*(a + b*tan(e + f*x)^2)^(1/2)*1i + b^2*(a + b*tan(e + f*x)^2)^(1/
2)*1i - a*b*(a + b*tan(e + f*x)^2)^(1/2)*2i)/(a - b)^(5/2))*1i)/(f*(a - b)^
(5/2)) + (a^2/(3*(a - b)) + ((a + b*tan(e + f*x)^2)*(2*a*b - a^2))/(a - b)^
2)/(b^2*f*(a + b*tan(e + f*x)^2)^(3/2))
```

3.347 $\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=103

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}-\frac{a}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}}-\frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-1/(a-b)^2/f/(a+
b*tan(f*x+e)^2)^(1/2)-1/3*a/(a-b)/b/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3751, 457, 79, 53, 65, 214}

$$-\frac{a}{3bf(a-b)\,(a+b\tan^2(e+fx))^{3/2}}-\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - a/(3*(a
 - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) - 1/((a - b)^2*f*Sqrt[a + b*Tan[e +
 f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{a}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+\right.}{2(a-b)f}$$

$$= -\frac{a}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}} - \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \text{Su}$$

$$= -\frac{a}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}} - \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \text{Su}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{a}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.20, size = 84, normalized size = 0.82

$$\frac{a(-a+b) - 3b\,{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{a+b\tan^2(e+fx)}{a-b}\right)(a+b\tan^2(e+fx))}{3(a-b)^2bf\,(a+b\tan^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*(-a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a
 - b)]*(a + b*Tan[e + f*x]^2))/(3*(a - b)^2*b*f*(a + b*Tan[e + f*x]^2)^(3/2
))
```

Maple [A]
time = 0.06, size = 110, normalized size = 1.07

method	result

derivativedivides	$\frac{-\frac{1}{3b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{1}{3(a-b)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{a+b(\tan^2(fx+e))}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}-\frac{}{(a-b)^2\sqrt{a+b}}}{f}$
default	$\frac{-\frac{1}{3b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{1}{3(a-b)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{a+b(\tan^2(fx+e))}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}-\frac{}{(a-b)^2\sqrt{a+b}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3/b/(a+b*tan(f*x+e)^2)^(3/2)-1/3/(a-b)/(a+b*tan(f*x+e)^2)^(3/2)-1/(
a-b)^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/(a-b)^2
/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(94) = 188.
time = 4.03, size = 592, normalized size = 5.75

$$\left[\frac{3\left(b^3\tan(fx+e)^4+2ab^2\tan(fx+e)^2+a^2b\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2(4ab-3b^2)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)-4\left(a^3+a^2b-2ab^2+3(ab^2-b^3)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{12\left((a^3b^3-3a^2b^4+3ab^5-b^6)f\tan(fx+e)^4+2(a^4b^2-3a^3b^3+3a^2b^4-ab^5)f\tan(fx+e)^2+(a^5b-3a^4b^2+3a^3b^3-a^2b^4)f\right)}, -\frac{3\left(b^3\tan(fx+e)^4+2ab^2\tan(fx+e)^2+a^2b\right)\sqrt{-a+b}\arctan\left(\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{b\tan(fx+e)^2+2a-b}\right)+2\left(a^3+a^2b-2ab^2+3(ab^2-b^3)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{6\left((a^3b^3-3a^2b^4+3ab^5-b^6)f\tan(fx+e)^4+2(a^4b^2-3a^3b^3+3a^2b^4-ab^5)f\tan(fx+e)^2+(a^5b-3a^4b^2+3a^3b^3-a^2b^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*tan(f*x + e)^4 + 2*a*b^2*tan(f*x + e)^2 + a^2*b)*sqrt(a - b)*
log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x
+ e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b +
b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(a^3 + a^2*b - 2*a*b^2 +
3*(a*b^2 - b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^3 - 3*a
```

```
^2*b^4 + 3*a*b^5 - b^6)*f*tan(f*x + e)^4 + 2*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b
^4 - a*b^5)*f*tan(f*x + e)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f)
, -1/6*(3*(b^3*tan(f*x + e)^4 + 2*a*b^2*tan(f*x + e)^2 + a^2*b)*sqrt(-a + b
)*arctan(2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a
- b)) + 2*(a^3 + a^2*b - 2*a*b^2 + 3*(a*b^2 - b^3)*tan(f*x + e)^2)*sqrt(b*t
an(f*x + e)^2 + a))/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*f*tan(f*x + e)^4
 + 2*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(f*x + e)^2 + (a^5*b -
3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [A]
time = 0.58, size = 116, normalized size = 1.13

$$-\frac{\frac{3\,b\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^2+a}}{\sqrt{-a+b}}\right)}{\left(a^2f-2\,abf+b^2f\right)\sqrt{-a+b}}+\frac{a^2+3\,\left(b\tan(fx+e)^2+a\right)b-ab}{\left(a^2f-2\,abf+b^2f\right)\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a^2*f - 2*a*b*f
 + b^2*f)*sqrt(-a + b)) + (a^2 + 3*(b*tan(f*x + e)^2 + a)*b - a*b)/((a^2*f
- 2*a*b*f + b^2*f)*(b*tan(f*x + e)^2 + a)^(3/2)))/b
```

Mupad [B]
time = 15.72, size = 138, normalized size = 1.34

$$-\frac{\frac{a}{3\,(a-b)}+\frac{b\,\left(b\tan(e+f\,x)^2+a\right)}{(a-b)^2}}{b\,f\,\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}-\frac{\mathrm{atan}\left(\frac{a^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}+b^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1\mathrm{i}-a\,b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,2\mathrm{i}}{(a-b)^{5/2}}\right)\,1\mathrm{i}}{f\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] - (atan((a^2*(a + b*tan(e + f*x)^2)^(1/2)*1i + b^2*(a + b*tan(e + f*x)^2)^(
1/2)*1i - a*b*(a + b*tan(e + f*x)^2)^(1/2)*2i)/(a - b)^(5/2))*1i)/(f*(a - b
)^(5/2)) - (a/(3*(a - b)) + (b*(a + b*tan(e + f*x)^2))/(a - b)^2)/(b*f*(a +
 b*tan(e + f*x)^2)^(3/2))
```

3.348 $\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}+\frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/(a-b)^2/f/(a
+b*tan(f*x+e)^2)^(1/2)+1/3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3751, 455, 53, 65, 214}

$$\frac{1}{f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + 1/(3
*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) + 1/((a - b)^2*f*Sqrt[a + b*Tan[e
+ f*x]^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e+fx\right.}{2(a-b)f}$$

$$= \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 58, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)]/(3*(a - b)
*f*(a + b*Tan[e + f*x]^2)^(3/2))
```

Maple [A]
time = 0.05, size = 89, normalized size = 0.90

method	result	si

derivativedivides	$\frac{\frac{1}{3(a-b)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}+\frac{1}{(a-b)^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}}{f}$	
default	$\frac{\frac{1}{3(a-b)\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a+b\left(\tan^2(fx+e)\right)}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}+\frac{1}{(a-b)^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}}{f}$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3/(a-b)/(a+b*tan(f*x+e)^2)^(3/2)+1/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*
tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/(a-b)^2/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(90) = 180$.
time = 4.15, size = 564, normalized size = 5.70

$$\left[\frac{3\left(b^2\tan(fx+e)^4+2ab\tan(fx+e)^2+a^2\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2(4ab-3b^2)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)+4\left(3(ab-b^2)\tan(fx+e)^2+4a^2-5ab+b^2\right)\sqrt{b\tan(fx+e)^2+a}}{12\left((a^3b^2-3a^2b^3+3ab^4-b^5)f\tan(fx+e)^4+2(a^4b-3a^3b^2+3a^2b^3-ab^4)f\tan(fx+e)^2+(a^5-3a^4b+3a^3b^2-a^2b^3)f\right)}, \frac{3\left(b^2\tan(fx+e)^4+2ab\tan(fx+e)^2+a^2\right)\sqrt{-a+b}\arctan\left(\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{b\tan(fx+e)^2+2a-b}\right)+2\left(3(ab-b^2)\tan(fx+e)^2+4a^2-5ab+b^2\right)\sqrt{b\tan(fx+e)^2+a}}{6\left((a^3b^2-3a^2b^3+3ab^4-b^5)f\tan(fx+e)^4+2(a^4b-3a^3b^2+3a^2b^3-ab^4)f\tan(fx+e)^2+(a^5-3a^4b+3a^3b^2-a^2b^3)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*log(
-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)
^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)
/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*(3*(a*b - b^2)*tan(f*x + e)^2
 + 4*a^2 - 5*a*b + b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^2 - 3*a^2*b^3 +
 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4
)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f), 1/6*(3*(b^2*
tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*arctan(2*sqrt(b*t
an(f*x + e)^2 + a)*sqrt(-a + b)/(b*tan(f*x + e)^2 + 2*a - b)) + 2*(3*(a*b -
```

```
b^2)*tan(f*x + e)^2 + 4*a^2 - 5*a*b + b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a
^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b
^3)*f)]
```

Sympy [A]
time = 14.45, size = 83, normalized size = 0.84

$$\frac{1}{3f\left(a-b\right)\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}+\frac{1}{f\left(a-b\right)^{2}\sqrt{a+b\tan^{2}\left(e+fx\right)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tan^{2}\left(e+fx\right)}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}\left(a-b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] 1/(3*f*(a - b)*(a + b*tan(e + f*x)**2)**(3/2)) + 1/(f*(a - b)**2*sqrt(a + b
*tan(e + f*x)**2)) + atan(sqrt(a + b*tan(e + f*x)**2)/sqrt(-a + b))/(f*sqrt
(-a + b)*(a - b)**2)
```

Giac [A]
time = 0.51, size = 105, normalized size = 1.06

$$\frac{\arctan\left(\frac{\sqrt{b\tan\left(fx+e\right)^{2}+a}}{\sqrt{-a+b}}\right)}{\left(a^{2}f-2\,abf+b^{2}f\right)\sqrt{-a+b}}+\frac{3\,b\tan\left(fx+e\right)^{2}+4\,a-b}{3\,\left(a^{2}f-2\,abf+b^{2}f\right)\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*tan(f*x + e)^2 + a)/sqrt(-a + b))/((a^2*f - 2*a*b*f + b^2*f)*
sqrt(-a + b)) + 1/3*(3*b*tan(f*x + e)^2 + 4*a - b)/((a^2*f - 2*a*b*f + b^2*
f)*(b*tan(f*x + e)^2 + a)^(3/2))
```

Mupad [B]
time = 15.70, size = 131, normalized size = 1.32

$$\frac{\frac{b\tan\left(e+f\,x\right)^2+a}{{\left(a-b\right)}^2}+\frac{1}{3\,\left(a-b\right)}}{f\,{\left(b\tan\left(e+f\,x\right)^2+a\right)}^{3/2}}+\frac{\mathrm{atan}\left(\frac{a^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1{}\mathrm{i}+b^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1{}\mathrm{i}-a\,b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,2{}\mathrm{i}}{{\left(a-b\right)}^{5/2}}\right)\,1{}\mathrm{i}}{f\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] ((a + b*tan(e + f*x)^2)/(a - b)^2 + 1/(3*(a - b)))/(f*(a + b*tan(e + f*x)^2
)^(3/2)) + (atan((a^2*(a + b*tan(e + f*x)^2)^(1/2)*1i + b^2*(a + b*tan(e +
f*x)^2)^(1/2)*1i - a*b*(a + b*tan(e + f*x)^2)^(1/2)*2i)/(a - b)^(5/2))*1i)/
(f*(a - b)^(5/2))
```

3.349 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}-\frac{b}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] -arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+arctanh((a+b*tan(f*x+e
)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-(2*a-b)*b/a^2/(a-b)^2/f/(a+b*tan(f*x+
e)^2)^(1/2)-1/3*b/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3751, 457, 87, 157, 162, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b(2a-b)}{a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{b}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + ArcTanh[Sqrt[a
 + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - b/(3*a*(a - b)*f*(a +
 b*Tan[e + f*x]^2)^(3/2)) - ((2*a - b)*b)/(a^2*(a - b)^2*f*Sqrt[a + b*Tan[e
 + f*x]^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && LtQ[p, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-b-bx}{x(1+x)(a+bx)^{3/2}}\,dx, x, \tan^2(e\right.}{2a(a-b)f}$$

$$= -\frac{b}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{b}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{b}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.25, size = 94, normalized size = 0.64

$$\frac{-a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\tan^2(e+fx)}{a}\right)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)]) + (a
 - b)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[e + f*x]^2)/a])/(3*a*(a -
 b)*f*(a + b*Tan[e + f*x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 331596 vs. $2(129) = 258$.
time = 47.96, size = 331597, normalized size = 2255.76

method	result	size
default	Expression too large to display	331597

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(133) = 266$.
time = 2.62, size = 1697, normalized size = 11.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)*sqrt(a - b)
*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b
)/(tan(f*x + e)^2 + 1)) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2
 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(7*a^4*b - 11*a^3*b^2 +
4*a^2*b^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*
x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^
4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3
*a^7*b + 3*a^6*b^2 - a^5*b^3)*f), 1/6*(6*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*
tan(f*x + e)^2 + a^5)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)/(a - b)) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^
2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 -
a*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e
```

```
)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(7*a^4*b - 11*a^3*b^2 + 4*a^2*b
^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^
2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(
a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b
+ 3*a^6*b^2 - a^5*b^3)*f), 1/6*(6*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a
^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 +
 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(-a)/a) + 3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)
*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - 2*(7*a^4*b - 11*a^3*b^2 + 4*a^2*b^3 +
 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 +
a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(a^7*
b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b + 3*
a^6*b^2 - a^5*b^3)*f), 1/3*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b
^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a
^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*
sqrt(-a)/a) + 3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)*sqr
t(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) - (7*a^4
*b - 11*a^3*b^2 + 4*a^2*b^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e
)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^
5)*f*tan(f*x + e)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x
 + e)^2 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot (e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 12.46, size = 2788, normalized size = 18.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] (b/(3*(a*b - a^2)) - (b*(a + b*tan(e + f*x)^2)*(2*a - b))/(a*b - a^2)^2)/(f
*(a + b*tan(e + f*x)^2)^(3/2)) - atanh((2*a^5*b^13*f^2*(a + b*tan(e + f*x)^
2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2
 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 -
 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2))
- (22*a^6*b^12*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f
^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^
2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2
 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (110*a^7*b^11*f^2*(a + b*tan(e + f
*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11
*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f
^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^
2)) - (330*a^8*b^10*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b
^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b
^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^
5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (660*a^9*b^9*f^2*(a + b*tan(e
 + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*
b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b
^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^
3*f^2)) - (922*a^10*b^8*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a
^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a
^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^1
1*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (912*a^11*b^7*f^2*(a + b*
tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110
*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*
a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^
13*b^3*f^2)) - (630*a^12*b^6*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)
*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 +
660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 29
0*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (290*a^13*b^5*f^2*(a
 + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2
+ 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 +
 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 +
10*a^13*b^3*f^2)) - (80*a^14*b^4*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(
1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^
2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2
+ 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (10*a^15*b^3*f^2
```

```
*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f
^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^
2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2
 + 10*a^13*b^3*f^2)))/(f*(a^5)^(1/2)) - (atan((a^12*f^3*(a + b*tan(e + f*x)
^2)^(1/2)*2i - a^7*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*
b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*2i + b^7*f*(a + b*
tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a
^2*b^3*f^2 + 10*a^3*b^2*f^2)*1i + a^4*b^8*f^3*(a + b*tan(e + f*x)^2)^(1/2)*
1i - a^5*b^7*f^3*(a + b*tan(e + f*x)^2)^(1/2)*9i + a^6*b^6*f^3*(a + b*tan(e
 + f*x)^2)^(1/2)*35i - a^7*b^5*f^3*(a + b*tan(e + f*x)^2)^(1/2)*77i + a^8*b
^4*f^3*(a + b*tan(e + f*x)^2)^(1/2)*105i - a^9*b^3*f^3*(a + b*tan(e + f*x)^
2)^(1/2)*91i + a^10*b^2*f^3*(a + b*tan(e + f*x)^2)^(1/2)*49i - a^11*b*f^3*(
a + b*tan(e + f*x)^2)^(1/2)*15i + a^2*b^5*f*(a + b*tan(e + f*x)^2)^(1/2)*(a
^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*
f^2)*21i - a^3*b^4*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*
b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*35i + a^4*b^3*f*(a
 + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 -
 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*36i - a^5*b^2*f*(a + b*tan(e + f*x)^2)^(1
/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^
3*b^2*f^2)*24i - a*b^6*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 +
5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*7i + a^6*b*f*(
a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2
- 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*10i)/((5*a^4*b - 5*a*b^4 + b^5 + 10*a^2*
b^3 - 10*a^3*b^2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b
^3*f^2 + 10*a^3*b^2*f^2)^(3/2)))*1i)/(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a
^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)^(1/2)
```

3.350 $\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=206

$$\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}-\frac{(3a-5b)b}{6a^2(a-b)f\,(a+b\tan^2(e-}$$

[Out] 1/2*(2*a+5*b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2)/f-arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-1/2*b*(a^2-8*a*b+5*b^2)/a^3/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/6*(3*a-5*b)*b/a^2/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-1/2*cot(f*x+e)^2/a/f/(a+b*tan(f*x+e)^2)^(3/2)

Rubi [A]
time = 0.23, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$,
Rules used = {3751, 457, 105, 157, 162, 65, 214}

$$\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{b(3a-5b)}{6a^2f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(a^2-8ab+5b^2)}{2a^3f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}-\frac{\cot^2(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]

[Out] ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) - ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f) - ((3*a - 5*b)*b)/(6*a^2*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - Cot[e + f*x]^2/(2*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (b*(a^2 - 8*a*b + 5*b^2))/(2*a^3*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2af\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2a+5b)+\frac{5bx}{2}}{x(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2af}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2af\,(a+b\tan^2(e+fx))^{3/2}} + \text{Subs}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2af\,(a+b\tan^2(e+fx))^{3/2}} - \frac{}{2a^3(a}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2af\,(a+b\tan^2(e+fx))^{3/2}} - \frac{}{2a^3(a}$$

$$= -\frac{(3a-5b)b}{6a^2(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2af\,(a+b\tan^2(e+fx))^{3/2}} - \frac{}{2a^3(a}$$

$$= \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+f}}{\sqrt{a-b}}\right.}{(a-b)^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.40, size = 138, normalized size = 0.67

$$\frac{\cot^2(e+fx)\left(-2a^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(3a\cot^2(e+fx)+(2a+5b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\tan^2(e+fx)}{a}\right)\right)\right)}{6a^2(-a+b)f\,(b+a\cot^2(e+fx))\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cot[e + f*x]^2*(-2*a^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x
]^2)/(a - b)] + (a - b)*(3*a*Cot[e + f*x]^2 + (2*a + 5*b)*Hypergeometric2F1
```

```
[-3/2, 1, -1/2, 1 + (b*Tan[e + f*x]^2)/a])))/(6*a^2*(-a + b)*f*(b + a*Cot[e
 + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2])
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 76.06, size = 531573, normalized size = 2580.45

method	result	size
default	Expression too large to display	531573

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(186) = 372.
time = 3.28, size = 2147, normalized size = 10.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x + e
)^2)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + 3*((2*a^4*b^2 - a^3*b^3 - 9*a^2*b
^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b^3 +
13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13*a^3*
b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*t
an(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(3*a^6 - 9*a^5*b + 9*
a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*
x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26*a^3*b^3 - 10*a^2*b^4)*tan(f*x + e)^
2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)
*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x +
 e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2), -1/12*(12*
(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x + e)^2)*sqrt
```

```
(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) - 3*((2*a
^4*b^2 - a^3*b^3 - 9*a^2*b^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*
b - a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a
^5*b - 9*a^4*b^2 + 13*a^3*b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*t
an(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)
 + 2*(3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^3*b^3 + 13
*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26*a^3*b^3 -
 10*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*
b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*
b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*t
an(f*x + e)^2), -1/6*(3*((2*a^4*b^2 - a^3*b^3 - 9*a^2*b^4 + 13*a*b^5 - 5*b^
6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5
)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13*a^3*b^3 - 5*a^2*b^4)*tan
(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) - 3*(a^
4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x + e)^2)*sqrt(a
- b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a
 - b)/(tan(f*x + e)^2 + 1)) + (3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*
(a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b -
19*a^4*b^2 + 26*a^3*b^3 - 10*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2
 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 + 2*(a
^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8*b +
 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2), -1/6*(3*((2*a^4*b^2 - a^3*b^3 - 9*
a^2*b^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b
^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13
*a^3*b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2
 + a)*sqrt(-a)/a) + 6*(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^
6*tan(f*x + e)^2)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
 b)/(a - b)) + (3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^
3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26
*a^3*b^3 - 10*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^
2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^
2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^
6*b^3)*f*tan(f*x + e)^2)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e + fx)}{(a + b\tan^2(e + fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**B**]
time = 13.10, size = 2500, normalized size = 12.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] (atan(((((a + b*tan(e + f*x)^2)^(1/2)*(400*a^9*b^14*f^3 - 3680*a^10*b^13*f^
3 + 14864*a^11*b^12*f^3 - 34240*a^12*b^11*f^3 + 48480*a^13*b^10*f^3 - 41280
*a^14*b^9*f^3 + 16864*a^15*b^8*f^3 + 2688*a^16*b^7*f^3 - 6000*a^17*b^6*f^3
+ 1440*a^18*b^5*f^3 + 1040*a^19*b^4*f^3 - 704*a^20*b^3*f^3 + 128*a^21*b^2*f
^3) - ((2*a + 5*b)*(320*a^12*b^14*f^4 - 3392*a^13*b^13*f^4 + 16192*a^14*b^1
2*f^4 - 45760*a^15*b^11*f^4 + 84608*a^16*b^10*f^4 - 106624*a^17*b^9*f^4 + 9
2288*a^18*b^8*f^4 - 53632*a^19*b^7*f^4 + 19520*a^20*b^6*f^4 - 3648*a^21*b^5
*f^4 + 64*a^22*b^4*f^4 + 64*a^23*b^3*f^4 - ((a + b*tan(e + f*x)^2)^(1/2)*(2
*a + 5*b)*(256*a^15*b^13*f^5 - 3072*a^16*b^12*f^5 + 16640*a^17*b^11*f^5 - 5
3760*a^18*b^10*f^5 + 115200*a^19*b^9*f^5 - 172032*a^20*b^8*f^5 + 182784*a^2
1*b^7*f^5 - 138240*a^22*b^6*f^5 + 72960*a^23*b^5*f^5 - 25600*a^24*b^4*f^5 +
 5376*a^25*b^3*f^5 - 512*a^26*b^2*f^5))/(4*f*(a^7)^(1/2))))/(4*f*(a^7)^(1/2
)))*(2*a + 5*b)*1i)/(4*f*(a^7)^(1/2)) + (((a + b*tan(e + f*x)^2)^(1/2)*(400
*a^9*b^14*f^3 - 3680*a^10*b^13*f^3 + 14864*a^11*b^12*f^3 - 34240*a^12*b^11*
f^3 + 48480*a^13*b^10*f^3 - 41280*a^14*b^9*f^3 + 16864*a^15*b^8*f^3 + 2688*
a^16*b^7*f^3 - 6000*a^17*b^6*f^3 + 1440*a^18*b^5*f^3 + 1040*a^19*b^4*f^3 -
704*a^20*b^3*f^3 + 128*a^21*b^2*f^3) + ((2*a + 5*b)*(320*a^12*b^14*f^4 - 33
92*a^13*b^13*f^4 + 16192*a^14*b^12*f^4 - 45760*a^15*b^11*f^4 + 84608*a^16*b
^10*f^4 - 106624*a^17*b^9*f^4 + 92288*a^18*b^8*f^4 - 53632*a^19*b^7*f^4 + 1
9520*a^20*b^6*f^4 - 3648*a^21*b^5*f^4 + 64*a^22*b^4*f^4 + 64*a^23*b^3*f^4 +
 ((a + b*tan(e + f*x)^2)^(1/2)*(2*a + 5*b)*(256*a^15*b^13*f^5 - 3072*a^16*b
^12*f^5 + 16640*a^17*b^11*f^5 - 53760*a^18*b^10*f^5 + 115200*a^19*b^9*f^5 -
 172032*a^20*b^8*f^5 + 182784*a^21*b^7*f^5 - 138240*a^22*b^6*f^5 + 72960*a^
23*b^5*f^5 - 25600*a^24*b^4*f^5 + 5376*a^25*b^3*f^5 - 512*a^26*b^2*f^5))/(4
```

f(a^7)^(1/2))))/(4*f*(a^7)^(1/2)))*(2*a + 5*b)*1i)/(4*f*(a^7)^(1/2)))/(40
0*a^9*b^12*f^2 - 2880*a^10*b^11*f^2 + 8704*a^11*b^10*f^2 - 14112*a^12*b^9*f
^2 + 12768*a^13*b^8*f^2 - 5600*a^14*b^7*f^2 + 1056*a^16*b^5*f^2 - 368*a^17*
b^4*f^2 + 32*a^18*b^3*f^2 + (((a + b*tan(e + f*x)^2)^(1/2)*(400*a^9*b^14*f^
3 - 3680*a^10*b^13*f^3 + 14864*a^11*b^12*f^3 - 34240*a^12*b^11*f^3 + 48480*
a^13*b^10*f^3 - 41280*a^14*b^9*f^3 + 16864*a^15*b^8*f^3 + 2688*a^16*b^7*f^3
 - 6000*a^17*b^6*f^3 + 1440*a^18*b^5*f^3 + 1040*a^19*b^4*f^3 - 704*a^20*b^3
*f^3 + 128*a^21*b^2*f^3) - ((2*a + 5*b)*(320*a^12*b^14*f^4 - 3392*a^13*b^13
*f^4 + 16192*a^14*b^12*f^4 - 45760*a^15*b^11*f^4 + 84608*a^16*b^10*f^4 - 10
6624*a^17*b^9*f^4 + 92288*a^18*b^8*f^4 - 53632*a^19*b^7*f^4 + 19520*a^20*b^
6*f^4 - 3648*a^21*b^5*f^4 + 64*a^22*b^4*f^4 + 64*a^23*b^3*f^4 - ((a + b*tan
(e + f*x)^2)^(1/2)*(2*a + 5*b)*(256*a^15*b^13*f^5 - 3072*a^16*b^12*f^5 + 16
640*a^17*b^11*f^5 - 53760*a^18*b^10*f^5 + 115200*a^19*b^9*f^5 - 172032*a^20
*b^8*f^5 + 182784*a^21*b^7*f^5 - 138240*a^22*b^6*f^5 + 72960*a^23*b^5*f^5 -
 25600*a^24*b^4*f^5 + 5376*a^25*b^3*f^5 - 512*a^26*b^2*f^5))/(4*f*(a^7)^(1/
2))))/(4*f*(a^7)^(1/2)))*(2*a + 5*b))/(4*f*(a^7)^(1/2)) - (((a + b*tan(e +
f*x)^2)^(1/2)*(400*a^9*b^14*f^3 - 3680*a^10*b^13*f^3 + 14864*a^11*b^12*f^3
- 34240*a^12*b^11*f^3 + 48480*a^13*b^10*f^3 - 41280*a^14*b^9*f^3 + 16864*a^
15*b^8*f^3 + 2688*a^16*b^7*f^3 - 6000*a^17*b^6*f^3 + 1440*a^18*b^5*f^3 + 10
40*a^19*b^4*f^3 - 704*a^20*b^3*f^3 + 128*a^21*b^2*f^3) + ((2*a + 5*b)*(320*
a^12*b^14*f^4 - 3392*a^13*b^13*f^4 + 16192*a^14*b^12*f^4 - 45760*a^15*b^11*
f^4 + 84608*a^16*b^10*f^4 - 106624*a^17*b^9*f^4 + 92288*a^18*b^8*f^4 - 5363
2*a^19*b^7*f^4 + 19520*a^20*b^6*f^4 - 3648*a^21*b^5*f^4 + 64*a^22*b^4*f^4 +
 64*a^23*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*(2*a + 5*b)*(256*a^15*b^13
*f^5 - 3072*a^16*b^12*f^5 + 16640*a^17*b^11*f^5 - 53760*a^18*b^10*f^5 + 115
200*a^19*b^9*f^5 - 172032*a^20*b^8*f^5 + 182784*a^21*b^7*f^5 - 138240*a^22*
b^6*f^5 + 72960*a^23*b^5*f^5 - 25600*a^24*b^4*f^5 + 5376*a^25*b^3*f^5 - 512
*a^26*b^2*f^5))/(4*f*(a^7)^(1/2))))/(4*f*(a^7)^(1/2)))*(2*a + 5*b))/(4*f*(a
^7)^(1/2))))*(2*a + 5*b)*1i)/(2*f*(a^7)^(1/2)) - (b^2/(3*a*(a - b)) + (b*(a
 + b*tan(e + f*x)^2)*(8*a*b - 5*b^2))/(3*(a^4 - 2*a^3*b + a^2*b^2)) + (b*(a
 + b*tan(e + f*x)^2)^2*(a^2 - 8*a*b + 5*b^2))/(2*(a^5 - 2*a^4*b + a^3*b^2))
)/(f*(a + b*tan(e + f*x)^2)^(5/2) - a*f*(a + b*tan(e + f*x)^2)^(3/2)) - (at
an((a^14*f^3*(a + b*tan(e + f*x)^2)^(1/2)*8i - a^9*f*(a + b*tan(e + f*x)^2)
^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10
*a^3*b^2*f^2)*8i + b^9*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 +
5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*25i + a^6*b^8*
f^3*(a + b*tan(e + f*x)^2)^(1/2)*4i - a^7*b^7*f^3*(a + b*tan(e + f*x)^2)^(1
/2)*36i + a^8*b^6*f^3*(a + b*tan(e + f*x)^2)^(1/2)*140i - a^9*b^5*f^3*(a +
b*tan(e + f*x)^2)^(1/2)*308i + a^10*b^4*f^3*(a + b*tan(e + f*x)^2)^(1/2)*42
0i - a^11*b^3*f^3*(a + b*tan(e + f*x)^2)^(1/2)*364i + a^12*b^2*f^3*(a + b*t
an(e + f*x)^2)^(1/2)*196i - a^13*b*f^3*(a + b*tan(e + f*x)^2)^(1/2)*60i + a
^2*b^7*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*
a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*38...

3.351 $\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=272

$$-\frac{(8a^2+20ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}+\frac{b(12a}{24a^3(a-b)f}$$

```
[Out] -1/8*(8*a^2+20*a*b+35*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(9/2
)/f+arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/8*b*(4*a^
3+3*a^2*b-50*a*b^2+35*b^3)/a^4/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/24*b*(1
2*a^2+15*a*b-35*b^2)/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)+1/8*(4*a+7*b)*cot
(f*x+e)^2/a^2/f/(a+b*tan(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^4/a/f/(a+b*tan(f*x+
e)^2)^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 457, 105, 156, 157, 162, 65, 214}

$$\frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(8a^2+20ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{b(12a^2+15ab-35b^2)}{24a^3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{b(4a^3+3a^2b-50ab^2+35b^3)}{8a^4f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}-\frac{\cot^4(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/8*((8*a^2 + 20*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]
)/(a^(9/2)*f) + ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5
/2)*f) + (b*(12*a^2 + 15*a*b - 35*b^2))/(24*a^3*(a - b)*f*(a + b*Tan[e + f*
x]^2)^(3/2)) + ((4*a + 7*b)*Cot[e + f*x]^2)/(8*a^2*f*(a + b*Tan[e + f*x]^2)
^(3/2)) - Cot[e + f*x]^4/(4*a*f*(a + b*Tan[e + f*x]^2)^(3/2)) + (b*(4*a^3 +
 3*a^2*b - 50*a*b^2 + 35*b^3))/(8*a^4*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+7b)+\frac{7bx}{2}}{x^2(1+x)(a+bx)^{5/2}}\,dx, x, \tan^2(e+fx)\right)}{4af}$$

$$= \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{4}(8a^2 \ldots\right.}{\ldots}$$

$$= \frac{b(12a^2+15ab-35b^2)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\ldots}{4af\,(\ldots}$$

$$= \frac{b(12a^2+15ab-35b^2)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\ldots}{4af\,(\ldots}$$

$$= \frac{b(12a^2+15ab-35b^2)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\ldots}{4af\,(\ldots}$$

$$= \frac{b(12a^2+15ab-35b^2)}{24a^3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\ldots}{4af\,(\ldots}$$

$$= -\frac{(8a^2+20ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+\ldots}}{\ldots}\right.}{(a-\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 1.31, size = 165, normalized size = 0.61

$$\frac{\cot^2(e+fx)\left(8a^3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(3a\cot^2(e+fx)\left(-4a-7b+2a\cot^2(e+fx)\right)-\left(8a^2+20ab+35b^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\tan^2(e+fx)}{a}\right)\right)\right)}{24a^3(-a+b)f\left(b+a\cot^2(e+fx)\right)\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cot[e + f*x]^2*(8*a^3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]
^2)/(a - b)] + (a - b)*(3*a*Cot[e + f*x]^2*(-4*a - 7*b + 2*a*Cot[e + f*x]^2
) - (8*a^2 + 20*a*b + 35*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[e
 + f*x]^2)/a])))/(24*a^3*(-a + b)*f*(b + a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e
 + f*x]^2])
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 107.50, size = 790286, normalized size = 2905.46

method	result	size
default	Expression too large to display	790286

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(250) = 500$.
time = 3.50, size = 2501, normalized size = 9.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(24*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x +
e)^4)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + 3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*
b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b
^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7
- 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)
*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a
)/tan(f*x + e)^2) - 2*(6*a^7 - 18*a^6*b + 18*a^5*b^2 - 6*a^4*b^3 - 3*(4*a^5
*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 - 4*(6*
a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4 -
```

3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^4), 1/48*(48*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x + e)^4)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) + 3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7 - 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(6*a^7 - 18*a^6*b + 18*a^5*b^2 - 6*a^4*b^3 - 3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^4), 1/24*(3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7 - 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 12*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x + e)^4)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - (6*a^7 - 18*a^6*b + 18*a^5*b^2 - 6*a^4*b^3 - 3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^4), 1/24*(3*((8*a^5*b^2 - 4*a^4*b^3 - a^3*b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8*a^7 - 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a)/a) + 24*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x + e)^4)*sqrt(-a + b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(a - b)) - (6*a^7 - 18*a^6*b + 18*a^5*b^2 - 6*a^4*b^3 - 3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^4)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [B]
time = 14.01, size = 2500, normalized size = 9.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] ((b*(a + b*tan(e + f*x)^2)^2*(15*a^2*b - 250*a*b^2 + 12*a^3 + 175*b^3))/(24
*(a^3*b - a^4)*(a - b)) - b^3/(3*a*(a - b)) + (b*(a + b*tan(e + f*x)^2)^3*(
3*a^2*b - 50*a*b^2 + 4*a^3 + 35*b^3))/(8*(a^3*b - a^4)*(a*b - a^2)) + (b*(1
0*a*b^2 - 7*b^3)*(a + b*tan(e + f*x)^2))/(3*a*(a - b)*(a*b - a^2)))/(f*(a +
 b*tan(e + f*x)^2)^(7/2) + a^2*f*(a + b*tan(e + f*x)^2)^(3/2) - 2*a*f*(a +
b*tan(e + f*x)^2)^(5/2)) - (atan((a^16*f^3*(a + b*tan(e + f*x)^2)^(1/2)*128
i - a^11*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 -
5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*128i + b^11*f*(a + b*tan(e +
 f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*
f^2 + 10*a^3*b^2*f^2)*1225i + a^8*b^8*f^3*(a + b*tan(e + f*x)^2)^(1/2)*64i
- a^9*b^7*f^3*(a + b*tan(e + f*x)^2)^(1/2)*576i + a^10*b^6*f^3*(a + b*tan(e
 + f*x)^2)^(1/2)*2240i - a^11*b^5*f^3*(a + b*tan(e + f*x)^2)^(1/2)*4928i +
a^12*b^4*f^3*(a + b*tan(e + f*x)^2)^(1/2)*6720i - a^13*b^3*f^3*(a + b*tan(e
```

+ f*x)^2)^(1/2)*5824i + a^14*b^2*f^3*(a + b*tan(e + f*x)^2)^(1/2)*3136i - a^15*b*f^3*(a + b*tan(e + f*x)^2)^(1/2)*960i + a^2*b^9*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*16885i - a^3*b^8*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*19875i + a^4*b^7*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*11859i - a^5*b^6*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*4053i + a^6*b^5*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*2919i - a^7*b^4*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*2625i + a^8*b^3*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*904i - a^9*b^2*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*256i - a*b^10*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*7175i + a^10*b*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*320i)/((a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)^(3/2)*(1225*b^9 - 4725*a*b^8 + 6210*a^2*b^7 - 2730*a^3*b^6 + 189*a^4*b^5 - 945*a^5*b^4 + 840*a^6*b^3)))*1i)/(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)^(1/2) - (atanh((156800*a^2*b^17*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) - (1545600*a^3*b^16*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) + (6775680*a^4*b^15*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 174993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 - 15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 860160*a^21*b^5*f^2) - (17326720*a^5*b^14*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(5488000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491

200*a^9*b^17*f^2 - 483286400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 6684
07040*a^12*b^14*f^2 + 435855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 + 1
74993280*a^15*b^11*f^2 - 118823040*a^16*b^10*f^2 + 51799680*a^17*b^9*f^2 -
15232896*a^18*b^8*f^2 + 7343616*a^19*b^7*f^2 - 3978240*a^20*b^6*f^2 + 86016
0*a^21*b^5*f^2) + (28492032*a^6*b^13*f^2*(a + b*tan(e + f*x)^2)^(1/2)*(320*
a^12*b + 64*a^13 + 1225*a^9*b^4 + 1400*a^10*b^3 + 960*a^11*b^2)^(1/2))/(548
8000*a^7*b^19*f^2 - 50960000*a^8*b^18*f^2 + 207491200*a^9*b^17*f^2 - 483286
400*a^10*b^16*f^2 + 704892160*a^11*b^15*f^2 - 668407040*a^12*b^14*f^2 + 435
855616*a^13*b^13*f^2 - 248036096*a^14*b^12*f^2 ...

3.352 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f}-\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f+arct
anh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-a*(a-2*b)*tan(f*
x+e)/(a-b)^2/b^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*a*tan(f*x+e)^3/(a-b)/b/f/(a
+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3751, 481, 592, 537, 223, 212, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f}-\frac{a(a-2b)\tan(e+fx)}{b^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\tan^3(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(b^(5/
2)*f) - (a*Tan[e + f*x]^3)/(3*(a - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (
a*(a - 2*b)*Tan[e + f*x])/((a - b)^2*b^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2(3a+3(a-b)x^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{3(a-b)bf}$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.96, size = 295, normalized size = 1.73

$$-\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(a^2(a-b)(2ab+(3a-7b)(a+b+(a-b)\cos(2(e+fx))))\sin(2(e+fx))-\frac{3a^2b\left(\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}\right)^{3/2}\left((a^2-3ab+2b^2)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)+b^2\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sin^2(e+fx)\sin(2(e+fx))}{\sqrt{2}}\right)}{3\sqrt{2}\,a(a-b)^3b^2f(a+b+(a-b)\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(a^2*(a - b)*
(2*a*b + (3*a - 7*b)*(a + b + (a - b)*Cos[2*(e + f*x)]))*Sin[2*(e + f*x)] -
 (3*a^2*b*(((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*((a
^2 - 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)
])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[-(b/(a - b)), ArcSin[Sq
```

```
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin
[e + f*x]^2*Sin[2*(e + f*x)])/Sqrt[2]))/(Sqrt[2]*a*(a - b)^3*b^2*f*(a + b +
 (a - b)*Cos[2*(e + f*x)])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(153) = 306$.
time = 0.10, size = 381, normalized size = 2.23

method	result
derivativedivides	$-\frac{\tan^3(fx+e)}{3b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{-\frac{\tan(fx+e)}{b\sqrt{a+b\left(\tan^2(fx+e)\right)}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right.}{b^{\frac{3}{2}}}}{b}$
default	$-\frac{\tan^3(fx+e)}{3b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{-\frac{\tan(fx+e)}{b\sqrt{a+b\left(\tan^2(fx+e)\right)}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\left(\tan^2(fx+e)\right)}\right.}{b^{\frac{3}{2}}}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/3*tan(f*x+e)^3/b/(a+b*tan(f*x+e)^2)^(3/2)+1/b*(-tan(f*x+e)/b/(a+b*t
an(f*x+e)^2)^(1/2)+1/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2)
))+1/2*tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(3/2)-1/2*a/b*(1/3*tan(f*x+e)/a/(a+b
*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))+1/3*tan(f
*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2
)+1/(a-b)*b*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+e)/(
a+b*tan(f*x+e)^2)^(1/2))-1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(
b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/(a-b)^2*b*tan(f*x+e
)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(161) = 322$.
time = 7.27, size = 1772, normalized size = 10.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [1/6*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 3*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2 + a^2*b^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((4*a^3*b^2 - 11*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x + e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/6*(6*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + 3*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2 + a^2*b^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((4*a^3*b^2 - 11*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x + e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/6*(6*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2 + a^2*b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) - 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*((4*a^3*b^2 - 11*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x + e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/3*(3*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2 + a^2*b^3)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt(b*tan(f*x + e)^2 + a)*sqrt(-b)/(b*tan(f*x + e))) + ((4*a^3*b^2 - 11*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*x + e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f)]

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^6}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.353 $\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f}-\frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f+1/3*(
a-4*b)*tan(f*x+e)/(a-b)^2/b/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*a*tan(f*x+e)/(a-
b)/b/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 481, 541, 12, 385, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}+\frac{(a-4b)\tan(e+fx)}{3bf(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\tan(e+fx)}{3bf(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (a*Tan[e + f*x])/(3*(a - b)*b*f*(a + b*Tan[e + f*x]^2)^(3/2)) + ((a
- 4*b)*Tan[e + f*x])/(3*(a - b)^2*b*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+(a-3b)x^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{3(a-b)bf}$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{a\tan(e+fx)}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}} +$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{a\tan(e+fx)}{3(a-b)bf\,(a+b\tan^2(e+fx))^{3/2}} + \frac{}{3(\cdots}$$

Mathematica [A]
time = 6.04, size = 260, normalized size = 1.98

$$\frac{\tan^5(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(\frac{\tanh^{-1}\left(\frac{\sqrt{-\tan^2(e+fx)+\frac{b\tan^2(e+fx)}{a}}}{\sqrt{1+\frac{b\tan^2(e+fx)}{a}}}\right)\sqrt{-\tan^2(e+fx)+\frac{b\tan^2(e+fx)}{a}}}{\sqrt{1+\frac{b\tan^2(e+fx)}{a}}} - \frac{-\tan^2(e+fx)+\frac{b\tan^2(e+fx)}{a}}{1+\frac{b\tan^2(e+fx)}{a}} - \frac{\left(-\tan^2(e+fx)+\frac{b\tan^2(e+fx)}{a}\right)^2}{3\left(1+\frac{b\tan^2(e+fx)}{a}\right)^2}\right)}{a^2f\sqrt{a+b\tan^2(e+fx)}\left(-\tan^2(e+fx)+\frac{b\tan^2(e+fx)}{a}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Tan[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*((ArcTanh[Sqrt[-Tan[e + f*x]^2 +
 (b*Tan[e + f*x]^2)/a]/Sqrt[1 + (b*Tan[e + f*x]^2)/a]]*Sqrt[-Tan[e + f*x]^2
 + (b*Tan[e + f*x]^2)/a])/Sqrt[1 + (b*Tan[e + f*x]^2)/a] - (-Tan[e + f*x]^2
 + (b*Tan[e + f*x]^2)/a)/(1 + (b*Tan[e + f*x]^2)/a) - (-Tan[e + f*x]^2 + (b
```

```
*Tan[e + f*x]^2)/a)^2/(3*(1 + (b*Tan[e + f*x]^2)/a)^2)))/(a^2*f*Sqrt[a + b*
Tan[e + f*x]^2]*(-Tan[e + f*x]^2 + (b*Tan[e + f*x]^2)/a)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(117) = 234$.
time = 0.10, size = 295, normalized size = 2.25

method	result
derivativedivides	$\frac{-\frac{\tan(fx+e)}{2b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{a\left(\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{2b}-\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}$
default	$\frac{-\frac{\tan(fx+e)}{2b\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{a\left(\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{2b}-\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(3/2)+1/2*a/b*(1/3*tan(f*x+e)/a/(
a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))-1/3*ta
n(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)-2/3/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(
1/2)-1/(a-b)*b*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+e
)/(a+b*tan(f*x+e)^2)^(1/2))-1/(a-b)^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/
2)+1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*
tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 1.60, size = 518, normalized size = 3.95

$$\left[-\frac{3\left(b^2\tan(fx+e)^4+2ab\tan(fx+e)^2+a^2\right)\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)-2\left((a^2-5ab+4b^2)\tan(fx+e)^3-3(a^2-ab)\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}}{6\left((a^3b^2-3a^2b^3+3ab^4-b^5)f\tan(fx+e)^4+2(a^4b-3a^3b^2+3a^2b^3-ab^4)f\tan(fx+e)^2+(a^5-3a^4b+3a^3b^2-a^2b^3)f\right)}, \frac{3\left(b^2\tan(fx+e)^4+2ab\tan(fx+e)^2+a^2\right)\sqrt{a-b}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{a-b}\tan(fx+e)}\right)+\left((a^2-5ab+4b^2)\tan(fx+e)^3-3(a^2-ab)\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}}{3\left((a^3b^2-3a^2b^3+3ab^4-b^5)f\tan(fx+e)^4+2(a^4b-3a^3b^2+3a^2b^3-ab^4)f\tan(fx+e)^2+(a^5-3a^4b+3a^3b^2-a^2b^3)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log
(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan
(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((a^2 - 5*a*b + 4*b^2)*tan(f*x + e
)^3 - 3*(a^2 - a*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^2 - 3
*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f), 1/
3*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(-
sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) + ((a^2 - 5*a*b + 4*
b^2)*tan(f*x + e)^3 - 3*(a^2 - a*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a
))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 -
 a^2*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^4}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.354 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f}+\frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f+1/3*
(2*a+b)*tan(f*x+e)/a/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/(a-b
)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]

time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3751, 482, 541, 12, 385, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}+\frac{(2a+b)\tan(e+fx)}{3af(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan(e+fx)}{3f(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) + Tan[e + f*x]/(3*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) + ((2*a +
 b)*Tan[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1-2x^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e+\right.}{3(a-b)f}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \text{S}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \text{S}$$

$$= \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \text{S}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} + \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.13, size = 365, normalized size = 2.85

$$\frac{\cos^4(e+fx)\cot(e+fx)\left(12(a-b)^3\,{}_2F_1\left(2,2;\frac{9}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\sqrt{\frac{\cos^2(e+fx)\sin^2(e+fx)\left(a^2-b^2\tan^2(e+fx)+ab\left(-1+\tan^2(e+fx)\right)\right)}{a^2}}+35a\sec^2(e+fx)\left(5a+2b\tan^2(e+fx)\right)\left(3\text{ArcSin}\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\right)\left(a+b\tan^2(e+fx)\right)^2+a\sec^2(e+fx)\left(-4b\tan^2(e+fx)+a(-3+\tan^2(e+fx))\right)\sqrt{\frac{\cos^2(e+fx)\sin^2(e+fx)\left(a^2-b^2\tan^2(e+fx)+ab\left(-1+\tan^2(e+fx)\right)\right)}{a^2}}\right)\right)}{315a^4(a-b)^2f\sqrt{a+b\tan^2(e+fx)}\sqrt{\frac{(a-b)\cos^2(e+fx)\sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)}{a^2}}\left(1+\frac{b\tan^2(e+fx)}{a}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*Cot[e + f*x]*(12*(a - b)^3*Hypergeometric2F1[2, 2, 9/2, ((a
 - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)*Sqrt[(Cos[e
+ f*x]^2*Sin[e + f*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^
2)))/a^2] + 35*a*Sec[e + f*x]^2*(5*a + 2*b*Tan[e + f*x]^2)*(3*ArcSin[Sqrt[(
(a - b)*Sin[e + f*x]^2)/a]]*(a + b*Tan[e + f*x]^2)^2 + a*Sec[e + f*x]^2*(-4
*b*Tan[e + f*x]^2 + a*(-3 + Tan[e + f*x]^2))*Sqrt[(Cos[e + f*x]^2*Sin[e + f
*x]^2*(a^2 - b^2*Tan[e + f*x]^2 + a*b*(-1 + Tan[e + f*x]^2)))/a^2])))/(315*
```

```
a^4*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2]*Sqrt[((a - b)*Cos[e + f*x]^2*Sin
[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]*(1 + (b*Tan[e + f*x]^2)/a))
```

Maple [A]
time = 0.09, size = 212, normalized size = 1.66

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}+\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx\right.}}\right.}{a-b}}{}$
default	$\frac{\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}+\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx\right.}}\right.}{a-b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2)+1/(a-b)*b*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^
2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))-1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arct
an(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/(a-b)
^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(120) = 240.
time = 2.77, size = 549, normalized size = 4.29

$$\left[-\frac{3\left(ab^2\tan(fx+e)^4+2a^2b\tan(fx+e)^2+a^3\right)\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)-2\left((2a^2b-ab^2-b^3)\tan(fx+e)^3+3(a^3-a^2b)\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}}{6\left((a^4b^2-3a^3b^3+3a^2b^4-ab^5)f\tan(fx+e)^4+2(a^5b-3a^4b^2+3a^3b^3-a^2b^4)f\tan(fx+e)^2+(a^6-3a^5b+3a^4b^2-a^3b^3)f\right)}, -\frac{3\left(ab^2\tan(fx+e)^4+2a^2b\tan(fx+e)^2+a^3\right)\sqrt{a-b}\arctan\left(-\frac{\sqrt{b\tan(fx+e)^2+a}}{\sqrt{a-b}\tan(fx+e)}\right)-\left((2a^2b-ab^2-b^3)\tan(fx+e)^3+3(a^3-a^2b)\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}}{3\left((a^4b^2-3a^3b^3+3a^2b^4-ab^5)f\tan(fx+e)^4+2(a^5b-3a^4b^2+3a^3b^3-a^2b^4)f\tan(fx+e)^2+(a^6-3a^5b+3a^4b^2-a^3b^3)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a*b^2*tan(f*x + e)^4 + 2*a^2*b*tan(f*x + e)^2 + a^3)*sqrt(-a + b)
*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)
*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((2*a^2*b - a*b^2 - b^3)*tan(f
*x + e)^3 + 3*(a^3 - a^2*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^4
*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(f*x + e)^4 + 2*(a^5*b - 3*a^4*b
^2 + 3*a^3*b^3 - a^2*b^4)*f*tan(f*x + e)^2 + (a^6 - 3*a^5*b + 3*a^4*b^2 - a
^3*b^3)*f), -1/3*(3*(a*b^2*tan(f*x + e)^4 + 2*a^2*b*tan(f*x + e)^2 + a^3)*s
qrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(sqrt(a - b)*tan(f*x + e))) -
 ((2*a^2*b - a*b^2 - b^3)*tan(f*x + e)^3 + 3*(a^3 - a^2*b)*tan(f*x + e))*sq
rt(b*tan(f*x + e)^2 + a))/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(
f*x + e)^4 + 2*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f*tan(f*x + e)^2 +
 (a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.355 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f}-\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*(
5*a-2*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+
e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3742, 425, 541, 12, 385, 209}

$$-\frac{b(5a-2b)\tan(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\tan(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Tan[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((5*
a - 2*b)*b*Tan[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e-\right.}{3a(a-b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{3a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.03, size = 1331, normalized size = 9.93

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]] -
 (3150*(a - b)*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2)/a +
 (1575*(a - b)^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4)/a
^2 + (2100*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/a - (
4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[
e + f*x]^2)/a^2 + (2100*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]
]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (840*b^2*ArcSin[Sqrt[((a - b)*Sin[e
+ f*x]^2)/a]]*Tan[e + f*x]^4)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*
```

Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 + (840*(a - b)^2*b^2
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4*Tan[e + f*x]^4)/a^
4 + 2100*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + 96*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)
/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*
x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan
[e + f*x]^2))/a] + (2800*b*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e + f*x]^
2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (168*b*Hypergeometri
c2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7
/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (48
*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*(((a
- b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Ta
n[e + f*x]^2))/a])/a + (1120*b^2*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Tan[e +
f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 + (72*b^2*Hype
rgeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]
^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]
)/a^2 + (24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x
]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^4*Sqrt[(Cos[e + f*x
]^2*(a + b*Tan[e + f*x]^2))/a])/a^2 - 1575*Sqrt[((a - b)*Cos[e + f*x]^2*Sin
[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (2100*b*Tan[e + f*x]^2*Sqrt[((a
- b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])/a - (840*b
^2*Tan[e + f*x]^4*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e
+ f*x]^2))/a^2])/a^2))/(315*a^2*f*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[a
+ b*Tan[e + f*x]^2]*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]*(1 + (
b*Tan[e + f*x]^2)/a))

Maple [A]
time = 0.10, size = 163, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{a-b}+\frac{\sqrt{b^4(a-b)}\ \arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}\sqrt{a}}\right.}{(a-b)^3b^2}}{f}$
default	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\left(\tan^2(fx+e)\right)}}\right)}{a-b}+\frac{\sqrt{b^4(a-b)}\ \arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}\sqrt{a}}\right.}{(a-b)^3b^2}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/f*(-1/(a-b)*b*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+
e)/(a+b*tan(f*x+e)^2)^(1/2))+1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-

```
b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/(a-b)^2*b*tan(f
*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(126) = 252$.
time = 1.37, size = 581, normalized size = 4.34

$$\left[-\frac{3\left(a^{2} b^{2} \tan\left(f x+e\right)^{4}+2 a^{3} b \tan\left(f x+e\right)^{2}+a^{4}\right) \sqrt{-a+b} \log\left(-\frac{(a-2b)\tan(fx+e)^{2}-2\sqrt{b \tan\left(f x+e\right)^{2}+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^{2}+1}\right)+2\left(\left(5 a^{2} b^{2}-7 a b^{3}+2 b^{4}\right) \tan\left(f x+e\right)^{3}+3\left(2 a^{3} b-3 a^{2} b^{2}+a b^{3}\right) \tan\left(f x+e\right)\right) \sqrt{b \tan\left(f x+e\right)^{2}+a}}{6\left(\left(a^{5} b^{2}-3 a^{4} b^{3}+3 a^{3} b^{4}-a^{2} b^{5}\right) f \tan\left(f x+e\right)^{4}+2\left(a^{6} b-3 a^{5} b^{2}+3 a^{4} b^{3}-a^{3} b^{4}\right) f \tan\left(f x+e\right)^{2}+\left(a^{7}-3 a^{6} b+3 a^{5} b^{2}-a^{4} b^{3}\right) f\right)}, \frac{3\left(a^{2} b^{2} \tan\left(f x+e\right)^{4}+2 a^{3} b \tan\left(f x+e\right)^{2}+a^{4}\right) \sqrt{a-b} \arctan\left(-\frac{\sqrt{b \tan\left(f x+e\right)^{2}+a}}{\sqrt{a-b}\tan(fx+e)}\right)-\left(\left(5 a^{2} b^{2}-7 a b^{3}+2 b^{4}\right) \tan\left(f x+e\right)^{3}+3\left(2 a^{3} b-3 a^{2} b^{2}+a b^{3}\right) \tan\left(f x+e\right)\right) \sqrt{b \tan\left(f x+e\right)^{2}+a}}{3\left(\left(a^{5} b^{2}-3 a^{4} b^{3}+3 a^{3} b^{4}-a^{2} b^{5}\right) f \tan\left(f x+e\right)^{4}+2\left(a^{6} b-3 a^{5} b^{2}+3 a^{4} b^{3}-a^{3} b^{4}\right) f \tan\left(f x+e\right)^{2}+\left(a^{7}-3 a^{6} b+3 a^{5} b^{2}-a^{4} b^{3}\right) f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(-a +
b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((5*a^2*b^2 - 7*a*b^3 + 2*b^
4)*tan(f*x + e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*ta
n(f*x + e)^2 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x +
 e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7
 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/3*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^
3*b*tan(f*x + e)^2 + a^4)*sqrt(a - b)*arctan(-sqrt(b*tan(f*x + e)^2 + a)/(s
qrt(a - b)*tan(f*x + e))) - ((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*tan(f*x + e)^3 +
 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/
((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b -
3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*
b^2 - a^4*b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**(-5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.356 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f}-\frac{b\cot(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*
(7*a-4*b)*b*cot(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*(a-4*b)*(
3*a-2*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f-1/3*b*cot(f*x+e)
/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]

time = 0.18, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3751, 483, 593, 597, 12, 385, 209}

$$-\frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)^2}-\frac{b(7a-4b)\cot(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\cot(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) - (b*Cot[e + f*x])/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - (
(7*a - 4*b)*b*Cot[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2])
- ((a - 4*b)*(3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(3*a^3*(a
 - b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-4b-4bx^2}{x^2(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e\right.}{3a(a-b)f}$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2 f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2 f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2 f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{b\cot(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2 f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 12.08, size = 1890, normalized size = 10.16

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -((Cos[e + f*x]^2*Cot[e + f*x]*((20*a*Csc[e + f*x]^2)/(3*(a - b)) - (5*a^2*
Csc[e + f*x]^4)/(a - b)^2 + (40*b*Sec[e + f*x]^2)/(a - b) - (30*a*b*Csc[e +
 f*x]^2*Sec[e + f*x]^2)/(a - b)^2 - (40*b^2*Sec[e + f*x]^4)/(a - b)^2 + (92
*(a - b)*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f
```

*x]^2)/(105*a) + (24*(a - b)*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b
)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(35*a) + (16*(a - b)*HypergeometricPFQ
[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(10
5*a) + (160*b^2*Sec[e + f*x]^2*Tan[e + f*x]^2)/(3*a*(a - b)) + (124*(a - b)
*b*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*
Tan[e + f*x]^2)/(35*a^2) + (16*(a - b)*b*HypergeometricPFQ[{2, 2, 2}, {1, 9
/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(7*a^2) + (
16*(a - b)*b*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e +
f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(35*a^2) + (64*b^3*Sec[e + f*x]^2
*Tan[e + f*x]^4)/(3*a^2*(a - b)) + (152*(a - b)*b^2*Hypergeometric2F1[2, 2,
 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(35*a^3) +
 (88*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*
x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(35*a^3) + (16*(a - b)*b^2*Hypergeo
metricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*
x]^2*Tan[e + f*x]^4)/(35*a^3) + (176*(a - b)*b^3*Hypergeometric2F1[2, 2, 9/
2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/(105*a^4) + (
32*(a - b)*b^3*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]
^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/(35*a^4) + (16*(a - b)*b^3*Hypergeome
tricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]
^2*Tan[e + f*x]^6)/(105*a^4) + (5*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])
/((((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*
x]^2))/a]) + (30*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)
/(a*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a]) + (40*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x
]^4)/(a^2*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Ta
n[e + f*x]^2))/a]) + (16*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e
 + f*x]^6)/(a^3*(((a - b)*Sin[e + f*x]^2)/a)^(5/2)*Sqrt[(Cos[e + f*x]^2*(a
+ b*Tan[e + f*x]^2))/a]) + (5*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/Sqr
t[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] - (10
*a*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Csc[e + f*x]^2)/((a - b)*Sqrt[(
(a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (60*b
*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sec[e + f*x]^2)/((a - b)*Sqrt[((a
 - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) + (30*b*A
rcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*Sqrt[((a - b)*Co
s[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (80*b^2*ArcSin[
Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)
*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])
+ (40*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*Sqr
t[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]) - (3
2*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sec[e + f*x]^2*Tan[e + f*x]^
4)/(a^2*(a - b)*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e +
f*x]^2))/a^2]) + (16*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f
*x]^6)/(a^3*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]
^2))/a^2]) - (16*b^3*(Tan[e + f*x] + Tan[e + f*x]^3)^2)/(a*(a - b)^2)))/(a^
2*f*Sqrt[a + b*Tan[e + f*x]^2]*(1 + (b*Tan[e + f*x]^2)/a)))

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(fx+e)}{(a+b(\tan^2(fx+e)))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(176) = 352.
time = 3.57, size = 781, normalized size = 4.20

$$\left[-\frac{3\left(a^3b^2\tan(fx+e)^5+2a^4b\tan(fx+e)^3+a^5\tan(fx+e)\right)\sqrt{-a+b}\log\left(-\frac{(a^2-8ab+8b^2)\tan(fx+e)^4-2(3a^2-4ab)\tan(fx+e)^2+a^2+4\left((a-2b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)+4\left(3a^5-9a^4b+9a^3b^2-3a^2b^3+(3a^3b^2-17a^2b^3+22ab^4-8b^5)\tan(fx+e)^4+3(2a^4b-9a^3b^2+11a^2b^3-4ab^4)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{12\left((a^6b^2-3a^5b^3+3a^4b^4-a^3b^5)f\tan(fx+e)^5+2(a^7b-3a^6b^2+3a^5b^3-a^4b^4)f\tan(fx+e)^3+(a^8-3a^7b+3a^6b^2-a^5b^3)f\tan(fx+e)\right)},\ -\frac{3\left(a^3b^2\tan(fx+e)^5+2a^4b\tan(fx+e)^3+a^5\tan(fx+e)\right)\sqrt{a-b}\arctan\left(-\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}\tan(fx+e)}{(a-2b)\tan(fx+e)^2-a}\right)+2\left(3a^5-9a^4b+9a^3b^2-3a^2b^3+(3a^3b^2-17a^2b^3+22ab^4-8b^5)\tan(fx+e)^4+3(2a^4b-9a^3b^2+11a^2b^3-4ab^4)\tan(fx+e)^2\right)\sqrt{b\tan(fx+e)^2+a}}{6\left((a^6b^2-3a^5b^3+3a^4b^4-a^3b^5)f\tan(fx+e)^5+2(a^7b-3a^6b^2+3a^5b^3-a^4b^4)f\tan(fx+e)^3+(a^8-3a^7b+3a^6b^2-a^5b^3)f\tan(fx+e)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^3*b^2*tan(f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*tan(f*x +
e))*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*
a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2
+ 1)) + 4*(3*a^5 - 9*a^4*b + 9*a^3*b^2 - 3*a^2*b^3 + (3*a^3*b^2 - 17*a^2*b^
3 + 22*a*b^4 - 8*b^5)*tan(f*x + e)^4 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3
- 4*a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^
3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^
3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan
(f*x + e)), -1/6*(3*(a^3*b^2*tan(f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*
tan(f*x + e))*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*
tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*(3*a^5 - 9*a^4*b + 9*a^3*b
^2 - 3*a^2*b^3 + (3*a^3*b^2 - 17*a^2*b^3 + 22*a*b^4 - 8*b^5)*tan(f*x + e)^4
 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3 - 4*a*b^4)*tan(f*x + e)^2)*sqrt(b*ta
n(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x +
```

```
e)^5 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8
- 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cot(e+f\,x)}^2}{{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2), x)
```

3.357 $\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=249

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f}-\frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}+$$

```
[Out] arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-(3*a-
2*b)*b*cot(f*x+e)^3/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*(a-2*b)*(3*a
^2+8*a*b-8*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^4/(a-b)^2/f-1/3*(a^2-
12*a*b+8*b^2)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f-1/3*b*cot
(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)
```

Rubi [A]
time = 0.24, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3751, 483, 593, 597, 12, 385, 209}

$$-\frac{b(3a-2b)\cot^3(e+fx)}{a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^4f(a-b)^2}-\frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f(a-b)^2}+\frac{\text{ArcTan}\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\cot^3(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5/2
)*f) - (b*Cot[e + f*x]^3)/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) - ((
3*a - 2*b)*b*Cot[e + f*x]^3)/(a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]^2]) +
 ((a - 2*b)*(3*a^2 + 8*a*b - 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2]
)/(3*a^4*(a - b)^2*f) - ((a^2 - 12*a*b + 8*b^2)*Cot[e + f*x]^3*Sqrt[a + b*T
an[e + f*x]^2])/(3*a^3*(a - b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{3(a-2b)-6bx^2}{x^4(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e\right.}{3a(a-b)f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\cot^3(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot^3(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\quad}{a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 16.39, size = 871, normalized size = 3.50

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] (-((b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[
-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f
*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)])))
 - (4*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/
(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[
2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)
*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqr
t[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((
a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)
]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(
e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*C
os[2*(e + f*x)]])/((a - b)^2*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos
[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((4*(a*Cos[e + f*x] + 2*b*Cos[e + f*
x])*Csc[e + f*x])/(3*a^4) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^3) + (2*b^4*
Sin[2*(e + f*x)])/(3*a^3*(a - b)^2*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e
 + f*x)])^2) - (4*(3*a*b^3*Sin[2*(e + f*x)] - 2*b^4*Sin[2*(e + f*x)]))/(3*a
^4*(a - b)^2*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)]))))/f
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(fx+e)}{\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

```
[Out] int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 1.61, size = 909, normalized size = 3.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [-1/12*(3*(a^4*b^2*tan(f*x + e)^7 + 2*a^5*b*tan(f*x + e)^5 + a^6*tan(f*x + e)^3)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*((3*a^4*b^2 - a^3*b^3 - 26*a^2*b^4 + 40*a*b^5 - 16*b^6)*tan(f*x + e)^6 - a^6 + 3*a^5*b - 3*a^4*b^2 + a^3*b^3 + 3*(2*a^5*b - a^4*b^2 - 13*a^3*b^3 + 20*a^2*b^4 - 8*a*b^5)*tan(f*x + e)^4 + 3*(a^6 - a^5*b - 3*a^4*b^2 + 5*a^3*b^3 - 2*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^3), 1/6*(3*(a^4*b^2*tan(f*x + e)^7 + 2*a^5*b*tan(f*x + e)^5 + a^6*tan(f*x + e)^3)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((3*a^4*b^2 - a^3*b^3 - 26*a^2*b^4 + 40*a*b^5 - 16*b^6)*tan(f*x + e)^6 - a^6 + 3*a^5*b - 3*a^4*b^2 + a^3*b^3 + 3*(2*a^5*b - a^4*b^2 - 13*a^3*b^3 + 20*a^2*b^4 - 8*a*b^5)*tan(f*x + e)^4 + 3*(a^6 - a^5*b - 3*a^4*b^2 + 5*a^3*b^3 - 2*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)

[Out] Integral(cot(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] integrate(cot(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

3.358 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=327

$$-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f}-\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*
(11*a-8*b)*b*cot(f*x+e)^5/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a
^4+10*a^3*b+8*a^2*b^2-176*a*b^3+128*b^4)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2
)/a^5/(a-b)^2/f+1/15*(5*a^3+4*a^2*b-88*a*b^2+64*b^3)*cot(f*x+e)^3*(a+b*tan(
f*x+e)^2)^(1/2)/a^4/(a-b)^2/f-1/5*(a^2-22*a*b+16*b^2)*cot(f*x+e)^5*(a+b*tan
(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f-1/3*b*cot(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)
^2)^(3/2)
```

Rubi [A]
time = 0.31, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3751, 483, 593, 597, 12, 385, 209}

$$-\frac{b(11a-8b)\cot^5(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{(a^2-22ab+16b^2)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^3f(a-b)^2}+\frac{(5a^3+4a^2b-88ab^2+64b^3)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^4f(a-b)^2}-\frac{(15a^4+10a^3b+8a^2b^2-176ab^3+128b^4)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^5f(a-b)^2}-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f(a-b)^{5/2}}-\frac{b\cot^5(e+fx)}{3af(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/((a - b)^(5
/2)*f)) - (b*Cot[e + f*x]^5)/(3*a*(a - b)*f*(a + b*Tan[e + f*x]^2)^(3/2)) -
 ((11*a - 8*b)*b*Cot[e + f*x]^5)/(3*a^2*(a - b)^2*f*Sqrt[a + b*Tan[e + f*x]
^2]) - ((15*a^4 + 10*a^3*b + 8*a^2*b^2 - 176*a*b^3 + 128*b^4)*Cot[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/(15*a^5*(a - b)^2*f) + ((5*a^3 + 4*a^2*b - 88*a
*b^2 + 64*b^3)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(15*a^4*(a - b)^2
*f) - ((a^2 - 22*a*b + 16*b^2)*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/(
5*a^3*(a - b)^2*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

`alQ[n]))`

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-8b-8bx^2}{x^6(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \tan(e\right.}{3a(a-b)f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} +$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot^5(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 14.49, size = 441, normalized size = 1.35

$$\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-\frac{15a^5b\left(\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}\right)^{3/2}\left(2(a-b)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)-2a\Pi\left(-\frac{b}{a-b};\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\middle|1\right)\right)\sin^2(e+fx)\sin(2(e+fx))}{2\sqrt{2}}-(a-b)\left((a-b)^2\left(23a^2+54ab+73b^2\right)(a+b+(a-b)\cos(2(e+fx)))^2\cot(e+fx)-a(a-b)^2(11a+14b)(a+b+(a-b)\cos(2(e+fx)))^2\cot(e+fx)\csc^2(e+fx)+3a^2(a-b)^2(a+b+(a-b)\cos(2(e+fx)))^2\cot(e+fx)\csc^4(e+fx)+10ab^5\sin(2(e+fx))-5(15a-11b)b^4(a+b+(a-b)\cos(2(e+fx)))\sin(2(e+fx))\right)\right)}{15\sqrt{2}a^5(a-b)^3f(a+b+(a-b)\cos(2(e+fx)))^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]

[Out] (Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((-15*a^5*b*(((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*(2*(a - b)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 2*a*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e + f*x]^2*Sin[2*(e + f*x)])/(2*Sqrt[2]) - (a - b)*((a - b)^2*(23*a^2 + 54*a*b + 73*b^2)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x] - a*(a - b)^2*(11*a + 14*b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^2 + 3*a^2*(a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^4 + 10*a*b^5*Sin[2*(e + f*x)] - 5*(15*a - 11*b)*b^4*(a + b + (a - b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])))/(15*Sqrt[2]*a^5*(a - b)^3*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(fx+e)}{\left(a+b\left(\tan^2(fx+e)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)

[Out] int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [A]
time = 2.65, size = 1055, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a^5*b^2*tan(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(f*x +
 e)^5)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e)
)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)
^2 + 1)) + 4*((15*a^5*b^2 - 5*a^4*b^3 - 2*a^3*b^4 - 184*a^2*b^5 + 304*a*b^6
 - 128*b^7)*tan(f*x + e)^8 + 3*a^7 - 9*a^6*b + 9*a^5*b^2 - 3*a^4*b^3 + 3*(1
0*a^6*b - 5*a^5*b^2 - a^4*b^3 - 92*a^3*b^4 + 152*a^2*b^5 - 64*a*b^6)*tan(f*
x + e)^6 + 3*(5*a^7 - 5*a^6*b + a^5*b^2 - 23*a^4*b^3 + 38*a^3*b^4 - 16*a^2*
b^5)*tan(f*x + e)^4 - (5*a^7 - 7*a^6*b - 9*a^5*b^2 + 19*a^4*b^3 - 8*a^3*b^4
)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6
*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b
^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e
)^5), -1/30*(15*(a^5*b^2*tan(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(
f*x + e)^5)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*ta
n(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((15*a^5*b^2 - 5*a^4*b^3 - 2
*a^3*b^4 - 184*a^2*b^5 + 304*a*b^6 - 128*b^7)*tan(f*x + e)^8 + 3*a^7 - 9*a^
6*b + 9*a^5*b^2 - 3*a^4*b^3 + 3*(10*a^6*b - 5*a^5*b^2 - a^4*b^3 - 92*a^3*b^
4 + 152*a^2*b^5 - 64*a*b^6)*tan(f*x + e)^6 + 3*(5*a^7 - 5*a^6*b + a^5*b^2 -
 23*a^4*b^3 + 38*a^3*b^4 - 16*a^2*b^5)*tan(f*x + e)^4 - (5*a^7 - 7*a^6*b -
9*a^5*b^2 + 19*a^4*b^3 - 8*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 +
 a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9
*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b +
3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^5)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.359 $\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx$

Optimal. Leaf size=72

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+m+2p);\frac{1}{2}(3+m+2p);-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m\left(b\tan^2(e+fx)\right)^p}{f(1+m+2p)}$$

```
[Out] hypergeom([1, 1/2+1/2*m+p],[3/2+1/2*m+p],-tan(f*x+e)^2)*tan(f*x+e)*(d*tan(f
*x+e))^m*(b*tan(f*x+e)^2)^p/f/(1+m+2*p)
```

Rubi [A]

time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3659, 20, 3557, 371}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(m+2p+1);\frac{1}{2}(m+2p+3);-\tan^2(e+fx)\right)}{f(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Tan[e + f*x
```

```
])^p)^FracPart[n]/(d*Tan[e + f*x])^(p*FracPart[n])), Int[(a + b*Tan[e + f*x
])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{2p}(e+fx)(d\tan(e+$$
$$= \left(\tan^{-m-2p}(e+fx)(d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p\right)\int \tan^m$$
$$= \frac{\left(\tan^{-m-2p}(e+fx)(d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p\right)\text{Subst}\Big(}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+m+2p);\frac{1}{2}(3+m+2p);-\tan^2(e+fx)\right)\tan(e+}{f(1+m+2p)}$$

Mathematica [A]
time = 0.07, size = 74, normalized size = 1.03

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+m+2p);1+\frac{1}{2}(1+m+2p);-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m\left(b\tan^2(e+fx)\right)^p}{f(1+m+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
[Out] (Hypergeometric2F1[1, (1 + m + 2*p)/2, 1 + (1 + m + 2*p)/2, -Tan[e + f*x]^2
]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int (d\tan(fx+e))^m \left(b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
[Out] int((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{2}\left(e+fx\right)\right)^{p}\left(d\tan\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*tan(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

```
[Out] int((d*tan(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

3.360 $\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=100

$$\frac{F_1\left(\frac{1+m}{2};1,-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\tan(e+fx))^{1+m}\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right.}{df(1+m)}$$

```
[Out] AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*tan(f
*x+e))^(1+m)*(a+b*tan(f*x+e)^2)^p/d/f/(1+m)/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3751, 525, 524}

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Tan[e + f*x])^(1 + m)*(a + b*Tan[e + f*x]^2)^p)/(d*f*(1 + m)*(1 +
(b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \frac{(dx)^m (a+bx^2)^p}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left((a+b\tan^2(e+fx))^p \left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right) \text{Subst}\left(\int \frac{(dx)^m (}{1\text{-}}\right.}{f}$$
$$= \frac{F_1\left(\frac{1+m}{2};1,-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\tan(e+}{df(1+m)}$$

Mathematica [A]
time = 0.21, size = 101, normalized size = 1.01

$$\frac{F_1\left(\frac{1+m}{2};-p,1;\frac{3+m}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*
x]^2]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + m)*
(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int (d\tan(fx+e))^m \left(a+b\left(\tan^2(fx+e)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(e+fx\right)\right)^{m}\left(a+b\tan^{2}\left(e+fx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((d*tan(e + f*x))**m*(a + b*tan(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int((d*tan(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

3.361 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=129

$$-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b^2f(1+p)}-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}+\frac{(a+b\tan^2(e}{2b^2f(2}$$

```
[Out] -1/2*(a+b)*(a+b*tan(f*x+e)^2)^(1+p)/b^2/f/(1+p)-1/2*hypergeom([1, 1+p],[2+p
],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(1+p)/(a-b)/f/(1+p)+1/2*(a+b
*tan(f*x+e)^2)^(2+p)/b^2/f/(2+p)
```

Rubi [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3751, 457, 90, 70}

$$-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+2}}{2b^2f(p+2)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/2*((a + b)*(a + b*Tan[e + f*x]^2)^(1 + p))/(b^2*f*(1 + p)) - (Hypergeome
tric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x
]^2)^(1 + p))/(2*(a - b)*f*(1 + p)) + (a + b*Tan[e + f*x]^2)^(2 + p)/(2*b^2
*f*(2 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^p}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-b)(a+bx)^p}{b}+\frac{(a+bx)^p}{1+x}+\frac{(a+bx)^{1+p}}{b}\right)\,dx, x, \tan^2(e+f$$
$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b^2f(1+p)}+\frac{\left(a+b\tan^2(e+fx)\right)^{2+p}}{2b^2f(2+p)}+$$
$$= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b^2f(1+p)}-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2}{a-}\right.}{2(a-}$$

Mathematica [A]
time = 0.59, size = 106, normalized size = 0.82

$$\frac{\left(a+b\tan^2(e+fx)\right)^{1+p}\left(b^2(2+p)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(a+b(2+p)-b(1+p)\tan^2(e+fx)\right)\right)}{2b^2(-a+b)f(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
[Out] ((a + b*Tan[e + f*x]^2)^(1 + p)*(b^2*(2 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (a + b*Tan[e + f*x]^2)/(a - b)] + (a - b)*(a + b*(2 + p) - b*(1 + p)*T
an[e + f*x]^2)))/(2*b^2*(-a + b)*f*(1 + p)*(2 + p))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\tan^5(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^{2}\left(e + fx\right)\right)^{p}\tan^{5}\left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^5\,{\left(b\tan(e+f\,x)^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^p, x)
```

3.362 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=95

$$\frac{\left(a+b\tan^2(e+fx)\right)^{1+p}}{2bf(1+p)}+\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

```
[Out] 1/2*(a+b*tan(f*x+e)^2)^(1+p)/b/f/(1+p)+1/2*hypergeom([1, 1+p],[2+p],(a+b*ta
n(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(1+p)/(a-b)/f/(1+p)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {3751, 457, 81, 70}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}}{2bf(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (a + b*Tan[e + f*x]^2)^(1 + p)/(2*b*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^(1 + p))/(
2*(a - b)*f*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^p}{1+x}\,dx, x, \tan^2(e+fx)\right)}{2f}\\ &= \frac{\left(a+b\tan^2(e+fx)\right)^{1+p}}{2bf(1+p)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx, x, \tan^2(e+fx)\right.}{2f}\\ &= \frac{\left(a+b\tan^2(e+fx)\right)^{1+p}}{2bf(1+p)} + \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)(}{2(a-b)f(1+}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 0.77

$$-\frac{\left(a-b+b\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b(-a+b)f(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/2*((a - b + b*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/
(a - b)])*(a + b*Tan[e + f*x]^2)^(1 + p))/(b*(-a + b)*f*(1 + p))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \left(\tan^3(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{2}\left(e+fx\right)\right)^{p}\tan^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^3\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^p, x)
```

3.363 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=63

$$-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^
(1+p)/(a-b)/f/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3751, 455, 70}

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a
 + b*Tan[e + f*x]^2)^(1 + p))/((a - b)*f*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

```
alQ[n]))
```

Rubi steps

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int \frac{x(a+bx^2)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx,x,\tan^2(e+fx)\right)}{2f}$$
$$=-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 1.00

$$-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a
 + b*Tan[e + f*x]^2)^(1 + p))/((a - b)*f*(1 + p))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \tan\left(fx+e\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^{2}\left(e + fx\right)\right)^{p} \tan\left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^p, x)
```

3.364 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=118

$$\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2\right.}{2af(1+p)}$$

```
[Out] 1/2*hypergeom([1, 1+p],[2+p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(
1+p)/(a-b)/f/(1+p)-1/2*hypergeom([1, 1+p],[2+p],1+b*tan(f*x+e)^2/a)*(a+b*ta
n(f*x+e)^2)^(1+p)/a/f/(1+p)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3751, 457, 88, 67, 70}

$$\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*
Tan[e + f*x]^2)^(1 + p))/(2*(a - b)*f*(1 + p)) - (Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a*f*
(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
p}, x] &&  !IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx^2)^p}{x(1+x^2)}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{x(1+x)}\,dx,x,\tan^2(e+fx)\right)}{2f}$$
$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,\tan^2(e+fx)\right)}{2f}-\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{1+x}\,dx,x,\tan\right.}{2f}$$
$$=\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}-\frac{{}_2F_1}{}$$

Mathematica [A]
time = 0.12, size = 98, normalized size = 0.83

$$\frac{\left(a\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)+(-a+b)\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\tan^2(e+fx)}{a}\right)\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2a(a-b)f(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((a*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)] + (-
a + b)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a])*(a + b
*Tan[e + f*x]^2)^(1 + p))/(2*a*(a - b)*f*(1 + p))
```

Maple [**F**]
time = 0.26, size = 0, normalized size = 0.00

$$\int \cot (fx+e) \left(a+b\big(\tan^2 (fx+e)\big)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2 (e+fx)\right)^p \cot (e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p*cot(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^p, x)
```

3.365 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=158

$$-\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af}-\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}+\frac{(a-b$$

```
[Out] -1/2*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1+p)/a/f-1/2*hypergeom([1, 1+p],[2+p]
,(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(1+p)/(a-b)/f/(1+p)+1/2*(-b*p
+a)*hypergeom([1, 1+p],[2+p],1+b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^2)^(1+p)/a
^2/f/(1+p)
```

Rubi [A]

time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3751, 457, 105, 162, 67, 70}

$$\frac{(a-bp)\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)}{2a^2f(p+1)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}-\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/2*(Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(1 + p))/(a*f) - (Hypergeometri
c2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2
)^(1 + p))/(2*(a - b)*f*(1 + p)) + ((a - b*p)*Hypergeometric2F1[1, 1 + p, 2
 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*a^2*f*(1
 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^3(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x^2(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f} \\
&= -\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(a-bp-}{x(1+x)}\right.}{} \\
&= -\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af} + \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx, x\right.}{2f} \\
&= -\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af} - \frac{{}_2F_1\left(1, 1+p; 2+p; \right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.47, size = 142, normalized size = 0.90

$$
\frac{\left(b+a\cot^2(e+fx)\right)\left(-a^2\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)-(a-b)\left(a(1+p)\cot^2(e+fx)+(-a+bp)\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\tan^2(e+fx)}{a}\right)\right)\right)\tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p}{2a^2(a-b)f(1+p)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((b + a*Cot[e + f*x]^2)*(-(a^2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Ta
n[e + f*x]^2)/(a - b)]) - (a - b)*(a*(1 + p)*Cot[e + f*x]^2 + (-a + b*p)*Hy
pergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]))*Tan[e + f*x]^2
*(a + b*Tan[e + f*x]^2)^p)/(2*a^2*(a - b)*f*(1 + p))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$
\int \left(\cot^3(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^p\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^3\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^p, x)
```

3.366 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=217

$$\frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f}-\frac{\cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4af}+\frac{{}_2F_1\Big(1,1+p;2}{}$$

[Out] 1/4*(-b*p+2*a+b)*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1+p)/a^2/f-1/4*cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1+p)/a/f+1/2*hypergeom([1, 1+p],[2+p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(1+p)/(a-b)/f/(1+p)-1/4*(2*a^2-2*a*b*p-b^2*(1-p)*p)*hypergeom([1, 1+p],[2+p],1+b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^2)^(1+p)/a^3/f/(1+p)

Rubi [A]
time = 0.17, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3751, 457, 105, 156, 162, 67, 70}

$$\frac{(2a-bp+b)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{4a^2f}-\frac{\left(2a^2-2abp-b^2(1-p)p\right)\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)}{a}+1\right)}{4a^3f(p+1)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}-\frac{\cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{4af}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]

[Out] ((2*a + b - b*p)*Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(1 + p))/(4*a^2*f) - (Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(1 + p))/(4*a*f) + (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^(1 + p))/(2*(a - b)*f*(1 + p)) - ((2*a^2 - 2*a*b*p - b^2*(1 - p)*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(4*a^3*f*(1 + p))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^5(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x^3(1+x)}\,dx, x, \tan^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4af} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(2a+b-}{x^2(1+}\right.}{}$$

$$= \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+}{}$$

$$= \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+}{}$$

$$= \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+}{}$$

Mathematica [A]
time = 1.73, size = 172, normalized size = 0.79

$$-\frac{(b+a\cot^2(e+fx))\left(-2a^3\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(a(1+p)\cot^2(e+fx)\left(-2a+b(-1+p)+a\cot^2(e+fx)\right)+\left(2a^2-2abp+b^2(-1+p)p\right)\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\tan^2(e+fx)}{a}\right)\right)\right)\tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p}{4a^3(a-b)f(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/4*((b + a*Cot[e + f*x]^2)*(-2*a^3*Hypergeometric2F1[1, 1 + p, 2 + p, (a
+ b*Tan[e + f*x]^2)/(a - b)] + (a - b)*(a*(1 + p)*Cot[e + f*x]^2*(-2*a + b*
(-1 + p) + a*Cot[e + f*x]^2) + (2*a^2 - 2*a*b*p + b^2*(-1 + p)*p)*Hypergeom
etric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]))*Tan[e + f*x]^2*(a + b
*Tan[e + f*x]^2)^p)/(a^3*(a - b)*f*(1 + p))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\cot^5\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(e+f\,x)^5\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^p, x)
```

3.367 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{7}{2};1,-p;\frac{9}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^7(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{7f}$$

```
[Out] 1/7*AppellF1(7/2,1,-p,9/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^7*(a+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 525, 524}

$$\frac{\tan^7(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{7}{2};1,-p;\frac{9}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[7/2, 1, -p, 9/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^7*(a + b*Tan[e + f*x]^2)^p)/(7*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^6\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\right.}{f}$$

$$= \frac{F_1\left(\frac{7}{2};1,-p;\frac{9}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^7(e+fx)\,(a+b}{7f}$$

Mathematica [**F**]
time = 2.32, size = 0, normalized size = 0.00

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
[Out] Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p, x]
```

Maple [**F**]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(\tan^6\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^6\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^p, x)
```

3.368 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{5f}$$

```
[Out] 1/5*AppellF1(5/2,1,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^5*(a+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 525, 524}

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a}\right.}{1+x^2}\right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^5(e+fx)\,(a+}{5f}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1896 vs. 2(83) = 166.
time = 14.71, size = 1896, normalized size = 22.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p) + (Tan[e + f*x]*(
a + b*Tan[e + f*x]^2)^p*((-a + b*(3 + 2*p))*Hypergeometric2F1[1/2, -p, 3/2,
 -((b*Tan[e + f*x]^2)/a)] + (a + b*Tan[e + f*x]^2)*(1 + (b*Tan[e + f*x]^2)/
a)^p))/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*
x]*(a + b*Tan[e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan
[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((
b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*T
an[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)*((6*a*b*p*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Tan[e + f*x]^2*(a +
 b*Tan[e + f*x]^2)^(-1 + p))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
 + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (3*a*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(a + b*Tan[e +
 f*x]^2)^p)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
```

```
+ f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -T
an[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) - (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/(3*a*
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] -
a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[
e + f*x]^2) + (3*a*Cos[e + f*x]*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x
])/(3*a) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3)*(a + b*Tan[e + f*x]^2)^p)/(3*a*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*App
ellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f
*x]^2) - (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2]*Cos[e + f*x]*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(4*(b*p*AppellF1[
3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[
3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x]^2*
Tan[e + f*x] + 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[
e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b
*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b
*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x
]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a)) - a*((6*b*p*AppellF1[5/2, 1 - p, 2
, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x
])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \left(\tan^4 (fx+e)\right) \left(a+b\left(\tan^2 (fx+e)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^p \tan^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e + f\,x)^4\,\left(b\tan(e + f\,x)^2 + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^p, x)
```

3.369 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{3f}$$

```
[Out] 1/3*AppellF1(3/2,1,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^3*(a+
b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 525, 524}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right.}{1+x^2}\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\,(a+}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1992 vs. $2(83) = 166$.
time = 13.92, size = 1992, normalized size = 24.00

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(2*p)*(Hypergeometric2F1[1/2, -p, 3/
2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/
2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-
3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2
*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]
^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2
])*Tan[e + f*x]^2)))/(f*(2*b*p*Sec[e + f*x]^2*Tan[e + f*x]^2*(a + b*Tan[e +
 f*x]^2)^(-1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)]
/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-3*a*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p,
2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + Sec[e
 + f*x]^2*(a + b*Tan[e + f*x]^2)^p*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Ta
n[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/2, -p, 1,
3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-3*a*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*Ap
```

```
pellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*A
ppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e +
 f*x]^2)) + Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((-2*b*p*Hypergeometric2F
1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (
b*Tan[e + f*x]^2)/a)^(-1 - p))/a - (6*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(-3*a*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appe
llF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*
x]^2) + (3*a*Cos[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
 + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*
x]^2*Tan[e + f*x])/3))/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Csc[e + f*x]*Sec[e + f*x]*(-
Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)] + (1 + (b*Tan[e +
f*x]^2)/a)^p))/(1 + (b*Tan[e + f*x]^2)/a)^p - (3*a*AppellF1[1/2, -p, 1, 3/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(4*(-(b*p*Appell
F1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x
]^2*Tan[e + f*x] - 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1
[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*
Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(-(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5
- (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
 + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a))) + a*((6*b*p*AppellF1[5/2, 1
 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[
e + f*x])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*a*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2,
-p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\tan^2\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^{2}{\left(e + f x \right)}\right)^{p} \tan^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
[Out] Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(e+f\,x\right)}^2\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^p, x)
```

3.370 $\int (a+b\tan^2(e+fx))^p\,dx$

Optimal. Leaf size=78

$$\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)*(a+b*tan(
f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 441, 440}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
+ f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx, x, \tan(e-$$
$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan(e+fx)\left(a+b\tan^2(e+fx\right.}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 192 vs. 2(78) = 156.
time = 0.30, size = 192, normalized size = 2.46

$$\frac{3aF_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)\sin(2(e+fx))\left(a+b\tan^2(e+fx)\right)^p}{6afF_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)+4f\left(bpF_1\left(\frac{3}{2};1-p,1;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)-aF_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)\right)\tan^2(e+fx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Si
n[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p)/(6*a*f*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 4*f*(b*p*AppellF1[3/2, 1 - p, 1,
 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(e + fx\right)\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)**2)**p,x)
[Out] Integral((a + b*tan(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p,x)
[Out] int((a + b*tan(e + f*x)^2)^p, x)
```

3.371 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=79

$$-\frac{F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{f}$$

```
[Out] -AppellF1(-1/2,1,-p,1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*cot(f*x+e)*(a+b*ta
n(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 525, 524}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot
[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2(1+x^2)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\left((a+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2(1+x^2)}\,d\right.}{f}$$

$$=-\frac{F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot(e+fx)\,(a}{f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1989 vs. 2(79) = 158.
time = 13.85, size = 1989, normalized size = 25.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(2*p)*(-(Hypergeometric2F1[-1/2, -p,
 1/2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2
)/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]
 + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*
x]^2])*Tan[e + f*x]^2)))/(f*(2*b*p*Sec[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-
1 + p)*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b
*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b
*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - Csc[e + f*x]
^2*(a + b*Tan[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e
+ f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/2, -p, 1, 3/2
, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(-3*a*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appe
```

```
llF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*
x]^2)) + Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((2*b*p*Hypergeometric2F1[-1
/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b*T
an[e + f*x]^2)/a)^(-1 - p))/a + (6*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(-3*a*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1
[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^
2) + (3*a*Sin[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*Appell
F1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^
2*Tan[e + f*x])/3))/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a)
, -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x
]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^
2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (Csc[e + f*x]*Sec[e + f*x]*(Hype
rgeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)] - (1 + (b*Tan[e + f*x
]^2)/a)^p))/(1 + (b*Tan[e + f*x]^2)/a)^p - (3*a*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(4*(-(b*p*AppellF1[
3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1
[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x]^2
*Tan[e + f*x] - 3*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^
2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/
2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan
[e + f*x])/3) + 2*Tan[e + f*x]^2*(-(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (
6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a))) + a*((6*b*p*AppellF1[5/2, 1 -
p, 2, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e +
 f*x])/(5*a) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*a*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1 -
p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2, -p,
 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)))
```

Maple [F]

time = 0.27, size = 0, normalized size = 0.00

$$\int \left(\cot^2(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^2\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^p, x)
```

3.372 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{3f}$$

```
[Out] -1/3*AppellF1(-3/2,1,-p,-1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*cot(f*x+e)^3*
(a+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 525, 524}

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/3*(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]
*Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\mathrm{Subst}\left(\int \frac{(a+bx^2)^p}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left((a+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^4(1+x^2)}\,d\right.}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^3(e+fx)\,(}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1887 vs. $2(83) = 166$.
time = 6.53, size = 1887, normalized size = 22.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (2*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*(
a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p) + (Cot[e + f*x]^3
*(a + b*Tan[e + f*x]^2)^p*(-a - b*Tan[e + f*x]^2 - ((3*a + b*(-1 + 2*p))*Hy
pergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1
+ (b*Tan[e + f*x]^2)/a)^p))/(3*a*f) + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x]*(a + b*Tan[e
+ f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a),
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a
), -Tan[e + f*x]^2])*Tan[e + f*x]^2)*((6*a*b*p*AppellF1[1/2, -p, 1, 3/2, -(
(b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^
2)^(-1 + p))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
 + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -
Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan
[e + f*x]^2])*Tan[e + f*x]^2) + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/(3*a
*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b
*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] -
```

```
 a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan
[e + f*x]^2]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2,
-p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (3
*a*Cos[e + f*x]*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f
*x]^2*Tan[e + f*x])/3)*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, -p, 1,
3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2
, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*a*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f
*x]*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5
/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3
*a*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*AppellF1[3/2, -p, 2, 5/2, -(
(b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2
*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)
/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF
1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x
]^2*Tan[e + f*x])/(5*a)) - a*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a) - (12*A
ppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sec[e +
f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
 f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \left(\cot^4\left(fx+e\right)\right)\left(a+b\left(\tan^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^4\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^p, x)
```

3.373 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{F_1\left(-\frac{5}{2};1,-p;-\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{5f}$$

```
[Out] -1/5*AppellF1(-5/2,1,-p,-3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*cot(f*x+e)^5*
(a+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3751, 525, 524}

$$-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{5}{2};1,-p;-\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -1/5*(AppellF1[-5/2, 1, -p, -3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]
*Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
```

```
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^6(1+x^2)}\,d\right.}{f}$$

$$= -\frac{F_1\left(-\frac{5}{2};1,-p;-\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^5(e+fx)\,(}{5f}$$

Mathematica [F]
time = 3.04, size = 0, normalized size = 0.00

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
[Out] Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p, x]
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\cot^6(fx+e)\right)\left(a+b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
[Out] int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^6\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^p,x)
[Out] int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^p, x)
```

3.374 $\int (a+b\tan^3(c+dx))^4\,dx$

Optimal. Leaf size=255

$$(a^4-6a^2b^2+b^4)\,x+\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}+\frac{b^2(6a^2-b^2)\tan(c+dx)}{d}+\frac{2ab(a^2-b^2)\tan^2(c+dx)}{d}-\frac{b^2}{}$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x+4*a*b*(a^2-b^2)*ln(cos(d*x+c))/d+b^2*(6*a^2-b^2)*tan(
d*x+c)/d+2*a*b*(a^2-b^2)*tan(d*x+c)^2/d-1/3*b^2*(6*a^2-b^2)*tan(d*x+c)^3/d+
a*b^3*tan(d*x+c)^4/d+1/5*b^2*(6*a^2-b^2)*tan(d*x+c)^5/d-2/3*a*b^3*tan(d*x+c
)^6/d+1/7*b^4*tan(d*x+c)^7/d+1/2*a*b^3*tan(d*x+c)^8/d-1/9*b^4*tan(d*x+c)^9/
d+1/11*b^4*tan(d*x+c)^11/d
```

Rubi [A]

time = 0.10, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3742, 1824, 649, 209, 266}

$$\frac{b^2(6a^2-b^2)\tan^5(c+dx)}{5d}-\frac{b^2(6a^2-b^2)\tan^3(c+dx)}{3d}+\frac{2ab(a^2-b^2)\tan^2(c+dx)}{d}+\frac{b^2(6a^2-b^2)\tan(c+dx)}{d}+\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}+x\left(a^4-6a^2b^2+b^4\right)+\frac{ab^3\tan^8(c+dx)}{2d}-\frac{2ab^3\tan^6(c+dx)}{3d}+\frac{ab^3\tan^4(c+dx)}{d}+\frac{b^4\tan^{11}(c+dx)}{11d}-\frac{b^4\tan^9(c+dx)}{9d}+\frac{b^4\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^4,x]
```

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x + (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2*
(6*a^2 - b^2)*Tan[c + d*x])/d + (2*a*b*(a^2 - b^2)*Tan[c + d*x]^2)/d - (b^2
*(6*a^2 - b^2)*Tan[c + d*x]^3)/(3*d) + (a*b^3*Tan[c + d*x]^4)/d + (b^2*(6*a
^2 - b^2)*Tan[c + d*x]^5)/(5*d) - (2*a*b^3*Tan[c + d*x]^6)/(3*d) + (b^4*Tan
[c + d*x]^7)/(7*d) + (a*b^3*Tan[c + d*x]^8)/(2*d) - (b^4*Tan[c + d*x]^9)/(9
*d) + (b^4*Tan[c + d*x]^11)/(11*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^3(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^3\right)^4}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int\left(6a^2b^2-b^4+4ab\left(a^2-b^2\right)x-b^2\left(6a^2-b^2\right)x^2+4ab^3x^3+b^2\left(6a^2-\right.\right.\right.}{}$$

$$=\frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d}+\frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d}-\frac{b^2\left(6a^2-b^2\right)\tan^3(c-}{3d}$$

$$=\frac{b^2\left(6a^2-b^2\right)\tan(c+dx)}{d}+\frac{2ab\left(a^2-b^2\right)\tan^2(c+dx)}{d}-\frac{b^2\left(6a^2-b^2\right)\tan^3(c-}{3d}$$

$$=\left(a^4-6a^2b^2+b^4\right)x+\frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{b^2\left(6a^2-b^2\right)\tan(c+d}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.63, size = 224, normalized size = 0.88

$$\frac{-3465i\left((a-ib)^4\log(i-\tan(c+dx))-(a+ib)^4\log(i+\tan(c+dx))\right)-6930b^2\left(-6a^2+b^2\right)\tan(c+dx)+13860ab\left(a^2-b^2\right)\tan^2(c+dx)+2310b^2\left(-6a^2+b^2\right)\tan^3(c+dx)+6930ab^3\tan^4(c+dx)-1386b^2\left(-6a^2+b^2\right)\tan^5(c+dx)-4620ab^3\tan^6(c+dx)+990b^4\tan^7(c+dx)+3465ab^3\tan^8(c+dx)-770b^4\tan^9(c+dx)+630b^4\tan^{11}(c+dx)}{6930d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^4,x]
```

```
[Out] ((-3465*I)*((a - I*b)^4*Log[I - Tan[c + d*x]] - (a + I*b)^4*Log[I + Tan[c +
 d*x]]) - 6930*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 13860*a*b*(a^2 - b^2)*Tan[
c + d*x]^2 + 2310*b^2*(-6*a^2 + b^2)*Tan[c + d*x]^3 + 6930*a*b^3*Tan[c + d*
x]^4 - 1386*b^2*(-6*a^2 + b^2)*Tan[c + d*x]^5 - 4620*a*b^3*Tan[c + d*x]^6 +
 990*b^4*Tan[c + d*x]^7 + 3465*a*b^3*Tan[c + d*x]^8 - 770*b^4*Tan[c + d*x]^
9 + 630*b^4*Tan[c + d*x]^11)/(6930*d)
```

Maple [A]
time = 0.16, size = 246, normalized size = 0.96 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^3)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/11*b^4*tan(d*x+c)^11-1/9*b^4*tan(d*x+c)^9+1/2*a*b^3*tan(d*x+c)^8+1/7
*b^4*tan(d*x+c)^7-2/3*a*b^3*tan(d*x+c)^6+6/5*a^2*b^2*tan(d*x+c)^5-1/5*b^4*t
an(d*x+c)^5+a*b^3*tan(d*x+c)^4-2*a^2*b^2*tan(d*x+c)^3+1/3*b^4*tan(d*x+c)^3+
2*a^3*b*tan(d*x+c)^2-2*a*b^3*tan(d*x+c)^2+6*a^2*b^2*tan(d*x+c)-b^4*tan(d*x+
c)+1/2*(-4*a^3*b+4*a*b^3)*ln(1+tan(d*x+c)^2)+(a^4-6*a^2*b^2+b^4)*arctan(tan
(d*x+c)))
```

Maxima [A]
time = 0.51, size = 260, normalized size = 1.02

$$a^4x+\frac{2\,(3\,\tan(dx+c)^5-5\,\tan(dx+c)^3-15\,dx-15\,c+15\,\tan(dx+c))a^2b^2}{5\,d}+\frac{(315\,\tan(dx+c)^{11}-385\,\tan(dx+c)^9+495\,\tan(dx+c)^7-693\,\tan(dx+c)^5+1155\,\tan(dx+c)^3+3465\,dx+3465\,c-3465\,\tan(dx+c))b^4}{3465\,d}+\frac{ab^3\left(\frac{48\,\sin(dx+c)^6-108\,\sin(dx+c)^4+88\,\sin(dx+c)^2-25}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-12\,\log\left(\sin(dx+c)^2-1\right)\right)}{6\,d}-\frac{2\,a^3b\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2/5*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d
*x + c))*a^2*b^2/d + 1/3465*(315*tan(d*x + c)^11 - 385*tan(d*x + c)^9 + 495
*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3465*d*x + 346
5*c - 3465*tan(d*x + c))*b^4/d + 1/6*a*b^3*((48*sin(d*x + c)^6 - 108*sin(d*
x + c)^4 + 88*sin(d*x + c)^2 - 25)/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*s
in(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 12*log(sin(d*x + c)^2 - 1))/d - 2*a
^3*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 2.74, size = 225, normalized size = 0.88

$$\frac{630\,b^4\tan(dx+c)^{11}-770\,b^4\tan(dx+c)^9+3465\,ab^3\tan(dx+c)^8+990\,b^4\tan(dx+c)^7-4620\,ab^3\tan(dx+c)^6+6930\,ab^3\tan(dx+c)^4+1386\,(6\,a^2b^2-b^4)\tan(dx+c)^5-2310\,(6\,a^2b^2-b^4)\tan(dx+c)^3+6930\,(a^4-6\,a^2b^2+b^4)dx+13860\,(a^3b-ab^3)\tan(dx+c)^2+13860\,(a^3b-ab^3)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+6930\,(6\,a^2b^2-b^4)\tan(dx+c)}{6930\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="fricas")
```

```
[Out] 1/6930*(630*b^4*tan(d*x + c)^11 - 770*b^4*tan(d*x + c)^9 + 3465*a*b^3*tan(d
*x + c)^8 + 990*b^4*tan(d*x + c)^7 - 4620*a*b^3*tan(d*x + c)^6 + 6930*a*b^3
*tan(d*x + c)^4 + 1386*(6*a^2*b^2 - b^4)*tan(d*x + c)^5 - 2310*(6*a^2*b^2 -
 b^4)*tan(d*x + c)^3 + 6930*(a^4 - 6*a^2*b^2 + b^4)*d*x + 13860*(a^3*b - a*
b^3)*tan(d*x + c)^2 + 13860*(a^3*b - a*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 6
930*(6*a^2*b^2 - b^4)*tan(d*x + c))/d
```

Sympy [A]
time = 0.49, size = 301, normalized size = 1.18

$$\begin{cases} a^4x-\frac{2a^3b\log\left(\tan^2(c+dx)+1\right)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}-6a^2b^2x+\frac{6a^2b^2\tan^5(c+dx)}{5d}-\frac{2a^2b^2\tan^3(c+dx)}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\log\left(\tan^2(c+dx)+1\right)}{d}+\frac{ab^3\tan^8(c+dx)}{2d}-\frac{2ab^3\tan^6(c+dx)}{3d}+\frac{ab^3\tan^4(c+dx)}{d}-\frac{2ab^3\tan^2(c+dx)}{d}+b^4x+\frac{b^4\tan^{11}(c+dx)}{11d}-\frac{b^4\tan^9(c+dx)}{9d}+\frac{b^4\tan^7(c+dx)}{7d}-\frac{b^4\tan^5(c+dx)}{5d}+\frac{b^4\tan^3(c+dx)}{3d}-\frac{b^4\tan(c+dx)}{d} & \text{for } d\neq 0 \\ x\left(a+b\tan^3(c)\right)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**3)**4,x)
```

```
[Out] Piecewise((a**4*x - 2*a**3*b*log(tan(c + d*x)**2 + 1)/d + 2*a**3*b*tan(c +
d*x)**2/d - 6*a**2*b**2*x + 6*a**2*b**2*tan(c + d*x)**5/(5*d) - 2*a**2*b**2
*tan(c + d*x)**3/d + 6*a**2*b**2*tan(c + d*x)/d + 2*a*b**3*log(tan(c + d*x)
**2 + 1)/d + a*b**3*tan(c + d*x)**8/(2*d) - 2*a*b**3*tan(c + d*x)**6/(3*d)
+ a*b**3*tan(c + d*x)**4/d - 2*a*b**3*tan(c + d*x)**2/d + b**4*x + b**4*tan
(c + d*x)**11/(11*d) - b**4*tan(c + d*x)**9/(9*d) + b**4*tan(c + d*x)**7/(7
*d) - b**4*tan(c + d*x)**5/(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c
+ d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**3)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5949 vs. $2(241) = 482$.
time = 29.92, size = 5949, normalized size = 23.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="giac")
```

```
[Out] 1/6930*(6930*a^4*d*x*tan(d*x)^11*tan(c)^11 - 41580*a^2*b^2*d*x*tan(d*x)^11*
tan(c)^11 + 6930*b^4*d*x*tan(d*x)^11*tan(c)^11 + 13860*a^3*b*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^11*tan(c)^11 - 13860*a*b^3*log
(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^11*tan(c)^11 - 7623
0*a^4*d*x*tan(d*x)^10*tan(c)^10 + 457380*a^2*b^2*d*x*tan(d*x)^10*tan(c)^10
- 76230*b^4*d*x*tan(d*x)^10*tan(c)^10 + 13860*a^3*b*tan(d*x)^11*tan(c)^11 -
 28875*a*b^3*tan(d*x)^11*tan(c)^11 - 152460*a^3*b*log(4*(tan(d*x)^4*tan(c)^
2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan
(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^10*tan(c)^10 + 152460*a*b^3*log(4*(tan(d*
x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^10*tan(c)^10 - 41580*a^2*b^2*
tan(d*x)^11*tan(c)^10 + 6930*b^4*tan(d*x)^11*tan(c)^10 - 41580*a^2*b^2*tan(
d*x)^10*tan(c)^11 + 6930*b^4*tan(d*x)^10*tan(c)^11 + 381150*a^4*d*x*tan(d*x
)^9*tan(c)^9 - 2286900*a^2*b^2*d*x*tan(d*x)^9*tan(c)^9 + 381150*b^4*d*x*tan
(d*x)^9*tan(c)^9 + 13860*a^3*b*tan(d*x)^11*tan(c)^9 - 13860*a*b^3*tan(d*x)^
11*tan(c)^9 - 124740*a^3*b*tan(d*x)^10*tan(c)^10 + 289905*a*b^3*tan(d*x)^10
*tan(c)^10 + 13860*a^3*b*tan(d*x)^9*tan(c)^11 - 13860*a*b^3*tan(d*x)^9*tan(
c)^11 + 13860*a^2*b^2*tan(d*x)^11*tan(c)^8 - 2310*b^4*tan(d*x)^11*tan(c)^8
+ 762300*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^9
*tan(c)^9 - 762300*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
 tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*
```

```
tan(d*x)^9*tan(c)^9 + 457380*a^2*b^2*tan(d*x)^10*tan(c)^9 - 76230*b^4*tan(d
*x)^10*tan(c)^9 + 457380*a^2*b^2*tan(d*x)^9*tan(c)^10 - 76230*b^4*tan(d*x)^
9*tan(c)^10 + 13860*a^2*b^2*tan(d*x)^8*tan(c)^11 - 2310*b^4*tan(d*x)^8*tan(
c)^11 + 6930*a*b^3*tan(d*x)^11*tan(c)^7 - 1143450*a^4*d*x*tan(d*x)^8*tan(c)
^8 + 6860700*a^2*b^2*d*x*tan(d*x)^8*tan(c)^8 - 1143450*b^4*d*x*tan(d*x)^8*t
an(c)^8 - 124740*a^3*b*tan(d*x)^10*tan(c)^8 + 152460*a*b^3*tan(d*x)^10*tan(
c)^8 + 512820*a^3*b*tan(d*x)^9*tan(c)^9 - 1297065*a*b^3*tan(d*x)^9*tan(c)^9
 - 124740*a^3*b*tan(d*x)^8*tan(c)^10 + 152460*a*b^3*tan(d*x)^8*tan(c)^10 +
6930*a*b^3*tan(d*x)^7*tan(c)^11 - 8316*a^2*b^2*tan(d*x)^11*tan(c)^6 + 1386*
b^4*tan(d*x)^11*tan(c)^6 - 152460*a^2*b^2*tan(d*x)^10*tan(c)^7 + 25410*b^4*
tan(d*x)^10*tan(c)^7 - 2286900*a^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x
)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan
(c)^2 + 1))*tan(d*x)^8*tan(c)^8 + 2286900*a*b^3*log(4*(tan(d*x)^4*tan(c)^2
- 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c
) + 1)/(tan(c)^2 + 1))*tan(d*x)^8*tan(c)^8 - 2286900*a^2*b^2*tan(d*x)^9*tan
(c)^8 + 381150*b^4*tan(d*x)^9*tan(c)^8 - 2286900*a^2*b^2*tan(d*x)^8*tan(c)^
9 + 381150*b^4*tan(d*x)^8*tan(c)^9 - 152460*a^2*b^2*tan(d*x)^7*tan(c)^10 +
25410*b^4*tan(d*x)^7*tan(c)^10 - 8316*a^2*b^2*tan(d*x)^6*tan(c)^11 + 1386*b
^4*tan(d*x)^6*tan(c)^11 - 4620*a*b^3*tan(d*x)^11*tan(c)^5 - 76230*a*b^3*tan
(d*x)^10*tan(c)^6 + 2286900*a^4*d*x*tan(d*x)^7*tan(c)^7 - 13721400*a^2*b^2*
d*x*tan(d*x)^7*tan(c)^7 + 2286900*b^4*d*x*tan(d*x)^7*tan(c)^7 + 498960*a^3*
b*tan(d*x)^9*tan(c)^7 - 762300*a*b^3*tan(d*x)^9*tan(c)^7 - 1288980*a^3*b*ta
n(d*x)^8*tan(c)^8 + 3382995*a*b^3*tan(d*x)^8*tan(c)^8 + 498960*a^3*b*tan(d*
x)^7*tan(c)^9 - 762300*a*b^3*tan(d*x)^7*tan(c)^9 - 76230*a*b^3*tan(d*x)^6*t
an(c)^10 - 4620*a*b^3*tan(d*x)^5*tan(c)^11 - 990*b^4*tan(d*x)^11*tan(c)^4 +
 49896*a^2*b^2*tan(d*x)^10*tan(c)^5 - 15246*b^4*tan(d*x)^10*tan(c)^5 + 6375
60*a^2*b^2*tan(d*x)^9*tan(c)^6 - 127050*b^4*tan(d*x)^9*tan(c)^6 + 4573800*a
^3*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^7*tan(c)^7
- 4573800*a*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^
7*tan(c)^7 + 6652800*a^2*b^2*tan(d*x)^8*tan(c)^7 - 1143450*b^4*tan(d*x)^8*t
an(c)^7 + 6652800*a^2*b^2*tan(d*x)^7*tan(c)^8 - 1143450*b^4*tan(d*x)^7*tan(
c)^8 + 637560*a^2*b^2*tan(d*x)^6*tan(c)^9 - 127050*b^4*tan(d*x)^6*tan(c)^9
+ 49896*a^2*b^2*tan(d*x)^5*tan(c)^10 - 15246*b^4*tan(d*x)^5*tan(c)^10 - 990
*b^4*tan(d*x)^4*tan(c)^11 + 3465*a*b^3*tan(d*x)^11*tan(c)^3 + 50820*a*b^3*t
an(d*x)^10*tan(c)^4 + 381150*a*b^3*tan(d*x)^9*tan(c)^5 - 3201660*a^4*d*x*ta
n(d*x)^6*tan(c)^6 + 19209960*a^2*b^2*d*x*tan(d*x)^6*tan(c)^6 - 3201660*b^4*
d*x*tan(d*x)^6*tan(c)^6 - 1164240*a^3*b*tan(d*x)^8*tan(c)^6 + 2286900*a*b^3
*tan(d*x)^8*tan(c)^6 + 2245320*a^3*b*tan(d*x)^7*tan(c)^7 - 5622540*a*b^3*ta
n(d*x)^7*tan(c)^7 - 1164240*a^3*b*tan(d*x)^6*tan(c)^8 + 2286900*a*b^3*tan(d
*x)^6*tan(c)^8 + 381150*a*b^3*tan(d*x)^5*tan(c)^9 + 50820*a*b^3*tan(d*x)^4*
tan(c)^10 + 3465*a*b^3*tan(d*x)^3*tan(c)^11 + 7...
```

Mupad [B]

time = 11.85, size = 310, normalized size = 1.22

$$\frac{\ln \left(\tan(c+dx)^2+1\right)\,\left(2\,a\,b^3-2\,a^3\,b\right)}{d}+\frac{\tan(c+dx)^3\,\left(\frac{b^4}{3}-2\,a^2\,b^2\right)}{d}-\frac{\tan(c+dx)^5\,\left(\frac{b^4}{5}-\frac{6\,a^2\,b^2}{5}\right)}{d}-\frac{\tan(c+dx)^2\,\left(2\,a\,b^3-2\,a^3\,b\right)}{d}-\frac{\tan(c+dx)\,\left(b^4-6\,a^2\,b^2\right)}{d}+\frac{b^4\tan(c+dx)^7}{7\,d}-\frac{b^4\tan(c+dx)^9}{9\,d}+\frac{b^4\tan(c+dx)^{11}}{11\,d}+\frac{\mathrm{atan}\left(\frac{\tan(c+dx)\,\left(-a^2+2\,a\,b+b^2\right)\,\left(a^2+2\,a\,b-b^2\right)}{a^4-6\,a^2\,b^2+b^4}\right)\,\left(-a^2+2\,a\,b+b^2\right)\,\left(a^2+2\,a\,b-b^2\right)}{d}+\frac{a\,b^3\tan(c+dx)^4}{d}-\frac{2\,a\,b^3\tan(c+dx)^6}{3\,d}+\frac{a\,b^3\tan(c+dx)^8}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^3)^4,x)
```

```
[Out] (log(tan(c + d*x)^2 + 1)*(2*a*b^3 - 2*a^3*b))/d + (tan(c + d*x)^3*(b^4/3 -
2*a^2*b^2))/d - (tan(c + d*x)^5*(b^4/5 - (6*a^2*b^2)/5))/d - (tan(c + d*x)^
2*(2*a*b^3 - 2*a^3*b))/d - (tan(c + d*x)*(b^4 - 6*a^2*b^2))/d + (b^4*tan(c
+ d*x)^7)/(7*d) - (b^4*tan(c + d*x)^9)/(9*d) + (b^4*tan(c + d*x)^11)/(11*d)
 + (atan((tan(c + d*x)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(a^4 + b^4
- 6*a^2*b^2))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/d + (a*b^3*tan(c + d
*x)^4)/d - (2*a*b^3*tan(c + d*x)^6)/(3*d) + (a*b^3*tan(c + d*x)^8)/(2*d)
```

3.375 $\int (a+b\tan^3(c+dx))^3 \, dx$

Optimal. Leaf size=168

$$a(a^2-3b^2)\,x+\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b(3a^2-b^2)\tan^2(c+dx)}{2d}-\frac{ab^2\tan^3(c+dx)}{d}+$$

```
[Out] a*(a^2-3*b^2)*x+b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b*(
3*a^2-b^2)*tan(d*x+c)^2/d-a*b^2*tan(d*x+c)^3/d+1/4*b^3*tan(d*x+c)^4/d+3/5*a
*b^2*tan(d*x+c)^5/d-1/6*b^3*tan(d*x+c)^6/d+1/8*b^3*tan(d*x+c)^8/d
```

Rubi [A]
time = 0.07, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3742, 1824, 649, 209, 266}

$$\frac{b(3a^2-b^2)\tan^2(c+dx)}{2d}+\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+ax(a^2-3b^2)+\frac{3ab^2\tan^5(c+dx)}{5d}-\frac{ab^2\tan^3(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^8(c+dx)}{8d}-\frac{b^3\tan^6(c+dx)}{6d}+\frac{b^3\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^3,x]
```

```
[Out] a*(a^2 - 3*b^2)*x + (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*Tan[c
+ d*x]])/d + (b*(3*a^2 - b^2)*Tan[c + d*x]^2)/(2*d) - (a*b^2*Tan[c + d*x]^3)
/d + (b^3*Tan[c + d*x]^4)/(4*d) + (3*a*b^2*Tan[c + d*x]^5)/(5*d) - (b^3*Tan
[c + d*x]^6)/(6*d) + (b^3*Tan[c + d*x]^8)/(8*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^3\right)^3}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(3ab^2+b\left(3a^2-b^2\right)x-3ab^2x^2+b^3x^3+3ab^2x^4-b^3x^5+b^3x^7+\frac{a^3}{}\right.\right.}{d}$$

$$= \frac{3ab^2\tan(c+dx)}{d}+\frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d}-\frac{ab^2\tan^3(c+dx)}{d}+\frac{b^3\tan^4(c}{4d}$$

$$= \frac{3ab^2\tan(c+dx)}{d}+\frac{b\left(3a^2-b^2\right)\tan^2(c+dx)}{2d}-\frac{ab^2\tan^3(c+dx)}{d}+\frac{b^3\tan^4(c}{4d}$$

$$= a\left(a^2-3b^2\right)x+\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b(3a^2-b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 160, normalized size = 0.95

$$\frac{60\left(-i(a-ib)^3\log(i-\tan(c+dx))+i(a+ib)^3\log(i+\tan(c+dx))\right)+360ab^2\tan(c+dx)-60b\left(-3a^2+b^2\right)\tan^2(c+dx)-120ab^2\tan^3(c+dx)+30b^3\tan^4(c+dx)+72ab^2\tan^5(c+dx)-20b^3\tan^6(c+dx)+15b^3\tan^8(c+dx)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^3,x]
```

```
[Out] (60*((-I)*(a - I*b)^3*Log[I - Tan[c + d*x]] + I*(a + I*b)^3*Log[I + Tan[c +
 d*x]]) + 360*a*b^2*Tan[c + d*x] - 60*b*(-3*a^2 + b^2)*Tan[c + d*x]^2 - 120
*a*b^2*Tan[c + d*x]^3 + 30*b^3*Tan[c + d*x]^4 + 72*a*b^2*Tan[c + d*x]^5 - 2
0*b^3*Tan[c + d*x]^6 + 15*b^3*Tan[c + d*x]^8)/(120*d)
```

Maple [A]
time = 0.11, size = 153, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{b^3\left(\tan^8(dx+c)\right)}{8}-\frac{b^3\left(\tan^6(dx+c)\right)}{6}+\frac{3a\,b^2\left(\tan^5(dx+c)\right)}{5}+\frac{b^3\left(\tan^4(dx+c)\right)}{4}-a\,b^2\left(\tan^3(dx+c)\right)+\frac{3a^2b\left(\tan^2(dx+c)\right)}{2}-\frac{b^3\left(\tan^2(\right.}{2}}{d}$

default	$\frac{\frac{b^3\left(\tan^8(dx+c)\right)}{8}-\frac{b^3\left(\tan^6(dx+c)\right)}{6}+\frac{3a\,b^2\left(\tan^5(dx+c)\right)}{5}+\frac{b^3\left(\tan^4(dx+c)\right)}{4}-a\,b^2\left(\tan^3(dx+c)\right)+\frac{3a^2b\left(\tan^2(dx+c)\right)}{2}-\frac{b^3\left(\tan^2(dx}{2}}{d}$
norman	$(a^3-3a\,b^2)\,x+\frac{b^3\left(\tan^4(dx+c)\right)}{4d}-\frac{b^3\left(\tan^6(dx+c)\right)}{6d}+\frac{b^3\left(\tan^8(dx+c)\right)}{8d}+\frac{3a\,b^2\tan(dx+c)}{d}-\frac{a\,b^2\left(\tan^3(dx+c\right.}{d}$
risch	$-3ia^2bx+ib^3x+a^3x-3a\,b^2x-\frac{6ib\,a^2c}{d}+\frac{2ib^3c}{d}-\frac{2b\left(-2229iab\,e^{6i(dx+c)}-45a^2e^{14i(dx+c)}+60b^2e^{14i(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*b^3*tan(d*x+c)^8-1/6*b^3*tan(d*x+c)^6+3/5*a*b^2*tan(d*x+c)^5+1/4*b
^3*tan(d*x+c)^4-a*b^2*tan(d*x+c)^3+3/2*a^2*b*tan(d*x+c)^2-1/2*b^3*tan(d*x+c
)^2+3*a*b^2*tan(d*x+c)+1/2*(-3*a^2*b+b^3)*ln(1+tan(d*x+c)^2)+(a^3-3*a*b^2)*
arctan(tan(d*x+c)))
```

Maxima [A]

time = 0.52, size = 183, normalized size = 1.09

$$a^3x+\frac{\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)ab^2}{5\,d}+\frac{b^3\left(\frac{48\,\sin(dx+c)^6-108\,\sin(dx+c)^4+88\,\sin(dx+c)^2-25}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-12\,\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{24\,d}-\frac{3\,a^2b\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/5*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d
*x + c))*a*b^2/d + 1/24*b^3*((48*sin(d*x + c)^6 - 108*sin(d*x + c)^4 + 88*s
in(d*x + c)^2 - 25)/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 -
 4*sin(d*x + c)^2 + 1) - 12*log(sin(d*x + c)^2 - 1))/d - 3/2*a^2*b*(1/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A]

time = 1.78, size = 148, normalized size = 0.88

$$\frac{15\,b^3\tan\left(dx+c\right)^8-20\,b^3\tan\left(dx+c\right)^6+72\,ab^2\tan\left(dx+c\right)^5+30\,b^3\tan\left(dx+c\right)^4-120\,ab^2\tan\left(dx+c\right)^3+360\,ab^2\tan\left(dx+c\right)+120\left(a^3-3\,ab^2\right)dx+60\left(3\,a^2b-b^3\right)\tan\left(dx+c\right)^2+60\left(3\,a^2b-b^3\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*b^3*tan(d*x + c)^8 - 20*b^3*tan(d*x + c)^6 + 72*a*b^2*tan(d*x + c
)^5 + 30*b^3*tan(d*x + c)^4 - 120*a*b^2*tan(d*x + c)^3 + 360*a*b^2*tan(d*x
+ c) + 120*(a^3 - 3*a*b^2)*d*x + 60*(3*a^2*b - b^3)*tan(d*x + c)^2 + 60*(3*
a^2*b - b^3)*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A]

time = 0.30, size = 194, normalized size = 1.15

$$\begin{cases} a^3x-\frac{3a^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{3a^2b\tan^2\left(c+dx\right)}{2d}-3ab^2x+\frac{3ab^2\tan^5\left(c+dx\right)}{5d}-\frac{ab^2\tan^3\left(c+dx\right)}{d}+\frac{3ab^2\tan\left(c+dx\right)}{d}+\frac{b^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{b^3\tan^8\left(c+dx\right)}{8d}-\frac{b^3\tan^6\left(c+dx\right)}{6d}+\frac{b^3\tan^4\left(c+dx\right)}{4d}-\frac{b^3\tan^2\left(c+dx\right)}{2d} & \text{for}\ d\neq 0 \\ x\left(a+b\tan^3\left(c\right)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c)**3)**3,x)

[Out] Piecewise((a**3*x - 3*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*a**2*b*tan(c + d*x)**2/(2*d) - 3*a*b**2*x + 3*a*b**2*tan(c + d*x)**5/(5*d) - a*b**2*tan(c + d*x)**3/d + 3*a*b**2*tan(c + d*x)/d + b**3*log(tan(c + d*x)**2 + 1)/(2*d) + b**3*tan(c + d*x)**8/(8*d) - b**3*tan(c + d*x)**6/(6*d) + b**3*tan(c + d*x)**4/(4*d) - b**3*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c)**3)**3, True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3125 vs. $2(158) = 316$.

time = 8.07, size = 3125, normalized size = 18.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="giac")

[Out] 1/120*(120*a^3*d*x*tan(d*x)^8*tan(c)^8 - 360*a*b^2*d*x*tan(d*x)^8*tan(c)^8 + 180*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^8*tan(c)^8 - 60*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^8*tan(c)^8 - 960*a^3*d*x*tan(d*x)^7*tan(c)^7 + 2880*a*b^2*d*x*tan(d*x)^7*tan(c)^7 + 180*a^2*b*tan(d*x)^8*tan(c)^8 - 125*b^3*tan(d*x)^8*tan(c)^8 - 1440*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^7*tan(c)^7 + 480*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^7*tan(c)^7 - 360*a*b^2*tan(d*x)^8*tan(c)^7 - 360*a*b^2*tan(d*x)^7*tan(c)^8 + 3360*a^3*d*x*tan(d*x)^6*tan(c)^6 - 10080*a*b^2*d*x*tan(d*x)^6*tan(c)^6 + 180*a^2*b*tan(d*x)^8*tan(c)^6 - 60*b^3*tan(d*x)^8*tan(c)^6 - 1080*a^2*b*tan(d*x)^7*tan(c)^7 + 880*b^3*tan(d*x)^7*tan(c)^7 + 180*a^2*b*tan(d*x)^6*tan(c)^8 - 60*b^3*tan(d*x)^6*tan(c)^8 + 120*a*b^2*tan(d*x)^8*tan(c)^5 + 5040*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^6*tan(c)^6 - 1680*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^6*tan(c)^6 + 2880*a*b^2*tan(d*x)^7*tan(c)^6 + 2880*a*b^2*tan(d*x)^6*tan(c)^7 + 120*a*b^2*tan(d*x)^5*tan(c)^8 + 30*b^3*tan(d*x)^8*tan(c)^4 - 6720*a^3*d*x*tan(d*x)^5*tan(c)^5 + 20160*a*b^2*d*x*tan(d*x)^5*tan(c)^5 - 1080*a^2*b*tan(d*x)^7*tan(c)^5 + 480*b^3*tan(d*x)^7*tan(c)^5 + 2880*a^2*b*tan(d*x)^6*tan(c)^6 - 2600*b^3*tan(d*x)^6*tan(c)^6 - 1080*a^2*b*tan(d*x)^5*tan(c)^7 + 480*b^3*tan(d*x)^5*tan(c)^7 + 30*b^3*tan(d*x)^4*tan(c)^8 - 72*a*b^2*tan(d*x)^8*tan(c)^3

```
- 960*a*b^2*tan(d*x)^7*tan(c)^4 - 10080*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 3360*b^3*log(4*(tan(d*x)^4*tan(c
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
an(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 - 10080*a*b^2*tan(d*x)^6*tan
(c)^5 - 10080*a*b^2*tan(d*x)^5*tan(c)^6 - 960*a*b^2*tan(d*x)^4*tan(c)^7 - 7
2*a*b^2*tan(d*x)^3*tan(c)^8 - 20*b^3*tan(d*x)^8*tan(c)^2 - 240*b^3*tan(d*x)
^7*tan(c)^3 + 8400*a^3*d*x*tan(d*x)^4*tan(c)^4 - 25200*a*b^2*d*x*tan(d*x)^4
*tan(c)^4 + 2700*a^2*b*tan(d*x)^6*tan(c)^4 - 1680*b^3*tan(d*x)^6*tan(c)^4 -
 4680*a^2*b*tan(d*x)^5*tan(c)^5 + 4080*b^3*tan(d*x)^5*tan(c)^5 + 2700*a^2*b
*tan(d*x)^4*tan(c)^6 - 1680*b^3*tan(d*x)^4*tan(c)^6 - 240*b^3*tan(d*x)^3*ta
n(c)^7 - 20*b^3*tan(d*x)^2*tan(c)^8 + 216*a*b^2*tan(d*x)^7*tan(c)^2 + 2280*
a*b^2*tan(d*x)^6*tan(c)^3 + 12600*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(
d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(
tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 4200*b^3*log(4*(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 18360*a*b^2*tan(d*x)^5*tan(c)^4
+ 18360*a*b^2*tan(d*x)^4*tan(c)^5 + 2280*a*b^2*tan(d*x)^3*tan(c)^6 + 216*a*
b^2*tan(d*x)^2*tan(c)^7 + 15*b^3*tan(d*x)^8 + 160*b^3*tan(d*x)^7*tan(c) + 8
40*b^3*tan(d*x)^6*tan(c)^2 - 6720*a^3*d*x*tan(d*x)^3*tan(c)^3 + 20160*a*b^2
*d*x*tan(d*x)^3*tan(c)^3 - 3600*a^2*b*tan(d*x)^5*tan(c)^3 + 3360*b^3*tan(d*
x)^5*tan(c)^3 + 5400*a^2*b*tan(d*x)^4*tan(c)^4 - 3420*b^3*tan(d*x)^4*tan(c)
^4 - 3600*a^2*b*tan(d*x)^3*tan(c)^5 + 3360*b^3*tan(d*x)^3*tan(c)^5 + 840*b^
3*tan(d*x)^2*tan(c)^6 + 160*b^3*tan(d*x)*tan(c)^7 + 15*b^3*tan(c)^8 - 216*a
*b^2*tan(d*x)^6*tan(c) - 2280*a*b^2*tan(d*x)^5*tan(c)^2 - 10080*a^2*b*log(4
*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 3360*b^3
*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 18
360*a*b^2*tan(d*x)^4*tan(c)^3 - 18360*a*b^2*tan(d*x)^3*tan(c)^4 - 2280*a*b^
2*tan(d*x)^2*tan(c)^5 - 216*a*b^2*tan(d*x)*tan(c)^6 - 20*b^3*tan(d*x)^6 - 2
40*b^3*tan(d*x)^5*tan(c) + 3360*a^3*d*x*tan(d*x)^2*tan(c)^2 - 10080*a*b^2*d
*x*tan(d*x)^2*tan(c)^2 + 2700*a^2*b*tan(d*x)^4*tan(c)^2 - 1680*b^3*tan(d*x)
^4*tan(c)^2 - 4680*a^2*b*tan(d*x)^3*tan(c)^3 + 4080*b^3*tan(d*x)^3*tan(c)^3
 + 2700*a^2*b*tan(d*x)^2*tan(c)^4 - 1680*b^3*tan(d*x)^2*tan(c)^4 - 240*b^3*
tan(d*x)*tan(c)^5 - 20*b^3*tan(c)^6 + 72*a*b^2*tan(d*x)^5 + 960*a*b^2*tan(d
*x)^4*tan(c) + 5040*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))
*tan(d*x)^2*tan(c)^2 - 1680*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*t
an(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
 + 1))*tan(d*x)^2*tan(c)^2 + 10080*a*b^2*tan(d*...
```

Mupad [B]

time = 11.60, size = 174, normalized size = 1.04

$$\frac{\tan(c+d\,x)^2\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)+\frac{b^3\,\tan(c+d\,x)^4}{4}-\frac{b^3\,\tan(c+d\,x)^6}{6}+\frac{b^3\,\tan(c+d\,x)^8}{8}-\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)-a\,\mathrm{atan}\left(\frac{a\,\tan(c+d\,x)\,\left(a^2-3\,b^2\right)}{3\,a\,b^2-a^3}\right)\,\left(a^2-3\,b^2\right)-a\,b^2\,\tan(c+d\,x)^3+\frac{3\,a\,b^2\,\tan(c+d\,x)^5}{5}+3\,a\,b^2\,\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^3)^3,x)
```

```
[Out] (tan(c + d*x)^2*((3*a^2*b)/2 - b^3/2) + (b^3*tan(c + d*x)^4)/4 - (b^3*tan(c
 + d*x)^6)/6 + (b^3*tan(c + d*x)^8)/8 - log(tan(c + d*x)^2 + 1)*((3*a^2*b)/
2 - b^3/2) - a*atan((a*tan(c + d*x)*(a^2 - 3*b^2))/(3*a*b^2 - a^3))*(a^2 -
3*b^2) - a*b^2*tan(c + d*x)^3 + (3*a*b^2*tan(c + d*x)^5)/5 + 3*a*b^2*tan(c
+ d*x))/d
```

3.376 $\int \left(a+b\tan^3(c+dx)\right)^2 dx$

Optimal. Leaf size=89

$$\left(a^2-b^2\right)x+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] (a^2-b^2)*x+2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d+a*b*tan(d*x+c)^2/d-1/3*
b^2*tan(d*x+c)^3/d+1/5*b^2*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3742, 1824, 649, 209, 266}

$$x\left(a^2-b^2\right)+\frac{ab\tan^2(c+dx)}{d}+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan^5(c+dx)}{5d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^2,x]
```

```
[Out] (a^2 - b^2)*x + (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (a*b*T
an[c + d*x]^2)/d - (b^2*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int (a+b\tan^3(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^3)^2}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+2abx-b^2x^2+b^2x^4+\frac{a^2-b^2-2abx}{1+x^2}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}+\frac{\text{Sub}}{}$$
$$= \frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}-\frac{(2a}{}$$
$$= (a^2-b^2)\,x+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 107, normalized size = 1.20

$$\frac{-15i((a-ib)^2\log(i-\tan(c+dx))-(a+ib)^2\log(i+\tan(c+dx)))+30b^2\tan(c+dx)+30ab\tan^2(c+dx)-10b^2\tan^3(c+dx)+6b^2\tan^5(c+dx)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^2,x]
[Out] ((-15*I)*((a - I*b)^2*Log[I - Tan[c + d*x]] - (a + I*b)^2*Log[I + Tan[c + d
*x]]) + 30*b^2*Tan[c + d*x] + 30*a*b*Tan[c + d*x]^2 - 10*b^2*Tan[c + d*x]^3
 + 6*b^2*Tan[c + d*x]^5)/(30*d)
```

Maple [A]
time = 0.08, size = 85, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{b^2(\tan^5(dx+c))}{5}-\frac{b^2(\tan^3(dx+c))}{3}+ab(\tan^2(dx+c))+b^2\tan(dx+c)-ab\ln(1+\tan^2(dx+c))+(a^2-b^2)\arctan(\tan(dx+c))}{d}$
default	$\frac{\frac{b^2(\tan^5(dx+c))}{5}-\frac{b^2(\tan^3(dx+c))}{3}+ab(\tan^2(dx+c))+b^2\tan(dx+c)-ab\ln(1+\tan^2(dx+c))+(a^2-b^2)\arctan(\tan(dx+c))}{d}$

norman	$(a^2-b^2)\,x+\frac{b^2\tan(dx+c)}{d}+\frac{ab(\tan^2(dx+c))}{d}-\frac{b^2(\tan^3(dx+c))}{3d}+\frac{b^2(\tan^5(dx+c))}{5d}-\frac{ab\ln(1+\tan^2(dx+c))}{d}$
risch	$-2iabx+x\,a^2-x\,b^2-\frac{4iabc}{d}+\frac{2b(45ib\,e^{8i(dx+c)}+30a\,e^{8i(dx+c)}+90ib\,e^{6i(dx+c)}+90a\,e^{6i(dx+c)}+140ib\,e^{4i(dx+c)}}{15d(e^{2i(dx+c)}+1)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b^2*tan(d*x+c)^5-1/3*b^2*tan(d*x+c)^3+a*b*tan(d*x+c)^2+b^2*tan(d*x
+c)-a*b*ln(1+tan(d*x+c)^2)+(a^2-b^2)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 83, normalized size = 0.93

$$a^2x+\frac{(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right))b^2}{15\,d}-\frac{ab\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(
d*x + c))*b^2/d - a*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 1.56, size = 85, normalized size = 0.96

$$\frac{3\,b^2\tan\left(dx+c\right)^5-5\,b^2\tan\left(dx+c\right)^3+15\,ab\tan\left(dx+c\right)^2+15\,(a^2-b^2)dx+15\,ab\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+15\,b^2\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*tan(d*x + c)^5 - 5*b^2*tan(d*x + c)^3 + 15*a*b*tan(d*x + c)^2 +
 15*(a^2 - b^2)*d*x + 15*a*b*log(1/(tan(d*x + c)^2 + 1)) + 15*b^2*tan(d*x +
 c))/d
```

Sympy [A]
time = 0.14, size = 94, normalized size = 1.06

$$\begin{cases} a^2x-\frac{ab\log\left(\tan^2\left(c+dx\right)+1\right)}{d}+\frac{ab\tan^2\left(c+dx\right)}{d}-b^2x+\frac{b^2\tan^5\left(c+dx\right)}{5d}-\frac{b^2\tan^3\left(c+dx\right)}{3d}+\frac{b^2\tan\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\left(a+b\tan^3\left(c\right)\right)^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x - a*b*log(tan(c + d*x)**2 + 1)/d + a*b*tan(c + d*x)**2/d
- b**2*x + b**2*tan(c + d*x)**5/(5*d) - b**2*tan(c + d*x)**3/(3*d) + b**2*t
an(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**3)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(85) = 170$.
time = 1.62, size = 1065, normalized size = 11.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*a^2*d*x*tan(d*x)^5*tan(c)^5 - 15*b^2*d*x*tan(d*x)^5*tan(c)^5 + 15*
a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 -
 75*a^2*d*x*tan(d*x)^4*tan(c)^4 + 75*b^2*d*x*tan(d*x)^4*tan(c)^4 + 15*a*b*t
an(d*x)^5*tan(c)^5 - 75*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
))*tan(d*x)^4*tan(c)^4 - 15*b^2*tan(d*x)^5*tan(c)^4 - 15*b^2*tan(d*x)^4*tan
(c)^5 + 150*a^2*d*x*tan(d*x)^3*tan(c)^3 - 150*b^2*d*x*tan(d*x)^3*tan(c)^3 +
 15*a*b*tan(d*x)^5*tan(c)^3 - 45*a*b*tan(d*x)^4*tan(c)^4 + 15*a*b*tan(d*x)^
3*tan(c)^5 + 5*b^2*tan(d*x)^5*tan(c)^2 + 150*a*b*log(4*(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 75*b^2*tan(d*x)^4*tan(c)^3 +
75*b^2*tan(d*x)^3*tan(c)^4 + 5*b^2*tan(d*x)^2*tan(c)^5 - 150*a^2*d*x*tan(d*
x)^2*tan(c)^2 + 150*b^2*d*x*tan(d*x)^2*tan(c)^2 - 45*a*b*tan(d*x)^4*tan(c)^
2 + 60*a*b*tan(d*x)^3*tan(c)^3 - 45*a*b*tan(d*x)^2*tan(c)^4 - 3*b^2*tan(d*x
)^5 - 25*b^2*tan(d*x)^4*tan(c) - 150*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/
(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 150*b^2*tan(d*x)^3*tan(c)^2 - 150*b^2
*tan(d*x)^2*tan(c)^3 - 25*b^2*tan(d*x)*tan(c)^4 - 3*b^2*tan(c)^5 + 75*a^2*d
*x*tan(d*x)*tan(c) - 75*b^2*d*x*tan(d*x)*tan(c) + 45*a*b*tan(d*x)^3*tan(c)
- 60*a*b*tan(d*x)^2*tan(c)^2 + 45*a*b*tan(d*x)*tan(c)^3 + 5*b^2*tan(d*x)^3
+ 75*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)
+ 75*b^2*tan(d*x)^2*tan(c) + 75*b^2*tan(d*x)*tan(c)^2 + 5*b^2*tan(c)^3 - 15
*a^2*d*x + 15*b^2*d*x - 15*a*b*tan(d*x)^2 + 45*a*b*tan(d*x)*tan(c) - 15*a*b
*tan(c)^2 - 15*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 15*b
^2*tan(d*x) - 15*b^2*tan(c) - 15*a*b)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)
^4*tan(c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan
(d*x)*tan(c) - d)
```

Mupad [B]

time = 11.60, size = 117, normalized size = 1.31

$$\frac{b^2\tan(c+d\,x)}{d}-\frac{b^2\tan(c+d\,x)^3}{3\,d}+\frac{b^2\tan(c+d\,x)^5}{5\,d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)\,(a-b)}{a^2-b^2}\right)\,(a+b)\,(a-b)}{d}-\frac{a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{d}+\frac{a\,b\tan(c+d\,x)^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^3)^2,x)
```

```
[Out] (b^2*tan(c + d*x))/d - (b^2*tan(c + d*x)^3)/(3*d) + (b^2*tan(c + d*x)^5)/(5
*d) + (atan((tan(c + d*x)*(a + b)*(a - b))/(a^2 - b^2))*(a + b)*(a - b))/d
- (a*b*log(tan(c + d*x)^2 + 1))/d + (a*b*tan(c + d*x)^2)/d
```

3.377 $\int (a+b\tan^3(c+dx))\ dx$

Optimal. Leaf size=32

$$ax+\frac{b\log(\cos(c+dx))}{d}+\frac{b\tan^2(c+dx)}{2d}$$

```
[Out] a*x+b*ln(cos(d*x+c))/d+1/2*b*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3554, 3556}

$$ax+\frac{b\tan^2(c+dx)}{2d}+\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[c + d*x]^3,x]
```

```
[Out] a*x + (b*Log[Cos[c + d*x]])/d + (b*Tan[c + d*x]^2)/(2*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\tan^3(c+dx))\ dx &= ax+b\int \tan^3(c+dx)\,dx\\ &= ax+\frac{b\tan^2(c+dx)}{2d}-b\int \tan(c+dx)\,dx\\ &= ax+\frac{b\log(\cos(c+dx))}{d}+\frac{b\tan^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.94

$$ax+\frac{b(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[c + d*x]^3,x]
[Out] a*x + (b*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.04, size = 36, normalized size = 1.12

method	result	size
default	$ax+\frac{b\left(\tan^{2}\left(dx+c\right)\right)}{2d}-\frac{b\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	36
norman	$ax+\frac{b\left(\tan^{2}\left(dx+c\right)\right)}{2d}-\frac{b\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	36
derivativedivides	$\frac{\frac{b\left(\tan^{2}\left(dx+c\right)\right)}{2}-\frac{b\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2}+a\arctan\left(\tan\left(dx+c\right)\right)}{d}$	40
risch	$ax-ixb-\frac{2ibc}{d}+\frac{2b\,\mathrm{e}^{2i\left(dx+c\right)}}{d\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)^{2}}+\frac{b\ln\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)}{d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b*tan(d*x+c)^2/d-1/2*b/d*ln(1+tan(d*x+c)^2)
```

Maxima [A]
time = 0.31, size = 36, normalized size = 1.12

$$a x - \frac{b \left(\frac{1}{\sin\left(d x + c\right)^{2} - 1} - \log\left(\sin\left(d x + c\right)^{2} - 1\right)\right)}{2 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^3,x, algorithm="maxima")
[Out] a*x - 1/2*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 2.04, size = 36, normalized size = 1.12

$$\frac{2 \, a d x + b \tan\left(d x + c\right)^{2} + b \log\left(\frac{1}{\tan\left(d x + c\right)^{2} + 1}\right)}{2 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^3,x, algorithm="fricas")
[Out] 1/2*(2*a*d*x + b*tan(d*x + c)^2 + b*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A]
time = 0.07, size = 37, normalized size = 1.16

$$ax + b\left(\begin{cases} -\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{\tan^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\tan^3\left(c\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)**3,x)
```

```
[Out] a*x + b*Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d),
 Ne(d, 0)), (x*tan(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(30) = 60.
time = 0.63, size = 251, normalized size = 7.84

$$ax + \frac{\left(\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right)\tan(dx)^2\tan(c)^2 + \tan(dx)^2\tan(c)^2 - 2\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right)\tan(dx)\tan(c) + \tan(dx)^2 + \tan(c)^2 + \log\left(\frac{4\left(\tan(dx)^4\tan(c)^2 - 2\tan(dx)^3\tan(c) + \tan(dx)^2\tan(c)^2 + \tan(dx)^2 - 2\tan(dx)\tan(c) + 1\right)}{\tan(c)^2+1}\right) + 1\right)b}{2\left(d\tan(dx)^2\tan(c)^2 - 2d\tan(dx)\tan(c) + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] a*x + 1/2*(log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan
(c)^2 + tan(d*x)^2*tan(c)^2 - 2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*t
an(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
 + 1))*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2 + log(4*(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1)/(tan(c)^2 + 1)) + 1)*b/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c)
 + d)
```

Mupad [B]
time = 11.53, size = 34, normalized size = 1.06

$$\frac{\frac{b\tan\left(c+d\,x\right)^2}{2} - \frac{b\,\ln\left(\tan\left(c+d\,x\right)^2+1\right)}{2} + a\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(c + d*x)^3,x)
```

```
[Out] ((b*tan(c + d*x)^2)/2 - (b*log(tan(c + d*x)^2 + 1))/2 + a*d*x)/d
```

3.378 $\int \frac{1}{a+b\tan^3(c+dx)}\,dx$

Optimal. Leaf size=256

$$\frac{ax}{a^2+b^2}+\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a^2+b^2\right)d}-\frac{b\log\left(a\cos^3(c+dx)+b\sin^3(c+dx)\right)}{3\left(a^2+b^2\right)d}+\frac{\sqrt[3]{b}\left(a^{4/3}\right.}{}$$

```
[Out] a*x/(a^2+b^2)-1/3*b*ln(a*cos(d*x+c)^3+b*sin(d*x+c)^3)/(a^2+b^2)/d+1/3*b^(1/
3)*(a^(4/3)+b^(4/3))*ln(a^(1/3)+b^(1/3)*tan(d*x+c))/a^(2/3)/(a^2+b^2)/d-1/6
*b^(1/3)*(a^(4/3)+b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*tan(d*x+c)+b^(2/3)*ta
n(d*x+c)^2)/a^(2/3)/(a^2+b^2)/d+1/3*b^(1/3)*(a^(4/3)-b^(4/3))*arctan(1/3*(a
^(1/3)-2*b^(1/3)*tan(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^2+b^2)/d*3^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3742, 6857, 649, 209, 266, 1885, 1874, 31, 648, 631, 210, 642}

$$-\frac{b\log\left(a\cos^3(c+dx)+b\sin^3(c+dx)\right)}{3d\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}+\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d\left(a^2+b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)}{6a^{2/3}d\left(a^2+b^2\right)}+\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{3a^{2/3}d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^(-1),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b^(1/3)*(a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3
)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 + b^2)*d) - (b*Lo
g[a*Cos[c + d*x]^3 + b*Sin[c + d*x]^3])/(3*(a^2 + b^2)*d) + (b^(1/3)*(a^(4/
3) + b^(4/3))*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(3*a^(2/3)*(a^2 + b^2)*d
) - (b^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x]
 + b^(2/3)*Tan[c + d*x]^2])/(6*a^(2/3)*(a^2 + b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\tan^3(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^3)}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a+bx}{(a^2+b^2)(1+x^2)} - \frac{b(-b+ax+bx^2)}{(a^2+b^2)(a+bx^3)}\right)\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+bx}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} - \frac{b\text{Subst}\left(\int \frac{-b+ax+bx^2}{a+bx^3}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d}\\
&= \frac{a\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} + \frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)\,d} - \frac{b\text{Subs}}{}\\
&= \frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} - \frac{b\log\left(a+b\tan^3(c+dx)\right)}{3\left(a^2+b^2\right)d} - \frac{b^{2/3}\text{Subst}\left(\int \frac{\sqrt[3]{a}\,(a}{a}\right.}{}\\
&= \frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{3a^{2/3}\left(a^2+b^2\right)d}\\
&= \frac{ax}{a^2+b^2} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{3a^{2/3}\left(a^2+b^2\right)d}\\
&= \frac{ax}{a^2+b^2} + \frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tan(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\left(a^2+b^2\right)d} - \frac{b\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{\sqrt[3]{}}{}
\end{aligned}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.41, size = 278, normalized size = 1.09

$$\frac{-2\sqrt{3}\,b^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 3ia^{5/3}\log(i-\tan(c+dx)) + 3a^{2/3}b\log(i-\tan(c+dx)) + 3ia^{5/3}\log(i+\tan(c+dx)) + 3a^{2/3}b\log(i+\tan(c+dx)) + 2b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right) - b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right) - 2a^{2/3}b\log\left(a+b\tan^3(c+dx)\right) - 3a^{2/3}b\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\tan^3(c+dx)}{a}\right)\tan^2(c+dx)}{6a^{2/3}\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1
/3))] - (3*I)*a^(5/3)*Log[I - Tan[c + d*x]] + 3*a^(2/3)*b*Log[I - Tan[c + d
*x]] + (3*I)*a^(5/3)*Log[I + Tan[c + d*x]] + 3*a^(2/3)*b*Log[I + Tan[c + d*
x]] + 2*b^(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]] - b^(5/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2] - 2*a^(2/3)*b*Log[a
```

```
 + b*Tan[c + d*x]^3] - 3*a^(2/3)*b*Hypergeometric2F1[2/3, 1, 5/3, -((b*Tan[
c + d*x]^3)/a)]*Tan[c + d*x]^2)/(6*a^(2/3)*(a^2 + b^2)*d)
```

Maple [A]

time = 0.37, size = 293, normalized size = 1.14

method	result
risch	$\frac{x}{ib+a}+\frac{2ia^2bd^3x}{a^4d^3+a^2b^2d^3}+\frac{2ia^2bd^2c}{a^4d^3+a^2b^2d^3}+\left(\sum_{\_R=\mathrm{RootOf}\left((27a^4d^3+27a^2b^2d^3)\_Z^3+27\_Z^2a^2bd^2-b\right)}\_R\ln\left(\mathrm{e}^{2i(\right.\right.$
derivativedivides	$\frac{\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}-\frac{\left(-b\left(\frac{\ln\left(\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\tan^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\mathrm{ar}}{}\right.\right.}{}$
default	$\frac{\frac{b\ln\left(1+\tan^2(dx+c)\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}-\frac{\left(-b\left(\frac{\ln\left(\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\tan^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\mathrm{ar}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)*(1/2*b*ln(1+tan(d*x+c)^2)+a*arctan(tan(d*x+c)))-(-b*(1/3/b
/(a/b)^(2/3)*ln(tan(d*x+c)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(tan(d*x+c)^2-(
a/b)^(1/3)*tan(d*x+c)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*tan(d*x+c)-1)))+a*(-1/3/b/(a/b)^(1/3)*ln(tan(d*x+c)+(a/
b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(tan(d*x+c)^2-(a/b)^(1/3)*tan(d*x+c)+(a/b)^(2
/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)
-1)))+1/3*ln(a+b*tan(d*x+c)^3))/(a^2+b^2)*b)
```

Maxima [A]

time = 0.51, size = 291, normalized size = 1.14

$$-\frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\right)-b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\tan(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{18(dx+c)a}{a^2+b^2}+\frac{3\left(b\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)+a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\tan(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{9b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{6\left(b\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)-a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\tan(dx+c)\right)}{a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{18d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="maxima")

[Out] -1/18*(2*sqrt(3)*(a*(3*(a/b)^(2/3) - 2) - b*(3*(a/b)^(1/3) - 2*a/b))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*tan(d*x + c))/(a/b)^(1/3))/((a^2*(a/b)^(2/3) + b^2*(a/b)^(2/3))*(a/b)^(1/3)) - 18*(d*x + c)*a/(a^2 + b^2) + 3*(b*(2*(a/b)^(2/3) + 1) + a*(a/b)^(1/3))*log(tan(d*x + c)^2 - (a/b)^(1/3)*tan(d*x + c) + (a/b)^(2/3))/(a^2*(a/b)^(2/3) + b^2*(a/b)^(2/3)) - 9*b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + 6*(b*((a/b)^(2/3) - 1) - a*(a/b)^(1/3))*log((a/b)^(1/3) + tan(d*x + c))/(a^2*(a/b)^(2/3) + b^2*(a/b)^(2/3)))/d

Fricas [C] Result contains complex when optimal does not.
time = 7.75, size = 4817, normalized size = 18.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="fricas")

[Out] -1/24*(2*(a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))*d*log(-1/4*(4*b^2*tan(d*x + c)^2 - ((a^4 + a^2*b^2)*d^2*tan(d*x + c)^2 - (a^4 + a^2*b^2)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2 + 2*(a^2*b*d*tan(d*x + c)^2 - a^2*b*d + 2*(a^3 - a*b^2)*d*tan(d*x + c))*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d)) - 4*a^2)/(tan(d*x + c)^2 + 1)) - 24*a*d*x - ((a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))*d - 3*sqrt(1/3)*(a^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b

```
^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*
(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2
*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d
+ b^2*d))*d - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 6*b)*log(1/4*(8*a^4
- 16*a^2*b^2 - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(d*x + c)^2 - (a^6 + 2*a
^4*b^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d
^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3
) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2
*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3)
)^(1/3)) + 2*b/(a^2*d + b^2*d))^2 + 8*(2*a^2*b^2 - b^4)*tan(d*x + c)^2 + 2*
((a^4*b + a^2*b^3)*d*tan(d*x + c)^2 + 2*(a^5 - a*b^4)*d*tan(d*x + c) - (a^4
*b + a^2*b^3)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) -
2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*
(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d
^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3
)) + 2*b/(a^2*d + b^2*d)) + 3*sqrt(1/3)*(4*(a^4*b + a^2*b^3)*d*tan(d*x + c)
^2 - 4*(a^5 - a*b^4)*d*tan(d*x + c) - ((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*tan(
d*x + c)^2 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*
(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2
+ b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2
*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/
((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d)) - 4*(a^4*b + a^2*b^3
)*d)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^
3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a
^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b/(
a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^
2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))^2*d^2 - 4*(a^2*b + b^3)*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3
 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqr
t(3) + 1)/((a^2*d + b^2*d)^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^
2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d
))*d - 12*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 24*(a^3*b - a*b^3)*tan(d*x
+ c))/(tan(d*x + c)^2 + 1)) - ((a^2 + b^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/
(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b
^2)^2*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)
^2*(b/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a
^2 + b^2)^2*a^2*d^3))^(1/3)) + 2*b/(a^2*d + b^2*d))*d + 3*sqrt(1/3)*(a^2 +
b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*
d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d)^3 - (a^2 - b^2)*b/((a^2 + b^2)^2
*a^2*d^3))^(1/3) + 2*(1/2)^(2/3)*b^2*(-I*sqrt(3) + 1)/((a^2*d + b^2*d)^2*(b
/(a^4*d^3 + a^2*b^2*d^3) - 2*b^3/(a^2*d + b^2*d...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\tan^3(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*tan(c + d*x)**3), x)
```

Giac [A]
time = 0.68, size = 334, normalized size = 1.30

$$\frac{\frac{2\left(a^3b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} + ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b^3 - b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \tan(dx+c)\right|\right)}{a^5b + 2a^3b^3 + ab^5} + \frac{6\left(\pi\left\lfloor\frac{dx+c}{\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) + \arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2\tan(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)\left(\left(-ab^2\right)^{\frac{1}{3}}b^2 + \left(-ab^2\right)^{\frac{2}{3}}a\right)}{\sqrt{3}\,a^3b + \sqrt{3}\,ab^3} + \frac{6(dx+c)a}{a^2+b^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2 - \left(-ab^2\right)^{\frac{2}{3}}a\right)\log\left(\tan(dx+c)^2 + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c) + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^3b + ab^3} + \frac{3b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} - \frac{2b\log\left(\left|b\tan(dx+c)^3 + a\right|\right)}{a^2+b^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*(a^3*b^2*(-a/b)^(1/3) + a*b^4*(-a/b)^(1/3) - a^2*b^3 - b^5)*(-a/b)^(
1/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)))/(a^5*b + 2*a^3*b^3 + a*b^5) + 6
*(pi*floor((d*x + c)/pi + 1/2)*sgn((-a/b)^(1/3)) + arctan(1/3*sqrt(3)*((-a/
b)^(1/3) + 2*tan(d*x + c))/(-a/b)^(1/3)))*((-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2
/3)*a)/(sqrt(3)*a^3*b + sqrt(3)*a*b^3) + 6*(d*x + c)*a/(a^2 + b^2) + ((-a*b
^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*log(tan(d*x + c)^2 + (-a/b)^(1/3)*tan(d*x
 + c) + (-a/b)^(2/3))/(a^3*b + a*b^3) + 3*b*log(tan(d*x + c)^2 + 1)/(a^2 +
b^2) - 2*b*log(abs(b*tan(d*x + c)^3 + a))/(a^2 + b^2))/d
```

Mupad [B]
time = 12.65, size = 342, normalized size = 1.34

$$\frac{\sum_{k=1}^{3}\ln\left(\mathrm{root}(27a^2b^2z^3 + 27a^4z^3 + 27a^2bz^2 - b, z, k)\left(\mathrm{root}(27a^2b^2z^3 + 27a^4z^3 + 27a^2bz^2 - b, z, k)\left(\mathrm{root}(27a^2b^2z^3 + 27a^4z^3 + 27a^2bz^2 - b, z, k)\left(\tan(c+dx)\left(12b^6 - 69a^2b^4\right) + \mathrm{root}(27a^2b^2z^3 + 27a^4z^3 + 27a^2bz^2 - b, z, k)\left(36ab^6 - 180a^3b^4 + \tan(c+dx)\left(162a^2b^5 - 54a^4b^3\right)\right) - 36ab^5 + 27a^3b^3\right) + 13ab^4 - 16b^5\tan(c+dx)\right) + 5b^4\tan(c+dx)\right)\right)\mathrm{root}(27a^2b^2z^3 + 27a^4z^3 + 27a^2bz^2 - b, z, k)}{d} + \frac{\ln\left(\tan(c+dx) - \mathrm{i}\right)}{2d\left(b + a\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+dx) + 1\mathrm{i}\right)1\mathrm{i}}{2d\left(a + b\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^3),x)
```

```
[Out] symsum(log(root(27*a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(root
(27*a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(root(27*a^2*b^2*z^3
 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(tan(c + d*x)*(12*b^6 - 69*a^2*b^4)
 + root(27*a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(36*a*b^6 - 1
80*a^3*b^4 + tan(c + d*x)*(162*a^2*b^5 - 54*a^4*b^3)) - 36*a*b^5 + 27*a^3*b
^3) + 13*a*b^4 - 16*b^5*tan(c + d*x)) + 5*b^4*tan(c + d*x)))*root(27*a^2*b^
2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k), k, 1, 3)/d + log(tan(c + d*x)
 - 1i)/(2*d*(a*1i + b)) + (log(tan(c + d*x) + 1i)*1i)/(2*d*(a + b*1i))
```

3.379 $\int \frac{1}{(a+b\tan^3(c+dx))^2}\,dx$

Optimal. Leaf size=558

$$\frac{(a^2-b^2)\,x}{(a^2+b^2)^2}+\frac{\sqrt[3]{b}\,(a^2-2a^{2/3}b^{4/3}-b^2)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\tan(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,(a^2+b^2)^2\,d}+\frac{\sqrt[3]{b}\,(a^{4/3}-2b^{4/3})\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\,\sqrt[3]{}}\right.}{3\sqrt{3}\,a^{5/3}\,(a^2+b^2)\,d}$$

```
[Out] (a^2-b^2)*x/(a^2+b^2)^2-2/3*a*b*ln(a*cos(d*x+c)^3+b*sin(d*x+c)^3)/(a^2+b^2)
^2/d+1/3*b^(1/3)*(a^2+2*a^(2/3)*b^(4/3)-b^2)*ln(a^(1/3)+b^(1/3)*tan(d*x+c))
/a^(1/3)/(a^2+b^2)^2/d+1/9*b^(1/3)*(a^(4/3)+2*b^(4/3))*ln(a^(1/3)+b^(1/3)*t
an(d*x+c))/a^(5/3)/(a^2+b^2)/d-1/6*b^(1/3)*(a^2+2*a^(2/3)*b^(4/3)-b^2)*ln(a
^(2/3)-a^(1/3)*b^(1/3)*tan(d*x+c)+b^(2/3)*tan(d*x+c)^2)/a^(1/3)/(a^2+b^2)^2
/d-1/18*b^(1/3)*(a^(4/3)+2*b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*tan(d*x+c)+b
^(2/3)*tan(d*x+c)^2)/a^(5/3)/(a^2+b^2)/d+1/3*b^(1/3)*(a^2-2*a^(2/3)*b^(4/3)
-b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tan(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/(a
^2+b^2)^2/d*3^(1/2)+1/9*b^(1/3)*(a^(4/3)-2*b^(4/3))*arctan(1/3*(a^(1/3)-2*b
^(1/3)*tan(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/(a^2+b^2)/d*3^(1/2)+1/3*b*(a+ta
n(d*x+c)*(b-a*tan(d*x+c)))/a/(a^2+b^2)/d/(a+b*tan(d*x+c)^3)
```

Rubi [A]
time = 0.48, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3742, 6857, 649, 209, 266, 1868, 1874, 31, 648, 631, 210, 642, 1885}

$$\frac{b(\tan(c+dx)(b-a\tan(c+dx))+a)}{3ad\,(a^2+b^2)\,(a+b\tan^3(c+dx))}-\frac{2ab\log\left(a\cos^3(c+dx)+b\sin^3(c+dx)\right)}{3d\,(a^2+b^2)^2}+\frac{x(a^2-b^2)}{(a^2+b^2)^2}+\frac{\sqrt[3]{b}\,(a^{4/3}-2b^{4/3})\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\tan(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}d\,(a^2+b^2)}+\frac{\sqrt[3]{b}\,(-2a^{2/3}b^{4/3}+a^2-b^2)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\tan(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,d\,(a^2+b^2)^2}-\frac{\sqrt[3]{b}\,(a^{4/3}+2b^{4/3})\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)}{18a^{5/3}d\,(a^2+b^2)}-\frac{\sqrt[3]{b}\,(2a^{2/3}b^{4/3}+a^2-b^2)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)}{6\sqrt[3]{a}\,d\,(a^2+b^2)^2}+\frac{\sqrt[3]{b}\,(a^{4/3}+2b^{4/3})\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\tan(c+dx)\right)}{9a^{5/3}d\,(a^2+b^2)}+\frac{\sqrt[3]{b}\,(2a^{2/3}b^{4/3}+a^2-b^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\tan(c+dx)\right)}{3\sqrt[3]{a}\,d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^3)^(-2),x]
```

```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (b^(1/3)*(a^2 - 2*a^(2/3)*b^(4/3) - b^2)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3
)*(a^2 + b^2)^2*d) + (b^(1/3)*(a^(4/3) - 2*b^(4/3))*ArcTan[(a^(1/3) - 2*b^(
1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(a^2 + b^2)*d) -
(2*a*b*Log[a*Cos[c + d*x]^3 + b*Sin[c + d*x]^3])/(3*(a^2 + b^2)^2*d) + (b^(
1/3)*(a^2 + 2*a^(2/3)*b^(4/3) - b^2)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(
3*a^(1/3)*(a^2 + b^2)^2*d) + (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(1/3) + b
^(1/3)*Tan[c + d*x]])/(9*a^(5/3)*(a^2 + b^2)*d) - (b^(1/3)*(a^2 + 2*a^(2/3)
*b^(4/3) - b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c
+ d*x]^2])/(6*a^(1/3)*(a^2 + b^2)^2*d) - (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log
[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2])/(18*a^(5
/3)*(a^2 + b^2)*d) + (b*(a + Tan[c + d*x]*(b - a*Tan[c + d*x])))/(3*a*(a^2
+ b^2)*d*(a + b*Tan[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\tan^3(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^3)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-b^2+2abx}{(a^2+b^2)^2(1+x^2)} - \frac{b(-b+ax+bx^2)}{(a^2+b^2)(a+bx^3)^2} + \frac{b(2ab-(a^2-b^2)x-2abx^2)}{(a^2+b^2)^2(a+bx^3)}\right)\,dx, x, \tan(c-\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2-b^2+2abx}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a^2+b^2)^2 d} + \frac{b\text{Subst}\left(\int \frac{2ab-(a^2-b^2)x-2abx^2}{a+bx^3}\,dx, x, \text{t}\right.}{(a^2+b^2)^2 d}$$

$$= \frac{b(a+\tan(c+dx)(b-a\tan(c+dx)))}{3a(a^2+b^2)d(a+b\tan^3(c+dx))} + \frac{b\text{Subst}\left(\int \frac{2ab+(-a^2+b^2)x}{a+bx^3}\,dx, x, \tan(c\right.}{(a^2+b^2)^2 d}$$

$$= \frac{(a^2-b^2)x}{(a^2+b^2)^2} - \frac{2ab\log(\cos(c+dx))}{(a^2+b^2)^2 d} - \frac{2ab\log(a+b\tan^3(c+dx))}{3(a^2+b^2)^2 d} + \frac{b(a+\text{ta}}{3a(a^2}$$

$$= \frac{(a^2-b^2)x}{(a^2+b^2)^2} - \frac{2ab\log(\cos(c+dx))}{(a^2+b^2)^2 d} + \frac{\sqrt[3]{b}\,(a^2+2a^{2/3}b^{4/3}-b^2)\log\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{3\sqrt[3]{a}\,(a^2+b^2)^2 d}$$

$$= \frac{(a^2-b^2)x}{(a^2+b^2)^2} - \frac{2ab\log(\cos(c+dx))}{(a^2+b^2)^2 d} + \frac{\sqrt[3]{b}\,(a^2+2a^{2/3}b^{4/3}-b^2)\log\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{3\sqrt[3]{a}\,(a^2+b^2)^2 d}$$

$$= \frac{(a^2-b^2)x}{(a^2+b^2)^2} + \frac{\sqrt[3]{b}\,(a^2-2a^{2/3}b^{4/3}-b^2)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tan(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,(a^2+b^2)^2 d} + \frac{\sqrt[3]{b}\,(a^{4/3}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.06, size = 504, normalized size = 0.90

$$-\frac{\frac{9i\log(i-\tan(c+dx))}{(a-ib)^2} - \frac{9i\log(i+\tan(c+dx))}{(a+ib)^2} - \frac{12\sqrt[3]{a}\,b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{(a^2+b^2)^2} + \frac{6\sqrt[3]{a}\,b^{5/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)\right)}{(a^2+b^2)^2} + \frac{2b^{5/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)\right)}{a^{5/3}(a^2+b^2)} + \frac{12ab\log(a+b\tan^3(c+dx))}{(a^2+b^2)^2} + \frac{9(a-b)b(a+b)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\tan^3(c+dx)}{a}\right)\tan^2(c+dx)}{a(a^2+b^2)^2} + \frac{9b\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{b\tan^3(c+dx)}{a}\right)\tan^2(c+dx)}{a(a^2+b^2)} - \frac{6b}{(a^2+b^2)(a+b\tan^3(c+dx))} - \frac{6b^2\tan(c+dx)}{a(a^2+b^2)(a+b\tan^3(c+dx))}}{18d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^3)^(-2),x]
```

```
[Out] -1/18*(((9*I)*Log[I - Tan[c + d*x]])/(a - I*b)^2 - ((9*I)*Log[I + Tan[c + d
*x]])/(a + I*b)^2 - (12*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]]
)/(a^2 + b^2)^2 + (6*a^(1/3)*b^(5/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)
```

```
*Tan[c + d*x])/(Sqrt[3]*a^(1/3))] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d
*x] + b^(2/3)*Tan[c + d*x]^2]))/(a^2 + b^2)^2 + (2*b^(5/3)*(2*Sqrt[3]*ArcTa
n[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))] - 2*Log[a^(1/3) + b
^(1/3)*Tan[c + d*x]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)
*Tan[c + d*x]^2]))/(a^(5/3)*(a^2 + b^2)) + (12*a*b*Log[a + b*Tan[c + d*x]^3
])/(a^2 + b^2)^2 + (9*(a - b)*b*(a + b)*Hypergeometric2F1[2/3, 1, 5/3, -((b
*Tan[c + d*x]^3)/a)]*Tan[c + d*x]^2)/(a*(a^2 + b^2)^2) + (9*b*Hypergeometri
c2F1[2/3, 2, 5/3, -((b*Tan[c + d*x]^3)/a)]*Tan[c + d*x]^2)/(a*(a^2 + b^2))
- (6*b)/((a^2 + b^2)*(a + b*Tan[c + d*x]^3)) - (6*b^2*Tan[c + d*x])/(a*(a^2
 + b^2)*(a + b*Tan[c + d*x]^3)))/d
```

Maple [A]
time = 0.93, size = 411, normalized size = 0.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^4+2*a^2*b^2+b^4)*(a*b*ln(1+tan(d*x+c)^2)+(a^2-b^2)*arctan(tan(d*x
+c)))-b/(a^4+2*a^2*b^2+b^4)*(((1/3*a^2+1/3*b^2)*tan(d*x+c)^2-1/3*b*(a^2+b^2
)/a*tan(d*x+c)-1/3*a^2-1/3*b^2)/(a+b*tan(d*x+c)^3)+2/3/a*((-4*a^2*b-b^3)*(1
/3/b/(a/b)^(2/3)*ln(tan(d*x+c)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(tan(d*x+c)
^2-(a/b)^(1/3)*tan(d*x+c)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)-1)))+(2*a^3-a*b^2)*(-1/3/b/(a/b)^(1/3)*l
n(tan(d*x+c)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(tan(d*x+c)^2-(a/b)^(1/3)*tan
(d*x+c)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*tan(d*x+c)-1)))+a^2*ln(a+b*tan(d*x+c)^3))))
```

Maxima [A]
time = 0.52, size = 502, normalized size = 0.90

$$\frac{\frac{9\,ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,\sqrt{3}\left(2\,a^3\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)-2\,a^2b\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{a}{b}\right)-ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\tan(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{9\left(a^2-b^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{\left(2\,a^2b\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\right)+2\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+b^3\right)\log\left(\tan(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\left(a^2b\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\right)-2\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}+ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-b^3\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\tan(dx+c)\right)}{a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{3\left(ab\tan(dx+c)^2-b^2\tan(dx+c)-ab\right)}{a^4+a^2b^2+\left(a^3b+ab^3\right)\tan(dx+c)^3}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(9*a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*sqrt(3)*(2*a
^3*((a/b)^(2/3) - 1) - 2*a^2*b*(2*(a/b)^(1/3) - a/b) - a*b^2*(a/b)^(2/3) -
b^3*(a/b)^(1/3))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*tan(d*x + c))/(a/b)^(
1/3))/((a^5*(a/b)^(2/3) + 2*a^3*b^2*(a/b)^(2/3) + a*b^4*(a/b)^(2/3))*(a/b)^
(1/3)) + 9*(a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (2*a^2*b*(3*(a/b
)^(2/3) + 2) + 2*a^3*(a/b)^(1/3) - a*b^2*(a/b)^(1/3) + b^3)*log(tan(d*x + c
)^2 - (a/b)^(1/3)*tan(d*x + c) + (a/b)^(2/3))/(a^5*(a/b)^(2/3) + 2*a^3*b^2*
(a/b)^(2/3) + a*b^4*(a/b)^(2/3)) - 2*(a^2*b*(3*(a/b)^(2/3) - 4) - 2*a^3*(a/
b)^(1/3) + a*b^2*(a/b)^(1/3) - b^3)*log((a/b)^(1/3) + tan(d*x + c))/(a^5*(a
/b)^(2/3) + 2*a^3*b^2*(a/b)^(2/3) + a*b^4*(a/b)^(2/3)) - 3*(a*b*tan(d*x + c
```

```
)^2 - b^2*tan(d*x + c) - a*b)/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*tan(d*x + c)
^3))/d
```

Fricas [C] Result contains complex when optimal does not.
time = 7.50, size = 11554, normalized size = 20.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/648*(216*a^3*b - 432*a*b^3 + 216*(2*a^2*b^2 - b^4 - 3*(a^3*b - a*b^3)*d*
x)*tan(d*x + c)^3 - 648*(a^4 - a^2*b^2)*d*x + 2*((a^5*b + 2*a^3*b^3 + a*b^5
)*d*tan(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*(4*(9*a^2*b^2/(a^4*d +
2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*s
qrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a
^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/72
9*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 -
28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27
*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2
*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/
(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^
2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^
4*d + 2*a^2*b^2*d + b^4*d))*log(1/324*(10368*a^6 - 12960*a^4*b^2 - 3888*a^2
*b^4 + ((2*a^10 + 5*a^8*b^2 + 4*a^6*b^4 + a^4*b^6)*d^2*tan(d*x + c)^2 - 4*(
a^9*b + 2*a^7*b^3 + a^5*b^5)*d^2*tan(d*x + c) - (2*a^10 + 5*a^8*b^2 + 4*a^6
*b^4 + a^4*b^6)*d^2)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a
^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4
*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^
4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*
a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*
b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d +
 b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*
a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*
b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a
^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2 -
 1296*(18*a^4*b^2 + 7*a^2*b^4 + b^6)*tan(d*x + c)^2 - 36*((8*a^7*b - 2*a^5*
b^3 - a^3*b^5)*d*tan(d*x + c)^2 + 2*(4*a^8 - 20*a^6*b^2 - 7*a^4*b^4 - a^2*b
^6)*d*tan(d*x + c) - (8*a^7*b - 2*a^5*b^3 - a^3*b^5)*d)*(4*(9*a^2*b^2/(a^4*
d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(
-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3
/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) +
4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^
6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-
8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4
```

```
*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b
^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 1
0*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b
/(a^4*d + 2*a^2*b^2*d + b^4*d)) + 2592*(4*a^5*b + 2*a^3*b^3 + a*b^5)*tan(d*
x + c))/(tan(d*x + c)^2 + 1)) + 216*(a^3*b + a*b^3)*tan(d*x + c)^2 + (324*a
^2*b^2*tan(d*x + c)^3 + 324*a^3*b - ((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x
+ c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d
+ b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/
(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a
^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b +
 b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 -
 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^
4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b
^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2
*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)
*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d + 2*a^2*
b^2*d + b^4*d)) + 3*sqrt(1/3)*((a^5*b + 2*a^3*b^3 + a*b^5)*d*tan(d*x + c)^3
 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*sqrt((29808*a^4*b^2 - 10368*a^2*b^4 - 518
4*b^6 - (a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/
(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^
2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*
a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d
)) + 4/729*(8*a^2*b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*
(8*a^6 - 28*a^4*b^2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3) +
81*(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 +
2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*
b + b^3)/(a^9*d^3 + 2*a^7*b^2*d^3 + a^5*b^4*d^3) - 4/729*(8*a^6 - 28*a^4*b^
2 - 10*a^2*b^4 - b^6)*b/((a^2 + b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 10
8*a*b/(a^4*d + 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b + 2*a^5*b^3 + a^3*b
^5)*(4*(9*a^2*b^2/(a^4*d + 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 + 2*a^4*b^
2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d + 2*a^2*b^2*d
+ b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d + 2
*a^2*b^2*d + b^4*d)) + 4/729*(8*a^2*b + b^3)/(a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.75, size = 605, normalized size = 1.08

$$\frac{\frac{9ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2a^2b^2+b^4}-\frac{6ab\log\left(\left|b\tan(dx+c)^3+a\right|\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(2a^8b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3a^6b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^8\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4a^7b^3-9a^5b^5-6a^3b^7-ab^9\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\tan(dx+c)\right|\right)}{a^{11}b+4a^9b^3+6a^7b^5+4a^5b^7+a^3b^9}+\frac{9\left(a^2-b^2\right)(dx+c)}{a^4+2a^2b^2+b^4}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\tan(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)\left(\left(2\sqrt{3}\,a^3-\sqrt{3}\,ab^2\right)\left(-ab^2\right)^{\frac{2}{3}}+\left(4\sqrt{3}\,a^2b^2+\sqrt{3}\,b^4\right)\left(-ab^2\right)^{\frac{1}{3}}\right)}{a^6b+2a^4b^3+a^2b^5}-\frac{\left(\left(2a^3-ab^2\right)\left(-ab^2\right)^{\frac{2}{3}}-\left(4a^2b^2+b^4\right)\left(-ab^2\right)^{\frac{1}{3}}\right)\log\left(\tan(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^6b+2a^4b^3+a^2b^5}+\frac{3\left(2a^2b^2\tan(dx+c)^3-a^3b\tan(dx+c)^2-ab^3\tan(dx+c)^2+a^2b^2\tan(dx+c)+b^4\tan(dx+c)+3a^3b+ab^3\right)}{\left(a^5+2a^3b^2+ab^4\right)\left(b\tan(dx+c)^3+a\right)}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/9*(9*a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 6*a*b*log(abs(
b*tan(d*x + c)^3 + a))/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a^8*b^2*(-a/b)^(1/3)
+ 3*a^6*b^4*(-a/b)^(1/3) - a^2*b^8*(-a/b)^(1/3) - 4*a^7*b^3 - 9*a^5*b^5 - 6
*a^3*b^7 - a*b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)))/(a^11
*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9) + 9*(a^2 - b^2)*(d*x + c)
/(a^4 + 2*a^2*b^2 + b^4) + 2*(pi*floor((d*x + c)/pi + 1/2)*sgn((-a/b)^(1/3)
) + arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*tan(d*x + c))/(-a/b)^(1/3)))*((2*s
qrt(3)*a^3 - sqrt(3)*a*b^2)*(-a*b^2)^(2/3) + (4*sqrt(3)*a^2*b^2 + sqrt(3)*b
^4)*(-a*b^2)^(1/3))/(a^6*b + 2*a^4*b^3 + a^2*b^5) - ((2*a^3 - a*b^2)*(-a*b^
2)^(2/3) - (4*a^2*b^2 + b^4)*(-a*b^2)^(1/3))*log(tan(d*x + c)^2 + (-a/b)^(1
/3)*tan(d*x + c) + (-a/b)^(2/3))/(a^6*b + 2*a^4*b^3 + a^2*b^5) + 3*(2*a^2*b
^2*tan(d*x + c)^3 - a^3*b*tan(d*x + c)^2 - a*b^3*tan(d*x + c)^2 + a^2*b^2*t
an(d*x + c) + b^4*tan(d*x + c) + 3*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4
)*(b*tan(d*x + c)^3 + a)))/d
```

Mupad [B]
time = 12.52, size = 988, normalized size = 1.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^3)^2,x)
```

```
[Out] symsum(log(root(1458*a^7*b^2*z^3 + 729*a^5*b^4*z^3 + 729*a^9*z^3 + 1458*a^6
*b*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z, k)*(((32*a*b^7)/27 - (128*a^3
*b^5)/27)/(a^7 + a^3*b^4 + 2*a^5*b^2) - root(1458*a^7*b^2*z^3 + 729*a^5*b^4
*z^3 + 729*a^9*z^3 + 1458*a^6*b*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z,
k)*(root(1458*a^7*b^2*z^3 + 729*a^5*b^4*z^3 + 729*a^9*z^3 + 1458*a^6*b*z^2
+ 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z, k)*((16*a^3*b^9 + 77*a^5*b^7 + 34*a^
7*b^5 - 27*a^9*b^3)/(a^7 + a^3*b^4 + 2*a^5*b^2) + root(1458*a^7*b^2*z^3 + 7
29*a^5*b^4*z^3 + 729*a^9*z^3 + 1458*a^6*b*z^2 + 108*a^3*b^2*z - 64*a^2*b -
8*b^3, z, k)*((108*a^6*b^8 - 36*a^4*b^10 + 324*a^8*b^6 + 180*a^10*b^4)/(a^7
 + a^3*b^4 + 2*a^5*b^2) - (tan(c + d*x)*(4374*a^5*b^9 + 7290*a^7*b^7 + 1458
*a^9*b^5 - 1458*a^11*b^3))/(27*(a^7 + a^3*b^4 + 2*a^5*b^2))) - (tan(c + d*x
)*(216*a^2*b^10 + 864*a^4*b^8 - 1836*a^6*b^6 - 2484*a^8*b^4))/(27*(a^7 + a^
3*b^4 + 2*a^5*b^2))) - ((64*a^2*b^8)/9 + (353*a^4*b^6)/9 + (388*a^6*b^4)/9)
/(a^7 + a^3*b^4 + 2*a^5*b^2) + (tan(c + d*x)*(96*a*b^9 + 408*a^3*b^7 + 447*
a^5*b^5))/(27*(a^7 + a^3*b^4 + 2*a^5*b^2))) + (tan(c + d*x)*(134*a^2*b^6 -
```

```
16*b^8 + 236*a^4*b^4))/(27*(a^7 + a^3*b^4 + 2*a^5*b^2))) - ((8*b^6)/27 + (1
6*a^2*b^4)/27)/(a^7 + a^3*b^4 + 2*a^5*b^2) - (8*a*b^5*tan(c + d*x))/(9*(a^7
 + a^3*b^4 + 2*a^5*b^2)))*root(1458*a^7*b^2*z^3 + 729*a^5*b^4*z^3 + 729*a^9
*z^3 + 1458*a^6*b*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z, k), k, 1, 3)/d
 + (log(tan(c + d*x) - 1i)*1i)/(2*d*(a*b*2i - a^2 + b^2)) + log(tan(c + d*x
) + 1i)/(2*d*(2*a*b - a^2*1i + b^2*1i)) + (b/(3*(a^2 + b^2)) - (b*tan(c + d
*x)^2)/(3*(a^2 + b^2)) + (b^2*tan(c + d*x))/(3*a*(a^2 + b^2)))/(d*(a + b*ta
n(c + d*x)^3))
```

3.380 $\int \frac{1}{1+\tan^3(x)} dx$

Optimal. Leaf size=37

$$\frac{x}{2}-\frac{1}{2}\log(\cos(x))+\frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)$$

```
[Out] 1/2*x-1/2*ln(cos(x))+1/6*ln(1+tan(x))-1/3*ln(1-tan(x)+tan(x)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3742, 2099, 649, 209, 266, 642}

$$\frac{x}{2}-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\frac{1}{6}\log(\tan(x)+1)-\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^3)^(-1),x]
```

```
[Out] x/2 - Log[Cos[x]]/2 + Log[1 + Tan[x]]/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\tan^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)\,(1+x^3)}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\,(1+x^2)}+\frac{1-2x}{3\,(1-x+x^2)}\right)\,dx, x, \tan(x)\right)\\
&= \frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\text{Subst}\left(\int \frac{1-2x}{1-x+x^2}\,dx, x, \tan(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx, x, \text{t}\right.\\
&= \frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)+\\
&= \frac{x}{2}-\frac{1}{2}\log(\cos(x))+\frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.16

$$\frac{1}{2}\text{ArcTan}(\tan(x))+\frac{1}{6}\log(1+\tan(x))+\frac{1}{4}\log\left(1+\tan^2(x)\right)-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^3)^(-1),x]
```

```
[Out] ArcTan[Tan[x]]/2 + Log[1 + Tan[x]]/6 + Log[1 + Tan[x]^2]/4 - Log[1 - Tan[x]
 + Tan[x]^2]/3
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.97

method	result	size

norman	$\frac{x}{2}+\frac{\ln(1+\tan(x))}{6}+\frac{\ln(1+\tan^2(x))}{4}-\frac{\ln(1-\tan(x)+\tan^2(x))}{3}$	34
derivativedivides	$-\frac{\ln(1-\tan(x)+\tan^2(x))}{3}+\frac{\ln(1+\tan^2(x))}{4}+\frac{\arctan(\tan(x))}{2}+\frac{\ln(1+\tan(x))}{6}$	36
default	$-\frac{\ln(1-\tan(x)+\tan^2(x))}{3}+\frac{\ln(1+\tan^2(x))}{4}+\frac{\arctan(\tan(x))}{2}+\frac{\ln(1+\tan(x))}{6}$	36
risch	$\frac{x}{2}+\frac{ix}{2}+\frac{\ln(e^{2ix}+i)}{6}-\frac{\ln(e^{4ix}-4ie^{2ix}-1)}{3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(1-tan(x)+tan(x)^2)+1/4*ln(1+tan(x)^2)+1/2*arctan(tan(x))+1/6*ln(1+t
an(x))
```

Maxima [A]
time = 0.50, size = 33, normalized size = 0.89

$$\frac{1}{2}x-\frac{1}{3}\log\left(\tan(x)^2-\tan(x)+1\right)+\frac{1}{4}\log\left(\tan(x)^2+1\right)+\frac{1}{6}\log\left(\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^3),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(ta
n(x) + 1)
```

Fricas [A]
time = 2.00, size = 48, normalized size = 1.30

$$\frac{1}{2}x+\frac{1}{12}\log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right)-\frac{1}{3}\log\left(\frac{\tan(x)^2-\tan(x)+1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2*x + 1/12*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/3*log((tan(x
)^2 - tan(x) + 1)/(tan(x)^2 + 1))
```

Sympy [A]
time = 0.06, size = 34, normalized size = 0.92

$$\frac{x}{2}+\frac{\log(\tan(x)+1)}{6}+\frac{\log(\tan^2(x)+1)}{4}-\frac{\log(\tan^2(x)-\tan(x)+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**3),x)
```

```
[Out] x/2 + log(tan(x) + 1)/6 + log(tan(x)**2 + 1)/4 - log(tan(x)**2 - tan(x) + 1
)/3
```

Giac [A]
time = 0.42, size = 34, normalized size = 0.92

$$\frac{1}{2}x - \frac{1}{3}\log\left(\tan\left(x\right)^2 - \tan\left(x\right) + 1\right) + \frac{1}{4}\log\left(\tan\left(x\right)^2 + 1\right) + \frac{1}{6}\log\left(\left|\tan\left(x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(ab
s(tan(x) + 1))
```

Mupad [B]
time = 11.62, size = 41, normalized size = 1.11

$$\frac{\ln\left(\tan(x) + 1\right)}{6} - \frac{\ln\left(\tan(x)^2 - \tan(x) + 1\right)}{3} + \ln\left(\tan(x) - \mathrm{i}\right)\left(\frac{1}{4} - \frac{1}{4}\mathrm{i}\right) + \ln\left(\tan(x) + 1\mathrm{i}\right)\left(\frac{1}{4} + \frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^3 + 1),x)
```

```
[Out] log(tan(x) + 1)/6 - log(tan(x)^2 - tan(x) + 1)/3 + log(tan(x) - 1i)*(1/4 -
1i/4) + log(tan(x) + 1i)*(1/4 + 1i/4)
```

3.381 $\int (a+b\tan^4(c+dx))^4\,dx$

Optimal. Leaf size=216

$$(a+b)^4x-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b^2(6a^2+4ab+$$

```
[Out] (a+b)^4*x-b*(2*a+b)*(2*a^2+2*a*b+b^2)*tan(d*x+c)/d+1/3*b*(2*a+b)*(2*a^2+2*a
*b+b^2)*tan(d*x+c)^3/d-1/5*b^2*(6*a^2+4*a*b+b^2)*tan(d*x+c)^5/d+1/7*b^2*(6*
a^2+4*a*b+b^2)*tan(d*x+c)^7/d-1/9*b^3*(4*a+b)*tan(d*x+c)^9/d+1/11*b^3*(4*a+
b)*tan(d*x+c)^11/d-1/13*b^4*tan(d*x+c)^13/d+1/15*b^4*tan(d*x+c)^15/d
```

Rubi [A]

time = 0.08, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 1168, 209}

$$\frac{b^2\left(6a^2+4ab+b^2\right)\tan^7(c+dx)}{7d}-\frac{b^2\left(6a^2+4ab+b^2\right)\tan^5(c+dx)}{5d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+dx)}{3d}-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^3(4a+b)\tan^{11}(c+dx)}{11d}-\frac{b^3(4a+b)\tan^9(c+dx)}{9d}+x(a+b)^4+\frac{b^4\tan^{15}(c+dx)}{15d}-\frac{b^4\tan^{13}(c+dx)}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^4,x]
```

```
[Out] (a + b)^4*x - (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x])/d + (b*(2*a
+ b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x]^3)/(3*d) - (b^2*(6*a^2 + 4*a*b + b^
2)*Tan[c + d*x]^5)/(5*d) + (b^2*(6*a^2 + 4*a*b + b^2)*Tan[c + d*x]^7)/(7*d)
 - (b^3*(4*a + b)*Tan[c + d*x]^9)/(9*d) + (b^3*(4*a + b)*Tan[c + d*x]^11)/(
11*d) - (b^4*Tan[c + d*x]^13)/(13*d) + (b^4*Tan[c + d*x]^15)/(15*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
```

```
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^4}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)\left(2a^2+2ab+b^2\right)+b(2a+b)\left(2a^2+2ab+b^2\right)x^2-b^2(6a\right.\right.}{}$$

$$= -\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+}{3d}$$

$$= (a+b)^4x-\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b(2a+b)\left(2a^2+2ab+b\right.}{3d}$$

Mathematica [A]
time = 3.21, size = 196, normalized size = 0.91

$$\frac{(a+b)^4\text{ArcTan}(\tan(c+dx))}{d}+\frac{b\tan(c+dx)\left(-45045\left(4a^3+6a^2b+4ab^2+b^3\right)+15015\left(4a^3+6a^2b+4ab^2+b^3\right)\tan^2(c+dx)-9009b\left(6a^2+4ab+b^2\right)\tan^4(c+dx)+6435b\left(6a^2+4ab+b^2\right)\tan^6(c+dx)-5005b^2(4a+b)\tan^8(c+dx)+4095b^2(4a+b)\tan^{10}(c+dx)-3465b^3\tan^{12}(c+dx)+3003b^3\tan^{14}(c+dx)\right)}{45045d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^4,x]
```

```
[Out] ((a + b)^4*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x]*(-45045*(4*a^3 + 6*a^2
*b + 4*a*b^2 + b^3) + 15015*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*Tan[c + d*x]^
2 - 9009*b*(6*a^2 + 4*a*b + b^2)*Tan[c + d*x]^4 + 6435*b*(6*a^2 + 4*a*b + b
^2)*Tan[c + d*x]^6 - 5005*b^2*(4*a + b)*Tan[c + d*x]^8 + 4095*b^2*(4*a + b)
*Tan[c + d*x]^10 - 3465*b^3*Tan[c + d*x]^12 + 3003*b^3*Tan[c + d*x]^14))/(4
5045*d)
```

Maple [A]
time = 0.19, size = 313, normalized size = 1.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/15*b^4*tan(d*x+c)^15-1/13*b^4*tan(d*x+c)^13+4/11*a*b^3*tan(d*x+c)^11
+1/11*b^4*tan(d*x+c)^11-4/9*a*b^3*tan(d*x+c)^9-1/9*b^4*tan(d*x+c)^9+6/7*a^2
*b^2*tan(d*x+c)^7+4/7*a*b^3*tan(d*x+c)^7+1/7*b^4*tan(d*x+c)^7-6/5*a^2*b^2*t
an(d*x+c)^5-4/5*a*b^3*tan(d*x+c)^5-1/5*b^4*tan(d*x+c)^5+4/3*a^3*b*tan(d*x+c
)^3+2*a^2*b^2*tan(d*x+c)^3+4/3*a*b^3*tan(d*x+c)^3+1/3*b^4*tan(d*x+c)^3-4*a^
3*b*tan(d*x+c)-6*a^2*b^2*tan(d*x+c)-4*a*b^3*tan(d*x+c)-b^4*tan(d*x+c)+(a^4+
4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 265, normalized size = 1.23

$$a^4x + \frac{4\left(\tan\left(dx+c\right)^3 + 3dx + 3c - 3\tan\left(dx+c\right)\right)a^3b}{3d} + \frac{2\left(15\tan\left(dx+c\right)^7 - 21\tan\left(dx+c\right)^5 + 35\tan\left(dx+c\right)^3 + 105dx + 105c - 105\tan\left(dx+c\right)\right)a^2b^2}{35d} + \frac{4\left(315\tan\left(dx+c\right)^{11} - 385\tan\left(dx+c\right)^9 + 495\tan\left(dx+c\right)^7 - 693\tan\left(dx+c\right)^5 + 1155\tan\left(dx+c\right)^3 + 3465dx + 3465c - 3465\tan\left(dx+c\right)\right)ab^3}{3465d} + \frac{\left(3003\tan\left(dx+c\right)^{15} - 3465\tan\left(dx+c\right)^{13} + 4095\tan\left(dx+c\right)^{11} - 5005\tan\left(dx+c\right)^9 + 6435\tan\left(dx+c\right)^7 - 9009\tan\left(dx+c\right)^5 + 15015\tan\left(dx+c\right)^3 + 45045dx + 45045c - 45045\tan\left(dx+c\right)\right)b^4}{45045d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 4/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3*b/d + 2/35*
(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*
c - 105*tan(d*x + c))*a^2*b^2/d + 4/3465*(315*tan(d*x + c)^11 - 385*tan(d*x
 + c)^9 + 495*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3
465*d*x + 3465*c - 3465*tan(d*x + c))*a*b^3/d + 1/45045*(3003*tan(d*x + c)^
15 - 3465*tan(d*x + c)^13 + 4095*tan(d*x + c)^11 - 5005*tan(d*x + c)^9 + 64
35*tan(d*x + c)^7 - 9009*tan(d*x + c)^5 + 15015*tan(d*x + c)^3 + 45045*d*x
+ 45045*c - 45045*tan(d*x + c))*b^4/d
```

Fricas [A]
time = 2.69, size = 225, normalized size = 1.04

$$\frac{3003b^4\tan\left(dx+c\right)^{15} - 3465b^4\tan\left(dx+c\right)^{13} + 4095\left(4ab^3+b^4\right)\tan\left(dx+c\right)^{11} - 5005\left(4ab^3+b^4\right)\tan\left(dx+c\right)^9 + 6435\left(6a^2b^2+4ab^3+b^4\right)\tan\left(dx+c\right)^7 - 9009\left(6a^2b^2+4ab^3+b^4\right)\tan\left(dx+c\right)^5 + 15015\left(4a^3b+6a^2b^2+4ab^3+b^4\right)\tan\left(dx+c\right)^3 + 45045\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)dx - 45045\left(4a^3b+6a^2b^2+4ab^3+b^4\right)\tan\left(dx+c\right)}{45045d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="fricas")
```

```
[Out] 1/45045*(3003*b^4*tan(d*x + c)^15 - 3465*b^4*tan(d*x + c)^13 + 4095*(4*a*b^
3 + b^4)*tan(d*x + c)^11 - 5005*(4*a*b^3 + b^4)*tan(d*x + c)^9 + 6435*(6*a^
2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^7 - 9009*(6*a^2*b^2 + 4*a*b^3 + b^4)*ta
n(d*x + c)^5 + 15015*(4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^3 +
 45045*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x - 45045*(4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c))/d
```

Sympy [A]
time = 0.97, size = 386, normalized size = 1.79

$$\begin{cases} a^4x + 4a^3bx + \frac{4a^3b\tan^3(c+dx)}{3d} - \frac{4a^3b\tan(c+dx)}{d} + 6a^2b^2x + \frac{6a^2b^2\tan^7(c+dx)}{7d} - \frac{6a^2b^2\tan^5(c+dx)}{5d} + \frac{2a^2b^2\tan^3(c+dx)}{d} - \frac{6a^2b^2\tan(c+dx)}{d} + 4ab^3x + \frac{4ab^3\tan^{11}(c+dx)}{11d} - \frac{4ab^3\tan^9(c+dx)}{9d} + \frac{4ab^3\tan^7(c+dx)}{7d} - \frac{4ab^3\tan^5(c+dx)}{5d} + \frac{4ab^3\tan^3(c+dx)}{3d} - \frac{4ab^3\tan(c+dx)}{d} + b^4x + \frac{b^4\tan^{15}(c+dx)}{15d} - \frac{b^4\tan^{13}(c+dx)}{13d} + \frac{b^4\tan^{11}(c+dx)}{11d} - \frac{b^4\tan^9(c+dx)}{9d} + \frac{b^4\tan^7(c+dx)}{7d} - \frac{b^4\tan^5(c+dx)}{5d} + \frac{b^4\tan^3(c+dx)}{3d} - \frac{b^4\tan(c+dx)}{d} & \text{for}\: d \neq 0 \\ x\left(a + b\tan^4(c)\right)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*x + 4*a**3*b*tan(c + d*x)**3/(3*d) - 4*a**3*b*
tan(c + d*x)/d + 6*a**2*b**2*x + 6*a**2*b**2*tan(c + d*x)**7/(7*d) - 6*a**2
*b**2*tan(c + d*x)**5/(5*d) + 2*a**2*b**2*tan(c + d*x)**3/d - 6*a**2*b**2*t
an(c + d*x)/d + 4*a*b**3*x + 4*a*b**3*tan(c + d*x)**11/(11*d) - 4*a*b**3*ta
n(c + d*x)**9/(9*d) + 4*a*b**3*tan(c + d*x)**7/(7*d) - 4*a*b**3*tan(c + d*x
```

```
)**5/(5*d) + 4*a*b**3*tan(c + d*x)**3/(3*d) - 4*a*b**3*tan(c + d*x)/d + b**
4*x + b**4*tan(c + d*x)**15/(15*d) - b**4*tan(c + d*x)**13/(13*d) + b**4*ta
n(c + d*x)**11/(11*d) - b**4*tan(c + d*x)**9/(9*d) + b**4*tan(c + d*x)**7/(
7*d) - b**4*tan(c + d*x)**5/(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c
 + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**4)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7741 vs. $2(202) = 404$.
time = 48.79, size = 7741, normalized size = 35.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="giac")
```

```
[Out] 1/45045*(45045*a^4*d*x*tan(d*x)^15*tan(c)^15 + 180180*a^3*b*d*x*tan(d*x)^15
*tan(c)^15 + 270270*a^2*b^2*d*x*tan(d*x)^15*tan(c)^15 + 180180*a*b^3*d*x*ta
n(d*x)^15*tan(c)^15 + 45045*b^4*d*x*tan(d*x)^15*tan(c)^15 - 675675*a^4*d*x*
tan(d*x)^14*tan(c)^14 - 2702700*a^3*b*d*x*tan(d*x)^14*tan(c)^14 - 4054050*a
^2*b^2*d*x*tan(d*x)^14*tan(c)^14 - 2702700*a*b^3*d*x*tan(d*x)^14*tan(c)^14
- 675675*b^4*d*x*tan(d*x)^14*tan(c)^14 + 180180*a^3*b*tan(d*x)^15*tan(c)^14
 + 270270*a^2*b^2*tan(d*x)^15*tan(c)^14 + 180180*a*b^3*tan(d*x)^15*tan(c)^1
4 + 45045*b^4*tan(d*x)^15*tan(c)^14 + 180180*a^3*b*tan(d*x)^14*tan(c)^15 +
270270*a^2*b^2*tan(d*x)^14*tan(c)^15 + 180180*a*b^3*tan(d*x)^14*tan(c)^15 +
 45045*b^4*tan(d*x)^14*tan(c)^15 + 4729725*a^4*d*x*tan(d*x)^13*tan(c)^13 +
18918900*a^3*b*d*x*tan(d*x)^13*tan(c)^13 + 28378350*a^2*b^2*d*x*tan(d*x)^13
*tan(c)^13 + 18918900*a*b^3*d*x*tan(d*x)^13*tan(c)^13 + 4729725*b^4*d*x*tan
(d*x)^13*tan(c)^13 - 60060*a^3*b*tan(d*x)^15*tan(c)^12 - 90090*a^2*b^2*tan(
d*x)^15*tan(c)^12 - 60060*a*b^3*tan(d*x)^15*tan(c)^12 - 15015*b^4*tan(d*x)^
15*tan(c)^12 - 2702700*a^3*b*tan(d*x)^14*tan(c)^13 - 4054050*a^2*b^2*tan(d*
x)^14*tan(c)^13 - 2702700*a*b^3*tan(d*x)^14*tan(c)^13 - 675675*b^4*tan(d*x)
^14*tan(c)^13 - 2702700*a^3*b*tan(d*x)^13*tan(c)^14 - 4054050*a^2*b^2*tan(d
*x)^13*tan(c)^14 - 2702700*a*b^3*tan(d*x)^13*tan(c)^14 - 675675*b^4*tan(d*x
)^13*tan(c)^14 - 60060*a^3*b*tan(d*x)^12*tan(c)^15 - 90090*a^2*b^2*tan(d*x)
^12*tan(c)^15 - 60060*a*b^3*tan(d*x)^12*tan(c)^15 - 15015*b^4*tan(d*x)^12*t
an(c)^15 - 20495475*a^4*d*x*tan(d*x)^12*tan(c)^12 - 81981900*a^3*b*d*x*tan(
d*x)^12*tan(c)^12 - 122972850*a^2*b^2*d*x*tan(d*x)^12*tan(c)^12 - 81981900*
a*b^3*d*x*tan(d*x)^12*tan(c)^12 - 20495475*b^4*d*x*tan(d*x)^12*tan(c)^12 +
54054*a^2*b^2*tan(d*x)^15*tan(c)^10 + 36036*a*b^3*tan(d*x)^15*tan(c)^10 + 9
009*b^4*tan(d*x)^15*tan(c)^10 + 720720*a^3*b*tan(d*x)^14*tan(c)^11 + 135135
0*a^2*b^2*tan(d*x)^14*tan(c)^11 + 900900*a*b^3*tan(d*x)^14*tan(c)^11 + 2252
25*b^4*tan(d*x)^14*tan(c)^11 + 18558540*a^3*b*tan(d*x)^13*tan(c)^12 + 28378
350*a^2*b^2*tan(d*x)^13*tan(c)^12 + 18918900*a*b^3*tan(d*x)^13*tan(c)^12 +
4729725*b^4*tan(d*x)^13*tan(c)^12 + 18558540*a^3*b*tan(d*x)^12*tan(c)^13 +
28378350*a^2*b^2*tan(d*x)^12*tan(c)^13 + 18918900*a*b^3*tan(d*x)^12*tan(c)^
```

```
13 + 4729725*b^4*tan(d*x)^12*tan(c)^13 + 720720*a^3*b*tan(d*x)^11*tan(c)^14
 + 1351350*a^2*b^2*tan(d*x)^11*tan(c)^14 + 900900*a*b^3*tan(d*x)^11*tan(c)^
14 + 225225*b^4*tan(d*x)^11*tan(c)^14 + 54054*a^2*b^2*tan(d*x)^10*tan(c)^15
 + 36036*a*b^3*tan(d*x)^10*tan(c)^15 + 9009*b^4*tan(d*x)^10*tan(c)^15 + 614
86425*a^4*d*x*tan(d*x)^11*tan(c)^11 + 245945700*a^3*b*d*x*tan(d*x)^11*tan(c
)^11 + 368918550*a^2*b^2*d*x*tan(d*x)^11*tan(c)^11 + 245945700*a*b^3*d*x*ta
n(d*x)^11*tan(c)^11 + 61486425*b^4*d*x*tan(d*x)^11*tan(c)^11 - 38610*a^2*b^
2*tan(d*x)^15*tan(c)^8 - 25740*a*b^3*tan(d*x)^15*tan(c)^8 - 6435*b^4*tan(d*
x)^15*tan(c)^8 - 810810*a^2*b^2*tan(d*x)^14*tan(c)^9 - 540540*a*b^3*tan(d*x
)^14*tan(c)^9 - 135135*b^4*tan(d*x)^14*tan(c)^9 - 3963960*a^3*b*tan(d*x)^13
*tan(c)^10 - 9459450*a^2*b^2*tan(d*x)^13*tan(c)^10 - 6306300*a*b^3*tan(d*x)
^13*tan(c)^10 - 1576575*b^4*tan(d*x)^13*tan(c)^10 - 77477400*a^3*b*tan(d*x)
^12*tan(c)^11 - 122972850*a^2*b^2*tan(d*x)^12*tan(c)^11 - 81981900*a*b^3*ta
n(d*x)^12*tan(c)^11 - 20495475*b^4*tan(d*x)^12*tan(c)^11 - 77477400*a^3*b*t
an(d*x)^11*tan(c)^12 - 122972850*a^2*b^2*tan(d*x)^11*tan(c)^12 - 81981900*a
*b^3*tan(d*x)^11*tan(c)^12 - 20495475*b^4*tan(d*x)^11*tan(c)^12 - 3963960*a
^3*b*tan(d*x)^10*tan(c)^13 - 9459450*a^2*b^2*tan(d*x)^10*tan(c)^13 - 630630
0*a*b^3*tan(d*x)^10*tan(c)^13 - 1576575*b^4*tan(d*x)^10*tan(c)^13 - 810810*
a^2*b^2*tan(d*x)^9*tan(c)^14 - 540540*a*b^3*tan(d*x)^9*tan(c)^14 - 135135*b
^4*tan(d*x)^9*tan(c)^14 - 38610*a^2*b^2*tan(d*x)^8*tan(c)^15 - 25740*a*b^3*
tan(d*x)^8*tan(c)^15 - 6435*b^4*tan(d*x)^8*tan(c)^15 - 135270135*a^4*d*x*ta
n(d*x)^10*tan(c)^10 - 541080540*a^3*b*d*x*tan(d*x)^10*tan(c)^10 - 811620810
*a^2*b^2*d*x*tan(d*x)^10*tan(c)^10 - 541080540*a*b^3*d*x*tan(d*x)^10*tan(c)
^10 - 135270135*b^4*d*x*tan(d*x)^10*tan(c)^10 + 20020*a*b^3*tan(d*x)^15*tan
(c)^6 + 5005*b^4*tan(d*x)^15*tan(c)^6 + 308880*a^2*b^2*tan(d*x)^14*tan(c)^7
 + 386100*a*b^3*tan(d*x)^14*tan(c)^7 + 96525*b^4*tan(d*x)^14*tan(c)^7 + 459
4590*a^2*b^2*tan(d*x)^13*tan(c)^8 + 3783780*a*b^3*tan(d*x)^13*tan(c)^8 + 94
5945*b^4*tan(d*x)^13*tan(c)^8 + 13213200*a^3*b*tan(d*x)^12*tan(c)^9 + 38468
430*a^2*b^2*tan(d*x)^12*tan(c)^9 + 27327300*a*b^3*tan(d*x)^12*tan(c)^9 + 68
31825*b^4*tan(d*x)^12*tan(c)^9 + 219999780*a^3*b*tan(d*x)^11*tan(c)^10 + 36
5134770*a^2*b^2*tan(d*x)^11*tan(c)^10 + 245945700*a*b^3*tan(d*x)^11*tan(c)^
10 + 61486425*b^4*tan(d*x)^11*tan(c)^10 + 219999780*a^3*b*tan(d*x)^10*tan(c
)^11 + 365134770*a^2*b^2*tan(d*x)^10*tan(c)^11 + 245945700*a*b^3*tan(d*x)^1
0*tan(c)^11 + 61486425*b^4*tan(d*x)^10*tan(c)^11 + 13213200*a^3*b*tan(d*x)^
9*tan(c)^12 + 38468430*a^2*b^2*tan(d*x)^9*tan(c)^12 + 27327300*a*b^3*tan(d*
x)^9*tan(c)^12 + 6831825*b^4*tan(d*x)^9*tan(c)^12 + 4594590*a^2*b^2*tan(d*x
)^8*tan(c)^13 + 3783780*a*b^3*tan(d*x)^8*tan(c)...
```

Mupad [B]

time = 11.59, size = 271, normalized size = 1.25

$$\frac{\tan(c+d\,x)^3\,\left(\frac{4\,a^3\,b}{3}+2\,a^2\,b^2+\frac{4\,a\,b^3}{3}+\frac{b^4}{3}\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,{(a+b)}^4}{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}\right)\,{(a+b)}^4}{d}-\frac{\tan(c+d\,x)\,\left(4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)}{d}-\frac{b^4\,\tan(c+d\,x)^{13}}{13\,d}+\frac{b^4\,\tan(c+d\,x)^{15}}{15\,d}-\frac{\tan(c+d\,x)^5\,\left(\frac{6\,a^2\,b^2}{5}+\frac{4\,a\,b^3}{5}+\frac{b^4}{5}\right)}{d}+\frac{\tan(c+d\,x)^7\,\left(\frac{6\,a^2\,b^2}{7}+\frac{4\,a\,b^3}{7}+\frac{b^4}{7}\right)}{d}-\frac{\tan(c+d\,x)^9\,\left(\frac{b^4}{9}+\frac{4\,a\,b^3}{9}\right)}{d}+\frac{\tan(c+d\,x)^{11}\,\left(\frac{b^4}{11}+\frac{4\,a\,b^3}{11}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^4)^4,x)
```

```
[Out] (tan(c + d*x)^3*((4*a*b^3)/3 + (4*a^3*b)/3 + b^4/3 + 2*a^2*b^2))/d + (atan(
(tan(c + d*x)*(a + b)^4)/(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))*(a +
b)^4)/d - (tan(c + d*x)*(4*a*b^3 + 4*a^3*b + b^4 + 6*a^2*b^2))/d - (b^4*tan
(c + d*x)^13)/(13*d) + (b^4*tan(c + d*x)^15)/(15*d) - (tan(c + d*x)^5*((4*a
*b^3)/5 + b^4/5 + (6*a^2*b^2)/5))/d + (tan(c + d*x)^7*((4*a*b^3)/7 + b^4/7
+ (6*a^2*b^2)/7))/d - (tan(c + d*x)^9*((4*a*b^3)/9 + b^4/9))/d + (tan(c + d
*x)^11*((4*a*b^3)/11 + b^4/11))/d
```

3.382 $\int (a+b\tan^4(c+dx))^3 \, dx$

Optimal. Leaf size=144

$$(a+b)^3x-\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b(3a^2+3ab+b^2)\tan^3(c+dx)}{3d}-\frac{b^2(3a+b)\tan^5(c+dx)}{5d}+\frac{b^2(3a+$$

```
[Out] (a+b)^3*x-b*(3*a^2+3*a*b+b^2)*tan(d*x+c)/d+1/3*b*(3*a^2+3*a*b+b^2)*tan(d*x+
c)^3/d-1/5*b^2*(3*a+b)*tan(d*x+c)^5/d+1/7*b^2*(3*a+b)*tan(d*x+c)^7/d-1/9*b^
3*tan(d*x+c)^9/d+1/11*b^3*tan(d*x+c)^11/d
```

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3742, 1168, 209}

$$\frac{b(3a^2+3ab+b^2)\tan^3(c+dx)}{3d}-\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b^2(3a+b)\tan^7(c+dx)}{7d}-\frac{b^2(3a+b)\tan^5(c+dx)}{5d}+x(a+b)^3+\frac{b^3\tan^{11}(c+dx)}{11d}-\frac{b^3\tan^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^3,x]
```

```
[Out] (a + b)^3*x - (b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x])/d + (b*(3*a^2 + 3*a*b
+ b^2)*Tan[c + d*x]^3)/(3*d) - (b^2*(3*a + b)*Tan[c + d*x]^5)/(5*d) + (b^2*
(3*a + b)*Tan[c + d*x]^7)/(7*d) - (b^3*Tan[c + d*x]^9)/(9*d) + (b^3*Tan[c +
 d*x]^11)/(11*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^3}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-b(3a^2+3ab+b^2)+b(3a^2+3ab+b^2)\,x^2-b^2(3a+b)x^4+b^2(3a\right.\right.}{d}$$

$$= -\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b(3a^2+3ab+b^2)\tan^3(c+dx)}{3d}-\frac{b^2(3a+b}{}$$

$$= (a+b)^3x-\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}+\frac{b(3a^2+3ab+b^2)\tan^3(c+dx)}{3d}-$$

Mathematica [A]

time = 0.67, size = 128, normalized size = 0.89

$$\frac{(a+b)^3\text{ArcTan}(\tan(c+dx))}{d}+\frac{b\tan(c+dx)\left(-3465(3a^2+3ab+b^2)+1155(3a^2+3ab+b^2)\tan^2(c+dx)-693b(3a+b)\tan^4(c+dx)+495b(3a+b)\tan^6(c+dx)-385b^2\tan^8(c+dx)+315b^2\tan^{10}(c+dx)\right)}{3465d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x]*(-3465*(3*a^2 + 3*a*b
+ b^2) + 1155*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x]^2 - 693*b*(3*a + b)*Tan[c
+ d*x]^4 + 495*b*(3*a + b)*Tan[c + d*x]^6 - 385*b^2*Tan[c + d*x]^8 + 315*b^
2*Tan[c + d*x]^10))/(3465*d)
```

Maple [A]

time = 0.12, size = 187, normalized size = 1.30

method	result
norman	$(a^3+3a^2b+3a\,b^2+b^3)\,x-\frac{b^3(\tan^9(dx+c))}{9d}+\frac{b^3(\tan^{11}(dx+c))}{11d}-\frac{b(3a^2+3ab+b^2)\tan(dx+c)}{d}+\frac{b(3a^2+}{}$
derivativedivides	$\frac{b^3\left(\tan^{11}(dx+c)\right)}{11}-\frac{b^3\left(\tan^9(dx+c)\right)}{9}+\frac{3a\,b^2\left(\tan^7(dx+c)\right)}{7}+\frac{b^3\left(\tan^7(dx+c)\right)}{7}-\frac{3a\,b^2\left(\tan^5(dx+c)\right)}{5}-\frac{b^3\left(\tan^5(dx+c)\right)}{5}+a^2b(\tan^3(d$
default	$\frac{b^3\left(\tan^{11}(dx+c)\right)}{11}-\frac{b^3\left(\tan^9(dx+c)\right)}{9}+\frac{3a\,b^2\left(\tan^7(dx+c)\right)}{7}+\frac{b^3\left(\tan^7(dx+c)\right)}{7}-\frac{3a\,b^2\left(\tan^5(dx+c)\right)}{5}-\frac{b^3\left(\tan^5(dx+c)\right)}{5}+a^2b(\tan^3(d$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x-\frac{4ib(751674b^2\mathrm{e}^{12i(dx+c)}+751674b^2\mathrm{e}^{10i(dx+c)}+1358280a^2\mathrm{e}^{8i(dx+c)}+6930a^2+3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/11*b^3*tan(d*x+c)^11-1/9*b^3*tan(d*x+c)^9+3/7*a*b^2*tan(d*x+c)^7+1/7
*b^3*tan(d*x+c)^7-3/5*a*b^2*tan(d*x+c)^5-1/5*b^3*tan(d*x+c)^5+a^2*b*tan(d*x
```

```
+c)^3+a*b^2*tan(d*x+c)^3+1/3*b^3*tan(d*x+c)^3-3*a^2*b*tan(d*x+c)-3*a*b^2*ta
n(d*x+c)-b^3*tan(d*x+c)+(a^3+3*a^2*b+3*a*b^2+b^3)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 167, normalized size = 1.16

$$a^3x+\frac{\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2b}{d}+\frac{\left(15\,\tan\left(dx+c\right)^7-21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+105\,dx+105\,c-105\,\tan\left(dx+c\right)\right)ab^2}{35\,d}+\frac{\left(315\,\tan\left(dx+c\right)^{11}-385\,\tan\left(dx+c\right)^9+495\,\tan\left(dx+c\right)^7-693\,\tan\left(dx+c\right)^5+1155\,\tan\left(dx+c\right)^3+3465\,dx+3465\,c-3465\,\tan\left(dx+c\right)\right)b^3}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2*b/d + 1/35*(15*
tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c -
105*tan(d*x + c))*a*b^2/d + 1/3465*(315*tan(d*x + c)^11 - 385*tan(d*x + c)^
9 + 495*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3465*d*
x + 3465*c - 3465*tan(d*x + c))*b^3/d
```

Fricas [A]
time = 3.67, size = 145, normalized size = 1.01

$$\frac{315\,b^3\tan\left(dx+c\right)^{11}-385\,b^3\tan\left(dx+c\right)^9+495\,\left(3\,ab^2+b^3\right)\tan\left(dx+c\right)^7-693\,\left(3\,ab^2+b^3\right)\tan\left(dx+c\right)^5+1155\,\left(3\,a^2b+3\,ab^2+b^3\right)\tan\left(dx+c\right)^3+3465\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)dx-3465\,\left(3\,a^2b+3\,ab^2+b^3\right)\tan\left(dx+c\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^3*tan(d*x + c)^11 - 385*b^3*tan(d*x + c)^9 + 495*(3*a*b^2 + b
^3)*tan(d*x + c)^7 - 693*(3*a*b^2 + b^3)*tan(d*x + c)^5 + 1155*(3*a^2*b + 3
*a*b^2 + b^3)*tan(d*x + c)^3 + 3465*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 3
465*(3*a^2*b + 3*a*b^2 + b^3)*tan(d*x + c))/d
```

Sympy [A]
time = 0.50, size = 224, normalized size = 1.56

$$\begin{cases} a^3x+3a^2bx+\frac{a^2b\tan^3\left(c+dx\right)}{d}-\frac{3a^2b\tan\left(c+dx\right)}{d}+3ab^2x+\frac{3ab^2\tan^7\left(c+dx\right)}{7d}-\frac{3ab^2\tan^5\left(c+dx\right)}{5d}+\frac{ab^2\tan^3\left(c+dx\right)}{d}-\frac{3ab^2\tan\left(c+dx\right)}{d}+b^3x+\frac{b^3\tan^{11}\left(c+dx\right)}{11d}-\frac{b^3\tan^9\left(c+dx\right)}{9d}+\frac{b^3\tan^7\left(c+dx\right)}{7d}-\frac{b^3\tan^5\left(c+dx\right)}{5d}+\frac{b^3\tan^3\left(c+dx\right)}{3d}-\frac{b^3\tan\left(c+dx\right)}{d} & \text{for}\: d\neq 0\\ x\left(a+b\tan^4\left(c\right)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x + a**2*b*tan(c + d*x)**3/d - 3*a**2*b*tan(c
+ d*x)/d + 3*a*b**2*x + 3*a*b**2*tan(c + d*x)**7/(7*d) - 3*a*b**2*tan(c + d
*x)**5/(5*d) + a*b**2*tan(c + d*x)**3/d - 3*a*b**2*tan(c + d*x)/d + b**3*x
+ b**3*tan(c + d*x)**11/(11*d) - b**3*tan(c + d*x)**9/(9*d) + b**3*tan(c +
d*x)**7/(7*d) - b**3*tan(c + d*x)**5/(5*d) + b**3*tan(c + d*x)**3/(3*d) - b
**3*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**4)**3, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3499 vs. $2(134) = 268$.
time = 15.36, size = 3499, normalized size = 24.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="giac")
```

```
[Out] 1/3465*(3465*a^3*d*x*tan(d*x)^11*tan(c)^11 + 10395*a^2*b*d*x*tan(d*x)^11*ta
n(c)^11 + 10395*a*b^2*d*x*tan(d*x)^11*tan(c)^11 + 3465*b^3*d*x*tan(d*x)^11*
tan(c)^11 - 38115*a^3*d*x*tan(d*x)^10*tan(c)^10 - 114345*a^2*b*d*x*tan(d*x)
^10*tan(c)^10 - 114345*a*b^2*d*x*tan(d*x)^10*tan(c)^10 - 38115*b^3*d*x*tan(
d*x)^10*tan(c)^10 + 10395*a^2*b*tan(d*x)^11*tan(c)^10 + 10395*a*b^2*tan(d*x
)^11*tan(c)^10 + 3465*b^3*tan(d*x)^11*tan(c)^10 + 10395*a^2*b*tan(d*x)^10*t
an(c)^11 + 10395*a*b^2*tan(d*x)^10*tan(c)^11 + 3465*b^3*tan(d*x)^10*tan(c)^
11 + 190575*a^3*d*x*tan(d*x)^9*tan(c)^9 + 571725*a^2*b*d*x*tan(d*x)^9*tan(c
)^9 + 571725*a*b^2*d*x*tan(d*x)^9*tan(c)^9 + 190575*b^3*d*x*tan(d*x)^9*tan(
c)^9 - 3465*a^2*b*tan(d*x)^11*tan(c)^8 - 3465*a*b^2*tan(d*x)^11*tan(c)^8 -
1155*b^3*tan(d*x)^11*tan(c)^8 - 114345*a^2*b*tan(d*x)^10*tan(c)^9 - 114345*
a*b^2*tan(d*x)^10*tan(c)^9 - 38115*b^3*tan(d*x)^10*tan(c)^9 - 114345*a^2*b*
tan(d*x)^9*tan(c)^10 - 114345*a*b^2*tan(d*x)^9*tan(c)^10 - 38115*b^3*tan(d*
x)^9*tan(c)^10 - 3465*a^2*b*tan(d*x)^8*tan(c)^11 - 3465*a*b^2*tan(d*x)^8*ta
n(c)^11 - 1155*b^3*tan(d*x)^8*tan(c)^11 - 571725*a^3*d*x*tan(d*x)^8*tan(c)^
8 - 1715175*a^2*b*d*x*tan(d*x)^8*tan(c)^8 - 1715175*a*b^2*d*x*tan(d*x)^8*ta
n(c)^8 - 571725*b^3*d*x*tan(d*x)^8*tan(c)^8 + 2079*a*b^2*tan(d*x)^11*tan(c)
^6 + 693*b^3*tan(d*x)^11*tan(c)^6 + 27720*a^2*b*tan(d*x)^10*tan(c)^7 + 3811
5*a*b^2*tan(d*x)^10*tan(c)^7 + 12705*b^3*tan(d*x)^10*tan(c)^7 + 550935*a^2*
b*tan(d*x)^9*tan(c)^8 + 571725*a*b^2*tan(d*x)^9*tan(c)^8 + 190575*b^3*tan(d
*x)^9*tan(c)^8 + 550935*a^2*b*tan(d*x)^8*tan(c)^9 + 571725*a*b^2*tan(d*x)^8
*tan(c)^9 + 190575*b^3*tan(d*x)^8*tan(c)^9 + 27720*a^2*b*tan(d*x)^7*tan(c)^
10 + 38115*a*b^2*tan(d*x)^7*tan(c)^10 + 12705*b^3*tan(d*x)^7*tan(c)^10 + 20
79*a*b^2*tan(d*x)^6*tan(c)^11 + 693*b^3*tan(d*x)^6*tan(c)^11 + 1143450*a^3*
d*x*tan(d*x)^7*tan(c)^7 + 3430350*a^2*b*d*x*tan(d*x)^7*tan(c)^7 + 3430350*a
*b^2*d*x*tan(d*x)^7*tan(c)^7 + 1143450*b^3*d*x*tan(d*x)^7*tan(c)^7 - 1485*a
*b^2*tan(d*x)^11*tan(c)^4 - 495*b^3*tan(d*x)^11*tan(c)^4 - 22869*a*b^2*tan(
d*x)^10*tan(c)^5 - 7623*b^3*tan(d*x)^10*tan(c)^5 - 97020*a^2*b*tan(d*x)^9*t
an(c)^6 - 190575*a*b^2*tan(d*x)^9*tan(c)^6 - 63525*b^3*tan(d*x)^9*tan(c)^6
- 1538460*a^2*b*tan(d*x)^8*tan(c)^7 - 1715175*a*b^2*tan(d*x)^8*tan(c)^7 - 5
71725*b^3*tan(d*x)^8*tan(c)^7 - 1538460*a^2*b*tan(d*x)^7*tan(c)^8 - 1715175
*a*b^2*tan(d*x)^7*tan(c)^8 - 571725*b^3*tan(d*x)^7*tan(c)^8 - 97020*a^2*b*t
an(d*x)^6*tan(c)^9 - 190575*a*b^2*tan(d*x)^6*tan(c)^9 - 63525*b^3*tan(d*x)^
6*tan(c)^9 - 22869*a*b^2*tan(d*x)^5*tan(c)^10 - 7623*b^3*tan(d*x)^5*tan(c)^
10 - 1485*a*b^2*tan(d*x)^4*tan(c)^11 - 495*b^3*tan(d*x)^4*tan(c)^11 - 16008
30*a^3*d*x*tan(d*x)^6*tan(c)^6 - 4802490*a^2*b*d*x*tan(d*x)^6*tan(c)^6 - 48
```

```
02490*a*b^2*d*x*tan(d*x)^6*tan(c)^6 - 1600830*b^3*d*x*tan(d*x)^6*tan(c)^6 +
 385*b^3*tan(d*x)^11*tan(c)^2 + 5940*a*b^2*tan(d*x)^10*tan(c)^3 + 5445*b^3*
tan(d*x)^10*tan(c)^3 + 72765*a*b^2*tan(d*x)^9*tan(c)^4 + 38115*b^3*tan(d*x)
^9*tan(c)^4 + 194040*a^2*b*tan(d*x)^8*tan(c)^5 + 474705*a*b^2*tan(d*x)^8*ta
n(c)^5 + 190575*b^3*tan(d*x)^8*tan(c)^5 + 2765070*a^2*b*tan(d*x)^7*tan(c)^6
 + 3284820*a*b^2*tan(d*x)^7*tan(c)^6 + 1143450*b^3*tan(d*x)^7*tan(c)^6 + 27
65070*a^2*b*tan(d*x)^6*tan(c)^7 + 3284820*a*b^2*tan(d*x)^6*tan(c)^7 + 11434
50*b^3*tan(d*x)^6*tan(c)^7 + 194040*a^2*b*tan(d*x)^5*tan(c)^8 + 474705*a*b^
2*tan(d*x)^5*tan(c)^8 + 190575*b^3*tan(d*x)^5*tan(c)^8 + 72765*a*b^2*tan(d*
x)^4*tan(c)^9 + 38115*b^3*tan(d*x)^4*tan(c)^9 + 5940*a*b^2*tan(d*x)^3*tan(c
)^10 + 5445*b^3*tan(d*x)^3*tan(c)^10 + 385*b^3*tan(d*x)^2*tan(c)^11 + 16008
30*a^3*d*x*tan(d*x)^5*tan(c)^5 + 4802490*a^2*b*d*x*tan(d*x)^5*tan(c)^5 + 48
02490*a*b^2*d*x*tan(d*x)^5*tan(c)^5 + 1600830*b^3*d*x*tan(d*x)^5*tan(c)^5 -
 315*b^3*tan(d*x)^11 - 4235*b^3*tan(d*x)^10*tan(c) - 8910*a*b^2*tan(d*x)^9*
tan(c)^2 - 27225*b^3*tan(d*x)^9*tan(c)^2 - 103950*a*b^2*tan(d*x)^8*tan(c)^3
 - 114345*b^3*tan(d*x)^8*tan(c)^3 - 242550*a^2*b*tan(d*x)^7*tan(c)^4 - 6375
60*a*b^2*tan(d*x)^7*tan(c)^4 - 381150*b^3*tan(d*x)^7*tan(c)^4 - 3347190*a^2
*b*tan(d*x)^6*tan(c)^5 - 4074840*a*b^2*tan(d*x)^6*tan(c)^5 - 1600830*b^3*ta
n(d*x)^6*tan(c)^5 - 3347190*a^2*b*tan(d*x)^5*tan(c)^6 - 4074840*a*b^2*tan(d
*x)^5*tan(c)^6 - 1600830*b^3*tan(d*x)^5*tan(c)^6 - 242550*a^2*b*tan(d*x)^4*
tan(c)^7 - 637560*a*b^2*tan(d*x)^4*tan(c)^7 - 381150*b^3*tan(d*x)^4*tan(c)^
7 - 103950*a*b^2*tan(d*x)^3*tan(c)^8 - 114345*b^3*tan(d*x)^3*tan(c)^8 - 891
0*a*b^2*tan(d*x)^2*tan(c)^9 - 27225*b^3*tan(d*x)^2*tan(c)^9 - 4235*b^3*tan(
d*x)*tan(c)^10 - 315*b^3*tan(c)^11 - 1143450*a^3*d*x*tan(d*x)^4*tan(c)^4 -
3430350*a^2*b*d*x*tan(d*x)^4*tan(c)^4 - 3430350*a*b^2*d*x*tan(d*x)^4*tan(c)
^4 - 1143450*b^3*d*x*tan(d*x)^4*tan(c)^4 + 385*b^3*tan(d*x)^9 + 5940*a*b^2*
tan(d*x)^8*tan(c) + 5445*b^3*tan(d*x)^8*tan(c) + 72765*a*b^2*tan(d*x)^7*tan
(c)^2 + 38115*b^3*tan(d*x)^7*tan(c)^2 + 194040*a^2*b*tan(d*x)^6*tan(c)^3 +
474705*a*b^2*tan(d*x)^6*tan(c)^3 + 190575*b^3*tan(d*x)^6*tan(c)^3 + 2765070
*a^2*b*tan(d*x)^5*tan(c)^4 + 3284820*a*b^2*tan(d*x)^5*tan(c)^4 + 1143450*b^
3*tan(d*x)^5*tan(c)^4 + 2765070*a^2*b*tan(d*x)^...
```

Mupad [B]

time = 11.66, size = 180, normalized size = 1.25

$$\frac{\tan(c+d\,x)^3\,\left(a^2\,b+a\,b^2+\frac{b^3}{3}\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)^3}{a^3+3\,a^2\,b+3\,a\,b^2+b^3}\right)\,(a+b)^3}{d}-\frac{b^3\tan(c+d\,x)^9}{9\,d}+\frac{b^3\tan(c+d\,x)^{11}}{11\,d}-\frac{\tan(c+d\,x)\,(3\,a^2\,b+3\,a\,b^2+b^3)}{d}-\frac{\tan(c+d\,x)^5\,\left(\frac{b^3}{5}+\frac{3\,a\,b^2}{5}\right)}{d}+\frac{\tan(c+d\,x)^7\,\left(\frac{b^3}{7}+\frac{3\,a\,b^2}{7}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^4)^3,x)
```

```
[Out] (tan(c + d*x)^3*(a*b^2 + a^2*b + b^3/3))/d + (atan((tan(c + d*x)*(a + b)^3)
/(3*a*b^2 + 3*a^2*b + a^3 + b^3))*(a + b)^3)/d - (b^3*tan(c + d*x)^9)/(9*d)
 + (b^3*tan(c + d*x)^11)/(11*d) - (tan(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/
d - (tan(c + d*x)^5*((3*a*b^2)/5 + b^3/5))/d + (tan(c + d*x)^7*((3*a*b^2)/7
 + b^3/7))/d
```

3.383 $\int (a+b\tan^4(c+dx))^2\,dx$

Optimal. Leaf size=82

$$(a+b)^2x-\frac{b(2a+b)\tan(c+dx)}{d}+\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b^2\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}$$

```
[Out] (a+b)^2*x-b*(2*a+b)*tan(d*x+c)/d+1/3*b*(2*a+b)*tan(d*x+c)^3/d-1/5*b^2*tan(d
*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d
```

Rubi [A]

time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3742, 1168, 209}

$$\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b(2a+b)\tan(c+dx)}{d}+x(a+b)^2+\frac{b^2\tan^7(c+dx)}{7d}-\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^2,x]
```

```
[Out] (a + b)^2*x - (b*(2*a + b)*Tan[c + d*x])/d + (b*(2*a + b)*Tan[c + d*x]^3)/(
3*d) - (b^2*Tan[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \left(a+b\tan^4(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^2}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-b(2a+b)+b(2a+b)x^2-b^2x^4+b^2x^6+\frac{(a+b)^2}{1+x^2}\right)\,dx,x,\tan(c+d$$
$$= -\frac{b(2a+b)\tan(c+dx)}{d}+\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b^2\tan^5(c+dx)}{5d}+\frac{b^2\tan$$
$$= (a+b)^2x-\frac{b(2a+b)\tan(c+dx)}{d}+\frac{b(2a+b)\tan^3(c+dx)}{3d}-\frac{b^2\tan^5(c+dx}{5d}$$

Mathematica [A]
time = 0.38, size = 75, normalized size = 0.91

$$\frac{105(a+b)^2\text{ArcTan}(\tan(c+dx))+b\tan(c+dx)\left(-105(2a+b)+35(2a+b)\tan^2(c+dx)-21b\tan^4(c+dx)+15b\tan^6(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^2,x]
```

```
[Out] (105*(a + b)^2*ArcTan[Tan[c + d*x]] + b*Tan[c + d*x]*(-105*(2*a + b) + 35*(
2*a + b)*Tan[c + d*x]^2 - 21*b*Tan[c + d*x]^4 + 15*b*Tan[c + d*x]^6))/(105*
d)
```

Maple [A]
time = 0.07, size = 97, normalized size = 1.18

method	result
norman	$\left(a^2+2ab+b^2\right)x-\frac{b^2\left(\tan^5(dx+c)\right)}{5d}+\frac{b^2\left(\tan^7(dx+c)\right)}{7d}-\frac{b(2a+b)\tan(dx+c)}{d}+\frac{b(2a+b)\left(\tan^3(dx+c)\right)}{3d}$
derivativedivides	$\frac{\frac{b^2\left(\tan^7(dx+c)\right)}{7}-\frac{b^2\left(\tan^5(dx+c)\right)}{5}+\frac{2ab\left(\tan^3(dx+c)\right)}{3}+\frac{b^2\left(\tan^3(dx+c)\right)}{3}-2ab\tan(dx+c)-b^2\tan(dx+c)+\left(a^2+2ab+b^2\right)\text{arc}}{d}$
default	$\frac{\frac{b^2\left(\tan^7(dx+c)\right)}{7}-\frac{b^2\left(\tan^5(dx+c)\right)}{5}+\frac{2ab\left(\tan^3(dx+c)\right)}{3}+\frac{b^2\left(\tan^3(dx+c)\right)}{3}-2ab\tan(dx+c)-b^2\tan(dx+c)+\left(a^2+2ab+b^2\right)\text{arc}}{d}$
risch	$x\,a^2+2xab+x\,b^2-\frac{8ib\left(105a\,e^{12i(dx+c)}+105b\,e^{12i(dx+c)}+525a\,e^{10i(dx+c)}+315b\,e^{10i(dx+c)}+1120a\,e^{8i(dx+c)}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*b^2*tan(d*x+c)^7-1/5*b^2*tan(d*x+c)^5+2/3*a*b*tan(d*x+c)^3+1/3*b^2
*tan(d*x+c)^3-2*a*b*tan(d*x+c)-b^2*tan(d*x+c)+(a^2+2*a*b+b^2)*arctan(tan(d*
x+c)))
```

Maxima [A]
time = 0.52, size = 91, normalized size = 1.11

$$a^2x+\frac{2\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)ab}{3\,d}+\frac{\left(15\,\tan\left(dx+c\right)^7-21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+105\,dx+105\,c-105\,\tan\left(dx+c\right)\right)b^2}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a*b/d + 1/105*(
15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c
 - 105*tan(d*x + c))*b^2/d
```

Fricas [A]
time = 2.31, size = 81, normalized size = 0.99

$$\frac{15\,b^2\tan\left(dx+c\right)^7-21\,b^2\tan\left(dx+c\right)^5+35\left(2\,ab+b^2\right)\tan\left(dx+c\right)^3+105\left(a^2+2\,ab+b^2\right)dx-105\left(2\,ab+b^2\right)\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 - 21*b^2*tan(d*x + c)^5 + 35*(2*a*b + b^2)*tan
(d*x + c)^3 + 105*(a^2 + 2*a*b + b^2)*d*x - 105*(2*a*b + b^2)*tan(d*x + c))
/d
```

Sympy [A]
time = 0.24, size = 116, normalized size = 1.41

$$\begin{cases} a^2x+2abx+\frac{2ab\tan^3\left(c+dx\right)}{3d}-\frac{2ab\tan\left(c+dx\right)}{d}+b^2x+\frac{b^2\tan^7\left(c+dx\right)}{7d}-\frac{b^2\tan^5\left(c+dx\right)}{5d}+\frac{b^2\tan^3\left(c+dx\right)}{3d}-\frac{b^2\tan\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x\left(a+b\tan^4\left(c\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x + 2*a*b*tan(c + d*x)**3/(3*d) - 2*a*b*tan(c + d
*x)/d + b**2*x + b**2*tan(c + d*x)**7/(7*d) - b**2*tan(c + d*x)**5/(5*d) +
b**2*tan(c + d*x)**3/(3*d) - b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(
c)**4)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. $2(76) = 152$.
time = 1.85, size = 1181, normalized size = 14.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^2,x, algorithm="giac")
```

```
[Out] 1/105*(105*a^2*d*x*tan(d*x)^7*tan(c)^7 + 210*a*b*d*x*tan(d*x)^7*tan(c)^7 +
105*b^2*d*x*tan(d*x)^7*tan(c)^7 - 735*a^2*d*x*tan(d*x)^6*tan(c)^6 - 1470*a*
b*d*x*tan(d*x)^6*tan(c)^6 - 735*b^2*d*x*tan(d*x)^6*tan(c)^6 + 210*a*b*tan(d
*x)^7*tan(c)^6 + 105*b^2*tan(d*x)^7*tan(c)^6 + 210*a*b*tan(d*x)^6*tan(c)^7
+ 105*b^2*tan(d*x)^6*tan(c)^7 + 2205*a^2*d*x*tan(d*x)^5*tan(c)^5 + 4410*a*b
*d*x*tan(d*x)^5*tan(c)^5 + 2205*b^2*d*x*tan(d*x)^5*tan(c)^5 - 70*a*b*tan(d*
x)^7*tan(c)^4 - 35*b^2*tan(d*x)^7*tan(c)^4 - 1470*a*b*tan(d*x)^6*tan(c)^5 -
 735*b^2*tan(d*x)^6*tan(c)^5 - 1470*a*b*tan(d*x)^5*tan(c)^6 - 735*b^2*tan(d
*x)^5*tan(c)^6 - 70*a*b*tan(d*x)^4*tan(c)^7 - 35*b^2*tan(d*x)^4*tan(c)^7 -
3675*a^2*d*x*tan(d*x)^4*tan(c)^4 - 7350*a*b*d*x*tan(d*x)^4*tan(c)^4 - 3675*
b^2*d*x*tan(d*x)^4*tan(c)^4 + 21*b^2*tan(d*x)^7*tan(c)^2 + 280*a*b*tan(d*x)
^6*tan(c)^3 + 245*b^2*tan(d*x)^6*tan(c)^3 + 3990*a*b*tan(d*x)^5*tan(c)^4 +
2205*b^2*tan(d*x)^5*tan(c)^4 + 3990*a*b*tan(d*x)^4*tan(c)^5 + 2205*b^2*tan(
d*x)^4*tan(c)^5 + 280*a*b*tan(d*x)^3*tan(c)^6 + 245*b^2*tan(d*x)^3*tan(c)^6
 + 21*b^2*tan(d*x)^2*tan(c)^7 + 3675*a^2*d*x*tan(d*x)^3*tan(c)^3 + 7350*a*b
*d*x*tan(d*x)^3*tan(c)^3 + 3675*b^2*d*x*tan(d*x)^3*tan(c)^3 - 15*b^2*tan(d*
x)^7 - 147*b^2*tan(d*x)^6*tan(c) - 420*a*b*tan(d*x)^5*tan(c)^2 - 735*b^2*ta
n(d*x)^5*tan(c)^2 - 5460*a*b*tan(d*x)^4*tan(c)^3 - 3675*b^2*tan(d*x)^4*tan(
c)^3 - 5460*a*b*tan(d*x)^3*tan(c)^4 - 3675*b^2*tan(d*x)^3*tan(c)^4 - 420*a*
b*tan(d*x)^2*tan(c)^5 - 735*b^2*tan(d*x)^2*tan(c)^5 - 147*b^2*tan(d*x)*tan(
c)^6 - 15*b^2*tan(c)^7 - 2205*a^2*d*x*tan(d*x)^2*tan(c)^2 - 4410*a*b*d*x*ta
n(d*x)^2*tan(c)^2 - 2205*b^2*d*x*tan(d*x)^2*tan(c)^2 + 21*b^2*tan(d*x)^5 +
280*a*b*tan(d*x)^4*tan(c) + 245*b^2*tan(d*x)^4*tan(c) + 3990*a*b*tan(d*x)^3
*tan(c)^2 + 2205*b^2*tan(d*x)^3*tan(c)^2 + 3990*a*b*tan(d*x)^2*tan(c)^3 + 2
205*b^2*tan(d*x)^2*tan(c)^3 + 280*a*b*tan(d*x)*tan(c)^4 + 245*b^2*tan(d*x)*
tan(c)^4 + 21*b^2*tan(c)^5 + 735*a^2*d*x*tan(d*x)*tan(c) + 1470*a*b*d*x*tan
(d*x)*tan(c) + 735*b^2*d*x*tan(d*x)*tan(c) - 70*a*b*tan(d*x)^3 - 35*b^2*tan
(d*x)^3 - 1470*a*b*tan(d*x)^2*tan(c) - 735*b^2*tan(d*x)^2*tan(c) - 1470*a*b
*tan(d*x)*tan(c)^2 - 735*b^2*tan(d*x)*tan(c)^2 - 70*a*b*tan(c)^3 - 35*b^2*t
an(c)^3 - 105*a^2*d*x - 210*a*b*d*x - 105*b^2*d*x + 210*a*b*tan(d*x) + 105*
b^2*tan(d*x) + 210*a*b*tan(c) + 105*b^2*tan(c))/(d*tan(d*x)^7*tan(c)^7 - 7*
d*tan(d*x)^6*tan(c)^6 + 21*d*tan(d*x)^5*tan(c)^5 - 35*d*tan(d*x)^4*tan(c)^4
 + 35*d*tan(d*x)^3*tan(c)^3 - 21*d*tan(d*x)^2*tan(c)^2 + 7*d*tan(d*x)*tan(c
) - d)
```

Mupad [B]
time = 11.47, size = 109, normalized size = 1.33

$$\frac{\tan(c+d\,x)^3\,\left(\frac{b^2}{3}+\frac{2\,a\,b}{3}\right)}{d}-\frac{b^2\tan(c+d\,x)^5}{5\,d}+\frac{b^2\tan(c+d\,x)^7}{7\,d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)^2}{a^2+2\,a\,b+b^2}\right)\,(a+b)^2}{d}-\frac{\tan(c+d\,x)\,\left(b^2+2\,a\,b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^4)^2,x)
```

```
[Out] (tan(c + d*x)^3*((2*a*b)/3 + b^2/3))/d - (b^2*tan(c + d*x)^5)/(5*d) + (b^2*
tan(c + d*x)^7)/(7*d) + (atan((tan(c + d*x)*(a + b)^2)/(2*a*b + a^2 + b^2))
*(a + b)^2)/d - (tan(c + d*x)*(2*a*b + b^2))/d
```

3.384 $\int (a+b\tan^4(c+dx))\,dx$

Optimal. Leaf size=35

$$ax+bx-\frac{b\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

```
[Out] a*x+b*x-b*tan(d*x+c)/d+1/3*b*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3554, 8}

$$ax+\frac{b\tan^3(c+dx)}{3d}-\frac{b\tan(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Tan[c + d*x]^4,x]
```

```
[Out] a*x + b*x - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^4(c+dx)\right)\,dx &= ax+b\int \tan^4(c+dx)\,dx\\
&= ax+\frac{b\tan^3(c+dx)}{3d}-b\int \tan^2(c+dx)\,dx\\
&= ax-\frac{b\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}+b\int 1\,dx\\
&= ax+bx-\frac{b\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.26

$$ax + \frac{b\text{ArcTan}(\tan(c+dx))}{d} - \frac{b\tan(c+dx)}{d} + \frac{b\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Tan[c + d*x]^4,x]
```

```
[Out] a*x + (b*ArcTan[Tan[c + d*x]])/d - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/
(3*d)
```

Maple [A]
time = 0.04, size = 43, normalized size = 1.23

method	result	size
norman	$(a+b)\,x - \frac{b\tan(dx+c)}{d} + \frac{b\left(\tan^3(dx+c)\right)}{3d}$	33
derivativedivides	$\frac{\frac{b\left(\tan^3(dx+c)\right)}{3} - b\tan(dx+c) + (a+b)\arctan(\tan(dx+c))}{d}$	37
default	$ax + \frac{b\left(\tan^3(dx+c)\right)}{3d} - \frac{b\tan(dx+c)}{d} + \frac{b\arctan(\tan(dx+c))}{d}$	43
risch	$ax + bx - \frac{4ib\left(3\,\mathrm{e}^{4i(dx+c)} + 3\,\mathrm{e}^{2i(dx+c)} + 2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/3*b*tan(d*x+c)^3/d-b*tan(d*x+c)/d+b/d*arctan(tan(d*x+c))
```

Maxima [A]
time = 0.53, size = 34, normalized size = 0.97

$$ax + \frac{\left(\tan\left(dx+c\right)^3 + 3\,dx + 3\,c - 3\,\tan\left(dx+c\right)\right)b}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)^4*b,x, algorithm="maxima")
```

```
[Out] a*x + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*b/d
```

Fricas [A]
time = 2.92, size = 32, normalized size = 0.91

$$\frac{b\tan\left(dx+c\right)^3 + 3\left(a+b\right)dx - 3\,b\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)^4*b,x, algorithm="fricas")
```

```
[Out] 1/3*(b*tan(d*x + c)^3 + 3*(a + b)*d*x - 3*b*tan(d*x + c))/d
```

Sympy [A]
time = 0.08, size = 32, normalized size = 0.91

$$ax + b\left(\begin{cases} x + \frac{\tan^{3}(c+dx)}{3d} - \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^{4}(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)**4*b,x)
```

```
[Out] a*x + b*Piecewise((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (
x*tan(c)**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(33) = 66.
time = 0.89, size = 590, normalized size = 16.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+tan(d*x+c)^4*b,x, algorithm="giac")
```

```
[Out] a*x + 1/12*(3*pi + 12*d*x*tan(d*x)^3*tan(c)^3 - 3*pi*sgn(2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 - 3*pi
*tan(d*x)^3*tan(c)^3 + 6*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*
tan(d*x)^3*tan(c)^3 + 6*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*t
an(d*x)^3*tan(c)^3 - 36*d*x*tan(d*x)^2*tan(c)^2 + 9*pi*sgn(2*tan(d*x)^2*tan
(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 9*
pi*tan(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)
))*tan(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1
))*tan(d*x)^2*tan(c)^2 + 12*tan(d*x)^3*tan(c)^2 + 12*tan(d*x)^2*tan(c)^3 +
36*d*x*tan(d*x)*tan(c) - 9*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - 4*tan(d*x)^3 - 9*pi*tan(d*x)*ta
n(c) + 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c)
 + 18*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) - 3
6*tan(d*x)^2*tan(c) - 36*tan(d*x)*tan(c)^2 - 4*tan(c)^3 - 12*d*x + 3*pi*sgn
(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 6*arc
tan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 6*arctan((tan(d*x) + tan(c
))/(tan(d*x)*tan(c) - 1)) + 12*tan(d*x) + 12*tan(c))*b/(d*tan(d*x)^3*tan(c)
^3 - 3*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 11.59, size = 31, normalized size = 0.89

$$\frac{\frac{b\,{\tan\left(c+d\,x\right)}^3}{3}-b\,\tan\left(c+d\,x\right)+d\,x\,\left(a+b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*tan(c + d*x)^4,x)
[Out] ((b*tan(c + d*x)^3)/3 - b*tan(c + d*x) + d*x*(a + b))/d
```

3.385 $\int \frac{1}{a+b\tan^4(c+dx)}\,dx$

Optimal. Leaf size=302

$$\frac{x}{a+b}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(a+b)d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(a+b)d}$$

```
[Out] x/(a+b)+1/4*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b
^(1/2))/a^(3/4)/(a+b)/d*2^(1/2)-1/4*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*tan(d*
x+c)/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4)/(a+b)/d*2^(1/2)-1/8*b^(1/4)*ln(a^(1
/2)-a^(1/4)*b^(1/4)*2^(1/2)*tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1/2)+b^(1/
2))/a^(3/4)/(a+b)/d*2^(1/2)+1/8*b^(1/4)*ln(a^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*
tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1/2)+b^(1/2))/a^(3/4)/(a+b)/d*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3742, 1185, 209, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\tan(c+dx)+\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}{4\sqrt{2}\,a^{3/4}d(a+b)}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\tan(c+dx)+\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}{4\sqrt{2}\,a^{3/4}d(a+b)}+\frac{x}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^(-1),x]
```

```
[Out] x/(a + b) + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c
+ d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)*d) - ((Sqrt[a] - Sqrt[b])*b^(1
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(a + b)*d) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)*d)
+ ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c
+ d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+b\tan^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^4)}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(1+x^2)}+\frac{b-bx^2}{(a+b)(a+bx^4)}\right)dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)d}+\frac{\text{Subst}\left(\int \frac{b-bx^2}{a+bx^4}\,dx, x, \tan(c+dx)\right)}{(a+b)d}\\
&= \frac{x}{a+b}-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx, x, \tan(c+dx)\right)}{2(a+b)d}+\frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{S}\cdots}{\cdots}\\
&= \frac{x}{a+b}-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \tan(c+dx)\right)}{4(a+b)d}-\frac{\left(1-\cdots\right.}{\cdots}\\
&= \frac{x}{a+b}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+\cdots\right.}{4\sqrt{2}\,a^{3/4}(a+b)d}\\
&= \frac{x}{a+b}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(a+b)d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\cdots}{2\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [A]
time = 0.37, size = 228, normalized size = 0.75

$$
\frac{8a^{3/4}\text{ArcTan}(\tan(c+dx))+\sqrt{2}\sqrt[4]{b}\left(2\left(\sqrt{a}-\sqrt{b}\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)-2\left(\sqrt{a}-\sqrt{b}\right)\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)-\left(\sqrt{a}+\sqrt{b}\right)\left(\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)-\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)\right)\right)}{8a^{3/4}(a+b)d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^(-1),x]
```

```
[Out] (8*a^(3/4)*ArcTan[Tan[c + d*x]] + Sqrt[2]*b^(1/4)*(2*(Sqrt[a] - Sqrt[b])*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)] - 2*(Sqrt[a] - Sqrt[b])*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)] - (Sqrt[a] + Sqrt[b])*(Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2] -
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^
2])))/(8*a^(3/4)*(a + b)*d)
```

Maple [A]
time = 0.32, size = 292, normalized size = 0.97

method	result
risch	$\frac{x}{a+b}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(256a^5d^4+512a^4b\,d^4+256a^3b^2d^4\right)\_Z^4-64\_Z^2a^2b\,d^2+b\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(-\frac{32a^3d^2}{a-b}\right.\right.\right.$
derivativedivides	$-\frac{b\left(-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\tan^2(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}{\tan^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\tan(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\tan(d}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8a}\right.}{}$
default	$-\frac{b\left(-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\tan^2(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}{\tan^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\tan(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\tan(d}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b/(a+b)*(-1/8*(a/b)^(1/4)/a*2^(1/2)*(ln((tan(d*x+c)^2+(a/b)^(1/4)*tan
(d*x+c)*2^(1/2)+(a/b)^(1/2))/(tan(d*x+c)^2-(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(
a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)-2*arctan(-2^(1/2)/(
a/b)^(1/4)*tan(d*x+c)+1))+1/8/b/(a/b)^(1/4)*2^(1/2)*(ln((tan(d*x+c)^2-(a/b)
^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2))/(tan(d*x+c)^2+(a/b)^(1/4)*tan(d*x+c)
*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)-2*arctan(
-2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)))+1/(a+b)*arctan(tan(d*x+c)))
```

Maxima [A]

time = 0.51, size = 261, normalized size = 0.86

$$-\frac{\frac{b\left(\frac{2\sqrt{2}\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}\tan(dx+c)^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\tan(dx+c)+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}\tan(dx+c)^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\tan(dx+c)+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}\right)}{a+b}-\frac{8(dx+c)}{a+b}}{8d}$$

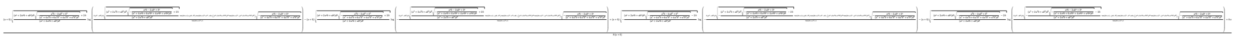
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(2*sqrt(2)*(sqrt(a) - sqrt(b))*arctan(1/2*sqrt(2)*(2*sqrt(b)*tan(d*
x + c) + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*(sqrt(a) - sqrt(b))*arctan(1/2*sqrt(2)*(2
*sqrt(b)*tan(d*x + c) - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*(sqrt(a) + sqrt(b))*log(sqrt
(b)*tan(d*x + c)^2 + sqrt(2)*a^(1/4)*b^(1/4)*tan(d*x + c) + sqrt(a))/(a^(3/
```

```
4)*b^(3/4)) + sqrt(2)*(sqrt(a) + sqrt(b))*log(sqrt(b)*tan(d*x + c)^2 - sqrt
(2)*a^(1/4)*b^(1/4)*tan(d*x + c) + sqrt(a))/(a^(3/4)*b^(3/4)))/(a + b) - 8*
(d*x + c)/(a + b))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1541 vs. $2(222) = 444$.
time = 3.56, size = 1541, normalized size = 5.10



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b),x, algorithm="fricas")
```

```
[Out] 1/8*((a + b)*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3
)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 + 2
*a^2*b + a*b^2)*d^2))*log((2*(a^3 - a*b^2)*d*sqrt(((a^3 + 2*a^2*b + a*b^2)*
d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
 a^3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) + (a*b -
 b^2)*tan(d*x + c)^2 + a^2 - a*b + ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x +
 c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7
+ 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) -
 (a + b)*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2
*b + a*b^2)*d^2))*log(-(2*(a^3 - a*b^2)*d*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2
*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^
3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) - (a*b - b^
2)*tan(d*x + c)^2 - a^2 + a*b - ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x + c)
^2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4
*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) + (a
 + b)*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^
7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b
 + a*b^2)*d^2))*log(-(2*(a^3 - a*b^2)*d*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*
sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3
*b^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) + (a*b - b^2
)*tan(d*x + c)^2 + a^2 - a*b - ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x + c)^
2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*
a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) - (a
+ b)*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7
 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b
+ a*b^2)*d^2))*log((2*(a^3 - a*b^2)*d*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sq
rt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4)*d^4)) - 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) - (a*b - b^2)*
tan(d*x + c)^2 - a^2 + a*b + ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan(d*x + c)^2
- (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^
6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2 + 1)) + 8*x)/
(a + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)**4*b),x)
```

```
[Out] Integral(1/(a + b*tan(c + d*x)**4), x)
```

Giac [A]
time = 0.94, size = 354, normalized size = 1.17

$$\frac{\frac{2\left((ab^3)^{\frac{1}{4}}b^2-(ab^3)^{\frac{3}{4}}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{2\left((ab^3)^{\frac{1}{4}}b^2-(ab^3)^{\frac{3}{4}}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{\left((ab^3)^{\frac{1}{4}}b^2+(ab^3)^{\frac{3}{4}}\right)\log\left(\tan(dx+c)^2+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)+\sqrt{\frac{a}{b}}\right)}{\sqrt{2}a^2b^2+\sqrt{2}ab^3}-\frac{\left((ab^3)^{\frac{1}{4}}b^2+(ab^3)^{\frac{3}{4}}\right)\log\left(\tan(dx+c)^2-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)+\sqrt{\frac{a}{b}}\right)}{\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{4(dx+c)}{a+b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b),x, algorithm="giac")
```

```
[Out] 1/4*(2*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*(pi*floor((d*x + c)/pi + 1/2) +
arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*tan(d*x + c))/(a/b)^(1/4)))/(sq
rt(2)*a^2*b^2 + sqrt(2)*a*b^3) + 2*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*(pi*
floor((d*x + c)/pi + 1/2) + arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*ta
n(d*x + c))/(a/b)^(1/4)))/(sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) + ((a*b^3)^(1/4
)*b^2 + (a*b^3)^(3/4))*log(tan(d*x + c)^2 + sqrt(2)*(a/b)^(1/4)*tan(d*x + c
) + sqrt(a/b))/(sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) - ((a*b^3)^(1/4)*b^2 + (a*
b^3)^(3/4))*log(tan(d*x + c)^2 - sqrt(2)*(a/b)^(1/4)*tan(d*x + c) + sqrt(a/
b))/(sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) + 4*(d*x + c)/(a + b))/d
```

Mupad [B]
time = 15.04, size = 2500, normalized size = 8.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^4),x)
```

```
[Out] (2*atan(((((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 2
56*a^4*b^4 - (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a
^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + tan(c + d*x)*(32*a*b^6 + 16*b^7
- 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - 6*b^5*tan(c + d*x))/(2*a
 + 2*b) - (((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 +
256*a^4*b^4 + (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*
a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - tan(c + d*x)*(32*a*b^6 + 16*b^7
 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + 6*b^5*tan(c + d*x))/(2*
```

```
a + 2*b))/(((((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5
+ 256*a^4*b^4 - (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 51
2*a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + tan(c + d*x)*(32*a*b^6 + 16*b
^7 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - 6*b^5*tan(c + d*x))*1
i)/(2*a + 2*b) + ((((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^
3*b^5 + 256*a^4*b^4 + (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^
5 - 512*a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - tan(c + d*x)*(32*a*b^6
+ 16*b^7 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + 6*b^5*tan(c + d
*x))*1i)/(2*a + 2*b))))/(d*(2*a + 2*b)) - (atan((((20*a*b^5 - (((2*a^2*b +
a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*
(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4 + tan(c + d*x)*((2*a^2*
b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1
/2)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a^5*b^4)) - tan(c + d*x)
*(32*a*b^6 + 16*b^7 - 240*a^2*b^5))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*
b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 4*b^6)*((2*a^2*b + a*(-a^
3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 6*b^
5*tan(c + d*x))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4
*b + a^5 + a^3*b^2)))^(1/2)*1i - ((20*a*b^5 - (((2*a^2*b + a*(-a^3*b)^(1/2)
 - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(128*a^2*b^6 - 6
4*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4 - tan(c + d*x)*((2*a^2*b + a*(-a^3*b)^(
1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(512*a^2*b^7
 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a^5*b^4)) + tan(c + d*x)*(32*a*b^6 + 16*
b^7 - 240*a^2*b^5))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2
*a^4*b + a^5 + a^3*b^2)))^(1/2) + 4*b^6)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(
-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) - 6*b^5*tan(c + d*x))*
((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b
^2)))^(1/2)*1i)/(((20*a*b^5 - (((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1
/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a*b^7 + 448*a^
3*b^5 + 256*a^4*b^4 + tan(c + d*x)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b
)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(512*a^2*b^7 + 512*a^3*b^6 -
 512*a^4*b^5 - 512*a^5*b^4)) - tan(c + d*x)*(32*a*b^6 + 16*b^7 - 240*a^2*b^
5))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a
^3*b^2)))^(1/2) + 4*b^6)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(
16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 6*b^5*tan(c + d*x))*((2*a^2*b + a*(-
a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + ((
20*a*b^5 - (((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b +
 a^5 + a^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4
 - tan(c + d*x)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4
*b + a^5 + a^3*b^2)))^(1/2)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*
a^5*b^4)) + tan(c + d*x)*(32*a*b^6 + 16*b^7 - 240*a^2*b^5))*((2*a^2*b + a*(
-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2) + 4
*b^6)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 +
 a^3*b^2)))^(1/2) - 6*b^5*tan(c + d*x))*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-
a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)))*((2*a^2*b + a*(-a^3*b
)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*2i)/d - (
```

```
atan((((20*a*b^5 - (((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2
*a^4*b + a^5 + a^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256
*a^4*b^4 + tan(c + d*x)*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(1
6*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^
5 - 512*a^5*b^4)) - tan(c + d*x)*(32*a*b^6 + 16*b^7 - 240*a^2*b^5))*((2*a^2
*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(
1/2) + 4*b^6)*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b
 + a^5 + a^3*b^2)))^(1/2) + 6*b^5*tan(c + d*x))*((2*a^2*b - a*(-a^3*b)^(1/2
) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a^3*b^2)))^(1/2)*1i - ((20*a*b^5
 - (((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 + a
^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4 - tan(c
 + d*x)*((2*a^2*b - a*(-a^3*b)^(1/2) + b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5
 + a^3*b^2)))^(1/2)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512*a^5*b^4)
) + tan(c + d*x)*(32*a*b^6 + 16*b^7 - 240*a^2*b...
```

3.386 $\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=648

$$\frac{x}{(a+b)^2}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(a+b)^2d}+\frac{\left(\sqrt{a}-3\sqrt{b}\right)\sqrt[4]{b}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan}{\sqrt[4]{a}}\right.}{8\sqrt{2}\,a^{7/4}(a+b)d}$$

```
[Out] x/(a+b)^2+1/16*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2
)-3*b^(1/2))/a^(7/4)/(a+b)/d*2^(1/2)-1/16*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*
tan(d*x+c)/a^(1/4))*(a^(1/2)-3*b^(1/2))/a^(7/4)/(a+b)/d*2^(1/2)+1/4*b^(1/4)
*arctan(1-b^(1/4)*2^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4)/(a+
b)^2/d*2^(1/2)-1/4*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*tan(d*x+c)/a^(1/4))*(a^
(1/2)-b^(1/2))/a^(3/4)/(a+b)^2/d*2^(1/2)-1/8*b^(1/4)*ln(a^(1/2)-a^(1/4)*b^(
1/4)*2^(1/2)*tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1/2)+b^(1/2))/a^(3/4)/(a+
b)^2/d*2^(1/2)+1/8*b^(1/4)*ln(a^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*tan(d*x+c)+b^
(1/2)*tan(d*x+c)^2)*(a^(1/2)+b^(1/2))/a^(3/4)/(a+b)^2/d*2^(1/2)-1/32*b^(1/4
)*ln(a^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1
/2)+3*b^(1/2))/a^(7/4)/(a+b)/d*2^(1/2)+1/32*b^(1/4)*ln(a^(1/2)+a^(1/4)*b^(1
/4)*2^(1/2)*tan(d*x+c)+b^(1/2)*tan(d*x+c)^2)*(a^(1/2)+3*b^(1/2))/a^(7/4)/(a
+b)/d*2^(1/2)+1/4*b*tan(d*x+c)*(1-tan(d*x+c)^2)/a/(a+b)/d/(a+tan(d*x+c)^4*b
)
```

Rubi [A]

time = 0.45, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3742, 1253, 209, 1193, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}d(a+b)}+\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}d(a+b)^2}-\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}d(a+b)^2}-\frac{\sqrt[4]{b}\left(\sqrt{a}+3\sqrt{b}\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\tan(c+dx)+\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}{16\sqrt{2}\,a^{7/4}d(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\tan(c+dx)+\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}{4\sqrt{2}\,a^{3/4}d(a+b)^2}+\frac{\sqrt[4]{b}\left(\sqrt{a}+3\sqrt{b}\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\tan(c+dx)+\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}{16\sqrt{2}\,a^{7/4}d(a+b)}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\tan(c+dx)+\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}{4\sqrt{2}\,a^{3/4}d(a+b)^2}+\frac{b\tan(c+dx)\left(1-\tan^2(c+dx)\right)}{4ad(a+b)\left(a+b\tan^4(c+dx)\right)}+\frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x]^4)^(-2),x]
```

```
[Out] x/(a + b)^2 + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[
c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2*d) + ((Sqrt[a] - 3*Sqrt[b]
)*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*(a + b)*d) - ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2*d) - ((Sqrt[a] - 3*Sq
rt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[
2]*a^(7/4)*(a + b)*d) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*
(a + b)^2*d) - ((Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a + b
)*d) + ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*T
```

```
an[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^2*d) + ((
Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c +
d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a + b)*d) + (b*Tan[c +
 d*x]*(1 - Tan[c + d*x]^2))/(4*a*(a + b)*d*(a + b*Tan[c + d*x]^4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1253

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^4)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2(1+x^2)} + \frac{b-bx^2}{(a+b)(a+bx^4)^2} + \frac{b-bx^2}{(a+b)^2(a+bx^4)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)^2d} + \frac{\text{Subst}\left(\int \frac{b-bx^2}{a+bx^4}\,dx, x, \tan(c+dx)\right)}{(a+b)^2d} + \frac{\text{Subs}}{}$$

$$= \frac{x}{(a+b)^2} + \frac{b\tan(c+dx)\left(1-\tan^2(c+dx)\right)}{4a(a+b)d\left(a+b\tan^4(c+dx)\right)} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+}{a+bx^4}\right.}{2(a+b)^2}$$

$$= \frac{x}{(a+b)^2} + \frac{b\tan(c+dx)\left(1-\tan^2(c+dx)\right)}{4a(a+b)d\left(a+b\tan^4(c+dx)\right)} - \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}-\sqrt{}}\right.}{4(a+}$$

$$= \frac{x}{(a+b)^2} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2\right.}{4\sqrt{2}\,a^{3/4}(a+b)^2d}$$

$$= \frac{x}{(a+b)^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(a+b)^2d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)}{}$$

$$= \frac{x}{(a+b)^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(a+b)^2d} + \frac{\left(\sqrt{a}-3\sqrt{b}\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.18, size = 598, normalized size = 0.92

$$\frac{\text{ArcTan}(\tan(c+dx))}{(a+b)^2d} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\left(\frac{\sqrt{2}\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} - \frac{\sqrt{2}\sqrt[4]{b}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}\right)}{4\sqrt{a}(a+b)^2d} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\frac{\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{\sqrt[4]{a}} - \frac{\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{\sqrt[4]{a}}\right)}{8\sqrt{a}(a+b)^2d} - \frac{3\left(\frac{2\left(\frac{\sqrt{2}b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} - \frac{\sqrt{2}b^{3/4}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}\right)}{\sqrt{a}} + \frac{\frac{\sqrt{2}b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{\sqrt[4]{a}} - \frac{\sqrt{2}b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{\sqrt[4]{a}}}{\sqrt{a}}\right)}{32a(a+b)d} - \frac{b\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{b\tan^4(c+dx)}{a}\right)\tan^3(c+dx)}{3a^2(a+b)d} + \frac{b\tan(c+dx)}{4a(a+b)d\left(a+b\tan^4(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x]^4)^(-2),x]
```

```
[Out] ArcTan[Tan[c + d*x]]/((a + b)^2*d) + ((Sqrt[a] - Sqrt[b])*((Sqrt[2]*b^(1/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4) - (Sqrt[2]*b^(
```

```
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4)))/(4*Sqrt[
a]*(a + b)^2*d) - ((Sqrt[a] + Sqrt[b])*((Sqrt[2]*b^(1/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/a^(1/4) - (Sqrt
[2]*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Ta
n[c + d*x]^2])/a^(1/4)))/(8*Sqrt[a]*(a + b)^2*d) - (3*((2*((Sqrt[2]*b^(3/4)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4) - (Sqrt[2]*b^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/a^(1/4)))/Sqrt[a]
+ ((Sqrt[2]*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sq
rt[b]*Tan[c + d*x]^2])/a^(1/4) - (Sqrt[2]*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/a^(1/4))/Sqrt[a]))/(32
*a*(a + b)*d) - (b*Hypergeometric2F1[3/4, 2, 7/4, -((b*Tan[c + d*x]^4)/a)]*
Tan[c + d*x]^3)/(3*a^2*(a + b)*d) + (b*Tan[c + d*x])/(4*a*(a + b)*d*(a + b*
Tan[c + d*x]^4))
```

Maple [A]
time = 1.04, size = 356, normalized size = 0.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^2*b*((1/4*(a+b)/a*tan(d*x+c)^3-1/4*(a+b)/a*tan(d*x+c))/(a+b*t
an(d*x+c)^4)+1/4/a*(1/8*(-7*a-3*b)*(a/b)^(1/4)/a*2^(1/2)*(ln((tan(d*x+c)^2+
(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2))/(tan(d*x+c)^2-(a/b)^(1/4)*tan(d
*x+c)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)-2*ar
ctan(-2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1))+1/8*(5*a+b)/b/(a/b)^(1/4)*2^(1/2)*
(ln((tan(d*x+c)^2-(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2))/(tan(d*x+c)^2
+(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*
tan(d*x+c)+1)-2*arctan(-2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1))))+1/(a+b)^2*arct
an(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 394, normalized size = 0.61

$$-\frac{\frac{b\left(\frac{2\sqrt{2}\left(b\left(\sqrt{a}-3\sqrt{b}\right)+5a^{\frac{3}{2}}-7a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\left(b\left(\sqrt{a}-3\sqrt{b}\right)+5a^{\frac{3}{2}}-7a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\left(b\left(\sqrt{a}+3\sqrt{b}\right)+5a^{\frac{3}{2}}+7a\sqrt{b}\right)\log\left(\sqrt{b}\tan(dx+c)^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\tan(dx+c)+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b\left(\sqrt{a}+3\sqrt{b}\right)+5a^{\frac{3}{2}}+7a\sqrt{b}\right)\log\left(\sqrt{b}\tan(dx+c)^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\tan(dx+c)+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}\right)}{a^3+2a^2b+ab^2}+\frac{8\left(b\tan(dx+c)^3-b\tan(dx+c)\right)}{(a^2b+ab^2)\tan(dx+c)^4+a^3+a^2b}-\frac{32(dx+c)}{a^2+2ab+b^2}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(b*(2*sqrt(2)*(b*(sqrt(a) - 3*sqrt(b)) + 5*a^(3/2) - 7*a*sqrt(b))*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*tan(d*x + c) + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*(b*(sqrt
(a) - 3*sqrt(b)) + 5*a^(3/2) - 7*a*sqrt(b))*arctan(1/2*sqrt(2)*(2*sqrt(b)*t
an(d*x + c) - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt
(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*(b*(sqrt(a) + 3*sqrt(b)) + 5*a^(3/2) +
 7*a*sqrt(b))*log(sqrt(b)*tan(d*x + c)^2 + sqrt(2)*a^(1/4)*b^(1/4)*tan(d*x
```

```
+ c) + sqrt(a))/(a^(3/4)*b^(3/4)) + sqrt(2)*(b*(sqrt(a) + 3*sqrt(b)) + 5*a^
(3/2) + 7*a*sqrt(b))*log(sqrt(b)*tan(d*x + c)^2 - sqrt(2)*a^(1/4)*b^(1/4)*t
an(d*x + c) + sqrt(a))/(a^(3/4)*b^(3/4)))/(a^3 + 2*a^2*b + a*b^2) + 8*(b*ta
n(d*x + c)^3 - b*tan(d*x + c))/((a^2*b + a*b^2)*tan(d*x + c)^4 + a^3 + a^2*
b) - 32*(d*x + c)/(a^2 + 2*a*b + b^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4291 vs. $2(484) = 968$.
time = 4.57, size = 4291, normalized size = 6.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*a*b*d*x*tan(d*x + c)^4 + 32*a^2*d*x - 8*(a*b + b^2)*tan(d*x + c)^3
 + ((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4 + (a^4 + 2*a^3*b + a^2*b^2
)*d)*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625
*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b
^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 +
56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) + 70*a^2*b + 44*a*b^2
 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2))*log((625
*a^5 - 750*a^4*b - 1376*a^3*b^2 - 594*a^2*b^3 - 81*a*b^4 + (625*a^4*b - 750
*a^3*b^2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b^5)*tan(d*x + c)^2 + 2*(2*(a^11 +
 5*a^10*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^7*b^4 + a^6*b^5)*d^3*sqrt(-(625*a
^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6
 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56
*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4))*tan(d*x + c) + (125*a^7
 + 5*a^6*b - 442*a^5*b^2 - 490*a^4*b^3 - 195*a^3*b^4 - 27*a^2*b^5)*d*tan(d*
x + c))*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(
625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*
a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4
 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) + 70*a^2*b + 44*a*
b^2 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2)) + ((2
5*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2
*tan(d*x + c)^2 - (25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^5*
b^4 + 9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*
a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^
2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*
b^8)*d^4)))/(tan(d*x + c)^2 + 1)) - ((a^3*b + 2*a^2*b^2 + a*b^3)*d*tan(d*x
+ c)^4 + (a^4 + 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748
*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b
^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7
*b^8)*d^4)) + 70*a^2*b + 44*a*b^2 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
```

```
a^4*b^3 + a^3*b^4)*d^2))*log((625*a^5 - 750*a^4*b - 1376*a^3*b^2 - 594*a^2*
b^3 - 81*a*b^4 + (625*a^4*b - 750*a^3*b^2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b
^5)*tan(d*x + c)^2 - 2*(2*(a^11 + 5*a^10*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^
7*b^4 + a^6*b^5)*d^3*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a
^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2
 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b
^8)*d^4))*tan(d*x + c) + (125*a^7 + 5*a^6*b - 442*a^5*b^2 - 490*a^4*b^3 - 1
95*a^3*b^4 - 27*a^2*b^5)*d*tan(d*x + c))*sqrt(((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2
748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^7)/((a^15 + 8*a^14*b + 28*a^1
3*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 +
a^7*b^8)*d^4)) + 70*a^2*b + 44*a*b^2 + 6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*d^2)) + ((25*a^9 + 109*a^8*b + 186*a^7*b^2 + 154*a^6*
b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2*tan(d*x + c)^2 - (25*a^9 + 109*a^8*b + 18
6*a^7*b^2 + 154*a^6*b^3 + 61*a^5*b^4 + 9*a^4*b^5)*d^2)*sqrt(-(625*a^6*b - 1
950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b^
7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b^
5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)))/(tan(d*x + c)^2 + 1)) - ((a^3*
b + 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^4 + (a^4 + 2*a^3*b + a^2*b^2)*d)*sqrt
(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^6*b -
 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*
b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*
b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) - 70*a^2*b - 44*a*b^2 - 6*b^3
)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2))*log(-(625*a^5 -
750*a^4*b - 1376*a^3*b^2 - 594*a^2*b^3 - 81*a*b^4 + (625*a^4*b - 750*a^3*b^
2 - 1376*a^2*b^3 - 594*a*b^4 - 81*b^5)*tan(d*x + c)^2 + 2*(2*(a^11 + 5*a^10
*b + 10*a^9*b^2 + 10*a^8*b^3 + 5*a^7*b^4 + a^6*b^5)*d^3*sqrt(-(625*a^6*b -
1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6 + 81*b
^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*a^10*b
^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4))*tan(d*x + c) - (125*a^7 + 5*a^
6*b - 442*a^5*b^2 - 490*a^4*b^3 - 195*a^3*b^4 - 27*a^2*b^5)*d*tan(d*x + c))
*sqrt(-((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2*sqrt(-(625*a^
6*b - 1950*a^5*b^2 - 529*a^4*b^3 + 2748*a^3*b^4 + 2383*a^2*b^5 + 738*a*b^6
+ 81*b^7)/((a^15 + 8*a^14*b + 28*a^13*b^2 + 56*a^12*b^3 + 70*a^11*b^4 + 56*
a^10*b^5 + 28*a^9*b^6 + 8*a^8*b^7 + a^7*b^8)*d^4)) - 70*a^2*b - 44*a*b^2 -
6*b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^2)) - ((25*a^9
+ 109*a^8*b + 186*a^7*b^2 + 154*a^6*b^3 + 61*a^...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\tan^4\left(c + dx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)**4*b)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**4)**(-2), x)
```

Giac [A]
time = 1.08, size = 517, normalized size = 0.80

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)\left((ab^3)^{\frac{3}{4}}(5a+b)-(ab^3)^{\frac{1}{4}}(7ab^2+3b^3)\right)}{\sqrt{2}a^4b^2+2\sqrt{2}a^3b^3+\sqrt{2}a^2b^4}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)\left((ab^3)^{\frac{3}{4}}(5a+b)-(ab^3)^{\frac{1}{4}}(7ab^2+3b^3)\right)}{\sqrt{2}a^4b^2+2\sqrt{2}a^3b^3+\sqrt{2}a^2b^4}-\frac{\left((ab^3)^{\frac{3}{4}}(5a+b)+(ab^3)^{\frac{1}{4}}(7ab^2+3b^3)\right)\log\left(\tan(dx+c)^2+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)+\sqrt{\frac{a}{b}}\right)}{\sqrt{2}a^4b^2+2\sqrt{2}a^3b^3+\sqrt{2}a^2b^4}+\frac{\left((ab^3)^{\frac{3}{4}}(5a+b)+(ab^3)^{\frac{1}{4}}(7ab^2+3b^3)\right)\log\left(\tan(dx+c)^2-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)+\sqrt{\frac{a}{b}}\right)}{\sqrt{2}a^4b^2+2\sqrt{2}a^3b^3+\sqrt{2}a^2b^4}-\frac{16(dx+c)}{a^2+2ab+b^2}+\frac{4\left(b\tan(dx+c)^3-b\tan(dx+c)\right)}{\left(b\tan(dx+c)^4+a\right)(a^2+ab)}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*(pi*floor((d*x + c)/pi + 1/2) + arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) + 2*tan(d*x + c))/(a/b)^(1/4)))*((a*b^3)^(3/4)*(5*a + b) - (a*b^3)^(1
/4)*(7*a*b^2 + 3*b^3))/(sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b
^4) + 2*(pi*floor((d*x + c)/pi + 1/2) + arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) - 2*tan(d*x + c))/(a/b)^(1/4)))*((a*b^3)^(3/4)*(5*a + b) - (a*b^3)^(1
/4)*(7*a*b^2 + 3*b^3))/(sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b
^4) - ((a*b^3)^(3/4)*(5*a + b) + (a*b^3)^(1/4)*(7*a*b^2 + 3*b^3))*log(tan(d
*x + c)^2 + sqrt(2)*(a/b)^(1/4)*tan(d*x + c) + sqrt(a/b))/(sqrt(2)*a^4*b^2
+ 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b^4) + ((a*b^3)^(3/4)*(5*a + b) + (a*b^3)
^(1/4)*(7*a*b^2 + 3*b^3))*log(tan(d*x + c)^2 - sqrt(2)*(a/b)^(1/4)*tan(d*x
+ c) + sqrt(a/b))/(sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b^4) -
 16*(d*x + c)/(a^2 + 2*a*b + b^2) + 4*(b*tan(d*x + c)^3 - b*tan(d*x + c))/(
(b*tan(d*x + c)^4 + a)*(a^2 + a*b)))/d
```

Mupad [B]
time = 15.69, size = 2500, normalized size = 3.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^4)^2,x)
```

```
[Out] ((b*tan(c + d*x))/(4*a*(a + b)) - (b*tan(c + d*x)^3)/(4*a*(a + b)))/(d*(a +
 b*tan(c + d*x)^4)) - (2*atan(((((((((((960*a^7*b^8 - 224*a^5*b^10 - 144*a^6
*b^9 - 48*a^4*b^11 + 2480*a^8*b^7 + 2592*a^9*b^6 + 1296*a^10*b^5 + 256*a^11
*b^4)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2) - (tan(c + d*x)
*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 3276
80*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*
a*b + 2*a^2 + 2*b^2)*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i
)/(4*a*b + 2*a^2 + 2*b^2) - (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 +
20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b
^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b
 + 2*a^2 + 2*b^2) - (((45*a*b^10)/16 + (305*a^2*b^9)/16 + (385*a^3*b^8)/8 +
 (657*a^4*b^7)/8 + (2081*a^5*b^6)/16 + (1277*a^6*b^5)/16)*1i)/(4*a^7*b + a^
8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))/(4*a*b + 2*a^2 + 2*b^2) - (tan(c + d*
```

x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*
(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(4*a*b + 2*a^2 + 2*b^2)
 - ((((((((960*a^7*b^8 - 224*a^5*b^10 - 144*a^6*b^9 - 48*a^4*b^11 + 2480*a^
8*b^7 + 2592*a^9*b^6 + 1296*a^10*b^5 + 256*a^11*b^4)*1i)/(4*a^7*b + a^8 + a
^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2) + (tan(c + d*x)*(65536*a^6*b^11 + 327680*a^
7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^
6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a*b + 2*a^2 + 2*b^2)*(4*a^7*
b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) +
(tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8
 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b^5)*1i)/(128*(4*a^7*b + a^8
+ a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) - (((45*a*
b^10)/16 + (305*a^2*b^9)/16 + (385*a^3*b^8)/8 + (657*a^4*b^7)/8 + (2081*a^5
*b^6)/16 + (1277*a^6*b^5)/16)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*
a^6*b^2))/(4*a*b + 2*a^2 + 2*b^2) + (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 189
4*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4
*a^5*b^3 + 6*a^6*b^2)))/(4*a*b + 2*a^2 + 2*b^2))/(((((((((960*a^7*b^8 - 22
4*a^5*b^10 - 144*a^6*b^9 - 48*a^4*b^11 + 2480*a^8*b^7 + 2592*a^9*b^6 + 1296
*a^10*b^5 + 256*a^11*b^4)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*
b^2) - (tan(c + d*x)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 3
27680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536
*a^13*b^4))/(128*(4*a*b + 2*a^2 + 2*b^2)*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b
^3 + 6*a^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) - (tan(c + d*x)*(1152*a^2*b^1
1 + 7936*a^3*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a
^7*b^6 - 49024*a^8*b^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a
^6*b^2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) - (((45*a*b^10)/16 + (305*a^2*b^9)/16
 + (385*a^3*b^8)/8 + (657*a^4*b^7)/8 + (2081*a^5*b^6)/16 + (1277*a^6*b^5)/1
6)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))*1i)/(4*a*b + 2*a^
2 + 2*b^2) - (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^
6 + 1425*a^4*b^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2
)))/(4*a*b + 2*a^2 + 2*b^2) + ((((((((960*a^7*b^8 - 224*a^5*b^10 - 144*a^6
*b^9 - 48*a^4*b^11 + 2480*a^8*b^7 + 2592*a^9*b^6 + 1296*a^10*b^5 + 256*a^11
*b^4)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2) + (tan(c + d*x)
*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 3276
80*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*
a*b + 2*a^2 + 2*b^2)*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i
)/(4*a*b + 2*a^2 + 2*b^2) + (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3*b^10 +
20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49024*a^8*b
^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(4*a*b
 + 2*a^2 + 2*b^2) - (((45*a*b^10)/16 + (305*a^2*b^9)/16 + (385*a^3*b^8)/8 +
 (657*a^4*b^7)/8 + (2081*a^5*b^6)/16 + (1277*a^6*b^5)/16)*1i)/(4*a^7*b + a^
8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))*1i)/(4*a*b + 2*a^2 + 2*b^2) + (tan(c
+ d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a^4*b^5)*1i
)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(4*a*b + 2*a^2 +
 2*b^2) + ((135*a*b^6)/64 + (81*b^7)/128 + (125*a^2*b^5)/128)/(4*a^7*b + a^
8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))))/(d*(4*a*b + 2*a^2 + 2*b^2)) + (atan

```
(((((((245760*a^7*b^8 - 57344*a^5*b^10 - 36864*a^6*b^9 - 12288*a^4*b^11 + 6
34880*a^8*b^7 + 663552*a^9*b^6 + 331776*a^10*b^5 + 65536*a^11*b^4)/(256*(4*
a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)) - (tan(c + d*x)*((9*b^3*(-a
^7*b)^(1/2) - 25*a^3*(-a^7*b)^(1/2) + 70*a^6*b + 6*a^4*b^3 + 44*a^5*b^2 + 4
1*a*b^2*(-a^7*b)^(1/2) + 39*a^2*b*(-a^7*b)^(1/2))/(256*(4*a^10*b + a^11 + a
^7*b^4 + 4*a^8*b^3 + 6*a^9*b^2)))^(1/2)*(65536*a^6*b^11 + 327680*a^7*b^10 +
 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b^7 - 589824*a^11*b^6 - 3276
80*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 +
6*a^6*b^2)))*((9*b^3*(-a^7*b)^(1/2) - 25*a^3*(-...
```

3.387 $\int \sqrt{a+b\tan^4(c+dx)}\,dx$

Optimal. Leaf size=650

$$\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d}+\frac{\sqrt{b}\,\tan(c+dx)\sqrt{a+b\tan^4(c+dx)}}{d\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}-\frac{\sqrt[4]{a}\,\sqrt[4]{b}\,E\Big(2\text{ArcTan}\Big(}{}$$

```
[Out] 1/2*arctan((a+b)^(1/2)*tan(d*x+c)/(a+tan(d*x+c)^4*b)^(1/2))*(a+b)^(1/2)/d+b
^(1/2)*(a+tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)/d/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)
-a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*tan(d*x+
c)/a^(1/4))),1/2*2^(1/2))*((a+tan(d*x+c)^4*b)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2
)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/d/(a+tan(d*x+c)^4*b)^(1/2)-1/2*b^
(1/4)*(a+b)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*tan(d*x+c)/a^
(1/4))),1/2*2^(1/2))*((a+tan(d*x+c)^4*b)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^
(1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/4)/d/(a^(1/2)-b^(1/2))/(a+tan(d*x
+c)^4*b)^(1/2)+1/2*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*tan(d*x+c)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)-b^(1/2))*((a+tan(d*x+c)^4*b)/(
a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/
4)/d/(a+tan(d*x+c)^4*b)^(1/2)+1/4*(a+b)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^
(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticPi(sin(2
*arctan(b^(1/4)*tan(d*x+c)/a^(1/4))),-1/4*(a^(1/2)-b^(1/2))^2/a^(1/2)/b^(1/
2),1/2*2^(1/2))*(a^(1/2)+b^(1/2))*((a+tan(d*x+c)^4*b)/(a^(1/2)+b^(1/2)*tan(
d*x+c)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/4)/b^(1/4)/d/(a^(1/2
)-b^(1/2))/(a+tan(d*x+c)^4*b)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3742, 1223, 1212, 226, 1210, 1231, 1721}

$$\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d}-\frac{\sqrt[4]{b}\,(a+b)\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}+\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,d\sqrt{a+b\tan^4(c+dx)}}-\frac{\sqrt[4]{a}\,\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{d\sqrt{a+b\tan^4(c+dx)}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)^2}}\,\Pi\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\,\sqrt{b}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\,\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}+\frac{\sqrt{b}\,\tan(c+dx)\sqrt{a+b\tan^4(c+dx)}}{d\left(\sqrt{a}+\sqrt{b}\,\tan^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^4]])
/(2*d) + (Sqrt[b]*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]^4])/(d*(Sqrt[a] + Sq
rt[b]*Tan[c + d*x]^2)) - (a^(1/4)*b^(1/4)*EllipticE[2*ArcTan[(b^(1/4)*Tan[c
 + d*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[
```

```
c + d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(d*Sqrt[a + b*Tan[c + d*
x]^4]) + ((Sqrt[a] - Sqrt[b])*b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d
*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c +
d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*d*Sqrt[a + b*Tan[
c + d*x]^4]) - (b^(1/4)*(a + b)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a
^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4
)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*d*S
qrt[a + b*Tan[c + d*x]^4]) + ((Sqrt[a] + Sqrt[b])*(a + b)*EllipticPi[-1/4*(
Sqrt[a] - Sqrt[b])^2/(Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(
1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/
(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/
4)*d*Sqrt[a + b*Tan[c + d*x]^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1223

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[-(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
```

```
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \sqrt{a+b\tan^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{b-bx^2}{\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^4}}\right.}{d}$$

$$= -\frac{\left(\sqrt{a}\sqrt{b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}\right.}{}$$

$$= \frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d} + \frac{\sqrt{b}\tan(c+dx)\sqrt{a+b\tan^4(c+}}{d\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.19, size = 219, normalized size = 0.34

$$\frac{\left(\sqrt{a}\,\sqrt{b}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\tan(c+dx)\right)\middle|-1\right)+\left(\sqrt{a}-i\sqrt{b}\right)\left(-\sqrt{b}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\tan(c+dx)\right)\middle|-1\right)+\left(-i\sqrt{a}+\sqrt{b}\right)\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\tan(c+dx)\right)\middle|-1\right)\right)\right)\sqrt{1+\frac{b\tan^4(c+dx)}{a}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,d\sqrt{a+b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x
]], -1] + (Sqrt[a] - I*Sqrt[b])*(-(Sqrt[b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt
[b])/Sqrt[a]]*Tan[c + d*x]], -1]) + ((-I)*Sqrt[a] + Sqrt[b])*EllipticPi[((-
I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], -1]
))*Sqrt[1 + (b*Tan[c + d*x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*
Tan[c + d*x]^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 531, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{b\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(dx+c)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\,(\tan^4(dx+c))}}+\frac{i\sqrt{b}\,\sqrt{a}\,\sqrt{1-}}{}}{}$
default	$\frac{-\frac{b\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(dx+c)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\,(\tan^4(dx+c))}}+\frac{i\sqrt{b}\,\sqrt{a}\,\sqrt{1-}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*
(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticF
(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan
(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticF(tan(d*x+c)*(I/a^(1/2)*b
^(1/2))^(1/2),I)-I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*ta
```

```
n(d*x+c)^4)^(1/2)*EllipticE(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I)+a/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticPi(tan(d*x+c)*
(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a
^(1/2)*b^(1/2))^(1/2))+b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan
(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4
)^(1/2)*EllipticPi(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(
-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c)^4 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*tan(d*x + c)^4 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan^4 (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)**4*b)**(1/2),x)
[Out] Integral(sqrt(a + b*tan(c + d*x)**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\tan\left(c+d\,x\right)^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^4)^(1/2),x)
```

```
[Out] int((a + b*tan(c + d*x)^4)^(1/2), x)
```

3.388 $\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx$

Optimal. Leaf size=348

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2\sqrt{a+b}\,d}-\frac{\sqrt[4]{b}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{\ \ }{\left(\sqrt{\ }\right.}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)d\sqrt{a+b\tan^4(c+dx)}}$$

```
[Out] 1/2*arctan((a+b)^(1/2)*tan(d*x+c)/(a+tan(d*x+c)^4*b)^(1/2))/d/(a+b)^(1/2)-1
/2*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*tan(d*x+c)/a^(
1/4))),1/2*2^(1/2))*((a+tan(d*x+c)^4*b)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^(
1/2)*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/4)/d/(a^(1/2)-b^(1/2))/(a+tan(d*x+
c)^4*b)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*tan(d*
x+c)/a^(1/4))),-1/4*(a^(1/2)-b^(1/2))^2/a^(1/2)/b^(1/2),1/2*2^(1/2))*(a^(1/
2)+b^(1/2))*((a+tan(d*x+c)^4*b)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^
(1/2)+b^(1/2)*tan(d*x+c)^2)/a^(1/4)/b^(1/4)/d/(a^(1/2)-b^(1/2))/(a+tan(d*x+
c)^4*b)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3742, 1231, 226, 1721}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d\sqrt{a+b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}\,\Pi\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^4]]/(2*Sqrt[a + b
]*d) - (b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(S
qrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqr
t[b]*Tan[c + d*x]^2)^2])/(2*a^(1/4)*(Sqrt[a] - Sqrt[b])*d*Sqrt[a + b*Tan[c
+ d*x]^4]) + ((Sqrt[a] + Sqrt[b])*EllipticPi[-1/4*(Sqrt[a] - Sqrt[b])^2/(Sq
rt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(Sqrt[a] + S
qrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c
 + d*x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*d*Sqrt[a + b*Tan[c +
d*x]^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 3742

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[c*(ff/f), Subst[Int[(a + b*(
ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(c+dx)\right)}{\left(\sqrt{a}-\sqrt{b}\right)d} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}}\right.}{\left(\sqrt{a}-\right.}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2\sqrt{a+b}\,d} - \frac{\sqrt[4]{b}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}\right.}{2\sqrt[4]{a}\left(\sqrt{a}-\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.26, size = 106, normalized size = 0.30

$$-\frac{i\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}}; i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle| -1\right)\sqrt{1+\frac{b\tan^4(c+dx)}{a}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,d\sqrt{a+b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Tan[c + d*x]^4],x]
```

```
[Out] ((-I)*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]
]*Tan[c + d*x]], -1]*Sqrt[1 + (b*Tan[c + d*x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt
[a]]*d*Sqrt[a + b*Tan[c + d*x]^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 123, normalized size = 0.35

method	result

derivativedivides	$\frac{\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(\tan(dx+c)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\frac{i\sqrt{a}}{\sqrt{b}},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\left(\tan^4(dx+c)\right)}}$
default	$\frac{\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,(\tan^2(dx+c))}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(\tan(dx+c)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\frac{i\sqrt{a}}{\sqrt{b}},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\left(\tan^4(dx+c)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tan(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticPi(ta
n(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*tan(d*x + c)^4 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)**4*b)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*tan(c + d*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(d*x + c)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\tan\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(c + d*x)^4)^(1/2),x)
```

```
[Out] int(1/(a + b*tan(c + d*x)^4)^(1/2), x)
```

3.389 $\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=103

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}}+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{4}(2-\tan^2(x))\sqrt{a+\ldots}$$

```
[Out] 1/4*(a+2*b)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2*arct
anh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))*(a+b)^(1/2)-1/4*(a+b*t
an(x)^4)^(1/2)*(2-tan(x)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3751, 1266, 829, 858, 223, 212, 739}

$$-\frac{1}{4}(2-\tan^2(x))\sqrt{a+b\tan^4(x)}+\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}}+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/(4*Sqrt[b]) +
(Sqrt[a + b]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/
2 - ((2 - Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Subst}\left(\int \frac{x^3\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx^2}}{1+x}\,dx, x, \tan^2(x)\right)$$

$$= -\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{\text{Subst}\left(\int \frac{-ab+b(a+2b)x}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x\right.}{4b}$$

$$= -\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\right.$$

$$= -\frac{1}{4}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{2}(a+b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, -\right.$$

$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}} + \frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2}{\sqrt{a+b}\sqrt{a+b}}\right.$$

Mathematica [A]
time = 2.56, size = 145, normalized size = 1.41

$$\frac{1}{4}\left(2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+2\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)+\frac{\left(-2+\tan^2(x)\right)\left(a+b\tan^4(x)\right)+\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan^4(x)}{a}}}{\sqrt{b}}}{\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2*Sqrt[a + b]
*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + ((-2 + Tan[
x]^2)*(a + b*Tan[x]^4) + (a^(3/2)*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[
1 + (b*Tan[x]^4)/a])/Sqrt[b])/Sqrt[a + b*Tan[x]^4])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(83) = 166.
time = 0.14, size = 181, normalized size = 1.76

method	result

derivativedivides	$\frac{\sqrt{a+b\left(\tan^4(x)\right)}\,(\tan^2(x))}{4}+\frac{a\ln\left(\sqrt{b}\,(\tan^2(x))+\sqrt{a+b\left(\tan^4(x)\right)}\right)}{4\sqrt{b}}-\sqrt{b\left(1+\tan^2(x\right.}$
default	$\frac{\sqrt{a+b\left(\tan^4(x)\right)}\,(\tan^2(x))}{4}+\frac{a\ln\left(\sqrt{b}\,(\tan^2(x))+\sqrt{a+b\left(\tan^4(x)\right)}\right)}{4\sqrt{b}}-\sqrt{b\left(1+\tan^2(x\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(x)^4)^(1/2)*tan(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a+b*tan(x)^4)^(1/2)*tan(x)^2+1/4*a/b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*ta
n(x)^4)^(1/2))-1/2*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2)+1/2*b^(1/2
)*ln((b*(1+tan(x)^2)-b)/b^(1/2)+(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/
2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)
^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x)^3, x)
```

Fricas [A]
time = 4.71, size = 555, normalized size = 5.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*
tan(x)^2 - a) + 2*sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^
2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan
(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/b, -1
/4*((a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) -
 sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(
x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^
```

```
2 + 1)) - sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/b, 1/8*(4*sqrt(-a - b)*b
*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan
(x)^4 + a^2 + a*b)) + (a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)
^4 + a)*sqrt(b)*tan(x)^2 - a) + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/
b, 1/4*(2*sqrt(-a - b)*b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(
-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) - (a + 2*b)*sqrt(-b)*arctan(sqr
t(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + sqrt(b*tan(x)^4 + a)*(b*tan(x)^2
 - 2*b))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^4{(x)}}\,\tan^3{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)**4)**(1/2)*tan(x)**3,x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*tan(x)**3, x)
```

Giac [A]
time = 0.45, size = 107, normalized size = 1.04

$$\frac{1}{4}\sqrt{b\tan\left(x\right)^{4}+a}\,\left(\tan\left(x\right)^{2}-2\right)-\frac{\left(a+b\right)\arctan\left(-\frac{\sqrt{b}\tan(x)^{2}-\sqrt{b\tan\left(x\right)^{4}+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{\left(a\sqrt{b}+2\,b^{\frac{3}{2}}\right)\log\left(\left|-\sqrt{b}\tan\left(x\right)^{2}+\sqrt{b\tan\left(x\right)^{4}+a}\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*tan(x)^4 + a)*(tan(x)^2 - 2) - (a + b)*arctan(-(sqrt(b)*tan(x)^2
 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/sqrt(-a - b) - 1/4*(a*sqrt
(b) + 2*b^(3/2))*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 + a)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(x\right)}^3\,\sqrt{b\,{\mathrm{tan}\left(x\right)}^4+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)^3*(a + b*tan(x)^4)^(1/2), x)
```

3.390 $\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=90

$$-\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\sqrt{a+b\tan^4(x)}$$

```
[Out] -1/2*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))*b^(1/2)-1/2*arctanh((a-
b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))*(a+b)^(1/2)+1/2*(a+b*tan(x)^4
)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3751, 1262, 749, 858, 223, 212, 739}

$$\frac{1}{2}\sqrt{a+b\tan^4(x)}-\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -1/2*(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]) - (Sqrt[a +
 b]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/2 + Sqrt[
a + b*Tan[x]^4]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Subst}\left(\int \frac{x\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x}\,dx, x, \tan^2(x)\right)$$
$$= \frac{1}{2}\sqrt{a+b\tan^4(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{a-bx}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)$$
$$= \frac{1}{2}\sqrt{a+b\tan^4(x)} - \frac{1}{2}b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right) + \frac{1}{2}(a+b)\text{Sul}$$
$$= \frac{1}{2}\sqrt{a+b\tan^4(x)} + \frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right.$$
$$= -\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2}{\sqrt{a+b}\sqrt{a+b}}\right.$$

Mathematica [A]
time = 0.03, size = 86, normalized size = 0.96

$$\frac{1}{2}\left(-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) + \sqrt{a+b\tan^4(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]) - Sqrt[a + b]*
ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + Sqrt[a + b*T
an[x]^4])/2
```

Maple [A]
time = 0.09, size = 139, normalized size = 1.54

method	result
derivativedivides	$\frac{\sqrt{b\left(1+\tan^2(x)\right)^2-2b\left(1+\tan^2(x)\right)+a+b}}{2} - \frac{\sqrt{b}\ln\left(\frac{b\left(1+\tan^2(x)\right)-b}{\sqrt{b}}+\sqrt{b\left(1+\tan^2(x\right.}\right.}{2}$

default	$\frac{\sqrt{b\left(1+\tan^2(x)\right)^2-2b\left(1+\tan^2(x)\right)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b\left(1+\tan^2(x)\right)-b}{\sqrt{b}}+\sqrt{b\left(1+\tan^2(x)\right.}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(x)^4)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(1+tan(
x)^2)-b)/b^(1/2)+(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))-1/2*(a+b)^(
1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+ta
n(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x), x)
```

Fricas [A]
time = 2.63, size = 475, normalized size = 5.28

$$\left[\frac{1}{4}\sqrt{b}\log\left(-2b\tan(x)^4+2\sqrt{b\tan(x)^4+a}\sqrt{b}\tan(x)^2-a\right)+\frac{1}{4}\sqrt{a+b}\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}(b\tan(x)^2-a)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+\frac{1}{2}\sqrt{b\tan(x)^4+a},\ \frac{1}{2}\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\sqrt{-b}}{b\tan(x)^2}\right)+\frac{1}{4}\sqrt{a+b}\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}(b\tan(x)^2-a)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+\frac{1}{2}\sqrt{b\tan(x)^4+a},\ -\frac{1}{2}\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}(b\tan(x)^2-a)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)+\frac{1}{4}\sqrt{b}\log\left(-2b\tan(x)^4+2\sqrt{b\tan(x)^4+a}\sqrt{b}\tan(x)^2-a\right)+\frac{1}{2}\sqrt{b\tan(x)^4+a},\ -\frac{1}{2}\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}(b\tan(x)^2-a)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)+\frac{1}{2}\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\sqrt{-b}}{b\tan(x)^2}\right)+\frac{1}{2}\sqrt{b\tan(x)^4+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 -
a) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(
b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*t
an(x)^2 + 1)) + 1/2*sqrt(b*tan(x)^4 + a), 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)
^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^
4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) +
2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/2*sqrt(b*tan(x)^4 + a), -1/2*
sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*
b + b^2)*tan(x)^4 + a^2 + a*b)) + 1/4*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*
tan(x)^4 + a)*sqrt(b)*tan(x)^2 - a) + 1/2*sqrt(b*tan(x)^4 + a), -1/2*sqrt(-
a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^
2)*tan(x)^4 + a^2 + a*b)) + 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-
b)/(b*tan(x)^2)) + 1/2*sqrt(b*tan(x)^4 + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan^4(x)}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)**4)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*tan(x), x)
```

Giac [A]
time = 0.45, size = 89, normalized size = 0.99

$$\frac{(a+b)\arctan\left(-\frac{\sqrt{b}\,\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}+\frac{1}{2}\sqrt{b}\,\log\left(\left|-\sqrt{b}\,\tan(x)^2+\sqrt{b\tan(x)^4+a}\right|\right)+\frac{1}{2}\sqrt{b\tan(x)^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] (a + b)*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a
 - b))/sqrt(-a - b) + 1/2*sqrt(b)*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)
^4 + a))) + 1/2*sqrt(b*tan(x)^4 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(x)\,\sqrt{b\tan(x)^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*tan(x)^4)^(1/2), x)
```

3.391 $\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=102

$$\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{}}\right.$$

```
[Out] -1/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))*a^(1/2)+1/2*arctanh(b^(1/2)*tan(
x)^2/(a+b*tan(x)^4)^(1/2))*b^(1/2)+1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(
a+b*tan(x)^4)^(1/2))*(a+b)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3751, 1266, 910, 272, 65, 214, 858, 223, 212, 739}

$$-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 + (Sqrt[a + b]
*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/2 - (Sqrt[a]
*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 910

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[Simp[c*d*f + a*e*g - c*(e*
f - d*g)*x, x]*((a + c*x^2)^(p - 1)/(f + g*x)), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
```

alQ[n]))

Rubi steps

$$\begin{aligned}
\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{x\left(1+x^2\right)}\,dx,x,\tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(1+x)}\,dx,x,\tan^2(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a-bx}{(1+x)\sqrt{a+bx^2}}\,dx,x,\tan^2(x)\right)\right)+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+b}}\right.\\
&= \frac{1}{4}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\tan^4(x)\right)+\frac{1}{2}b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\tan\right.\\
&= -\left(\frac{1}{2}(-a-b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx,x,\frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right)+\frac{a\text{Subst}\left(\int\right.}{}\\
&= \frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}\sqrt{a+b}\,\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan}}\right.
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 98, normalized size = 0.96

$$\frac{1}{2}\left(\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\sqrt{a+b}\,\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + Sqrt[a + b]*Arc
Tanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] - Sqrt[a]*ArcTanh
[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \cot\left(x\right)\sqrt{a+b\left(\tan^4\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)*(a+b*tan(x)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*cot(x), x)
```

Fricas [A]
time = 3.07, size = 1021, normalized size = 10.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a
) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b
*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*ta
n(x)^2 + 1)) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a)
 + 2*a)/tan(x)^4), -1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*ta
n(x)^2)) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2
*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4
 + 2*tan(x)^2 + 1)) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*
sqrt(a) + 2*a)/tan(x)^4), 1/2*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)
/a) + 1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^
2 + a) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*s
qrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 +
 2*tan(x)^2 + 1)), 1/2*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) - 1
/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(a
 + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)
*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)),
1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a))
 + 1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 +
 a) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/t
an(x)^4), 1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(
x)^2 - a)) - 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)
) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan
(x)^4), 1/2*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*sqrt(-a
- b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a)) + 1/4*sqrt(
```

```
b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a), 1/2*sqr
t(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*sqrt(-a - b)*arctan(sqr
t(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a)) - 1/2*sqrt(-b)*arctan(sqrt
(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^4(x)}\,\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*cot(x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(x)\,\sqrt{b\tan(x)^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)*(a + b*tan(x)^4)^(1/2), x)
```

3.392 $\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal. Leaf size=643

$$-\frac{1}{2}\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)}-\frac{\sqrt{b}\,\tan(x)\sqrt{a+b\tan^4(x)}}{\sqrt{a}+\sqrt{b}\,\tan^2(x)}+\frac{\sqrt[4]{a}\,\sqrt[4]{b}}{}$$

```
[Out] -1/2*arctan((a+b)^(1/2)*tan(x)/(a+b*tan(x)^4)^(1/2))*(a+b)^(1/2)+1/3*(a+b*t
an(x)^4)^(1/2)*tan(x)-b^(1/2)*(a+b*tan(x)^4)^(1/2)*tan(x)/(a^(1/2)+b^(1/2)*
tan(x)^2)+a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*tan(x)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*tan(x)/
a^(1/4))),1/2*2^(1/2))*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)*
(a^(1/2)+b^(1/2)*tan(x)^2)/(a+b*tan(x)^4)^(1/2)+1/3*a^(3/4)*(cos(2*arctan(b
^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))*Elli
pticF(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),1/2*2^(1/2))*((a+b*tan(x)^4)/(a
^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/b^(1/4)/(a+b*t
an(x)^4)^(1/2)+1/2*b^(1/4)*(a+b)*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
tan(x)/a^(1/4))),1/2*2^(1/2))*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)^2)
^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/a^(1/4)/(a^(1/2)-b^(1/2))/(a+b*tan(x)^4)^
(1/2)-1/2*b^(1/4)*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*tan(x)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)-b^(1/2))*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)
^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/a^(1/4)/(a+b*tan(x)^4)^(1/2)-1/4*(a+b)
*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(x
)/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),-1/4*(a^(1/2)-
b^(1/2))^2/a^(1/2)/b^(1/2),1/2*2^(1/2))*(a^(1/2)+b^(1/2))*((a+b*tan(x)^4)/(
a^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/a^(1/4)/b^(1/
4)/(a^(1/2)-b^(1/2))/(a+b*tan(x)^4)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3751, 1350, 201, 226, 1223, 1212, 1210, 1231, 1721}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{a+b\tan^4(x)}}-\frac{1}{2}\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{\sqrt[4]{b}\,(a+b)\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}-\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt{a+b\tan^4(x)}}+\frac{\sqrt[4]{a}\,\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+b\tan^4(x)}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\,\tan^2(x)\right)^2}}\,\Pi\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\,\sqrt{b}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\,\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}+\frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)}-\frac{\sqrt{b}\,\tan(x)\sqrt{a+b\tan^4(x)}}{\sqrt{a}+\sqrt{b}\,\tan^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -1/2*(Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a + b*Tan[x]^4]]) + (Tan
[x]*Sqrt[a + b*Tan[x]^4])/3 - (Sqrt[b]*Tan[x]*Sqrt[a + b*Tan[x]^4])/(Sqrt[a
```

```
] + Sqrt[b]*Tan[x]^2) + (a^(1/4)*b^(1/4)*EllipticE[2*ArcTan[(b^(1/4)*Tan[x]
)/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a
] + Sqrt[b]*Tan[x]^2)^2])/Sqrt[a + b*Tan[x]^4] + (a^(3/4)*EllipticF[2*ArcTa
n[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*
Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(3*b^(1/4)*Sqrt[a + b*Tan[x]^4])
 - ((Sqrt[a] - Sqrt[b])*b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)
], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[
b]*Tan[x]^2)^2])/(2*a^(1/4)*Sqrt[a + b*Tan[x]^4]) + (b^(1/4)*(a + b)*Ellipt
icF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*S
qrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(2*a^(1/4)*(Sqrt[a] -
 Sqrt[b])*Sqrt[a + b*Tan[x]^4]) - ((Sqrt[a] + Sqrt[b])*(a + b)*EllipticPi[-
1/4*(Sqrt[a] - Sqrt[b])^2/(Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[x])/a^(1
/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sq
rt[b]*Tan[x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x
]^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1223

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[-(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Subst}\left(\int \frac{x^2\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\sqrt{a+bx^4} - \frac{\sqrt{a+bx^4}}{1+x^2}\right)dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \sqrt{a+bx^4}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)} + \frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \tan(x)\right) - (a$$

$$= \frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)} + \frac{a^{3/4}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\,\text{ta}\right.}{3\sqrt[4]{b}\sqrt{a+b\tan^4}}$$

$$= -\frac{1}{2}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)} - \frac{\sqrt{b}\,\text{t}}{\sqrt{}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 12.48, size = 404, normalized size = 0.63

$$\sqrt{\frac{3a+3b+4a\cos(2x)-4b\cos(2x)+a\cos(4x)+b\cos(4x)}{3+4\cos(2x)+\cos(4x)}}\left(-\frac{1}{2}\sin(2x)+\frac{\tan(x)}{3}\right)+\frac{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\cos(x)\sin(x)+3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b\sin^2(x)\tan^3(x)-3\sqrt{a}\sqrt{b}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)\sqrt{1+\frac{b\tan^4(x)}{a}}+\left(-2ia+3\sqrt{a}\sqrt{b}-3ib\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)\sqrt{1+\frac{b\tan^4(x)}{a}}+3ia\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)\sqrt{1+\frac{b\tan^4(x)}{a}}+3ib\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)\sqrt{1+\frac{b\tan^4(x)}{a}}}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2*Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] Sqrt[(3*a + 3*b + 4*a*Cos[2*x] - 4*b*Cos[2*x] + a*Cos[4*x] + b*Cos[4*x])/(3
 + 4*Cos[2*x] + Cos[4*x])]*(-1/2*Sin[2*x] + Tan[x]/3) + (3*a*Sqrt[(I*Sqrt[b
])/Sqrt[a]]*Cos[x]*Sin[x] + 3*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Sin[x]^2*Tan[x]^3
 - 3*Sqrt[a]*Sqrt[b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]],
 -1]*Sqrt[1 + (b*Tan[x]^4)/a] + ((-2*I)*a + 3*Sqrt[a]*Sqrt[b] - (3*I)*b)*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*Sqrt[1 + (b*Tan[x]
^4)/a] + (3*I)*a*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[
b])/Sqrt[a]]*Tan[x]], -1]*Sqrt[1 + (b*Tan[x]^4)/a] + (3*I)*b*EllipticPi[((-
I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*Sqrt[
1 + (b*Tan[x]^4)/a])/(3*Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.10, size = 537, normalized size = 0.84

method	result
derivativedivides	$\frac{\sqrt{a+b\left(\tan^4\left(x\right)\right)}\,\tan\left(x\right)}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,\left(\tan^2\left(x\right)\right)}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\left(\tan^2\left(x\right)\right)}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan\left(x\right)\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\left(\tan^4\left(x\right)\right)}}$
default	$\frac{\sqrt{a+b\left(\tan^4\left(x\right)\right)}\,\tan\left(x\right)}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,\left(\tan^2\left(x\right)\right)}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\left(\tan^2\left(x\right)\right)}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan\left(x\right)\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\left(\tan^4\left(x\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(x)^4)^(1/2)*tan(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a+b*tan(x)^4)^(1/2)*tan(x)+2/3*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^
4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)+b/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)
^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-I
*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(ta
n(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2
)/(a+b*tan(x)^4)^(1/2)*EllipticE(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-a/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticPi(tan(x)*(I/a^(1/2)*b^(1
/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))
^(1/2))-b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1
+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticPi(tan(x)*(
I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^
(1/2)*b^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\tan^4(x)}\,\tan^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)**4)**(1/2)*tan(x)**2,x)
```

```
[Out] Integral(sqrt(a + b*tan(x)**4)*tan(x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(x)^4)^(1/2)*tan(x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(x)^2\,\sqrt{b\tan(x)^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2*(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)^2*(a + b*tan(x)^4)^(1/2), x)
```

3.393 $\int \tan^3(x) \left(a + b\tan^4(x)\right)^{3/2} dx$

Optimal. Leaf size=148

$$\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}}+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{16}(8(a$$

```
[Out] 1/2*(a+b)^(3/2)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))+1/
16*(3*a^2+12*a*b+8*b^2)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))/b^(1
/2)-1/16*(a+b*tan(x)^4)^(1/2)*(8*a+8*b-(3*a+4*b)*tan(x)^2)-1/24*(4-3*tan(x)
^2)*(a+b*tan(x)^4)^(3/2)
```

Rubi [A]
time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3751, 1266, 829, 858, 223, 212, 739}

$$\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}}-\frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2}-\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])
/(16*Sqrt[b]) + ((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a
 + b*Tan[x]^4])])/2 - ((8*(a + b) - (3*a + 4*b)*Tan[x]^2)*Sqrt[a + b*Tan[x]
^4])/16 - ((4 - 3*Tan[x]^2)*(a + b*Tan[x]^4)^(3/2))/24
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \tan^3(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx=\text{Subst}\left(\int \frac{x^3(a+bx^4)^{3/2}}{1+x^2}\,dx,x,\tan(x)\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx^2)^{3/2}}{1+x}\,dx,x,\tan^2(x)\right)$$

$$=-\frac{1}{24}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2}+\frac{\text{Subst}\left(\int \frac{(-ab+b(3a+4b)x)\sqrt{a+b}}{1+x}\right.}{8b}$$

$$=-\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)}-\frac{1}{24}\left(4-3\tan^2(x)\right.$$

$$=-\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)}-\frac{1}{24}\left(4-3\tan^2(x)\right.$$

$$=-\frac{1}{16}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\sqrt{a+b\tan^4(x)}-\frac{1}{24}\left(4-3\tan^2(x)\right.$$

$$=\frac{(3a^2+12ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}}+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\right.$$

Mathematica [A]
time = 4.22, size = 189, normalized size = 1.28

$$\frac{1}{48}\left(24\sqrt{b}(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+24(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)+\frac{3\sqrt{a}(3a+4b)\sinh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a}}\right)\sqrt{a+b\tan^4(x)}}{\sqrt{b}\sqrt{1+\frac{b\tan^4(x)}{a}}}+\sqrt{a+b\tan^4(x)}\left(-8(4a+3b)+3(5a+4b)\tan^2(x)-8b\tan^4(x)+6b\tan^6(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (24*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 24*(
a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] +
 (3*Sqrt[a]*(3*a + 4*b)*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[a + b*Tan[
x]^4])/(Sqrt[b]*Sqrt[1 + (b*Tan[x]^4)/a]) + Sqrt[a + b*Tan[x]^4]*(-8*(4*a +
 3*b) + 3*(5*a + 4*b)*Tan[x]^2 - 8*b*Tan[x]^4 + 6*b*Tan[x]^6))/48
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(125) = 250$.
time = 0.11, size = 374, normalized size = 2.53

method	result
derivativedivides	$\frac{b(\tan^6(x))\sqrt{a+b(\tan^4(x))}}{8} + \frac{5a(\tan^2(x))\sqrt{a+b(\tan^4(x))}}{16} + \frac{3a^2\ln\left(\sqrt{b}\,(\tan^2(x))+\sqrt{a+b(\right.}}{16\sqrt{b}}$
default	$\frac{b(\tan^6(x))\sqrt{a+b(\tan^4(x))}}{8} + \frac{5a(\tan^2(x))\sqrt{a+b(\tan^4(x))}}{16} + \frac{3a^2\ln\left(\sqrt{b}\,(\tan^2(x))+\sqrt{a+b(\right.}}{16\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a+b*tan(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b*tan(x)^6*(a+b*tan(x)^4)^(1/2)+5/16*a*tan(x)^2*(a+b*tan(x)^4)^(1/2)+3/
16*a^2*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))/b^(1/2)-1/6*b*tan(x)^4*(a+
b*tan(x)^4)^(1/2)-2/3*a*(a+b*tan(x)^4)^(1/2)+1/4*b*tan(x)^2*(a+b*tan(x)^4)^
(1/2)+3/4*a*b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))-1/2*b*(a+b*ta
n(x)^4)^(1/2)+1/2*b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))+1/2/(a+
b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(
1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))*a^2+1/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+
tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+t
an(x)^2))*a*b+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b
*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))*b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*tan(x)^3, x)
```

Fricas [A]
time = 4.19, size = 758, normalized size = 5.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(
x)^4 + a)*sqrt(b)*tan(x)^2 - a) + 24*(a*b + b^2)*sqrt(a + b)*log(((a*b + 2*
```

```
b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sq
rt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*(6*b^2*tan(x)^6 -
 8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(
x)^4 + a))/b, -1/48*(3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(b*tan(
x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) - 12*(a*b + b^2)*sqrt(a + b)*log(((a*b + 2
*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*s
qrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) - (6*b^2*tan(x)^6 -
8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x
)^4 + a))/b, 1/96*(48*(a*b + b^2)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*
(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + 3*(3*a^
2 + 12*a*b + 8*b^2)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt
(b)*tan(x)^2 - a) + 2*(6*b^2*tan(x)^6 - 8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*
tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x)^4 + a))/b, 1/48*(24*(a*b + b^2)*s
qrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b
 + b^2)*tan(x)^4 + a^2 + a*b)) - 3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(-b)*arctan
(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + (6*b^2*tan(x)^6 - 8*b^2*tan(
x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x)^4 + a))/
b]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^{4}\left(x\right)\right)^{\frac{3}{2}}\tan^{3}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3*(a+b*tan(x)**4)**(3/2),x)
[Out] Integral((a + b*tan(x)**4)**(3/2)*tan(x)**3, x)
```

Giac [**A**]

time = 0.46, size = 176, normalized size = 1.19

$$\frac{1}{48}\,\sqrt{b\tan\left(x\right)^{4}+a}\left(\left(2\left(3\,b\tan\left(x\right)^{2}-4\,b\right)\tan\left(x\right)^{2}+\frac{3\left(5\,ab^{2}+4\,b^{3}\right)}{b^{2}}\right)\tan\left(x\right)^{2}-\frac{8\left(4\,ab^{2}+3\,b^{3}\right)}{b^{2}}\right)-\frac{\left(a^{2}+2\,ab+b^{2}\right)\arctan\left(-\frac{\sqrt{b}\,\tan(x)^{2}-\sqrt{b\tan\left(x\right)^{4}+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{\left(3\,a^{2}\sqrt{b}+12\,ab^{\frac{3}{2}}+8\,b^{\frac{5}{2}}\right)\log\left(\left|-\sqrt{b}\,\tan\left(x\right)^{2}+\sqrt{b\tan\left(x\right)^{4}+a}\right|\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
[Out] 1/48*sqrt(b*tan(x)^4 + a)*((2*(3*b*tan(x)^2 - 4*b)*tan(x)^2 + 3*(5*a*b^2 +
4*b^3)/b^2)*tan(x)^2 - 8*(4*a*b^2 + 3*b^3)/b^2) - (a^2 + 2*a*b + b^2)*arcta
n(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/sqrt(-
a - b) - 1/16*(3*a^2*sqrt(b) + 12*a*b^(3/2) + 8*b^(5/2))*log(abs(-sqrt(b)*t
an(x)^2 + sqrt(b*tan(x)^4 + a)))/b
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(x\right)}^3\,{\left(b\,{\mathrm{tan}\left(x\right)}^4+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3*(a + b*tan(x)^4)^(3/2),x)
[Out] int(tan(x)^3*(a + b*tan(x)^4)^(3/2), x)
```

3.394 $\int \tan(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{4}(2(a+b)$$

```
[Out] -1/2*(a+b)^(3/2)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))-1
/4*(3*a+2*b)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))*b^(1/2)+1/4*(a+
b*tan(x)^4)^(1/2)*(2*a+2*b-b*tan(x)^2)+1/6*(a+b*tan(x)^4)^(3/2)
```

Rubi [A]

time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3751, 1262, 749, 829, 858, 223, 212, 739}

$$\frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}+\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}-\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])
 - ((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4
])])/2 + ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4 + (a + b*Tan[x]^
4)^(3/2)/6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}\int \tan(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx &= \text{Subst}\left(\int \frac{x(a+bx^4)^{3/2}}{1+x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{1+x}\,dx, x, \tan^2(x)\right)\\ &= \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}+\frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)\sqrt{a+bx^2}}{1+x}\,dx, x, \tan^2(x)\right)\\ &= \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}+\frac{\text{Subst}(}{}\\ &= \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}+\frac{1}{2}(a+\\ &= \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}-\frac{1}{2}(a+\\ &= -\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{}{\sqrt{a}}\right.\end{aligned}$$

Mathematica [A]
time = 3.04, size = 166, normalized size = 1.32

$$\frac{1}{12}\left(-6\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-6(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)+\sqrt{a+b\tan^4(x)}\left(8a+6b-3b\tan^2(x)+2b\tan^4(x)\right)-\frac{3\sqrt{a}\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a}}\right)\sqrt{a+b\tan^4(x)}}{\sqrt{1+\frac{b\tan^4(x)}{a}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] - 6*(a
 + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] +
Sqrt[a + b*Tan[x]^4]*(8*a + 6*b - 3*b*Tan[x]^2 + 2*b*Tan[x]^4) - (3*Sqrt[a]
*Sqrt[b]*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])/Sqrt[1 +
 (b*Tan[x]^4)/a])/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(103) = 206.
time = 0.09, size = 313, normalized size = 2.48

method	result

derivativedivides	$\frac{b(\tan^4(x))\sqrt{a+b(\tan^4(x))}}{6}+\frac{2a\sqrt{a+b(\tan^4(x))}}{3}-\frac{b(\tan^2(x))\sqrt{a+b(\tan^4(x))}}{4}-\frac{3a\sqrt{}}{}$
default	$\frac{b(\tan^4(x))\sqrt{a+b(\tan^4(x))}}{6}+\frac{2a\sqrt{a+b(\tan^4(x))}}{3}-\frac{b(\tan^2(x))\sqrt{a+b(\tan^4(x))}}{4}-\frac{3a\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a+b*tan(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b*tan(x)^4*(a+b*tan(x)^4)^(1/2)+2/3*a*(a+b*tan(x)^4)^(1/2)-1/4*b*tan(x)
^2*(a+b*tan(x)^4)^(1/2)-3/4*a*b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1
/2))+1/2*b*(a+b*tan(x)^4)^(1/2)-1/2*b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)
^4)^(1/2))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1
+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))*a^2-1/(a+b)^(1/2)*l
n((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2
)+a+b)^(1/2))/(1+tan(x)^2))*a*b-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2
)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2)
)*b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*tan(x), x)
```

Fricas [A]
time = 2.91, size = 593, normalized size = 4.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)
*tan(x)^2 - a) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(
x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(
```

```
tan(x)^4 + 2*tan(x)^2 + 1)) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b
)*sqrt(b*tan(x)^4 + a), 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a
)*sqrt(-b)/(b*tan(x)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 -
2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^
2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 +
 8*a + 6*b)*sqrt(b*tan(x)^4 + a), -1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*t
an(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*
b)) + 1/8*(3*a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sq
rt(b)*tan(x)^2 - a) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b)*sqrt(b
*tan(x)^4 + a), -1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*ta
n(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + 1/4*(3*a + 2
*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/12*(2*b
*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b)*sqrt(b*tan(x)^4 + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^{4}\left(x\right)\right)^{\frac{3}{2}} \tan\left(x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)**4)**(3/2),x)
[Out] Integral((a + b*tan(x)**4)**(3/2)*tan(x), x)
```

Giac [A]
time = 0.53, size = 138, normalized size = 1.10

$$\frac{1}{4}\left(3\,a\sqrt{b} + 2\,b^{\frac{3}{2}}\right)\log\left(\left|-\sqrt{b}\,\tan\left(x\right)^{2} + \sqrt{b\tan\left(x\right)^{4} + a}\right|\right) + \frac{1}{12}\,\sqrt{b\tan\left(x\right)^{4} + a}\,\left(\left(2\,b\tan\left(x\right)^{2} - 3\,b\right)\tan\left(x\right)^{2} + \frac{2\,\left(4\,ab + 3\,b^{2}\right)}{b}\right) + \frac{\left(a^{2} + 2\,ab + b^{2}\right)\arctan\left(-\frac{\sqrt{b}\,\tan(x)^{2} - \sqrt{b\tan\left(x\right)^{4} + a} + \sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
[Out] 1/4*(3*a*sqrt(b) + 2*b^(3/2))*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 +
 a))) + 1/12*sqrt(b*tan(x)^4 + a)*((2*b*tan(x)^2 - 3*b)*tan(x)^2 + 2*(4*a*b
 + 3*b^2)/b) + (a^2 + 2*a*b + b^2)*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x
)^4 + a) + sqrt(b))/sqrt(-a - b))/sqrt(-a - b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(x)\,\left(b\tan(x)^4 + a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*tan(x)^4)^(3/2),x)
[Out] int(tan(x)*(a + b*tan(x)^4)^(3/2), x)
```

3.395 $\int \cot(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=155

$$\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\right.$$

```
[Out] -1/2*a^(3/2)*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))+1/2*(a+b)^(3/2)*arctanh(
(a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))+1/4*(3*a+2*b)*arctanh(b^(1
/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))*b^(1/2)+1/2*a*(a+b*tan(x)^4)^(1/2)-1/4*(
a+b*tan(x)^4)^(1/2)*(2*a+2*b-b*tan(x)^2)
```

Rubi [A]
time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3751, 1266, 910, 272, 52, 65, 214, 829, 858, 223, 212, 739}

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\frac{1}{2}a\sqrt{a+b\tan^4(x)}-\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}+\frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/4 +
((a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]
)/2 - (a^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4/Sqrt[a]]])/2 + (a*Sqrt[a + b*Tan
[x]^4])/2 - ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/4
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 910

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[Simp[c*d*f + a*e*g - c*(e*
f - d*g)*x, x]*((a + c*x^2)^(p - 1)/(f + g*x)), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \cot(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx = \text{Subst}\left(\int \frac{(a+bx^4)^{3/2}}{x\,(1+x^2)}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x(1+x)}\,dx, x, \tan^2(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)\sqrt{a+bx^2}}{1+x}\,dx, x, \tan^2(x)\right)\right) + \frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{a}}{}\right.$$

$$= -\frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{4}a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,\right.$$

$$= \frac{1}{2}a\sqrt{a+b\tan^4(x)} - \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{1}{4}a^2\text{Sub}$$

$$= \frac{1}{2}a\sqrt{a+b\tan^4(x)} - \frac{1}{4}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)} + \frac{a^2\text{Subs}}{}$$

$$= \frac{1}{4}\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\tanh^{-1}\left(\frac{}{\sqrt{a+}}\right.$$

Mathematica [A]
time = 2.05, size = 190, normalized size = 1.23

$$\frac{1}{4}\left(2\sqrt{b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + 2(a+b)^{3/2}\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right) - 2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right) - 2b\sqrt{a+b\tan^4(x)} + b\tan^2(x)\sqrt{a+b\tan^4(x)} + \frac{\sqrt{a}\,\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a}}\right)\sqrt{a+b\tan^4(x)}}{\sqrt{1+\frac{b\tan^4(x)}{a}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2*(a
+ b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] - 2
*a^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]] - 2*b*Sqrt[a + b*Tan[x]^4] +
 b*Tan[x]^2*Sqrt[a + b*Tan[x]^4] + (Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tan[x]
^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])/Sqrt[1 + (b*Tan[x]^4)/a])/4
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \cot\left(x\right)\left(a+b\left(\tan^4\left(x\right)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)*(a+b*tan(x)^4)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(x)^4 + a)^(3/2)*cot(x), x)
```

Fricas [A]
time = 36.69, size = 1269, normalized size = 8.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*
tan(x)^2 + a) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x
)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(t
an(x)^4 + 2*tan(x)^2 + 1)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^
4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*
b), -1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x
)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*
sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4
+ 2*tan(x)^2 + 1)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*s
qrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), 1/2*
sqrt(-a)*a*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/8*(3*a + 2*b)*sqrt(b
)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a) + 1/4*(a
+ b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4
 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 +
1)) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), 1/2*sqrt(-a)*a*arctan(sq
rt(b*tan(x)^4 + a)*sqrt(-a)/a) - 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan
(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*ta
n(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a +
b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/4*sqrt(b*tan(x)^4 + a)*(
b*tan(x)^2 - 2*b), 1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqr
t(-a - b)/(b*tan(x)^2 - a)) + 1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*
sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a) + 1/4*a^(3/2)*log((b*tan(x)^4 -
```

```
2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*
(b*tan(x)^2 - 2*b), 1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sq
rt(-a - b)/(b*tan(x)^2 - a)) - 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x
)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*ta
n(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2
 - 2*b), 1/2*sqrt(-a)*a*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*(a +
b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a))
+ 1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)
*tan(x)^2 + a) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), 1/2*sqrt(-a)*
a*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 1/2*(a + b)*sqrt(-a - b)*arctan
(sqrt(b*tan(x)^4 + a)*sqrt(-a - b)/(b*tan(x)^2 - a)) - 1/4*(3*a + 2*b)*sqrt
(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(b*tan(x)
^4 + a)*(b*tan(x)^2 - 2*b)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^4\left(x\right)\right)^{\frac{3}{2}} \cot\left(x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral((a + b*tan(x)**4)**(3/2)*cot(x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(x)\,\left(b\tan(x)^4 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)*(a + b*tan(x)^4)^(3/2), x)
```

3.396 $\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[Out] 1/2*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2*arctanh((a-b
*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 1266, 858, 223, 212, 739}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[b]) + ArcTanh[(a -
 b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx &= \text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan\right.\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \right.\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}
\end{aligned}
$$

Mathematica [A]

time = 0.04, size = 74, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[b]) + ArcTanh[(a -
 b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b])
```

Maple [A]
time = 0.11, size = 91, normalized size = 1.23

method	result
derivativedivides	$\frac{\ln\left(\sqrt{b}\,(\tan^2(x))+\sqrt{a+b\,(\tan^4(x))}\right)}{2\sqrt{b}}+\frac{\ln\left(\frac{2a+2b-2b\left(1+\tan^2(x)\right)+2\sqrt{a+b}\,\sqrt{b\,(1+\tan^2(x))^2-}}{1+\tan^2(x)}\right)}{2\sqrt{a+b}}$
default	$\frac{\ln\left(\sqrt{b}\,(\tan^2(x))+\sqrt{a+b\,(\tan^4(x))}\right)}{2\sqrt{b}}+\frac{\ln\left(\frac{2a+2b-2b\left(1+\tan^2(x)\right)+2\sqrt{a+b}\,\sqrt{b\,(1+\tan^2(x))^2-}}{1+\tan^2(x)}\right)}{2\sqrt{a+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2/(a+b)^(1/2)*ln((2
*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+
b)^(1/2))/(1+tan(x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^3/sqrt(b*tan(x)^4 + a), x)
```

Fricas [A]
time = 1.99, size = 483, normalized size = 6.53

$$\left[\frac{(a+b)\sqrt{b}\log\left(-2\,b\tan(x)^4-2\sqrt{b\tan(x)^4+a}\,\sqrt{b}\tan(x)^2-a\right)+\sqrt{a+b}\,b\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)}{4(ab+b^2)},\ -\frac{2(a+b)\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,\sqrt{-b}}{b\tan(x)^2}\right)-\sqrt{a+b}\,b\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)}{4(ab+b^2)},\ \frac{2\sqrt{-a-b}\,b\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)+(a+b)\sqrt{b}\log\left(-2\,b\tan(x)^4-2\sqrt{b\tan(x)^4+a}\,\sqrt{b}\tan(x)^2-a\right)}{4(ab+b^2)},\ \frac{\sqrt{-a-b}\,b\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)-(a+b)\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,\sqrt{-b}}{b\tan(x)^2}\right)}{2(ab+b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*ta
n(x)^2 - a) + sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 -
2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^
4 + 2*tan(x)^2 + 1)))/(a*b + b^2), -1/4*(2*(a + b)*sqrt(-b)*arctan(sqrt(b*t
an(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) - sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(
x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b)
 + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)))/(a*b + b^2), 1/4*(2*sqrt(-a -
 b)*b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2
)*tan(x)^4 + a^2 + a*b)) + (a + b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan
(x)^4 + a)*sqrt(b)*tan(x)^2 - a))/(a*b + b^2), 1/2*(sqrt(-a - b)*b*arctan(s
qrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a
^2 + a*b)) - (a + b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x
)^2)))/(a*b + b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)**3/sqrt(a + b*tan(x)**4), x)
```

Giac [A]
time = 0.43, size = 75, normalized size = 1.01

$$-\frac{\arctan\left(-\frac{\sqrt{b}\,\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}-\frac{\log\left(\left|-\sqrt{b}\,\tan(x)^2+\sqrt{b\tan(x)^4+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/
sqrt(-a - b) - 1/2*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 + a)))/sqrt(
b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^3}{\sqrt{b\tan(x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)^3/(a + b*tan(x)^4)^(1/2), x)
```

3.397 $\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

```
[Out] -1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3751, 1262, 739, 212}

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/Sqrt[a + b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$
-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/Sqrt[a +
b]
```

Maple [A]
time = 0.20, size = 65, normalized size = 1.59

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a+2b-2b\left(1+\tan^2(x)\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan^2(x)\right)^2-2b\left(1+\tan^2(x)\right)+a+b}}{1+\tan^2(x)}\right)}{2\sqrt{a+b}}$	65
default	$-\frac{\ln\left(\frac{2a+2b-2b\left(1+\tan^2(x)\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan^2(x)\right)^2-2b\left(1+\tan^2(x)\right)+a+b}}{1+\tan^2(x)}\right)}{2\sqrt{a+b}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)
^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/sqrt(b*tan(x)^4 + a), x)
```

Fricas [A]
time = 2.35, size = 150, normalized size = 3.66

$$\left[\frac{\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)}{4\sqrt{a+b}},\ -\frac{\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)}{2(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*
(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1))/sq
rt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*s
qrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b))/(a + b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a + b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*tan(x)**4), x)
```

Giac [A]
time = 0.46, size = 46, normalized size = 1.12

$$\frac{\arctan\left(-\frac{\sqrt{b}\tan(x)^2 - \sqrt{b\tan(x)^4 + a} + \sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/s
qrt(-a - b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(x)}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)/(a + b*tan(x)^4)^(1/2), x)
```

3.398 $\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))/a^(1/2)+1/2*arctanh((a-b*tan(x)^
2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3751, 1266, 975, 739, 212, 272, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b]
) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\,(1+x^2)\,\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a+bx^2}} + \frac{1}{x\sqrt{a+bx^2}}\right)\,dx, x, \tan^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x,\right.$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \tan^4(x)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a+b\,\text{ta}}{\sqrt{a+b\,\text{ta}}}\right.$$
$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\tan^4(x)}\right)}{2b}$$
$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b]
) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a+b\left(\tan^4(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*tan(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)/(a+b*tan(x)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)/sqrt(b*tan(x)^4 + a), x)
```

Fricas [A]
time = 3.92, size = 475, normalized size = 6.79

$$\left[\frac{\sqrt{a+b}\,a\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+(a+b)\sqrt{a}\log\left(-\frac{b\tan(x)^4-2\sqrt{b\tan(x)^4+a}\,\sqrt{a}+2a}{\tan(x)^4}\right)}{4\,(a^2+ab)}, \frac{2\sqrt{-a}\,(a+b)\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,\sqrt{-a}}{a}\right)+\sqrt{a+b}\,a\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)}{4\,(a^2+ab)}, \frac{2a\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)+(a+b)\sqrt{a}\log\left(-\frac{b\tan(x)^4-2\sqrt{b\tan(x)^4+a}\,\sqrt{a}+2a}{\tan(x)^4}\right)}{4\,(a^2+ab)}, \frac{a\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,(b\tan(x)^2-a)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)+\sqrt{-a}\,(a+b)\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,\sqrt{-a}}{a}\right)}{2\,(a^2+ab)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*a*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b
*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*ta
n(x)^2 + 1)) + (a + b)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sq
rt(a) + 2*a)/tan(x)^4))/(a^2 + a*b), 1/4*(2*sqrt(-a)*(a + b)*arctan(sqrt(b*
tan(x)^4 + a)*sqrt(-a)/a) + sqrt(a + b)*a*log(((a*b + 2*b^2)*tan(x)^4 - 2*a
*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 +
 a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)))/(a^2 + a*b), 1/4*(2*a*sqrt(-a - b)*arct
an(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4
 + a^2 + a*b)) + (a + b)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*
sqrt(a) + 2*a)/tan(x)^4))/(a^2 + a*b), 1/2*(a*sqrt(-a - b)*arctan(sqrt(b*ta
n(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b
)) + sqrt(-a)*(a + b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a))/(a^2 + a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a + b\tan^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)**4)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*tan(x)**4), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*tan(x)^4)^(1/2),x)
```

```
[Out] int(cot(x)/(a + b*tan(x)^4)^(1/2), x)
```

3.399 $\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal. Leaf size=291

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\sqrt[4]{a}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(}{\left(\sqrt{a}+\sqrt{b}\tan\right.}}}{2\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}$$

```
[Out] -1/2*arctan((a+b)^(1/2)*tan(x)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)+1/2*a^(1/4
)*(cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(
x)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),1/2*2^(1/2))*(
(a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^
2)/b^(1/4)/(a^(1/2)-b^(1/2))/(a+b*tan(x)^4)^(1/2)-1/4*(cos(2*arctan(b^(1/4)
*tan(x)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*tan(x)/a^(1/4)))*EllipticPi
(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),-1/4*(a^(1/2)-b^(1/2))^2/a^(1/2)/b^(
1/2),1/2*2^(1/2))*(a^(1/2)+b^(1/2))*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)
^2)^2)^(1/2)*(a^(1/2)+b^(1/2)*tan(x)^2)/a^(1/4)/b^(1/4)/(a^(1/2)-b^(1/2))/(
a+b*tan(x)^4)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3751, 1334, 226, 1721}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\Pi\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}};2\text{ArcTan}\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] -1/2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a + b*Tan[x]^4]]/Sqrt[a + b] + (a^(1/
4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Ta
n[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(2*(Sqrt[a]
- Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]^4]) - ((Sqrt[a] + Sqrt[b])*EllipticPi[
-1/4*(Sqrt[a] - Sqrt[b])^2/(Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[x])/a^(
1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + S
qrt[b]*Tan[x]^2)^2])/(4*a^(1/4)*(Sqrt[a] - Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[
x]^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1334

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2]}, Dist[(-a)*((e + d*q)/(c*d^2 - a*e^2)), Int[1/Sqrt[a
+ c*x^4], x], x] + Dist[a*d*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)$$

$$= \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \tan(x)\right)}{\sqrt{a}-\sqrt{b}} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{(1+x^2)\sqrt{a+bx^4}}\,dx, x, \text{ta}\right.}{\sqrt{a}-\sqrt{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}} + \frac{\sqrt[4]{a}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\,\text{tan}\right.}{2\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.47, size = 122, normalized size = 0.42

$$-\frac{i\left(F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)-\Pi\left(-\frac{i\sqrt{a}}{\sqrt{b}};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)\right)\sqrt{1+\frac{b\tan^4(x)}{a}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/Sqrt[a + b*Tan[x]^4],x]
```

```
[Out] ((-I)*(EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1] - Ellipti
cPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1
])*Sqrt[1 + (b*Tan[x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*Tan[x]^4
])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 179, normalized size = 0.62

method	result
derivativedivides	$\frac{\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(x))}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,(\tan^2(x))}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(x)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\,(\tan^4(x))}}-\frac{\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(x))}{\sqrt{a}}}}{}$
default	$\frac{\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(x))}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,(\tan^2(x))}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(x)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\,(\tan^4(x))}}-\frac{\sqrt{1-\frac{i\sqrt{b}\,(\tan^2(x))}{\sqrt{a}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)
*b^(1/2))^(1/2),I)-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*Elliptic
Pi(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^
(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
[Out] integral(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)}{\sqrt{a + b\tan^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+b*tan(x)**4)**(1/2),x)
[Out] Integral(tan(x)**2/sqrt(a + b*tan(x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(x)^2}{\sqrt{b\tan(x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + b*tan(x)^4)^(1/2),x)
[Out] int(tan(x)^2/(a + b*tan(x)^4)^(1/2), x)
```

3.400 $\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}} dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}$$

```
[Out] 1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(3/2)+1/
2*(-1+tan(x)^2)/(a+b)/(a+b*tan(x)^4)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 1266, 837, 12, 739, 212}

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3/
2)) - (1 - Tan[x]^2)/(2*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \text{Subst}\left(\int \frac{x^3}{(1+x^2)\,(a+bx^4)^{3/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\,(a+bx^2)^{3/2}}\,dx, x, \tan^2(x)\right)$$
$$= -\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{ab}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2ab(a+b)}$$
$$= -\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2(a+b)}$$
$$= -\frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)}$$
$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}$$

Mathematica [A]
time = 0.21, size = 67, normalized size = 0.94

$$\frac{1}{2}\left(\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}} + \frac{-1+\tan^2(x)}{(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + b*Tan[x]^4)^(3/2),x]
[Out] (ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2)
 + (-1 + Tan[x]^2)/((a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(57) = 114.
time = 0.92, size = 267, normalized size = 3.76

method	result
derivativedivides	$\frac{\tan^2(x)}{2\sqrt{a+b\left(\tan^4(x)\right)}\, a}+\frac{\sqrt{b\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)^2-2\sqrt{-ab}\,\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}}{4\left(\sqrt{-ab}-b\right)a\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}$
default	$\frac{\tan^2(x)}{2\sqrt{a+b\left(\tan^4(x)\right)}\, a}+\frac{\sqrt{b\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)^2-2\sqrt{-ab}\,\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}}{4\left(\sqrt{-ab}-b\right)a\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*tan(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b*tan(x)^4)^(1/2)/a*tan(x)^2+1/4/((-a*b)^(1/2)-b)/a/(tan(x)^2+(-a*b)
^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1
/2)/b))^(1/2)-1/4/((-a*b)^(1/2)+b)/a/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2
-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/2*b/((
-a*b)^(1/2)+b)/((-a*b)^(1/2)-b)/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*
(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^3/(b*tan(x)^4 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(59) = 118.
time = 3.18, size = 292, normalized size = 4.11

$$\left[\frac{\left(b\tan\left(x\right)^4+a\right)\sqrt{a+b}\,\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan\left(x\right)^4+a}\,\left(b\tan(x)^2-a\right)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+2\sqrt{b\tan\left(x\right)^4+a}\,\left(\left(a+b\right)\tan\left(x\right)^2-a-b\right)}{4\left(\left(a^2b+2ab^2+b^3\right)\tan\left(x\right)^4+a^3+2a^2b+ab^2\right)}, \frac{\left(b\tan\left(x\right)^4+a\right)\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{b\tan\left(x\right)^4+a}\,\left(b\tan(x)^2-a\right)\sqrt{-a-b}}{\left(ab+b^2\right)\tan(x)^4+a^2+ab}\right)+\sqrt{b\tan\left(x\right)^4+a}\,\left(\left(a+b\right)\tan\left(x\right)^2-a-b\right)}{2\left(\left(a^2b+2ab^2+b^3\right)\tan\left(x\right)^4+a^3+2a^2b+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*tan(x)^4 + a)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(
x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(
```

```
tan(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*((a + b)*tan(x)^2 - a
- b))/((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2), 1/2*((b*t
an(x)^4 + a)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt
(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + sqrt(b*tan(x)^4 + a)*((a + b
)*tan(x)^2 - a - b))/((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*
b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(x)}{(a + b\tan^4(x))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*tan(x)**4)**(3/2),x)
[Out] Integral(tan(x)**3/(a + b*tan(x)**4)**(3/2), x)
```

Giac [A]
time = 0.42, size = 103, normalized size = 1.45

$$\frac{\frac{(a+b)\tan(x)^2}{a^2+2\,ab+b^2} - \frac{a+b}{a^2+2\,ab+b^2}}{2\,\sqrt{b\tan(x)^4 + a}} + \frac{\arctan\left(\frac{\sqrt{b}\,\tan(x)^2 - \sqrt{b\tan(x)^4 + a} + \sqrt{b}}{\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
[Out] 1/2*((a + b)*tan(x)^2/(a^2 + 2*a*b + b^2) - (a + b)/(a^2 + 2*a*b + b^2))/sq
rt(b*tan(x)^4 + a) + arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt
(b))/sqrt(-a - b))/((a + b)*sqrt(-a - b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^3}{\left(b\tan(x)^4 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + b*tan(x)^4)^(3/2),x)
[Out] int(tan(x)^3/(a + b*tan(x)^4)^(3/2), x)
```

3.401 $\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}+\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}$$

```
[Out] -1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(3/2)+1
/2*(a+b*tan(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3751, 1262, 755, 12, 739, 212}

$$\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] -1/2*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(
3/2) + (a + b*Tan[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{x}{(1+x^2)\left(a+bx^4\right)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{a}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2a(a+b)}\\
&= \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx^2}}\,dx, x, \tan^2(x)\right)}{2(a+b)}\\
&= \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{a-b\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)}\\
&= -\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}} + \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 73, normalized size = 0.99

$$
\frac{1}{2}\left(-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}} + \frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (-(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/
2)) + (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(62) = 124.
time = 0.09, size = 248, normalized size = 3.35

method	result
derivativedivides	$-\frac{\sqrt{b\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)^2-2\sqrt{-ab}\,\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}}{4\left(\sqrt{-ab}-b\right)a\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}+\frac{\sqrt{b\left(\tan^2(x)-\frac{\sqrt{-}}{b}\right.}}{4\left(\sqrt{-}\right.}$
default	$-\frac{\sqrt{b\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)^2-2\sqrt{-ab}\,\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}}{4\left(\sqrt{-ab}-b\right)a\left(\tan^2(x)+\frac{\sqrt{-ab}}{b}\right)}+\frac{\sqrt{b\left(\tan^2(x)-\frac{\sqrt{-}}{b}\right.}}{4\left(\sqrt{-}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*tan(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/((-a*b)^(1/2)-b)/a/(tan(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2)
/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/4/((-a*b)^(1/2)+b)/
a/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(
tan(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/2*b/((-a*b)^(1/2)+b)/((-a*b)^(1/2)-b)/(a+
b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(
1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b*tan(x)^4 + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(64) = 128.
time = 2.68, size = 319, normalized size = 4.31

$$\left[\frac{\left(ab\tan(x)^4+a^2\right)\sqrt{a+b}\,\log\left(\frac{\left(ab+2b^2\right)\tan(x)^4-2ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}\,\left(b\tan(x)^2-a\right)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+2\sqrt{b\tan(x)^4+a}\,\left(\left(ab+b^2\right)\tan(x)^2+a^2+ab\right)}{4\left(\left(a^3b+2a^2b^2+ab^3\right)\tan(x)^4+a^4+2a^3b+a^2b^2\right)},-\frac{\left(ab\tan(x)^4+a^2\right)\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\,\left(b\tan(x)^2-a\right)\sqrt{-a-b}}{\left(ab+b^2\right)\tan(x)^4+a^2+ab}\right)-\sqrt{b\tan(x)^4+a}\,\left(\left(ab+b^2\right)\tan(x)^2+a^2+ab\right)}{2\left(\left(a^3b+2a^2b^2+ab^3\right)\tan(x)^4+a^4+2a^3b+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*tan(x)^4 + a^2)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*
tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*
```

```
b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*((a*b + b^2)*tan(x
)^2 + a^2 + a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*tan(x)^4 + a^4 + 2*a^3*b + a
^2*b^2), -1/2*((a*b*tan(x)^4 + a^2)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a
)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) - sqrt(
b*tan(x)^4 + a)*((a*b + b^2)*tan(x)^2 + a^2 + a*b))/((a^3*b + 2*a^2*b^2 + a
*b^3)*tan(x)^4 + a^4 + 2*a^3*b + a^2*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral(tan(x)/(a + b*tan(x)**4)**(3/2), x)
```

Giac [A]
time = 0.44, size = 119, normalized size = 1.61

$$\frac{\frac{(ab+b^2)\tan(x)^2}{a^3+2a^2b+ab^2}+\frac{a^2+ab}{a^3+2a^2b+ab^2}}{2\sqrt{b\tan(x)^4+a}}-\frac{\arctan\left(\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((a*b + b^2)*tan(x)^2/(a^3 + 2*a^2*b + a*b^2) + (a^2 + a*b)/(a^3 + 2*a^
2*b + a*b^2))/sqrt(b*tan(x)^4 + a) - arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(
x)^4 + a) + sqrt(b))/sqrt(-a - b))/((a + b)*sqrt(-a - b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)}{\left(b\tan(x)^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*tan(x)^4)^(3/2),x)
```

```
[Out] int(tan(x)/(a + b*tan(x)^4)^(3/2), x)
```

3.402 $\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{1}{2a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}$$

```
[Out] -1/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))/a^(3/2)+1/2*arctanh((a-b*tan(x)^
2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(3/2)+1/2/a/(a+b*tan(x)^4)^(1/2)
+1/2*(-a-b*tan(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3751, 1266, 975, 755, 12, 739, 212, 272, 53, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{1}{2a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(3/
2)) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*a^(3/2)) + 1/(2*a*Sqrt[a + b
*Tan[x]^4]) - (a + b*Tan[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\left(a+bx^4\right)^{3/2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\left(a+bx^2\right)^{3/2}} + \frac{1}{x\left(a+bx^2\right)^{3/2}}\right)dx, x, \tan^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\left(a+bx^2\right)^{3/2}}\,dx, x, \tan^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,d\right.$$

$$= -\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \tan^4(x)\right) + \frac{\text{Subst}}{}$$

$$= \frac{1}{2a\sqrt{a+b\tan^4(x)}} - \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{ta}\right.}{4a}$$

$$= \frac{1}{2a\sqrt{a+b\tan^4(x)}} - \frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+}\right.}{2ab}$$

$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{}{2a\sqrt{a+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.41, size = 108, normalized size = 0.89

$$\frac{1}{2}\left(\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}} + \frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{b\tan^4(x)}{a}\right)}{a\sqrt{a+b\tan^4(x)}} - \frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Tan[x]^4)^(3/2),x]
```

```
[Out] (ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2)
 + Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[x]^4)/a]/(a*Sqrt[a + b*Tan[x]
^4]) - (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(a + b(\tan^4(x)))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)/(a+b*tan(x)^4)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(99) = 198.
time = 4.25, size = 954, normalized size = 7.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*tan(x)^4 + a^3)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*
b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 +
a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + ((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3
 + 2*a^2*b + a*b^2)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(
a) + 2*a)/tan(x)^4) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^2 - (a^2*b + a*b^
2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(
x)^4), 1/4*(2*((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sq
rt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + (a^2*b*tan(x)^4 + a^3)*sqr
t(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 +
```

```
 a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)
) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5
 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(x)^4), 1/4*(2*(a^2
*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + ((a^2*b + 2*a*b^2 + b^
3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*ta
n(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^
2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^
2 + a^2*b^3)*tan(x)^4), 1/2*((a^2*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqr
t(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2
 + a*b)) + ((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sqrt(
-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + sqrt(b*tan(x)^4 + a)*(a^2*b +
 a*b^2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a
^3*b^2 + a^2*b^3)*tan(x)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(a + b\tan^4(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)**4)**(3/2),x)
```

```
[Out] Integral(cot(x)/(a + b*tan(x)**4)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(x)}{\left(b\,{\tan\left(x\right)}^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*tan(x)^4)^(3/2),x)
```

```
[Out] int(cot(x)/(a + b*tan(x)^4)^(3/2), x)
```

3.403 $\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}}-\frac{3a+(-2a+b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}}$$

```
[Out] 1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(5/2)+1/
6*(-3*a-(-2*a+b)*tan(x)^2)/a/(a+b)^2/(a+b*tan(x)^4)^(1/2)+1/6*(-1+tan(x)^2)
/(a+b)/(a+b*tan(x)^4)^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3751, 1266, 837, 12, 739, 212}

$$-\frac{(b-2a)\tan^2(x)+3a}{6a(a+b)^2\sqrt{a+b\tan^4(x)}}-\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5/
2)) - (1 - Tan[x]^2)/(6*(a + b)*(a + b*Tan[x]^4)^(3/2)) - (3*a + (-2*a + b)
*Tan[x]^2)/(6*a*(a + b)^2*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x^3}{(1+x^2)\,(a+bx^4)^{5/2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\,(a+bx^2)^{5/2}}\,dx, x, \tan^2(x)\right)$$

$$= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{ab-2abx}{(1+x)(a+bx^2)^{3/2}}\,dx, x, \tan^2(x)\right)}{6ab(a+b)}$$

$$= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int -\frac{3a^2 \cdots}{(1+x)\sqrt{a \cdots}}\right.}{6a^2b^2 \cdots}$$

$$= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+\cdots}}\right.}{2(a \cdots}$$

$$= -\frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, \cdots\right.}{2(a \cdots}$$

$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a \cdots}{6a(a+b)^2\sqrt{\cdots}}$$

Mathematica [A]
time = 0.51, size = 104, normalized size = 0.95

$$\frac{1}{6}\left(\frac{3\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\,\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}} + \frac{-a(4a+b)+3a^2\tan^2(x)-3ab\tan^4(x)+(2a-b)b\tan^6(x)}{a(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ((3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^(
5/2) + (-(a*(4*a + b)) + 3*a^2*Tan[x]^2 - 3*a*b*Tan[x]^4 + (2*a - b)*b*Tan[
x]^6)/(a*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(92) = 184$.
time = 0.94, size = 654, normalized size = 6.00

method	result
derivativedivides	$\frac{\sqrt{a+b(\tan^4(x))}\,(\tan^2(x))(2b(\tan^4(x))+3a)}{6a^2(b^2(\tan^8(x))+2ab(\tan^4(x))+a^2)} - \frac{\sqrt{b\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2+2\sqrt{-ab}\,(\tan}}{24\left(\sqrt{-ab}+b\right)a\sqrt{-ab}\,\left(\tan^2(x)-\sqrt{}\right.}$
default	$\frac{\sqrt{a+b(\tan^4(x))}\,(\tan^2(x))(2b(\tan^4(x))+3a)}{6a^2(b^2(\tan^8(x))+2ab(\tan^4(x))+a^2)} - \frac{\sqrt{b\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2+2\sqrt{-ab}\,(\tan}}{24\left(\sqrt{-ab}+b\right)a\sqrt{-ab}\,\left(\tan^2(x)-\sqrt{}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*tan(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a+b*tan(x)^4)^(1/2)*tan(x)^2*(2*b*tan(x)^4+3*a)/a^2/(b^2*tan(x)^8+2*a*
b*tan(x)^4+a^2)-1/24/((-a*b)^(1/2)+b)/a/(-a*b)^(1/2)/(tan(x)^2-(-a*b)^(1/2)
/b)^2*(b*(tan(x)^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/
b))^(1/2)-1/24/((-a*b)^(1/2)+b)/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2-
(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/8*(2*(-
a*b)^(1/2)+b)/((-a*b)^(1/2)+b)^2/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2
-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/8*(2*(
-a*b)^(1/2)-b)/((-a*b)^(1/2)-b)^2/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^
2+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/24/((
-a*b)^(1/2)-b)/a/(-a*b)^(1/2)/(tan(x)^2+(-a*b)^(1/2)/b)^2*(b*(tan(x)^2+(-a*
b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/24/((-a*b)^
(1/2)-b)/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2)/b)^2-2*(-a
*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/2*b^2/((-a*b)^(1/2)+b)^2/((-a*
b)^(1/2)-b)^2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+
tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)^3/(b*tan(x)^4 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(95) = 190.
time = 2.24, size = 556, normalized size = 5.10

$$\left[\frac{3\,(ab^{2}\tan(x)^{8}+2\,a^{2}b\tan(x)^{4}+a^{3})\sqrt{a+b}\log\left(\frac{(ab+2\,b^{2})\tan(x)^{4}-2\,ab\tan(x)^{2}-2\,\sqrt{b\tan(x)^{4}+a}\,(b\tan(x)^{2}-a)\sqrt{a+b}+2\,a^{2}+ab}{\tan(x)^{4}+2\,\tan(x)^{2}+1}\right)+2\,((2\,a^{2}b+ab^{2}-b^{3})\tan(x)^{6}-3\,(a^{2}b+ab^{2})\tan(x)^{4}-4\,a^{3}-5\,a^{2}b-ab^{2}+3\,(a^{3}+a^{2}b)\tan(x)^{2})\sqrt{b\tan(x)^{4}+a}}{12\,((a^{4}b^{2}+3\,a^{3}b^{3}+3\,a^{2}b^{4}+ab^{5})\tan(x)^{8}+a^{6}+3\,a^{5}b+3\,a^{4}b^{2}+a^{3}b^{3}+2\,(a^{5}b+3\,a^{4}b^{2}+3\,a^{3}b^{3}+a^{2}b^{4})\tan(x)^{4})},\ \frac{3\,(ab^{2}\tan(x)^{8}+2\,a^{2}b\tan(x)^{4}+a^{3})\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^{4}+a}\,(b\tan(x)^{2}-a)\sqrt{-a-b}}{(ab+b^{2})\tan(x)^{4}+a^{2}+ab}\right)+((2\,a^{2}b+ab^{2}-b^{3})\tan(x)^{6}-3\,(a^{2}b+ab^{2})\tan(x)^{4}-4\,a^{3}-5\,a^{2}b-ab^{2}+3\,(a^{3}+a^{2}b)\tan(x)^{2})\sqrt{b\tan(x)^{4}+a}}{6\,((a^{4}b^{2}+3\,a^{3}b^{3}+3\,a^{2}b^{4}+ab^{5})\tan(x)^{8}+a^{6}+3\,a^{5}b+3\,a^{4}b^{2}+a^{3}b^{3}+2\,(a^{5}b+3\,a^{4}b^{2}+3\,a^{3}b^{3}+a^{2}b^{4})\tan(x)^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*b^2*tan(x)^8 + 2*a^2*b*tan(x)^4 + a^3)*sqrt(a + b)*log(((a*b +
2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*
sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*((2*a^2*b + a*b
^2 - b^3)*tan(x)^6 - 3*(a^2*b + a*b^2)*tan(x)^4 - 4*a^3 - 5*a^2*b - a*b^2 +
 3*(a^3 + a^2*b)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^4*b^2 + 3*a^3*b^3 + 3*
a^2*b^4 + a*b^5)*tan(x)^8 + a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + 2*(a^5*b
+ 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(x)^4), 1/6*(3*(a*b^2*tan(x)^8 + 2*a^
2*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 -
a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + ((2*a^2*b + a*b^2 - b
^3)*tan(x)^6 - 3*(a^2*b + a*b^2)*tan(x)^4 - 4*a^3 - 5*a^2*b - a*b^2 + 3*(a^
3 + a^2*b)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^
4 + a*b^5)*tan(x)^8 + a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + 2*(a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(x)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}(x)}{(a+b\tan^{4}(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*tan(x)**4)**(5/2),x)
```

```
[Out] Integral(tan(x)**3/(a + b*tan(x)**4)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(95) = 190.
time = 0.44, size = 597, normalized size = 5.48

$$\frac{\left(\left(\frac{(2\,a^{7}b^{2}+11\,a^{6}b^{3}+24\,a^{5}b^{4}+25\,a^{4}b^{5}+10\,a^{3}b^{6}-3\,a^{2}b^{7}-4\,ab^{8}-b^{9})\tan(x)^{2}}{a^{9}b+8\,a^{8}b^{2}+28\,a^{7}b^{3}+56\,a^{6}b^{4}+70\,a^{5}b^{5}+56\,a^{4}b^{6}+28\,a^{3}b^{7}+8\,a^{2}b^{8}+ab^{9}}-\frac{3\,(a^{7}b^{2}+6\,a^{6}b^{3}+15\,a^{5}b^{4}+20\,a^{4}b^{5}+15\,a^{3}b^{6}+6\,a^{2}b^{7}+ab^{8})}{a^{9}b+8\,a^{8}b^{2}+28\,a^{7}b^{3}+56\,a^{6}b^{4}+70\,a^{5}b^{5}+56\,a^{4}b^{6}+28\,a^{3}b^{7}+8\,a^{2}b^{8}+ab^{9}}\right)\tan(x)^{2}+\frac{3\,(a^{8}b+6\,a^{7}b^{2}+15\,a^{6}b^{3}+20\,a^{5}b^{4}+15\,a^{4}b^{5}+6\,a^{3}b^{6}+a^{2}b^{7})}{a^{9}b+8\,a^{8}b^{2}+28\,a^{7}b^{3}+56\,a^{6}b^{4}+70\,a^{5}b^{5}+56\,a^{4}b^{6}+28\,a^{3}b^{7}+8\,a^{2}b^{8}+ab^{9}}\right)\tan(x)^{2}-\frac{4\,a^{8}b+25\,a^{7}b^{2}+66\,a^{6}b^{3}+95\,a^{5}b^{4}+80\,a^{4}b^{5}+39\,a^{3}b^{6}+10\,a^{2}b^{7}+ab^{8}}{a^{9}b+8\,a^{8}b^{2}+28\,a^{7}b^{3}+56\,a^{6}b^{4}+70\,a^{5}b^{5}+56\,a^{4}b^{6}+28\,a^{3}b^{7}+8\,a^{2}b^{8}+ab^{9}}}{6\,(b\tan(x)^{4}+a)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{b}\tan(x)^{2}-\sqrt{b\tan(x)^{4}+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a^{2}+2\,ab+b^{2})\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((((2*a^7*b^2 + 11*a^6*b^3 + 24*a^5*b^4 + 25*a^4*b^5 + 10*a^3*b^6 - 3*a
^2*b^7 - 4*a*b^8 - b^9)*tan(x)^2/(a^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^6*b
```

```
^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^9) - 3*(a^7*b^2
 + 6*a^6*b^3 + 15*a^5*b^4 + 20*a^4*b^5 + 15*a^3*b^6 + 6*a^2*b^7 + a*b^8)/(a
^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a
^3*b^7 + 8*a^2*b^8 + a*b^9))*tan(x)^2 + 3*(a^8*b + 6*a^7*b^2 + 15*a^6*b^3 +
 20*a^5*b^4 + 15*a^4*b^5 + 6*a^3*b^6 + a^2*b^7)/(a^9*b + 8*a^8*b^2 + 28*a^7
*b^3 + 56*a^6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^
9))*tan(x)^2 - (4*a^8*b + 25*a^7*b^2 + 66*a^6*b^3 + 95*a^5*b^4 + 80*a^4*b^5
 + 39*a^3*b^6 + 10*a^2*b^7 + a*b^8)/(a^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^
6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^9))/(b*tan(x
)^4 + a)^(3/2) + arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))
/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a - b))
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)^3}{\left(b\tan\left(x\right)^4+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + b*tan(x)^4)^(5/2),x)
```

```
[Out] int(tan(x)^3/(a + b*tan(x)^4)^(5/2), x)
```

3.404 $\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}+\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}$$

```
[Out] -1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(5/2)+1
/6*(3*a^2+b*(5*a+2*b)*tan(x)^2)/a^2/(a+b)^2/(a+b*tan(x)^4)^(1/2)+1/6*(a+b*t
an(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(3/2)
```

Rubi [A]

time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3751, 1262, 755, 837, 12, 739, 212}

$$\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] -1/2*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(
5/2) + (a + b*Tan[x]^2)/(6*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + (3*a^2 + b*(
5*a + 2*b)*Tan[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x}{(1+x^2)\left(a+bx^4\right)^{5/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)$$
$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a-2b-2bx}{(1+x)(a+bx^2)^{3/2}}\,dx, x, \tan^2(x)\right)}{6a(a+b)}$$
$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{3a^2b}{(1+x)\sqrt{a+}}\right.}{6a^2b(}$$
$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+}}\right.}{2(a}$$
$$= \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}} - \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx\right.}{2(a}$$
$$= -\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} + \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b}{6a^2(a+b}$$

Mathematica [A]
time = 0.53, size = 113, normalized size = 0.97

$$\frac{1}{6}\left(-\frac{3\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}} + \frac{a^2(4a+b)+3ab(2a+b)\tan^2(x)+3a^2b\tan^4(x)+b^2(5a+2b)\tan^6(x)}{a^2(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ((-3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^
(5/2) + (a^2*(4*a + b) + 3*a*b*(2*a + b)*Tan[x]^2 + 3*a^2*b*Tan[x]^4 + b^2*
(5*a + 2*b)*Tan[x]^6)/(a^2*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(101) = 202$.
time = 0.09, size = 602, normalized size = 5.15

method	result
derivativedivides	$\frac{\sqrt{b\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2+2\sqrt{-ab}\,\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)}}{24\left(\sqrt{-ab}+b\right)a\sqrt{-ab}\,\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2}+\frac{\sqrt{b\left(\tan^2(x)-\frac{\sqrt{-a}}{b}\right.}}{24\left(\sqrt{-\right.}}$
default	$\frac{\sqrt{b\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2+2\sqrt{-ab}\,\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)}}{24\left(\sqrt{-ab}+b\right)a\sqrt{-ab}\,\left(\tan^2(x)-\frac{\sqrt{-ab}}{b}\right)^2}+\frac{\sqrt{b\left(\tan^2(x)-\frac{\sqrt{-a}}{b}\right.}}{24\left(\sqrt{-\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*tan(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/((-a*b)^(1/2)+b)/a/(-a*b)^(1/2)/(tan(x)^2-(-a*b)^(1/2)/b)^2*(b*(tan(x)
^2-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/24/(
(-a*b)^(1/2)+b)/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2-(-a*b)^(1/2)/b)^
2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)+1/8*(2*(-a*b)^(1/2)+b)/((
-a*b)^(1/2)+b)^2/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*(b*(tan(x)^2-(-a*b)^(1/2)/b)
^2+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/8*(2*(-a*b)^(1/2)-b)/(
(-a*b)^(1/2)-b)^2/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2)/b
)^2-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)+1/24/((-a*b)^(1/2)-b)/a
/(-a*b)^(1/2)/(tan(x)^2+(-a*b)^(1/2)/b)^2*(b*(tan(x)^2+(-a*b)^(1/2)/b)^2-2*
(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/24/((-a*b)^(1/2)-b)/a^2/(ta
n(x)^2+(-a*b)^(1/2)/b)*(b*(tan(x)^2+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(tan(x
)^2+(-a*b)^(1/2)/b))^(1/2)-1/2*b^2/((-a*b)^(1/2)+b)^2/((-a*b)^(1/2)-b)^2/(a
+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*
(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b*tan(x)^4 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(103) = 206.
time = 2.97, size = 599, normalized size = 5.12

$$\left[\frac{3\left(a^{2} b^{2} \tan\left(x\right)^{8} + 2 a^{3} b \tan\left(x\right)^{4} + a^{4}\right) \sqrt{a + b} \log\left(\frac{\left(a b + 2 b^{2}\right) \tan\left(x\right)^{4} - 2 a b \tan\left(x\right)^{2} + 2 \sqrt{b \tan\left(x\right)^{4} + a} \left(b \tan\left(x\right)^{2} - a\right) \sqrt{a + b} + 2 a^{2} + a b}{\tan\left(x\right)^{4} + 2 \tan\left(x\right)^{2} + 1}\right) + 2\left(\left(5 a^{2} b^{2} + 7 a b^{3} + 2 b^{4}\right) \tan\left(x\right)^{6} + 3\left(a^{3} b + a^{2} b^{2}\right) \tan\left(x\right)^{4} + 4 a^{4} + 5 a^{3} b + a^{2} b^{2} + 3\left(2 a^{3} b + 3 a^{2} b^{2} + a b^{3}\right) \tan\left(x\right)^{2}\right) \sqrt{b \tan\left(x\right)^{4} + a}}{12\left(\left(a^{5} b^{2} + 3 a^{4} b^{3} + 3 a^{3} b^{4} + a^{2} b^{5}\right) \tan\left(x\right)^{8} + a^{7} + 3 a^{6} b + 3 a^{5} b^{2} + a^{4} b^{3} + 2\left(a^{6} b + 3 a^{5} b^{2} + 3 a^{4} b^{3} + a^{3} b^{4}\right) \tan\left(x\right)^{4}\right)}, -\frac{3\left(a^{2} b^{2} \tan\left(x\right)^{8} + 2 a^{3} b \tan\left(x\right)^{4} + a^{4}\right) \sqrt{-a - b} \arctan\left(\frac{\sqrt{b \tan\left(x\right)^{4} + a} \left(b \tan\left(x\right)^{2} - a\right) \sqrt{-a - b}}{\left(a b + b^{2}\right) \tan\left(x\right)^{4} + a^{2} + a b}\right) - \left(\left(5 a^{2} b^{2} + 7 a b^{3} + 2 b^{4}\right) \tan\left(x\right)^{6} + 3\left(a^{3} b + a^{2} b^{2}\right) \tan\left(x\right)^{4} + 4 a^{4} + 5 a^{3} b + a^{2} b^{2} + 3\left(2 a^{3} b + 3 a^{2} b^{2} + a b^{3}\right) \tan\left(x\right)^{2}\right) \sqrt{b \tan\left(x\right)^{4} + a}}{6\left(\left(a^{5} b^{2} + 3 a^{4} b^{3} + 3 a^{3} b^{4} + a^{2} b^{5}\right) \tan\left(x\right)^{8} + a^{7} + 3 a^{6} b + 3 a^{5} b^{2} + a^{4} b^{3} + 2\left(a^{6} b + 3 a^{5} b^{2} + 3 a^{4} b^{3} + a^{3} b^{4}\right) \tan\left(x\right)^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*tan(x)^8 + 2*a^3*b*tan(x)^4 + a^4)*sqrt(a + b)*log(((a*b
+ 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*((5*a^2*b^2 +
 7*a*b^3 + 2*b^4)*tan(x)^6 + 3*(a^3*b + a^2*b^2)*tan(x)^4 + 4*a^4 + 5*a^3*b
 + a^2*b^2 + 3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*tan(x)^2)*sqrt(b*tan(x)^4 + a)
)/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*tan(x)^8 + a^7 + 3*a^6*b + 3
*a^5*b^2 + a^4*b^3 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*tan(x)^4),
 -1/6*(3*(a^2*b^2*tan(x)^8 + 2*a^3*b*tan(x)^4 + a^4)*sqrt(-a - b)*arctan(sq
rt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^
2 + a*b)) - ((5*a^2*b^2 + 7*a*b^3 + 2*b^4)*tan(x)^6 + 3*(a^3*b + a^2*b^2)*t
an(x)^4 + 4*a^4 + 5*a^3*b + a^2*b^2 + 3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*tan(x
)^2)*sqrt(b*tan(x)^4 + a))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*tan
(x)^8 + a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*tan(x)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(x\right)}{\left(a + b \tan^{4}\left(x\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)**4)**(5/2),x)
```

```
[Out] Integral(tan(x)/(a + b*tan(x)**4)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 618 vs. 2(103) = 206.
time = 0.45, size = 618, normalized size = 5.28

$$\frac{\left(\left(\frac{\left(5 a^{7} b^{3} + 32 a^{6} b^{4} + 87 a^{5} b^{5} + 130 a^{4} b^{6} + 115 a^{3} b^{7} + 60 a^{2} b^{8} + 17 a b^{9} + 2 b^{10}\right) \tan\left(x\right)^{2}}{a^{10} b + 8 a^{9} b^{2} + 28 a^{8} b^{3} + 56 a^{7} b^{4} + 70 a^{6} b^{5} + 56 a^{5} b^{6} + 28 a^{4} b^{7} + 8 a^{3} b^{8} + a^{2} b^{9}} + \frac{3\left(a^{8} b^{2} + 6 a^{7} b^{3} + 15 a^{6} b^{4} + 20 a^{5} b^{5} + 15 a^{4} b^{6} + 6 a^{3} b^{7} + a^{2} b^{8}\right)}{a^{10} b + 8 a^{9} b^{2} + 28 a^{8} b^{3} + 56 a^{7} b^{4} + 70 a^{6} b^{5} + 56 a^{5} b^{6} + 28 a^{4} b^{7} + 8 a^{3} b^{8} + a^{2} b^{9}}\right) \tan\left(x\right)^{2} + \frac{3\left(2 a^{8} b^{2} + 13 a^{7} b^{3} + 36 a^{6} b^{4} + 55 a^{5} b^{5} + 50 a^{4} b^{6} + 27 a^{3} b^{7} + 8 a^{2} b^{8} + a b^{9}\right)}{a^{10} b + 8 a^{9} b^{2} + 28 a^{8} b^{3} + 56 a^{7} b^{4} + 70 a^{6} b^{5} + 56 a^{5} b^{6} + 28 a^{4} b^{7} + 8 a^{3} b^{8} + a^{2} b^{9}}\right) \tan\left(x\right)^{2} + \frac{4 a^{9} b + 25 a^{8} b^{2} + 66 a^{7} b^{3} + 95 a^{6} b^{4} + 80 a^{5} b^{5} + 39 a^{4} b^{6} + 10 a^{3} b^{7} + a^{2} b^{8}}{a^{10} b + 8 a^{9} b^{2} + 28 a^{8} b^{3} + 56 a^{7} b^{4} + 70 a^{6} b^{5} + 56 a^{5} b^{6} + 28 a^{4} b^{7} + 8 a^{3} b^{8} + a^{2} b^{9}}}{6\left(b \tan\left(x\right)^{4} + a\right)^{\frac{3}{2}}} - \frac{\arctan\left(\frac{\sqrt{b} \tan\left(x\right)^{2} - \sqrt{b \tan\left(x\right)^{4} + a} + \sqrt{b}}{\sqrt{-a - b}}\right)}{\left(a^{2} + 2 a b + b^{2}\right) \sqrt{-a - b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((((5*a^7*b^3 + 32*a^6*b^4 + 87*a^5*b^5 + 130*a^4*b^6 + 115*a^3*b^7 + 6
0*a^2*b^8 + 17*a*b^9 + 2*b^10)*tan(x)^2/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 +
```

```
56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9) +
3*(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 +
 a^2*b^8)/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a
^5*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9))*tan(x)^2 + 3*(2*a^8*b^2 + 13*a^
7*b^3 + 36*a^6*b^4 + 55*a^5*b^5 + 50*a^4*b^6 + 27*a^3*b^7 + 8*a^2*b^8 + a*b
^9)/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6
 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9))*tan(x)^2 + (4*a^9*b + 25*a^8*b^2 + 66
*a^7*b^3 + 95*a^6*b^4 + 80*a^5*b^5 + 39*a^4*b^6 + 10*a^3*b^7 + a^2*b^8)/(a^
10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6 + 28*a
^4*b^7 + 8*a^3*b^8 + a^2*b^9))/(b*tan(x)^4 + a)^(3/2) - arctan((sqrt(b)*tan
(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*
sqrt(-a - b))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(x)}{\left(b\tan\left(x\right)^4+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*tan(x)^4)^(5/2),x)
[Out] int(tan(x)/(a + b*tan(x)^4)^(5/2), x)
```

3.405 $\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\tan}{6a(a+b)\left(a+b\right.}$$

```
[Out] -1/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))/a^(5/2)+1/2*arctanh((a-b*tan(x)^
2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(5/2)+1/2/a^2/(a+b*tan(x)^4)^(1/
2)+1/6*(-3*a^2-b*(5*a+2*b)*tan(x)^2)/a^2/(a+b)^2/(a+b*tan(x)^4)^(1/2)+1/6/a
/(a+b*tan(x)^4)^(3/2)+1/6*(-a-b*tan(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3751, 1266, 975, 755, 837, 12, 739, 212, 272, 53, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{1}{2a^2\sqrt{a+b\tan^4(x)}}-\frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}}-\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*(a + b)^(5/
2)) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*a^(5/2)) + 1/(6*a*(a + b*Tan
[x]^4)^(3/2)) - (a + b*Tan[x]^2)/(6*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + 1/(
2*a^2*Sqrt[a + b*Tan[x]^4]) - (3*a^2 + b*(5*a + 2*b)*Tan[x]^2)/(6*a^2*(a +
b)^2*Sqrt[a + b*Tan[x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
```

```
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\left(a+bx^4\right)^{5/2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\left(a+bx^2\right)^{5/2}} + \frac{1}{x\left(a+bx^2\right)^{5/2}}\right)dx, x, \tan^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\left(a+bx^2\right)^{5/2}}\,dx, x, \tan^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,d\right.$$

$$= -\frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \tan^4(x)\right) - \frac{\text{Sub}}{}$$

$$= \frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}} - \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a^2+b(5a+2b)\tan^2(x}{6a^2(a+b)^2\sqrt{a+b\tan^4(x}}$$

$$= \frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}} - \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{1}{2a^2\sqrt{a+b\tan^4(x)}} - \frac{3}{6a}$$

$$= \frac{1}{6a\left(a+b\tan^4(x)\right)^{3/2}} - \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{1}{2a^2\sqrt{a+b\tan^4(x)}} - \frac{3}{6a}$$

$$= \frac{\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}} + \frac{}{6a(a+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.96, size = 149, normalized size = 0.81

$$\frac{1}{6}\left(\frac{3\tanh^{-1}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}} + \frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1+\frac{b\tan^4(x)}{a}\right)}{a\left(a+b\tan^4(x)\right)^{3/2}} - \frac{a^2(4a+b)+3ab(2a+b)\tan^2(x)+3a^2b\tan^4(x)+b^2(5a+2b)\tan^6(x)}{a^2(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Tan[x]^4)^(5/2),x]
```

```
[Out] ((3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^(
5/2) + Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[x]^4)/a]/(a*(a + b*Tan[x
]^4)^(3/2)) - (a^2*(4*a + b) + 3*a*b*(2*a + b)*Tan[x]^2 + 3*a^2*b*Tan[x]^4
+ b^2*(5*a + 2*b)*Tan[x]^6)/(a^2*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(a+b(\tan^4(x)))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*tan(x)^4)^(5/2),x)
```

```
[Out] int(cot(x)/(a+b*tan(x)^4)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(153) = 306$.
time = 4.90, size = 1749, normalized size = 9.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(a + b)*log(((a*b
+ 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 3*((a^3*b^2 + 3
*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 +
2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4)*sqrt(a)*log(-(b*tan(x)^
4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) - 2*((5*a^3*b^2 + 7*a^2
*b^3 + 2*a*b^4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2
+ 3*a^2*b^3 + a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)
```

```
*sqrt(b*tan(x)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^
8 + a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3
+ a^4*b^4)*tan(x)^4), 1/12*(6*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)
^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3
 + a*b^4)*tan(x)^4)*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + 3*(a
^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(a + b)*log(((a*b + 2*b^2)*ta
n(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a +
b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) - 2*((5*a^3*b^2 + 7*a^2*b^3
+ 2*a*b^4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2 + 3*a
^2*b^3 + a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)*sqrt
(b*tan(x)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^8 + a
^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4
*b^4)*tan(x)^4), 1/12*(6*(a^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(-
a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^
2)*tan(x)^4 + a^2 + a*b)) + 3*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)
^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3
 + a*b^4)*tan(x)^4)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(
a) + 2*a)/tan(x)^4) - 2*((5*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4)*tan(x)^6 - 7*a^4
*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4 +
3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^6*b^2
 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^8 + a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*tan(x)^4), 1/6*(3*(a^
3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^
4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) +
3*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)^8 + a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4)*sqrt(-a)
*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) - ((5*a^3*b^2 + 7*a^2*b^3 + 2*a*b^
4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a^3*b^2 + 3*a^2*b^3 +
 a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*tan(x)^2)*sqrt(b*tan(x
)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*tan(x)^8 + a^8 + 3*a
^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*ta
n(x)^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{(a + b\tan^4(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)**4)**(5/2),x)
```

```
[Out] Integral(cot(x)/(a + b*tan(x)**4)**(5/2), x)
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*tan(x)^4)^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.406 $\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 \, dx$

Optimal. Leaf size=212

$$\frac{\left(a^2-b^2\sqrt{-c^2}\right) {}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+fx))^m}{2f(1+m)}+\frac{\left(a^2+b^2\sqrt{-c^2}\right) {}_2F_1\Big(}{}$$

```
[Out] 1/2*hypergeom([1, 1+m],[2+m],-c*tan(f*x+e)/(-c^2)^(1/2))*(a^2-b^2*(-c^2)^(1
/2))*tan(f*x+e)*(d*tan(f*x+e))^m/f/(1+m)+1/2*hypergeom([1, 1+m],[2+m],c*tan
(f*x+e)/(-c^2)^(1/2))*(a^2+b^2*(-c^2)^(1/2))*tan(f*x+e)*(d*tan(f*x+e))^m/f/
(1+m)+4*a*b*hypergeom([1, 3/4+1/2*m],[7/4+1/2*m],-tan(f*x+e)^2)*(c*tan(f*x+
e))^(3/2)*(d*tan(f*x+e))^m/c/f/(3+2*m)
```

Rubi [A]
time = 0.46, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3751, 15, 1845, 371, 1300}

$$\frac{\left(a^2-b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(e+fx))^m {}_2F_1\left(1,m+1;m+2;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)}+\frac{\left(a^2+b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(e+fx))^m {}_2F_1\left(1,m+1;m+2;\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)}+\frac{4ab(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m {}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(2m+7);-\tan^2(e+fx)\right)}{cf(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] ((a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f*x
])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*f*(1 + m)) + ((a^2 + b^
2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, (c*Tan[e + f*x])/Sqrt[-c^2
]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*f*(1 + m)) + (4*a*b*Hypergeometric2F
1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2)*(d*T
an[e + f*x])^m)/(c*f*(3 + 2*m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1300

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (c_.)*(x_)^4), x_Sym
bol] :> With[{q = Rt[(-a)*c, 2]}, Dist[-(e/2 + c*(d/(2*q))), Int[(f*x)^m/(q
 - c*x^2), x], x] + Dist[e/2 - c*(d/(2*q)), Int[(f*x)^m/(q + c*x^2), x], x]
] /; FreeQ[{a, c, d, e, f, m}, x]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx = \frac{c\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2\left(\frac{dx}{c}\right)^m}{c^2+x^2}dx, x, c\tan(e+fx)\right)}{f}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m(a+bx)^2}{c^2+x^4}dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m)\text{Subst}\left(\int \frac{x^{1+2m}(}{c^2+}\right.}{f}$$

$$= \frac{(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m)\text{Subst}\left(\int \left(\frac{2abx^2}{c^2+}\right.\right.}{f}$$

$$= \frac{(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m)\text{Subst}\left(\int \frac{x^{1+2m}(}{c^2}\right.}{f}$$

$$= \frac{4ab\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(e+fx)\right)(c\text{ta}}{cf(3+2m)}$$

$$= \frac{\left(a^2-b^2\sqrt{-c^2}\right){}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\text{ta}}{2f(1+m)}$$

Mathematica [A]
time = 0.95, size = 151, normalized size = 0.71

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(\frac{a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(e+fx)\right)}{1+m}+b\left(\frac{bc\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{2+m}+\frac{4a\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(e+fx)\right)\sqrt{c\tan(e+fx)}}{3+2m}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] (Tan[e + f*x]*(d*Tan[e + f*x])^m*((a^2*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, -Tan[e + f*x]^2])/(1 + m) + b*((b*c*Hypergeometric2F1[1, (2 + m)/2,
(4 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(2 + m) + (4*a*Hypergeometric2F1[
1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*Sqrt[c*Tan[e + f*x]])/(3 + 2*
m))))/f
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \left(a+b\sqrt{c\tan(fx+e)}\right)^2 (d\tan(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x, algorithm="maxim
a")
```

```
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)^2*(d*tan(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x, algorithm="frica
s")
```

```
[Out] integral(2*sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*a*b + (b^2*c*tan(f*x + e
) + a^2)*(d*tan(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))**(1/2))**2*(d*tan(f*x+e))**m,x)
```

```
[Out] Integral((d*tan(e + f*x))**m*(a + b*sqrt(c*tan(e + f*x)))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))^2*(d*tan(f*x+e))^m,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)^2*(d*tan(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\sqrt{c\tan\left(e+f\,x\right)}\right)^2\,{\left(d\tan(e+f\,x)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^(1/2))^2*(d*tan(e + f*x))^m,x)
[Out] int((a + b*(c*tan(e + f*x))^(1/2))^2*(d*tan(e + f*x))^m, x)
```

3.407 $\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx$

Optimal. Leaf size=121

$$\frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m}{f(1+m)}+\frac{2b\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(e\right.}{cf(3}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(f*x+e)^2)*tan(f*x+e)*(d*tan(f*x
+e))^m/f/(1+m)+2*b*hypergeom([1, 3/4+1/2*m],[7/4+1/2*m],-tan(f*x+e)^2)*(c*t
an(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/f/(3+2*m)
```

Rubi [A]
time = 0.22, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3751, 15, 1845, 371}

$$\frac{a\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(e+fx)\right)}{f(m+1)}+\frac{2b(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(2m+7);-\tan^2(e+fx)\right)}{cf(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x]
*(d*Tan[e + f*x])^m)/(f*(1 + m)) + (2*b*Hypergeometric2F1[1, (3 + 2*m)/4, (
7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*
f*(3 + 2*m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int (d\tan(e+fx))^m\left(a+b\sqrt{c\tan(e+fx)}\right)\,dx=\frac{c\text{Subst}\left(\int\frac{\left(a+b\sqrt{x}\right)\left(\frac{dx}{c}\right)^m}{c^2+x^2}\,dx,x,c\tan(e+fx)\right)}{f}$$

$$=\frac{(2c)\text{Subst}\left(\int\frac{x\left(\frac{dx^2}{c}\right)^m(a+bx)}{c^2+x^4}\,dx,x,\sqrt{c\tan(e+fx)}\right)}{f}$$

$$=\frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int\frac{x^{1+2m}(a}{c^2+x}\right.}{f}$$

$$=\frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int\left(\frac{ax^{1+2m}}{c^2+x^4}\right.\right.}{f}$$

$$=\frac{\left(2ac(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int\frac{x^{1+2m}}{c^2+x^4}\right.}{f}$$

$$=\frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e}{f(1+m)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.42, size = 304, normalized size = 2.51

$$\frac{\left(\left(a-b\sqrt[4]{-c^2}\right){}_2F_1\left(1,2(1+m);3+2m;-\frac{\sqrt{c\tan(e+fx)}}{\sqrt[4]{-c^2}}\right)+\left(a+ib\sqrt[4]{-c^2}\right){}_2F_1\left(1,2(1+m);3+2m;-\frac{i\sqrt{c\tan(e+fx)}}{\sqrt[4]{-c^2}}\right)+a\,{}_2F_1\left(1,2(1+m);3+2m;\frac{i\sqrt{c\tan(e+fx)}}{\sqrt[4]{-c^2}}\right)-ib\sqrt[4]{-c^2}\,{}_2F_1\left(1,2(1+m);3+2m;\frac{i\sqrt{c\tan(e+fx)}}{\sqrt[4]{-c^2}}\right)+a\,{}_2F_1\left(1,2(1+m);3+2m;\frac{\sqrt{c\tan(e+fx)}}{\sqrt[4]{-c^2}}\right)+b\sqrt[4]{-c^2}\,{}_2F_1\left(1,2(1+m);3+2m;\frac{\sqrt{c\tan(e+fx)}}{\sqrt[4]{-c^2}}\right)\right)\tan(e+fx)(d\tan(e+fx))^m}{4f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (((a - b*(-c^2)^(1/4))*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -(Sqrt[c*Ta
n[e + f*x]]/(-c^2)^(1/4))] + (a + I*b*(-c^2)^(1/4))*Hypergeometric2F1[1, 2*
(1 + m), 3 + 2*m, ((-I)*Sqrt[c*Tan[e + f*x]])/(-c^2)^(1/4)] + a*Hypergeomet
ric2F1[1, 2*(1 + m), 3 + 2*m, (I*Sqrt[c*Tan[e + f*x]])/(-c^2)^(1/4)] - I*b*
(-c^2)^(1/4)*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, (I*Sqrt[c*Tan[e + f*x
```

```
]])/(-c^2)^(1/4)] + a*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, Sqrt[c*Tan[e
 + f*x]]/(-c^2)^(1/4)] + b*(-c^2)^(1/4)*Hypergeometric2F1[1, 2*(1 + m), 3 +
 2*m, Sqrt[c*Tan[e + f*x]]/(-c^2)^(1/4)])*Tan[e + f*x]*(d*Tan[e + f*x])^m)/
(4*f*(1 + m))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(a+b\sqrt{c\tan\left(fx+e\right)}\right)\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x)
[Out] int((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x, algorithm="maxima"
)
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)*(d*tan(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x, algorithm="fricas"
)
[Out] integral(sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*b + (d*tan(f*x + e))^m*a,
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(e+fx\right)\right)^{m}\left(a+b\sqrt{c\tan\left(e+fx\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))**(1/2))*(d*tan(f*x+e))**m,x)
```

```
[Out] Integral((d*tan(e + f*x))**m*(a + b*sqrt(c*tan(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^(1/2))*(d*tan(f*x+e))^m,x, algorithm="giac")
[Out] integrate((sqrt(c*tan(f*x + e))*b + a)*(d*tan(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\sqrt{c\,\tan\left(e+f\,x\right)}\right)\,{\left(d\,\tan(e+f\,x)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^(1/2))*(d*tan(e + f*x))^m,x)
[Out] int((a + b*(c*tan(e + f*x))^(1/2))*(d*tan(e + f*x))^m, x)
```

3.408 $\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx$

Optimal. Leaf size=460

$$\frac{a\left(a^2-b^2\sqrt{-c^2}\right)\,{}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+fx))^m}{2\left(a^4+b^4c^2\right)f(1+m)}+\frac{a\left(a^2+b^2\sqrt{-c^2}\right)\,{}_2F_1\left(\right.}{}$$

```
[Out] b^4*c^2*hypergeom([1, 2+2*m],[3+2*m],-b*(c*tan(f*x+e))^(1/2)/a)*tan(f*x+e)*
(d*tan(f*x+e))^m/a/(b^4*c^2+a^4)/f/(1+m)+1/2*a*hypergeom([1, 1+m],[2+m],-c*
tan(f*x+e)/(-c^2)^(1/2))*(a^2-b^2*(-c^2)^(1/2))*tan(f*x+e)*(d*tan(f*x+e))^m
/(b^4*c^2+a^4)/f/(1+m)+1/2*a*hypergeom([1, 1+m],[2+m],c*tan(f*x+e)/(-c^2)^(
1/2))*(a^2+b^2*(-c^2)^(1/2))*tan(f*x+e)*(d*tan(f*x+e))^m/(b^4*c^2+a^4)/f/(1
+m)-b*hypergeom([1, 3/2+m],[5/2+m],-c*tan(f*x+e)/(-c^2)^(1/2))*(a^2-b^2*(-c
^2)^(1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)/f/(3+2*m)-
b*hypergeom([1, 3/2+m],[5/2+m],c*tan(f*x+e)/(-c^2)^(1/2))*(a^2+b^2*(-c^2)^(
1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)/f/(3+2*m)
```

Rubi [A]
time = 0.86, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3751, 15, 6857, 66, 1845, 1300, 371}

$$\frac{b^4c^2\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)}{af(m+1)\left(a^4+b^4c^2\right)}-\frac{b\left(a^2-b^2\sqrt{-c^2}\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{cf(2m+3)\left(a^4+b^4c^2\right)}-\frac{b\left(a^2+b^2\sqrt{-c^2}\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{cf(2m+3)\left(a^4+b^4c^2\right)}+\frac{a\left(a^2-b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,m+1;m+2;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)\left(a^4+b^4c^2\right)}+\frac{a\left(a^2+b^2\sqrt{-c^2}\right)\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,m+1;m+2;\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)\left(a^4+b^4c^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (a*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f
*x])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*c^2)*f*(1
+ m)) + (a*(a^2 + b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2 + m, (c*Tan
[e + f*x])/Sqrt[-c^2]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*c^2)*
f*(1 + m)) + (b^4*c^2*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*
Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(a*(a^4 + b^4*c^2)*f*(1
 + m)) - (b*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1, (3 + 2*m)/2, (5 + 2
*m)/2, -((c*Tan[e + f*x])/Sqrt[-c^2])]*(c*Tan[e + f*x])^(3/2)*(d*Tan[e + f*
x])^m)/(c*(a^4 + b^4*c^2)*f*(3 + 2*m)) - (b*(a^2 + b^2*Sqrt[-c^2])*Hypergeo
metric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, (c*Tan[e + f*x])/Sqrt[-c^2]]*(c*Tan[
e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*(a^4 + b^4*c^2)*f*(3 + 2*m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
```

```
 &&  !IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1300

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (c_.)*(x_)^4), x_Sym
bol] :> With[{q = Rt[(-a)*c, 2]}, Dist[-(e/2 + c*(d/(2*q))), Int[(f*x)^m/(q
 - c*x^2), x], x] + Dist[e/2 - c*(d/(2*q)), Int[(f*x)^m/(q + c*x^2), x], x]
] /; FreeQ[{a, c, d, e, f, m}, x]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \frac{c\text{Subst}\left(\int \frac{\left(\frac{dx}{c}\right)^m}{\left(a+b\sqrt{x}\right)(c^2+x^2)}\,dx, x, c\tan(e+fx)\right)}{f}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m}{(a+bx)(c^2+x^4)}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m)\,\text{Subst}\Big(\int \frac{x^{1+2m}}{(a+bx)(c^2+x^4)}\,dx, x, \sqrt{c\tan(e}{f}$$

$$= \frac{(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m)\,\text{Subst}\Big(\int \Big(\frac{b^4x^{1+2m}}{(a^4+b^4c^2)(a+bx)} + \frac{x^{1+2m}(a^3-a^2}{(a^4+b^4}}{f}$$

$$= \frac{(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m)\,\text{Subst}\Big(\int \frac{x^{1+2m}(a^3-a^2bx+ab^2x^2-b^3x^3)}{c^2+x^4}\,dx,}{(a^4+b^4c^2)\,f}$$

$$= \frac{b^4c^2\,{}_2F_1\left(1, 2(1+m); 3+2m; -\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+f}{a\,(a^4+b^4c^2)\,f(1+m)}$$

$$= \frac{b^4c^2\,{}_2F_1\left(1, 2(1+m); 3+2m; -\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+f}{a\,(a^4+b^4c^2)\,f(1+m)}$$

$$= \frac{b^4c^2\,{}_2F_1\left(1, 2(1+m); 3+2m; -\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+f}{a\,(a^4+b^4c^2)\,f(1+m)}$$

$$= \frac{a\left(a^2-b^2\sqrt{-c^2}\right)\,{}_2F_1\left(1, 1+m; 2+m; -\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+}{2\,(a^4+b^4c^2)\,f(1+m)}$$

Mathematica [A]
time = 4.87, size = 265, normalized size = 0.58

$$\frac{c\tan(e+fx)(d\tan(e+fx))^m\left(\frac{a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(e+fx)\right)}{c+cm}+b\left(\frac{b^3c\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)}{a+am}+\frac{ab\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{2+m}+2\sqrt{c\tan(e+fx)}\left(-\frac{a^2\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(e+fx)\right)}{3c+2cm}-\frac{b^2\,{}_2F_1\left(1,\frac{1}{4}(5+2m);\frac{1}{4}(9+2m);-\tan^2(e+fx)\right)\tan(e+fx)}{5+2m}\right)\right)\right)}{(a^4+b^4c^2)\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

```
[Out] (c*Tan[e + f*x]*(d*Tan[e + f*x])^m*((a^3*Hypergeometric2F1[1, (1 + m)/2, (3
 + m)/2, -Tan[e + f*x]^2])/(c + c*m) + b*((b^3*c*Hypergeometric2F1[1, 2*(1
```

```
+ m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)])/(a + a*m) + (a*b*Hypergeomet
ric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(2 + m) + 2*
Sqrt[c*Tan[e + f*x]]*(-((a^2*Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4,
 -Tan[e + f*x]^2])/(3*c + 2*c*m)) - (b^2*Hypergeometric2F1[1, (5 + 2*m)/4,
(9 + 2*m)/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(5 + 2*m)))))/((a^4 + b^4*c^2)*
f)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^m}{a+b\sqrt{c\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x)
```

```
[Out] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="maxima"
)
```

```
[Out] integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="fricas"
)
```

```
[Out] integral((sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*b - (d*tan(f*x + e))^m*a)
/(b^2*c*tan(f*x + e) - a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m/(a+b*(c*tan(f*x+e))**(1/2)),x)
[Out] Integral((d*tan(e + f*x))**m/(a + b*sqrt(c*tan(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\tan(e+f\,x))^m}{a+b\,\sqrt{c\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2)),x)
[Out] int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2)), x)
```

3.409 $\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx$

Optimal. Leaf size=617

$$\frac{\left(a^6-3a^2b^4c^2-3a^4b^2\sqrt{-c^2}-b^6(-c^2)^{3/2}\right)\,{}_2F_1\left(1,1+m;2+m;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+fx))}{2\left(a^4+b^4c^2\right)^2f(1+m)}$$

```
[Out] 4*a^2*b^4*c^2*hypergeom([1, 2+2*m],[3+2*m],-b*(c*tan(f*x+e))^(1/2)/a)*tan(f
*x+e)*(d*tan(f*x+e))^m/(b^4*c^2+a^4)^2/f/(1+m)+b^4*c^2*hypergeom([2, 2+2*m]
,[3+2*m],-b*(c*tan(f*x+e))^(1/2)/a)*tan(f*x+e)*(d*tan(f*x+e))^m/a^2/(b^4*c^
2+a^4)/f/(1+m)+1/2*hypergeom([1, 1+m],[2+m],-c*tan(f*x+e)/(-c^2)^(1/2))*(a^
6-3*a^2*b^4*c^2-b^6*(-c^2)^(3/2)-3*a^4*b^2*(-c^2)^(1/2))*tan(f*x+e)*(d*tan(
f*x+e))^m/(b^4*c^2+a^4)^2/f/(1+m)+1/2*hypergeom([1, 1+m],[2+m],c*tan(f*x+e)
/(-c^2)^(1/2))*(a^6-3*a^2*b^4*c^2+b^6*(-c^2)^(3/2)+3*a^4*b^2*(-c^2)^(1/2))*
tan(f*x+e)*(d*tan(f*x+e))^m/(b^4*c^2+a^4)^2/f/(1+m)-2*a*b*hypergeom([1, 3/2
+m],[5/2+m],-c*tan(f*x+e)/(-c^2)^(1/2))*(a^4-b^4*c^2-2*a^2*b^2*(-c^2)^(1/2)
)*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)^2/f/(3+2*m)-2*a*b*h
ypergeom([1, 3/2+m],[5/2+m],c*tan(f*x+e)/(-c^2)^(1/2))*(a^4-b^4*c^2+2*a^2*b
^2*(-c^2)^(1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)^2/f/
(3+2*m)
```

Rubi [A]

time = 1.07, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3751, 15, 6857, 66, 1845, 1300, 371}

$$\frac{4a^2b^4c^2\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)}{f(m+1)\left(a^4+b^4c^2\right)^2}+\frac{b^4c^2\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(2,2(m+1);2m+3;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)}{a^2f(m+1)\left(a^4+b^4c^2\right)}-\frac{2ab\left(a^4-2a^2b^2\sqrt{-c^2}-b^4c^2\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{cf(2m+3)\left(a^4+b^4c^2\right)^2}-\frac{2ab\left(a^4+2a^2b^2\sqrt{-c^2}-b^4c^2\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{cf(2m+3)\left(a^4+b^4c^2\right)^2}+\frac{\left(a^6-3a^4b^2\sqrt{-c^2}-3a^2b^4c^2-b^6(-c^2)^{3/2}\right)\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,m+1;m+2;-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)\left(a^4+b^4c^2\right)^2}+\frac{\left(a^6+3a^4b^2\sqrt{-c^2}-3a^2b^4c^2+b^6(-c^2)^{3/2}\right)\tan(e+fx)(d\tan(e+fx))^m\,{}_2F_1\left(1,m+1;m+2;\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)}{2f(m+1)\left(a^4+b^4c^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] ((a^6 - 3*a^2*b^4*c^2 - 3*a^4*b^2*Sqrt[-c^2] - b^6*(-c^2)^(3/2))*Hypergeome
tric2F1[1, 1 + m, 2 + m, -((c*Tan[e + f*x])/Sqrt[-c^2])]*Tan[e + f*x]*(d*Ta
n[e + f*x])^m)/(2*(a^4 + b^4*c^2)^2*f*(1 + m)) + ((a^6 - 3*a^2*b^4*c^2 + 3*
a^4*b^2*Sqrt[-c^2] + b^6*(-c^2)^(3/2))*Hypergeometric2F1[1, 1 + m, 2 + m, (
c*Tan[e + f*x])/Sqrt[-c^2]]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(2*(a^4 + b^4*
c^2)^2*f*(1 + m)) + (4*a^2*b^4*c^2*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m,
 -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/((a^4 + b^
4*c^2)^2*f*(1 + m)) + (b^4*c^2*Hypergeometric2F1[2, 2*(1 + m), 3 + 2*m, -((
b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x]*(d*Tan[e + f*x])^m)/(a^2*(a^4 + b^
4*c^2)*f*(1 + m)) - (2*a*b*(a^4 - b^4*c^2 - 2*a^2*b^2*Sqrt[-c^2])*Hypergeom
etric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, -((c*Tan[e + f*x])/Sqrt[-c^2])]*(c*Ta
```

```
n[e + f*x])^(3/2)*(d*Tan[e + f*x])^m)/(c*(a^4 + b^4*c^2)^2*f*(3 + 2*m)) - (
2*a*b*(a^4 - b^4*c^2 + 2*a^2*b^2*Sqrt[-c^2])*Hypergeometric2F1[1, (3 + 2*m)
/2, (5 + 2*m)/2, (c*Tan[e + f*x])/Sqrt[-c^2]]*(c*Tan[e + f*x])^(3/2)*(d*Tan
[e + f*x])^m)/(c*(a^4 + b^4*c^2)^2*f*(3 + 2*m))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1300

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (c_.)*(x_)^4), x_Sym
bol] :> With[{q = Rt[(-a)*c, 2]}, Dist[-(e/2 + c*(d/(2*q))), Int[(f*x)^m/(q
 - c*x^2), x], x] + Dist[e/2 - c*(d/(2*q)), Int[(f*x)^m/(q + c*x^2), x], x]
] /; FreeQ[{a, c, d, e, f, m}, x]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \frac{c\text{Subst}\left(\int \frac{\left(\frac{dx}{c}\right)^m}{\left(a+b\sqrt{x}\right)^2(c^2+x^2)}\,dx, x, c\tan(e+fx)\right)}{f}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{x\left(\frac{dx^2}{c}\right)^m}{(a+bx)^2(c^2+x^4)}\,dx, x, \sqrt{c\tan(e+fx)}\right)}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}}{(a+bx)^2(c^2+x^4)}\,dx, x, \sqrt{c}\right.}{f}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \left(\frac{b^4x^{1+2m}}{(a^4+b^4c^2)(a+bx)^2} + \frac{4a}{(a^4+b}\right.\right.}{}$$

$$= \frac{\left(2c(c\tan(e+fx))^{-m}(d\tan(e+fx))^m\right)\text{Subst}\left(\int \frac{x^{1+2m}\left(a^2(a^4-3b^4c^2)-2ab(a^4\right.}{}\right.}{\left(a^4+b^4c^2\right)^2 f}$$

$$= \frac{4a^2b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\,\text{ta}}{\left(a^4+b^4c^2\right)^2 f(1+m)}$$

$$= \frac{4a^2b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\,\text{ta}}{\left(a^4+b^4c^2\right)^2 f(1+m)}$$

$$= \frac{4a^2b^4c^2\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\,\text{ta}}{\left(a^4+b^4c^2\right)^2 f(1+m)}$$

$$= \frac{\left(a^6-3a^2b^4c^2-3a^4b^2\sqrt{-c^2}-b^6\left(-c^2\right)^{3/2}\right)\,{}_2F_1\left(1,1+m;2+m;-\frac{c\tan}{\sqrt{}}\right.}{2\left(a^4+b^4c^2\right)^2 f(1+m)}$$

Mathematica [A]

time = 4.05, size = 381, normalized size = 0.62

$$\frac{c(d\tan(e+fx))^m\left(\frac{a^2(a^4-3b^4c^2)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{c(1+m)}+\frac{4a^2b^4c\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)}{1+m}+\frac{b^4c(a^4+b^4c^2)\,{}_2F_1\left(2,2(1+m);3+2m;-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)}{a^2(1+m)}+\frac{b^2(3a^4-b^4c^2)\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\tan^2(e+fx)\right)\tan^2(e+fx)}{2+m}+\frac{4ab(-a^4+b^4c^2)\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(e+fx)\right)(c\tan(e+fx))^{3/2}}{c^2(3+2m)}-\frac{8a^3b^3\,{}_2F_1\left(1,\frac{1}{4}(5+2m);\frac{1}{4}(9+2m);-\tan^2(e+fx)\right)(c\tan(e+fx))^{5/2}}{c^2(5+2m)}\right)}{\left(a^4+b^4c^2\right)^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

```
[Out] (c*(d*Tan[e + f*x])^m*((a^2*(a^4 - 3*b^4*c^2)*Hypergeometric2F1[1, (1 + m)/
2, (3 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(c*(1 + m)) + (4*a^2*b^4*c*Hyp
ergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e
+ f*x])/(1 + m) + (b^4*c*(a^4 + b^4*c^2)*Hypergeometric2F1[2, 2*(1 + m), 3
+ 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x])/(a^2*(1 + m)) + (b^2*(3
*a^4 - b^4*c^2)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2]
*Tan[e + f*x]^2)/(2 + m) + (4*a*b*(-a^4 + b^4*c^2)*Hypergeometric2F1[1, (3
+ 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2))/(c^2*(3 + 2
*m)) - (8*a^3*b^3*Hypergeometric2F1[1, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[e + f
*x]^2]*(c*Tan[e + f*x])^(5/2))/(c^2*(5 + 2*m))))/((a^4 + b^4*c^2)^2*f)
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(d\tan (fx+e))^{m}}{\left(a+b\sqrt{c\tan (fx+e)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

```
[Out] int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="frica
s")
```

```
[Out] integral(-(2*sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*a*b - (b^2*c*tan(f*x +
 e) + a^2)*(d*tan(f*x + e))^m)/(b^4*c^2*tan(f*x + e)^2 - 2*a^2*b^2*c*tan(f*
x + e) + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m/(a+b*(c*tan(f*x+e))**(1/2))**2,x)
```

```
[Out] Integral((d*tan(e + f*x))**m/(a + b*sqrt(c*tan(e + f*x)))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="giac"
)
```

```
[Out] integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\tan(e+f\,x))^m}{\left(a+b\,\sqrt{c\tan(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2))^2,x)
```

```
[Out] int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2))^2, x)
```

3.410 $\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=74

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+m+np);\frac{1}{2}(3+m+np);-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p}{f(1+m+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2*m+1/2],[1/2*n*p+1/2*m+3/2],-tan(f*x+e)^2)*tan(f*x
+e)*(d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p/f/(n*p+m+1)
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3659, 20, 3557, 371}

$$\frac{\tan(e+fx)(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p {}_2F_1\left(1,\frac{1}{2}(m+np+1);\frac{1}{2}(m+np+3);-\tan^2(e+fx)\right)}{f(m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n*p)/2, (3 + m + n*p)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[c^IntPart[n]*((c*(d*Tan[e + f*x
```

```
])^p)^FracPart[n]/(d*Tan[e + f*x])^(p*FracPart[n])), Int[(a + b*Tan[e + f*x
])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{n}$$

$$= \left((c\tan(e+fx))^{-m-np}(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\right)$$

$$= \frac{\left(c(c\tan(e+fx))^{-m-np}(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^{p}\right.}{f}$$

$$= \frac{{}_2F_1\left(1,\tfrac{1}{2}(1+m+np);\tfrac{1}{2}(3+m+np);-\tan^2(e+fx)\right)\tan(e}{f(1+m+np)}$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 1.03

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+m+np);1+\frac{1}{2}(1+m+np);-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p}{f(1+m+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
[Out] (Hypergeometric2F1[1, (1 + m + n*p)/2, 1 + (1 + m + n*p)/2, -Tan[e + f*x]^2
]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p
))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(fx+e\right)\right)^{m}\left(b\left(c\tan\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b(c\tan\left(e+fx\right))^{n}\right)^{p}\left(d\tan\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*tan(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^m\,{\left(b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((d*tan(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

3.411 $\int \tan^2(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=63

$$\frac{{}_2F_1\left(1,\frac{1}{2}(3+np);\frac{1}{2}(5+np);-\tan^2(e+fx)\right)\tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+3/2],[1/2*n*p+5/2],-tan(f*x+e)^2)*tan(f*x+e)^3*(b*(c*
tan(f*x+e))^n)^p/f/(n*p+3)
```

Rubi [A]
time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3740, 16, 3557, 371}

$$\frac{\tan^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+3);\frac{1}{2}(np+5);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
```

```
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \tan^2(e+fx)(c\tan(e$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{2+np}\,dx}{c^2}$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{2+np}}{c^2+x^2}\,dx, x, c\,\text{ta}\right.}{cf}$$

$$= \frac{{}_2F_1\left(1,\frac{1}{2}(3+np);\frac{1}{2}(5+np);-\tan^2(e+fx)\right)\tan^3(e+fx)\left(b(c\,\text{ta}\right.}{f(3+np)}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 1.03

$$\frac{{}_2F_1\left(1,\frac{1}{2}(3+np);1+\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (3 + n*p)/2, 1 + (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e
+ f*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\tan^2(fx+e)\right)\left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \tan^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\,{\left(b\,{\left(c\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(tan(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

3.412 $\int (b(c\tan(e+fx))^n)^p \, dx$

Optimal. Leaf size=61

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*ta
n(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A]

time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (b(c\tan(e+fx))^n)^p\,dx = \left((c\tan(e+fx))^{-np}(b(c\tan(e+fx))^n)^p\right)\int (c\tan(e+fx))^{np}\,dx$$
$$= \frac{(c(c\tan(e+fx))^{-np}(b(c\tan(e+fx))^n)^p)\,\text{Subst}\left(\int \frac{x^{np}}{c^2+x^2}\,dx, x, c\tan(e+fx)\right)}{f}$$
$$= \frac{{}_2F_1\left(1, \frac{1}{2}(1+np); \frac{1}{2}(3+np); -\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.97

$$\frac{{}_2F_1\left(1, \frac{1}{2}(1+np); \frac{1}{2}(3+np); -\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f+fnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f + f*n*p)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e+fx\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p, x)
```

3.413 $\int \cot^2(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=63

$$-\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-1+np);\frac{1}{2}(1+np);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(1-np)}$$

```
[Out] -cot(f*x+e)*hypergeom([1, 1/2*n*p-1/2],[1/2*n*p+1/2],-tan(f*x+e)^2)*(b*(c*t
an(f*x+e))^n)^p/f/(-n*p+1)
```

Rubi [A]
time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3740, 16, 3557, 371}

$$-\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-1);\frac{1}{2}(np+1);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n*p)/2, (1 + n*p)/2, -Tan[e + f*
x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
```

```
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^2(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \left((c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int \cot^2(e+fx)(c\tan(e-$$

$$= \left(c^2(c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int (c\tan(e+fx))^{-2+n}$$

$$= \frac{\left(c^3(c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\operatorname{Subst}\left(\int \frac{x^{-2+np}}{c^2+x^2}\,dx, x, c\right.}{f}$$

$$= -\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-1+np);\frac{1}{2}(1+np);-\tan^2(e+fx)\right)(b(c\mathrm{t}}{f(1-np)}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.97

$$\frac{\cot(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-1+np);\frac{1}{2}(1+np);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(-1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n*p)/2, (1 + n*p)/2, -Tan[e + f*x]
^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-1 + n*p))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \left(\cot^2(fx+e)\right)(b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan{(e+fx)})^n)^p\cot^2{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{cot}(e+f\,x)^2\,{\left(b\,{\left(c\,\mathrm{tan}(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

3.414 $\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=65

$$-\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-3+np);\frac{1}{2}(-1+np);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}$$

```
[Out] -cot(f*x+e)^3*hypergeom([1, 1/2*n*p-3/2],[1/2*n*p-1/2],-tan(f*x+e)^2)*(b*(c
*tan(f*x+e))^n)^p/f/(-n*p+3)
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3740, 16, 3557, 371}

$$-\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-3);\frac{1}{2}(np-1);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]^3*Hypergeometric2F1[1, (-3 + n*p)/2, (-1 + n*p)/2, -Tan[e +
 f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p)))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
```

```
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^4(e+fx)\,(b(c\tan(e+fx))^n)^p\;dx = \left((c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int \cot^4(e+fx)(c\tan(e$$

$$= \left(c^4(c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int (c\tan(e+fx))^{-4+}$$

$$= \frac{\left(c^5(c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\text{Subst}\left(\int \frac{x^{-4+np}}{c^2+x^2}\,dx, x\right.}{f}$$

$$= -\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-3+np);\frac{1}{2}(-1+np);-\tan^2(e+fx)\right)(}{f(3-np)}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 1.00

$$\frac{\cot^3(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-3+np);1+\frac{1}{2}(-3+np);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(-3+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]^3*Hypergeometric2F1[1, (-3 + n*p)/2, 1 + (-3 + n*p)/2, -Tan[e
 + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-3 + n*p))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(\cot^4(fx+e)\right)(b(c\tan(fx+e))^n)^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan{(e+fx)})^n)^p \cot^4{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)^4\,{\left(b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)^4*(b*(c*tan(e + f*x))^n)^p, x)
```

3.415 $\int \cot^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=65

$$-\frac{\cot^5(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-5+np);\frac{1}{2}(-3+np);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}$$

```
[Out] -cot(f*x+e)^5*hypergeom([1, 1/2*n*p-5/2],[1/2*n*p-3/2],-tan(f*x+e)^2)*(b*(c
*tan(f*x+e))^n)^p/f/(-n*p+5)
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3740, 16, 3557, 371}

$$-\frac{\cot^5(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np-5);\frac{1}{2}(np-3);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]^5*Hypergeometric2F1[1, (-5 + n*p)/2, (-3 + n*p)/2, -Tan[e +
 f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(5 - n*p)))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
```

```
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \cot^6(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx &= \left((c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int \cot^6(e+fx)(c\tan(e+\\ &= \left(c^6(c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int (c\tan(e+fx))^{-6+n\\ &= \frac{\left(c^7(c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\mathrm{Subst}\left(\int \frac{x^{-6+np}}{c^2+x^2}\,dx,x,c}{f}\\ &= -\frac{\cot^5(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-5+np);\frac{1}{2}(-3+np);-\tan^2(e+fx)\right)(b}{f(5-np)}\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 65, normalized size = 1.00

$$\frac{\cot^5(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-5+np);1+\frac{1}{2}(-5+np);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(-5+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]^5*Hypergeometric2F1[1, (-5 + n*p)/2, 1 + (-5 + n*p)/2, -Tan[e
 + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-5 + n*p))
```

Maple [**F**]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\cot^6(fx+e)\right)(b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan{(e+fx)})^n)^p \cot^6{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)^6\,{\left(b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cot(e + f*x)^6*(b*(c*tan(e + f*x))^n)^p, x)
```

3.416 $\int \tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=63

$$\frac{{}_2F_1\left(1,\frac{1}{2}(4+np);\frac{1}{2}(6+np);-\tan^2(e+fx)\right)\tan^4(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(4+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+2],[1/2*n*p+3],-tan(f*x+e)^2)*tan(f*x+e)^4*(b*(c*tan(
f*x+e))^n)^p/f/(n*p+4)
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3740, 16, 3557, 371}

$$\frac{\tan^4(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+4);\frac{1}{2}(np+6);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (4 + n*p)/2, (6 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(4 + n*p))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
```

```
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \tan^3(e+fx)(c\tan($$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{3+np}\,d}{c^3}$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int \frac{x^{3+np}}{c^2+x^2}\,dx, x, c\,t\right.}{c^2f}$$

$$= \frac{{}_2F_1\left(1,\frac{1}{2}(4+np);\frac{1}{2}(6+np);-\tan^2(e+fx)\right)\tan^4(e+fx)\left(b(c\,t\right.}{f(4+np)}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.97

$$\frac{{}_2F_1\left(1,2+\frac{np}{2};3+\frac{np}{2};-\tan^2(e+fx)\right)\tan^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(4+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
[Out] (Hypergeometric2F1[1, 2 + (n*p)/2, 3 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(4 + n*p))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\tan^3(fx+e)\right)\left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \tan^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^3\,{\left(b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int(tan(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

3.417 $\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=63

$$\frac{{}_2F_1\left(1,\frac{1}{2}(2+np);\frac{1}{2}(4+np);-\tan^2(e+fx)\right)\tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(2+np)}$$

[Out] hypergeom([1, 1/2*n*p+1],[1/2*n*p+2],-tan(f*x+e)^2)*tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p/f/(n*p+2)

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3740, 16, 3557, 371}

$$\frac{\tan^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+2);\frac{1}{2}(np+4);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]

[Out] (Hypergeometric2F1[1, (2 + n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p))

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
```

```
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \tan(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \left((c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int \tan(e+fx)(c\tan(e+$$
$$= \frac{\left((c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int (c\tan(e+fx))^{1+np}\,dx}{c}$$
$$= \frac{\left((c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\mathrm{Subst}\left(\int \frac{x^{1+np}}{c^2+x^2}\,dx, x, c\tan\right.}{f}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(2+np);\frac{1}{2}(4+np);-\tan^2(e+fx)\right)\tan^2(e+fx)\,(b(c\tan}{f(2+np)}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.97

$$\frac{{}_2F_1\left(1,1+\frac{np}{2};2+\frac{np}{2};-\tan^2(e+fx)\right)\tan^2(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(2+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
[Out] (Hypergeometric2F1[1, 1 + (n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \tan(fx+e)\,(b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \tan(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)\,\left(b\,{\left(c\tan(e+f\,x)\right)}^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(tan(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

3.418 $\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=50

$$\frac{{}_2F_1\left(1,\frac{np}{2};1+\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

```
[Out] hypergeom([1, 1/2*n*p],[1/2*n*p+1],-tan(f*x+e)^2)*(b*(c*tan(f*x+e))^n)^p/f/
n/p
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3740, 16, 3557, 371}

$$\frac{{}_2F_1\left(1,\frac{np}{2};\frac{np}{2}+1;-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (n*p)/2, 1 + (n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e +
f*x])^n)^p)/(f*n*p)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
```

```
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cot(e+fx)(c\tan(e+$$

$$= \left(c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (c\tan(e+fx))^{-1+np}$$

$$= \frac{\left(c^2(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\mathrm{Subst}\left(\int \frac{x^{-1+np}}{c^2+x^2}\,dx, x,\right.}{f}$$

$$= \frac{{}_2F_1\left(1,\frac{np}{2};1+\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$\frac{{}_2F_1\left(1,\frac{np}{2};1+\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (n*p)/2, 1 + (n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e +
f*x])^n)^p)/(f*n*p)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \cot(fx+e)\left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan (e+fx))^{n})^{p}\cot (e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)\,{\left(b\,{\left(c\tan(e+f\,x)\right)}^{n}\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int(cot(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

3.419 $\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=62

$$-\frac{\cot^2(e+fx)\,_2F_1\left(1,\frac{1}{2}(-2+np);\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

```
[Out] -cot(f*x+e)^2*hypergeom([1, 1/2*n*p-1],[1/2*n*p],-tan(f*x+e)^2)*(b*(c*tan(f
*x+e))^n)^p/f/(-n*p+2)
```

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3740, 16, 3557, 371}

$$-\frac{\cot^2(e+fx)\,_2F_1\left(1,\frac{1}{2}(np-2);\frac{np}{2};-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((Cot[e + f*x]^2*Hypergeometric2F1[1, (-2 + n*p)/2, (n*p)/2, -Tan[e + f*x]
^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 - n*p)))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
```

```
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^3(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \left((c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int \cot^3(e+fx)(c\tan(e -$$

$$= \left(c^3(c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int (c\tan(e+fx))^{-3+n}$$

$$= \frac{\left(c^4(c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\mathrm{Subst}\left(\int \frac{x^{-3+np}}{c^2+x^2}\,dx, x, c}{f}$$

$$= -\frac{\cot^2(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-2+np);\frac{np}{2};-\tan^2(e+fx)\right)(b(c\tan(e +}{f(2-np)}$$

Mathematica [**A**]
time = 0.04, size = 59, normalized size = 0.95

$$\frac{\cot^2(e+fx)\,{}_2F_1\left(1,-1+\frac{np}{2};\frac{np}{2};-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(-2+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]^2*Hypergeometric2F1[1, -1 + (n*p)/2, (n*p)/2, -Tan[e + f*x]^2
]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-2 + n*p))
```

Maple [**F**]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\cot^3(fx+e)\right)(b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan{(e+fx)})^n)^p\cot^3{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{cot}(e+f\,x)^3\,{\left(b\,{\left(c\,\mathrm{tan}(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cot(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

3.420 $\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left((d\tan(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\tan(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int (d\tan(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int (d\tan(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A]
time = 2.42, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int (d\tan(fx+e))^m\left(a+b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan (e+fx))^m (a+b(c\tan (e+fx))^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((d*tan(e + f*x))**m*(a + b*(c*tan(e + f*x))**n)**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (d\tan(e+f\,x))^m\,(a+b\,(c\tan(e+f\,x))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*tan(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p, x)
```

3.421 $\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=78

$$\frac{(d\cot(e+fx))^m\, {}_2F_1\left(1,\frac{1}{2}(1-m+2p);\frac{1}{2}(3-m+2p);-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^2(e+fx)\right)^p}{f(1-m+2p)}$$

```
[Out] (d*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+p],[3/2-1/2*m+p],-tan(f*x+e)^2)*ta
n(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1-m+2*p)
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3739, 2684, 3557, 371}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\cot(e+fx))^m\, {}_2F_1\left(1,\frac{1}{2}(-m+2p+1);\frac{1}{2}(-m+2p+3);-\tan^2(e+fx)\right)}{f(-m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + 2*p)/2, (3 - m + 2*p)/2,
-Tan[e + f*x]^2]*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 - m + 2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\cot(e+fx))^m \tan^{2p}(e+$$

$$= \left((d\cot(e+fx))^m \tan^{m-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int \tan^{-m+2}$$

$$= \frac{\left((d\cot(e+fx))^m \tan^{m-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\text{Subst}\left(\int \frac{x}{}\right.}{f}$$

$$= \frac{(d\cot(e+fx))^m \,{}_2F_1\left(1,\frac{1}{2}(1-m+2p);\frac{1}{2}(3-m+2p);-\tan^2(e\right.}{f(1-m+2p)}$$

Mathematica [A]
time = 0.11, size = 70, normalized size = 0.90

$$-\frac{d(d\cot(e+fx))^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}-\frac{m}{2}+p;\frac{3}{2}-\frac{m}{2}+p;-\tan^2(e+fx)\right)\left(b\tan^2(e+fx)\right)^p}{f(-1+m-2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
[Out] -((d*(d*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, 1/2 - m/2 + p, 3/2 - m/
2 + p, -Tan[e + f*x]^2]*(b*Tan[e + f*x]^2)^p)/(f*(-1 + m - 2*p)))
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int (d\cot(fx+e))^m \left(b\left(\tan^2(fx+e)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
[Out] int((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{2}\left(e+fx\right)\right)^{p}\left(d\cot\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*cot(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{cot}\left(e+f\,x\right)\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

```
[Out] int((d*cot(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

3.422 $\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=107

$$\frac{F_1\left(\frac{1-m}{2};1,-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\cot(e+fx))^m\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b}{a}\right.}{f(1-m)}$$

```
[Out] AppellF1(1/2-1/2*m,1,-p,3/2-1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*cot(f
*x+e))^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(1-m)/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3755, 3751, 525, 524}

$$\frac{\tan(e+fx)(d\cot(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1-m}{2};1,-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 - m)/2, 1, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(d*Cot[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 - m)*
(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
```

```
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 3755

```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(d*Cot[e + f*x])^FracPart[m]*(Tan
[e + f*x]/d)^FracPart[m], Int[(a + b*(c*Tan[e + f*x])^n)^p/(Tan[e + f*x]/d)
^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx = \left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m\right)\int \left(\frac{\tan(e+fx)}{d}\right)^{-m}$$

$$= \frac{\left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m\right)\mathrm{Subst}\left(\int \frac{\left(\frac{x}{d}\right)^{-m}(a+bx^2)^p}{1+x^2}\,dx\right.}{f}$$

$$= \frac{\left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m \left(a+b\tan^2(e+fx)\right)^p \left(1+\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1-m}{2};1,-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\cot(e}{f(1}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 265 vs. 2(107) = 214.

time = 1.73, size = 265, normalized size = 2.48

$$-\frac{a(-3+m)F_1\left(\frac{1-m}{2};-p,1;\frac{3-m}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)\cos^2(e+fx)\cot(e+fx)(d\cot(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p}{f(-1+m)\left(-2bpF_1\left(\frac{3-m}{2};1-p,1;\frac{5-m}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)+2aF_1\left(\frac{3-m}{2};-p,2;\frac{5-m}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)+a(-3+m)F_1\left(\frac{1-m}{2};-p,1;\frac{3-m}{2};-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)\cot^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cot[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((a*(-3 + m)*AppellF1[(1 - m)/2, -p, 1, (3 - m)/2, -((b*Tan[e + f*x]^2)/a)
, -Tan[e + f*x]^2]*Cos[e + f*x]^2*Cot[e + f*x]*(d*Cot[e + f*x])^m*(a + b*Ta
n[e + f*x]^2)^p)/(f*(-1 + m)*(-2*b*p*AppellF1[(3 - m)/2, 1 - p, 1, (5 - m)/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*a*AppellF1[(3 - m)/2, -p,
2, (5 - m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + a*(-3 + m)*Appell
F1[(1 - m)/2, -p, 1, (3 - m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*C
ot[e + f*x]^2)))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int (d\cot (fx+e))^m \left(a+b\left(\tan^2 (fx+e)\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
[Out] int((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cot(e+f\,x))^m\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
```

```
[Out] int((d*cot(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

3.423 $\int (d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=80

$$\frac{(d\cot(e+fx))^m \, _2F_1\left(1,\frac{1}{2}(1-m+np);\frac{1}{2}(3-m+np);-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-m+np)}$$

```
[Out] (d*cot(f*x+e))^m*hypergeom([1, 1/2*n*p-1/2*m+1/2],[1/2*n*p-1/2*m+3/2],-tan(
f*x+e)^2)*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p-m+1)
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3740, 2684, 3557, 371}

$$\frac{\tan(e+fx)(d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, _2F_1\left(1,\frac{1}{2}(-m+np+1);\frac{1}{2}(-m+np+3);-\tan^2(e+fx)\right)}{f(-m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n*p)/2, (3 - m + n*p)/2,
-Tan[e + f*x]^2]*Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
```

```
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\cot(e+fx))^m$$

$$= \left((d\cot(e+fx))^m(c\tan(e+fx))^{m-np}\left(b(c\tan(e+fx))^n\right)^p\right)$$

$$= \frac{\left(c(d\cot(e+fx))^m(c\tan(e+fx))^{m-np}\left(b(c\tan(e+fx))^n\right)^p\right)}{f}$$

$$= \frac{(d\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(1-m+np);\frac{1}{2}(3-m+np);-\tan\right.}{f(1-m+np)}$$

Mathematica [A]
time = 0.10, size = 77, normalized size = 0.96

$$\frac{d(d\cot(e+fx))^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(1-m+np);\frac{1}{2}(3-m+np);-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(1-m+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*(d*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, (1 - m + n*p)/2, (3 - m +
 n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int (d\cot(fx+e))^m \left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan{(e+fx)})^n)^p (d\cot{(e+fx)})^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*cot(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\mathrm{cot}\left(e+f\,x\right)\right)}^m\,{\left(b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((d*cot(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

3.424 $\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p\, dx$

Optimal. Leaf size=57

$$(d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m \text{Int}\left(\left(\frac{\tan(e+fx)}{d}\right)^{-m} (a+b(c\tan(e+fx))^n)^p, x\right)$$

```
[Out] (d*cot(f*x+e))^m*(tan(f*x+e)/d)^m*Unintegrable((a+b*(c*tan(f*x+e))^n)^p/((t
an(f*x+e)/d)^m),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p\, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*Cot[e + f*x])^m*(Tan[e + f*x]/d)^m*Defer[Int][(a + b*(c*Tan[e + f*x])^n)
^p/(Tan[e + f*x]/d)^m, x]
```

Rubi steps

$$\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p\, dx = \left((d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m\right) \int \left(\frac{\tan(e+fx)}{d}\right.$$

Mathematica [A]
time = 5.96, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int (d\cot(fx+e))^m (a+b(c\tan(fx+e))^n)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\cot(e+f\,x))^m\,(a+b\,(c\tan(e+f\,x))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d*cot(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p, x)
```

3.425 $\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{(4a-b)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a-b)\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 1/8*(4*a-b)*arctanh(sin(d*x+c))/d+1/8*(4*a-b)*sec(d*x+c)*tan(d*x+c)/d+1/4*b
*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]

time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3757, 393, 205, 212}

$$\frac{(4a-b)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((4*a - b)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a - b)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{(1-x^2)^3}\,dx,x,\sin(c+dx)\right)}{d}$$
$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{(4a-b)\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx,x,\sin(c$$
$$= \frac{(4a-b)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} +$$
$$= \frac{(4a-b)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(4a-b)\sec(c+dx)\tan(c+dx)}{8d}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 1.33

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} - \frac{b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} - \frac{b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) - (b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Sec[
c + d*x]*Tan[c + d*x])/(2*d) - (b*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec
[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A]
time = 0.24, size = 102, normalized size = 1.46

method	result
derivativedivides	$\frac{b\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{b\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-\frac{i\left(4a\,e^{7i(dx+c)}-b\,e^{7i(dx+c)}+4a\,e^{5i(dx+c)}+7b\,e^{5i(dx+c)}-4a\,e^{3i(dx+c)}-7b\,e^{3i(dx+c)}-4a\,e^{i(dx+c)}+b\,e^{i(dx+c)}\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}+\ln(e^i$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin
(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(
sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 95, normalized size = 1.36

$$\frac{(4\,a-b)\log\left(\sin\left(dx+c\right)+1\right)-(4\,a-b)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((4\,a-b)\sin(dx+c)^3-(4\,a+b)\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*((4*a - b)*log(sin(d*x + c) + 1) - (4*a - b)*log(sin(d*x + c) - 1) - 2
*((4*a - b)*sin(d*x + c)^3 - (4*a + b)*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 2.89, size = 95, normalized size = 1.36

$$\frac{(4\,a-b)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-(4\,a-b)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left((4\,a-b)\cos\left(dx+c\right)^2+2\,b\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*((4*a - b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (4*a - b)*cos(d*x +
c)^4*log(-sin(d*x + c) + 1) + 2*((4*a - b)*cos(d*x + c)^2 + 2*b)*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(c+dx\right)\right)\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**3, x)
```

Giac [A]
time = 0.61, size = 98, normalized size = 1.40

$$\frac{(4\,a-b)\log\left(|\sin\left(dx+c\right)+1|\right)-(4\,a-b)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(4\,a\sin(dx+c)^3-b\sin(dx+c)^3-4\,a\sin(dx+c)-b\sin(dx+c)\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*((4*a - b)*log(abs(sin(d*x + c) + 1)) - (4*a - b)*log(abs(sin(d*x + c)
 - 1)) - 2*(4*a*sin(d*x + c)^3 - b*sin(d*x + c)^3 - 4*a*sin(d*x + c) - b*si
n(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 14.41, size = 147, normalized size = 2.10

$$\frac{\left(a+\frac{b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{7\,b}{4}-a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{7\,b}{4}-a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(a+\frac{b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a-\frac{b}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*(a + b/4) - tan(c/2 + (d*x)/2)^3*(a - (7*b)/4) - tan(
c/2 + (d*x)/2)^5*(a - (7*b)/4) + tan(c/2 + (d*x)/2)*(a + b/4))/(d*(6*tan(c/
2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2
+ (d*x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(a - b/4))/d
```

3.426 $\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=42

$$\frac{(2a-b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*(2*a-b)*arctanh(sin(d*x+c))/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3757, 393, 212}

$$\frac{(2a-b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((2*a - b)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]*Tan[c + d*x])/(2*
d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{(1-x^2)^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{b\sec(c+dx)\tan(c+dx)}{2d} + \frac{(2a-b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+d$$
$$= \frac{(2a-b)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 1.14

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (b*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.14, size = 67, normalized size = 1.60

method	result	size
derivativedivides	$\frac{b\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	67
default	$\frac{b\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	67
risch	$-\frac{ib\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{i(dx+c)}-i\right)a}{d}+\frac{\ln\left(e^{i(dx+c)}-i\right)b}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)a}{d}-\frac{\ln\left(e^{i(dx+c)}+i\right)b}{2d}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.29, size = 62, normalized size = 1.48

$$\frac{(2a-b)\log(\sin(dx+c)+1)-(2a-b)\log(\sin(dx+c)-1)-\frac{2b\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*a - b)*log(sin(d*x + c) + 1) - (2*a - b)*log(sin(d*x + c) - 1) - 2*
b*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 2.52, size = 76, normalized size = 1.81

$$\frac{(2a-b)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2a-b)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2b\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a - b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - b)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*b*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2(c+dx)\right)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x), x)
```

Giac [A]
time = 0.60, size = 64, normalized size = 1.52

$$\frac{(2a-b)\log(|\sin(dx+c)+1|)-(2a-b)\log(|\sin(dx+c)-1|)-\frac{2b\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((2*a - b)*log(abs(sin(d*x + c) + 1)) - (2*a - b)*log(abs(sin(d*x + c)
- 1)) - 2*b*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B]
time = 12.29, size = 79, normalized size = 1.88

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(2\,a-b)}{d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x),x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(2*a - b))/d + (b*tan(c/2 + (d*x)/2) + b*tan(c/2
 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.427 $\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}+\frac{(a-b)\sin(c+dx)}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d+(a-b)*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3757, 396, 212}

$$\frac{(a-b)\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + ((a - b)*Sin[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{(a-b)\sin(c+dx)}{d} + \frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} + \frac{(a-b)\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.68

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d} + \frac{a\cos(dx)\sin(c)}{d} + \frac{a\cos(c)\sin(dx)}{d} - \frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]^2),x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
 - (b*Sin[c + d*x])/d
```

Maple [A]
time = 0.16, size = 39, normalized size = 1.39

method	result	size
derivativedivides	$\frac{b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+\sin(dx+c)a}{d}$	39
default	$\frac{b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+\sin(dx+c)a}{d}$	39
risch	$-\frac{i e^{i(dx+c)}a}{2d} + \frac{i e^{i(dx+c)}b}{2d} + \frac{i e^{-i(dx+c)}a}{2d} - \frac{i e^{-i(dx+c)}b}{2d} + \frac{\ln(e^{i(dx+c)}+i)b}{d} - \frac{\ln(e^{i(dx+c)}-i)b}{d}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+sin(d*x+c)*a)
```

Maxima [A]
time = 0.29, size = 46, normalized size = 1.64

$$\frac{b(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1) - 2\sin(dx+c)) + 2a\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) + 2
*a*sin(d*x + c))/d
```

Fricas [A]
time = 3.12, size = 44, normalized size = 1.57

$$\frac{b\log(\sin(dx+c)+1) - b\log(-\sin(dx+c)+1) + 2(a-b)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1) + 2*(a - b)*sin(d*x
 + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan^2(c+dx))\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x), x)
```

Giac [A]
time = 0.59, size = 48, normalized size = 1.71

$$\frac{b(\log(|\sin(dx+c)+1|) - \log(|\sin(dx+c)-1|) - 2\sin(dx+c)) + 2a\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*(log(abs(sin(d*x + c) + 1)) - log(abs(sin(d*x + c) - 1)) - 2*sin(d*x
 + c)) + 2*a*sin(d*x + c))/d
```

Mupad [B]
time = 11.87, size = 32, normalized size = 1.14

$$\frac{\sin(c+d\,x)\,(a-b)}{d} + \frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x)^2),x)
```

```
[Out] (sin(c + d*x)*(a - b))/d + (2*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.428 $\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$\frac{a\sin(c+dx)}{d}-\frac{(a-b)\sin^3(c+dx)}{3d}$$

```
[Out] a*sin(d*x+c)/d-1/3*(a-b)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = $\{3757\}$

$$\frac{a\sin(c+dx)}{d}-\frac{(a-b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - ((a - b)*Sin[c + d*x]^3)/(3*d)
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx&=\frac{\operatorname{Subst}\left(\int \left(a-(a-b)x^2\right)\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{a\sin(c+dx)}{d}-\frac{(a-b)\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.38

$$\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{b\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
[Out] (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.19, size = 36, normalized size = 1.12

method	result	size
derivativedivides	$\frac{\frac{b\left(\sin^3(dx+c)\right)}{3}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
default	$\frac{\frac{b\left(\sin^3(dx+c)\right)}{3}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
risch	$\frac{3a\sin(dx+c)}{4d}+\frac{\sin(dx+c)b}{4d}+\frac{\sin(3dx+3c)a}{12d}-\frac{\sin(3dx+3c)b}{12d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*b*sin(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 29, normalized size = 0.91

$$-\frac{(a-b)\sin{(dx+c)}^3-3\,a\sin{(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
[Out] -1/3*((a - b)*sin(d*x + c)^3 - 3*a*sin(d*x + c))/d
```

Fricas [A]
time = 2.29, size = 30, normalized size = 0.94

$$\frac{\left((a-b)\cos{(dx+c)}^2+2\,a+b\right)\sin{(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
[Out] 1/3*((a - b)*cos(d*x + c)^2 + 2*a + b)*sin(d*x + c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2{(c+dx)}\right)\cos^3{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**3, x)
```

Giac [A]
time = 0.60, size = 36, normalized size = 1.12

$$-\frac{a\sin\left(dx+c\right)^{3}-b\sin\left(dx+c\right)^{3}-3\,a\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*(a*sin(d*x + c)^3 - b*sin(d*x + c)^3 - 3*a*sin(d*x + c))/d
```

Mupad [B]
time = 12.10, size = 47, normalized size = 1.47

$$\frac{9\,a\,\sin\left(c+d\,x\right)+3\,b\,\sin\left(c+d\,x\right)+a\,\sin\left(3\,c+3\,d\,x\right)-b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x)^2),x)
```

```
[Out] (9*a*sin(c + d*x) + 3*b*sin(c + d*x) + a*sin(3*c + 3*d*x) - b*sin(3*c + 3*d
*x))/(12*d)
```

3.429 $\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{a\sin(c+dx)}{d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)\sin^5(c+dx)}{5d}$$

```
[Out] a*sin(d*x+c)/d-1/3*(2*a-b)*sin(d*x+c)^3/d+1/5*(a-b)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3757, 380}

$$\frac{(a-b)\sin^5(c+dx)}{5d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - ((2*a - b)*Sin[c + d*x]^3)/(3*d) + ((a - b)*Sin[c + d*
x]^5)/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a-(a-b)x^2\right)\,dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a-(2a-b)x^2+(a-b)x^4\right)\,dx,x,\sin(c+dx)\right)}{d}\\
&= \frac{a\sin(c+dx)}{d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)\sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 52, normalized size = 0.96

$$\frac{(89a + 11b + 4(7a - 2b)\cos(2(c + dx)) + 3(a - b)\cos(4(c + dx)))\sin(c + dx)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2),x]
[Out] ((89*a + 11*b + 4*(7*a - 2*b)*Cos[2*(c + d*x)] + 3*(a - b)*Cos[4*(c + d*x)]
)*Sin[c + d*x])/(120*d)
```

Maple [A]
time = 0.23, size = 72, normalized size = 1.33

method	result	size
derivativedivides	$\frac{b\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$	72
default	$\frac{b\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$	72
risch	$\frac{5a\sin(dx+c)}{8d}+\frac{\sin(dx+c)b}{8d}+\frac{\sin(5dx+5c)a}{80d}-\frac{\sin(5dx+5c)b}{80d}+\frac{5\sin(3dx+3c)a}{48d}-\frac{\sin(3dx+3c)b}{48d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))+1/5*
a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.87

$$\frac{3\,(a-b)\sin\left(dx+c\right)^5-5\,(2\,a-b)\sin\left(dx+c\right)^3+15\,a\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
[Out] 1/15*(3*(a - b)*sin(d*x + c)^5 - 5*(2*a - b)*sin(d*x + c)^3 + 15*a*sin(d*x
+ c))/d
```

Fricas [A]
time = 2.50, size = 47, normalized size = 0.87

$$\frac{\left(3\,(a-b)\cos\left(dx+c\right)^4+(4\,a+b)\cos\left(dx+c\right)^2+8\,a+2\,b\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*(a - b)*cos(d*x + c)^4 + (4*a + b)*cos(d*x + c)^2 + 8*a + 2*b)*sin(
d*x + c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^{2}\left(c+dx\right)\right)\cos^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2147 vs. 2(50) = 100.
time = 13.19, size = 2147, normalized size = 39.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/15*(15*a*tan(1/2*d*x)^10*tan(1/2*c)^9 + 15*a*tan(1/2*d*x)^9*tan(1/2*c)^1
0 + 20*a*tan(1/2*d*x)^10*tan(1/2*c)^7 + 20*b*tan(1/2*d*x)^10*tan(1/2*c)^7 -
 75*a*tan(1/2*d*x)^9*tan(1/2*c)^8 + 60*b*tan(1/2*d*x)^9*tan(1/2*c)^8 - 75*a
*tan(1/2*d*x)^8*tan(1/2*c)^9 + 60*b*tan(1/2*d*x)^8*tan(1/2*c)^9 + 20*a*tan(
1/2*d*x)^7*tan(1/2*c)^10 + 20*b*tan(1/2*d*x)^7*tan(1/2*c)^10 + 58*a*tan(1/2
*d*x)^10*tan(1/2*c)^5 - 8*b*tan(1/2*d*x)^10*tan(1/2*c)^5 + 150*a*tan(1/2*d*
x)^9*tan(1/2*c)^6 - 180*b*tan(1/2*d*x)^9*tan(1/2*c)^6 + 700*a*tan(1/2*d*x)^
8*tan(1/2*c)^7 - 500*b*tan(1/2*d*x)^8*tan(1/2*c)^7 + 700*a*tan(1/2*d*x)^7*t
an(1/2*c)^8 - 500*b*tan(1/2*d*x)^7*tan(1/2*c)^8 + 150*a*tan(1/2*d*x)^6*tan(
1/2*c)^9 - 180*b*tan(1/2*d*x)^6*tan(1/2*c)^9 + 58*a*tan(1/2*d*x)^5*tan(1/2*
c)^10 - 8*b*tan(1/2*d*x)^5*tan(1/2*c)^10 + 20*a*tan(1/2*d*x)^10*tan(1/2*c)^
3 + 20*b*tan(1/2*d*x)^10*tan(1/2*c)^3 - 150*a*tan(1/2*d*x)^9*tan(1/2*c)^4 +
 180*b*tan(1/2*d*x)^9*tan(1/2*c)^4 - 610*a*tan(1/2*d*x)^8*tan(1/2*c)^5 + 10
40*b*tan(1/2*d*x)^8*tan(1/2*c)^5 - 2200*a*tan(1/2*d*x)^7*tan(1/2*c)^6 + 236
0*b*tan(1/2*d*x)^7*tan(1/2*c)^6 - 2200*a*tan(1/2*d*x)^6*tan(1/2*c)^7 + 2360
*b*tan(1/2*d*x)^6*tan(1/2*c)^7 - 610*a*tan(1/2*d*x)^5*tan(1/2*c)^8 + 1040*b
*tan(1/2*d*x)^5*tan(1/2*c)^8 - 150*a*tan(1/2*d*x)^4*tan(1/2*c)^9 + 180*b*ta
n(1/2*d*x)^4*tan(1/2*c)^9 + 20*a*tan(1/2*d*x)^3*tan(1/2*c)^10 + 20*b*tan(1/
2*d*x)^3*tan(1/2*c)^10 + 15*a*tan(1/2*d*x)^10*tan(1/2*c) + 75*a*tan(1/2*d*x
)^9*tan(1/2*c)^2 - 60*b*tan(1/2*d*x)^9*tan(1/2*c)^2 + 700*a*tan(1/2*d*x)^8*
```

tan(1/2*c)^3 - 500*b*tan(1/2*d*x)^8*tan(1/2*c)^3 + 2200*a*tan(1/2*d*x)^7*ta
n(1/2*c)^4 - 2360*b*tan(1/2*d*x)^7*tan(1/2*c)^4 + 5380*a*tan(1/2*d*x)^6*tan
(1/2*c)^5 - 5000*b*tan(1/2*d*x)^6*tan(1/2*c)^5 + 5380*a*tan(1/2*d*x)^5*tan(
1/2*c)^6 - 5000*b*tan(1/2*d*x)^5*tan(1/2*c)^6 + 2200*a*tan(1/2*d*x)^4*tan(1
/2*c)^7 - 2360*b*tan(1/2*d*x)^4*tan(1/2*c)^7 + 700*a*tan(1/2*d*x)^3*tan(1/2
*c)^8 - 500*b*tan(1/2*d*x)^3*tan(1/2*c)^8 + 75*a*tan(1/2*d*x)^2*tan(1/2*c)^
9 - 60*b*tan(1/2*d*x)^2*tan(1/2*c)^9 + 15*a*tan(1/2*d*x)*tan(1/2*c)^10 - 15
*a*tan(1/2*d*x)^9 - 75*a*tan(1/2*d*x)^8*tan(1/2*c) + 60*b*tan(1/2*d*x)^8*ta
n(1/2*c) - 700*a*tan(1/2*d*x)^7*tan(1/2*c)^2 + 500*b*tan(1/2*d*x)^7*tan(1/2
*c)^2 - 2200*a*tan(1/2*d*x)^6*tan(1/2*c)^3 + 2360*b*tan(1/2*d*x)^6*tan(1/2*
c)^3 - 5380*a*tan(1/2*d*x)^5*tan(1/2*c)^4 + 5000*b*tan(1/2*d*x)^5*tan(1/2*c
)^4 - 5380*a*tan(1/2*d*x)^4*tan(1/2*c)^5 + 5000*b*tan(1/2*d*x)^4*tan(1/2*c)
^5 - 2200*a*tan(1/2*d*x)^3*tan(1/2*c)^6 + 2360*b*tan(1/2*d*x)^3*tan(1/2*c)^
6 - 700*a*tan(1/2*d*x)^2*tan(1/2*c)^7 + 500*b*tan(1/2*d*x)^2*tan(1/2*c)^7 -
75*a*tan(1/2*d*x)*tan(1/2*c)^8 + 60*b*tan(1/2*d*x)*tan(1/2*c)^8 - 15*a*tan
(1/2*c)^9 - 20*a*tan(1/2*d*x)^7 - 20*b*tan(1/2*d*x)^7 + 150*a*tan(1/2*d*x)^
6*tan(1/2*c) - 180*b*tan(1/2*d*x)^6*tan(1/2*c) + 610*a*tan(1/2*d*x)^5*tan(1
/2*c)^2 - 1040*b*tan(1/2*d*x)^5*tan(1/2*c)^2 + 2200*a*tan(1/2*d*x)^4*tan(1/
2*c)^3 - 2360*b*tan(1/2*d*x)^4*tan(1/2*c)^3 + 2200*a*tan(1/2*d*x)^3*tan(1/2
*c)^4 - 2360*b*tan(1/2*d*x)^3*tan(1/2*c)^4 + 610*a*tan(1/2*d*x)^2*tan(1/2*c
)^5 - 1040*b*tan(1/2*d*x)^2*tan(1/2*c)^5 + 150*a*tan(1/2*d*x)*tan(1/2*c)^6
- 180*b*tan(1/2*d*x)*tan(1/2*c)^6 - 20*a*tan(1/2*c)^7 - 20*b*tan(1/2*c)^7 -
58*a*tan(1/2*d*x)^5 + 8*b*tan(1/2*d*x)^5 - 150*a*tan(1/2*d*x)^4*tan(1/2*c)
+ 180*b*tan(1/2*d*x)^4*tan(1/2*c) - 700*a*tan(1/2*d*x)^3*tan(1/2*c)^2 + 50
0*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 700*a*tan(1/2*d*x)^2*tan(1/2*c)^3 + 500*b
*tan(1/2*d*x)^2*tan(1/2*c)^3 - 150*a*tan(1/2*d*x)*tan(1/2*c)^4 + 180*b*tan(
1/2*d*x)*tan(1/2*c)^4 - 58*a*tan(1/2*c)^5 + 8*b*tan(1/2*c)^5 - 20*a*tan(1/2
*d*x)^3 - 20*b*tan(1/2*d*x)^3 + 75*a*tan(1/2*d*x)^2*tan(1/2*c) - 60*b*tan(1
/2*d*x)^2*tan(1/2*c) + 75*a*tan(1/2*d*x)*tan(1/2*c)^2 - 60*b*tan(1/2*d*x)*t
an(1/2*c)^2 - 20*a*tan(1/2*c)^3 - 20*b*tan(1/2*c)^3 - 15*a*tan(1/2*d*x) - 1
5*a*tan(1/2*c))/(d*tan(1/2*d*x)^10*tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^10*tan(
1/2*c)^8 + 5*d*tan(1/2*d*x)^8*tan(1/2*c)^10 + 10*d*tan(1/2*d*x)^10*tan(1/2*
c)^6 + 25*d*tan(1/2*d*x)^8*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^6*tan(1/2*c)^10
+ 10*d*tan(1/2*d*x)^10*tan(1/2*c)^4 + 50*d*tan(1/2*d*x)^8*tan(1/2*c)^6 + 5
0*d*tan(1/2*d*x)^6*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^4*tan(1/2*c)^10 + 5*d*t
an(1/2*d*x)^10*tan(1/2*c)^2 + 50*d*tan(1/2*d*x)^8*tan(1/2*c)^4 + 100*d*tan(
1/2*d*x)^6*tan(1/2*c)^6 + 50*d*tan(1/2*d*x)^4*tan(1/2*c)^8 + 5*d*tan(1/2*d*
x)^2*tan(1/2*c)^10 + d*tan(1/2*d*x)^10 + 25*d*tan(1/2*d*x)^8*tan(1/2*c)^2 +
100*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 100*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + 25
*d*tan(1/2*d*x)^2*tan(1/2*c)^8 + d*tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^8 + 50*
d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 100*d*tan(1/2*d*x)^4*tan(1/2*c)^4 + 50*d*ta
n(1/2*d*x)^2*tan(1/2*c)^6 + 5*d*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^6 + 50*d*t
an(1/2*d*x)^4*tan(1/2*c)^2 + 50*d*tan(1/2*d*x)^2*tan(1/2*c)^4 + 10*d*tan(1/
2*c)^6 + 10*d*tan(1/2*d*x)^4 + 25*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 10*d*tan(
1/2*c)^4 + 5*d*tan(1/2*d*x)^2 + 5*d*tan(1/2*c)^2 + d)

Mupad [B]
time = 12.07, size = 71, normalized size = 1.31

$$\frac{\frac{5\,a\,\sin(c+d\,x)}{8}+\frac{b\,\sin(c+d\,x)}{8}+\frac{5\,a\,\sin(3\,c+3\,d\,x)}{48}+\frac{a\,\sin(5\,c+5\,d\,x)}{80}-\frac{b\,\sin(3\,c+3\,d\,x)}{48}-\frac{b\,\sin(5\,c+5\,d\,x)}{80}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((5*a*sin(c + d*x))/8 + (b*sin(c + d*x))/8 + (5*a*sin(3*c + 3*d*x))/48 + (a
*sin(5*c + 5*d*x))/80 - (b*sin(3*c + 3*d*x))/48 - (b*sin(5*c + 5*d*x))/80)/
d
```

3.430 $\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=76

$$\frac{a\sin(c+dx)}{d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\frac{(a-b)\sin^7(c+dx)}{7d}$$

```
[Out] a*sin(d*x+c)/d-1/3*(3*a-b)*sin(d*x+c)^3/d+1/5*(3*a-2*b)*sin(d*x+c)^5/d-1/7*
(a-b)*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3757, 380}

$$-\frac{(a-b)\sin^7(c+dx)}{7d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Sin[c + d*x])/d - ((3*a - b)*Sin[c + d*x]^3)/(3*d) + ((3*a - 2*b)*Sin[c
+ d*x]^5)/(5*d) - ((a - b)*Sin[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a-(a-b)x^2\right)\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a-(3a-b)x^2+(3a-2b)x^4-(a-b)x^6\right)\,dx,x,\sin(c\right.}{d}\\ &= \frac{a\sin(c+dx)}{d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\end{aligned}$$

Mathematica [A]

time = 0.20, size = 75, normalized size = 0.99

$$\frac{(2286a+206b+(897a-113b)\cos(2(c+dx))+6(27a-13b)\cos(4(c+dx))+15a\cos(6(c+dx))-15b\cos(6(c+dx)))\sin(c+dx)}{3360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2),x]
[Out] ((2286*a + 206*b + (897*a - 113*b)*Cos[2*(c + d*x)] + 6*(27*a - 13*b)*Cos[4
*(c + d*x)] + 15*a*Cos[6*(c + d*x)] - 15*b*Cos[6*(c + d*x)])*Sin[c + d*x])/
(3360*d)
```

Maple [A]

time = 0.23, size = 92, normalized size = 1.21

method	result
derivativedivides	$\frac{b\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+\right.}{5}\right.}{7}}{d}$
default	$\frac{b\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+\right.}{5}\right.}{7}}{d}$
risch	$\frac{35a\sin(dx+c)}{64d}+\frac{5\sin(dx+c)b}{64d}+\frac{\sin(7dx+7c)a}{448d}-\frac{\sin(7dx+7c)b}{448d}+\frac{7\sin(5dx+5c)a}{320d}-\frac{3\sin(5dx+5c)b}{320d}+\frac{7\sin(3dx}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*
sin(d*x+c))
```

Maxima [A]

time = 0.30, size = 64, normalized size = 0.84

$$-\frac{15\,(a-b)\sin\left(dx+c\right)^{7}-21\,(3\,a-2\,b)\sin\left(dx+c\right)^{5}+35\,(3\,a-b)\sin\left(dx+c\right)^{3}-105\,a\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
[Out] -1/105*(15*(a - b)*sin(d*x + c)^7 - 21*(3*a - 2*b)*sin(d*x + c)^5 + 35*(3*a
 - b)*sin(d*x + c)^3 - 105*a*sin(d*x + c))/d
```

Fricas [A]
time = 2.78, size = 63, normalized size = 0.83

$$\frac{\left(15\,(a-b)\cos\left(dx+c\right)^{6}+3\,(6\,a+b)\cos\left(dx+c\right)^{4}+4\,(6\,a+b)\cos\left(dx+c\right)^{2}+48\,a+8\,b\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*(a - b)*cos(d*x + c)^6 + 3*(6*a + b)*cos(d*x + c)^4 + 4*(6*a + b)
*cos(d*x + c)^2 + 48*a + 8*b)*sin(d*x + c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(c+dx\right)\right)\cos^{7}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29589 vs. 2(70) = 140.
time = 18.66, size = 29589, normalized size = 389.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/6720*(12705*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^13
+ 147*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(5/2*c)*tan(1/2*c)^14 + 12705*a*t
an(5/2*d*x)^2*tan(1/2*d*x)^13*tan(5/2*c)^2*tan(1/2*c)^14 + 147*a*tan(5/2*d*
x)*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^14 + 34230*a*tan(5/2*d*x)^2*tan(
1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^11 + 17920*b*tan(5/2*d*x)^2*tan(1/2*d*x
)^14*tan(5/2*c)^2*tan(1/2*c)^11 + 1029*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan
(5/2*c)*tan(1/2*c)^12 - 62475*a*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(5/2*c)^2
*tan(1/2*c)^12 + 53760*b*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(5/2*c)^2*tan(1/
2*c)^12 + 1029*a*tan(5/2*d*x)*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^12 +
12705*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(1/2*c)^13 - 62475*a*tan(5/2*d*x)
^2*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^13 + 53760*b*tan(5/2*d*x)^2*tan(
1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^13 + 12705*a*tan(1/2*d*x)^14*tan(5/2*c)
^2*tan(1/2*c)^13 + 12705*a*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(1/2*c)^14 - 1
47*a*tan(5/2*d*x)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 1029*a*tan(5/2*d*x)^2*tan
```

(1/2*d*x)^12*tan(5/2*c)*tan(1/2*c)^14 - 147*a*tan(1/2*d*x)^14*tan(5/2*c)*ta
n(1/2*c)^14 + 34230*a*tan(5/2*d*x)^2*tan(1/2*d*x)^11*tan(5/2*c)^2*tan(1/2*c
)^14 + 17920*b*tan(5/2*d*x)^2*tan(1/2*d*x)^11*tan(5/2*c)^2*tan(1/2*c)^14 +
1029*a*tan(5/2*d*x)*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^14 + 12705*a*ta
n(1/2*d*x)^13*tan(5/2*c)^2*tan(1/2*c)^14 + 113967*a*tan(5/2*d*x)^2*tan(1/2*
d*x)^14*tan(5/2*c)^2*tan(1/2*c)^9 - 14336*b*tan(5/2*d*x)^2*tan(1/2*d*x)^14*
tan(5/2*c)^2*tan(1/2*c)^9 + 3087*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(5/2*c
)*tan(1/2*c)^10 + 193305*a*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(5/2*c)^2*tan(
1/2*c)^10 - 268800*b*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(5/2*c)^2*tan(1/2*c)
^10 + 3087*a*tan(5/2*d*x)*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^10 + 3423
0*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(1/2*c)^11 + 17920*b*tan(5/2*d*x)^2*t
an(1/2*d*x)^14*tan(1/2*c)^11 + 1001070*a*tan(5/2*d*x)^2*tan(1/2*d*x)^12*tan
(5/2*c)^2*tan(1/2*c)^11 - 734720*b*tan(5/2*d*x)^2*tan(1/2*d*x)^12*tan(5/2*c
)^2*tan(1/2*c)^11 + 34230*a*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^11 + 17
920*b*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^11 - 62475*a*tan(5/2*d*x)^2*t
an(1/2*d*x)^13*tan(1/2*c)^12 + 53760*b*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(1
/2*c)^12 - 1029*a*tan(5/2*d*x)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 7203*a*tan(5
/2*d*x)^2*tan(1/2*d*x)^12*tan(5/2*c)*tan(1/2*c)^12 - 1029*a*tan(1/2*d*x)^14
*tan(5/2*c)*tan(1/2*c)^12 + 1001070*a*tan(5/2*d*x)^2*tan(1/2*d*x)^11*tan(5/
2*c)^2*tan(1/2*c)^12 - 734720*b*tan(5/2*d*x)^2*tan(1/2*d*x)^11*tan(5/2*c)^2
*tan(1/2*c)^12 + 7203*a*tan(5/2*d*x)*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c
)^12 - 62475*a*tan(1/2*d*x)^13*tan(5/2*c)^2*tan(1/2*c)^12 + 53760*b*tan(1/2
*d*x)^13*tan(5/2*c)^2*tan(1/2*c)^12 - 62475*a*tan(5/2*d*x)^2*tan(1/2*d*x)^1
2*tan(1/2*c)^13 + 53760*b*tan(5/2*d*x)^2*tan(1/2*d*x)^12*tan(1/2*c)^13 + 12
705*a*tan(1/2*d*x)^14*tan(1/2*c)^13 + 193305*a*tan(5/2*d*x)^2*tan(1/2*d*x)^
10*tan(5/2*c)^2*tan(1/2*c)^13 - 268800*b*tan(5/2*d*x)^2*tan(1/2*d*x)^10*tan
(5/2*c)^2*tan(1/2*c)^13 - 62475*a*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^1
3 + 53760*b*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^13 + 34230*a*tan(5/2*d*
x)^2*tan(1/2*d*x)^11*tan(1/2*c)^14 + 17920*b*tan(5/2*d*x)^2*tan(1/2*d*x)^11
*tan(1/2*c)^14 - 1029*a*tan(5/2*d*x)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 12705*
a*tan(1/2*d*x)^13*tan(1/2*c)^14 + 3087*a*tan(5/2*d*x)^2*tan(1/2*d*x)^10*tan
(5/2*c)*tan(1/2*c)^14 - 1029*a*tan(1/2*d*x)^12*tan(5/2*c)*tan(1/2*c)^14 + 1
13967*a*tan(5/2*d*x)^2*tan(1/2*d*x)^9*tan(5/2*c)^2*tan(1/2*c)^14 - 14336*b*
tan(5/2*d*x)^2*tan(1/2*d*x)^9*tan(5/2*c)^2*tan(1/2*c)^14 + 3087*a*tan(5/2*d
*x)*tan(1/2*d*x)^10*tan(5/2*c)^2*tan(1/2*c)^14 + 34230*a*tan(1/2*d*x)^11*ta
n(5/2*c)^2*tan(1/2*c)^14 + 17920*b*tan(1/2*d*x)^11*tan(5/2*c)^2*tan(1/2*c)^
14 + 62004*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^7 + 583
68*b*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^7 + 5145*a*tan(
5/2*d*x)^2*tan(1/2*d*x)^14*tan(5/2*c)*tan(1/2*c)^8 - 591675*a*tan(5/2*d*x)^
2*tan(1/2*d*x)^13*tan(5/2*c)^2*tan(1/2*c)^8 + 537600*b*tan(5/2*d*x)^2*tan(1
/2*d*x)^13*tan(5/2*c)^2*tan(1/2*c)^8 + 5145*a*tan(5/2*d*x)*tan(1/2*d*x)^14*
tan(5/2*c)^2*tan(1/2*c)^8 + 113967*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(1/2
*c)^9 - 14336*b*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(1/2*c)^9 - 1943781*a*tan
(5/2*d*x)^2*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^9 + 2856448*b*tan(5/2*d
*x)^2*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^9 + 113967*a*tan(1/2*d*x)^14*

```
tan(5/2*c)^2*tan(1/2*c)^9 - 14336*b*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)
^9 + 193305*a*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(1/2*c)^10 - 268800*b*tan(5
/2*d*x)^2*tan(1/2*d*x)^13*tan(1/2*c)^10 - 3087*a*tan(5/2*d*x)*tan(1/2*d*x)^
14*tan(1/2*c)^10 + 21609*a*tan(5/2*d*x)^2*tan(1/2*d*x)^12*tan(5/2*c)*tan(1/
2*c)^10 - 3087*a*tan(1/2*d*x)^14*tan(5/2*c)*tan(1/2*c)^10 - 6072570*a*tan(5
/2*d*x)^2*tan(1/2*d*x)^11*tan(5/2*c)^2*tan(1/2*c)^10 + 6289920*b*tan(5/2*d*
x)^2*tan(1/2*d*x)^11*tan(5/2*c)^2*tan(1/2*c)^10 + 21609*a*tan(5/2*d*x)*tan(
1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^10 + 193305...
```

Mupad [B]

time = 12.00, size = 95, normalized size = 1.25

$$\frac{\frac{35\,a\,\sin(c+d\,x)}{64}+\frac{5\,b\,\sin(c+d\,x)}{64}+\frac{7\,a\,\sin(3\,c+3\,d\,x)}{64}+\frac{7\,a\,\sin(5\,c+5\,d\,x)}{320}+\frac{a\,\sin(7\,c+7\,d\,x)}{448}-\frac{b\,\sin(3\,c+3\,d\,x)}{192}-\frac{3\,b\,\sin(5\,c+5\,d\,x)}{320}-\frac{b\,\sin(7\,c+7\,d\,x)}{448}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((35*a*sin(c + d*x))/64 + (5*b*sin(c + d*x))/64 + (7*a*sin(3*c + 3*d*x))/64
 + (7*a*sin(5*c + 5*d*x))/320 + (a*sin(7*c + 7*d*x))/448 - (b*sin(3*c + 3*d
*x))/192 - (3*b*sin(5*c + 5*d*x))/320 - (b*sin(7*c + 7*d*x))/448)/d
```

3.431 $\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=68

$$\frac{a\tan(c+dx)}{d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{(a+2b)\tan^5(c+dx)}{5d}+\frac{b\tan^7(c+dx)}{7d}$$

```
[Out] a*tan(d*x+c)/d+1/3*(2*a+b)*tan(d*x+c)^3/d+1/5*(a+2*b)*tan(d*x+c)^5/d+1/7*b*
tan(d*x+c)^7/d
```

Rubi [A]

time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3756, 380}

$$\frac{(a+2b)\tan^5(c+dx)}{5d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Tan[c + d*x])/d + ((2*a + b)*Tan[c + d*x]^3)/(3*d) + ((a + 2*b)*Tan[c +
d*x]^5)/(5*d) + (b*Tan[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+bx^2\right)\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a+(2a+b)x^2+(a+2b)x^4+bx^6\right)\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{a\tan(c+dx)}{d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{(a+2b)\tan^5(c+dx)}{5d}+\end{aligned}$$

Mathematica [A]
time = 0.18, size = 75, normalized size = 1.10

$$\frac{\tan(c+dx)\left(105a-8b-4b\sec^2(c+dx)-3b\sec^4(c+dx)+15b\sec^6(c+dx)+70a\tan^2(c+dx)+21a\tan^4(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (Tan[c + d*x]*(105*a - 8*b - 4*b*Sec[c + d*x]^2 - 3*b*Sec[c + d*x]^4 + 15*b
*Sec[c + d*x]^6 + 70*a*Tan[c + d*x]^2 + 21*a*Tan[c + d*x]^4))/(105*d)
```

Maple [A]
time = 0.21, size = 94, normalized size = 1.38

method	result
derivativedivides	$\frac{b\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}$
default	$\frac{b\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}$
risch	$\frac{16i\left(70a\,e^{8i(dx+c)}-70b\,e^{8i(dx+c)}+175a\,e^{6i(dx+c)}+35b\,e^{6i(dx+c)}+147a\,e^{4i(dx+c)}-21b\,e^{4i(dx+c)}+49a\,e^{2i(dx+c)}-7b\,e^{2i(d\ldots}\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*
sin(d*x+c)^3/cos(d*x+c)^3)-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan
(d*x+c))
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.82

$$\frac{15\,b\tan\left(dx+c\right)^7+21\,(a+2\,b)\tan\left(dx+c\right)^5+35\,(2\,a+b)\tan\left(dx+c\right)^3+105\,a\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(d*x + c)^7 + 21*(a + 2*b)*tan(d*x + c)^5 + 35*(2*a + b)*tan
(d*x + c)^3 + 105*a*tan(d*x + c))/d
```

Fricas [A]
time = 2.46, size = 74, normalized size = 1.09

$$\frac{\left(8\,(7\,a-b)\cos\left(dx+c\right)^6+4\,(7\,a-b)\cos\left(dx+c\right)^4+3\,(7\,a-b)\cos\left(dx+c\right)^2+15\,b\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(7*a - b)*cos(d*x + c)^6 + 4*(7*a - b)*cos(d*x + c)^4 + 3*(7*a - b
)*cos(d*x + c)^2 + 15*b)*sin(d*x + c)/(d*cos(d*x + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(c + dx\right)\right)\sec^6\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**6, x)
```

Giac [A]
time = 0.63, size = 70, normalized size = 1.03

$$\frac{15\,b\tan\left(dx+c\right)^{7} + 21\,a\tan\left(dx+c\right)^{5} + 42\,b\tan\left(dx+c\right)^{5} + 70\,a\tan\left(dx+c\right)^{3} + 35\,b\tan\left(dx+c\right)^{3} + 105\,a\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*b*tan(d*x + c)^7 + 21*a*tan(d*x + c)^5 + 42*b*tan(d*x + c)^5 + 70
*a*tan(d*x + c)^3 + 35*b*tan(d*x + c)^3 + 105*a*tan(d*x + c))/d
```

Mupad [B]
time = 12.01, size = 56, normalized size = 0.82

$$\frac{\frac{b\tan(c+d\,x)^7}{7} + \left(\frac{a}{5}+\frac{2\,b}{5}\right)\,\tan(c+d\,x)^5 + \left(\frac{2\,a}{3}+\frac{b}{3}\right)\,\tan(c+d\,x)^3 + a\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x)^6,x)
```

```
[Out] (tan(c + d*x)^3*((2*a)/3 + b/3) + tan(c + d*x)^5*(a/5 + (2*b)/5) + a*tan(c
+ d*x) + (b*tan(c + d*x)^7)/7)/d
```

3.432 $\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{a\tan(c+dx)}{d}+\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{b\tan^5(c+dx)}{5d}$$

```
[Out] a*tan(d*x+c)/d+1/3*(a+b)*tan(d*x+c)^3/d+1/5*b*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3756, 380}

$$\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Tan[c + d*x])/d + ((a + b)*Tan[c + d*x]^3)/(3*d) + (b*Tan[c + d*x]^5)/(5
*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+bx^2\right)\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2+bx^4\right)\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{a\tan(c+dx)}{d}+\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{b\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 53, normalized size = 1.15

$$\frac{\tan(c+dx)\left(15a-2b-b\sec^2(c+dx)+3b\sec^4(c+dx)+5a\tan^2(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (Tan[c + d*x]*(15*a - 2*b - b*Sec[c + d*x]^2 + 3*b*Sec[c + d*x]^4 + 5*a*Tan
[c + d*x]^2))/(15*d)
```

Maple [A]
time = 0.19, size = 66, normalized size = 1.43

method	result	size
derivativedivides	$\frac{b\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	66
default	$\frac{b\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	66
risch	$\frac{4i\left(15a\,e^{6i(dx+c)}-15b\,e^{6i(dx+c)}+35a\,e^{4i(dx+c)}+5b\,e^{4i(dx+c)}+25a\,e^{2i(dx+c)}-5b\,e^{2i(dx+c)}+5a-b\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)-a*(-2
/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 39, normalized size = 0.85

$$\frac{3\,b\tan\left(dx+c\right)^5+5\,(a+b)\tan\left(dx+c\right)^3+15\,a\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(d*x + c)^5 + 5*(a + b)*tan(d*x + c)^3 + 15*a*tan(d*x + c))/d
```

Fricas [A]
time = 2.26, size = 56, normalized size = 1.22

$$\frac{\left(2\,(5\,a-b)\cos\left(dx+c\right)^4+(5\,a-b)\cos\left(dx+c\right)^2+3\,b\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*a - b)*cos(d*x + c)^4 + (5*a - b)*cos(d*x + c)^2 + 3*b)*sin(d*x
+ c)/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2\left(c+dx\right)\right)\sec^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**4, x)
```

Giac [A]
time = 0.59, size = 48, normalized size = 1.04

$$\frac{3\,b\tan\left(dx+c\right)^5+5\,a\tan\left(dx+c\right)^3+5\,b\tan\left(dx+c\right)^3+15\,a\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(d*x + c)^5 + 5*a*tan(d*x + c)^3 + 5*b*tan(d*x + c)^3 + 15*a*t
an(d*x + c))/d
```

Mupad [B]
time = 11.94, size = 40, normalized size = 0.87

$$\frac{\frac{b\tan(c+d\,x)^5}{5}+\left(\frac{a}{3}+\frac{b}{3}\right)\,\tan(c+d\,x)^3+a\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x)^4,x)
```

```
[Out] (tan(c + d*x)^3*(a/3 + b/3) + a*tan(c + d*x) + (b*tan(c + d*x)^5)/5)/d
```

3.433 $\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

```
[Out] a*tan(d*x+c)/d+1/3*b*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3756}

$$\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (a*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.00

$$\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
[Out] (a*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.21, size = 33, normalized size = 1.18

method	result	size
derivativedivides	$\frac{\frac{b\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+a\tan(dx+c)}{d}$	33
default	$\frac{\frac{b\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+a\tan(dx+c)}{d}$	33
risch	$-\frac{2i\left(-3a\,\mathrm{e}^{4i(dx+c)}+3b\,\mathrm{e}^{4i(dx+c)}-6a\,\mathrm{e}^{2i(dx+c)}-3a+b\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*b*sin(d*x+c)^3/cos(d*x+c)^3+a*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.89

$$\frac{b\tan\left(dx+c\right)^3+3\,a\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
[Out] 1/3*(b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/d
```

Fricas [A]
time = 2.34, size = 37, normalized size = 1.32

$$\frac{\left((3\,a-b)\cos\left(dx+c\right)^2+b\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
[Out] 1/3*((3*a - b)*cos(d*x + c)^2 + b)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [A]
time = 0.96, size = 36, normalized size = 1.29

$$\begin{cases}\frac{a\tan\left(c+dx\right)+\frac{b\tan^3\left(c+dx\right)}{3}}{d} & \text{for}\ d\neq 0\\ x\left(a+b\tan^2\left(c\right)\right)\sec^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Piecewise(((a*tan(c + d*x) + b*tan(c + d*x)**3/3)/d, Ne(d, 0)), (x*(a + b*t
an(c)**2)*sec(c)**2, True))
```

Giac [A]
time = 0.58, size = 25, normalized size = 0.89

$$\frac{b\tan\left(dx+c\right)^{3}+3\,a\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/d
```

Mupad [B]
time = 11.86, size = 25, normalized size = 0.89

$$\frac{\tan(c+d\,x)\,\left(b\tan(c+d\,x)^2+3\,a\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c + d*x)*(3*a + b*tan(c + d*x)^2))/(3*d)
```

3.434 $\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}(a+b)x+\frac{(a-b)\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*(a+b)*x+1/2*(a-b)*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3756, 393, 209}

$$\frac{(a-b)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+b)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((a + b)*x)/2 + ((a - b)*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)\cos(c+dx)\sin(c+dx)}{2d} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c\right.}{2d}$$
$$= \frac{1}{2}(a+b)x + \frac{(a-b)\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.97

$$\frac{2(a+b)(c+dx)+(a-b)\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
[Out] (2*(a + b)*(c + d*x) + (a - b)*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.16, size = 54, normalized size = 1.64

method	result	size
risch	$\frac{ax}{2}+\frac{bx}{2}+\frac{\sin(2dx+2c)a}{4d}-\frac{\sin(2dx+2c)b}{4d}$	40
derivativedivides	$\frac{b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54
default	$\frac{b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 39, normalized size = 1.18

$$\frac{(dx+c)(a+b)+\frac{(a-b)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*(a + b) + (a - b)*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Fricas [**A**]
time = 2.85, size = 30, normalized size = 0.91

$$\frac{(a+b)dx + (a-b)\cos(dx+c)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*d*x + (a - b)*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2(c+dx)\right)\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(29) = 58.
time = 0.63, size = 169, normalized size = 5.12

$$\frac{adx\tan(dx)^2\tan(c)^2 + bdx\tan(dx)^2\tan(c)^2 + adx\tan(dx)^2 + bdx\tan(dx)^2 + adx\tan(c)^2 + bdx\tan(c)^2 - a\tan(dx)^2\tan(c) + b\tan(dx)^2\tan(c) - a\tan(dx)\tan(c)^2 + b\tan(dx)\tan(c)^2 + adx + bdx + a\tan(dx) - b\tan(dx) + a\tan(c) - b\tan(c)}{2\left(d\tan(dx)^2\tan(c)^2 + d\tan(dx)^2 + d\tan(c)^2 + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*d*x*tan(d*x)^2*tan(c)^2 + b*d*x*tan(d*x)^2*tan(c)^2 + a*d*x*tan(d*x)
^2 + b*d*x*tan(d*x)^2 + a*d*x*tan(c)^2 + b*d*x*tan(c)^2 - a*tan(d*x)^2*tan(
c) + b*tan(d*x)^2*tan(c) - a*tan(d*x)*tan(c)^2 + b*tan(d*x)*tan(c)^2 + a*d*
x + b*d*x + a*tan(d*x) - b*tan(d*x) + a*tan(c) - b*tan(c))/(d*tan(d*x)^2*ta
n(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

Mupad [**B**]
time = 11.94, size = 32, normalized size = 0.97

$$\frac{\sin(2\,c + 2\,d\,x)\,\left(\frac{a}{4} - \frac{b}{4}\right) + d\,x\,\left(\frac{a}{2} + \frac{b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x)^2),x)
```

```
[Out] (sin(2*c + 2*d*x)*(a/4 - b/4) + d*x*(a/2 + b/2))/d
```

3.435 $\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{1}{8}(3a+b)x+\frac{(3a+b)\cos(c+dx)\sin(c+dx)}{8d}+\frac{(a-b)\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*(3*a+b)*x+1/8*(3*a+b)*cos(d*x+c)*sin(d*x+c)/d+1/4*(a-b)*cos(d*x+c)^3*si
n(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3756, 393, 205, 209}

$$\frac{(a-b)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{(3a+b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a+b)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((3*a + b)*x)/8 + ((3*a + b)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((a - b)*Co
s[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{(3a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x\ldots\right.}{4d}$$
$$= \frac{(3a+b)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(c+dx)\sin(c+d\ldots}{4d}$$
$$= \frac{1}{8}(3a+b)x + \frac{(3a+b)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(c+\ldots}{4\ldots}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 0.75

$$\frac{4bdx+12a(c+dx)+8a\sin(2(c+dx))+(a-b)\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (4*b*d*x + 12*a*(c + d*x) + 8*a*Sin[2*(c + d*x)] + (a - b)*Sin[4*(c + d*x)]
)/(32*d)
```

Maple [A]
time = 0.19, size = 81, normalized size = 1.33

method	result	size
risch	$\frac{3ax}{8}+\frac{bx}{8}+\frac{\sin(4dx+4c)a}{32d}-\frac{\sin(4dx+4c)b}{32d}+\frac{\sin(2dx+2c)a}{4d}$	55
derivativedivides	$\frac{b\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	81

default	$\frac{b\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*
c)+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.51, size = 69, normalized size = 1.13

$$\frac{(dx+c)(3a+b)+\frac{(3a+b)\tan(dx+c)^3+(5a-b)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((d*x + c)*(3*a + b) + ((3*a + b)*tan(d*x + c)^3 + (5*a - b)*tan(d*x +
c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 4.05, size = 49, normalized size = 0.80

$$\frac{(3a+b)dx+\left(2(a-b)\cos(dx+c)^3+(3a+b)\cos(dx+c)\right)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((3*a + b)*d*x + (2*(a - b)*cos(d*x + c)^3 + (3*a + b)*cos(d*x + c))*si
n(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2(c+dx)\right)\cos^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. $2(55) = 110$.
time = 1.88, size = 2010, normalized size = 32.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*ta
n(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a*d*x*tan
(d*x)^4*tan(c)^4 + 8*b*d*x*tan(d*x)^4*tan(c)^4 + 3*pi*b*sgn(-2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 +
6*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)
*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 6*pi*b*sgn(2*tan(d
*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(
d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 6*b*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 6*b*arctan(-(tan(d*x) - tan(c))/(ta
n(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 48*a*d*x*tan(d*x)^4*tan(c)^2 + 16
*b*d*x*tan(d*x)^4*tan(c)^2 + 6*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 48*a*d*x*tan(d*x)^2*
tan(c)^4 + 16*b*d*x*tan(d*x)^2*tan(c)^4 + 6*pi*b*sgn(-2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 3*pi*b*
sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 12*pi*b*sgn(2*tan(d*x)^2*tan(c)^2
- 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)
)*tan(d*x)^2*tan(c)^2 + 12*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
1))*tan(d*x)^4*tan(c)^2 - 12*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c)
 + 1))*tan(d*x)^4*tan(c)^2 - 40*a*tan(d*x)^4*tan(c)^3 + 8*b*tan(d*x)^4*tan(
c)^3 + 3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 12*b*arctan((tan(d*x)
+ tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 12*b*arctan(-(tan(d*
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 40*a*tan(d*x)^3*t
an(c)^4 + 8*b*tan(d*x)^3*tan(c)^4 + 24*a*d*x*tan(d*x)^4 + 8*b*d*x*tan(d*x)^
4 + 3*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*
tan(c))*tan(d*x)^4 + 96*a*d*x*tan(d*x)^2*tan(c)^2 + 32*b*d*x*tan(d*x)^2*tan
(c)^2 + 12*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
 - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 24*a*d*x*tan(c)^4 + 8*b*d*x*tan(c)^4 + 3
*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c
))*tan(c)^4 + 6*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 6*b*arctan((t
an(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4 - 6*b*arctan(-(tan(d*x)
 - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4 - 24*a*tan(d*x)^4*tan(c) - 8*b
*tan(d*x)^4*tan(c) + 6*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^
2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 24*b*arc
```

```
tan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 24*b*a
rctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 48*
a*tan(d*x)^3*tan(c)^2 - 48*b*tan(d*x)^3*tan(c)^2 + 48*a*tan(d*x)^2*tan(c)^3
 - 48*b*tan(d*x)^2*tan(c)^3 + 6*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(
c) - 1))*tan(c)^4 - 6*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*
tan(c)^4 - 24*a*tan(d*x)*tan(c)^4 - 8*b*tan(d*x)*tan(c)^4 + 48*a*d*x*tan(d*
x)^2 + 16*b*d*x*tan(d*x)^2 + 6*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 48*a*d*x*tan(c)^2 + 16*b*d*x*
tan(c)^2 + 6*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*
x) - 2*tan(c))*tan(c)^2 + 3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 12*b*arctan(
(tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 12*b*arctan(-(tan(d
*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 24*a*tan(d*x)^3 + 8*b*tan
(d*x)^3 - 48*a*tan(d*x)^2*tan(c) + 48*b*tan(d*x)^2*tan(c) + 12*b*arctan((ta
n(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^2 - 12*b*arctan(-(tan(d*x) -
 tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2 - 48*a*tan(d*x)*tan(c)^2 + 48*b*ta
n(d*x)*tan(c)^2 + 24*a*tan(c)^3 + 8*b*tan(c)^3 + 24*a*d*x + 8*b*d*x + 3*pi*
b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) +
 6*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 6*b*arctan(-(tan(d
*x) - tan(c))/(tan(d*x)*tan(c) + 1)) + 40*a*tan(d*x) - 8*b*tan(d*x) + 40*a*
tan(c) - 8*b*tan(c))/(d*tan(d*x)^4*tan(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d
*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 + 4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4
+ 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

Mupad [B]
time = 12.06, size = 67, normalized size = 1.10

$$x\left(\frac{3\,a}{8}+\frac{b}{8}\right)+\frac{\left(\frac{3\,a}{8}+\frac{b}{8}\right)\,\tan(c+d\,x)^3+\left(\frac{5\,a}{8}-\frac{b}{8}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x)^2),x)
[Out] x*((3*a)/8 + b/8) + (tan(c + d*x)^3*((3*a)/8 + b/8) + tan(c + d*x)*((5*a)/8
 - b/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.436 $\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal. Leaf size=87

$$\frac{1}{16}(5a+b)x+\frac{(5a+b)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(5a+b)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{(a-b)\cos^5(c+dx)\sin(c$$

```
[Out] 1/16*(5*a+b)*x+1/16*(5*a+b)*cos(d*x+c)*sin(d*x+c)/d+1/24*(5*a+b)*cos(d*x+c)
^3*sin(d*x+c)/d+1/6*(a-b)*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3756, 393, 205, 209}

$$\frac{(a-b)\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{(5a+b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(5a+b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(5a+b)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((5*a + b)*x)/16 + ((5*a + b)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((5*a + b
)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + ((a - b)*Cos[c + d*x]^5*Sin[c + d*x
])/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{(a-b)\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{(5a+b)\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, \text{t}\right.}{6d}$$

$$= \frac{(5a+b)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{(a-b)\cos^5(c+dx)\sin(c+dx}{6d}$$

$$= \frac{(5a+b)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(5a+b)\cos^3(c+dx)\sin(c+dx}{24d}$$

$$= \frac{1}{16}(5a+b)x + \frac{(5a+b)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(5a+b)\cos^3(c}{2}$$

Mathematica [A]
time = 0.15, size = 74, normalized size = 0.85

$$\frac{60ac+60adx+12bdx+3(15a+b)\sin(2(c+dx))+(9a-3b)\sin(4(c+dx))+a\sin(6(c+dx))-b\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
[Out] (60*a*c + 60*a*d*x + 12*b*d*x + 3*(15*a + b)*Sin[2*(c + d*x)] + (9*a - 3*b)
*Sin[4*(c + d*x)] + a*Sin[6*(c + d*x)] - b*Sin[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 0.24, size = 102, normalized size = 1.17

method	result
risch	$\frac{5ax}{16}+\frac{bx}{16}+\frac{\sin(6dx+6c)a}{192d}-\frac{\sin(6dx+6c)b}{192d}+\frac{3\sin(4dx+4c)a}{64d}-\frac{\sin(4dx+4c)b}{64d}+\frac{15\sin(2dx+2c)a}{64d}+\frac{\sin(2dx+}{64d}$

derivativedivides	$\frac{b\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right.}{6}\right.}{d}$
default	$\frac{b\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right.}{6}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))

Maxima [A]
time = 0.52, size = 97, normalized size = 1.11

$$\frac{3\,(dx+c)(5\,a+b)+\frac{3\,(5\,a+b)\tan(dx+c)^5+8\,(5\,a+b)\tan(dx+c)^3+3\,(11\,a-b)\tan(dx+c)}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="maxima")

[Out] 1/48*(3*(d*x + c)*(5*a + b) + (3*(5*a + b)*tan(d*x + c)^5 + 8*(5*a + b)*tan(d*x + c)^3 + 3*(11*a - b)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d

Fricas [A]
time = 3.69, size = 66, normalized size = 0.76

$$\frac{3\,(5\,a+b)dx+\left(8\,(a-b)\cos\left(dx+c\right)^5+2\,(5\,a+b)\cos\left(dx+c\right)^3+3\,(5\,a+b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="fricas")

[Out] 1/48*(3*(5*a + b)*d*x + (8*(a - b)*cos(d*x + c)^5 + 2*(5*a + b)*cos(d*x + c)^3 + 3*(5*a + b)*cos(d*x + c))*sin(d*x + c))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(c+dx\right)\right)\cos^6\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*(a+b*tan(d*x+c)**2),x)

[Out] Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**6, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3757 vs. 2(79) = 158.
time = 2.26, size = 3757, normalized size = 43.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="giac")

```
[Out] 1/96*(3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*ta
n(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^6 + 30*a*d*x*tan
(d*x)^6*tan(c)^6 + 6*b*d*x*tan(d*x)^6*tan(c)^6 + 3*pi*b*sgn(-2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^6 +
9*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)
*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^4 + 9*pi*b*sgn(2*tan(d
*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(
d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^6 + 6*b*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^6 - 6*b*arctan(-(tan(d*x) - tan(c))/(ta
n(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^6 + 90*a*d*x*tan(d*x)^6*tan(c)^4 + 18
*b*d*x*tan(d*x)^6*tan(c)^4 + 9*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^4 + 90*a*d*x*tan(d*x)^4*
tan(c)^6 + 18*b*d*x*tan(d*x)^4*tan(c)^6 + 9*pi*b*sgn(-2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^6 + 9*pi*b*
sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^2 + 27*pi*b*sgn(2*tan(d*x)^2*
tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) -
 2*tan(c))*tan(d*x)^4*tan(c)^4 + 18*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*
tan(c) - 1))*tan(d*x)^6*tan(c)^4 - 18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*
x)*tan(c) + 1))*tan(d*x)^6*tan(c)^4 - 66*a*tan(d*x)^6*tan(c)^5 + 6*b*tan(d*
x)^6*tan(c)^5 + 9*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan
(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^6 + 18
*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^6 -
18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^6
 - 66*a*tan(d*x)^5*tan(c)^6 + 6*b*tan(d*x)^5*tan(c)^6 + 90*a*d*x*tan(d*x)^6
*tan(c)^2 + 18*b*d*x*tan(d*x)^6*tan(c)^2 + 9*pi*b*sgn(-2*tan(d*x)^2*tan(c)
+ 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^2 + 270*a*
d*x*tan(d*x)^4*tan(c)^4 + 54*b*d*x*tan(d*x)^4*tan(c)^4 + 27*pi*b*sgn(-2*tan
(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*ta
n(c)^4 + 90*a*d*x*tan(d*x)^2*tan(c)^6 + 18*b*d*x*tan(d*x)^2*tan(c)^6 + 9*pi
*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*
```

```
tan(d*x)^2*tan(c)^6 + 3*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)
^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6 + 27*pi
*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 18*b*arctan((tan(d*x)
+ tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^2 - 18*b*arctan(-(tan(d*
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan(c)^2 - 80*a*tan(d*x)^6*t
an(c)^3 - 16*b*tan(d*x)^6*tan(c)^3 + 27*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)
*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*ta
n(d*x)^2*tan(c)^4 + 54*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*
tan(d*x)^4*tan(c)^4 - 54*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1
))*tan(d*x)^4*tan(c)^4 + 90*a*tan(d*x)^5*tan(c)^4 - 78*b*tan(d*x)^5*tan(c)^
4 + 90*a*tan(d*x)^4*tan(c)^5 - 78*b*tan(d*x)^4*tan(c)^5 + 3*pi*b*sgn(2*tan(
d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan
(d*x) - 2*tan(c))*tan(c)^6 + 18*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(
c) - 1))*tan(d*x)^2*tan(c)^6 - 18*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*t
an(c) + 1))*tan(d*x)^2*tan(c)^6 - 80*a*tan(d*x)^3*tan(c)^6 - 16*b*tan(d*x)^
3*tan(c)^6 + 30*a*d*x*tan(d*x)^6 + 6*b*d*x*tan(d*x)^6 + 3*pi*b*sgn(-2*tan(d
*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6 + 27
0*a*d*x*tan(d*x)^4*tan(c)^2 + 54*b*d*x*tan(d*x)^4*tan(c)^2 + 27*pi*b*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^
4*tan(c)^2 + 270*a*d*x*tan(d*x)^2*tan(c)^4 + 54*b*d*x*tan(d*x)^2*tan(c)^4 +
 27*pi*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*ta
n(c))*tan(d*x)^2*tan(c)^4 + 30*a*d*x*tan(c)^6 + 6*b*d*x*tan(c)^6 + 3*pi*b*s
gn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(
c)^6 + 9*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 6*b*arctan((tan(d*x)
 + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6 - 6*b*arctan(-(tan(d*x) - tan(
c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6 - 30*a*tan(d*x)^6*tan(c) - 6*b*tan(d*
x)^6*tan(c) + 27*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(
c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 54*
b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^2 - 5
4*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2
+ 90*a*tan(d*x)^5*tan(c)^2 + 18*b*tan(d*x)^5*tan(c)^2 - 240*a*tan(d*x)^4*ta
n(c)^3 + 144*b*tan(d*x)^4*tan(c)^3 + 9*pi*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*
sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan
(c)^4 + 54*b*arctan((tan(d*x) + tan(c))/(tan(d*...
```

Mupad [B]

time = 12.54, size = 93, normalized size = 1.07

$$x\left(\frac{5\,a}{16}+\frac{b}{16}\right)+\frac{\left(\frac{5\,a}{16}+\frac{b}{16}\right)\,\tan(c+d\,x)^5+\left(\frac{5\,a}{6}+\frac{b}{6}\right)\,\tan(c+d\,x)^3+\left(\frac{11\,a}{16}-\frac{b}{16}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^6+3\tan\left(c+d\,x\right)^4+3\tan\left(c+d\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b*tan(c + d*x)^2),x)
```

```
[Out] x*((5*a)/16 + b/16) + (tan(c + d*x)^3*((5*a)/6 + b/6) + tan(c + d*x)^5*((5*
a)/16 + b/16) + tan(c + d*x)*((11*a)/16 - b/16))/(d*(3*tan(c + d*x)^2 + 3*t
an(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

3.437 $\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=128

$$\frac{(8a^2-4ab+b^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(8a^2-4ab+b^2)\sec(c+dx)\tan(c+dx)}{16d}+\frac{(8a-3b)b\sec^3(c+dx)\tan(c+dx)}{24d}$$

```
[Out] 1/16*(8*a^2-4*a*b+b^2)*arctanh(sin(d*x+c))/d+1/16*(8*a^2-4*a*b+b^2)*sec(d*x
+c)*tan(d*x+c)/d+1/24*(8*a-3*b)*b*sec(d*x+c)^3*tan(d*x+c)/d+1/6*b*sec(d*x+c
)^5*(a-(a-b)*sin(d*x+c)^2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {3757, 424, 393, 205, 212}

$$\frac{(8a^2-4ab+b^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(8a^2-4ab+b^2)\tan(c+dx)\sec(c+dx)}{16d}+\frac{b(8a-3b)\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{b\tan(c+dx)\sec^5(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 4*a*b + b^2)*ArcTanh[Sin[c + d*x]])/(16*d) + ((8*a^2 - 4*a*b + b^
2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((8*a - 3*b)*b*Sec[c + d*x]^3*Tan[c
+ d*x])/(24*d) + (b*Sec[c + d*x]^5*(a - (a - b)*Sin[c + d*x]^2)*Tan[c + d*x
])/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^4} dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{b\sec^5(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)\tan(c+dx)}{6d} - \frac{\text{Subst}\left(\int \right.}{}$$
$$= \frac{(8a-3b)b\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{b\sec^5(c+dx)\left(a-(a-b)\right.}{6d}$$
$$= \frac{\left(8a^2-4ab+b^2\right)\sec(c+dx)\tan(c+dx)}{16d} + \frac{(8a-3b)b\sec^3(c+dx}{24d}$$
$$= \frac{\left(8a^2-4ab+b^2\right)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{\left(8a^2-4ab+b^2\right)\sec(c+}{16d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 7.42, size = 875, normalized size = 6.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Sin[c + d*x]*(65625*a^2*ArcTanh[Sqrt[Sin[c + d*x]^2]] - 36855*a^2*ArcTanh[
Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^2 - 91875*a*(a - b)*ArcTanh[Sqrt[Sin[c +
 d*x]^2]]*Sin[c + d*x]^2 + 1680*a^2*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d
*x]^4 + 54180*a*(a - b)*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^4 + 3297
0*(a - b)^2*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^4 - 1365*a*(a - b)*A
rcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^6 - 19845*(a - b)^2*ArcTanh[Sqrt[
Sin[c + d*x]^2]]*Sin[c + d*x]^6 + 525*(a - b)^2*ArcTanh[Sqrt[Sin[c + d*x]^2
]]*Sin[c + d*x]^8 - 65625*a^2*Sqrt[Sin[c + d*x]^2] - 23555*a*(a - b)*Sin[c
+ d*x]^4*Sqrt[Sin[c + d*x]^2] - 32970*(a - b)^2*Sin[c + d*x]^4*Sqrt[Sin[c +
 d*x]^2] + 8855*(a - b)^2*Sin[c + d*x]^6*Sqrt[Sin[c + d*x]^2] + 620*a^2*Hyp
ergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*
Sqrt[Sin[c + d*x]^2] + 160*a^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*Sqrt[Sin[c + d*x]^2] + 16*a^2*Hyper
geometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c
 + d*x]^6*Sqrt[Sin[c + d*x]^2] - 968*a*(a - b)*HypergeometricPFQ[{3/2, 2, 2
, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^8*Sqrt[Sin[c + d*x]^2] - 28
8*a*(a - b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, Sin[c + d*
x]^2]*Sin[c + d*x]^8*Sqrt[Sin[c + d*x]^2] - 32*a*(a - b)*HypergeometricPFQ[
{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^8*Sqr
t[Sin[c + d*x]^2] + 380*(a - b)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1,
9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^10*Sqrt[Sin[c + d*x]^2] + 128*(a - b)^2*
HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c
+ d*x]^10*Sqrt[Sin[c + d*x]^2] + 16*(a - b)^2*HypergeometricPFQ[{3/2, 2, 2,
 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^10*Sqrt[Sin[c +
d*x]^2] + 14980*a^2*(Sin[c + d*x]^2)^(3/2) + 91875*a*(a - b)*(Sin[c + d*x]^
2)^(3/2)))/(2520*d*(Sin[c + d*x]^2)^(5/2))
```

Maple [A]
time = 0.32, size = 199, normalized size = 1.55

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^5(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^5(dx+c)}{24\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{48\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+2ab\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}\right.}{d}$
default	$\frac{b^2\left(\frac{\sin^5(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^5(dx+c)}{24\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{48\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+2ab\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}\right.}{d}$
risch	$-\frac{i\left(24a^2e^{11i(dx+c)}-12ab\,e^{11i(dx+c)}+3b^2e^{11i(dx+c)}+72a^2e^{9i(dx+c)}+60ab\,e^{9i(dx+c)}-47b^2e^{9i(dx+c)}+48a^2e^{7i(dx+c)}+7\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/6*sin(d*x+c)^5/cos(d*x+c)^6+1/24*sin(d*x+c)^5/cos(d*x+c)^4-1/48
*sin(d*x+c)^5/cos(d*x+c)^2-1/48*sin(d*x+c)^3-1/16*sin(d*x+c)+1/16*ln(sec(d*
x+c)+tan(d*x+c)))+2*a*b*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos
```

```
(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+a^2*(1/2*sec(d*x+c)
*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.30, size = 156, normalized size = 1.22

$$\frac{3\left(8a^2-4ab+b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(8a^2-4ab+b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\left(8a^2-4ab+b^2\right)\sin(dx+c)^5-8\left(6a^2-b^2\right)\sin(dx+c)^3+3\left(8a^2+4ab-b^2\right)\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(8*a^2 - 4*a*b + b^2)*log(sin(d*x + c) + 1) - 3*(8*a^2 - 4*a*b + b^
2)*log(sin(d*x + c) - 1) - 2*(3*(8*a^2 - 4*a*b + b^2)*sin(d*x + c)^5 - 8*(6
*a^2 - b^2)*sin(d*x + c)^3 + 3*(8*a^2 + 4*a*b - b^2)*sin(d*x + c))/(sin(d*x
 + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 3.29, size = 137, normalized size = 1.07

$$\frac{3\left(8a^2-4ab+b^2\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-3\left(8a^2-4ab+b^2\right)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\left(8a^2-4ab+b^2\right)\cos\left(dx+c\right)^4+2\left(12ab-7b^2\right)\cos\left(dx+c\right)^2+8b^2\right)\sin\left(dx+c\right)}{96d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*a^2 - 4*a*b + b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(8*a
^2 - 4*a*b + b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(3*(8*a^2 - 4*a
*b + b^2)*cos(d*x + c)^4 + 2*(12*a*b - 7*b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d
*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(c+dx\right)\right)^2\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**3, x)
```

Giac [A]
time = 0.78, size = 167, normalized size = 1.30

$$\frac{3\left(8a^2-4ab+b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\left(8a^2-4ab+b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(24a^2\sin(dx+c)^5-12ab\sin(dx+c)^5+3b^2\sin(dx+c)^5-48a^2\sin(dx+c)^3+8b^2\sin(dx+c)^3+24a^2\sin(dx+c)+12ab\sin(dx+c)-3b^2\sin(dx+c)\right)}{\left(\sin(dx+c)^2-1\right)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(8*a^2 - 4*a*b + b^2)*log(abs(sin(d*x + c) + 1)) - 3*(8*a^2 - 4*a*b
 + b^2)*log(abs(sin(d*x + c) - 1)) - 2*(24*a^2*sin(d*x + c)^5 - 12*a*b*sin(
d*x + c)^5 + 3*b^2*sin(d*x + c)^5 - 48*a^2*sin(d*x + c)^3 + 8*b^2*sin(d*x +
 c)^3 + 24*a^2*sin(d*x + c) + 12*a*b*sin(d*x + c) - 3*b^2*sin(d*x + c))/(si
n(d*x + c)^2 - 1)^3)/d
```

Mupad [B]
time = 15.64, size = 269, normalized size = 2.10

$$\frac{\left(a^2+\frac{a\,b}{2}-\frac{b^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(-3\,a^2+\frac{5\,a\,b}{2}+\frac{17\,b^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(2\,a^2-3\,a\,b+\frac{19\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(2\,a^2-3\,a\,b+\frac{19\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-3\,a^2+\frac{5\,a\,b}{2}+\frac{17\,b^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(a^2+\frac{a\,b}{2}-\frac{b^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\frac{a\,b}{2}+\frac{b^2}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(2*a^2 - 3*a*b + (19*b^2)/4) + tan(c/2 + (d*x)/2)^7*(
2*a^2 - 3*a*b + (19*b^2)/4) + tan(c/2 + (d*x)/2)^3*((5*a*b)/2 - 3*a^2 + (17
*b^2)/24) + tan(c/2 + (d*x)/2)^9*((5*a*b)/2 - 3*a^2 + (17*b^2)/24) + tan(c/
2 + (d*x)/2)*((a*b)/2 + a^2 - b^2/8) + tan(c/2 + (d*x)/2)^11*((a*b)/2 + a^2
 - b^2/8))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/
2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/
2 + (d*x)/2)^12 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(a^2 - (a*b)/2 + b^2/8))
/d
```

3.438 $\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{(8a^2-8ab+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3(2a-b)b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\left(a-(a-b)\sin^2(\right.}{4d}$$

```
[Out] 1/8*(8*a^2-8*a*b+3*b^2)*arctanh(sin(d*x+c))/d+3/8*(2*a-b)*b*sec(d*x+c)*tan(
d*x+c)/d+1/4*b*sec(d*x+c)^3*(a-(a-b)*sin(d*x+c)^2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3757, 424, 393, 212}

$$\frac{(8a^2-8ab+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3b(2a-b)\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (3*(2*a - b)*b*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a - (a - b)*Sin[c + d*x]^
2)*Tan[c + d*x])/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^3(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)\tan(c+dx)}{4d} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{3(2a-b)b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\left(a-(a-b)\,\text{s}\right.}{4d}$$

$$= \frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3(2a-b)b\sec(c+dx)\,\text{t}}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.48, size = 347, normalized size = 3.61

$$\frac{\csc^3(c+dx)\left(128\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{9}{2};\sin^2(c+dx)\right)\sin^6(c+dx)\left(a+(-a+b)\sin^2(c+dx)\right)^2+128\,{}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};\sin^2(c+dx)\right)\sin^6(c+dx)\left(\frac{1}{2}a^2\cos^2(c+dx)(9+5\cos(2(c+dx)))+b\sin^2(c+dx)(7a+5a\cos(2(c+dx))+5b\sin^2(c+dx))\right)+35\left(-3375a^2+3a(1969a-1750b)\sin^2(c+dx)+\left(-3161a^2+5108ab-1947b^2\right)\sin^4(c+dx)+485(a-b)^2\sin^6(c+dx)+\frac{3\tanh^{-1}\left(\sqrt{\sin^2(c+dx)}\right)\left(1125a^2-2a(1172a-875b)\sin^2(c+dx)+\left(1674a^2-2286ab+649b^2\right)\sin^4(c+dx)+\left(-400a^2+778ab-378b^2\right)\sin^6(c+dx)+9(a-b)^2\sin^8(c+dx)\right)}{\sqrt{\sin^2(c+dx)}}\right)\right)}{6720d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (Csc[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, S
in[c + d*x]^2]*Sin[c + d*x]^6*(a + (-a + b)*Sin[c + d*x]^2)^2 + 128*Hyperge
ometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*((a^
2*Cos[c + d*x]^2*(9 + 5*Cos[2*(c + d*x)]))/2 + b*Sin[c + d*x]^2*(7*a + 5*a*
Cos[2*(c + d*x)] + 5*b*Sin[c + d*x]^2)) + 35*(-3375*a^2 + 3*a*(1969*a - 175
0*b)*Sin[c + d*x]^2 + (-3161*a^2 + 5108*a*b - 1947*b^2)*Sin[c + d*x]^4 + 48
5*(a - b)^2*Sin[c + d*x]^6 + (3*ArcTanh[Sqrt[Sin[c + d*x]^2]]*(1125*a^2 - 2
*a*(1172*a - 875*b)*Sin[c + d*x]^2 + (1674*a^2 - 2286*a*b + 649*b^2)*Sin[c
+ d*x]^4 + (-400*a^2 + 778*a*b - 378*b^2)*Sin[c + d*x]^6 + 9*(a - b)^2*Sin[
c + d*x]^8))/Sqrt[Sin[c + d*x]^2])))/(6720*d)
```

Maple [A]
time = 0.20, size = 146, normalized size = 1.52

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2ab\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\ln\right.}{d}$
default	$\frac{b^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2ab\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\ln\right.}{d}$
risch	$\frac{ib\left(-8a\,e^{7i(dx+c)}+5b\,e^{7i(dx+c)}-8a\,e^{5i(dx+c)}-3b\,e^{5i(dx+c)}+8a\,e^{3i(dx+c)}+3b\,e^{3i(dx+c)}+8a\,e^{i(dx+c)}-5b\,e^{i(dx+c)}\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}-\ln(e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*s
in(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*(1/2*sin(d*
x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*ln(se
c(d*x+c)+tan(d*x+c)))
```

Maxima [A]

time = 0.30, size = 119, normalized size = 1.24

$$\frac{(8a^2-8ab+3b^2)\log(\sin(dx+c)+1)-(8a^2-8ab+3b^2)\log(\sin(dx+c)-1)-\frac{2\left((8ab-5b^2)\sin(dx+c)^3-(8ab-3b^2)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*((8*a^2 - 8*a*b + 3*b^2)*log(sin(d*x + c) + 1) - (8*a^2 - 8*a*b + 3*b^
2)*log(sin(d*x + c) - 1) - 2*((8*a*b - 5*b^2)*sin(d*x + c)^3 - (8*a*b - 3*b
^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]

time = 2.66, size = 116, normalized size = 1.21

$$\frac{(8a^2-8ab+3b^2)\cos(dx+c)^4\log(\sin(dx+c)+1)-(8a^2-8ab+3b^2)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left((8ab-5b^2)\cos(dx+c)^2+2b^2\right)\sin(dx+c)}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((8*a^2 - 8*a*b + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^2
 - 8*a*b + 3*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*((8*a*b - 5*b^2
)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2(c+dx)\right)^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x), x)
```

Giac [A]
time = 0.74, size = 120, normalized size = 1.25

$$\frac{(8a^2-8ab+3b^2)\log(|\sin(dx+c)+1|)-(8a^2-8ab+3b^2)\log(|\sin(dx+c)-1|)-\frac{2\left(8ab\sin(dx+c)^3-5b^2\sin(dx+c)^3-8ab\sin(dx+c)+3b^2\sin(dx+c)\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((8*a^2 - 8*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1)) - (8*a^2 - 8*a*b +
 3*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(8*a*b*sin(d*x + c)^3 - 5*b^2*sin(d*
x + c)^3 - 8*a*b*sin(d*x + c) + 3*b^2*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)
/d
```

Mupad [B]
time = 14.57, size = 177, normalized size = 1.84

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(2a^2-2ab+\frac{3b^2}{4}\right)}{d}+\frac{\left(2ab-\frac{3b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{11b^2}{4}-2ab\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{11b^2}{4}-2ab\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(2ab-\frac{3b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x),x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(2*a^2 - 2*a*b + (3*b^2)/4))/d + (tan(c/2 + (d*x
)/2)^7*(2*a*b - (3*b^2)/4) - tan(c/2 + (d*x)/2)^3*(2*a*b - (11*b^2)/4) - ta
n(c/2 + (d*x)/2)^5*(2*a*b - (11*b^2)/4) + tan(c/2 + (d*x)/2)*(2*a*b - (3*b^
2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d
*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.439 $\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=62

$$\frac{(4a-3b)b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(a-b)^2\sin(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*(4*a-3*b)*b*arctanh(sin(d*x+c))/d+(a-b)^2*sin(d*x+c)/d+1/2*b^2*sec(d*x+
c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3757, 398, 393, 212}

$$\frac{(a-b)^2\sin(c+dx)}{d}+\frac{b(4a-3b)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - 3*b)*b*ArcTanh[Sin[c + d*x]])/(2*d) + ((a - b)^2*Sin[c + d*x])/d +
(b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((a-b)^2+\frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}\right)\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{(a-b)^2\sin(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}\,dx, x, \sin(c+dx\right.}{d}$$
$$= \frac{(a-b)^2\sin(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{((4a-3b)b)}{}$$
$$= \frac{(4a-3b)b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(a-b)^2\sin(c+dx)}{d}+\frac{b^2\sec(c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 146 vs. $2(62) = 124$.
time = 0.49, size = 146, normalized size = 2.35

$$\frac{-2(4a-3b)b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(4a-3b)b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{b^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{b^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+4(a-b)^2\sin(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (-2*(4*a - 3*b)*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(4*a - 3*b)*
b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2 - b^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 4*(a - b)^2*S
in[c + d*x])/(4*d)
```

Maple [A]
time = 0.19, size = 100, normalized size = 1.61

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))}{d}$

default	$\frac{b^2\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-}{d}$
risch	$-\frac{ie^{i(dx+c)}a^2}{2d}+\frac{ie^{i(dx+c)}ab}{d}-\frac{ie^{i(dx+c)}b^2}{2d}+\frac{ie^{-i(dx+c)}a^2}{2d}-\frac{ie^{-i(dx+c)}ab}{d}+\frac{ie^{-i(dx+c)}b^2}{2d}-\frac{ib^2\left(e^{3i(dx+c)}-e^{i(}\right.}{d\left(e^{2i(dx+c)}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2
*ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a
^2*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 105, normalized size = 1.69

$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log(\sin(dx+c)+1)-3\log(\sin(dx+c)-1)-4\sin(dx+c)\right)-4ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-4a^2\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 4*a*b*(log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 4*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 1.76, size = 106, normalized size = 1.71

$$\frac{(4ab-3b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-(4ab-3b^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\left(2(a^2-2ab+b^2)\cos(dx+c)^2+b^2\right)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((4*a*b - 3*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (4*a*b - 3*b^2)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(a^2 - 2*a*b + b^2)*cos(d*x +
 c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2(c+dx)\right)^2\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x), x)
```

Giac [A]
time = 0.78, size = 104, normalized size = 1.68

$$\frac{4\,a^2\sin\left(dx+c\right)-8\,ab\sin\left(dx+c\right)+4\,b^2\sin\left(dx+c\right)+\left(4\,ab-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(4\,ab-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,b^2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*sin(d*x + c) - 8*a*b*sin(d*x + c) + 4*b^2*sin(d*x + c) + (4*a*b
- 3*b^2)*log(abs(sin(d*x + c) + 1)) - (4*a*b - 3*b^2)*log(abs(sin(d*x + c)
- 1)) - 2*b^2*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B]
time = 14.31, size = 148, normalized size = 2.39

$$\frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,a-3\,b\right)}{d}-\frac{\left(2\,a^2-4\,a\,b+3\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-4\,a^2+8\,a\,b-2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2-4\,a\,b+3\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] (b*atanh(tan(c/2 + (d*x)/2))*(4*a - 3*b))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2
- 4*a*b + 3*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^2 - 8*a*b + 2*b^2) + tan(c/2 +
 (d*x)/2)*(2*a^2 - 4*a*b + 3*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*
x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.440 $\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{(a^2-b^2)\sin(c+dx)}{d}-\frac{(a-b)^2\sin^3(c+dx)}{3d}$$

```
[Out] b^2*arctanh(sin(d*x+c))/d+(a^2-b^2)*sin(d*x+c)/d-1/3*(a-b)^2*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {3757, 398, 212}

$$\frac{(a^2-b^2)\sin(c+dx)}{d}-\frac{(a-b)^2\sin^3(c+dx)}{3d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d + ((a^2 - b^2)*Sin[c + d*x])/d - ((a - b)^2*S
in[c + d*x]^3)/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-b^2-(a-b)^2x^2+\frac{b^2}{1-x^2}\right)\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{(a^2-b^2)\sin(c+dx)}{d} - \frac{(a-b)^2\sin^3(c+dx)}{3d} + \frac{b^2\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{}$$
$$= \frac{b^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{(a^2-b^2)\sin(c+dx)}{d} - \frac{(a-b)^2\sin^3(c}{3d}$$

Mathematica [A]
time = 0.26, size = 71, normalized size = 1.27

$$\frac{\sin(c+dx)\left(\frac{3b^2\tanh^{-1}\left(\sqrt{\sin^2(c+dx)}\right)}{\sqrt{\sin^2(c+dx)}} - (a-b)\left(-3(a+b)+(a-b)\sin^2(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
[Out] (Sin[c + d*x]*((3*b^2*ArcTanh[Sqrt[Sin[c + d*x]^2]])/Sqrt[Sin[c + d*x]^2] -
 (a - b)*(-3*(a + b) + (a - b)*Sin[c + d*x]^2)))/(3*d)
```

Maple [A]
time = 0.20, size = 76, normalized size = 1.36

method	result
derivativedivides	$\frac{b^2\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{2ab\left(\sin^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{b^2\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{2ab\left(\sin^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
risch	$-\frac{3ie^{i(dx+c)}a^2}{8d}-\frac{ie^{i(dx+c)}ab}{4d}+\frac{5ie^{i(dx+c)}b^2}{8d}+\frac{3ie^{-i(dx+c)}a^2}{8d}+\frac{ie^{-i(dx+c)}ab}{4d}-\frac{5ie^{-i(dx+c)}b^2}{8d}+\frac{\ln\left(e^{i(dx+c)}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+2/3*a*b*s
in(d*x+c)^3+1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 1.29

$$-\frac{2\,(a^2-2\,ab+b^2)\sin{(dx+c)}^3-3\,b^2\log{(\sin{(dx+c)}+1)}+3\,b^2\log{(\sin{(dx+c)}-1)}-6\,(a^2-b^2)\sin{(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(a^2 - 2*a*b + b^2)*sin(d*x + c)^3 - 3*b^2*log(sin(d*x + c) + 1) +
3*b^2*log(sin(d*x + c) - 1) - 6*(a^2 - b^2)*sin(d*x + c))/d
```

Fricas [A]
time = 2.43, size = 79, normalized size = 1.41

$$\frac{3\,b^2\log{(\sin{(dx+c)}+1)}-3\,b^2\log{(-\sin{(dx+c)}+1)}+2\,\left((a^2-2\,ab+b^2)\cos{(dx+c)}^2+2\,a^2+2\,ab-4\,b^2\right)\sin{(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*log(sin(d*x + c) + 1) - 3*b^2*log(-sin(d*x + c) + 1) + 2*((a^2 -
 2*a*b + b^2)*cos(d*x + c)^2 + 2*a^2 + 2*a*b - 4*b^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2{(c+dx)}\right)^2\cos^3{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**3, x)
```

Giac [A]
time = 0.86, size = 96, normalized size = 1.71

$$-\frac{2\,a^2\sin{(dx+c)}^3-4\,ab\sin{(dx+c)}^3+2\,b^2\sin{(dx+c)}^3-3\,b^2\log{(|\sin{(dx+c)}+1|)}+3\,b^2\log{(|\sin{(dx+c)}-1|)}-6\,a^2\sin{(dx+c)}+6\,b^2\sin{(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*sin(d*x + c)^3 - 4*a*b*sin(d*x + c)^3 + 2*b^2*sin(d*x + c)^3 -
3*b^2*log(abs(sin(d*x + c) + 1)) + 3*b^2*log(abs(sin(d*x + c) - 1)) - 6*a^2
*sin(d*x + c) + 6*b^2*sin(d*x + c))/d
```

Mupad [B]
time = 14.04, size = 136, normalized size = 2.43

$$\frac{2\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\left(2\,a^2-2\,b^2\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(\frac{4\,a^2}{3}+\frac{16\,a\,b}{3}-\frac{20\,b^2}{3}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(2\,a^2-2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] (2*b^2*atanh(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*b^2)
 + tan(c/2 + (d*x)/2)^3*((16*a*b)/3 + (4*a^2)/3 - (20*b^2)/3) + tan(c/2 + (
d*x)/2)*(2*a^2 - 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^
4 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.441 $\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$\frac{a^2\sin(c+dx)}{d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}$$

```
[Out] a^2*sin(d*x+c)/d-2/3*a*(a-b)*sin(d*x+c)^3/d+1/5*(a-b)^2*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3757, 200}

$$\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d - (2*a*(a - b)*Sin[c + d*x]^3)/(3*d) + ((a - b)^2*Sin[
c + d*x]^5)/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(a-(a-b)x^2\right)^2\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^2-2a(a-b)x^2+(a-b)^2x^4\right)\,dx,x,\sin(c+dx)\right)}{d}\\&=\frac{a^2\sin(c+dx)}{d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 70, normalized size = 1.23

$$\frac{(89a^2+22ab+9b^2+4(7a^2-4ab-3b^2)\cos(2(c+dx))+3(a-b)^2\cos(4(c+dx)))\sin(c+dx)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((89*a^2 + 22*a*b + 9*b^2 + 4*(7*a^2 - 4*a*b - 3*b^2)*Cos[2*(c + d*x)] + 3*
(a - b)^2*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d)
```

Maple [A]
time = 0.24, size = 89, normalized size = 1.56

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^5(dx+c)\right)}{5}+2ab\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+}{5}}{d}$
default	$\frac{\frac{b^2\left(\sin^5(dx+c)\right)}{5}+2ab\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+}{5}}{d}$
risch	$\frac{5a^2\sin(dx+c)}{8d}+\frac{\sin(dx+c)ab}{4d}+\frac{\sin(dx+c)b^2}{8d}+\frac{\sin(5dx+5c)a^2}{80d}-\frac{\sin(5dx+5c)ab}{40d}+\frac{\sin(5dx+5c)b^2}{80d}+\frac{5\sin(3d}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b^2*sin(d*x+c)^5+2*a*b*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d
*x+c)^2)*sin(d*x+c))+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.98

$$\frac{3\,(a^2-2\,ab+b^2)\sin{(dx+c)}^5-10\,(a^2-ab)\sin{(dx+c)}^3+15\,a^2\sin{(dx+c)}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a^2 - 2*a*b + b^2)*sin(d*x + c)^5 - 10*(a^2 - a*b)*sin(d*x + c)^3
+ 15*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 2.42, size = 71, normalized size = 1.25

$$\frac{\left(3\,(a^2-2\,ab+b^2)\cos{(dx+c)}^4+2\,(2\,a^2+ab-3\,b^2)\cos{(dx+c)}^2+8\,a^2+4\,ab+3\,b^2\right)\sin{(dx+c)}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*(a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(2*a^2 + a*b - 3*b^2)*cos(d*
x + c)^2 + 8*a^2 + 4*a*b + 3*b^2)*sin(d*x + c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan^2\left(c + dx\right)\right)^2 \cos^5\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2946 vs. 2(53) = 106.
time = 118.78, size = 2946, normalized size = 51.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*tan(1/2*d*x)^10*tan(1/2*c)^9 + 15*a^2*tan(1/2*d*x)^9*tan(1/2*
c)^10 + 20*a^2*tan(1/2*d*x)^10*tan(1/2*c)^7 + 40*a*b*tan(1/2*d*x)^10*tan(1/
2*c)^7 - 75*a^2*tan(1/2*d*x)^9*tan(1/2*c)^8 + 120*a*b*tan(1/2*d*x)^9*tan(1/
2*c)^8 - 75*a^2*tan(1/2*d*x)^8*tan(1/2*c)^9 + 120*a*b*tan(1/2*d*x)^8*tan(1/
2*c)^9 + 20*a^2*tan(1/2*d*x)^7*tan(1/2*c)^10 + 40*a*b*tan(1/2*d*x)^7*tan(1/
2*c)^10 + 58*a^2*tan(1/2*d*x)^10*tan(1/2*c)^5 - 16*a*b*tan(1/2*d*x)^10*tan(
1/2*c)^5 + 48*b^2*tan(1/2*d*x)^10*tan(1/2*c)^5 + 150*a^2*tan(1/2*d*x)^9*tan
(1/2*c)^6 - 360*a*b*tan(1/2*d*x)^9*tan(1/2*c)^6 + 240*b^2*tan(1/2*d*x)^9*ta
n(1/2*c)^6 + 700*a^2*tan(1/2*d*x)^8*tan(1/2*c)^7 - 1000*a*b*tan(1/2*d*x)^8*
tan(1/2*c)^7 + 480*b^2*tan(1/2*d*x)^8*tan(1/2*c)^7 + 700*a^2*tan(1/2*d*x)^7
*tan(1/2*c)^8 - 1000*a*b*tan(1/2*d*x)^7*tan(1/2*c)^8 + 480*b^2*tan(1/2*d*x)
^7*tan(1/2*c)^8 + 150*a^2*tan(1/2*d*x)^6*tan(1/2*c)^9 - 360*a*b*tan(1/2*d*x
)^6*tan(1/2*c)^9 + 240*b^2*tan(1/2*d*x)^6*tan(1/2*c)^9 + 58*a^2*tan(1/2*d*x
)^5*tan(1/2*c)^10 - 16*a*b*tan(1/2*d*x)^5*tan(1/2*c)^10 + 48*b^2*tan(1/2*d*
x)^5*tan(1/2*c)^10 + 20*a^2*tan(1/2*d*x)^10*tan(1/2*c)^3 + 40*a*b*tan(1/2*d
*x)^10*tan(1/2*c)^3 - 150*a^2*tan(1/2*d*x)^9*tan(1/2*c)^4 + 360*a*b*tan(1/2
*d*x)^9*tan(1/2*c)^4 - 240*b^2*tan(1/2*d*x)^9*tan(1/2*c)^4 - 610*a^2*tan(1/
2*d*x)^8*tan(1/2*c)^5 + 2080*a*b*tan(1/2*d*x)^8*tan(1/2*c)^5 - 1200*b^2*tan
(1/2*d*x)^8*tan(1/2*c)^5 - 2200*a^2*tan(1/2*d*x)^7*tan(1/2*c)^6 + 4720*a*b*
tan(1/2*d*x)^7*tan(1/2*c)^6 - 2400*b^2*tan(1/2*d*x)^7*tan(1/2*c)^6 - 2200*a
```

^2*tan(1/2*d*x)^6*tan(1/2*c)^7 + 4720*a*b*tan(1/2*d*x)^6*tan(1/2*c)^7 - 240
0*b^2*tan(1/2*d*x)^6*tan(1/2*c)^7 - 610*a^2*tan(1/2*d*x)^5*tan(1/2*c)^8 + 2
080*a*b*tan(1/2*d*x)^5*tan(1/2*c)^8 - 1200*b^2*tan(1/2*d*x)^5*tan(1/2*c)^8
- 150*a^2*tan(1/2*d*x)^4*tan(1/2*c)^9 + 360*a*b*tan(1/2*d*x)^4*tan(1/2*c)^9
 - 240*b^2*tan(1/2*d*x)^4*tan(1/2*c)^9 + 20*a^2*tan(1/2*d*x)^3*tan(1/2*c)^1
0 + 40*a*b*tan(1/2*d*x)^3*tan(1/2*c)^10 + 15*a^2*tan(1/2*d*x)^10*tan(1/2*c)
 + 75*a^2*tan(1/2*d*x)^9*tan(1/2*c)^2 - 120*a*b*tan(1/2*d*x)^9*tan(1/2*c)^2
 + 700*a^2*tan(1/2*d*x)^8*tan(1/2*c)^3 - 1000*a*b*tan(1/2*d*x)^8*tan(1/2*c)
^3 + 480*b^2*tan(1/2*d*x)^8*tan(1/2*c)^3 + 2200*a^2*tan(1/2*d*x)^7*tan(1/2*
c)^4 - 4720*a*b*tan(1/2*d*x)^7*tan(1/2*c)^4 + 2400*b^2*tan(1/2*d*x)^7*tan(1
/2*c)^4 + 5380*a^2*tan(1/2*d*x)^6*tan(1/2*c)^5 - 10000*a*b*tan(1/2*d*x)^6*t
an(1/2*c)^5 + 4800*b^2*tan(1/2*d*x)^6*tan(1/2*c)^5 + 5380*a^2*tan(1/2*d*x)^
5*tan(1/2*c)^6 - 10000*a*b*tan(1/2*d*x)^5*tan(1/2*c)^6 + 4800*b^2*tan(1/2*d
*x)^5*tan(1/2*c)^6 + 2200*a^2*tan(1/2*d*x)^4*tan(1/2*c)^7 - 4720*a*b*tan(1/
2*d*x)^4*tan(1/2*c)^7 + 2400*b^2*tan(1/2*d*x)^4*tan(1/2*c)^7 + 700*a^2*tan(
1/2*d*x)^3*tan(1/2*c)^8 - 1000*a*b*tan(1/2*d*x)^3*tan(1/2*c)^8 + 480*b^2*ta
n(1/2*d*x)^3*tan(1/2*c)^8 + 75*a^2*tan(1/2*d*x)^2*tan(1/2*c)^9 - 120*a*b*ta
n(1/2*d*x)^2*tan(1/2*c)^9 + 15*a^2*tan(1/2*d*x)*tan(1/2*c)^10 - 15*a^2*tan(
1/2*d*x)^9 - 75*a^2*tan(1/2*d*x)^8*tan(1/2*c) + 120*a*b*tan(1/2*d*x)^8*tan(
1/2*c) - 700*a^2*tan(1/2*d*x)^7*tan(1/2*c)^2 + 1000*a*b*tan(1/2*d*x)^7*tan(
1/2*c)^2 - 480*b^2*tan(1/2*d*x)^7*tan(1/2*c)^2 - 2200*a^2*tan(1/2*d*x)^6*ta
n(1/2*c)^3 + 4720*a*b*tan(1/2*d*x)^6*tan(1/2*c)^3 - 2400*b^2*tan(1/2*d*x)^6
*tan(1/2*c)^3 - 5380*a^2*tan(1/2*d*x)^5*tan(1/2*c)^4 + 10000*a*b*tan(1/2*d*
x)^5*tan(1/2*c)^4 - 4800*b^2*tan(1/2*d*x)^5*tan(1/2*c)^4 - 5380*a^2*tan(1/2
*d*x)^4*tan(1/2*c)^5 + 10000*a*b*tan(1/2*d*x)^4*tan(1/2*c)^5 - 4800*b^2*tan
(1/2*d*x)^4*tan(1/2*c)^5 - 2200*a^2*tan(1/2*d*x)^3*tan(1/2*c)^6 + 4720*a*b*
tan(1/2*d*x)^3*tan(1/2*c)^6 - 2400*b^2*tan(1/2*d*x)^3*tan(1/2*c)^6 - 700*a^
2*tan(1/2*d*x)^2*tan(1/2*c)^7 + 1000*a*b*tan(1/2*d*x)^2*tan(1/2*c)^7 - 480*
b^2*tan(1/2*d*x)^2*tan(1/2*c)^7 - 75*a^2*tan(1/2*d*x)*tan(1/2*c)^8 + 120*a*
b*tan(1/2*d*x)*tan(1/2*c)^8 - 15*a^2*tan(1/2*c)^9 - 20*a^2*tan(1/2*d*x)^7 -
 40*a*b*tan(1/2*d*x)^7 + 150*a^2*tan(1/2*d*x)^6*tan(1/2*c) - 360*a*b*tan(1/
2*d*x)^6*tan(1/2*c) + 240*b^2*tan(1/2*d*x)^6*tan(1/2*c) + 610*a^2*tan(1/2*d
*x)^5*tan(1/2*c)^2 - 2080*a*b*tan(1/2*d*x)^5*tan(1/2*c)^2 + 1200*b^2*tan(1/
2*d*x)^5*tan(1/2*c)^2 + 2200*a^2*tan(1/2*d*x)^4*tan(1/2*c)^3 - 4720*a*b*tan
(1/2*d*x)^4*tan(1/2*c)^3 + 2400*b^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 2200*a^2*
tan(1/2*d*x)^3*tan(1/2*c)^4 - 4720*a*b*tan(1/2*d*x)^3*tan(1/2*c)^4 + 2400*b
^2*tan(1/2*d*x)^3*tan(1/2*c)^4 + 610*a^2*tan(1/2*d*x)^2*tan(1/2*c)^5 - 2080
*a*b*tan(1/2*d*x)^2*tan(1/2*c)^5 + 1200*b^2*tan(1/2*d*x)^2*tan(1/2*c)^5 + 1
50*a^2*tan(1/2*d*x)*tan(1/2*c)^6 - 360*a*b*tan(1/2*d*x)*tan(1/2*c)^6 + 240*
b^2*tan(1/2*d*x)*tan(1/2*c)^6 - 20*a^2*tan(1/2*c)^7 - 40*a*b*tan(1/2*c)^7 -
 58*a^2*tan(1/2*d*x)^5 + 16*a*b*tan(1/2*d*x)^5 - 48*b^2*tan(1/2*d*x)^5 - 15
0*a^2*tan(1/2*d*x)^4*tan(1/2*c) + 360*a*b*tan(1/2*d*x)^4*tan(1/2*c) - 240*b
^2*tan(1/2*d*x)^4*tan(1/2*c) - 700*a^2*tan(1/2*d*x)^3*tan(1/2*c)^2 + 1000*a
*b*tan(1/2*d*x)^3*tan(1/2*c)^2 - 480*b^2*tan(1/2*d*x)^3*tan(1/2*c)^2 - 700*
a^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 1000*a*b*tan(1/2*d*x)^2*tan(1/2*c)^3 - 48

```
0*b^2*tan(1/2*d*x)^2*tan(1/2*c)^3 - 150*a^2*tan(1/2*d*x)*tan(1/2*c)^4 + 360
*a*b*tan(1/2*d*x)*tan(1/2*c)^4 - 240*b^2*tan(1/...
```

Mupad [B]
time = 12.20, size = 119, normalized size = 2.09

$$\frac{\frac{5\,a^2\,\sin(c+d\,x)}{8}+\frac{b^2\,\sin(c+d\,x)}{8}+\frac{5\,a^2\,\sin(3\,c+3\,d\,x)}{48}+\frac{a^2\,\sin(5\,c+5\,d\,x)}{80}-\frac{b^2\,\sin(3\,c+3\,d\,x)}{16}+\frac{b^2\,\sin(5\,c+5\,d\,x)}{80}+\frac{a\,b\,\sin(c+d\,x)}{4}-\frac{a\,b\,\sin(3\,c+3\,d\,x)}{24}-\frac{a\,b\,\sin(5\,c+5\,d\,x)}{40}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((5*a^2*sin(c + d*x))/8 + (b^2*sin(c + d*x))/8 + (5*a^2*sin(3*c + 3*d*x))/4
8 + (a^2*sin(5*c + 5*d*x))/80 - (b^2*sin(3*c + 3*d*x))/16 + (b^2*sin(5*c +
5*d*x))/80 + (a*b*sin(c + d*x))/4 - (a*b*sin(3*c + 3*d*x))/24 - (a*b*sin(5*
c + 5*d*x))/40)/d
```

3.442 $\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=86

$$\frac{a^2\sin(c+dx)}{d}-\frac{a(3a-2b)\sin^3(c+dx)}{3d}+\frac{(a-b)(3a-b)\sin^5(c+dx)}{5d}-\frac{(a-b)^2\sin^7(c+dx)}{7d}$$

```
[Out] a^2*sin(d*x+c)/d-1/3*a*(3*a-2*b)*sin(d*x+c)^3/d+1/5*(a-b)*(3*a-b)*sin(d*x+c
)^5/d-1/7*(a-b)^2*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3757, 380}

$$\frac{a^2\sin(c+dx)}{d}-\frac{(a-b)^2\sin^7(c+dx)}{7d}+\frac{(a-b)(3a-b)\sin^5(c+dx)}{5d}-\frac{a(3a-2b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d - (a*(3*a - 2*b)*Sin[c + d*x]^3)/(3*d) + ((a - b)*(3*a
 - b)*Sin[c + d*x]^5)/(5*d) - ((a - b)^2*Sin[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a-(a-b)x^2\right)^2\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^2-a(3a-2b)x^2+\left(3a^2-4ab+b^2\right)x^4-(a-b)^2x^6\right.}{d}\\ &= \frac{a^2\sin(c+dx)}{d}-\frac{a(3a-2b)\sin^3(c+dx)}{3d}+\frac{(a-b)(3a-b)\sin^5}{5d}\end{aligned}$$

Mathematica [A]

time = 0.44, size = 90, normalized size = 1.05

$$\frac{105(35a^2+10ab+3b^2)\sin(c+dx)+35(7a-3b)(3a+b)\sin(3(c+dx))+21(a-b)(7a+b)\sin(5(c+dx))+15(a-b)^2\sin(7(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (105*(35*a^2 + 10*a*b + 3*b^2)*Sin[c + d*x] + 35*(7*a - 3*b)*(3*a + b)*Sin[
3*(c + d*x)] + 21*(a - b)*(7*a + b)*Sin[5*(c + d*x)] + 15*(a - b)^2*Sin[7*(
c + d*x)])/(6720*d)
```

Maple [A]

time = 0.28, size = 153, normalized size = 1.78

method	result
derivativedivides	$\frac{b^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{35}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\right.}{d}$
default	$\frac{b^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{35}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\right.}{d}$
risch	$\frac{35a^2\sin(dx+c)}{64d}+\frac{5\sin(dx+c)ab}{32d}+\frac{3\sin(dx+c)b^2}{64d}+\frac{\sin(7dx+7c)a^2}{448d}-\frac{\sin(7dx+7c)ab}{224d}+\frac{\sin(7dx+7c)b^2}{448d}+7\sin($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*
(2+cos(d*x+c)^2)*sin(d*x+c))+2*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]

time = 0.30, size = 81, normalized size = 0.94

$$-\frac{15\,(a^2-2\,ab+b^2)\sin{(dx+c)}^7-21\,(3\,a^2-4\,ab+b^2)\sin{(dx+c)}^5+35\,(3\,a^2-2\,ab)\sin{(dx+c)}^3-105\,a^2\sin{(dx+c)}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/105*(15*(a^2 - 2*a*b + b^2)*sin(d*x + c)^7 - 21*(3*a^2 - 4*a*b + b^2)*si
n(d*x + c)^5 + 35*(3*a^2 - 2*a*b)*sin(d*x + c)^3 - 105*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 2.58, size = 95, normalized size = 1.10

$$\frac{\left(15\left(a^2-2ab+b^2\right)\cos\left(dx+c\right)^6+6\left(3a^2+ab-4b^2\right)\cos\left(dx+c\right)^4+\left(24a^2+8ab+3b^2\right)\cos\left(dx+c\right)^2+48a^2+16ab+6b^2\right)\sin\left(dx+c\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*(a^2 - 2*a*b + b^2)*cos(d*x + c)^6 + 6*(3*a^2 + a*b - 4*b^2)*cos(
d*x + c)^4 + (24*a^2 + 8*a*b + 3*b^2)*cos(d*x + c)^2 + 48*a^2 + 16*a*b + 6*
b^2)*sin(d*x + c)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 12.27, size = 160, normalized size = 1.86

$$\frac{\frac{35a^2\sin(c+dx)}{64}+\frac{3b^2\sin(c+dx)}{64}+\frac{7a^2\sin(3c+3dx)}{64}+\frac{7a^2\sin(5c+5dx)}{320}+\frac{a^2\sin(7c+7dx)}{448}-\frac{b^2\sin(3c+3dx)}{64}-\frac{b^2\sin(5c+5dx)}{320}+\frac{b^2\sin(7c+7dx)}{448}+\frac{5ab\sin(c+dx)}{32}-\frac{ab\sin(3c+3dx)}{96}-\frac{3ab\sin(5c+5dx)}{160}-\frac{ab\sin(7c+7dx)}{224}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((35*a^2*sin(c + d*x))/64 + (3*b^2*sin(c + d*x))/64 + (7*a^2*sin(3*c + 3*d*
x))/64 + (7*a^2*sin(5*c + 5*d*x))/320 + (a^2*sin(7*c + 7*d*x))/448 - (b^2*s
in(3*c + 3*d*x))/64 - (b^2*sin(5*c + 5*d*x))/320 + (b^2*sin(7*c + 7*d*x))/4
48 + (5*a*b*sin(c + d*x))/32 - (a*b*sin(3*c + 3*d*x))/96 - (3*a*b*sin(5*c +
 5*d*x))/160 - (a*b*sin(7*c + 7*d*x))/224)/d
```

3.443 $\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=114

$$\frac{a^2\sin(c+dx)}{d}-\frac{2a(2a-b)\sin^3(c+dx)}{3d}+\frac{(6a^2-6ab+b^2)\sin^5(c+dx)}{5d}-\frac{2(a-b)(2a-b)\sin^7(c+dx)}{7d}+\frac{(a-$$

```
[Out] a^2*sin(d*x+c)/d-2/3*a*(2*a-b)*sin(d*x+c)^3/d+1/5*(6*a^2-6*a*b+b^2)*sin(d*x
+c)^5/d-2/7*(a-b)*(2*a-b)*sin(d*x+c)^7/d+1/9*(a-b)^2*sin(d*x+c)^9/d
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3757, 380}

$$\frac{(6a^2-6ab+b^2)\sin^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}+\frac{(a-b)^2\sin^9(c+dx)}{9d}-\frac{2(a-b)(2a-b)\sin^7(c+dx)}{7d}-\frac{2a(2a-b)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d - (2*a*(2*a - b)*Sin[c + d*x]^3)/(3*d) + ((6*a^2 - 6*a
*b + b^2)*Sin[c + d*x]^5)/(5*d) - (2*(a - b)*(2*a - b)*Sin[c + d*x]^7)/(7*d
) + ((a - b)^2*Sin[c + d*x]^9)/(9*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a-(a-b)x^2\right)^2\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^2-2a(2a-b)x^2+\left(6a^2-6ab+b^2\right)x^4-2\left(2a^2-3ab\right.\right.\right.}{d}\\ &= \frac{a^2\sin(c+dx)}{d}-\frac{2a(2a-b)\sin^3(c+dx)}{3d}+\frac{\left(6a^2-6ab+b^2\right)\sin^5(}{5d}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 116, normalized size = 1.02

$$\frac{630(63a^2+14ab+3b^2)\sin(c+dx)+420(21a^2-b^2)\sin(3(c+dx))+252(9a^2-4ab-b^2)\sin(5(c+dx))+45(a-b)(9a-b)\sin(7(c+dx))+35(a-b)^2\sin(9(c+dx))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (630*(63*a^2 + 14*a*b + 3*b^2)*Sin[c + d*x] + 420*(21*a^2 - b^2)*Sin[3*(c +
 d*x)] + 252*(9*a^2 - 4*a*b - b^2)*Sin[5*(c + d*x)] + 45*(a - b)*(9*a - b)*
Sin[7*(c + d*x)] + 35*(a - b)^2*Sin[9*(c + d*x)])/(80640*d)
```

Maple [A]
time = 0.29, size = 183, normalized size = 1.61

method	result
derivativedivides	$b^2\left(-\frac{(\sin^3(dx+c))(\cos^6(dx+c))}{9}-\frac{\sin(dx+c)(\cos^6(dx+c))}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{105}\right)+2ab\left(-\frac{\sin(dx+c}{}\right.$
default	$b^2\left(-\frac{(\sin^3(dx+c))(\cos^6(dx+c))}{9}-\frac{\sin(dx+c)(\cos^6(dx+c))}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{105}\right)+2ab\left(-\frac{\sin(dx+c}{}\right.$
risch	$\frac{63a^2\sin(dx+c)}{128d}+\frac{7\sin(dx+c)ab}{64d}+\frac{3\sin(dx+c)b^2}{128d}+\frac{\sin(9dx+9c)a^2}{2304d}-\frac{\sin(9dx+9c)ab}{1152d}+\frac{\sin(9dx+9c)b^2}{2304d}+\frac{9\sin}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+2*a*b*(-1/9*sin(d*x+c)*cos
(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c))+1/9*a^2*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/3
5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 104, normalized size = 0.91

$$\frac{35\,(a^2-2\,ab+b^2)\sin(dx+c)^9-90\,(2\,a^2-3\,ab+b^2)\sin(dx+c)^7+63\,(6\,a^2-6\,ab+b^2)\sin(dx+c)^5-210\,(2\,a^2-ab)\sin(dx+c)^3+315\,a^2\sin(dx+c)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(a^2 - 2*a*b + b^2)*sin(d*x + c)^9 - 90*(2*a^2 - 3*a*b + b^2)*sin
(d*x + c)^7 + 63*(6*a^2 - 6*a*b + b^2)*sin(d*x + c)^5 - 210*(2*a^2 - a*b)*s
in(d*x + c)^3 + 315*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 2.92, size = 117, normalized size = 1.03

$$\frac{\left(35\left(a^2-2ab+b^2\right)\cos\left(dx+c\right)^8+10\left(4a^2+ab-5b^2\right)\cos\left(dx+c\right)^6+3\left(16a^2+4ab+b^2\right)\cos\left(dx+c\right)^4+4\left(16a^2+4ab+b^2\right)\cos\left(dx+c\right)^2+128a^2+32ab+8b^2\right)\sin\left(dx+c\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*(a^2 - 2*a*b + b^2)*cos(d*x + c)^8 + 10*(4*a^2 + a*b - 5*b^2)*cos
(d*x + c)^6 + 3*(16*a^2 + 4*a*b + b^2)*cos(d*x + c)^4 + 4*(16*a^2 + 4*a*b +
 b^2)*cos(d*x + c)^2 + 128*a^2 + 32*a*b + 8*b^2)*sin(d*x + c)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 12.40, size = 188, normalized size = 1.65

$$\frac{\frac{63a^2\sin(c+dx)}{128}+\frac{3b^2\sin(c+dx)}{128}+\frac{7a^2\sin(3c+3dx)}{64}+\frac{9a^2\sin(5c+5dx)}{320}+\frac{9a^2\sin(7c+7dx)}{1792}+\frac{a^2\sin(9c+9dx)}{2304}-\frac{b^2\sin(3c+3dx)}{192}-\frac{b^2\sin(5c+5dx)}{320}+\frac{b^2\sin(7c+7dx)}{1792}+\frac{b^2\sin(9c+9dx)}{2304}+\frac{7ab\sin(c+dx)}{64}-\frac{ab\sin(5c+5dx)}{80}-\frac{5ab\sin(7c+7dx)}{896}-\frac{ab\sin(9c+9dx)}{1152}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((63*a^2*sin(c + d*x))/128 + (3*b^2*sin(c + d*x))/128 + (7*a^2*sin(3*c + 3*
d*x))/64 + (9*a^2*sin(5*c + 5*d*x))/320 + (9*a^2*sin(7*c + 7*d*x))/1792 + (
a^2*sin(9*c + 9*d*x))/2304 - (b^2*sin(3*c + 3*d*x))/192 - (b^2*sin(5*c + 5*
d*x))/320 + (b^2*sin(7*c + 7*d*x))/1792 + (b^2*sin(9*c + 9*d*x))/2304 + (7*
a*b*sin(c + d*x))/64 - (a*b*sin(5*c + 5*d*x))/80 - (5*a*b*sin(7*c + 7*d*x))
/896 - (a*b*sin(9*c + 9*d*x))/1152)/d
```

3.444 $\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{a^2\tan(c+dx)}{d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{(a^2+4ab+b^2)\tan^5(c+dx)}{5d}+\frac{2b(a+b)\tan^7(c+dx)}{7d}+\frac{b^2\tan^9(c+dx)}{9d}$$

```
[Out] a^2*tan(d*x+c)/d+2/3*a*(a+b)*tan(d*x+c)^3/d+1/5*(a^2+4*a*b+b^2)*tan(d*x+c)^
5/d+2/7*b*(a+b)*tan(d*x+c)^7/d+1/9*b^2*tan(d*x+c)^9/d
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3756, 380}

$$\frac{(a^2+4ab+b^2)\tan^5(c+dx)}{5d}+\frac{a^2\tan(c+dx)}{d}+\frac{2b(a+b)\tan^7(c+dx)}{7d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*(a + b)*Tan[c + d*x]^3)/(3*d) + ((a^2 + 4*a*b +
 b^2)*Tan[c + d*x]^5)/(5*d) + (2*b*(a + b)*Tan[c + d*x]^7)/(7*d) + (b^2*Tan
[c + d*x]^9)/(9*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)^2\left(a+bx^2\right)^2\,dx,x,\tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int\left(a^2+2a(a+b)x^2+\left(a^2+4ab+b^2\right)x^4+2b(a+b)x^6+b\right.}{d}$$
$$= \frac{a^2\tan(c+dx)}{d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{\left(a^2+4ab+b^2\right)\tan^5(c}{5d}$$

Mathematica [A]
time = 0.26, size = 106, normalized size = 1.10

$$\frac{\left(8\left(21a^2-6ab+b^2\right)+4\left(21a^2-6ab+b^2\right)\sec^2(c+dx)+3\left(21a^2-6ab+b^2\right)\sec^4(c+dx)+10(9a-5b)b\sec^6(c+dx)+35b^2\sec^8(c+dx)\right)\tan(c+dx)}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((8*(21*a^2 - 6*a*b + b^2) + 4*(21*a^2 - 6*a*b + b^2)*Sec[c + d*x]^2 + 3*(2
1*a^2 - 6*a*b + b^2)*Sec[c + d*x]^4 + 10*(9*a - 5*b)*b*Sec[c + d*x]^6 + 35*
b^2*Sec[c + d*x]^8)*Tan[c + d*x])/(315*d)
```

Maple [A]
time = 0.27, size = 157, normalized size = 1.64

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^5(dx+c)}{9\cos(dx+c)^9}+\frac{4\left(\sin^5(dx+c)\right)}{63\cos(dx+c)^7}+\frac{8\left(\sin^5(dx+c)\right)}{315\cos(dx+c)^5}\right)+2ab\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4\right.}{}\right.}{d}$
default	$\frac{b^2\left(\frac{\sin^5(dx+c)}{9\cos(dx+c)^9}+\frac{4\left(\sin^5(dx+c)\right)}{63\cos(dx+c)^7}+\frac{8\left(\sin^5(dx+c)\right)}{315\cos(dx+c)^5}\right)+2ab\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4\right.}{}\right.}{d}$
risch	$\frac{16i\left(210a^2e^{12i(dx+c)}-420ab\,e^{12i(dx+c)}+210b^2e^{12i(dx+c)}+945a^2e^{10i(dx+c)}-630ab\,e^{10i(dx+c)}-315b^2e^{10i(dx+c)}+1701a^2e^{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/31
5*sin(d*x+c)^5/cos(d*x+c)^5)+2*a*b*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(
d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)-a^2*(-8/15-1/5*sec(d
*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.30, size = 85, normalized size = 0.89

$$\frac{35\,b^2\tan\left(dx+c\right)^9+90\left(ab+b^2\right)\tan\left(dx+c\right)^7+63\left(a^2+4\,ab+b^2\right)\tan\left(dx+c\right)^5+210\left(a^2+ab\right)\tan\left(dx+c\right)^3+315\,a^2\tan\left(dx+c\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(d*x + c)^9 + 90*(a*b + b^2)*tan(d*x + c)^7 + 63*(a^2 + 4*
a*b + b^2)*tan(d*x + c)^5 + 210*(a^2 + a*b)*tan(d*x + c)^3 + 315*a^2*tan(d*
x + c))/d
```

Fricas [A]
time = 1.53, size = 114, normalized size = 1.19

$$\frac{\left(8\left(21a^{2}-6ab+b^{2}\right)\cos\left(dx+c\right)^{8}+4\left(21a^{2}-6ab+b^{2}\right)\cos\left(dx+c\right)^{6}+3\left(21a^{2}-6ab+b^{2}\right)\cos\left(dx+c\right)^{4}+10\left(9ab-5b^{2}\right)\cos\left(dx+c\right)^{2}+35b^{2}\right)\sin\left(dx+c\right)}{315d\cos\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(21*a^2 - 6*a*b + b^2)*cos(d*x + c)^8 + 4*(21*a^2 - 6*a*b + b^2)*c
os(d*x + c)^6 + 3*(21*a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 10*(9*a*b - 5*b^2
)*cos(d*x + c)^2 + 35*b^2)*sin(d*x + c)/(d*cos(d*x + c)^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^{2}\left(c+dx\right)\right)^{2}\sec^{6}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**6, x)
```

Giac [A]
time = 0.83, size = 118, normalized size = 1.23

$$\frac{35b^{2}\tan\left(dx+c\right)^{9}+90ab\tan\left(dx+c\right)^{7}+90b^{2}\tan\left(dx+c\right)^{7}+63a^{2}\tan\left(dx+c\right)^{5}+252ab\tan\left(dx+c\right)^{5}+63b^{2}\tan\left(dx+c\right)^{5}+210a^{2}\tan\left(dx+c\right)^{3}+210ab\tan\left(dx+c\right)^{3}+315a^{2}\tan\left(dx+c\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(d*x + c)^9 + 90*a*b*tan(d*x + c)^7 + 90*b^2*tan(d*x + c)^
7 + 63*a^2*tan(d*x + c)^5 + 252*a*b*tan(d*x + c)^5 + 63*b^2*tan(d*x + c)^5
+ 210*a^2*tan(d*x + c)^3 + 210*a*b*tan(d*x + c)^3 + 315*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 12.22, size = 80, normalized size = 0.83

$$\frac{a^{2}\tan(c+d\,x)+\frac{b^{2}\tan(c+d\,x)^{9}}{9}+\tan(c+d\,x)^{5}\left(\frac{a^{2}}{5}+\frac{4\,a\,b}{5}+\frac{b^{2}}{5}\right)+\frac{2\,a\tan(c+d\,x)^{3}\,(a+b)}{3}+\frac{2\,b\tan(c+d\,x)^{7}\,(a+b)}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^6,x)
```

```
[Out] (a^2*tan(c + d*x) + (b^2*tan(c + d*x)^9)/9 + tan(c + d*x)^5*((4*a*b)/5 + a^
2/5 + b^2/5) + (2*a*tan(c + d*x)^3*(a + b))/3 + (2*b*tan(c + d*x)^7*(a + b)
)/7)/d
```

3.445 $\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=74

$$\frac{a^2\tan(c+dx)}{d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}$$

```
[Out] a^2*tan(d*x+c)/d+1/3*a*(a+2*b)*tan(d*x+c)^3/d+1/5*b*(2*a+b)*tan(d*x+c)^5/d+
1/7*b^2*tan(d*x+c)^7/d
```

Rubi [A]

time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {3756, 380}

$$\frac{a^2\tan(c+dx)}{d}+\frac{b(2a+b)\tan^5(c+dx)}{5d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*(a + 2*b)*Tan[c + d*x]^3)/(3*d) + (b*(2*a + b)*Ta
n[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^2\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^2+a(a+2b)x^2+b(2a+b)x^4+b^2x^6\right)\,dx,x,\tan(c+\right.}{d}\\ &= \frac{a^2\tan(c+dx)}{d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\tan^5(c+d}{5d}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 83, normalized size = 1.12

$$\frac{(70a^2-28ab+6b^2+(35a^2-14ab+3b^2)\sec^2(c+dx)+6(7a-4b)b\sec^4(c+dx)+15b^2\sec^6(c+dx))\tan(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((70*a^2 - 28*a*b + 6*b^2 + (35*a^2 - 14*a*b + 3*b^2)*Sec[c + d*x]^2 + 6*(7
*a - 4*b)*b*Sec[c + d*x]^4 + 15*b^2*Sec[c + d*x]^6)*Tan[c + d*x])/(105*d)
```

Maple [A]
time = 0.24, size = 111, normalized size = 1.50

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(dx+c)\right)}{35\cos(dx+c)^5}\right)+2ab\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{b^2\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(dx+c)\right)}{35\cos(dx+c)^5}\right)+2ab\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
risch	$4i(105a^2\mathrm{e}^{10i(dx+c)}-210ab\,\mathrm{e}^{10i(dx+c)}+105b^2\mathrm{e}^{10i(dx+c)}+455a^2\mathrm{e}^{8i(dx+c)}-350ab\,\mathrm{e}^{8i(dx+c)}-105b^2\mathrm{e}^{8i(dx+c)}+770a^2\mathrm{e}^{6i(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)+2*a
*b*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)-a^2*(-2/3
-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.30, size = 66, normalized size = 0.89

$$\frac{15\,b^2\tan\left(dx+c\right)^7+21\,(2\,ab+b^2)\tan\left(dx+c\right)^5+35\,(a^2+2\,ab)\tan\left(dx+c\right)^3+105\,a^2\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 21*(2*a*b + b^2)*tan(d*x + c)^5 + 35*(a^2 +
2*a*b)*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 1.26, size = 94, normalized size = 1.27

$$\frac{\left(2\,(35\,a^2-14\,ab+3\,b^2)\cos\left(dx+c\right)^6+(35\,a^2-14\,ab+3\,b^2)\cos\left(dx+c\right)^4+6\,(7\,ab-4\,b^2)\cos\left(dx+c\right)^2+15\,b^2\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(35*a^2 - 14*a*b + 3*b^2)*cos(d*x + c)^6 + (35*a^2 - 14*a*b + 3*b^
2)*cos(d*x + c)^4 + 6*(7*a*b - 4*b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c)
/(d*cos(d*x + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(c + dx\right)\right)^2 \sec^4\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**4, x)
```

Giac [A]
time = 0.80, size = 80, normalized size = 1.08

$$\frac{15\,b^2\tan\left(dx+c\right)^7 + 42\,ab\tan\left(dx+c\right)^5 + 21\,b^2\tan\left(dx+c\right)^5 + 35\,a^2\tan\left(dx+c\right)^3 + 70\,ab\tan\left(dx+c\right)^3 + 105\,a^2\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 42*a*b*tan(d*x + c)^5 + 21*b^2*tan(d*x + c)^
5 + 35*a^2*tan(d*x + c)^3 + 70*a*b*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 12.28, size = 60, normalized size = 0.81

$$\frac{a^2\tan(c+d\,x) + \frac{b^2\tan(c+d\,x)^7}{7} + \frac{a\tan(c+d\,x)^3\,(a+2\,b)}{3} + \frac{b\tan(c+d\,x)^5\,(2\,a+b)}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c + d*x) + (b^2*tan(c + d*x)^7)/7 + (a*tan(c + d*x)^3*(a + 2*b))/3
 + (b*tan(c + d*x)^5*(2*a + b))/5)/d
```

3.446 $\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] a^2*tan(d*x+c)/d+2/3*a*b*tan(d*x+c)^3/d+1/5*b^2*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 200}

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*b*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/
(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)^2\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^2+2abx^2+b^2x^4\right)\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 49, normalized size = 1.00

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (2*a*b*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)/
(5*d)
```

Maple [A]
time = 0.21, size = 57, normalized size = 1.16

method	result
derivativedivides	$\frac{\frac{b^2\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}+\frac{2ab\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+a^2\tan(dx+c)}{d}$
default	$\frac{\frac{b^2\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}+\frac{2ab\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+a^2\tan(dx+c)}{d}$
risch	$\frac{2i(15a^2e^{8i(dx+c)}-30ab\,e^{8i(dx+c)}+15b^2e^{8i(dx+c)}+60a^2e^{6i(dx+c)}-60ab\,e^{6i(dx+c)}+90a^2e^{4i(dx+c)}-40ab\,e^{4i(dx+c)}+30b^2}{15d\left(e^{2i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b^2*sin(d*x+c)^5/cos(d*x+c)^5+2/3*a*b*sin(d*x+c)^3/cos(d*x+c)^3+a^
2*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 42, normalized size = 0.86

$$\frac{3\,b^2\tan\left(dx+c\right)^5+10\,ab\tan\left(dx+c\right)^3+15\,a^2\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^2*tan(d*x + c)^5 + 10*a*b*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 1.60, size = 69, normalized size = 1.41

$$\frac{\left((15\,a^2-10\,ab+3\,b^2)\cos\left(dx+c\right)^4+2\,(5\,ab-3\,b^2)\cos\left(dx+c\right)^2+3\,b^2\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^2 - 10*a*b + 3*b^2)*cos(d*x + c)^4 + 2*(5*a*b - 3*b^2)*cos(d*x
+ c)^2 + 3*b^2)*sin(d*x + c)/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\tan^2\left(c + dx\right)\right)^2 \sec^2\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.77, size = 42, normalized size = 0.86

$$\frac{3\,b^2\tan\left(dx + c\right)^5 + 10\,ab\tan\left(dx + c\right)^3 + 15\,a^2\tan\left(dx + c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^2*tan(d*x + c)^5 + 10*a*b*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 12.13, size = 40, normalized size = 0.82

$$\frac{a^2\tan(c + d\,x) + \frac{2\,a\,b\tan(c+d\,x)^3}{3} + \frac{b^2\tan(c+d\,x)^5}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^2,x)
```

```
[Out] (a^2*tan(c + d*x) + (b^2*tan(c + d*x)^5)/5 + (2*a*b*tan(c + d*x)^3)/3)/d
```

3.447 $\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}(a-b)(a+3b)x+\frac{(a-b)^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] 1/2*(a-b)*(a+3*b)*x+1/2*(a-b)^2*cos(d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {3756, 398, 393, 209}

$$\frac{(a-b)^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(a+3b)(a-b)+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)*(a + 3*b)*x)/2 + ((a - b)^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^
2*Tan[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2-b^2+2(a-b)bx^2}{(1+x^2)^2}\right)\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{b^2\tan(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}+\frac{((a-b)(a+$$
$$= \frac{1}{2}(a-b)(a+3b)x+\frac{(a-b)^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c-}{d}$$

Mathematica [A]
time = 0.27, size = 55, normalized size = 1.00

$$\frac{2(a^2+2ab-3b^2)(c+dx)+(a-b)^2\sin(2(c+dx))+4b^2\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
[Out] (2*(a^2 + 2*a*b - 3*b^2)*(c + d*x) + (a - b)^2*Sin[2*(c + d*x)] + 4*b^2*Tan
[c + d*x])/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(51) = 102.
time = 0.19, size = 111, normalized size = 2.02

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2ab\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\left(\frac{\cos(dx+c)\sin}{2}\right.}{d}$
default	$\frac{b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2ab\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\left(\frac{\cos(dx+c)\sin}{2}\right.}{d}$

risch	$\frac{x a^2}{2} + xab - \frac{3x b^2}{2} - \frac{ie^{2i(dx+c)} a^2}{8d} + \frac{ie^{2i(dx+c)} ab}{4d} - \frac{ie^{2i(dx+c)} b^2}{8d} + \frac{ie^{-2i(dx+c)} a^2}{8d} - \frac{ie^{-2i(dx+c)} ab}{4d} + ie^{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+2*a*b*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*(1/2*co
s(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.52, size = 66, normalized size = 1.20

$$\frac{2\,b^2\tan\left(dx+c\right)+\left(a^2+2\,ab-3\,b^2\right)\left(dx+c\right)+\frac{\left(a^2-2\,ab+b^2\right)\tan\left(dx+c\right)}{\tan\left(dx+c\right)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*tan(d*x + c) + (a^2 + 2*a*b - 3*b^2)*(d*x + c) + (a^2 - 2*a*b +
b^2)*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 2.30, size = 69, normalized size = 1.25

$$\frac{\left(a^2+2\,ab-3\,b^2\right)dx\cos\left(dx+c\right)+\left(\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^2+2\,b^2\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 2*a*b - 3*b^2)*d*x*cos(d*x + c) + ((a^2 - 2*a*b + b^2)*cos(d*x
+ c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan^2\left(c+dx\right)\right)^2\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(51) = 102.
time = 0.86, size = 594, normalized size = 10.80

$$\frac{a^{2}dx\tan(dx)^{3}\tan(c)^{3}+2\,abdx\tan(dx)^{3}\tan(c)^{3}-3\,b^{2}dx\tan(dx)^{3}\tan(c)^{3}+a^{2}dx\tan(dx)^{3}\tan(c)+2\,abdx\tan(dx)^{3}\tan(c)-3\,b^{2}dx\tan(dx)^{3}\tan(c)-a^{2}dx\tan(dx)^{2}\tan(c)^{2}-2\,abdx\tan(dx)^{2}\tan(c)^{2}+3\,b^{2}dx\tan(dx)^{2}\tan(c)^{2}+a^{2}dx\tan(dx)\tan(c)^{3}+2\,abdx\tan(dx)\tan(c)^{3}-3\,b^{2}dx\tan(dx)\tan(c)^{3}-a^{2}\tan(dx)^{3}\tan(c)^{2}+2\,ab\tan(dx)^{3}\tan(c)^{2}-3\,b^{2}\tan(dx)^{3}\tan(c)^{2}-a^{2}\tan(dx)^{2}\tan(c)^{3}+2\,ab\tan(dx)^{2}\tan(c)^{3}-3\,b^{2}\tan(dx)^{2}\tan(c)^{3}-a^{2}dx\tan(dx)^{2}-2\,abdx\tan(dx)^{2}+3\,b^{2}dx\tan(dx)^{2}+a^{2}dx\tan(dx)\tan(c)+2\,abdx\tan(dx)\tan(c)-3\,b^{2}dx\tan(dx)\tan(c)-a^{2}dx\tan(c)^{2}-2\,abdx\tan(c)^{2}+3\,b^{2}dx\tan(c)^{2}-2\,b^{2}\tan(dx)^{3}+2\,a^{2}\tan(dx)^{2}\tan(c)-4\,ab\tan(dx)^{2}\tan(c)+2\,a^{2}\tan(dx)\tan(c)^{2}-4\,ab\tan(dx)\tan(c)^{2}-2\,b^{2}\tan(c)^{3}-a^{2}dx-2\,abdx+3\,b^{2}dx-a^{2}\tan(dx)+2\,ab\tan(dx)-3\,b^{2}\tan(dx)-a^{2}\tan(c)+2\,ab\tan(c)-3\,b^{2}\tan(c)}{2\,\left(d\tan(dx)^{3}\tan(c)^{3}+d\tan(dx)^{3}\tan(c)-d\tan(dx)^{2}\tan(c)^{2}+d\tan(dx)\tan(c)^{3}-d\tan(dx)^{2}+d\tan(dx)\tan(c)-d\tan(c)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^3*tan(c)^3 + 2*a*b*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*
x*tan(d*x)^3*tan(c)^3 + a^2*d*x*tan(d*x)^3*tan(c) + 2*a*b*d*x*tan(d*x)^3*ta
n(c) - 3*b^2*d*x*tan(d*x)^3*tan(c) - a^2*d*x*tan(d*x)^2*tan(c)^2 - 2*a*b*d*
x*tan(d*x)^2*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*tan(d*x)*ta
n(c)^3 + 2*a*b*d*x*tan(d*x)*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(c)^3 - a^2*ta
n(d*x)^3*tan(c)^2 + 2*a*b*tan(d*x)^3*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(c)^2 -
 a^2*tan(d*x)^2*tan(c)^3 + 2*a*b*tan(d*x)^2*tan(c)^3 - 3*b^2*tan(d*x)^2*tan
(c)^3 - a^2*d*x*tan(d*x)^2 - 2*a*b*d*x*tan(d*x)^2 + 3*b^2*d*x*tan(d*x)^2 +
a^2*d*x*tan(d*x)*tan(c) + 2*a*b*d*x*tan(d*x)*tan(c) - 3*b^2*d*x*tan(d*x)*ta
n(c) - a^2*d*x*tan(c)^2 - 2*a*b*d*x*tan(c)^2 + 3*b^2*d*x*tan(c)^2 - 2*b^2*t
an(d*x)^3 + 2*a^2*tan(d*x)^2*tan(c) - 4*a*b*tan(d*x)^2*tan(c) + 2*a^2*tan(d
*x)*tan(c)^2 - 4*a*b*tan(d*x)*tan(c)^2 - 2*b^2*tan(c)^3 - a^2*d*x - 2*a*b*d
*x + 3*b^2*d*x - a^2*tan(d*x) + 2*a*b*tan(d*x) - 3*b^2*tan(d*x) - a^2*tan(c
) + 2*a*b*tan(c) - 3*b^2*tan(c))/(d*tan(d*x)^3*tan(c)^3 + d*tan(d*x)^3*tan(
c) - d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d*x)^2 + d*tan(d*x
)*tan(c) - d*tan(c)^2 - d)
```

Mupad [B]
time = 12.25, size = 91, normalized size = 1.65

$$\frac{b^2\tan(c+d\,x)}{d}+\frac{\sin\left(2\,c+2\,d\,x\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{2\,d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)\,(a+3\,b)}{2\,\left(\frac{a^2}{2}+a\,b-\frac{3\,b^2}{2}\right)}\right)\,(a-b)\,(a+3\,b)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] (b^2*tan(c + d*x))/d + (sin(2*c + 2*d*x)*(a^2/2 - a*b + b^2/2))/(2*d) + (at
an((tan(c + d*x)*(a - b)*(a + 3*b))/(2*(a*b + a^2/2 - (3*b^2)/2)))*(a - b)*
(a + 3*b))/(2*d)
```

3.448 $\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=87

$$\frac{1}{8}\left(3a^2+2ab+3b^2\right)x+\frac{3(a^2-b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{(a-b)\cos^3(c+dx)\sin(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d}$$

```
[Out] 1/8*(3*a^2+2*a*b+3*b^2)*x+3/8*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)/d+1/4*(a-b)*c
os(d*x+c)^3*sin(d*x+c)*(a+b*tan(d*x+c)^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3756, 424, 393, 209}

$$\frac{3(a^2-b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2+2ab+3b^2\right)+\frac{(a-b)\sin(c+dx)\cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*x)/8 + (3*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + ((a - b)*Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x]^2))/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{(a-b)\cos^3(c+dx)\sin(c+dx)\left(a+b\tan^2(c+dx)\right)}{4d} + \frac{\text{Subst}\Big(\int$$

$$= \frac{3(a^2-b^2)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-b)\cos^3(c+dx)\sin(c+}{4d}$$

$$= \frac{1}{8}\left(3a^2+2ab+3b^2\right)x + \frac{3(a^2-b^2)\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a-}{}$$

Mathematica [A]
time = 0.21, size = 65, normalized size = 0.75

$$\frac{4(3a^2+2ab+3b^2)(c+dx)+8(a^2-b^2)\sin(2(c+dx))+(a-b)^2\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 + 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sin[2*(c + d*x)] + (a
- b)^2*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.23, size = 122, normalized size = 1.40

method	result
risch	$\frac{3x\,a^2}{8}+\frac{xab}{4}+\frac{3x\,b^2}{8}+\frac{\sin(4dx+4c)a^2}{32d}-\frac{\sin(4dx+4c)ab}{16d}+\frac{\sin(4dx+4c)b^2}{32d}+\frac{\sin(2dx+2c)a^2}{4d}-\frac{\sin(2dx+2c)b^2}{4d}$
derivativedivides	$\frac{b^2\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a^2\Big(}{d}$

default	$\frac{b^2\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a^2}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a*
b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+a^
2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.51, size = 97, normalized size = 1.11

$$\frac{(3a^2+2ab+3b^2)(dx+c)+\frac{(3a^2+2ab-5b^2)\tan(dx+c)^3+(5a^2-2ab-3b^2)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 2*a*b + 3*b^2)*(d*x + c) + ((3*a^2 + 2*a*b - 5*b^2)*tan(d*x +
 c)^3 + (5*a^2 - 2*a*b - 3*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x +
 c)^2 + 1))/d
```

Fricas [A]
time = 2.09, size = 75, normalized size = 0.86

$$\frac{(3a^2+2ab+3b^2)dx+\left(2(a^2-2ab+b^2)\cos(dx+c)^3+(3a^2+2ab-5b^2)\cos(dx+c)\right)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((3*a^2 + 2*a*b + 3*b^2)*d*x + (2*(a^2 - 2*a*b + b^2)*cos(d*x + c)^3 +
(3*a^2 + 2*a*b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tan^2(c+dx)\right)^2\cos^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3916 vs. 2(81) = 162.
time = 21.83, size = 3916, normalized size = 45.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/32*(3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 - 5*pi*b^2*s
gn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^
2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 12*a^2*d*x*tan(d*x)^4*tan(
c)^4 + 8*a*b*d*x*tan(d*x)^4*tan(c)^4 + 12*b^2*d*x*tan(d*x)^4*tan(c)^4 + 3*p
i*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c
))*tan(d*x)^4*tan(c)^4 - 5*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 6*pi*a*b*sgn(2*tan(d*x
)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*
x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 - 10*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 -
2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*
tan(d*x)^4*tan(c)^2 + 6*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*
x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c
)^4 - 10*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2
*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 6*a*b*arc
tan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 10*b^2
*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 6*
a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4
+ 10*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(
c)^4 + 24*a^2*d*x*tan(d*x)^4*tan(c)^2 + 16*a*b*d*x*tan(d*x)^4*tan(c)^2 + 24
*b^2*d*x*tan(d*x)^4*tan(c)^2 + 6*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 - 10*pi*b^2*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^
4*tan(c)^2 + 24*a^2*d*x*tan(d*x)^2*tan(c)^4 + 16*a*b*d*x*tan(d*x)^2*tan(c)^
4 + 24*b^2*d*x*tan(d*x)^2*tan(c)^4 + 6*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 - 10*pi*b^2*
sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan
(d*x)^2*tan(c)^4 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^
2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 - 5*pi*b
^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 12*pi*a*b*sgn(2*tan(d*x)^2*tan(
c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*t
an(c))*tan(d*x)^2*tan(c)^2 - 20*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-
2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)
^2*tan(c)^2 + 12*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(
d*x)^4*tan(c)^2 - 20*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*
```

```
tan(d*x)^4*tan(c)^2 - 12*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) +
 1))*tan(d*x)^4*tan(c)^2 + 20*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan
(c) + 1))*tan(d*x)^4*tan(c)^2 - 20*a^2*tan(d*x)^4*tan(c)^3 + 8*a*b*tan(d*x)
^4*tan(c)^3 + 12*b^2*tan(d*x)^4*tan(c)^3 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)
^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan
(c))*tan(c)^4 - 5*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 12*a*b*arct
an((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 20*b^2*
arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 12*
a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4
+ 20*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(
c)^4 - 20*a^2*tan(d*x)^3*tan(c)^4 + 8*a*b*tan(d*x)^3*tan(c)^4 + 12*b^2*tan(
d*x)^3*tan(c)^4 + 12*a^2*d*x*tan(d*x)^4 + 8*a*b*d*x*tan(d*x)^4 + 12*b^2*d*x
*tan(d*x)^4 + 3*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*t
an(d*x) - 2*tan(c))*tan(d*x)^4 - 5*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 48*a^2*d*x*tan(d*x)^2*t
an(c)^2 + 32*a*b*d*x*tan(d*x)^2*tan(c)^2 + 48*b^2*d*x*tan(d*x)^2*tan(c)^2 +
 12*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*
tan(c))*tan(d*x)^2*tan(c)^2 - 20*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 12*a^2*d*x*tan(c
)^4 + 8*a*b*d*x*tan(c)^4 + 12*b^2*d*x*tan(c)^4 + 3*pi*a*b*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 - 5*pi*b^2*
sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan
(c)^4 + 6*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 - 10*pi*b^2*sgn(2*t
an(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*
tan(d*x) - 2*tan(c))*tan(d*x)^2 + 6*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x
)*tan(c) - 1))*tan(d*x)^4 - 10*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan
(c) - 1))*tan(d*x)^4 - 6*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) +
 1))*tan(d*x)^4 + 10*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))
*tan(d*x)^4 - 12*a^2*tan(d*x)^4*tan(c) - 8*a*b*...
```

Mupad [B]
time = 12.24, size = 93, normalized size = 1.07

$$x\left(\frac{3\,a^2}{8}+\frac{a\,b}{4}+\frac{3\,b^2}{8}\right)-\frac{\tan(c+d\,x)\,\left(-\frac{5\,a^2}{8}+\frac{a\,b}{4}+\frac{3\,b^2}{8}\right)-\tan(c+d\,x)^3\,\left(\frac{3\,a^2}{8}+\frac{a\,b}{4}-\frac{5\,b^2}{8}\right)}{d\,\left({\tan\left(c+d\,x\right)}^4+2\,{\tan\left(c+d\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] x*((a*b)/4 + (3*a^2)/8 + (3*b^2)/8) - (tan(c + d*x)*((a*b)/4 - (5*a^2)/8 +
(3*b^2)/8) - tan(c + d*x)^3*((a*b)/4 + (3*a^2)/8 - (5*b^2)/8))/(d*(2*tan(c
+ d*x)^2 + tan(c + d*x)^4 + 1))
```

3.449 $\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=122

$$\frac{1}{16}\left(5a^2+2ab+b^2\right)x+\frac{\left(5a^2+2ab+b^2\right)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(a-b)(5a+3b)\cos^3(c+dx)\sin(c+dx)}{24d}+$$

```
[Out] 1/16*(5*a^2+2*a*b+b^2)*x+1/16*(5*a^2+2*a*b+b^2)*cos(d*x+c)*sin(d*x+c)/d+1/2
4*(a-b)*(5*a+3*b)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*(a-b)*cos(d*x+c)^5*sin(d*x+
c)*(a+b*tan(d*x+c)^2)/d
```

Rubi [A]

time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3756, 424, 393, 205, 209}

$$\frac{\left(5a^2+2ab+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(5a^2+2ab+b^2\right)+\frac{(a-b)(5a+3b)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(a-b)\sin(c+dx)\cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 2*a*b + b^2)*x)/16 + ((5*a^2 + 2*a*b + b^2)*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (
(a - b)*Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Tan[c + d*x]^2))/(6*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)\cos^5(c+dx)\sin(c+dx)\left(a+b\tan^2(c+dx)\right)}{6d} + \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{(a-b)(5a+3b)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{(a-b)\cos^5(c+dx)}{}$$
$$= \frac{(5a^2+2ab+b^2)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(a-b)(5a+3b)\cos^3}{24}$$
$$= \frac{1}{16}\left(5a^2+2ab+b^2\right)x + \frac{(5a^2+2ab+b^2)\cos(c+dx)\sin(c+dx)}{16d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 87, normalized size = 0.71

$$\frac{12((1-2i)a+b)((1+2i)a+b)(c+dx)+3(5a-b)(3a+b)\sin(2(c+dx))+3(a-b)(3a+b)\sin(4(c+dx))+(a-b)^2\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (12*((1 - 2*I)*a + b)*((1 + 2*I)*a + b)*(c + d*x) + 3*(5*a - b)*(3*a + b)*S
in[2*(c + d*x)] + 3*(a - b)*(3*a + b)*Sin[4*(c + d*x)] + (a - b)^2*Sin[6*(c
 + d*x)])/(192*d)
```

Maple [A]
time = 0.25, size = 166, normalized size = 1.36

method	result
derivativedivides	$\frac{b^{2}\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^{3}(dx+c)\right)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{6}+\right.}{d}$
default	$\frac{b^{2}\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^{3}(dx+c)\right)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{6}+\right.}{d}$
risch	$\frac{5x\,a^{2}}{16}+\frac{xab}{8}+\frac{x\,b^{2}}{16}+\frac{\sin(6dx+6c)a^{2}}{192d}-\frac{\sin(6dx+6c)ab}{96d}+\frac{\sin(6dx+6c)b^{2}}{192d}+\frac{3\sin(4dx+4c)a^{2}}{64d}-\frac{\sin(4dx+4c)ab}{32d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1
/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*(1/6*(cos
(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.53, size = 131, normalized size = 1.07

$$\frac{3\,(5\,a^{2}+2\,ab+b^{2})(dx+c)+\frac{3\,(5\,a^{2}+2\,ab+b^{2})\tan(dx+c)^{5}+8\,(5\,a^{2}+2\,ab-b^{2})\tan(dx+c)^{3}+3\,(11\,a^{2}-2\,ab-b^{2})\tan(dx+c)}{\tan(dx+c)^{6}+3\,\tan(dx+c)^{4}+3\,\tan(dx+c)^{2}+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^2 + 2*a*b + b^2)*(d*x + c) + (3*(5*a^2 + 2*a*b + b^2)*tan(d*x
+ c)^5 + 8*(5*a^2 + 2*a*b - b^2)*tan(d*x + c)^3 + 3*(11*a^2 - 2*a*b - b^2)*
tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 1.82, size = 98, normalized size = 0.80

$$\frac{3\,(5\,a^{2}+2\,ab+b^{2})dx+\left(8\,(a^{2}-2\,ab+b^{2})\cos(dx+c)^{5}+2\,(5\,a^{2}+2\,ab-7\,b^{2})\cos(dx+c)^{3}+3\,(5\,a^{2}+2\,ab+b^{2})\cos(dx+c)\right)\sin(dx+c)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a^2 + 2*a*b + b^2)*d*x + (8*(a^2 - 2*a*b + b^2)*cos(d*x + c)^5 +
 2*(5*a^2 + 2*a*b - 7*b^2)*cos(d*x + c)^3 + 3*(5*a^2 + 2*a*b + b^2)*cos(d*x
 + c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4487 vs. 2(114) = 228.
time = 22.43, size = 4487, normalized size = 36.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^6 + 15*a^2*d*x
*tan(d*x)^6*tan(c)^6 + 6*a*b*d*x*tan(d*x)^6*tan(c)^6 + 3*b^2*d*x*tan(d*x)^6
*tan(c)^6 + 3*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan
(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^6 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2
- 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)
)*tan(d*x)^6*tan(c)^4 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan
(c)^6 + 6*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6*
tan(c)^6 - 6*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x
)^6*tan(c)^6 + 45*a^2*d*x*tan(d*x)^6*tan(c)^4 + 18*a*b*d*x*tan(d*x)^6*tan(c
)^4 + 9*b^2*d*x*tan(d*x)^6*tan(c)^4 + 9*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2
*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan(c)^4 + 45*a^2*d*
x*tan(d*x)^4*tan(c)^6 + 18*a*b*d*x*tan(d*x)^4*tan(c)^6 + 9*b^2*d*x*tan(d*x)
^4*tan(c)^6 + 9*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*t
an(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^6 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^
2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(
c))*tan(d*x)^6*tan(c)^2 + 27*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*t
an(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*
tan(c)^4 + 18*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x
)^6*tan(c)^4 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*ta
n(d*x)^6*tan(c)^4 - 33*a^2*tan(d*x)^6*tan(c)^5 + 6*a*b*tan(d*x)^6*tan(c)^5
+ 3*b^2*tan(d*x)^6*tan(c)^5 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-
2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)
^2*tan(c)^6 + 18*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(
d*x)^4*tan(c)^6 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))
```

```
*tan(d*x)^4*tan(c)^6 - 33*a^2*tan(d*x)^5*tan(c)^6 + 6*a*b*tan(d*x)^5*tan(c)
^6 + 3*b^2*tan(d*x)^5*tan(c)^6 + 45*a^2*d*x*tan(d*x)^6*tan(c)^2 + 18*a*b*d*
x*tan(d*x)^6*tan(c)^2 + 9*b^2*d*x*tan(d*x)^6*tan(c)^2 + 9*pi*a*b*sgn(-2*tan
(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*ta
n(c)^2 + 135*a^2*d*x*tan(d*x)^4*tan(c)^4 + 54*a*b*d*x*tan(d*x)^4*tan(c)^4 +
 27*b^2*d*x*tan(d*x)^4*tan(c)^4 + 27*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*ta
n(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 45*a^2*d*x*t
an(d*x)^2*tan(c)^6 + 18*a*b*d*x*tan(d*x)^2*tan(c)^6 + 9*b^2*d*x*tan(d*x)^2*
tan(c)^6 + 9*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(
d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^6 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 -
 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))
*tan(d*x)^6 + 27*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 1
8*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^2
 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6*tan
(c)^2 - 40*a^2*tan(d*x)^6*tan(c)^3 - 16*a*b*tan(d*x)^6*tan(c)^3 + 8*b^2*tan
(d*x)^6*tan(c)^3 + 27*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)
^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^
4 + 54*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan
(c)^4 - 54*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^
4*tan(c)^4 + 45*a^2*tan(d*x)^5*tan(c)^4 - 78*a*b*tan(d*x)^5*tan(c)^4 + 9*b^
2*tan(d*x)^5*tan(c)^4 + 45*a^2*tan(d*x)^4*tan(c)^5 - 78*a*b*tan(d*x)^4*tan(
c)^5 + 9*b^2*tan(d*x)^4*tan(c)^5 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*
sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan
(c)^6 + 18*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2
*tan(c)^6 - 18*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d
*x)^2*tan(c)^6 - 40*a^2*tan(d*x)^3*tan(c)^6 - 16*a*b*tan(d*x)^3*tan(c)^6 +
8*b^2*tan(d*x)^3*tan(c)^6 + 15*a^2*d*x*tan(d*x)^6 + 6*a*b*d*x*tan(d*x)^6 +
3*b^2*d*x*tan(d*x)^6 + 3*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^6 + 135*a^2*d*x*tan(d*x)^4*tan(c)^2 +
 54*a*b*d*x*tan(d*x)^4*tan(c)^2 + 27*b^2*d*x*tan(d*x)^4*tan(c)^2 + 27*pi*a*
b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*t
an(d*x)^4*tan(c)^2 + 135*a^2*d*x*tan(d*x)^2*tan(c)^4 + 54*a*b*d*x*tan(d*x)^
2*tan(c)^4 + 27*b^2*d*x*tan(d*x)^2*tan(c)^4 + 27*pi*a*b*sgn(-2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 +
15*a^2*d*x*tan(c)^6 + 6*a*b*d*x*tan(c)^6 + 3*b^2*d*x*tan(c)^6 + 3*pi*a*b*sg
n(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c
)^6 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 6*a*b*arctan((tan(d
*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6 - 6*a*b*arctan(-(tan(d*x) -
 tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^6 - 15...
```

Mupad [B]

time = 13.24, size = 126, normalized size = 1.03

$$x\left(\frac{5\,a^2}{16}+\frac{a\,b}{8}+\frac{b^2}{16}\right)+\frac{\left(\frac{5\,a^2}{16}+\frac{a\,b}{8}+\frac{b^2}{16}\right)\,\tan(c+d\,x)^5+\left(\frac{5\,a^2}{6}+\frac{a\,b}{3}-\frac{b^2}{6}\right)\,\tan(c+d\,x)^3+\left(\frac{11\,a^2}{16}-\frac{a\,b}{8}-\frac{b^2}{16}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^6+3\tan\left(c+d\,x\right)^4+3\tan\left(c+d\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] x*((a*b)/8 + (5*a^2)/16 + b^2/16) + (tan(c + d*x)^3*((a*b)/3 + (5*a^2)/6 -
b^2/6) - tan(c + d*x)*((a*b)/8 - (11*a^2)/16 + b^2/16) + tan(c + d*x)^5*((a
*b)/8 + (5*a^2)/16 + b^2/16))/(d*(3*tan(c + d*x)^2 + 3*tan(c + d*x)^4 + tan
(c + d*x)^6 + 1))
```

3.450 $\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{(2a-3b)\tanh^{-1}(\sin(c+dx))}{2b^2d}+\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}+\frac{\sec(c+dx)\tan(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a-3*b)*arctanh(sin(d*x+c))/b^2/d+(a-b)^(3/2)*arctanh(sin(d*x+c)*(a-
b)^(1/2)/a^(1/2))/b^2/d/a^(1/2)+1/2*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3757, 425, 536, 212, 214}

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}-\frac{(2a-3b)\tanh^{-1}(\sin(c+dx))}{2b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -1/2*((2*a - 3*b)*ArcTanh[Sin[c + d*x]])/(b^2*d) + ((a - b)^(3/2)*ArcTanh[(
Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Sec[c + d*x]*Tan[c +
 d*x])/(2*b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-(a-b)x^2)}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{-a+2b+(-a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \sin(c+dx)\right)}{2bd}\\ &= \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{(2a-3b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{2b^2d} + \frac{(a-b)^2}{\cdots}\\ &= -\frac{(2a-3b)\tanh^{-1}(\sin(c+dx))}{2b^2d} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d} + \frac{\sec(\cdots}{\cdots}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 207 vs. 2(90) = 180.

time = 0.88, size = 207, normalized size = 2.30

$$\frac{2(2a-3b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(-2a+3b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{2(a-b)^{3/2}\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)}{\sqrt{a}}+\frac{2(a-b)^{3/2}\log\left(\sqrt{a}+\sqrt{a-b}\,\sin(c+dx)\right)}{\sqrt{a}}+\frac{b}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{b}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (2*(2*a - 3*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-2*a + 3*b)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*(a - b)^(3/2)*Log[Sqrt[a] - Sqr
```

```
t[a - b]*Sin[c + d*x]])/Sqrt[a] + (2*(a - b)^(3/2)*Log[Sqrt[a] + Sqrt[a - b
]*Sin[c + d*x]])/Sqrt[a] + b/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - b/(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(4*b^2*d)
```

Maple [A]
time = 0.41, size = 129, normalized size = 1.43

method	result
derivativedivides	$\frac{-\frac{1}{4b(\sin(dx+c)-1)}+\frac{(2a-3b)\ln(\sin(dx+c)-1)}{4b^2}-\frac{(-a^2+2ab-b^2)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{b^2\sqrt{a(a-b)}}-\frac{1}{4b(\sin(dx+c)+1)}+\frac{(-2a+3b)\ln(\sin}{4b^2}}{d}$
default	$\frac{-\frac{1}{4b(\sin(dx+c)-1)}+\frac{(2a-3b)\ln(\sin(dx+c)-1)}{4b^2}-\frac{(-a^2+2ab-b^2)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{b^2\sqrt{a(a-b)}}-\frac{1}{4b(\sin(dx+c)+1)}+\frac{(-2a+3b)\ln(\sin}{4b^2}}{d}$
risch	$-\frac{i(e^{3i(dx+c)}-e^{i(dx+c)})}{db(e^{2i(dx+c)}+1)^2}+\frac{\ln(e^{i(dx+c)}-i)a}{db^2}-\frac{3\ln(e^{i(dx+c)}-i)}{2db}-\frac{\ln(e^{i(dx+c)}+i)a}{db^2}+\frac{3\ln(e^{i(dx+c)}+i)}{2db}+\frac{\sqrt{a(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/b/(sin(d*x+c)-1)+1/4*(2*a-3*b)/b^2*ln(sin(d*x+c)-1)-1/b^2*(-a^2+2
*a*b-b^2)/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))-1/4/b/(
sin(d*x+c)+1)+1/4/b^2*(-2*a+3*b)*ln(sin(d*x+c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.84, size = 292, normalized size = 3.24

$$\left[-\frac{2\,(a-b)\sqrt{\frac{a-b}{a}}\cos\left(dx+c\right)^2\log\left(-\frac{(a-b)\cos(dx+c)^2+2\,a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)+(2\,a-3\,b)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(2\,a-3\,b)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)-2\,b\sin\left(dx+c\right)}{4\,b^2d\cos\left(dx+c\right)^2},\ -\frac{4\,(a-b)\sqrt{-\frac{a-b}{a}}\arctan\left(\sqrt{-\frac{a-b}{a}}\sin\left(dx+c\right)\right)\cos\left(dx+c\right)^2+(2\,a-3\,b)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(2\,a-3\,b)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)-2\,b\sin\left(dx+c\right)}{4\,b^2d\cos\left(dx+c\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a - b)*sqrt((a - b)/a)*cos(d*x + c)^2*log(-((a - b)*cos(d*x + c)^
2 + 2*a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b
)) + (2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - 3*b)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) - 2*b*sin(d*x + c))/(b^2*d*cos(d*x + c)^2),
-1/4*(4*(a - b)*sqrt(-(a - b)/a)*arctan(sqrt(-(a - b)/a)*sin(d*x + c))*cos(
d*x + c)^2 + (2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - 3*b)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*b*sin(d*x + c))/(b^2*d*cos(d*x +
 c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)**2), x)
```

Giac [A]
time = 0.65, size = 131, normalized size = 1.46

$$-\frac{\frac{(2a-3b)\log(|\sin(dx+c)+1|)}{b^2}-\frac{(2a-3b)\log(|\sin(dx+c)-1|)}{b^2}-\frac{4\left(a^2-2ab+b^2\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^2}+\frac{2\sin(dx+c)}{\left(\sin(dx+c)^2-1\right)b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/4*((2*a - 3*b)*log(abs(sin(d*x + c) + 1))/b^2 - (2*a - 3*b)*log(abs(sin(
d*x + c) - 1))/b^2 - 4*(a^2 - 2*a*b + b^2)*arctan(-(a*sin(d*x + c) - b*sin(
d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*b^2) + 2*sin(d*x + c)/((sin(d
*x + c)^2 - 1)*b))/d
```

Mupad [B]
time = 13.76, size = 268, normalized size = 2.98

$$-\frac{\left(\frac{\mathrm{atanh}\left(\frac{\sin(c+dx)\sqrt{a-b}}{\sqrt{a}}\right)(a-b)^{3/2}\,1\mathrm{i}}{2}-a^{3/2}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-a^{3/2}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(2c+2dx)+\frac{\cos(2c+2dx)\,\mathrm{atanh}\left(\frac{\sin(c+dx)\sqrt{a-b}}{\sqrt{a}}\right)(a-b)^{3/2}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{\sqrt{a}\,b^2\,d\left(\frac{\cos(2c+2dx)}{2}+\frac{1}{2}\right)}-\frac{\left(\frac{\sqrt{a}\,\sin(c+dx)\,1\mathrm{i}}{2}+\frac{3\sqrt{a}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}+\frac{3\sqrt{a}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(2c+2dx)}{2}\right)1\mathrm{i}}{\sqrt{a}\,b\,d\left(\frac{\cos(2c+2dx)}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x)^2)),x)

[Out] - (((atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))*(a - b)^(3/2)*1i)/2 - a^(3
/2)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)) - a^(3/2)*atan((sin(c/
2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) + (cos(2*c + 2*d*x)*a
tanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))*(a - b)^(3/2)*1i)/2)*1i)/(a^(1/2
)*b^2*d*(cos(2*c + 2*d*x)/2 + 1/2)) - (((a^(1/2)*sin(c + d*x)*1i)/2 + (3*a^
(1/2)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)))/2 + (3*a^(1/2)*atan
((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2)*1i)/(a^(1
/2)*b*d*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.451 $\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

```
[Out] arctanh(sin(d*x+c))/b/d-arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))*(a-b)^(1/2)
/b/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3757, 400, 212, 214}

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b*d) - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sin[c + d*x
])/Sqrt[a]])/(Sqrt[a]*b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{bd} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{bd}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 136 vs. 2(59) = 118.
time = 0.18, size = 136, normalized size = 2.31

$$\frac{-2\sqrt{a}\,\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{a}\,\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{a-b}\left(\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)-\log\left(\sqrt{a}+\sqrt{a-b}\,\sin(c+dx)\right)\right)}{2\sqrt{a}\,bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
[Out] (-2*Sqrt[a]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*Sqrt[a]*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + Sqrt[a - b]*(Log[Sqrt[a] - Sqrt[a - b]*Sin
[c + d*x]] - Log[Sqrt[a] + Sqrt[a - b]*Sin[c + d*x]]))/(2*Sqrt[a]*b*d)
```

Maple [A]
time = 0.30, size = 75, normalized size = 1.27

method	result
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2b}-\frac{(a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{b\sqrt{a(a-b)}}+\frac{\ln(\sin(dx+c)+1)}{2b}}{d}$
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2b}-\frac{(a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{b\sqrt{a(a-b)}}+\frac{\ln(\sin(dx+c)+1)}{2b}}{d}$

risch	$-\frac{\ln(\mathrm{e}^{i(dx+c)}-i)}{db}+\frac{\ln(\mathrm{e}^{i(dx+c)}+i)}{db}+\frac{\sqrt{a(a-b)}\,\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{2i\sqrt{a(a-b)}\,\mathrm{e}^{i(dx+c)}}{a-b}-1\right)}{2adb}-\frac{\sqrt{a(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/b*ln(sin(d*x+c)-1)-(a-b)/b/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+
c)/(a*(a-b))^(1/2))+1/2/b*ln(sin(d*x+c)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 1.88, size = 169, normalized size = 2.86

$$\left[\frac{\sqrt{\frac{a-b}{a}}\log\left(-\frac{(a-b)\cos(dx+c)^2+2a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+\log(\sin(dx+c)+1)-\log(-\sin(dx+c)+1)}{2bd},\ \frac{2\sqrt{-\frac{a-b}{a}}\arctan\left(\sqrt{-\frac{a-b}{a}}\sin(dx+c)\right)+\log(\sin(dx+c)+1)-\log(-\sin(dx+c)+1)}{2bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt((a - b)/a)*log(-((a - b)*cos(d*x + c)^2 + 2*a*sqrt((a - b)/a)*si
n(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + log(sin(d*x + c) + 1)
 - log(-sin(d*x + c) + 1))/(b*d), 1/2*(2*sqrt(-(a - b)/a)*arctan(sqrt(-(a -
 b)/a)*sin(d*x + c)) + log(sin(d*x + c) + 1) - log(-sin(d*x + c) + 1))/(b*d
)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)**2), x)
```

Giac [A]
time = 0.64, size = 88, normalized size = 1.49

$$-\frac{\frac{2\,(a-b)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b}-\frac{\log(|\sin(dx+c)+1|)}{b}+\frac{\log(|\sin(dx+c)-1|)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a - b)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))
/(sqrt(-a^2 + a*b)*b) - log(abs(sin(d*x + c) + 1))/b + log(abs(sin(d*x + c)
 - 1))/b)/d
```

Mupad [B]
time = 12.67, size = 67, normalized size = 1.14

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}-\frac{\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,\sqrt{a-b}}{\sqrt{a}\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] (2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) - (atanh((sin(c + d*
x)*(a - b)^(1/2))/a^(1/2))*(a - b)^(1/2))/(a^(1/2)*b*d)
```

3.452 $\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}\,d}$$

```
[Out] arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/d/a^(1/2)/(a-b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3757, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a-b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Maple [A]
time = 0.24, size = 36, normalized size = 0.90

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{d\sqrt{a(a-b)}}$	36
default	$\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{d\sqrt{a(a-b)}}$	36
risch	$\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,d}-\frac{\ln\left(e^{2i(dx+c)}-\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,d}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.41, size = 122, normalized size = 3.05

$$\left[\frac{\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)}{2\sqrt{a^2-ab}\,d}, -\frac{\sqrt{-a^2+ab}\,\arctan\left(\frac{\sqrt{-a^2+ab}\,\sin(dx+c)}{a}\right)}{(a^2-ab)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a +
b)/((a - b)*cos(d*x + c)^2 + b))/(sqrt(a^2 - a*b)*d), -sqrt(-a^2 + a*b)*arc
tan(sqrt(-a^2 + a*b)*sin(d*x + c)/a)/((a^2 - a*b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x)**2), x)
```

Giac [A]
time = 0.62, size = 47, normalized size = 1.18

$$-\frac{\arctan\left(\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -arctan((a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*
b)*d)
```

Mupad [B]
time = 12.57, size = 32, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)}{\sqrt{a}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))/(a^(1/2)*d*(a - b)^(1/2))
```

3.453 $\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{3/2}d}+\frac{\sin(c+dx)}{(a-b)d}$$

```
[Out] sin(d*x+c)/(a-b)/d-b*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/(a-b)^(3/2)/d/
a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3757, 396, 214}

$$\frac{\sin(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d)
) + Sin[c + d*x]/((a - b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sin(c+dx)}{(a-b)d} - \frac{b\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{(a-b)d}$$
$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{3/2}d} + \frac{\sin(c+dx)}{(a-b)d}$$

Mathematica [A]
time = 0.16, size = 89, normalized size = 1.48

$$\frac{\frac{b\left(\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)-\log\left(\sqrt{a}+\sqrt{a-b}\,\sin(c+dx)\right)\right)}{\sqrt{a}\,(a-b)^{3/2}}+\frac{2\sin(c+dx)}{a-b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]^2),x]
[Out] ((b*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a] + Sqrt[a - b]*Si
n[c + d*x]]))/(Sqrt[a]*(a - b)^(3/2)) + (2*Sin[c + d*x])/(a - b))/(2*d)
```

Maple [A]
time = 0.32, size = 61, normalized size = 1.02

method	result	size
derivativedivides	$\frac{\frac{\sin(dx+c)}{a-b}-\frac{b\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)\sqrt{a(a-b)}}}{d}$	61
default	$\frac{\frac{\sin(dx+c)}{a-b}-\frac{b\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)\sqrt{a(a-b)}}}{d}$	61
risch	$-\frac{i\mathrm{e}^{i(dx+c)}}{2(a-b)d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2(a-b)d}+\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{2ia\,\mathrm{e}^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,(a-b)d}-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\,\mathrm{e}^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,(a-b)d}$	162

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a-b)*sin(d*x+c)-1/(a-b)*b/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/
(a*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.56, size = 182, normalized size = 3.03

$$\left[-\frac{\sqrt{a^2-ab}\,b\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\,(a^2-ab)\sin\left(dx+c\right)}{2\,(a^3-2\,a^2b+ab^2)d},\ \frac{\sqrt{-a^2+ab}\,b\arctan\left(\frac{\sqrt{-a^2+ab}\,\sin(dx+c)}{a}\right)+(a^2-ab)\sin\left(dx+c\right)}{(a^3-2\,a^2b+ab^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - a*b)*b*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*s
in(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(a^2 - a*b)*sin(d*
x + c))/((a^3 - 2*a^2*b + a*b^2)*d), (sqrt(-a^2 + a*b)*b*arctan(sqrt(-a^2 +
 a*b)*sin(d*x + c)/a) + (a^2 - a*b)*sin(d*x + c))/((a^3 - 2*a^2*b + a*b^2)*
d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{a+b\tan^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x)**2), x)
```

Giac [A]
time = 0.64, size = 73, normalized size = 1.22

$$-\frac{\frac{b\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,(a-b)}-\frac{\sin(dx+c)}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*(a - b)) - sin(d*x + c)/(a - b))/d
```

Mupad [B]
time = 12.32, size = 61, normalized size = 1.02

$$\frac{\sin\left(c+d\,x\right)}{d\,\left(a-b\right)}+\frac{b\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,(a-b)^{3/2}}{\sqrt{a}\,b-a^{3/2}}\right)}{\sqrt{a}\,d\,\left(a-b\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x)^2),x)
```

```
[Out] sin(c + d*x)/(d*(a - b)) + (b*atanh((sin(c + d*x)*(a - b)^(3/2))/(a^(1/2)*b
 - a^(3/2))))/(a^(1/2)*d*(a - b)^(3/2))
```

3.454 $\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2}d}+\frac{(a-2b)\sin(c+dx)}{(a-b)^2d}-\frac{\sin^3(c+dx)}{3(a-b)d}$$

```
[Out] (a-2*b)*sin(d*x+c)/(a-b)^2/d-1/3*sin(d*x+c)^3/(a-b)/d+b^2*arctanh(sin(d*x+c
)*(a-b)^(1/2)/a^(1/2))/(a-b)^(5/2)/d/a^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3757, 398, 214}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{5/2}}-\frac{\sin^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\sin(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d)
 + ((a - 2*b)*Sin[c + d*x])/((a - b)^2*d) - Sin[c + d*x]^3/(3*(a - b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{(a-b)^2}-\frac{x^2}{a-b}+\frac{b^2}{(a-b)^2(a-(a-b)x^2)}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{(a-2b)\sin(c+dx)}{(a-b)^2d}-\frac{\sin^3(c+dx)}{3(a-b)d}+\frac{b^2\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{(a-b)^2d}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2}d}+\frac{(a-2b)\sin(c+dx)}{(a-b)^2d}-\frac{\sin^3(c+dx)}{3(a-b)d}$$

Mathematica [A]
time = 0.32, size = 115, normalized size = 1.31

$$\frac{\frac{6b^2\left(-\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)+\log\left(\sqrt{a}+\sqrt{a-b}\,\sin(c+dx)\right)\right)}{\sqrt{a}\,(a-b)^{5/2}}+\frac{3(3a-7b)\sin(c+dx)}{(a-b)^2}+\frac{\sin(3(c+dx))}{a-b}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((6*b^2*(-Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] + Log[Sqrt[a] + Sqrt[a -
b]*Sin[c + d*x]]))/(Sqrt[a]*(a - b)^(5/2)) + (3*(3*a - 7*b)*Sin[c + d*x])/(
a - b)^2 + Sin[3*(c + d*x)]/(a - b))/(12*d)
```

Maple [A]
time = 0.36, size = 98, normalized size = 1.11

method	result
derivativedivides	$\frac{-\frac{\frac{a\left(\sin^3(dx+c)\right)}{3}-\frac{b\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)a+2\sin(dx+c)b}{(a-b)^2}+\frac{b^2\,\text{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)^2\sqrt{a(a-b)}}}{d}$
default	$\frac{-\frac{\frac{a\left(\sin^3(dx+c)\right)}{3}-\frac{b\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)a+2\sin(dx+c)b}{(a-b)^2}+\frac{b^2\,\text{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)^2\sqrt{a(a-b)}}}{d}$

risch	$-\frac{3i\mathrm{e}^{i(dx+c)}a}{8(a-b)^2d}+\frac{7i\mathrm{e}^{i(dx+c)}b}{8(a-b)^2d}+\frac{3i\mathrm{e}^{-i(dx+c)}a}{8(a-b)^2d}-\frac{7i\mathrm{e}^{-i(dx+c)}b}{8(a-b)^2d}+\frac{b^2\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\,\mathrm{e}^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,(a-b)^2d}-\frac{b^2\ln\left(\mathrm{e}^{2i\right.}}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a-b)^2*(1/3*a*sin(d*x+c)^3-1/3*b*sin(d*x+c)^3-sin(d*x+c)*a+2*sin(d
*x+c)*b)+b^2/(a-b)^2/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/
2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 2.06, size = 276, normalized size = 3.14

$$\left[\frac{3\sqrt{a^2-ab}\,b^2\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(2a^3-7a^2b+5ab^2+(a^3-2a^2b+ab^2)\cos(dx+c)^2\right)\sin(dx+c)}{6(a^4-3a^3b+3a^2b^2-ab^3)d},\ -\frac{3\sqrt{-a^2+ab}\,b^2\arctan\left(\frac{\sqrt{-a^2+ab}\,\sin(dx+c)}{a}\right)-\left(2a^3-7a^2b+5ab^2+(a^3-2a^2b+ab^2)\cos(dx+c)^2\right)\sin(dx+c)}{3(a^4-3a^3b+3a^2b^2-ab^3)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 - a*b)*b^2*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b
)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + 2*(2*a^3 - 7*a^2*
b + 5*a*b^2 + (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 -
 3*a^3*b + 3*a^2*b^2 - a*b^3)*d), -1/3*(3*sqrt(-a^2 + a*b)*b^2*arctan(sqrt(
-a^2 + a*b)*sin(d*x + c)/a) - (2*a^3 - 7*a^2*b + 5*a*b^2 + (a^3 - 2*a^2*b +
 a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*
d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(78) = 156.
time = 0.64, size = 161, normalized size = 1.83

$$\frac{\frac{3\,b^2\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{-a^2+ab}}-\frac{a^2\sin(dx+c)^3-2\,ab\sin(dx+c)^3+b^2\sin(dx+c)^3-3\,a^2\sin(dx+c)+9\,ab\sin(dx+c)-6\,b^2\sin(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/((a^
2 - 2*a*b + b^2)*sqrt(-a^2 + a*b)) - (a^2*sin(d*x + c)^3 - 2*a*b*sin(d*x +
c)^3 + b^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c) + 9*a*b*sin(d*x + c) - 6*b^2
*sin(d*x + c))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

Mupad [B]
time = 15.44, size = 251, normalized size = 2.85

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a-2\,b)}{a^2-2\,a\,b+b^2}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a-4\,b)}{3\,(a^2-2\,a\,b+b^2)}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(a-2\,b)}{a^2-2\,a\,b+b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{b^2\,\mathrm{atan}\left(\frac{2\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+6\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2-2\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{\sqrt{a}\,(a-b)^{5/2}\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}\right)\,1\mathrm{i}}{\sqrt{a}\,d\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)*(a - 2*b))/(a^2 - 2*a*b + b^2) + (4*tan(c/2 + (d*x)/
2)^3*(a - 4*b))/(3*(a^2 - 2*a*b + b^2)) + (2*tan(c/2 + (d*x)/2)^5*(a - 2*b)
)/(a^2 - 2*a*b + b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4
+ tan(c/2 + (d*x)/2)^6 + 1)) - (b^2*atan((a^3*tan(c/2 + (d*x)/2)*2i - b^3*t
an(c/2 + (d*x)/2)*2i + a*b^2*tan(c/2 + (d*x)/2)*6i - a^2*b*tan(c/2 + (d*x)/
2)*6i)/(a^(1/2)*(a - b)^(5/2)*(tan(c/2 + (d*x)/2)^2 + 1)))*1i)/(a^(1/2)*d*(
a - b)^(5/2))
```

3.455 $\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{7/2}d}+\frac{(a^2-3ab+3b^2)\sin(c+dx)}{(a-b)^3d}-\frac{(2a-3b)\sin^3(c+dx)}{3(a-b)^2d}+\frac{\sin^5(c+dx)}{5(a-b)d}$$

```
[Out] (a^2-3*a*b+3*b^2)*sin(d*x+c)/(a-b)^3/d-1/3*(2*a-3*b)*sin(d*x+c)^3/(a-b)^2/d
+1/5*sin(d*x+c)^5/(a-b)/d-b^3*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/(a-b)
^(7/2)/d/a^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3757, 398, 214}

$$\frac{(a^2-3ab+3b^2)\sin(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{7/2}}+\frac{\sin^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\sin^3(c+dx)}{3d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2)*
d)) + ((a^2 - 3*a*b + 3*b^2)*Sin[c + d*x])/((a - b)^3*d) - ((2*a - 3*b)*Sin
[c + d*x]^3)/(3*(a - b)^2*d) + Sin[c + d*x]^5/(5*(a - b)*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-(a-b)x^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{(a-b)^3} - \frac{(2a-3b)x^2}{(a-b)^2} + \frac{x^4}{a-b} - \frac{b^3}{(a-b)^3(a-(a-b)x^2)}\right)\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{(a^2-3ab+3b^2)\sin(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\sin^3(c+dx)}{3(a-b)^2 d} + \frac{\sin^5(c+dx)}{5(a-b)d} - \frac{b^3\text{Subs}}{}$$

$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{7/2}d} + \frac{(a^2-3ab+3b^2)\sin(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\sin^3}{3(a-b}$$

Mathematica [A]
time = 1.17, size = 148, normalized size = 1.17

$$\frac{\frac{120b^3\left(\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)-\log\left(\sqrt{a}+\sqrt{a-b}\,\sin(c+dx)\right)\right)}{\sqrt{a}\,(a-b)^{7/2}} + \frac{30(5a^2-16ab+19b^2)\sin(c+dx)}{(a-b)^3} + \frac{5(5a-9b)\sin(3(c+dx))}{(a-b)^2} + \frac{3\sin(5(c+dx))}{a-b}}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
[Out] ((120*b^3*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a] + Sqrt[a -
 b]*Sin[c + d*x]]))/(Sqrt[a]*(a - b)^(7/2)) + (30*(5*a^2 - 16*a*b + 19*b^2)
*Sin[c + d*x])/(a - b)^3 + (5*(5*a - 9*b)*Sin[3*(c + d*x)])/(a - b)^2 + (3*
Sin[5*(c + d*x)])/(a - b))/(240*d)
```

Maple [A]
time = 0.43, size = 165, normalized size = 1.31

method	result
derivativedivides	$$\frac{\frac{\frac{a^2(\sin^5(dx+c))}{5} - \frac{2ab(\sin^5(dx+c))}{5} + \frac{b^2(\sin^5(dx+c))}{5} - \frac{2a^2(\sin^3(dx+c))}{3} + \frac{5ab(\sin^3(dx+c))}{3} - b^2(\sin^3(dx+c)) + a^2\sin(dx+c) - 3ab}{(a-b)^3}}{d}$$

default	$\frac{\frac{\frac{a^2\left(\sin^5(dx+c)\right)}{5}-\frac{2ab\left(\sin^5(dx+c)\right)}{5}+\frac{b^2\left(\sin^5(dx+c)\right)}{5}-\frac{2a^2\left(\sin^3(dx+c)\right)}{3}+\frac{5ab\left(\sin^3(dx+c)\right)}{3}-b^2\left(\sin^3(dx+c)\right)+a^2\sin(dx+c)-3ab\sin}{(a-b)^3}}{d}$
risch	$-\frac{5ie^{i(dx+c)}a^2}{16(a-b)^3d}+\frac{ie^{i(dx+c)}ab}{(a-b)^3d}-\frac{19ie^{i(dx+c)}b^2}{16(a-b)^3d}+\frac{5ie^{-i(dx+c)}a^2}{16(a-b)(a^2-2ab+b^2)d}-\frac{ie^{-i(dx+c)}ab}{(a-b)(a^2-2ab+b^2)d}+\frac{19ie^{-i(dx}}{16(a-b)(a^2-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a-b)^3*(1/5*a^2*sin(d*x+c)^5-2/5*a*b*sin(d*x+c)^5+1/5*b^2*sin(d*x+c
)^5-2/3*a^2*sin(d*x+c)^3+5/3*a*b*sin(d*x+c)^3-b^2*sin(d*x+c)^3+a^2*sin(d*x+
c)-3*a*b*sin(d*x+c)+3*b^2*sin(d*x+c))-b^3/(a-b)^3/(a*(a-b))^(1/2)*arctanh((
a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.38, size = 395, normalized size = 3.13

$$\left[-\frac{15\sqrt{a^2-ab}\,b^3\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\left(3\left(a^4-3a^3b+3a^2b^2-ab^3\right)\cos(dx+c)^4+8a^4-34a^3b+59a^2b^2-33ab^3+\left(4a^4-17a^3b+22a^2b^2-9ab^3\right)\cos(dx+c)^2\right)\sin(dx+c)}{30\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4\right)d},\ \frac{15\sqrt{-a^2+ab}\,b^3\arctan\left(\frac{\sqrt{-a^2+ab}\sin(dx+c)}{a}\right)+\left(3\left(a^4-3a^3b+3a^2b^2-ab^3\right)\cos(dx+c)^4+8a^4-34a^3b+59a^2b^2-33ab^3+\left(4a^4-17a^3b+22a^2b^2-9ab^3\right)\cos(dx+c)^2\right)\sin(dx+c)}{15\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(a^2 - a*b)*b^3*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 -
a*b)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(3*(a^4 - 3*
a^3*b + 3*a^2*b^2 - a*b^3)*cos(d*x + c)^4 + 8*a^4 - 34*a^3*b + 59*a^2*b^2 -
 33*a*b^3 + (4*a^4 - 17*a^3*b + 22*a^2*b^2 - 9*a*b^3)*cos(d*x + c)^2)*sin(d
*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d), 1/15*(15*sqrt
(-a^2 + a*b)*b^3*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (3*(a^4 - 3*a^3*
b + 3*a^2*b^2 - a*b^3)*cos(d*x + c)^4 + 8*a^4 - 34*a^3*b + 59*a^2*b^2 - 33*
```

```
a*b^3 + (4*a^4 - 17*a^3*b + 22*a^2*b^2 - 9*a*b^3)*cos(d*x + c)^2)*sin(d*x +
 c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(114) = 228.
time = 0.66, size = 319, normalized size = 2.53

$$-\frac{\frac{15\,b^{3}\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^{2}+ab}}\right)}{(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3})\sqrt{-a^{2}+ab}}-\frac{3\,a^{4}\sin(dx+c)^{5}-12\,a^{3}b\sin(dx+c)^{5}+18\,a^{2}b^{2}\sin(dx+c)^{5}-12\,ab^{3}\sin(dx+c)^{5}+3\,b^{4}\sin(dx+c)^{5}-10\,a^{4}\sin(dx+c)^{3}+45\,a^{3}b\sin(dx+c)^{3}-75\,a^{2}b^{2}\sin(dx+c)^{3}+55\,ab^{3}\sin(dx+c)^{3}-15\,b^{4}\sin(dx+c)^{3}+15\,a^{4}\sin(dx+c)-75\,a^{3}b\sin(dx+c)+150\,a^{2}b^{2}\sin(dx+c)-135\,ab^{3}\sin(dx+c)+45\,b^{4}\sin(dx+c)}{a^{5}-5\,a^{4}b+10\,a^{3}b^{2}-10\,a^{2}b^{3}+5\,ab^{4}-b^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^3*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a^2 + a*b)) - (3*a^4*sin(d*x + c)^5 -
 12*a^3*b*sin(d*x + c)^5 + 18*a^2*b^2*sin(d*x + c)^5 - 12*a*b^3*sin(d*x + c
)^5 + 3*b^4*sin(d*x + c)^5 - 10*a^4*sin(d*x + c)^3 + 45*a^3*b*sin(d*x + c)^
3 - 75*a^2*b^2*sin(d*x + c)^3 + 55*a*b^3*sin(d*x + c)^3 - 15*b^4*sin(d*x +
c)^3 + 15*a^4*sin(d*x + c) - 75*a^3*b*sin(d*x + c) + 150*a^2*b^2*sin(d*x +
c) - 135*a*b^3*sin(d*x + c) + 45*b^4*sin(d*x + c))/(a^5 - 5*a^4*b + 10*a^3*
b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5))/d
```

Mupad [**B**]
time = 15.08, size = 1493, normalized size = 11.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)*(a^2 - 3*a*b + 3*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^
3) + (tan(c/2 + (d*x)/2)^9*(2*a^2 - 6*a*b + 6*b^2))/(3*a*b^2 - 3*a^2*b + a^
3 - b^3) + (tan(c/2 + (d*x)/2)^3*((8*a^2)/3 - (32*a*b)/3 + 16*b^2))/(3*a*b^
2 - 3*a^2*b + a^3 - b^3) + (tan(c/2 + (d*x)/2)^7*((8*a^2)/3 - (32*a*b)/3 +
16*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(c/2 + (d*x)/2)^5*((116*a^2)
/15 - (332*a*b)/15 + (132*b^2)/5))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(d*(5*t
```

```
an(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5
*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (b^3*atan(((b^3*((tan
(c/2 + (d*x)/2)*(16*a*b^10 - 96*a^2*b^9 + 240*a^3*b^8 - 320*a^4*b^7 + 240*a
^5*b^6 - 96*a^6*b^5 + 16*a^7*b^4))/2 + (b^3*(tan(c/2 + (d*x)/2)^2*(4*a^12 -
 44*a^11*b + 8*a^2*b^10 - 76*a^3*b^9 + 324*a^4*b^8 - 816*a^5*b^7 + 1344*a^6
*b^6 - 1512*a^7*b^5 + 1176*a^8*b^4 - 624*a^9*b^3 + 216*a^10*b^2) + 36*a^11*
b - 4*a^12 + 4*a^3*b^9 - 36*a^4*b^8 + 144*a^5*b^7 - 336*a^6*b^6 + 504*a^7*b
^5 - 504*a^8*b^4 + 336*a^9*b^3 - 144*a^10*b^2))/(a^(1/2)*(a - b)^(7/2)))*1i
)/(a^(1/2)*(a - b)^(7/2)) + (b^3*((tan(c/2 + (d*x)/2)*(16*a*b^10 - 96*a^2*b
^9 + 240*a^3*b^8 - 320*a^4*b^7 + 240*a^5*b^6 - 96*a^6*b^5 + 16*a^7*b^4))/2
- (b^3*(tan(c/2 + (d*x)/2)^2*(4*a^12 - 44*a^11*b + 8*a^2*b^10 - 76*a^3*b^9
+ 324*a^4*b^8 - 816*a^5*b^7 + 1344*a^6*b^6 - 1512*a^7*b^5 + 1176*a^8*b^4 -
624*a^9*b^3 + 216*a^10*b^2) + 36*a^11*b - 4*a^12 + 4*a^3*b^9 - 36*a^4*b^8 +
 144*a^5*b^7 - 336*a^6*b^6 + 504*a^7*b^5 - 504*a^8*b^4 + 336*a^9*b^3 - 144*
a^10*b^2))/(a^(1/2)*(a - b)^(7/2)))*1i)/(a^(1/2)*(a - b)^(7/2)))/(tan(c/2 +
 (d*x)/2)^2*(8*a*b^9 - 24*a^2*b^8 + 24*a^3*b^7 - 8*a^4*b^6) - 8*a*b^9 + 24*
a^2*b^8 - 24*a^3*b^7 + 8*a^4*b^6 + (b^3*((tan(c/2 + (d*x)/2)*(16*a*b^10 - 9
6*a^2*b^9 + 240*a^3*b^8 - 320*a^4*b^7 + 240*a^5*b^6 - 96*a^6*b^5 + 16*a^7*b
^4))/2 + (b^3*(tan(c/2 + (d*x)/2)^2*(4*a^12 - 44*a^11*b + 8*a^2*b^10 - 76*a
^3*b^9 + 324*a^4*b^8 - 816*a^5*b^7 + 1344*a^6*b^6 - 1512*a^7*b^5 + 1176*a^8
*b^4 - 624*a^9*b^3 + 216*a^10*b^2) + 36*a^11*b - 4*a^12 + 4*a^3*b^9 - 36*a^
4*b^8 + 144*a^5*b^7 - 336*a^6*b^6 + 504*a^7*b^5 - 504*a^8*b^4 + 336*a^9*b^3
 - 144*a^10*b^2))/(a^(1/2)*(a - b)^(7/2))))/(a^(1/2)*(a - b)^(7/2)) - (b^3*
((tan(c/2 + (d*x)/2)*(16*a*b^10 - 96*a^2*b^9 + 240*a^3*b^8 - 320*a^4*b^7 +
240*a^5*b^6 - 96*a^6*b^5 + 16*a^7*b^4))/2 - (b^3*(tan(c/2 + (d*x)/2)^2*(4*a
^12 - 44*a^11*b + 8*a^2*b^10 - 76*a^3*b^9 + 324*a^4*b^8 - 816*a^5*b^7 + 134
4*a^6*b^6 - 1512*a^7*b^5 + 1176*a^8*b^4 - 624*a^9*b^3 + 216*a^10*b^2) + 36*
a^11*b - 4*a^12 + 4*a^3*b^9 - 36*a^4*b^8 + 144*a^5*b^7 - 336*a^6*b^6 + 504*
a^7*b^5 - 504*a^8*b^4 + 336*a^9*b^3 - 144*a^10*b^2))/(a^(1/2)*(a - b)^(7/2)
)))/(a^(1/2)*(a - b)^(7/2))))*1i)/(a^(1/2)*d*(a - b)^(7/2))
```

3.456 $\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(a-b)^3\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d}+\frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d}-\frac{(a-3b)\tan^3(c+dx)}{3b^2d}+\frac{\tan^5(c+dx)}{5bd}$$

```
[Out] -(a-b)^3*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/b^(7/2)/d/a^(1/2)+(a^2-3*a*b+3*
b^2)*tan(d*x+c)/b^3/d-1/3*(a-3*b)*tan(d*x+c)^3/b^2/d+1/5*tan(d*x+c)^5/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3756, 398, 211}

$$\frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d}-\frac{(a-b)^3\mathrm{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d}-\frac{(a-3b)\tan^3(c+dx)}{3b^2d}+\frac{\tan^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d)) +
 ((a^2 - 3*a*b + 3*b^2)*Tan[c + d*x])/(b^3*d) - ((a - 3*b)*Tan[c + d*x]^3)/
(3*b^2*d) + Tan[c + d*x]^5/(5*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
```

```
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{b^3} - \frac{(a-3b)x^2}{b^2} + \frac{x^4}{b} + \frac{-a^3+3a^2b-3ab^2+b^3}{b^3(a+bx^2)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d} - \frac{(a-3b)\tan^3(c+dx)}{3b^2d} + \frac{\tan^5(c+dx)}{5bd} - \frac{(a-b)^3 \cdots}{\cdots}$$

$$= -\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d} + \frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d} - \frac{(a-3b)\tan^3 \cdots}{3b^2d}$$

Mathematica [A]
time = 0.59, size = 103, normalized size = 0.95

$$\frac{-\frac{15(a-b)^3\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} + \sqrt{b}\,(15a^2-40ab+33b^2-(5a-9b)b\sec^2(c+dx)+3b^2\sec^4(c+dx))\tan(c+dx)}{15b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-15*(a - b)^3*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a] + Sqrt[b]*(
15*a^2 - 40*a*b + 33*b^2 - (5*a - 9*b)*b*Sec[c + d*x]^2 + 3*b^2*Sec[c + d*x
]^4)*Tan[c + d*x])/(15*b^(7/2)*d)
```

Maple [A]
time = 0.40, size = 123, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{\frac{\left(\tan^5(dx+c)\right)b^2}{5} - \frac{ab\left(\tan^3(dx+c)\right)}{3} + b^2\left(\tan^3(dx+c)\right) + a^2\tan(dx+c) - 3ab\tan(dx+c) + 3b^2\tan(dx+c)}{b^3} + \frac{\left(-a^3+3a^2b-3a\,b^2+b^3\right)\arctan \cdots}{b^3\sqrt{ab}}}{d}$
default	$\frac{\frac{\frac{\left(\tan^5(dx+c)\right)b^2}{5} - \frac{ab\left(\tan^3(dx+c)\right)}{3} + b^2\left(\tan^3(dx+c)\right) + a^2\tan(dx+c) - 3ab\tan(dx+c) + 3b^2\tan(dx+c)}{b^3} + \frac{\left(-a^3+3a^2b-3a\,b^2+b^3\right)\arctan \cdots}{b^3\sqrt{ab}}}{d}$

risch	$\frac{2i(15a^2 e^{8i(dx+c)}-30ab\, e^{8i(dx+c)}+15b^2 e^{8i(dx+c)}+60a^2 e^{6i(dx+c)}-150ab\, e^{6i(dx+c)}+90b^2 e^{6i(dx+c)}+90a^2 e^{4i(dx+c)}-250}{15d\, b^3 (e^{2i(dx+c)}+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*(1/5*tan(d*x+c)^5*b^2-1/3*a*b*tan(d*x+c)^3+b^2*tan(d*x+c)^3+a^2*
tan(d*x+c)-3*a*b*tan(d*x+c)+3*b^2*tan(d*x+c))+(-a^3+3*a^2*b-3*a*b^2+b^3)/b^
3/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 110, normalized size = 1.02

$$-\frac{\frac{15\,(a^3-3\,a^2b+3\,ab^2-b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}-\frac{3\,b^2\tan(dx+c)^5-5\,(ab-3\,b^2)\tan(dx+c)^3+15\,(a^2-3\,ab+3\,b^2)\tan(dx+c)}{b^3}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/
(sqrt(a*b)*b^3) - (3*b^2*tan(d*x + c)^5 - 5*(a*b - 3*b^2)*tan(d*x + c)^3 +
15*(a^2 - 3*a*b + 3*b^2)*tan(d*x + c))/b^3)/d
```

Fricas [A]
time = 4.00, size = 425, normalized size = 3.94

$$\left[\frac{15\,(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{-ab}\,\cos(dx+c)^5\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)+4\left((15\,a^3b-40\,a^2b^2+33\,ab^3)\cos(dx+c)^4+3\,ab^3-(5\,a^2b^2-9\,ab^3)\cos(dx+c)^2\right)\sin(dx+c)}{60\,ab^4d\cos(dx+c)^5},\ \frac{15\,(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab}\,\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)\cos(dx+c)^5+2\left((15\,a^3b-40\,a^2b^2+33\,ab^3)\cos(dx+c)^4+3\,ab^3-(5\,a^2b^2-9\,ab^3)\cos(dx+c)^2\right)\sin(dx+c)}{30\,ab^4d\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a*b)*cos(d*x + c)^5*log(((a
^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a +
 b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 -
 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + 4*((1
5*a^3*b - 40*a^2*b^2 + 33*a*b^3)*cos(d*x + c)^4 + 3*a*b^3 - (5*a^2*b^2 - 9*
a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^5), 1/30*(15*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 -
 b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^5 + 2*((15*a^3*
b - 40*a^2*b^2 + 33*a*b^3)*cos(d*x + c)^4 + 3*a*b^3 - (5*a^2*b^2 - 9*a*b^3)
*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**8/(a + b*tan(c + d*x)**2), x)
```

Giac [A]
time = 0.67, size = 151, normalized size = 1.40

$$-\frac{\frac{15\,(a^3-3\,a^2b+3\,ab^2-b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,b^3}-\frac{3\,b^4\tan(dx+c)^5-5\,ab^3\tan(dx+c)^3+15\,b^4\tan(dx+c)^3+15\,a^2b^2\tan(dx+c)-45\,ab^3\tan(dx+c)+45\,b^4\tan(dx+c)}{b^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn
(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(sqrt(a*b)*b^3) - (3*b^4*tan(d*x +
c)^5 - 5*a*b^3*tan(d*x + c)^3 + 15*b^4*tan(d*x + c)^3 + 15*a^2*b^2*tan(d*x
+ c) - 45*a*b^3*tan(d*x + c) + 45*b^4*tan(d*x + c))/b^5)/d
```

Mupad [B]
time = 12.29, size = 136, normalized size = 1.26

$$\frac{\tan(c+d\,x)\,\left(\frac{3}{b}+\frac{a\,\left(\frac{a}{b^2}-\frac{3}{b}\right)}{b}\right)}{d}+\frac{{\tan(c+d\,x)}^5}{5\,b\,d}-\frac{{\tan(c+d\,x)}^3\,\left(\frac{a}{3\,b^2}-\frac{1}{b}\right)}{d}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)\,{(a-b)}^3}{\sqrt{a}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}\right)\,{(a-b)}^3}{\sqrt{a}\,b^{7/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] (tan(c + d*x)*(3/b + (a*(a/b^2 - 3/b))/b))/d + tan(c + d*x)^5/(5*b*d) - (ta
n(c + d*x)^3*(a/(3*b^2) - 1/b))/d - (atan((b^(1/2)*tan(c + d*x)*(a - b)^3)/
(a^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(a - b)^3)/(a^(1/2)*b^(7/2)*d)
```

3.457 $\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{(a-b)^2\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}$$

```
[Out] (a-b)^2*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/b^(5/2)/d/a^(1/2)-(a-2*b)*tan(d*
x+c)/b^2/d+1/3*tan(d*x+c)^3/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3756, 398, 211}

$$\frac{(a-b)^2\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - ((
a - 2*b)*Tan[c + d*x])/(b^2*d) + Tan[c + d*x]^3/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
```

```
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a-2b}{b^2}+\frac{x^2}{b}+\frac{a^2-2ab+b^2}{b^2(a+bx^2)}\right)dx, x, \tan(c+dx)\right)}{d}\\ &= -\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}+\frac{(a-b)^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx\right.}{b^2d}\\ &= \frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\tan(c+dx)}{b^2d}+\frac{\tan^3(c+dx)}{3bd}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 74, normalized size = 0.96

$$\frac{\frac{3(a-b)^2\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}+\sqrt{b}\,(-3a+5b+b\sec^2(c+dx))\tan(c+dx)}{3b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2),x]
[Out] ((3*(a - b)^2*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a] + Sqrt[b]*(-3
*a + 5*b + b*Sec[c + d*x]^2)*Tan[c + d*x])/(3*b^(5/2)*d)
```

Maple [A]
time = 0.35, size = 74, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^3(dx+c)\right)}{3}+a\tan(dx+c)-2b\tan(dx+c)}{b^2}+\frac{\left(a^2-2ab+b^2\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^3(dx+c)\right)}{3}+a\tan(dx+c)-2b\tan(dx+c)}{b^2}+\frac{\left(a^2-2ab+b^2\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}}{d}$

risch	$-\frac{2i\left(3a\,e^{4i(dx+c)}-3b\,e^{4i(dx+c)}+6a\,e^{2i(dx+c)}-12b\,e^{2i(dx+c)}+3a-5b\right)}{3d\,b^2\left(e^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right.}{2\sqrt{-ab}\,d\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2*(-1/3*b*tan(d*x+c)^3+a*tan(d*x+c)-2*b*tan(d*x+c))+(a^2-2*a*b+b^
2)/b^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 69, normalized size = 0.90

$$\frac{\frac{3\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{b\tan(dx+c)^3-3\,(a-2\,b)\tan(dx+c)}{b^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2 - 2*a*b + b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*b^2)
 + (b*tan(d*x + c)^3 - 3*(a - 2*b)*tan(d*x + c))/b^2)/d
```

Fricas [A]
time = 2.93, size = 339, normalized size = 4.40

$$\left[-\frac{3\left(a^2-2\,ab+b^2\right)\sqrt{-ab}\,\cos\left(dx+c\right)^3\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)-4\left(ab^2-\left(3\,a^2b-5\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{12\,ab^3d\cos\left(dx+c\right)^3},\ -\frac{3\left(a^2-2\,ab+b^2\right)\sqrt{ab}\,\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)\cos\left(dx+c\right)^3-2\left(ab^2-\left(3\,a^2b-5\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{6\,ab^3d\cos\left(dx+c\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^2 - 2*a*b + b^2)*sqrt(-a*b)*cos(d*x + c)^3*log(((a^2 + 6*a*b +
 b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x
+ c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2
)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) - 4*(a*b^2 - (3*a^2
*b - 5*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^3), -1/6*
(3*(a^2 - 2*a*b + b^2)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sq
rt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^3 - 2*(a*b^2 - (3*a^2
*b - 5*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^6\left(c+dx\right)}{a+b\tan^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)**2), x)
```

Giac [A]
time = 0.65, size = 96, normalized size = 1.25

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(a^2-2\,ab+b^2)}{\sqrt{ab}\,b^2}+\frac{b^2\tan(dx+c)^3-3\,ab\tan(dx+c)+6\,b^2\tan(dx+c)}{b^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*
b)))*(a^2 - 2*a*b + b^2)/(sqrt(a*b)*b^2) + (b^2*tan(d*x + c)^3 - 3*a*b*tan(
d*x + c) + 6*b^2*tan(d*x + c))/b^3)/d
```

Mupad [B]
time = 12.23, size = 90, normalized size = 1.17

$$\frac{\tan(c+d\,x)^3}{3\,b\,d}-\frac{\tan(c+d\,x)\,\left(\frac{a}{b^2}-\frac{2}{b}\right)}{d}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)\,(a-b)^2}{\sqrt{a}\,(a^2-2\,a\,b+b^2)}\right)\,(a-b)^2}{\sqrt{a}\,b^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] tan(c + d*x)^3/(3*b*d) - (tan(c + d*x)*(a/b^2 - 2/b))/d + (atan((b^(1/2)*ta
n(c + d*x)*(a - b)^2)/(a^(1/2)*(a^2 - 2*a*b + b^2)))*(a - b)^2)/(a^(1/2)*b^
(5/2)*d)
```

3.458 $\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}+\frac{\tan(c+dx)}{bd}$$

```
[Out] -(a-b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/b^(3/2)/d/a^(1/2)+tan(d*x+c)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3756, 396, 211}

$$\frac{\tan(c+dx)}{bd}-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) + T
an[c + d*x]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan(c+dx)}{bd} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{bd}\\ &= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d} + \frac{\tan(c+dx)}{bd}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 52, normalized size = 1.00

$$-\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d} + \frac{\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) + T
an[c + d*x]/(b*d)
```

Maple [A]
time = 0.29, size = 44, normalized size = 0.85

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b}+\frac{(-a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{b\sqrt{ab}}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b}+\frac{(-a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{b\sqrt{ab}}}{d}$
risch	$\frac{2i}{db\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{2i(dx+c)}-\frac{2iab-\sqrt{-ab}\,a-\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)a}{2\sqrt{-ab}\,db}+\frac{\ln\left(e^{2i(dx+c)}-\frac{2iab-\sqrt{-ab}\,a-\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b*tan(d*x+c)+(-a+b)/b/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.55, size = 45, normalized size = 0.87

$$-\frac{\frac{(a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{\tan(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -((a - b)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*b) - tan(d*x + c)/b)/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(44) = 88.
time = 3.00, size = 267, normalized size = 5.13

$$\left[\frac{\sqrt{-ab}\,(a-b)\cos(dx+c)\log\left(\frac{(a^2+6ab+b^2)\cos(dx+c)^4-2\,(3ab+b^2)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{(a^2-2ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)+4ab\sin(dx+c)}{4ab^2d\cos(dx+c)},\ \frac{\sqrt{ab}\,(a-b)\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2ab\cos(dx+c)\sin(dx+c)}\right)\cos(dx+c)+2ab\sin(dx+c)}{2ab^2d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a*b)*(a - b)*cos(d*x + c)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)
^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x
 + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 +
 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + 4*a*b*sin(d*x + c))/(a*b^2*d*cos(d*
x + c)), 1/2*(sqrt(a*b)*(a - b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqr
t(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c) + 2*a*b*sin(d*x + c))/
(a*b^2*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)**2), x)
```

Giac [A]
time = 0.64, size = 62, normalized size = 1.19

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(a-b)}{\sqrt{ab}\,b}-\frac{\tan(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))*
(a - b)/(sqrt(a*b)*b) - tan(d*x + c)/b)/d
```

Mupad [B]
time = 12.40, size = 44, normalized size = 0.85

$$\frac{\tan(c+d\,x)}{b\,d}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)}{\sqrt{a}}\right)\,(a-b)}{\sqrt{a}\,b^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] tan(c + d*x)/(b*d) - (atan((b^(1/2)*tan(c + d*x))/a^(1/2))*(a - b))/(a^(1/2
)*b^(3/2)*d)
```

3.459 $\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

```
[Out] arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A]
time = 0.31, size = 24, normalized size = 0.75

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
default	$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
risch	$-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{-2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{\sqrt{-ab}\,(a-b)}\right)}{2\sqrt{-ab}\,d}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Maxima [A]
time = 0.55, size = 23, normalized size = 0.72

$$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.

time = 2.56, size = 205, normalized size = 6.41

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2+4\,\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)}{4\,abd},\ -\frac{\sqrt{ab}\,\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)}{2\,abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/4*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2))/(a*b*d), -1/2*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))/(a*b*d)]

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)

[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)

Giac [A]

time = 0.63, size = 40, normalized size = 1.25

$$\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")

[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(sqrt(a*b)*d)

Mupad [B]

time = 12.50, size = 24, normalized size = 0.75

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)

[Out] atan((b^(1/2)*tan(c + d*x))/a^(1/2))/(a^(1/2)*b^(1/2)*d)

3.460 $\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{(a-3b)x}{2(a-b)^2}+\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d}$$

```
[Out] 1/2*(a-3*b)*x/(a-b)^2+1/2*cos(d*x+c)*sin(d*x+c)/(a-b)/d+b^(3/2)*arctan(b^(1
/2)*tan(d*x+c)/a^(1/2))/(a-b)^2/d/a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {3756, 425, 536, 209, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^2}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)}+\frac{x(a-3b)}{2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((a - 3*b)*x)/(2*(a - b)^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a
]])/(Sqrt[a]*(a - b)^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*(a - b)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d} - \frac{\text{Subst}\left(\int \frac{-a+2b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{2(a-b)d}\\ &= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d} + \frac{(a-3b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2(a-b)^2d} + \frac{b^2\text{Subst}\left(\right.}{}\\ &= \frac{(a-3b)x}{2(a-b)^2} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 78, normalized size = 0.94

$$\frac{4b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right) + \sqrt{a}\,(2(a-3b)(c+dx) + (a-b)\sin(2(c+dx)))}{4\sqrt{a}\,(a-b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*(2*(a - 3*b)*(c
 + d*x) + (a - b)*Sin[2*(c + d*x)]))/(4*Sqrt[a]*(a - b)^2*d)
```

Maple [A]
time = 0.40, size = 85, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)^2\sqrt{ab}}+\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(dx+c)}{1+\tan^2(dx+c)}+\frac{(a-3b)\arctan(\tan(dx+c))}{2}}{(a-b)^2}}{d}$
default	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)^2\sqrt{ab}}+\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(dx+c)}{1+\tan^2(dx+c)}+\frac{(a-3b)\arctan(\tan(dx+c))}{2}}{(a-b)^2}}{d}$
risch	$\frac{xa}{2(a-b)^2}-\frac{3xb}{2(a-b)^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8(a-b)d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8(a-b)d}+\frac{\sqrt{-ab}\,b\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)^2d}-\frac{\sqrt{-ab}\,b\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/(a-b)^2*((1
/2*a-1/2*b)*tan(d*x+c)/(1+tan(d*x+c)^2)+1/2*(a-3*b)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.52, size = 95, normalized size = 1.14

$$\frac{\frac{2\,b^2 \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{ab}}+\frac{(dx+c)(a-3\,b)}{a^2-2\,ab+b^2}+\frac{\tan(dx+c)}{(a-b)\tan(dx+c)^2+a-b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^2 - 2*a*b + b^2)*sqrt(a*b))
 + (d*x + c)*(a - 3*b)/(a^2 - 2*a*b + b^2) + tan(d*x + c)/((a - b)*tan(d*x
+ c)^2 + a - b))/d
```

Fricas [A]
time = 2.31, size = 290, normalized size = 3.49

$$\left[\frac{2\,(a-3\,b)dx+2\,(a-b)\cos(dx+c)\sin(dx+c)+b\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2-4\left((a^2+ab)\cos(dx+c)^3-ab\cos(dx+c)\right)\sqrt{-\frac{b}{a}}\sin(dx+c)+b^2}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)}{4\,(a^2-2\,ab+b^2)d},\ \frac{(a-3\,b)dx+(a-b)\cos(dx+c)\sin(dx+c)-b\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(dx+c)\sin(dx+c)}\right)}{2\,(a^2-2\,ab+b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a - 3*b)*d*x + 2*(a - b)*cos(d*x + c)*sin(d*x + c) + b*sqrt(-b/a)*
log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 -
4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*sqrt(-b/a)*sin(d*x + c) +
 b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 +
b^2)))/((a^2 - 2*a*b + b^2)*d), 1/2*((a - 3*b)*d*x + (a - b)*cos(d*x + c)*s
in(d*x + c) - b*sqrt(b/a)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)
/(b*cos(d*x + c)*sin(d*x + c))))/((a^2 - 2*a*b + b^2)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.64, size = 110, normalized size = 1.33

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)b^2}{(a^2-2\,ab+b^2)\sqrt{ab}}+\frac{(dx+c)(a-3\,b)}{a^2-2\,ab+b^2}+\frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)(a-b)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*
b)))*b^2/((a^2 - 2*a*b + b^2)*sqrt(a*b)) + (d*x + c)*(a - 3*b)/(a^2 - 2*a*b
 + b^2) + tan(d*x + c)/((tan(d*x + c)^2 + 1)*(a - b)))/d
```

Mupad [**B**]
time = 13.89, size = 254, normalized size = 3.06

$$-\frac{6\,a\,b\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)-a^2\,\sin\left(2\,c+2\,d\,x\right)-2\,a^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)+a\,b\,\sin\left(2\,c+2\,d\,x\right)+\mathrm{atan}\left(\frac{a^2\,b^3\,\sin(c+d\,x)\,\sqrt{-a\,b^3}\,9\mathrm{i}-a^3\,b^2\,\sin(c+d\,x)\,\sqrt{-a\,b^3}\,6\mathrm{i}-a\,b^4\,\sin(c+d\,x)\,\sqrt{-a\,b^3}\,4\mathrm{i}+a^4\,b\,\sin(c+d\,x)\,\sqrt{-a\,b^3}\,1\mathrm{i}}{-\cos(c+d\,x)\,a^5\,b^2+6\,\cos(c+d\,x)\,a^4\,b^3-9\,\cos(c+d\,x)\,a^3\,b^4+4\,\cos(c+d\,x)\,a^2\,b^5}\right)\,\sqrt{-a\,b^3}\,4\mathrm{i}}{4\,d\,a^3-8\,d\,a^2\,b+4\,d\,a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x)^2),x)
```

```
[Out] -(atan((a^2*b^3*sin(c + d*x)*(-a*b^3)^(1/2)*9i - a^3*b^2*sin(c + d*x)*(-a*b
^3)^(1/2)*6i - a*b^4*sin(c + d*x)*(-a*b^3)^(1/2)*4i + a^4*b*sin(c + d*x)*(-
a*b^3)^(1/2)*1i)/(4*a^2*b^5*cos(c + d*x) - 9*a^3*b^4*cos(c + d*x) + 6*a^4*b
^3*cos(c + d*x) - a^5*b^2*cos(c + d*x)))*(-a*b^3)^(1/2)*4i - 2*a^2*atan(sin
(c + d*x)/cos(c + d*x)) - a^2*sin(2*c + 2*d*x) + 6*a*b*atan(sin(c + d*x)/co
s(c + d*x)) + a*b*sin(2*c + 2*d*x))/(4*a^3*d + 4*a*b^2*d - 8*a^2*b*d)
```

3.461 $\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=129

$$\frac{(3a^2-10ab+15b^2)\,x}{8(a-b)^3}-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\ \tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^3d}+\frac{(3a-7b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d}$$

```
[Out] 1/8*(3*a^2-10*a*b+15*b^2)*x/(a-b)^3+1/8*(3*a-7*b)*cos(d*x+c)*sin(d*x+c)/(a-
b)^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/(a-b)/d-b^(5/2)*arctan(b^(1/2)*tan(d*x+c
)/a^(1/2))/(a-b)^3/d/a^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3756, 425, 541, 536, 209, 211}

$$\frac{x(3a^2-10ab+15b^2)}{8(a-b)^3}-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\ \tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^3}+\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\sin(c+dx)\cos(c+dx)}{8d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 10*a*b + 15*b^2)*x)/(8*(a - b)^3) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[
c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^3*d) + ((3*a - 7*b)*Cos[c + d*x]*Sin[c
 + d*x])/(8*(a - b)^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*(a - b)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d} - \frac{\text{Subst}\left(\int \frac{-3a+4b-3bx^2}{(1+x^2)^2(a+bx^2)}\,dx, x, \tan(c+dx)\right)}{4(a-b)d}$$

$$= \frac{(3a-7b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d} + \frac{\text{Subst}\left(\int \frac{3a^2-7ab}{(1\cdots}\right.}{}$$

$$= \frac{(3a-7b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d} - \frac{b^3\text{Subst}\left(\int \frac{1}{a+bx^2}\right.}{(a\cdots}$$

$$= \frac{(3a^2-10ab+15b^2)\,x}{8(a-b)^3} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^3d} + \frac{(3a-7b)\cos(c+dx)\sin\cdots}{8(a-b)^2d}$$

Mathematica [A]
time = 0.28, size = 113, normalized size = 0.88

$$\frac{-32b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a}\left(4(3a^2-10ab+15b^2)(c+dx)+8(a^2-3ab+2b^2)\sin(2(c+dx))+(a-b)^2\sin(4(c+dx))\right)}{32\sqrt{a}(a-b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (-32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*(4*(3*a^2 - 1
0*a*b + 15*b^2)*(c + d*x) + 8*(a^2 - 3*a*b + 2*b^2)*Sin[2*(c + d*x)] + (a -
 b)^2*Sin[4*(c + d*x)]))/(32*Sqrt[a]*(a - b)^3*d)
```

Maple [A]
time = 0.39, size = 130, normalized size = 1.01

method	result
derivativedivides	$\frac{-\frac{b^3\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)^3\sqrt{ab}}+\frac{\frac{\left(\frac{3}{8}a^2-\frac{5}{4}ab+\frac{7}{8}b^2\right)\left(\tan^3(dx+c)\right)+\left(-\frac{7}{4}ab+\frac{9}{8}b^2+\frac{5}{8}a^2\right)\tan(dx+c)}{\left(1+\tan^2(dx+c)\right)^2}+\frac{\left(3a^2-10ab+15b^2\right)\arctan(\tan(dx+c}{8}}{(a-b)^3}}{d}$
default	$\frac{-\frac{b^3\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)^3\sqrt{ab}}+\frac{\frac{\left(\frac{3}{8}a^2-\frac{5}{4}ab+\frac{7}{8}b^2\right)\left(\tan^3(dx+c)\right)+\left(-\frac{7}{4}ab+\frac{9}{8}b^2+\frac{5}{8}a^2\right)\tan(dx+c)}{\left(1+\tan^2(dx+c)\right)^2}+\frac{\left(3a^2-10ab+15b^2\right)\arctan(\tan(dx+c}{8}}{(a-b)^3}}{d}$
risch	$\frac{3xa^2}{8(a-b)^3}-\frac{5xab}{4(a-b)^3}+\frac{15xb^2}{8(a-b)^3}-\frac{ie^{2i(dx+c)}a}{8(a-b)^2d}+\frac{ie^{2i(dx+c)}b}{4(a-b)^2d}+\frac{ie^{-2i(dx+c)}a}{8(a^2-2ab+b^2)d}-\frac{ie^{-2i(dx+c)}b}{4(a^2-2ab+b^2)d}+\frac{\sqrt{-ab}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^3/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/(a-b)^3*((
(3/8*a^2-5/4*a*b+7/8*b^2)*tan(d*x+c)^3+(-7/4*a*b+9/8*b^2+5/8*a^2)*tan(d*x+c
))/(1+tan(d*x+c)^2)^2+1/8*(3*a^2-10*a*b+15*b^2)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.52, size = 185, normalized size = 1.43

$$-\frac{\frac{8\,b^3\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab}}-\frac{(3\,a^2-10\,ab+15\,b^2)(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(3\,a-7\,b)\tan(dx+c)^3+(5\,a-9\,b)\tan(dx+c)}{(a^2-2\,ab+b^2)\tan(dx+c)^4+2\,(a^2-2\,ab+b^2)\tan(dx+c)^2+a^2-2\,ab+b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^3*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*sqrt(a*b)) - (3*a^2 - 10*a*b + 15*b^2)*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) - ((3*a - 7*b)*tan(d*x + c)^3 + (5*a - 9*b)*tan(d*x + c))/((a^2 -
2*a*b + b^2)*tan(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*tan(d*x + c)^2 + a^2 -
2*a*b + b^2))/d
```

Fricas [A]
time = 3.05, size = 401, normalized size = 3.11

$$\left[-\frac{2\,b^{2}\sqrt{-\frac{b}{a}}\,\log\left(\frac{(a^{2}+6\,ab+b^{2})\cos(dx+c)^{4}-2\,(3\,ab+b^{2})\cos(dx+c)^{2}-4\,\left((a^{2}+ab)\cos(dx+c)^{3}-ab\cos(dx+c)\right)\sqrt{-\frac{b}{a}}\,\sin(dx+c)+b^{2}}{(a^{2}-2\,ab+b^{2})\cos(dx+c)^{4}+2\,(ab-b^{2})\cos(dx+c)^{2}+b^{2}}\right)-(3\,a^{2}-10\,ab+15\,b^{2})dx-\left(2\,(a^{2}-2\,ab+b^{2})\cos(dx+c)^{3}+(3\,a^{2}-10\,ab+7\,b^{2})\cos(dx+c)\right)\sin(dx+c)}{8\,(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3})d},\ \frac{4\,b^{2}\sqrt{\frac{b}{a}}\,\arctan\left(\frac{\left((a+b)\cos(dx+c)^{2}-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(dx+c)\sin(dx+c)}\right)+(3\,a^{2}-10\,ab+15\,b^{2})dx+\left(2\,(a^{2}-2\,ab+b^{2})\cos(dx+c)^{3}+(3\,a^{2}-10\,ab+7\,b^{2})\cos(dx+c)\right)\sin(dx+c)}{8\,(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3})d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*b^2*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b
+ b^2)*cos(d*x + c)^2 - 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*s
qrt(-b/a)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b
- b^2)*cos(d*x + c)^2 + b^2)) - (3*a^2 - 10*a*b + 15*b^2)*d*x - (2*(a^2 - 2
*a*b + b^2)*cos(d*x + c)^3 + (3*a^2 - 10*a*b + 7*b^2)*cos(d*x + c))*sin(d*x
 + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d), 1/8*(4*b^2*sqrt(b/a)*arctan(1/2
*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d*x + c)*sin(d*x + c))) + (3
*a^2 - 10*a*b + 15*b^2)*d*x + (2*(a^2 - 2*a*b + b^2)*cos(d*x + c)^3 + (3*a^
2 - 10*a*b + 7*b^2)*cos(d*x + c))*sin(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 183, normalized size = 1.42

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)b^{3}}{(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3})\sqrt{ab}}-\frac{(3\,a^{2}-10\,ab+15\,b^{2})(dx+c)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{3\,a\tan(dx+c)^{3}-7\,b\tan(dx+c)^{3}+5\,a\tan(dx+c)-9\,b\tan(dx+c)}{(a^{2}-2\,ab+b^{2})\left(\tan(dx+c)^{2}+1\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a
*b)))*b^3/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)) - (3*a^2 - 10*a*b + 1
5*b^2)*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a*tan(d*x + c)^3 - 7*
b*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 9*b*tan(d*x + c))/((a^2 - 2*a*b + b^2
)*(tan(d*x + c)^2 + 1)^2))/d
```

Mupad [B]
time = 15.87, size = 2500, normalized size = 19.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x)^2),x)
```

```
[Out] ((tan(c + d*x)*(5*a - 9*b))/(8*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)^3*(3*a
 - 7*b))/(8*(a^2 - 2*a*b + b^2)))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)
) - (atan(((((tan(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4
+ 9*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((256*b^1
0 - 1760*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5
+ 3040*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b
^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (tan(c + d*x)*(3*a^2 - 10*a*b
 + 15*b^2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b
^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i
+ a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(3*a^2 - 1
0*a*b + 15*b^2))/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(3*a^2 - 10*
a*b + 15*b^2)*1i)/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) + (((tan(c +
 d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b^3))/(32*(a^
4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((256*b^10 - 1760*a*b^9 + 5280
*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5 + 3040*a^6*b^4 - 800*
a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*
a^3*b^3 + 15*a^4*b^2)) + (tan(c + d*x)*(3*a^2 - 10*a*b + 15*b^2)*(1280*a*b^
8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1
280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a
^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(3*a^2 - 10*a*b + 15*b^2))/(16*
(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(3*a^2 - 10*a*b + 15*b^2)*1i)/(16
*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))/((115*a*b^7 - 105*b^8 - 51*a^2*b
^6 + 9*a^3*b^5)/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^
3 + 15*a^4*b^2)) - (((tan(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*
a^3*b^4 + 9*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + ((
(256*b^10 - 1760*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*
a^5*b^5 + 3040*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a
*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (tan(c + d*x)*(3*a^2
- 10*a*b + 15*b^2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 12
80*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a
^2*b*3i + a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(3
```

*a^2 - 10*a*b + 15*b^2))/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(3*a
^2 - 10*a*b + 15*b^2))/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) + (((ta
n(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b^3))/(3
2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((256*b^10 - 1760*a*b^9 +
 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5 + 3040*a^6*b^4 -
 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4
- 20*a^3*b^3 + 15*a^4*b^2)) + (tan(c + d*x)*(3*a^2 - 10*a*b + 15*b^2)*(1280
*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^
4 + 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1
i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(3*a^2 - 10*a*b + 15*b^2))
/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(3*a^2 - 10*a*b + 15*b^2))/(
16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i))))*(3*a^2 - 10*a*b + 15*b^2)*1i)
/(8*d*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - (atan((((-a*b^5)^(1/2)*((t
an(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b^3))/(
32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + ((-a*b^5)^(1/2)*((256*b^1
0 - 1760*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5
+ 3040*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b
^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (tan(c + d*x)*(-a*b^5)^(1/2)*
(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a
^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(64*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^
2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(2*(a*b^3 + 3*a^3*b - a^4
 - 3*a^2*b^2)))*1i)/(2*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)) + ((-a*b^5)^(1/
2)*((tan(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b
^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - ((-a*b^5)^(1/2)*((2
56*b^10 - 1760*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^
5*b^5 + 3040*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b
^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(c + d*x)*(-a*b^5)^
(1/2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 -
2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(64*(a*b^3 + 3*a^3*b - a^4 - 3*
a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(2*(a*b^3 + 3*a^3*b
 - a^4 - 3*a^2*b^2)))*1i)/(2*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))/((115*a*
b^7 - 105*b^8 - 51*a^2*b^6 + 9*a^3*b^5)/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6
+ 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - ((-a*b^5)^(1/2)*((tan(c + d*x)*(
289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*b^3))/(32*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + ((-a*b^5)^(1/2)*((256*b^10 - 1760*a*b^
9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 ...

3.462 $\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=167

$$-\frac{(4a-5b)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(a-b)(2a-b)\sin(c+d}{2ab^2d\left(a-(a-b)\sin^2(c+\right.}$$

```
[Out] -1/2*(4*a-5*b)*arctanh(sin(d*x+c))/b^3/d+1/2*(a-b)^(3/2)*(4*a+b)*arctanh(si
n(d*x+c)*(a-b)^(1/2)/a^(1/2))/a^(3/2)/b^3/d+1/2*(a-b)*(2*a-b)*sin(d*x+c)/a/
b^2/d/(a-(a-b)*sin(d*x+c)^2)+1/2*sec(d*x+c)*tan(d*x+c)/b/d/(a-(a-b)*sin(d*x
+c)^2)
```

Rubi [A]
time = 0.18, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3757, 425, 541, 536, 212, 214}

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}-\frac{(4a-5b)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{(2a-b)(a-b)\sin(c+dx)}{2ab^2d\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\tan(c+dx)\sec(c+dx)}{2bd\left(a-(a-b)\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((4*a - 5*b)*ArcTanh[Sin[c + d*x]])/(b^3*d) + ((a - b)^(3/2)*(4*a + b)
*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + ((a - b)*
(2*a - b)*Sin[c + d*x])/(2*a*b^2*d*(a - (a - b)*Sin[c + d*x]^2)) + (Sec[c +
 d*x]*Tan[c + d*x])/(2*b*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-(a-b)x^2)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sec(c+dx)\tan(c+dx)}{2bd\,(a-(a-b)\sin^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{-a+2b-3(a-b)x^2}{(1-x^2)(a+(-a+b)x^2)^2}\,dx, x, \sin(c+dx)\right)}{2bd}$$

$$= \frac{(a-b)(2a-b)\sin(c+dx)}{2ab^2d\,(a-(a-b)\sin^2(c+dx))} + \frac{\sec(c+dx)\tan(c+dx)}{2bd\,(a-(a-b)\sin^2(c+dx))} - \frac{\text{Subst}\left(\int \frac{2}{\ldots}\right.}{\ldots}$$

$$= \frac{(a-b)(2a-b)\sin(c+dx)}{2ab^2d\,(a-(a-b)\sin^2(c+dx))} + \frac{\sec(c+dx)\tan(c+dx)}{2bd\,(a-(a-b)\sin^2(c+dx))} - \frac{(4a-5b)\text{S}\ldots}{\ldots}$$

$$= -\frac{(4a-5b)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}$$

Mathematica [A]

time = 2.56, size = 254, normalized size = 1.52

$$\frac{2(4a-5b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(-4a+5b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{(a-b)^{3/2}(4a+b)\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)}{a^{3/2}}+\frac{(a-b)^{3/2}(4a+b)\log\left(\sqrt{a}+\sqrt{a-b}\,\sin(c+dx)\right)}{a^{3/2}}+\frac{b}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{b}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(a-b)^2b\sin(c+dx)}{a(a+b+(a-b)\cos(2(c+dx)))}}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (2*(4*a - 5*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-4*a + 5*b)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - ((a - b)^(3/2)*(4*a + b)*Log[Sqrt[
a] - Sqrt[a - b]*Sin[c + d*x]])/a^(3/2) + ((a - b)^(3/2)*(4*a + b)*Log[Sqrt
[a] + Sqrt[a - b]*Sin[c + d*x]])/a^(3/2) + b/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2 - b/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(a - b)^2*b*Sin[
c + d*x])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))/(4*b^3*d)
```

Maple [A]

time = 0.52, size = 175, normalized size = 1.05

method	result
derivativedivides	$\frac{-\frac{1}{4b^2(\sin(dx+c)+1)}+\frac{(-4a+5b)\ln(\sin(dx+c)+1)}{4b^3}-\frac{1}{4b^2(\sin(dx+c)-1)}+\frac{(4a-5b)\ln(\sin(dx+c)-1)}{4b^3}-\frac{(a^2-2ab+b^2)\left(\frac{b\sin\ldots}{2a\left(a\left(\sin^2(dx+c)\ldots\right.\right.}\ldots\right.}{\ldots}}{d}$

default	$\dfrac{-\frac{1}{4b^2(\sin(dx+c)+1)}+\frac{(-4a+5b)\ln(\sin(dx+c)+1)}{4b^3}-\frac{1}{4b^2(\sin(dx+c)-1)}+\frac{(4a-5b)\ln(\sin(dx+c)-1)}{4b^3}-\frac{(a^2-2ab+b^2)\left(\frac{b}{2a\left(a\left(\sin^2(dx+\cdots\right.\right.}\right.}{}}{d}$
risch	$\dfrac{i(2a^2e^{7i(dx+c)}-3ab\,e^{7i(dx+c)}+b^2e^{7i(dx+c)}+2a^2e^{5i(dx+c)}+ab\,e^{5i(dx+c)}+b^2e^{5i(dx+c)}-2a^2e^{3i(dx+c)}-ab\,e^{3i(dx+c)}-b^2e^{3\cdots}}{d\,b^2\left(e^{2i(dx+c)}+1\right)^2a\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/b^2/(sin(d*x+c)+1)+1/4/b^3*(-4*a+5*b)*ln(sin(d*x+c)+1)-1/4/b^2/(s
in(d*x+c)-1)+1/4*(4*a-5*b)/b^3*ln(sin(d*x+c)-1)-(a^2-2*a*b+b^2)/b^3*(1/2*b/
a*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)-1/2*(4*a+b)/a/(a*(a-b))^(1/2
)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see 'assume?' for more det
ails)Is
```

Fricas [A]
time = 2.70, size = 635, normalized size = 3.80

$$\left[-\frac{\left((4a^3-7a^2b+2ab^2+b^3)\cos(dx+c)^4+(4a^2b-3ab^2-b^3)\cos(dx+c)^2\right)\sqrt{\frac{a-b}{a}}\log\left(-\frac{(a-b)\cos(dx+c)^2+2a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+\left((4a^3-9a^2b+5ab^2)\cos(dx+c)^4+(4a^2b-5ab^2)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-\left((4a^3-9a^2b+5ab^2)\cos(dx+c)^4+(4a^2b-5ab^2)\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)-2\left(ab^2+(2a^2b-3ab^2+b^3)\cos(dx+c)^2\right)\sin(dx+c)}{4\left(ab^4d\cos(dx+c)^2+(a^2b^3-ab^4)d\cos(dx+c)^4\right)},\ -\frac{2\left((4a^3-7a^2b+2ab^2+b^3)\cos(dx+c)^4+(4a^2b-3ab^2-b^3)\cos(dx+c)^2\right)\sqrt{-\frac{a-b}{a}}\arctan\left(\sqrt{-\frac{a-b}{a}}\sin(dx+c)\right)+\left((4a^3-9a^2b+5ab^2)\cos(dx+c)^4+(4a^2b-5ab^2)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-\left((4a^3-9a^2b+5ab^2)\cos(dx+c)^4+(4a^2b-5ab^2)\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)-2\left(ab^2+(2a^2b-3ab^2+b^3)\cos(dx+c)^2\right)\sin(dx+c)}{4\left(ab^4d\cos(dx+c)^2+(a^2b^3-ab^4)d\cos(dx+c)^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(((4*a^3 - 7*a^2*b + 2*a*b^2 + b^3)*cos(d*x + c)^4 + (4*a^2*b - 3*a*b
^2 - b^3)*cos(d*x + c)^2)*sqrt((a - b)/a)*log(-((a - b)*cos(d*x + c)^2 + 2*
a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + (
(4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) - ((4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 +
 (4*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a*b^2 + (2
```

```
*a^2*b - 3*a*b^2 + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)
^2 + (a^2*b^3 - a*b^4)*d*cos(d*x + c)^4), -1/4*(2*((4*a^3 - 7*a^2*b + 2*a*b
^2 + b^3)*cos(d*x + c)^4 + (4*a^2*b - 3*a*b^2 - b^3)*cos(d*x + c)^2)*sqrt(-
(a - b)/a)*arctan(sqrt(-(a - b)/a)*sin(d*x + c)) + ((4*a^3 - 9*a^2*b + 5*a*
b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) - ((4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*co
s(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a*b^2 + (2*a^2*b - 3*a*b^2 + b^3)
*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^2 + (a^2*b^3 - a*b^4)*
d*cos(d*x + c)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^7(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**7/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.78, size = 245, normalized size = 1.47

$$-\frac{\frac{(4a-5b)\log(|\sin(dx+c)+1|)}{b^3}-\frac{(4a-5b)\log(|\sin(dx+c)-1|)}{b^3}-\frac{2\,(4a^3-7a^2b+2ab^2+b^3)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,ab^3}+\frac{2\left(2a^2\sin(dx+c)^3-3ab\sin(dx+c)^3+b^2\sin(dx+c)^3-2a^2\sin(dx+c)+2ab\sin(dx+c)-b^2\sin(dx+c)\right)}{\left(a\sin(dx+c)^4-b\sin(dx+c)^4-2a\sin(dx+c)^2+b\sin(dx+c)^2+a\right)ab^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/4*((4*a - 5*b)*log(abs(sin(d*x + c) + 1))/b^3 - (4*a - 5*b)*log(abs(sin(
d*x + c) - 1))/b^3 - 2*(4*a^3 - 7*a^2*b + 2*a*b^2 + b^3)*arctan(-(a*sin(d*x
 + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*b^3) + 2*(2*a
^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^3 + b^2*sin(d*x + c)^3 - 2*a^2*sin(d
*x + c) + 2*a*b*sin(d*x + c) - b^2*sin(d*x + c))/((a*sin(d*x + c)^4 - b*sin
(d*x + c)^4 - 2*a*sin(d*x + c)^2 + b*sin(d*x + c)^2 + a)*a*b^2))/d
```

Mupad [B]
time = 15.24, size = 2500, normalized size = 14.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(2*a^2 - 2*a*b + b^2))/(a*b^2) - (tan(c/2 + (d*x)/2)^3
*(2*a^2 - 6*a*b + b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^7*(2*a^2 - 2*a*b + b^
```

2))/(a*b^2) - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 6*a*b + b^2))/(a*b^2))/(d*(a - tan(c/2 + (d*x)/2)^2*(4*a - 4*b) - tan(c/2 + (d*x)/2)^6*(4*a - 4*b) + tan(c/2 + (d*x)/2)^4*(6*a - 8*b) + a*tan(c/2 + (d*x)/2)^8)) - (atan((((4*a - 5*b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 - 2236*a^4*b^12 + 768*a^5*b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*b^8 - 13552*a^9*b^7 + 64*a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + (((((256*(256*a^4*b^16 + 192*a^5*b^15 - 1088*a^6*b^14 - 192*a^7*b^13 + 1600*a^8*b^12 - 768*a^9*b^11))/(a^3*b^10) - (256*tan(c/2 + (d*x)/2)*(4*a - 5*b)*(1024*a^5*b^15 - 2304*a^6*b^14 + 1664*a^7*b^13 - 384*a^8*b^12))/(a^3*b^11))*(4*a - 5*b))/(2*b^3) - (512*tan(c/2 + (d*x)/2)*(64*a^2*b^14 + 160*a^3*b^13 - 984*a^4*b^12 - 6560*a^5*b^11 + 28720*a^6*b^10 - 42400*a^7*b^9 + 29512*a^8*b^8 - 9664*a^9*b^7 + 1152*a^10*b^6))/(a^3*b^8))*(4*a - 5*b))/(2*b^3))*(4*a - 5*b))/(2*b^3) + (512*tan(c/2 + (d*x)/2)*(8*a*b^11 - 8960*a^11*b + 768*a^12 + b^12 + 396*a^2*b^10 + 440*a^3*b^9 - 7144*a^4*b^8 + 6656*a^5*b^7 + 34712*a^6*b^6 - 106784*a^7*b^5 + 138675*a^8*b^4 - 100016*a^9*b^3 + 41248*a^10*b^2))/(a^3*b^8))*1i)/(2*b^3) - ((4*a - 5*b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 - 2236*a^4*b^12 + 768*a^5*b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*b^8 - 13552*a^9*b^7 + 64*a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + (((((256*(256*a^4*b^16 + 192*a^5*b^15 - 1088*a^6*b^14 - 192*a^7*b^13 + 1600*a^8*b^12 - 768*a^9*b^11))/(a^3*b^10) + (256*tan(c/2 + (d*x)/2)*(4*a - 5*b)*(1024*a^5*b^15 - 2304*a^6*b^14 + 1664*a^7*b^13 - 384*a^8*b^12))/(a^3*b^11))*(4*a - 5*b))/(2*b^3) + (512*tan(c/2 + (d*x)/2)*(64*a^2*b^14 + 160*a^3*b^13 - 984*a^4*b^12 - 6560*a^5*b^11 + 28720*a^6*b^10 - 42400*a^7*b^9 + 29512*a^8*b^8 - 9664*a^9*b^7 + 1152*a^10*b^6))/(a^3*b^8))*(4*a - 5*b))/(2*b^3))*(4*a - 5*b))/(2*b^3) - (512*tan(c/2 + (d*x)/2)*(8*a*b^11 - 8960*a^11*b + 768*a^12 + b^12 + 396*a^2*b^10 + 440*a^3*b^9 - 7144*a^4*b^8 + 6656*a^5*b^7 + 34712*a^6*b^6 - 106784*a^7*b^5 + 138675*a^8*b^4 - 100016*a^9*b^3 + 41248*a^10*b^2))/(a^3*b^8))*1i)/(2*b^3))/(((4*a - 5*b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 - 2236*a^4*b^12 + 768*a^5*b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*b^8 - 13552*a^9*b^7 + 64*a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + (((((256*(256*a^4*b^16 + 192*a^5*b^15 - 1088*a^6*b^14 - 192*a^7*b^13 + 1600*a^8*b^12 - 768*a^9*b^11))/(a^3*b^10) - (256*tan(c/2 + (d*x)/2)*(4*a - 5*b)*(1024*a^5*b^15 - 2304*a^6*b^14 + 1664*a^7*b^13 - 384*a^8*b^12))/(a^3*b^11))*(4*a - 5*b))/(2*b^3) - (512*tan(c/2 + (d*x)/2)*(64*a^2*b^14 + 160*a^3*b^13 - 984*a^4*b^12 - 6560*a^5*b^11 + 28720*a^6*b^10 - 42400*a^7*b^9 + 29512*a^8*b^8 - 9664*a^9*b^7 + 1152*a^10*b^6))/(a^3*b^8))*(4*a - 5*b))/(2*b^3))*(4*a - 5*b))/(2*b^3) + (512*tan(c/2 + (d*x)/2)*(8*a*b^11 - 8960*a^11*b + 768*a^12 + b^12 + 396*a^2*b^10 + 440*a^3*b^9 - 7144*a^4*b^8 + 6656*a^5*b^7 + 34712*a^6*b^6 - 106784*a^7*b^5 + 138675*a^8*b^4 - 100016*a^9*b^3 + 41248*a^10*b^2))/(a^3*b^8)))/(2*b^3) - (512*(64*a*b^11 - 27904*a^11*b + 3584*a^12 - 5*b^12 + 467*a^2*b^10 - 1322*a^3*b^9 - 4957*a^4*b^8 + 18148*a^5*b^7 + 165*a^6*b^6 - 81226*a^7*b^5 + 165322*a^8*b^4 - 164368*a^9*b^3 + 92032*a^10*b^2))/(a^3*b^10) + ((4*a - 5*b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 - 2236*a^4*b^12 + 768*a^5*b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*b^8 - 13552*a^9*b^7 + 64*a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + (((((256*(256*a^4*b^16 + 192*a^5*b^15

```
 - 1088*a^6*b^14 - 192*a^7*b^13 + 1600*a^8*b^12 - 768*a^9*b^11))/(a^3*b^10)
 + (256*tan(c/2 + (d*x)/2)*(4*a - 5*b)*(1024*a^5*b^15 - 2304*a^6*b^14 + 166
4*a^7*b^13 - 384*a^8*b^12))/(a^3*b^11))*(4*a - 5*b))/(2*b^3) + (512*tan(c/2
 + (d*x)/2)*(64*a^2*b^14 + 160*a^3*b^13 - 984*a^4*b^12 - 6560*a^5*b^11 + 28
720*a^6*b^10 - 42400*a^7*b^9 + 29512*a^8*b^8 - 9664*a^9*b^7 + 1152*a^10*b^6
))/(a^3*b^8))*(4*a - 5*b))/(2*b^3))*(4*a - 5*b))/(2*b^3) - (512*tan(c/2 + (
d*x)/2)*(8*a*b^11 - 8960*a^11*b + 768*a^12 + b^12 + 396*a^2*b^10 + 440*a^3*
b^9 - 7144*a^4*b^8 + 6656*a^5*b^7 + 34712*a^6*b^6 - 106784*a^7*b^5 + 138675
*a^8*b^4 - 100016*a^9*b^3 + 41248*a^10*b^2))/(a^3*b^8)))/(2*b^3)))*(4*a - 5
*b)*1i)/(b^3*d) + (atan((((a - b)^(3/2)*(4*a + b)*((128*tan(c/2 + (d*x)/2)*
(26*a*b^8 - 1232*a^8*b + 192*a^9 + b^9 + 40*a^2*b^7 - 376*a^3*b^6 + 44*a^4*
b^5 + 1964*a^5*b^4 - 3767*a^6*b^3 + 3108*a^7*b^2))/(a^2*b^6) - ((a - b)^(3/
2)*(4*a + b)*((16*(80*a^3*b^11 - 52*a^4*b^10 - 912*a^5*b^9 + 488*a^6*b^8 +
3008*a^7*b^7 - 4836*a^8*b^6 + 2784*a^9*b^5 - 576*a^10*b^4))/(a^3*b^8) + (16
*tan(c/2 + (d*x)/2)^2*(128*a^2*b^12 + 368*a^3*b^11 - 1164*a^4*b^10 - 7344*a
^5*b^9 + 24216*a^6*b^8 - 27808*a^7*b^7 + 15332*a^8*b^6 - 4320*a^9*b^5 + 576
*a^10*b^4))/(a^3*b^8) - (((128*tan(c/2 + (d*x)/2)*(16*a^3*b^10 + 340*a^4*b^
9 - 952*a^5*b^8 + 836*a^6*b^7 - 240*a^7*b^6))/(a^2*b^6) - (((16*(1024*a^6*b
^12 - 1536*a^7*b^11 + 576*a^8*b^10))/(a^3*b^8) + (16*tan(c/2 + (d*x)/2)^2*(
2048*a^5*b^13 - 4096*a^6*b^12 + 2688*a^7*b^11 -...
```

3.463 $\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=109

$$\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d-1/2*(a-b)*sin(d*x+c)/a/b/d/(a-(a-b)*sin(d*x+c)^2)
-1/2*(2*a+b)*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))*(a-b)^(1/2)/a^(3/2)/b^
2/d
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3757, 425, 536, 212, 214}

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]
*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) - ((a - b)*Sin[c + d*x])/(2*a*b*
d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= -\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a-b+(-a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \sin(c+dx\right.}{2abd}\\ &= -\frac{(a-b)\sin(c+dx)}{2abd\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{b^2d} - \frac{((a-b}{}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(}{2abd\,(a}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 191, normalized size = 1.75

$$\frac{-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sqrt{a-b}\,(2a+b)\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)}{a^{3/2}}+\frac{(-2a^2+ab+b^2)\log\left(\sqrt{a}+\sqrt{a-b}\,\sin(c+dx)\right)}{a^{3/2}\sqrt{a-b}}+\frac{4b(-a+b)\sin(c+dx)}{a(a+b+(a-b)\cos(2(c+dx)))}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (-4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + (Sqrt[a - b]*(2*a + b)*Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d
```

```
*x]])/a^(3/2) + ((-2*a^2 + a*b + b^2)*Log[Sqrt[a] + Sqrt[a - b]*Sin[c + d*x
]])/(a^(3/2)*Sqrt[a - b]) + (4*b*(-a + b)*Sin[c + d*x])/(a*(a + b + (a - b)
*Cos[2*(c + d*x)])))/(4*b^2*d)
```

Maple [A]
time = 0.43, size = 124, normalized size = 1.14

method	result
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2b^2}+\frac{\ln(\sin(dx+c)+1)}{2b^2}+\frac{(a-b)\left(\frac{b\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}-\frac{(2a+b)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{b^2}}{d}$
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{2b^2}+\frac{\ln(\sin(dx+c)+1)}{2b^2}+\frac{(a-b)\left(\frac{b\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}-\frac{(2a+b)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{b^2}}{d}$
risch	$\frac{i(a-b)\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{bda\left(a\,e^{4i(dx+c)}-b\,e^{4i(dx+c)}+2a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+a-b\right)}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{d\,b^2}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{d\,b^2}+\frac{\sqrt{a(a-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/b^2*ln(sin(d*x+c)-1)+1/2/b^2*ln(sin(d*x+c)+1)+(a-b)/b^2*(1/2*b/a*
sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)-1/2*(2*a+b)/a/(a*(a-b))^(1/2)*
arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.54, size = 407, normalized size = 3.73

$$\left[\frac{\left((2a^2-ab-b^2)\cos(dx+c)^2+2ab+b^2\right)\sqrt{\frac{a-b}{a}}\log\left(-\frac{(a-b)\cos(dx+c)^2+2a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left((a^2-ab)\cos(dx+c)^2+ab\right)\log(\sin(dx+c)+1)-2\left((a^2-ab)\cos(dx+c)^2+ab\right)\log(-\sin(dx+c)+1)-2(ab-b^2)\sin(dx+c)}{4\left(ab^3d+(a^2b^2-ab^3)d\cos(dx+c)^2\right)}, \frac{\left((2a^2-ab-b^2)\cos(dx+c)^2+2ab+b^2\right)\sqrt{-\frac{a-b}{a}}\arctan\left(\sqrt{-\frac{a-b}{a}}\sin(dx+c)\right)+\left((a^2-ab)\cos(dx+c)^2+ab\right)\log(\sin(dx+c)+1)-\left((a^2-ab)\cos(dx+c)^2+ab\right)\log(-\sin(dx+c)+1)-(ab-b^2)\sin(dx+c)}{2\left(ab^3d+(a^2b^2-ab^3)d\cos(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a^2 - a*b - b^2)*cos(d*x + c)^2 + 2*a*b + b^2)*sqrt((a - b)/a)*lo
g(-((a - b)*cos(d*x + c)^2 + 2*a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((
a - b)*cos(d*x + c)^2 + b)) + 2*((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(sin(
d*x + c) + 1) - 2*((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(-sin(d*x + c) + 1)
 - 2*(a*b - b^2)*sin(d*x + c))/(a*b^3*d + (a^2*b^2 - a*b^3)*d*cos(d*x + c)^
2), 1/2*(((2*a^2 - a*b - b^2)*cos(d*x + c)^2 + 2*a*b + b^2)*sqrt(-(a - b)/a
)*arctan(sqrt(-(a - b)/a)*sin(d*x + c)) + ((a^2 - a*b)*cos(d*x + c)^2 + a*b
)*log(sin(d*x + c) + 1) - ((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(-sin(d*x +
 c) + 1) - (a*b - b^2)*sin(d*x + c))/(a*b^3*d + (a^2*b^2 - a*b^3)*d*cos(d*x
 + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.74, size = 153, normalized size = 1.40

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{b^2}-\frac{\log(|\sin(dx+c)-1|)}{b^2}-\frac{(2a^2-ab-b^2)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,ab^2}+\frac{a\sin(dx+c)-b\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)ab}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(d*x + c) + 1))/b^2 - log(abs(sin(d*x + c) - 1))/b^2 - (2*a
^2 - a*b - b^2)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))
/(sqrt(-a^2 + a*b)*a*b^2) + (a*sin(d*x + c) - b*sin(d*x + c))/((a*sin(d*x +
 c)^2 - b*sin(d*x + c)^2 - a)*a*b))/d
```

Mupad [B]
time = 14.37, size = 946, normalized size = 8.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x)^2)^2),x)

[Out] ((a^2*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos(c + d*
x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*(b - a)^(1/2)*1i - a^(
5/2)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i + (b^2*atan((sin(c/2 +
 (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos(c + d*x)))/(2*a^(1/2)*cos(c
/2 + (d*x)/2)^3*(b - a)^(1/2)))*(b - a)^(1/2)*1i)/2 - a^(3/2)*b*atanh(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i - a^(1/2)*b^2*sin(c + d*x)*1i + a^2*at
an((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)))*(b -
a)^(1/2)*1i + (b^2*atan((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*
(b - a)^(1/2)))*(b - a)^(1/2)*1i)/2 - a^(5/2)*atanh(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*cos(2*c + 2*d*x)*2i + a^(3/2)*b*sin(c + d*x)*1i + a^2*atan(
(a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)))*cos(2*c
+ 2*d*x)*(b - a)^(1/2)*1i - (b^2*atan((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c
/2 + (d*x)/2)*(b - a)^(1/2)))*cos(2*c + 2*d*x)*(b - a)^(1/2)*1i)/2 + (a*b*a
tan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos(c + d*x)))/(2
*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*(b - a)^(1/2)*3i)/2 + (a*b*at
an((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)))*(b -
a)^(1/2)*3i)/2 + a^2*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) -
 2*b*cos(c + d*x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*cos(2*c
 + 2*d*x)*(b - a)^(1/2)*1i - (b^2*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*c
os(c + d*x) - 2*b*cos(c + d*x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1
/2)))*cos(2*c + 2*d*x)*(b - a)^(1/2)*1i)/2 + a^(3/2)*b*atanh(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*2i - (a*b*atan((a^(1/2)*sin(c/2 +
 (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)))*cos(2*c + 2*d*x)*(b - a)^(
1/2)*1i)/2 - (a*b*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*
b*cos(c + d*x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*cos(2*c +
2*d*x)*(b - a)^(1/2)*1i)/2)*1i)/(2*a^(3/2)*b^2*d*(a/2 + b/2 + (a*cos(2*c +
2*d*x))/2 - (b*cos(2*c + 2*d*x))/2))
```

3.464 $\int \frac{\sec^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}\,d}+\frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

```
[Out] 1/2*sin(d*x+c)/a/d/(a-(a-b)*sin(d*x+c)^2)+1/2*arctanh(sin(d*x+c)*(a-b)^(1/2
)/a^(1/2))/a^(3/2)/d/(a-b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3757, 205, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Sin
[c + d*x]/(2*a*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a-(a-b)x^2)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \sin(c+dx)\right)}{2ad}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}\,d} + \frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

Mathematica [A]
time = 0.17, size = 75, normalized size = 0.95

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a}\sin(c+dx)}{a+(-a+b)\sin^2(c+dx)}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
[Out] (ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*Sin[c +
 d*x])/(a + (-a + b)*Sin[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [A]
time = 0.29, size = 80, normalized size = 1.01

method	result
derivativedivides	$\frac{-\frac{\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}+\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}}{d}$
default	$\frac{-\frac{\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}+\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}}{d}$

risch	$-\frac{i\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{ad\left(a\,e^{4i(dx+c)}-b\,e^{4i(dx+c)}+2a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+a-b\right)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{4\sqrt{a^2-ab}\,da}-\frac{\ln\left(e^{2i(dx+c)}\right.}{4\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*sin(d*x+c)/a/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/2/a/(a*(a-b))^(1
/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.90, size = 266, normalized size = 3.37

$$\left[\frac{\left((a-b)\cos\left(dx+c\right)^2+b\right)\sqrt{a^2-ab}\,\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(a^2-ab\right)\sin\left(dx+c\right)}{4\left(\left(a^4-2\,a^3b+a^2b^2\right)d\cos\left(dx+c\right)^2+\left(a^3b-a^2b^2\right)d\right)},-\frac{\left((a-b)\cos\left(dx+c\right)^2+b\right)\sqrt{-a^2+ab}\,\arctan\left(\frac{\sqrt{-a^2+ab}\,\sin(dx+c)}{a}\right)-\left(a^2-ab\right)\sin\left(dx+c\right)}{2\left(\left(a^4-2\,a^3b+a^2b^2\right)d\cos\left(dx+c\right)^2+\left(a^3b-a^2b^2\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((a - b)*cos(d*x + c)^2 + b)*sqrt(a^2 - a*b)*log(-((a - b)*cos(d*x +
c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 +
b)) + 2*(a^2 - a*b)*sin(d*x + c))/((a^4 - 2*a^3*b + a^2*b^2)*d*cos(d*x + c)
^2 + (a^3*b - a^2*b^2)*d), -1/2*(((a - b)*cos(d*x + c)^2 + b)*sqrt(-a^2 + a
*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) - (a^2 - a*b)*sin(d*x + c))/((a
^4 - 2*a^3*b + a^2*b^2)*d*cos(d*x + c)^2 + (a^3*b - a^2*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(c+dx\right)}{\left(a+b\tan^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)**2)**2,x)
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.73, size = 91, normalized size = 1.15

$$\frac{\frac{\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a}-\frac{\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/2*(arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2
 + a*b)*a) - sin(d*x + c)/((a*sin(d*x + c)^2 - b*sin(d*x + c)^2 - a)*a))/d
```

Mupad [B]
time = 12.94, size = 187, normalized size = 2.37

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(4b-2a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}-\frac{\operatorname{atanh}\left(\frac{4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3/2}\sqrt{a-b}\left(\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a-b}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a}+\frac{2}{a}-\frac{2}{a-b}+\frac{4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{ab-a^2}\right)}\right)}{2a^{3/2}d\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x)^2)^2),x)
[Out] (tan(c/2 + (d*x)/2)^3/a + tan(c/2 + (d*x)/2)/a)/(d*(a - tan(c/2 + (d*x)/2)^
2*(2*a - 4*b) + a*tan(c/2 + (d*x)/2)^4)) - atanh((4*b*tan(c/2 + (d*x)/2))/(
a^(3/2)*(a - b)^(1/2)*((2*tan(c/2 + (d*x)/2)^2)/(a - b) - (2*tan(c/2 + (d*x
)/2)^2)/a + 2/a - 2/(a - b) + (4*b*tan(c/2 + (d*x)/2)^2)/(a*b - a^2))))/(2*
a^(3/2)*d*(a - b)^(1/2))
```

3.465 $\int \frac{\sec(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal. Leaf size=94

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d}-\frac{b\sin(c+dx)}{2a(a-b)d\,(a-(a-b)\sin^2(c+dx))}$$

```
[Out] 1/2*(2*a-b)*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/a^(3/2)/(a-b)^(3/2)/d-1
/2*b*sin(d*x+c)/a/(a-b)/d/(a-(a-b)*sin(d*x+c)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3757, 393, 214}

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sin(c+dx)}{2ad(a-b)\,(a-(a-b)\sin^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^
(3/2)*d) - (b*Sin[c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
```

```
x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{b\sin(c+dx)}{2a(a-b)d\left(a-(a-b)\sin^2(c+dx)\right)} + \frac{(2a-b)\operatorname{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \mathrm{s}\right.}{2a(a-b)d}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d} - \frac{b\sin(c+dx)}{2a(a-b)d\left(a-(a-b)\sin^2(c+dx)\right)}$$

Mathematica [A]
time = 0.17, size = 92, normalized size = 0.98

$$\frac{\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{a}\,b\sin(c+dx)}{(a-b)\left(-a+(a-b)\sin^2(c+dx)\right)}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
[Out] (((2*a - b)*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(a - b)^(3/2) + (S
qrt[a]*b*Sin[c + d*x])/((a - b)*(-a + (a - b)*Sin[c + d*x]^2)))/(2*a^(3/2)*
d)
```

Maple [A]
time = 0.42, size = 102, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{b\sin(dx+c)}{2a(a-b)\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}+\frac{(2a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a(a-b)\sqrt{a(a-b)}}}{d}$
default	$\frac{\frac{b\sin(dx+c)}{2a(a-b)\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}+\frac{(2a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a(a-b)\sqrt{a(a-b)}}}{d}$

risch	$\frac{ib\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{ad(-a+b)\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-a+b\right)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,(a-b)d}-\frac{\ln\left(e^{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b/a/(a-b)*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/2*(2*a-b)
/a/(a-b)/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.98, size = 337, normalized size = 3.59

$$\left[\frac{\left(\left(2a^2-3ab+b^2\right)\cos\left(dx+c\right)^2+2ab-b^2\right)\sqrt{a^2-ab}\,\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\left(a^2b-ab^2\right)\sin\left(dx+c\right)}{4\left(\left(a^5-3a^4b+3a^3b^2-a^2b^3\right)d\cos\left(dx+c\right)^2+\left(a^4b-2a^3b^2+a^2b^3\right)d\right)},-\frac{\left(\left(2a^2-3ab+b^2\right)\cos\left(dx+c\right)^2+2ab-b^2\right)\sqrt{-a^2+ab}\,\arctan\left(\frac{\sqrt{-a^2+ab}\,\sin(dx+c)}{a}\right)+\left(a^2b-ab^2\right)\sin\left(dx+c\right)}{2\left(\left(a^5-3a^4b+3a^3b^2-a^2b^3\right)d\cos\left(dx+c\right)^2+\left(a^4b-2a^3b^2+a^2b^3\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a^2 - 3*a*b + b^2)*cos(d*x + c)^2 + 2*a*b - b^2)*sqrt(a^2 - a*b)*
log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a + b)/((
a - b)*cos(d*x + c)^2 + b)) - 2*(a^2*b - a*b^2)*sin(d*x + c))/((a^5 - 3*a^4
*b + 3*a^3*b^2 - a^2*b^3)*d*cos(d*x + c)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*
d), -1/2*(((2*a^2 - 3*a*b + b^2)*cos(d*x + c)^2 + 2*a*b - b^2)*sqrt(-a^2 +
a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (a^2*b - a*b^2)*sin(d*x + c)
)/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cos(d*x + c)^2 + (a^4*b - 2*a^3*
b^2 + a^2*b^3)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\tan^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.69, size = 112, normalized size = 1.19

$$-\frac{\frac{(2\,a-b)\arctan\left(\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{(a^2-ab)\sqrt{-a^2+ab}}-\frac{b\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)(a^2-ab)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a - b)*arctan((a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/
((a^2 - a*b)*sqrt(-a^2 + a*b)) - b*sin(d*x + c)/((a*sin(d*x + c)^2 - b*sin(
d*x + c)^2 - a)*(a^2 - a*b)))/d
```

Mupad [B]
time = 12.78, size = 239, normalized size = 2.54

$$-\frac{\left(a^2\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}-\frac{b^2\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}}{2}+a^2\,\cos\left(2\,c+2\,d\,x\right)\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}+\frac{b^2\,\cos(2\,c+2\,d\,x)\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}}{2}+\frac{a\,b\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}}{2}-\frac{a\,b\,\cos(2\,c+2\,d\,x)\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,3\mathrm{i}}{2}-\sqrt{a}\,b\,\sin\left(c+d\,x\right)\,\sqrt{a-b}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,a^{3/2}\,d\,{\left(a-b\right)}^{3/2}\,\left(\frac{a}{2}+\frac{b}{2}+\frac{a\,\cos(2\,c+2\,d\,x)}{2}-\frac{b\,\cos(2\,c+2\,d\,x)}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] -((a^2*atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))*1i - (b^2*atanh((sin(c +
 d*x)*(a - b)^(1/2))/a^(1/2))*1i)/2 + a^2*cos(2*c + 2*d*x)*atanh((sin(c + d
*x)*(a - b)^(1/2))/a^(1/2))*1i + (b^2*cos(2*c + 2*d*x)*atanh((sin(c + d*x)*
(a - b)^(1/2))/a^(1/2))*1i)/2 + (a*b*atanh((sin(c + d*x)*(a - b)^(1/2))/a^(
1/2))*1i)/2 - (a*b*cos(2*c + 2*d*x)*atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1
/2))*3i)/2 - a^(1/2)*b*sin(c + d*x)*(a - b)^(1/2)*1i)*1i)/(2*a^(3/2)*d*(a -
 b)^(3/2)*(a/2 + b/2 + (a*cos(2*c + 2*d*x))/2 - (b*cos(2*c + 2*d*x))/2))
```

3.466 $\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=114

$$-\frac{(4a-b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d}+\frac{\sin(c+dx)}{(a-b)^2d}+\frac{b^2\sin(c+dx)}{2a(a-b)^2d\,(a-(a-b)\sin^2(c+dx))}$$

```
[Out] -1/2*(4*a-b)*b*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/a^(3/2)/(a-b)^(5/2)/
d+sin(d*x+c)/(a-b)^2/d+1/2*b^2*sin(d*x+c)/a/(a-b)^2/d/(a-(a-b)*sin(d*x+c)^2
)
```

Rubi [A]
time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3757, 398, 393, 214}

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\sin(c+dx)}{2ad(a-b)^2\left(a-(a-b)\sin^2(c+dx)\right)}+\frac{\sin(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(a^(3/2)*(a
- b)^(5/2)*d) + Sin[c + d*x]/((a - b)^2*d) + (b^2*Sin[c + d*x])/(2*a*(a - b
)^2*d*(a - (a - b)*Sin[c + d*x]^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a-(a-b)x^2)^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2}-\frac{(2a-b)b-2(a-b)bx^2}{(a-b)^2(a+(-a+b)x^2)^2}\right)dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\sin(c+dx)}{(a-b)^2d}-\frac{\text{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{(a+(-a+b)x^2)^2}\,dx, x, \sin(c+dx)\right)}{(a-b)^2d}\\ &= \frac{\sin(c+dx)}{(a-b)^2d}+\frac{b^2\sin(c+dx)}{2a(a-b)^2d\,(a-(a-b)\sin^2(c+dx))}-\frac{((4a-b)b)\text{Subst}\left(\int \ldots\right.}{2a\ldots}\\ &= -\frac{(4a-b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d}+\frac{\sin(c+dx)}{(a-b)^2d}+\frac{b^2\sin(c\ldots}{2a(a-b)^2d\,(a-(a\ldots}\end{aligned}$$

Mathematica [A]
time = 0.73, size = 124, normalized size = 1.09

$$\frac{\frac{(4a-b)b\left(\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)-\log\left(\sqrt{a}+\sqrt{a-b}\,\sin(c+dx)\right)\right)}{a^{3/2}(a-b)^{5/2}}+\frac{4\left(1+\frac{b^2}{a(a+b+(a-b)\cos(2(c+dx)))}\right)\sin(c+dx)}{(a-b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
[Out] (((4*a - b)*b*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a] + Sqrt
[a - b]*Sin[c + d*x]]))/(a^(3/2)*(a - b)^(5/2)) + (4*(1 + b^2/(a*(a + b + (
a - b)*Cos[2*(c + d*x)])))*Sin[c + d*x])/(a - b)^2)/(4*d)
```

Maple [A]
time = 0.48, size = 118, normalized size = 1.04

method	result
derivativedivides	$\frac{\frac{\sin(dx+c)}{a^2-2ab+b^2}+\frac{b\left(-\frac{b\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}-\frac{(4a-b)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{(a-b)^2}}{d}$
default	$\frac{\frac{\sin(dx+c)}{a^2-2ab+b^2}+\frac{b\left(-\frac{b\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}-\frac{(4a-b)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{(a-b)^2}}{d}$
risch	$-\frac{ie^{i(dx+c)}}{2d(a^2-2ab+b^2)}+\frac{ie^{-i(dx+c)}}{2d(a^2-2ab+b^2)}+\frac{ib^2\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d(-a+b)^2a\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-a+b\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2-2*a*b+b^2)*sin(d*x+c)+b/(a-b)^2*(-1/2*b/a*sin(d*x+c)/(a*sin(d*x
+c)^2-b*sin(d*x+c)^2-a)-1/2*(4*a-b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x
+c)/(a*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(103) = 206.
time = 3.78, size = 451, normalized size = 3.96

$$\left[-\frac{(4\,ab^2-b^3+(4\,a^2b-5\,ab^2+b^3)\cos(dx+c)^2)\sqrt{a^2-ab}\,\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\,(2\,a^3b-a^2b^2-ab^3+2\,(a^4-2\,a^3b+a^2b^2)\cos(dx+c)^2)\sin(dx+c)}{4\,((a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3+a^2b^4)d\cos(dx+c)^2+(a^5b-3\,a^4b^2+3\,a^3b^3-a^2b^4)d)},\ \frac{(4\,ab^2-b^3+(4\,a^2b-5\,ab^2+b^3)\cos(dx+c)^2)\sqrt{-a^2+ab}\,\arctan\left(\frac{\sqrt{-a^2+ab}\,\sin(dx+c)}{a}\right)+(2\,a^3b-a^2b^2-ab^3+2\,(a^4-2\,a^3b+a^2b^2)\cos(dx+c)^2)\sin(dx+c)}{2\,((a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3+a^2b^4)d\cos(dx+c)^2+(a^5b-3\,a^4b^2+3\,a^3b^3-a^2b^4)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(a^2
- a*b)*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a
+ b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(2*a^3*b - a^2*b^2 - a*b^3 + 2*(a^4
- 2*a^3*b + a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 4*a^5*b + 6*a^4*
b^2 - 4*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^
3 - a^2*b^4)*d), 1/2*((4*a*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cos(d*x +
c)^2)*sqrt(-a^2 + a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (2*a^3*b -
 a^2*b^2 - a*b^3 + 2*(a^4 - 2*a^3*b + a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c)
)/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^2 + (a^
5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.75, size = 152, normalized size = 1.33

$$-\frac{\frac{b^2\sin(dx+c)}{(a^3-2a^2b+ab^2)\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}+\frac{(4ab-b^2)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{(a^3-2a^2b+ab^2)\sqrt{-a^2+ab}}-\frac{2\sin(dx+c)}{a^2-2ab+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*sin(d*x + c)/((a^3 - 2*a^2*b + a*b^2)*(a*sin(d*x + c)^2 - b*sin(d
*x + c)^2 - a)) + (4*a*b - b^2)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/s
qrt(-a^2 + a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) - 2*sin(d*x + c
)/(a^2 - 2*a*b + b^2))/d
```

Mupad [B]
time = 15.55, size = 269, normalized size = 2.36

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^2+b^2)}{a(a-b)^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^2+b^2)}{a(a-b)^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(-2a^2+4ab+b^2)}{a(a-b)^2}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+(4b-a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(4b-a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}+\frac{b\,\mathrm{atan}\left(\frac{2\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^3-6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b+6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)ab^2-2\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)b^3}{\sqrt{a}\,(a-b)^{5/2}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}\right)(4a-b)\,1\mathrm{i}}{2a^{3/2}d(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(a*(a - b)^2) + (tan(c/2 + (d*x)/2)^5*(
2*a^2 + b^2))/(a*(a - b)^2) + (2*tan(c/2 + (d*x)/2)^3*(4*a*b - 2*a^2 + b^2)
)/(a*(a - b)^2))/(d*(a - tan(c/2 + (d*x)/2)^2*(a - 4*b) - tan(c/2 + (d*x)/2
)^4*(a - 4*b) + a*tan(c/2 + (d*x)/2)^6)) + (b*atan((a^3*tan(c/2 + (d*x)/2)*
2i - b^3*tan(c/2 + (d*x)/2)*2i + a*b^2*tan(c/2 + (d*x)/2)*6i - a^2*b*tan(c/
2 + (d*x)/2)*6i)/(a^(1/2)*(a - b)^(5/2)*(tan(c/2 + (d*x)/2)^2 + 1)))*(4*a -
 b)*1i)/(2*a^(3/2)*d*(a - b)^(5/2))
```

3.467 $\int \frac{\cos^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal. Leaf size=143

$$\frac{(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d}+\frac{(a-3b)\sin(c+dx)}{(a-b)^3d}-\frac{\sin^3(c+dx)}{3(a-b)^2d}-\frac{b^3\sin(c+dx)}{2a(a-b)^3d\,(a-(a-b)\sin^2(c$$

```
[Out] 1/2*(6*a-b)*b^2*arctanh(sin(d*x+c)*(a-b)^(1/2)/a^(1/2))/a^(3/2)/(a-b)^(7/2)
/d+(a-3*b)*sin(d*x+c)/(a-b)^3/d-1/3*sin(d*x+c)^3/(a-b)^2/d-1/2*b^3*sin(d*x+
c)/a/(a-b)^3/d/(a-(a-b)*sin(d*x+c)^2)
```

Rubi [A]
time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3757, 398, 393, 214}

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\sin(c+dx)}{2ad(a-b)^3\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\sin^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\sin(c+dx)}{d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
 b)^(7/2)*d) + ((a - 3*b)*Sin[c + d*x])/((a - b)^3*d) - Sin[c + d*x]^3/(3*(
a - b)^2*d) - (b^3*Sin[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Sin[c + d*x]
^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3757

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
 Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*
x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-(a-b)x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{(a-b)^3}-\frac{x^2}{(a-b)^2}+\frac{(3a-b)b^2-3(a-b)b^2x^2}{(a-b)^3\left(a+(-a+b)x^2\right)^2}\right)\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{(a-3b)\sin(c+dx)}{(a-b)^3d}-\frac{\sin^3(c+dx)}{3(a-b)^2d}+\frac{\text{Subst}\left(\int \frac{(3a-b)b^2-3(a-b)b^2x^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \sin(c+
}{(a-b)^3d}\\
&= \frac{(a-3b)\sin(c+dx)}{(a-b)^3d}-\frac{\sin^3(c+dx)}{3(a-b)^2d}-\frac{b^3\sin(c+dx)}{2a(a-b)^3d\left(a-(a-b)\sin^2(c+dx)\right)}-\\
&= \frac{(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d}+\frac{(a-3b)\sin(c+dx)}{(a-b)^3d}-\frac{\sin^3(c+dx)}{3(a-b)^2d}-
\end{aligned}
$$

Mathematica [A]
time = 1.02, size = 147, normalized size = 1.03

$$
\frac{\frac{3b^2(-6a+b)\left(\log\left(\sqrt{a}-\sqrt{a-b}\,\sin(c+dx)\right)-\log\left(\sqrt{a}+\sqrt{a-b}\,\sin(c+dx)\right)\right)}{a^{3/2}(a-b)^{7/2}}+\frac{3\left(3a-11b-\frac{4b^3}{a(a+b+(a-b)\cos(2(c+dx)))}\right)\sin(c+dx)}{(a-b)^3}+\frac{\sin(3(c+dx))}{(a-b)^2}}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*b^2*(-6*a + b)*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a] +
 Sqrt[a - b]*Sin[c + d*x]]))/(a^(3/2)*(a - b)^(7/2)) + (3*(3*a - 11*b - (4*
b^3)/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))*Sin[c + d*x])/(a - b)^3 + Sin[
3*(c + d*x)]/(a - b)^2)/(12*d)
```

Maple [A]
time = 0.51, size = 164, normalized size = 1.15

method	result
derivativedivides	$\frac{-\frac{\frac{a\left(\sin^3(dx+c)\right)}{3}-\frac{b\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)a+3b\sin(dx+c)}{\left(a^2-2ab+b^2\right)(a-b)}-\frac{b^2\left(-\frac{b\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}-\frac{(6a-b)\operatorname{arctanh}\left(\frac{(}{\sqrt{}}\right)}{2a\sqrt{a(}}\right)}{(a-b)^3}}{d}$
default	$\frac{-\frac{\frac{a\left(\sin^3(dx+c)\right)}{3}-\frac{b\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)a+3b\sin(dx+c)}{\left(a^2-2ab+b^2\right)(a-b)}-\frac{b^2\left(-\frac{b\sin(dx+c)}{2a\left(a\left(\sin^2(dx+c)\right)-b\left(\sin^2(dx+c)\right)-a\right)}-\frac{(6a-b)\operatorname{arctanh}\left(\frac{(}{\sqrt{}}\right)}{2a\sqrt{a(}}\right)}{(a-b)^3}}{d}$
risch	$-\frac{i e^{3i(dx+c)}}{24(a^2-2ab+b^2)d}-\frac{3i e^{i(dx+c)}a}{8(a^2-2ab+b^2)(a-b)d}+\frac{11i e^{i(dx+c)}b}{8(a^2-2ab+b^2)(a-b)d}+\frac{3i e^{-i(dx+c)}a}{8d(a^3-3a^2b+3ab^2-b^3)}-\frac{11i e^{-i(}}{8d(a^3-3a^2b+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a^2-2*a*b+b^2)/(a-b)*(1/3*a*sin(d*x+c)^3-1/3*b*sin(d*x+c)^3-sin(d*
x+c)*a+3*b*sin(d*x+c))-b^2/(a-b)^3*(-1/2*b/a*sin(d*x+c)/(a*sin(d*x+c)^2-b*s
in(d*x+c)^2-a)-1/2*(6*a-b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a
-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(130) = 260$.
time = 3.32, size = 600, normalized size = 4.20

$$\left[\frac{3\left(6ab^3-b^4+\left(6a^2b^2-7ab^3+b^4\right)\cos\left(dx+c\right)^2\right)\sqrt{a^2-ab}\,\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\,\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(4a^4b-20a^3b^2+13a^2b^3+3ab^4+2\left(a^5-3a^4b+3a^3b^2-a^2b^3\right)\cos\left(dx+c\right)^4+2\left(2a^5-11a^4b+16a^3b^2-7a^2b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{12\left(\left(a^7-5a^6b+10a^5b^2-10a^4b^3+5a^3b^4-a^2b^5\right)d\cos\left(dx+c\right)^2+\left(a^6b-4a^5b^2+6a^4b^3-4a^3b^4+a^2b^5\right)d\right)},-\frac{3\left(6ab^3-b^4+\left(6a^2b^2-7ab^3+b^4\right)\cos\left(dx+c\right)^2\right)\sqrt{-a^2+ab}\,\arctan\left(\frac{\sqrt{-a^2+ab}\,\sin(dx+c)}{a}\right)-\left(4a^4b-20a^3b^2+13a^2b^3+3ab^4+2\left(a^5-3a^4b+3a^3b^2-a^2b^3\right)\cos\left(dx+c\right)^4+2\left(2a^5-11a^4b+16a^3b^2-7a^2b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{6\left(\left(a^7-5a^6b+10a^5b^2-10a^4b^3+5a^3b^4-a^2b^5\right)d\cos\left(dx+c\right)^2+\left(a^6b-4a^5b^2+6a^4b^3-4a^3b^4+a^2b^5\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(6*a*b^3 - b^4 + (6*a^2*b^2 - 7*a*b^3 + b^4)*cos(d*x + c)^2)*sqrt(
a^2 - a*b)*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) -
2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + 2*(4*a^4*b - 20*a^3*b^2 + 13*a^2*b
^3 + 3*a*b^4 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^4 + 2*(
2*a^5 - 11*a^4*b + 16*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((
a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(d*x +
c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d), -1/6*(3*(6
*a*b^3 - b^4 + (6*a^2*b^2 - 7*a*b^3 + b^4)*cos(d*x + c)^2)*sqrt(-a^2 + a*b)
*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) - (4*a^4*b - 20*a^3*b^2 + 13*a^2*b
^3 + 3*a*b^4 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^4 + 2*(
2*a^5 - 11*a^4*b + 16*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((
a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(d*x +
c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(130) = 260$.
time = 0.77, size = 329, normalized size = 2.30

$$\frac{\frac{3 \, b^{3} \sin\left(dx+c\right)}{{\left(a^{4}-3 \, a^{3} b+3 \, a^{2} b^{2}-a b^{3}\right)} {\left(a \sin\left(dx+c\right)^{2}-b \sin\left(dx+c\right)^{2}-a\right)}} + \frac{3 \, {\left(6 \, a b^{2}-b^{3}\right)} \arctan\left(-\frac{a \sin\left(dx+c\right)-b \sin\left(dx+c\right)}{\sqrt{-a^{2}+a b}}\right)}{{\left(a^{4}-3 \, a^{3} b+3 \, a^{2} b^{2}-a b^{3}\right)} \sqrt{-a^{2}+a b}} - \frac{2 \, {\left(a^{4} \sin\left(dx+c\right)^{3}-4 \, a^{3} b \sin\left(dx+c\right)^{3}+6 \, a^{2} b^{2} \sin\left(dx+c\right)^{3}-4 \, a b^{3} \sin\left(dx+c\right)^{3}+b^{4} \sin\left(dx+c\right)^{3}-3 \, a^{4} \sin\left(dx+c\right)+18 \, a^{3} b \sin\left(dx+c\right)-36 \, a^{2} b^{2} \sin\left(dx+c\right)+30 \, a b^{3} \sin\left(dx+c\right)-9 \, b^{4} \sin\left(dx+c\right)\right)}}{a^{6}-6 \, a^{5} b+15 \, a^{4} b^{2}-20 \, a^{3} b^{3}+15 \, a^{2} b^{4}-6 \, a b^{5}+b^{6}}}{6 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*sin(d*x + c)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(a*sin(d*x + c
)^2 - b*sin(d*x + c)^2 - a)) + 3*(6*a*b^2 - b^3)*arctan(-(a*sin(d*x + c) -
b*sin(d*x + c))/sqrt(-a^2 + a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt
(-a^2 + a*b)) - 2*(a^4*sin(d*x + c)^3 - 4*a^3*b*sin(d*x + c)^3 + 6*a^2*b^2*
sin(d*x + c)^3 - 4*a*b^3*sin(d*x + c)^3 + b^4*sin(d*x + c)^3 - 3*a^4*sin(d*
x + c) + 18*a^3*b*sin(d*x + c) - 36*a^2*b^2*sin(d*x + c) + 30*a*b^3*sin(d*x
 + c) - 9*b^4*sin(d*x + c))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a
^2*b^4 - 6*a*b^5 + b^6))/d
```

Mupad [B]
time = 16.01, size = 1690, normalized size = 11.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x)^2)^2,x)
[Out] - ((tan(c/2 + (d*x)/2)*(6*a^2*b - 2*a^3 + b^3))/(a*(3*a*b^2 - 3*a^2*b + a^3
 - b^3)) + (tan(c/2 + (d*x)/2)^9*(6*a^2*b - 2*a^3 + b^3))/(a*(3*a*b^2 - 3*a
^2*b + a^3 - b^3)) + (4*tan(c/2 + (d*x)/2)^3*(18*a*b^2 - 8*a^2*b + 2*a^3 +
3*b^3))/(3*a*(a - b)*(a^2 - 2*a*b + b^2)) + (4*tan(c/2 + (d*x)/2)^7*(18*a*b
^2 - 8*a^2*b + 2*a^3 + 3*b^3))/(3*a*(a - b)*(a^2 - 2*a*b + b^2)) + (2*tan(c
/2 + (d*x)/2)^5*(56*a*b^2 - 18*a^2*b - 2*a^3 + 9*b^3))/(3*a*(a - b)*(a^2 -
2*a*b + b^2)))/(d*(a + tan(c/2 + (d*x)/2)^2*(a + 4*b) + tan(c/2 + (d*x)/2)^
8*(a + 4*b) - tan(c/2 + (d*x)/2)^4*(2*a - 12*b) - tan(c/2 + (d*x)/2)^6*(2*a
 - 12*b) + a*tan(c/2 + (d*x)/2)^10)) - (b^2*atan(((b^2*(tan(c/2 + (d*x)/2)*
(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b^8 - 880*a^6*b^7 + 1080*a^7*b^6 - 768*a
^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3) - (b^2*(6*a - b)*(tan(c/2 + (d*x)/2)^2*
(16*a^15 - 176*a^14*b + 32*a^5*b^10 - 304*a^6*b^9 + 1296*a^7*b^8 - 3264*a^8
*b^7 + 5376*a^9*b^6 - 6048*a^10*b^5 + 4704*a^11*b^4 - 2496*a^12*b^3 + 864*a
^13*b^2) + 144*a^14*b - 16*a^15 + 16*a^6*b^9 - 144*a^7*b^8 + 576*a^8*b^7 -
1344*a^9*b^6 + 2016*a^10*b^5 - 2016*a^11*b^4 + 1344*a^12*b^3 - 576*a^13*b^2
))/(4*a^(3/2)*(a - b)^(7/2)))*(6*a - b)*1i)/(4*a^(3/2)*(a - b)^(7/2)) + (b^
2*(tan(c/2 + (d*x)/2)*(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b^8 - 880*a^6*b^7
+ 1080*a^7*b^6 - 768*a^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3) + (b^2*(6*a - b)*
(tan(c/2 + (d*x)/2)^2*(16*a^15 - 176*a^14*b + 32*a^5*b^10 - 304*a^6*b^9 + 1
296*a^7*b^8 - 3264*a^8*b^7 + 5376*a^9*b^6 - 6048*a^10*b^5 + 4704*a^11*b^4 -
 2496*a^12*b^3 + 864*a^13*b^2) + 144*a^14*b - 16*a^15 + 16*a^6*b^9 - 144*a^
7*b^8 + 576*a^8*b^7 - 1344*a^9*b^6 + 2016*a^10*b^5 - 2016*a^11*b^4 + 1344*a
^12*b^3 - 576*a^13*b^2))/(4*a^(3/2)*(a - b)^(7/2)))*(6*a - b)*1i)/(4*a^(3/2
)*(a - b)^(7/2)))/(2*tan(c/2 + (d*x)/2)^2*(a^2*b^9 - 15*a^3*b^8 + 75*a^4*b^
7 - 145*a^5*b^6 + 120*a^6*b^5 - 36*a^7*b^4) - 2*a^2*b^9 + 30*a^3*b^8 - 150*
a^4*b^7 + 290*a^5*b^6 - 240*a^6*b^5 + 72*a^7*b^4 - (b^2*(tan(c/2 + (d*x)/2)
*(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b^8 - 880*a^6*b^7 + 1080*a^7*b^6 - 768*
a^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3) - (b^2*(6*a - b)*(tan(c/2 + (d*x)/2)^2
*(16*a^15 - 176*a^14*b + 32*a^5*b^10 - 304*a^6*b^9 + 1296*a^7*b^8 - 3264*a^
8*b^7 + 5376*a^9*b^6 - 6048*a^10*b^5 + 4704*a^11*b^4 - 2496*a^12*b^3 + 864*
a^13*b^2) + 144*a^14*b - 16*a^15 + 16*a^6*b^9 - 144*a^7*b^8 + 576*a^8*b^7 -
 1344*a^9*b^6 + 2016*a^10*b^5 - 2016*a^11*b^4 + 1344*a^12*b^3 - 576*a^13*b^
2))/(4*a^(3/2)*(a - b)^(7/2)))*(6*a - b))/(4*a^(3/2)*(a - b)^(7/2)) + (b^2*
(tan(c/2 + (d*x)/2)*(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b^8 - 880*a^6*b^7 +
1080*a^7*b^6 - 768*a^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3) + (b^2*(6*a - b)*(t
an(c/2 + (d*x)/2)^2*(16*a^15 - 176*a^14*b + 32*a^5*b^10 - 304*a^6*b^9 + 129
6*a^7*b^8 - 3264*a^8*b^7 + 5376*a^9*b^6 - 6048*a^10*b^5 + 4704*a^11*b^4 - 2
```

```
496*a^12*b^3 + 864*a^13*b^2) + 144*a^14*b - 16*a^15 + 16*a^6*b^9 - 144*a^7*
b^8 + 576*a^8*b^7 - 1344*a^9*b^6 + 2016*a^10*b^5 - 2016*a^11*b^4 + 1344*a^1
2*b^3 - 576*a^13*b^2))/(4*a^(3/2)*(a - b)^(7/2)))*(6*a - b))/(4*a^(3/2)*(a
- b)^(7/2))))*(6*a - b)*1i)/(2*a^(3/2)*d*(a - b)^(7/2))
```

3.468 $\int \frac{\sec^8(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal. Leaf size=127

$$\frac{(a-b)^2(5a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d}-\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan(c+dx)}{2ab^3d\,(a+b\tan^2(c+dx))}$$

```
[Out] 1/2*(a-b)^2*(5*a+b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(7/2)/d-(2
*a-3*b)*tan(d*x+c)/b^3/d+1/3*tan(d*x+c)^3/b^2/d-1/2*(a-b)^3*tan(d*x+c)/a/b^
3/d/(a+b*tan(d*x+c)^2)
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3756, 398, 393, 211}

$$\frac{(5a+b)(a-b)^2\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d}-\frac{(a-b)^3\tan(c+dx)}{2ab^3d\,(a+b\tan^2(c+dx))}-\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)^2*(5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(
7/2)*d) - ((2*a - 3*b)*Tan[c + d*x])/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d) - (
(a - b)^3*Tan[c + d*x])/(2*a*b^3*d*(a + b*Tan[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(a+bx^2)^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{2a-3b}{b^3}+\frac{x^2}{b^2}+\frac{(a-b)^2(2a+b)+3(a-b)^2bx^2}{b^3(a+bx^2)^2}\right)dx,x,\tan(c+dx)\right)}{d}\\ &= -\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}+\frac{\text{Subst}\left(\int \frac{(a-b)^2(2a+b)+3(a-b)^2bx^2}{(a+bx^2)^2}\,dx,x\right.}{b^3d}\\ &= -\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan(c+dx)}{2ab^3d\,(a+b\tan^2(c+dx))}+\frac{((a-}{}\\ &= \frac{(a-b)^2(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d}-\frac{(2a-3b)\tan(c+dx)}{b^3d}+\frac{\tan^3(c+dx}{3b^2d}\end{aligned}$$

Mathematica [A]
time = 0.50, size = 135, normalized size = 1.06

$$\frac{\frac{3(a-b)^2(5a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{3\sqrt{b}\,(-a+b)^3\sin(2(c+dx))}{a(a+b+(a-b)\cos(2(c+dx)))}+4\sqrt{b}\,(-3a+4b)\tan(c+dx)+2b^{3/2}\sec^2(c+dx)\tan(c+dx)}{6b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*(a - b)^2*(5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2) + (
3*Sqrt[b]*(-a + b)^3*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)]
)) + 4*Sqrt[b]*(-3*a + 4*b)*Tan[c + d*x] + 2*b^(3/2)*Sec[c + d*x]^2*Tan[c +
 d*x])/(6*b^(7/2)*d)
```

Maple [A]
time = 0.43, size = 137, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^{3}(dx+c)\right)}{3}+2a\tan(dx+c)-3b\tan(dx+c)}{b^{3}}+\frac{-\frac{\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)\tan(dx+c)}{2a\left(a+b\left(\tan^{2}(dx+c)\right)\right)}+\frac{\left(5a^{3}-9a^{2}b+3a\,b^{2}+b^{3}\right)\arctan\left(\frac{b\tan(dx+}{\sqrt{ab}}\right.}{2a\sqrt{ab}}}{b^{3}}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^{3}(dx+c)\right)}{3}+2a\tan(dx+c)-3b\tan(dx+c)}{b^{3}}+\frac{-\frac{\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)\tan(dx+c)}{2a\left(a+b\left(\tan^{2}(dx+c)\right)\right)}+\frac{\left(5a^{3}-9a^{2}b+3a\,b^{2}+b^{3}\right)\arctan\left(\frac{b\tan(dx+}{\sqrt{ab}}\right.}{2a\sqrt{ab}}}{b^{3}}}{d}$
risch	$\frac{i\left(15a^{3}\mathrm{e}^{8i(dx+c)}-27a^{2}b\,\mathrm{e}^{8i(dx+c)}+9a\,b^{2}\mathrm{e}^{8i(dx+c)}+3b^{3}\mathrm{e}^{8i(dx+c)}+60a^{3}\mathrm{e}^{6i(dx+c)}-78a^{2}b\,\mathrm{e}^{6i(dx+c)}+12a\,b^{2}\mathrm{e}^{6i(dx+c)}+6b^{3}\right.}{3d\,b^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}a(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^3*(-1/3*b*tan(d*x+c)^3+2*a*tan(d*x+c)-3*b*tan(d*x+c))+1/b^3*(-1/2
*(a^3-3*a^2*b+3*a*b^2-b^3)/a*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*(5*a^3-9*a^2
*b+3*a*b^2+b^3)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.53, size = 137, normalized size = 1.08

$$-\frac{\frac{3\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)\tan(dx+c)}{ab^{4}\tan(dx+c)^{2}+a^{2}b^{3}}-\frac{2\left(b\tan(dx+c)^{3}-3\,(2\,a-3\,b)\tan(dx+c)\right)}{b^{3}}-\frac{3\left(5\,a^{3}-9\,a^{2}b+3\,ab^{2}+b^{3}\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(d*x + c)/(a*b^4*tan(d*x + c)^2
+ a^2*b^3) - 2*(b*tan(d*x + c)^3 - 3*(2*a - 3*b)*tan(d*x + c))/b^3 - 3*(5*a
^3 - 9*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*a
*b^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(113) = 226.
time = 3.15, size = 597, normalized size = 4.70

$$\left[-\frac{3\left(\left(5\,a^{4}-14\,a^{3}b+12\,a^{2}b^{2}-2\,ab^{3}-b^{4}\right)\cos(dx+c)^{5}+\left(5\,a^{3}b-9\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)\cos(dx+c)^{3}\right)\sqrt{-ab}\,\log\left(\frac{\left(a^{2}+6\,ab+b^{2}\right)\cos(dx+c)^{4}-2\left(3\,ab+b^{2}\right)\cos(dx+c)^{2}+4\left((a+b)\cos(dx+c)^{3}-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^{2}}{\left(a^{2}-2\,ab+b^{2}\right)\cos(dx+c)^{4}+2\left(ab-b^{2}\right)\cos(dx+c)^{2}+b^{2}}\right)-4\left(2\,a^{2}b^{3}-\left(15\,a^{4}b-37\,a^{3}b^{2}+25\,a^{2}b^{3}-3\,ab^{4}\right)\cos(dx+c)^{4}-2\left(5\,a^{3}b^{2}-7\,a^{2}b^{3}\right)\cos(dx+c)^{2}\right)\sin(dx+c)}{24\left(a^{2}b^{5}d\cos(dx+c)^{3}+\left(a^{3}b^{4}-a^{2}b^{5}\right)d\cos(dx+c)^{5}\right)},\ -\frac{3\left(\left(5\,a^{4}-14\,a^{3}b+12\,a^{2}b^{2}-2\,ab^{3}-b^{4}\right)\cos(dx+c)^{5}+\left(5\,a^{3}b-9\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)\cos(dx+c)^{3}\right)\sqrt{ab}\,\arctan\left(\frac{\left((a+b)\cos(dx+c)^{2}-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)-2\left(2\,a^{2}b^{3}-\left(15\,a^{4}b-37\,a^{3}b^{2}+25\,a^{2}b^{3}-3\,ab^{4}\right)\cos(dx+c)^{4}-2\left(5\,a^{3}b^{2}-7\,a^{2}b^{3}\right)\cos(dx+c)^{2}\right)\sin(dx+c)}{12\left(a^{2}b^{5}d\cos(dx+c)^{3}+\left(a^{3}b^{4}-a^{2}b^{5}\right)d\cos(dx+c)^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((5*a^4 - 14*a^3*b + 12*a^2*b^2 - 2*a*b^3 - b^4)*cos(d*x + c)^5 +
 (5*a^3*b - 9*a^2*b^2 + 3*a*b^3 + b^4)*cos(d*x + c)^3)*sqrt(-a*b)*log(((a^2
 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b
)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2
*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) - 4*(2*a^
2*b^3 - (15*a^4*b - 37*a^3*b^2 + 25*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4 - 2*(
5*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^5*d*cos(d*x + c
)^3 + (a^3*b^4 - a^2*b^5)*d*cos(d*x + c)^5), -1/12*(3*((5*a^4 - 14*a^3*b +
12*a^2*b^2 - 2*a*b^3 - b^4)*cos(d*x + c)^5 + (5*a^3*b - 9*a^2*b^2 + 3*a*b^3
 + b^4)*cos(d*x + c)^3)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*s
qrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))) - 2*(2*a^2*b^3 - (15*a^4*b - 37*a
^3*b^2 + 25*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4 - 2*(5*a^3*b^2 - 7*a^2*b^3)*c
os(d*x + c)^2)*sin(d*x + c))/(a^2*b^5*d*cos(d*x + c)^3 + (a^3*b^4 - a^2*b^5
)*d*cos(d*x + c)^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**8/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.76, size = 180, normalized size = 1.42

$$\frac{\frac{3\,(5\,a^3-9\,a^2b+3\,ab^2+b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{\sqrt{ab}\,ab^3}-\frac{3\,(a^3\tan(dx+c)-3\,a^2b\tan(dx+c)+3\,ab^2\tan(dx+c)-b^3\tan(dx+c))}{\left(b\tan(dx+c)^2+a\right)ab^3}+\frac{2\left(b^4\tan(dx+c)^3-6\,ab^3\tan(dx+c)+9\,b^4\tan(dx+c)\right)}{b^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(
b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(sqrt(a*b)*a*b^3) - 3*(a^3*tan(d*x +
 c) - 3*a^2*b*tan(d*x + c) + 3*a*b^2*tan(d*x + c) - b^3*tan(d*x + c))/((b*t
an(d*x + c)^2 + a)*a*b^3) + 2*(b^4*tan(d*x + c)^3 - 6*a*b^3*tan(d*x + c) +
9*b^4*tan(d*x + c))/b^6)/d
```

Mupad [B]
time = 12.17, size = 167, normalized size = 1.31

$$\frac{\tan(c+d\,x)^3}{3\,b^2\,d}-\frac{\tan(c+d\,x)\,\left(\frac{2\,a}{b^3}-\frac{3}{b^2}\right)}{d}-\frac{\tan(c+d\,x)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,a\,d\,\left(b^4\tan\left(c+d\,x\right)^2+a\,b^3\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)\,(a-b)^2\,(5\,a+b)}{\sqrt{a}\,(5\,a^3-9\,a^2\,b+3\,a\,b^2+b^3)}\right)\,(a-b)^2\,(5\,a+b)}{2\,a^{3/2}\,b^{7/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] tan(c + d*x)^3/(3*b^2*d) - (tan(c + d*x)*((2*a)/b^3 - 3/b^2))/d - (tan(c +
d*x)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*a*d*(a*b^3 + b^4*tan(c + d*x)^2))
+ (atan((b^(1/2)*tan(c + d*x)*(a - b)^2*(5*a + b))/(a^(1/2)*(3*a*b^2 - 9*a^
2*b + 5*a^3 + b^3)))*(a - b)^2*(5*a + b))/(2*a^(3/2)*b^(7/2)*d)
```

3.469 $\int \frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{(3a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{\tan(c+dx)}{b^2d}+\frac{(a-b)^2\tan(c+dx)}{2ab^2d\,(a+b\tan^2(c+dx))}$$

```
[Out] -1/2*(3*a^2-2*a*b-b^2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(5/2)/d
+tan(d*x+c)/b^2/d+1/2*(a-b)^2*tan(d*x+c)/a/b^2/d/(a+b*tan(d*x+c)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3756, 398, 393, 211}

$$-\frac{(3a^2-2ab-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\tan(c+dx)}{2ab^2d\,(a+b\tan^2(c+dx))}+\frac{\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] -1/2*((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2
)*b^(5/2)*d) + Tan[c + d*x]/(b^2*d) + ((a - b)^2*Tan[c + d*x])/(2*a*b^2*d*(
a + b*Tan[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+bx^2\right)^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2(a-b)bx^2}{b^2\left(a+bx^2\right)^2}\right)dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\tan(c+dx)}{b^2d}-\frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{\left(a+bx^2\right)^2}\,dx,x,\tan(c+dx)\right)}{b^2d}\\ &= \frac{\tan(c+dx)}{b^2d}+\frac{(a-b)^2\tan(c+dx)}{2ab^2d\left(a+b\tan^2(c+dx)\right)}-\frac{((a-b)(3a+b))\text{Subst}\left(\int \frac{1}{a+bx^2}\,c\right.}{2ab^2d}\\ &= -\frac{(a-b)(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{\tan(c+dx)}{b^2d}+\frac{(a-b)^2\tan(c+}{2ab^2d\left(a+b\tan^2(c\right.}\end{aligned}$$

Mathematica [A]
time = 0.42, size = 104, normalized size = 1.00

$$\frac{-\frac{(a-b)(3a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{(a-b)^2\sqrt{b}\,\sin(2(c+dx))}{a(a+b+(a-b)\cos(2(c+dx)))}+2\sqrt{b}\,\tan(c+dx)}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (-(((a - b)*(3*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2)) + ((
a - b)^2*Sqrt[b]*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])) +
 2*Sqrt[b]*Tan[c + d*x])/(2*b^(5/2)*d)
```

Maple [A]
time = 0.38, size = 97, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{-\frac{(a^2-2ab+b^2)\tan(dx+c)}{2a\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{(3a^2-2ab-b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{-\frac{(a^2-2ab+b^2)\tan(dx+c)}{2a\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{(3a^2-2ab-b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}}{d}$
risch	$\frac{i\left(-3a^2e^{4i(dx+c)}+2ab\,e^{4i(dx+c)}+b^2e^{4i(dx+c)}-6a^2e^{2i(dx+c)}-2ab\,e^{2i(dx+c)}-3a^2+4ab-b^2\right)}{a\,b^2d\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-a+b\right)\left(e^{2i(dx+c)}+1\right)}-\frac{3a\ln\left(e^{2i(dx+c)}-\frac{2iab-\sqrt{}}{}\right.}{4\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*tan(d*x+c)-1/b^2*(-1/2*(a^2-2*a*b+b^2)/a*tan(d*x+c)/(a+b*tan(d*x
+c)^2)+1/2*(3*a^2-2*a*b-b^2)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
))
```

Maxima [A]
time = 0.53, size = 100, normalized size = 0.96

$$\frac{\frac{(a^2-2\,ab+b^2)\tan(dx+c)}{ab^3\tan(dx+c)^2+a^2b^2}+\frac{2\,\tan(dx+c)}{b^2}-\frac{(3\,a^2-2\,ab-b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 - 2*a*b + b^2)*tan(d*x + c)/(a*b^3*tan(d*x + c)^2 + a^2*b^2) + 2*
tan(d*x + c)/b^2 - (3*a^2 - 2*a*b - b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/(
sqrt(a*b)*a*b^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(92) = 184.
time = 2.54, size = 479, normalized size = 4.61

$$\left[\frac{\left((3\,a^3-5\,a^2b+ab^2+b^3)\cos(dx+c)^3+(3\,a^2b-2\,ab^2-b^3)\cos(dx+c)\right)\sqrt{-ab}\,\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)+4\left(2\,a^2b^2+(3\,a^3b-4\,a^2b^2+ab^3)\cos(dx+c)^2\right)\sin(dx+c)}{8\left(a^2b^4d\cos(dx+c)+(a^3b^3-a^2b^4)d\cos(dx+c)^3\right)},\ \frac{\left((3\,a^3-5\,a^2b+ab^2+b^3)\cos(dx+c)^3+(3\,a^2b-2\,ab^2-b^3)\cos(dx+c)\right)\sqrt{ab}\,\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)+2\left(2\,a^2b^2+(3\,a^3b-4\,a^2b^2+ab^3)\cos(dx+c)^2\right)\sin(dx+c)}{4\left(a^2b^4d\cos(dx+c)+(a^3b^3-a^2b^4)d\cos(dx+c)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(d*x + c)^3 + (3*a^2*b - 2*a*b^2
- b^3)*cos(d*x + c))*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2
*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))
*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*
b - b^2)*cos(d*x + c)^2 + b^2)) + 4*(2*a^2*b^2 + (3*a^3*b - 4*a^2*b^2 + a*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^4*d*cos(d*x + c) + (a^3*b^3 - a^2*
b^4)*d*cos(d*x + c)^3), 1/4*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(d*x + c)^
3 + (3*a^2*b - 2*a*b^2 - b^3)*cos(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*c
os(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))) + 2*(2*a^2*b^
2 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^4*d*
cos(d*x + c) + (a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.73, size = 128, normalized size = 1.23

$$\frac{\frac{2\tan(dx+c)}{b^2} - \frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(3a^2-2ab-b^2)}{\sqrt{ab}\,ab^2} + \frac{a^2\tan(dx+c)-2ab\tan(dx+c)+b^2\tan(dx+c)}{\left(b\tan(dx+c)^2+a\right)ab^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*tan(d*x + c)/b^2 - (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*t
an(d*x + c)/sqrt(a*b)))*(3*a^2 - 2*a*b - b^2)/(sqrt(a*b)*a*b^2) + (a^2*tan(
d*x + c) - 2*a*b*tan(d*x + c) + b^2*tan(d*x + c))/((b*tan(d*x + c)^2 + a)*a
*b^2))/d
```

Mupad [B]
time = 12.41, size = 119, normalized size = 1.14

$$\frac{\tan(c+d\,x)}{b^2\,d}+\frac{\tan(c+d\,x)\,\left(a^2-2\,a\,b+b^2\right)}{2\,a\,d\,\left(b^3\,\tan\left(c+d\,x\right)^2+a\,b^2\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)\,(a-b)\,(3\,a+b)}{\sqrt{a}\,\left(-3\,a^2+2\,a\,b+b^2\right)}\right)\,(a-b)\,(3\,a+b)}{2\,a^{3/2}\,b^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] tan(c + d*x)/(b^2*d) + (tan(c + d*x)*(a^2 - 2*a*b + b^2))/(2*a*d*(a*b^2 + b
^3*tan(c + d*x)^2)) + (atan((b^(1/2)*tan(c + d*x)*(a - b)*(3*a + b))/(a^(1/
2)*(2*a*b - 3*a^2 + b^2)))*(a - b)*(3*a + b))/(2*a^(3/2)*b^(5/2)*d)
```

3.470 $\int \frac{\sec^4(c+dx)}{(a+b\tan^2(c+dx))^2} dx$

Optimal. Leaf size=77

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\tan(c+dx)}{2abd\,(a+b\tan^2(c+dx))}$$

```
[Out] 1/2*(a+b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(3/2)/d-1/2*(a-b)*ta
n(d*x+c)/a/b/d/(a+b*tan(d*x+c)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3756, 393, 211}

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\tan(c+dx)}{2abd\,(a+b\tan^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - ((
a - b)*Tan[c + d*x])/(2*a*b*d*(a + b*Tan[c + d*x]^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
```

```
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{2abd}$$
$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} - \frac{(a-b)\tan(c+dx)}{2abd\left(a+b\tan^2(c+dx)\right)}$$

Mathematica [A]
time = 0.21, size = 83, normalized size = 1.08

$$\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right) + \frac{\sqrt{a}\,\sqrt{b}\,(-a+b)\sin(2(c+dx))}{a+b+(a-b)\cos(2(c+dx))}}{2a^{3/2}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
[Out] ((a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(-a + b)
*Sin[2*(c + d*x)])/(a + b + (a - b)*Cos[2*(c + d*x)]))/(2*a^(3/2)*b^(3/2)*d
)
```

Maple [A]
time = 0.35, size = 69, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{(a-b)\tan(dx+c)}{2ab\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{(a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}}{d}$
default	$\frac{-\frac{(a-b)\tan(dx+c)}{2ab\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{(a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}}{d}$

risch	$-\frac{i\left(a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}+a-b\right)}{abd\left(a\,e^{4i(dx+c)}-b\,e^{4i(dx+c)}+2a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+a-b\right)}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,db}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*(a-b)/a/b*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*(a+b)/a/b/(a*b)^(1/2)
*arctan(b*tan(d*x+c)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.56, size = 69, normalized size = 0.90

$$-\frac{\frac{(a-b)\tan(dx+c)}{ab^2\tan(dx+c)^2+a^2b}-\frac{(a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*tan(d*x + c)/(a*b^2*tan(d*x + c)^2 + a^2*b) - (a + b)*arctan(
b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*a*b))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(65) = 130.
time = 3.76, size = 367, normalized size = 4.77

$$\left[-\frac{4\left(a^2b-ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2+ab+b^2\right)\sqrt{-ab}\,\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(dx+c)^4-2\left(3\,ab+b^2\right)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{\left(a^2-2\,ab+b^2\right)\cos(dx+c)^4+2\left(ab-b^2\right)\cos(dx+c)^2+b^2}\right)}{8\left(a^2b^3d+\left(a^3b^2-a^2b^3\right)d\cos\left(dx+c\right)^2\right)},\ -\frac{2\left(a^2b-ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2+ab+b^2\right)\sqrt{ab}\,\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)}{4\left(a^2b^3d+\left(a^3b^2-a^2b^3\right)d\cos\left(dx+c\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + ((a^2 - b^2)*cos(d*x +
 c)^2 + a*b + b^2)*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(
3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*s
qrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b
- b^2)*cos(d*x + c)^2 + b^2)))/(a^2*b^3*d + (a^3*b^2 - a^2*b^3)*d*cos(d*x +
 c)^2), -1/4*(2*(a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + ((a^2 - b^2)*co
s(d*x + c)^2 + a*b + b^2)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)
*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))))/(a^2*b^3*d + (a^3*b^2 - a^2*b^
3)*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^4\left(c+dx\right)}{\left(a+b\tan^2\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.71, size = 92, normalized size = 1.19

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)(a+b)}{\sqrt{ab}\,ab}-\frac{a\tan(dx+c)-b\tan(dx+c)}{\left(b\tan(dx+c)^2+a\right)ab}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)
))*(a + b)/(sqrt(a*b)*a*b) - (a*tan(d*x + c) - b*tan(d*x + c))/((b*tan(d*x
+ c)^2 + a)*a*b))/d
```

Mupad [B]
time = 12.19, size = 65, normalized size = 0.84

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)}{\sqrt{a}}\right)\,(a+b)}{2\,a^{3/2}\,b^{3/2}\,d}-\frac{\tan(c+d\,x)\,(a-b)}{2\,a\,b\,d\,\left(b\tan\left(c+d\,x\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] (atan((b^(1/2)*tan(c + d*x))/a^(1/2))*(a + b))/(2*a^(3/2)*b^(3/2)*d) - (tan
(c + d*x)*(a - b))/(2*a*b*d*(a + b*tan(c + d*x)^2))
```

3.471 $\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\tan(c+dx)}{2ad\,(a+b\tan^2(c+dx))}$$

```
[Out] 1/2*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/d/b^(1/2)+1/2*tan(d*x+c)/a/d
/(a+b*tan(d*x+c)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3756, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\tan(c+dx)}{2ad\,(a+b\tan^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Tan[c + d*x]
/(2*a*d*(a + b*Tan[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
```

```
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{2ad}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\tan(c+dx)}{2ad\left(a+b\tan^2(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 63, normalized size = 0.95

$$\frac{\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}\tan(c+dx)}{a+b\tan^2(c+dx)}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
[Out] (ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tan[c + d*x])/(a
 + b*Tan[c + d*x]^2))/(2*a^(3/2)*d)
```

Maple [A]
time = 0.28, size = 55, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{2a\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{d}$
default	$\frac{\frac{\tan(dx+c)}{2a\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{d}$

risch	$\frac{i\left(a\,\mathrm{e}^{2i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}+a-b\right)}{ad(a-b)\left(a\,\mathrm{e}^{4i(dx+c)}-b\,\mathrm{e}^{4i(dx+c)}+2a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}+a-b\right)}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*tan(d*x+c)/a/(a+b*tan(d*x+c)^2)+1/2/a/(a*b)^(1/2)*arctan(b*tan(d*x
+c)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.53, size = 53, normalized size = 0.80

$$\frac{\frac{\tan(dx+c)}{ab\tan(dx+c)^2+a^2}+\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(tan(d*x + c)/(a*b*tan(d*x + c)^2 + a^2) + arctan(b*tan(d*x + c)/sqrt(a
*b))/(sqrt(a*b)*a))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(54) = 108.
time = 2.70, size = 327, normalized size = 4.95

$$\left[\frac{4\,ab\cos(dx+c)\sin(dx+c)-\left((a-b)\cos(dx+c)^2+b\right)\sqrt{-ab}\,\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)}{8\left(a^2b^2d+(a^3b-a^2b^2)d\cos(dx+c)^2\right)},\ \frac{2\,ab\cos(dx+c)\sin(dx+c)-\left((a-b)\cos(dx+c)^2+b\right)\sqrt{ab}\,\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)}{4\left(a^2b^2d+(a^3b-a^2b^2)d\cos(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*b*cos(d*x + c)*sin(d*x + c) - ((a - b)*cos(d*x + c)^2 + b)*sqrt(-
a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)
^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) +
b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b
^2)))/(a^2*b^2*d + (a^3*b - a^2*b^2)*d*cos(d*x + c)^2), 1/4*(2*a*b*cos(d*x
+ c)*sin(d*x + c) - ((a - b)*cos(d*x + c)^2 + b)*sqrt(a*b)*arctan(1/2*((a +
 b)*cos(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))))/(a^2*b^
2*d + (a^3*b - a^2*b^2)*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2)**2, x)
```

Giac [A]
time = 0.70, size = 70, normalized size = 1.06

$$\frac{\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{\tan(dx+c)}{\left(b\tan(dx+c)^2+a\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)
))/(sqrt(a*b)*a) + tan(d*x + c)/((b*tan(d*x + c)^2 + a)*a))/d
```

Mupad [B]
time = 12.14, size = 54, normalized size = 0.82

$$\frac{\tan(c+d\,x)}{2\,a\,d\,\left(b\tan\left(c+d\,x\right)^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)^2),x)
```

```
[Out] tan(c + d*x)/(2*a*d*(a + b*tan(c + d*x)^2)) + atan((b^(1/2)*tan(c + d*x))/a
^(1/2))/(2*a^(3/2)*b^(1/2)*d)
```

3.472 $\int \frac{\cos^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$\frac{(a-5b)x}{2(a-b)^3}+\frac{(5a-b)b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\,(a+b\tan^2(c+dx))}+\frac{b(a+b)\tan(c+dx)}{2a(a-b)^2d\,(a+b\tan^2(c+dx))}$$

```
[Out] 1/2*(a-5*b)*x/(a-b)^3+1/2*(5*a-b)*b^(3/2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2)
)/a^(3/2)/(a-b)^3/d+1/2*cos(d*x+c)*sin(d*x+c)/(a-b)/d/(a+b*tan(d*x+c)^2)+1/
2*b*(a+b)*tan(d*x+c)/a/(a-b)^2/d/(a+b*tan(d*x+c)^2)
```

Rubi [A]

time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3756, 425, 541, 536, 209, 211}

$$\frac{b^{3/2}(5a-b)\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{b(a+b)\tan(c+dx)}{2ad(a-b)^2\,(a+b\tan^2(c+dx))}+\frac{\sin(c+dx)\cos(c+dx)}{2d(a-b)\,(a+b\tan^2(c+dx))}+\frac{x(a-5b)}{2(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((a - 5*b)*x)/(2*(a - b)^3) + ((5*a - b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*
x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*(a -
 b)*d*(a + b*Tan[c + d*x]^2)) + (b*(a + b)*Tan[c + d*x])/(2*a*(a - b)^2*d*(
a + b*Tan[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\,(a+b\tan^2(c+dx))} - \frac{\text{Subst}\left(\int \frac{-a+2b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{2(a-b)d}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\,(a+b\tan^2(c+dx))} + \frac{b(a+b)\tan(c+dx)}{2a(a-b)^2d\,(a+b\tan^2(c+dx))} - \frac{\text{Subst}\Big(\int}{}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\,(a+b\tan^2(c+dx))} + \frac{b(a+b)\tan(c+dx)}{2a(a-b)^2d\,(a+b\tan^2(c+dx))} + \frac{(a-5b)}{}$$

$$= \frac{(a-5b)x}{2(a-b)^3} + \frac{(5a-b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d} + \frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\,(a+b\tan^2(c+}$$

Mathematica [A]
time = 0.81, size = 116, normalized size = 0.78

$$\frac{2(a-5b)(c+dx) - \frac{2b^{3/2}(-5a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + (a-b)\sin(2(c+dx)) + \frac{2(a-b)b^2\sin(2(c+dx))}{a(a+b+(a-b)\cos(2(c+dx)))}}{4(a-b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
[Out] (2*(a - 5*b)*(c + d*x) - (2*b^(3/2)*(-5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x]
)/Sqrt[a]])/a^(3/2) + (a - b)*Sin[2*(c + d*x)] + (2*(a - b)*b^2*Sin[2*(c +
d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))/(4*(a - b)^3*d)
```

Maple [A]
time = 0.43, size = 128, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(dx+c)}{1+\tan^2(dx+c)}+\frac{(a-5b)\arctan(\tan(dx+c))}{2}}{(a-b)^3}+\frac{b^2\left(\frac{(a-b)\tan(dx+c)}{2a\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{(5a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^3}}{d}$

default	$\frac{\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(dx+c)}{1+\tan^2(dx+c)}+\frac{(a-5b)\arctan(\tan(dx+c))}{2}}{(a-b)^3}+\frac{b^2\left(\frac{(a-b)\tan(dx+c)}{2a\left(a+b\left(\tan^2(dx+c)\right)\right)}+\frac{(5a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^3}}{d}$
risch	$\frac{xa}{2(a^2-2ab+b^2)(a-b)}-\frac{5xb}{2(a^2-2ab+b^2)(a-b)}-\frac{i e^{2i(dx+c)}}{8d(a^2-2ab+b^2)}+\frac{i e^{-2i(dx+c)}}{8d(a^2-2ab+b^2)}+\frac{ib^2\left(a\,e^{2i(d}\right.}{d(-a+b)^3a\left(-a\,e^{4i(dx+c)}+b\,e\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a-b)^3*((1/2*a-1/2*b)*tan(d*x+c)/(1+tan(d*x+c)^2)+1/2*(a-5*b)*arcta
n(tan(d*x+c)))+b^2/(a-b)^3*(1/2/a*(a-b)*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*(
5*a-b)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.53, size = 209, normalized size = 1.41

$$\frac{\frac{(dx+c)(a-5\,b)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(5\,ab^2-b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\sqrt{ab}}+\frac{(ab+b^2)\tan(dx+c)^3+(a^2+b^2)\tan(dx+c)}{(a^3b-2\,a^2b^2+ab^3)\tan(dx+c)^4+a^4-2\,a^3b+a^2b^2+(a^4-a^3b-a^2b^2+ab^3)\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*(a - 5*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (5*a*b^2 - b^3)*
arctan(b*tan(d*x + c)/sqrt(a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(
a*b)) + ((a*b + b^2)*tan(d*x + c)^3 + (a^2 + b^2)*tan(d*x + c))/((a^3*b - 2
*a^2*b^2 + a*b^3)*tan(d*x + c)^4 + a^4 - 2*a^3*b + a^2*b^2 + (a^4 - a^3*b -
 a^2*b^2 + a*b^3)*tan(d*x + c)^2))/d
```

Fricas [A]
time = 3.92, size = 614, normalized size = 4.15

$$\left[\frac{4\,(a^3-6\,a^2b+5\,ab^2)dx\cos(dx+c)^2+4\,(a^2b-5\,ab^2)dx+(5\,ab^2-b^3+(5\,a^2b-6\,ab^2+b^3)\cos(dx+c)^2)\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2-4\left((a^2+ab)\cos(dx+c)^3-ab\cos(dx+c)\right)\sqrt{-\frac{b}{a}}\sin(dx+c)+b^2}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)+4\left((a^3-2\,a^2b+ab^2)\cos(dx+c)^3+(a^2b-b^3)\cos(dx+c)\right)\sin(dx+c)}{8\left((a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)d\cos(dx+c)^2+(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4)d\right)},\ \frac{2\,(a^3-6\,a^2b+5\,ab^2)dx\cos(dx+c)^2+2\,(a^2b-5\,ab^2)dx-(5\,ab^2-b^3+(5\,a^2b-6\,ab^2+b^3)\cos(dx+c)^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(dx+c)\sin(dx+c)}\right)+2\left((a^3-2\,a^2b+ab^2)\cos(dx+c)^3+(a^2b-b^3)\cos(dx+c)\right)\sin(dx+c)}{4\left((a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)d\cos(dx+c)^2+(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^3 - 6*a^2*b + 5*a*b^2)*d*x*cos(d*x + c)^2 + 4*(a^2*b - 5*a*b^2)*
d*x + (5*a*b^2 - b^3 + (5*a^2*b - 6*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(-b/a)
*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 -
 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*sqrt(-b/a)*sin(d*x + c)
+ b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 +
```

```
b^2)) + 4*((a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^3 + (a^2*b - b^3)*cos(d*x
+ c))*sin(d*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cos(
d*x + c)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d), 1/4*(2*(a^3 - 6*a^
2*b + 5*a*b^2)*d*x*cos(d*x + c)^2 + 2*(a^2*b - 5*a*b^2)*d*x - (5*a*b^2 - b^
3 + (5*a^2*b - 6*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(b/a)*arctan(1/2*((a + b)
*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d*x + c)*sin(d*x + c))) + 2*((a^3 - 2
*a^2*b + a*b^2)*cos(d*x + c)^3 + (a^2*b - b^3)*cos(d*x + c))*sin(d*x + c))/
((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cos(d*x + c)^2 + (a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.75, size = 211, normalized size = 1.43

$$\frac{\frac{(dx+c)(a-5b)}{a^3-3a^2b+3ab^2-b^3}+\frac{(5ab^2-b^3)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{(a^4-3a^3b+3a^2b^2-ab^3)\sqrt{ab}}+\frac{ab\tan(dx+c)^3+b^2\tan(dx+c)^3+a^2\tan(dx+c)+b^2\tan(dx+c)}{\left(b\tan(dx+c)^4+a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(a^3-2a^2b+ab^2)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*(a - 5*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (5*a*b^2 - b^3)*
(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/((
a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)) + (a*b*tan(d*x + c)^3 + b^2*t
an(d*x + c)^3 + a^2*tan(d*x + c) + b^2*tan(d*x + c))/((b*tan(d*x + c)^4 + a
*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a^3 - 2*a^2*b + a*b^2)))/d
```

Mupad [**B**]
time = 16.29, size = 2500, normalized size = 16.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x)^2)^2,x)
```

```
[Out] ((tan(c + d*x)*(a^2 + b^2))/(2*a*(a^2 - 2*a*b + b^2)) + (b*tan(c + d*x)^3*(
a + b))/(2*a*(a^2 - 2*a*b + b^2)))/(d*(a + tan(c + d*x)^2*(a + b) + b*tan(c
```

+ d*x)^4)) - (atan(((((((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) - (tan(c + d*x)*(a - 5*b)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - (tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b)*1i)/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - (((((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) + (tan(c + d*x)*(a - 5*b)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) + (tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b)*1i)/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))/((((((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) - (tan(c + d*x)*(a - 5*b)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - (tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - ((21*a*b^6)/4 - (5*b^7)/4 + (21*a^2*b^5)/4 - (5*a^3*b^4)/4)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) + (((((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) + (tan(c + d*x)*(a - 5*b)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) + (tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i))))*(a - 5*b)*1i)/(2*d*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - (atan((((5*a - b)*(-a^3*b^3)^(1/2)*((tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)) - (((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) - (tan(c + d*x)*(5*a - b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16 *a^9*b^2))/(8*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(5*a - b)*(-a^3*b^3)^(1/2))/(4*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*1i)/(4*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) + ((5*a - b)*(-a^3*b^3)^(1/2)*((tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)) + (((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) + (tan(c + d*x)*(5*a - b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(5*a - b)*(-a^3*b^3)^(1/2))/(4*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*1i)/(4*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))/(((21*a*b^6)/4 - (5*b^7)/4 + (21*a^2*b^5)/4 - (5*a^3*b^4)/4)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) + ((5*a - b)*(-a^3*b^3)^(1/2)*((tan(c + d*x)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)) - (((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) - (tan(c + d*x)*(5*a - b)*(-a^3*b^3)^(1/2)*(16*a^2...

3.473 $\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=212

$$\frac{(3a^2-14ab+35b^2)\,x}{8(a-b)^4}-\frac{(7a-b)b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^4d}+\frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\,(a+b\tan^2(c+dx))}+\frac{\cos^3(c+}{4(a-b)d\,(a}$$

```
[Out] 1/8*(3*a^2-14*a*b+35*b^2)*x/(a-b)^4-1/2*(7*a-b)*b^(5/2)*arctan(b^(1/2)*tan(
d*x+c)/a^(1/2))/a^(3/2)/(a-b)^4/d+3/8*(a-3*b)*cos(d*x+c)*sin(d*x+c)/(a-b)^2
/d/(a+b*tan(d*x+c)^2)+1/4*cos(d*x+c)^3*sin(d*x+c)/(a-b)/d/(a+b*tan(d*x+c)^2
)+1/8*(a-4*b)*b*(3*a+b)*tan(d*x+c)/a/(a-b)^3/d/(a+b*tan(d*x+c)^2)
```

Rubi [A]
time = 0.20, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3756, 425, 541, 536, 209, 211}

$$-\frac{b^{5/2}(7a-b)\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^4}+\frac{x(3a^2-14ab+35b^2)}{8(a-b)^4}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{8ad(a-b)^3\,(a+b\tan^2(c+dx))}+\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a-b)\,(a+b\tan^2(c+dx))}+\frac{3(a-3b)\sin(c+dx)\cos(c+dx)}{8d(a-b)^2\,(a+b\tan^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 14*a*b + 35*b^2)*x)/(8*(a - b)^4) - ((7*a - b)*b^(5/2)*ArcTan[(Sq
rt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^4*d) + (3*(a - 3*b)*Cos[c
+ d*x]*Sin[c + d*x])/(8*(a - b)^2*d*(a + b*Tan[c + d*x]^2)) + (Cos[c + d*x]
^3*Sin[c + d*x])/(4*(a - b)*d*(a + b*Tan[c + d*x]^2)) + ((a - 4*b)*b*(3*a +
 b)*Tan[c + d*x])/(8*a*(a - b)^3*d*(a + b*Tan[c + d*x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-3a+4b-5bx^2}{(1+x^2)^2(a+bx^2)^2}\,dx, x, \tan(c+dx)\right)}{4(a-b)d}$$

$$= \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{(a-4b\cdots}{8a(a-b\cdots}$$

$$= \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{(a-4b\cdots}{8a(a-b\cdots}$$

$$= \frac{(3a^2-14ab+35b^2)\,x}{8(a-b)^4} - \frac{(7a-b)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^4d} + \frac{3(a-3b)\cos(c\cdots}{8(a-b)^2d\,(a\cdots}$$

Mathematica [A]

time = 1.36, size = 148, normalized size = 0.70

$$\frac{4(3a^2-14ab+35b^2)(c+dx) + \frac{16b^{5/2}(-7a+b)\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + 8(a-3b)(a-b)\sin(2(c+dx)) - \frac{16(a-b)b^3\sin(2(c+dx))}{a(a+b+(a-b)\cos(2(c+dx)))} + (a-b)^2\sin(4(c+dx))}{32(a-b)^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 - 14*a*b + 35*b^2)*(c + d*x) + (16*b^(5/2)*(-7*a + b)*ArcTan[(Sqr
t[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2) + 8*(a - 3*b)*(a - b)*Sin[2*(c + d*x)]
 - (16*(a - b)*b^3*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)]))
 + (a - b)^2*Sin[4*(c + d*x)])/(32*(a - b)^4*d)
```

Maple [A]

time = 0.51, size = 173, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-\frac{7}{4}ab+\frac{11}{8}b^2\right)\left(\tan^3(dx+c)\right)+\left(-\frac{9}{4}ab+\frac{13}{8}b^2+\frac{5}{8}a^2\right)\tan(dx+c)}{\left(1+\tan^2(dx+c)\right)^2}+\frac{\left(3a^2-14ab+35b^2\right)\arctan(\tan(dx+c))}{8}}{(a-b)^4} - \frac{b^3\left(\frac{(a-b)\tan(dx+\cdots}{2a\left(a+b\left(\tan^2(dx+\cdots\right.\right.}\cdots\right.}{\cdots}}{d}$

default	$\frac{\frac{\left(\frac{3}{8}a^2-\frac{7}{4}ab+\frac{11}{8}b^2\right)\left(\tan^3(dx+c)\right)+\left(-\frac{9}{4}ab+\frac{13}{8}b^2+\frac{5}{8}a^2\right)\tan(dx+c)}{\left(1+\tan^2(dx+c)\right)^2}+\frac{\left(3a^2-14ab+35b^2\right)\arctan(\tan(dx+c))}{8}}{(a-b)^4}}{d} - \frac{b^3\left(\frac{(a-b)\tan(dx}{2a\left(a+b\left(\tan^2(d\right.\right.}\right.}{}$
risch	$\frac{3x\,a^2}{8(a^2-2ab+b^2)(a-b)^2}-\frac{7xab}{4(a^2-2ab+b^2)(a-b)^2}+\frac{35x\,b^2}{8(a^2-2ab+b^2)(a-b)^2}-\frac{ie^{4i(dx+c)}}{64(a-b)^2d}-\frac{ie^{2i(dx+c)}a}{8(a-b)^3d}+\frac{3ie^{2i(dx-}}{8(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a-b)^4*(((3/8*a^2-7/4*a*b+11/8*b^2)*tan(d*x+c)^3+(-9/4*a*b+13/8*b^2
+5/8*a^2)*tan(d*x+c))/(1+tan(d*x+c)^2)^2+1/8*(3*a^2-14*a*b+35*b^2)*arctan(t
an(d*x+c)))-b^3/(a-b)^4*(1/2/a*(a-b)*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*(7*a
-b)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))))
```

Maxima [A]
time = 0.52, size = 355, normalized size = 1.67

$$\frac{\frac{(3a^2-14ab+35b^2)(dx+c)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{4(7ab^3-b^4)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4)\sqrt{ab}}+\frac{(3a^2b-11ab^2-4b^3)\tan(dx+c)^5+(3a^3-6a^2b-13ab^2-8b^3)\tan(dx+c)^3+(5a^3-13a^2b-4b^3)\tan(dx+c)}{(a^4b-3a^3b^2+3a^2b^3-ab^4)\tan(dx+c)^6+a^5-3a^4b+3a^3b^2-a^2b^3+(a^5-a^4b-3a^3b^2+5a^2b^3-2ab^4)\tan(dx+c)^4+(2a^5-5a^4b+3a^3b^2+a^2b^3-ab^4)\tan(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 - 14*a*b + 35*b^2)*(d*x + c)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b
^3 + b^4) - 4*(7*a*b^3 - b^4)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^5 - 4*a^
4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sqrt(a*b)) + ((3*a^2*b - 11*a*b^2 - 4*
b^3)*tan(d*x + c)^5 + (3*a^3 - 6*a^2*b - 13*a*b^2 - 8*b^3)*tan(d*x + c)^3 +
 (5*a^3 - 13*a^2*b - 4*b^3)*tan(d*x + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 -
 a*b^4)*tan(d*x + c)^6 + a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^5 - a^4*b
 - 3*a^3*b^2 + 5*a^2*b^3 - 2*a*b^4)*tan(d*x + c)^4 + (2*a^5 - 5*a^4*b + 3*a
^3*b^2 + a^2*b^3 - a*b^4)*tan(d*x + c)^2))/d
```

Fricas [A]
time = 2.14, size = 801, normalized size = 3.78

$$\left[\frac{(3a^4-17a^3b+49a^2b^2-35ab^3)dx\cos(dx+c)^2+(3a^3b-14a^2b^2+35ab^3)dx-(7ab^3-b^4+(7a^2b^2-8ab^3+b^4)\cos(dx+c)^2)\sqrt{-\frac{b}{a}}\log\left(\text{[illegible]}\right)+(2(a^4-3a^3b+3a^2b^2-ab^3)\cos(dx+c)^5+3(a^4-5a^3b+7a^2b^2-3ab^3)\cos(dx+c)^3+(3a^3b-14a^2b^2+7ab^3+4b^4)\cos(dx+c))\sin(dx+c)}{8((a^6-5a^5b+10a^4b^2-10a^3b^3+5a^2b^4-ab^5)d\cos(dx+c)^2+(a^5b-4a^4b^2+6a^3b^3-4a^2b^4+ab^5)d)}, \frac{(3a^4-17a^3b+49a^2b^2-35ab^3)dx\cos(dx+c)^2+(3a^3b-14a^2b^2+35ab^3)dx+2(7ab^3-b^4+(7a^2b^2-8ab^3+b^4)\cos(dx+c)^2)\sqrt{\frac{b}{a}}\arctan\left(\text{[illegible]}\right)+(2(a^4-3a^3b+3a^2b^2-ab^3)\cos(dx+c)^5+3(a^4-5a^3b+7a^2b^2-3ab^3)\cos(dx+c)^3+(3a^3b-14a^2b^2+7ab^3+4b^4)\cos(dx+c))\sin(dx+c)}{8((a^6-5a^5b+10a^4b^2-10a^3b^3+5a^2b^4-ab^5)d\cos(dx+c)^2+(a^5b-4a^4b^2+6a^3b^3-4a^2b^4+ab^5)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^4 - 17*a^3*b + 49*a^2*b^2 - 35*a*b^3)*d*x*cos(d*x + c)^2 + (3*a^
3*b - 14*a^2*b^2 + 35*a*b^3)*d*x - (7*a*b^3 - b^4 + (7*a^2*b^2 - 8*a*b^3 +
```

```
b^4)*cos(d*x + c)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2
*(3*a*b + b^2)*cos(d*x + c)^2 - 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x
 + c))*sqrt(-b/a)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 +
 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + (2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b
^3)*cos(d*x + c)^5 + 3*(a^4 - 5*a^3*b + 7*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^3
 + (3*a^3*b - 14*a^2*b^2 + 7*a*b^3 + 4*b^4)*cos(d*x + c))*sin(d*x + c))/((a
^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d*cos(d*x + c)^
2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d), 1/8*((3*a^4 - 1
7*a^3*b + 49*a^2*b^2 - 35*a*b^3)*d*x*cos(d*x + c)^2 + (3*a^3*b - 14*a^2*b^2
 + 35*a*b^3)*d*x + 2*(7*a*b^3 - b^4 + (7*a^2*b^2 - 8*a*b^3 + b^4)*cos(d*x +
 c)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d
*x + c)*sin(d*x + c))) + (2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(d*x + c
)^5 + 3*(a^4 - 5*a^3*b + 7*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^3 + (3*a^3*b - 1
4*a^2*b^2 + 7*a*b^3 + 4*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6 - 5*a^5*b +
10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*
a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.79, size = 269, normalized size = 1.27

$$-\frac{\frac{4\,b^3\tan(dx+c)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\left(b\tan(dx+c)^2+a\right)}-\frac{(3\,a^2-14\,ab+35\,b^2)(dx+c)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{4\,(7\,ab^3-b^4)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)\right)}{(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)\sqrt{ab}}-\frac{3\,a\tan(dx+c)^3-11\,b\tan(dx+c)^3+5\,a\tan(dx+c)-13\,b\tan(dx+c)}{(a^3-3\,a^2b+3\,ab^2-b^3)\left(\tan(dx+c)^2+1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*tan(d*x + c)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(b*tan(d*x +
c)^2 + a)) - (3*a^2 - 14*a*b + 35*b^2)*(d*x + c)/(a^4 - 4*a^3*b + 6*a^2*b^2
 - 4*a*b^3 + b^4) + 4*(7*a*b^3 - b^4)*(pi*floor((d*x + c)/pi + 1/2)*sgn(b)
+ arctan(b*tan(d*x + c)/sqrt(a*b)))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
 + a*b^4)*sqrt(a*b)) - (3*a*tan(d*x + c)^3 - 11*b*tan(d*x + c)^3 + 5*a*tan(
d*x + c) - 13*b*tan(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(tan(d*x + c
)^2 + 1)^2))/d
```

Mupad [B]
time = 17.28, size = 2500, normalized size = 11.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x)^2)^2,x)

[Out] - ((tan(c + d*x)^5*(11*a*b^2 - 3*a^2*b + 4*b^3))/(8*a*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(c + d*x)^3*(13*a*b^2 + 6*a^2*b - 3*a^3 + 8*b^3))/(8*a*(a - b)*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)*(13*a^2*b - 5*a^3 + 4*b^3))/(8*a *(a - b)*(a^2 - 2*a*b + b^2)))/(d*(a + b*tan(c + d*x)^6 + tan(c + d*x)^2*(2 *a + b) + tan(c + d*x)^4*(a + 2*b))) - (atan((((((2*a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^8 + 1197*a^ 7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35*a^11*b^3)/2 - (3*a^12 *b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(c + d*x)*(a^2* 3i - a*b*14i + b^2*35i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168 *a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792 *a^10*b^3 + 256*a^11*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) *(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^ 2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a ^2*b^2)) - (tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a^2*b^7 - 980*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^ 3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i)*1i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((((2*a*b^13 - 28 *a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^ 8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35*a^11*b^3)/ 2 - (3*a^12*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 8 4*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) + (tan(c + d*x)*(a^2*3i - a*b*14i + b^2*35i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4 *b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9 *b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a^2*b^7 - 98 0*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b + a^2 *b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14 i + b^2*35i)*1i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))/((((((2* a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35 *a^11*b^3)/2 - (3*a^12*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36* a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(c + d*x)*(a^2*3i - a*b*14i + b^2*35i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(512*(a^4 - 4*a^3*b - 4*a* b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a

^2*b^7 - 980*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - ((651*a*b^9)/64 - (35*b^10)/16 + (1275*a^2*b^8)/32 - (451*a^3*b^7)/16 + (267*a^4*b^6)/32 - (63*a^5*b^5)/64)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) + (((((2*a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35*a^11*b^3)/2 - (3*a^12*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) + (tan(c + d*x)*(a^2*3i - a*b*14i + b^2*35i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a^2*b^7 - 980*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))))*(a^2*3i - a*b*14i + b^2*35i)*1i)/(8*d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (atan(((((tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a^2*b^7 - 980*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((7*a - b)*(-a^3*b^5)^(1/2)*((2*a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35*a^11*b^3)/2 - (3*a...

3.474 $\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx$

Optimal. Leaf size=95

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+2p)}\, {}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1+m+2p);\frac{1}{2}(3+2p);\sin^2(e+fx)\right)(d\sec(e+fx))^m\tan(e+fx)}{f(1+2p)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*m+p)*hypergeom([1/2+p, 1/2+1/2*m+p],[3/2+p],sin(f*x
+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1+2*p)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3739, 2697}

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\sec(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(m+2p+1)}\, {}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1);\frac{1}{2}(2p+3);\sin^2(e+fx)\right)}{f(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m + 2*p)/2)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m +
 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*Ta
n[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\sec(e+fx))^m \tan^{2p}(e+$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+2p)}\, {}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1+m+2p);\frac{1}{2}(3+2p);\text{s}\right.}{f(1+2}$$

Mathematica [A]
time = 0.13, size = 81, normalized size = 0.85

$$\frac{\cot(e+fx)\, {}_2F_1\left(\frac{m}{2},\frac{1}{2}-p;\frac{2+m}{2};\sec^2(e+fx)\right)(d\sec(e+fx))^m \left(-\tan^2(e+fx)\right)^{\frac{1}{2}-p}\left(b\tan^2(e+fx)\right)^p}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[m/2, 1/2 - p, (2 + m)/2, Sec[e + f*x]^2]*(d
*Sec[e + f*x])^m*(-Tan[e + f*x]^2)^(1/2 - p)*(b*Tan[e + f*x]^2)^p)/(f*m)
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^m \left(b\left(\tan^2(fx+e)\right)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{2}\left(e+fx\right)\right)^{p}\left(d\sec\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*sec(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

```
[Out] int((d/cos(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

3.475 $\int (d\sec(e+fx))^m (a+b\tan^2(e+fx))^p \, dx$

Optimal. Leaf size=108

$$\frac{F_1\left(\frac{1}{2};1-\frac{m}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx)\left(a+b\tan^2(\right.}{f}$$

```
[Out] AppellF1(1/2,1-1/2*m,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*sec(f*x+e))
^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/((sec(f*x+e)^2)^(1/2*m))/((1+b*tan(f*x
+e)^2/a)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3760, 441, 440}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1-\frac{m}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*
(d*Sec[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(Sec[e + f*x]^
2)^(m/2)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3760

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff
*((d*Sec[e + f*x])^m/(f*(Sec[e + f*x]^2)^(m/2))), Subst[Int[(1 + ff^2*x^2)^
(m/2 - 1)*(a + b*ff^2*x^2)^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b,
```

```
d, e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right) \text{Subst}\left(\int (1+x^2)^{-1+\frac{m}{2}}\right.}{f}$$

$$= \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2} \left(a+b\tan^2(e+fx)\right)^p \left(\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};1-\frac{m}{2},-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right) (d\sec(e+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2033 vs. $2(108) = 216$.
time = 14.72, size = 2033, normalized size = 18.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
a)]*(d*Sec[e + f*x])^m*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]*(a + b*Tan[
e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)*((6*a*b*p
*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*
(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p))/(3*a
*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]
+ (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (3*a*AppellF1[1/2, 1 - m/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(m/2)*(a + b
*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (6*a*(-1
+ m/2)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)]*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/
```

```
(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (3*a*(Sec[e + f*x]^2)^(-1 + m
/2)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*(1 - m/2
)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]
*Sec[e + f*x]^2*Tan[e + f*x])/3)*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/
2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*App
ellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] +
 a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/a)])*Tan[e + f*x]^2) - (3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]*
(a + b*Tan[e + f*x]^2)^p*(2*(2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Sec[e + f*x]^2*Tan[e + f*
x] + 3*a*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*a) - (2*(1 - m/2)*Appell
F1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e +
 f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/
2, 1 - m/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f
*x]^2*Tan[e + f*x])/(5*a) - (6*(1 - m/2)*AppellF1[5/2, 2 - m/2, 1 - p, 7/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/5)
+ a*(-2 + m)*((6*b*p*AppellF1[5/2, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/a)]*Sec[e + f*x]^2*Tan[e + f*x])/(5*a) - (6*(2 - m/2)*Ap
pellF1[5/2, 3 - m/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Sec
[e + f*x]^2*Tan[e + f*x])/5))))/(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2
 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)
^2))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x+e\right)\right)^{m} \left(a+b \left(\tan^{2}\left(f x+e\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan(e+f\,x)^2+a\right)^p \left(\frac{d}{\cos(e+f\,x)}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p*(d/cos(e + f*x))^m,x)
```

```
[Out] int((a + b*tan(e + f*x)^2)^p*(d/cos(e + f*x))^m, x)
```

3.476 $\int (d\sec(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx$

Optimal. Leaf size=97

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+np)}\, {}_2F_1\left(\frac{1}{2}(1+np), \frac{1}{2}(1+m+np); \frac{1}{2}(3+np); \sin^2(e+fx)\right)(d\sec(e+fx))^m \tan(e+fx)}{f(1+np)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2*m+1/2)*hypergeom([1/2*n*p+1/2, 1/2*n*p+1/2*m+1/
2],[1/2*n*p+3/2],sin(f*x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3740, 2697}

$$\frac{\tan(e+fx)(d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+np+1)} (b(c\tan(e+fx))^n)^p \, {}_2F_1\left(\frac{1}{2}(np+1), \frac{1}{2}(m+np+1); \frac{1}{2}(np+3); \sin^2(e+fx)\right)}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m +
 n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\sec(e+fx))^m$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1+m+np);\frac{1}{2}(3+}{f}$$

Mathematica [A]
time = 0.11, size = 89, normalized size = 0.92

$$\frac{\cot(e+fx)\,{}_2F_1\left(\frac{m}{2},\frac{1}{2}(1-np);\frac{2+m}{2};\sec^2(e+fx)\right)(d\sec(e+fx))^m\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\left(b(c\tan(e+fx))^n\right)^p}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[m/2, (1 - n*p)/2, (2 + m)/2, Sec[e + f*x]^2
]*(d*Sec[e + f*x])^m*(-Tan[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)
^p)/(f*m)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (d\sec(fx+e))^m \left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan{(e+fx)})^n)^p\,(d\sec{(e+fx)})^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*sec(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(b\,{\left(c\,\mathrm{tan}(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
[Out] int((d/cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

3.477 $\int \sec^6(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=99

$$\frac{\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}+\frac{2\tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3+np)}+\frac{\tan^5(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(5+np)}$$

```
[Out] tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)+2*tan(f*x+e)^3*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p+3)+tan(f*x+e)^5*(b*(c*tan(f*x+e))^n)^p/f/(n*p+5)
```

Rubi [A]
time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2687, 276}

$$\frac{\tan^5(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(np+5)}+\frac{2\tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(np+3)}+\frac{\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)) + (2*Tan[e + f*x]^3*(
b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p)) + (Tan[e + f*x]^5*(b*(c*Tan[e + f*x]
)^n)^p)/(f*(5 + n*p))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^6(e+fx)(c\tan(e-$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int (cx)^{np}\left(1+x^2\right)^2\right.}{f}$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\text{Subst}\left(\int\left((cx)^{np}+\frac{2(cx)^2}{c^2}\right.\right.}{f}$$
$$= \frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)} + \frac{2\tan^3(e+fx)\left(b(c\tan(e+fx\right.}{f(3+np)}$$

Mathematica [A]
time = 1.41, size = 122, normalized size = 1.23

$$\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\left((8+6np+n^2p^2+2(3+np)\cos(2(e+fx))+\cos(4(e+fx)))\sec^4(e+fx)\tan^2(e+fx)+8\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\right)}{f(1+np)(3+np)(5+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p*((8 + 6*n*p + n^2*p^2 + 2*(3 + n*p)*
Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Sec[e + f*x]^4*Tan[e + f*x]^2 + 8*(-Ta
n[e + f*x]^2)^((1 - n*p)/2)))/(f*(1 + n*p)*(3 + n*p)*(5 + n*p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.58, size = 70270, normalized size = 709.80

method	result	size
risch	Expression too large to display	70270

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.34, size = 112, normalized size = 1.13

$$\frac{\frac{b^p c^{np}\left(\tan(fx+e)^n\right)^p\tan(fx+e)^5}{np+5} + \frac{2\,b^p c^{np}\left(\tan(fx+e)^n\right)^p\tan(fx+e)^3}{np+3} + \frac{b^p c^{np}\left(\tan(fx+e)^n\right)^p\tan(fx+e)}{np+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*c^(n*p)*(tan(f*x + e)^n)^p*tan(f*x + e)^5/(n*p + 5) + 2*b^p*c^(n*p)*(t
an(f*x + e)^n)^p*tan(f*x + e)^3/(n*p + 3) + b^p*c^(n*p)*(tan(f*x + e)^n)^p*
tan(f*x + e)/(n*p + 1))/f
```

Fricas [**A**]
time = 3.21, size = 113, normalized size = 1.14

$$\frac{\left(n^2p^2+8\cos\left(fx+e\right)^4+4\left(np+1\right)\cos\left(fx+e\right)^2+4\,np+3\right)e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}\sin\left(fx+e\right)}{\left(fn^3p^3+9\,fn^2p^2+23\,fnp+15\,f\right)\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (n^2*p^2 + 8*cos(f*x + e)^4 + 4*(n*p + 1)*cos(f*x + e)^2 + 4*n*p + 3)*e^(n*
p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))*sin(f*x + e)/((f*n^3*p^3 + 9
*f*n^2*p^2 + 23*f*n*p + 15*f)*cos(f*x + e)^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b(c\tan\left(e+fx\right))^n\right)^p\sec^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**6, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.24Unable to divide
, perhaps due to rounding error%%%{1,[0,1,4,0,0]%%%}+%%%{2,[0,1,2,2,0]%%%}+
%%%{1,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\left(c\tan(e+f\,x)\right)^n\right)^p}{\cos\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6,x)
[Out] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6, x)
```

3.478 $\int \sec^4(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=65

$$\frac{\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}+\frac{\tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3+np)}$$

```
[Out] tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)+tan(f*x+e)^3*(b*(c*tan(f*x+e))^
n)^p/f/(n*p+3)
```

Rubi [A]

time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2687, 14}

$$\frac{\tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(np+3)}+\frac{\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)) + (Tan[e + f*x]^3*(b*
(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sec^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^4(e+fx)(c\tan(e-$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\mathrm{Subst}\left(\int (cx)^{np}\left(1+x^2\right)\, c\right.}{f}$$

$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\mathrm{Subst}\left(\int\left((cx)^{np}+\frac{(cx)^{2+}}{c^2}\right.\right.}{f}$$

$$= \frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)} + \frac{\tan^3(e+fx)\left(b(c\tan(e+fx)\right)}{f(3+np)}$$

Mathematica [A]
time = 1.36, size = 87, normalized size = 1.34

$$\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\left(\left(2+(1+np)\sec^2(e+fx)\right)\tan^2(e+fx)+2\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\right)}{f(1+np)(3+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p*((2 + (1 + n*p)*Sec[e + f*x]^2)*Tan[
e + f*x]^2 + 2*(-Tan[e + f*x]^2)^((1 - n*p)/2)))/(f*(1 + n*p)*(3 + n*p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.00, size = 34277, normalized size = 527.34

method	result	size
risch	Expression too large to display	34277

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.33, size = 75, normalized size = 1.15

$$\frac{\frac{b^p c^{np}\left(\tan(fx+e)^n\right)^p\tan(fx+e)^3}{np+3}+\frac{b^p c^{np}\left(\tan(fx+e)^n\right)^p\tan(fx+e)}{np+1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] (b^p*c^(n*p)*(tan(f*x + e)^n)^p*tan(f*x + e)^3/(n*p + 3) + b^p*c^(n*p)*(tan
(f*x + e)^n)^p*tan(f*x + e)/(n*p + 1))/f
```

Fricas [A]
time = 2.52, size = 80, normalized size = 1.23

$$\frac{\left(np + 2\,\cos\left(fx + e\right)^{2} + 1\right)e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right) + p\log(b)\right)}\sin\left(fx + e\right)}{\left(fn^{2}p^{2} + 4\,fnp + 3\,f\right)\cos\left(fx + e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] (n*p + 2*cos(f*x + e)^2 + 1)*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*lo
g(b))*sin(f*x + e)/((f*n^2*p^2 + 4*f*n*p + 3*f)*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\left(c\tan\left(e + fx\right)\right)^{n}\right)^{p}\sec^{4}\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.64Unable to divide
, perhaps due to rounding error%%%{1,[0,1,2,0,0]%%%}+%%%{1,[0,1,0,2,0]%%%}
/ %%%{
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\left(c\tan(e + f\,x)\right)^{n}\right)^{p}}{\cos\left(e + f\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4,x)
```

```
[Out] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4, x)
```

3.479 $\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=31

$$\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

```
[Out] tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2687, 32}

$$\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^2(e+fx)(c\tan(e$$
$$= \frac{\left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\mathrm{Subst}\left(\int (cx)^{np}\,dx, x, \mathrm{ta}\right.}{f}$$
$$= \frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.00

$$\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
[Out] (Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.16, size = 10286, normalized size = 331.81

method	result	size
risch	Expression too large to display	10286

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [A]
time = 0.32, size = 37, normalized size = 1.19

$$\frac{b^p c^{np}\left(\tan\left(fx+e\right)^n\right)^p \tan\left(fx+e\right)}{(np+1)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] b^p*c^(n*p)*(tan(f*x + e)^n)^p*tan(f*x + e)/((n*p + 1)*f)
```

Fricas [A]
time = 2.27, size = 53, normalized size = 1.71

$$\frac{e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}\sin\left(fx+e\right)}{(fnp+f)\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))*sin(f*x + e)/((f*n*p +
f)*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan{(e+fx)})^n)^p \sec^2{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.11Unable to divide
, perhaps due to rounding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Er
ror: B
```

Mupad [B]
time = 13.29, size = 31, normalized size = 1.00

$$\frac{\tan(e+f\,x)\,\left(b\,{\left(c\tan(e+f\,x)\right)}^n\right)^p}{f\,\left(n\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^2,x)
```

```
[Out] (tan(e + f*x)*(b*(c*tan(e + f*x))^n)^p)/(f*(n*p + 1))
```

3.480 $\int (b(c\tan(e+fx))^n)^p \, dx$

Optimal. Leaf size=61

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*ta
n(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{\tan(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (b(c\tan(e+fx))^n)^p\,dx &= \left((c\tan(e+fx))^{-np}(b(c\tan(e+fx))^n)^p\right)\int (c\tan(e+fx))^{np}\,dx\\ &= \frac{(c(c\tan(e+fx))^{-np}(b(c\tan(e+fx))^n)^p)\,\text{Subst}\left(\int \frac{x^{np}}{c^2+x^2}\,dx, x, c\tan(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 59, normalized size = 0.97

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f+fnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f + f*n*p)
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan{(e + fx)})^n)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p,x)
[Out] int((b*(c*tan(e + f*x))^n)^p, x)
```

3.481 $\int \cos^2(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=61

$$\frac{{}_2F_1\left(2,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}$$

```
[Out] hypergeom([2, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*ta
n(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3740, 2687, 371}

$$\frac{\tan(e+fx)\,{}_2F_1\left(2,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f*
x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cos^2(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \left((c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\int \cos^2(e+fx)(c\tan(e$$

$$= \frac{\left((c\tan(e+fx))^{-np}\,(b(c\tan(e+fx))^n)^p\right)\mathrm{Subst}\left(\int \frac{(cx)^{np}}{(1+x^2)^2}\,dx,x,\mathrm{t}\right.}{f}$$

$$= \frac{{}_2F_1\left(2,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\,\mathrm{ta}}{f(1+np)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 3.42, size = 1060, normalized size = 17.38

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (2*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[e + f*x]^2*Tan[(e + f*x)/2]*(b*
(c*Tan[e + f*x])^n)^p)/(f*((AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (3 +
 n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)/2,
 n*p, 3, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*
x)/2]^2 + n*p*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)/2, n*p, 3, (3 +
 n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Sec[e
 + f*x] + (2*(1 + n*p)*(-AppellF1[(3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 8*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 12*AppellF1[(3 + n*p)/2, n
*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1
[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] - 4*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 4*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2]^2)/(3 + n*p) - 2*n*p*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(1 + n*p)/2, n*p, 2, (
3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[(1 + n*p)
```

```
/2, n*p, 3, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[e +
f*x]*Tan[(e + f*x)/2]^2))
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(b(c\tan\left(fx+e\right))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b(c\tan\left(e+fx\right))^n\right)^p\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cos(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(e+fx)^2 \left(b\left(c\tan(e+fx)\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cos(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

3.482 $\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=93

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(4+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(4+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sec^3(e+fx)\tan(e+fx)\left(b(c\tan(e+\right.}{f(1+np)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+2)*hypergeom([1/2*n*p+2, 1/2*n*p+1/2],[1/2*n*p+3/2]
,sin(f*x+e)^2)*sec(f*x+e)^3*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3740, 2697}

$$\frac{\tan(e+fx)\sec^3(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+4)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((4 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2,
 (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(1 + n*p))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec^3(e+fx)(c\tan(e$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(4+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(4+np);\frac{1}{2}(3+np);\sin^2(e+\right.}{f(1+np)}$$

Mathematica [**A**]
time = 0.09, size = 81, normalized size = 0.87

$$\frac{\csc(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(1-np);\frac{5}{2};\sec^2(e+fx)\right)\sec^2(e+fx)\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\left(b(c\tan(e+fx))^n\right)^p}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Csc[e + f*x]*Hypergeometric2F1[3/2, (1 - n*p)/2, 5/2, Sec[e + f*x]^2]*Sec[
e + f*x]^2*(-Tan[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)^p)/(3*f)
```

Maple [**F**]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\sec^3(fx+e)\right)\left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \sec^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,(c\tan(e+f\,x))^n)^p}{\cos(e+f\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3,x)
[Out] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3, x)
```

3.483 $\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=91

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(2+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sec(e+fx)\tan(e+fx)\left(b(c\tan(e+\right.}{f(1+np)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1)*hypergeom([1/2*n*p+1, 1/2*n*p+1/2],[1/2*n*p+3/2]
,sin(f*x+e)^2)*sec(f*x+e)*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3740, 2697}

$$\frac{\tan(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+2)}\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((2 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2,
 (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x])
^n)^p)/(f*(1 + n*p))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \sec(e+fx)(c\tan(e+$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(2+np)}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np);\frac{1}{2}(3+np);\sin^2(e+f\right.}{f(1+np)}$$

Mathematica [A]
time = 0.05, size = 70, normalized size = 0.77

$$\frac{\csc(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{3}{2};\sec^2(e+fx)\right)\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\left(b(c\tan(e+fx))^n\right)^p}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Csc[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, 3/2, Sec[e + f*x]^2]*(-Ta
n[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)^p)/f
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \sec(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,(c\tan(e+f\,x))^n)^p}{\cos(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x),x)
[Out] int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x), x)
```

3.484 $\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=79

$$\frac{\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p)*hypergeom([1/2*n*p, 1/2*n*p+1/2],[1/2*n*p+3/2],sin
(f*x+e)^2)*sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3740, 2697}

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (3 + n*
p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos(e+fx)(c\tan(e+$$

$$= \frac{\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{np}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+}{f(1+np)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 2.29, size = 482, normalized size = 6.10

$$\frac{(3+np)\left(F_1\left(\frac{1}{2}(1+np);np,1;\frac{1}{2}(3+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2F_1\left(\frac{1}{2}(1+np);np,2;\frac{1}{2}(3+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(2(e+fx))\left(b(c\tan(e+fx))^n\right)^p}{2f(1+np)\left((3+np)F_1\left(\frac{1}{2}(1+np);np,1;\frac{1}{2}(3+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((3+np)F_1\left(\frac{1}{2}(1+np);np,2;\frac{1}{2}(3+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+\left(F_1\left(\frac{1}{2}(3+np);np,2;\frac{1}{2}(5+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4F_1\left(\frac{1}{2}(3+np);np,3;\frac{1}{2}(5+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-npF_1\left(\frac{1}{2}(3+np);1+np,1;\frac{1}{2}(5+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2npF_1\left(\frac{1}{2}(3+np);1+np,2;\frac{1}{2}(5+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((3 + n*p)*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - 2*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[2*(e + f*x)]*(b*(c*Tan[e + f*x])^n)^
p)/(2*f*(1 + n*p)*((3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((3 + n*p)*AppellF1[(1 + n*p)/2,
n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[(
3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 4*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5
 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \cos(fx+e)\left(b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b(c\tan(e+fx))^n)^p \cos(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)\,\left(b\,{\left(c\tan(e+f\,x)\right)}^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cos(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

3.485 $\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=82

$$\frac{\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx)\right)\sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p)*hypergeom([1/2*n*p-1, 1/2*n*p+1/2],[1/2*n*p+3/2],s
in(f*x+e)^2)*sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3740, 2697}

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((n*p)/2)*Hypergeometric2F1[(-2 + n*p)/2, (1 + n*p)/2, (3
 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n
*p))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int \cos^3(e+fx)(c\tan(e-$$
$$= \frac{\cos^2(e+fx)^{\frac{np}{2}}\,{}_2F_1\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(e+fx\right.}{f(1+np)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 4.28, size = 1552, normalized size = 18.93

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((6 + 2*n*p)*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - 6*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 12*AppellF1[(1 + n*p)/2, n*p, 3, (3 +
 n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 8*AppellF1[(1 + n*p)/2,
 n*p, 4, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*
x)/2]^3*Cos[e + f*x]^3*Sin[(e + f*x)/2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n
*p)*(-AppellF1[(3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 12*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] - 36*AppellF1[(3 + n*p)/2, n*p, 4, (5 + n*p)/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 32*AppellF1[(3 + n*p)/2, n*p,
5, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1[(3
+ n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 - 6*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + 12*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 8*n*p*AppellF1[(3 + n*p)/2, 1
 + n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + n*p
)*AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 18*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 6*n*p*Appe
llF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[(e + f*x)/2]^2 + 36*AppellF1[(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 12*n*p*AppellF1[
(1 + n*p)/2, n*p, 3, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Cos[(e + f*x)/2]^2 - 8*(3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 4, (3 + n*p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + AppellF1[(3
+ n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[e + f*x] - 12*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^
```

```
2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 36*AppellF1[(3 + n*p)/2, n*p, 4, (5
+ n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] - 32*Appell
F1[(3 + n*p)/2, n*p, 5, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Cos[e + f*x] - n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 6*n*p*AppellF1[(3 + n*p)/
2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e
+ f*x] - 12*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 3, (5 + n*p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 8*n*p*AppellF1[(3 + n*p)/2, 1 +
 n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]
))
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \left(\cos^{3}\left(f x+e\right)\right) \left(b (c \tan \left(f x+e\right))^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
[Out] int(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,{\left(b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int(cos(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

3.486 $\int (d\sec(e+fx))^m\, (a+b(c\tan(e+fx))^n)^p\, dx$

Optimal. Leaf size=30

$$\text{Int}\left((d\sec(e+fx))^m\, (a+b(c\tan(e+fx))^n)^p, x\right)$$

```
[Out] Unintegrable((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\sec(e+fx))^m\, (a+b(c\tan(e+fx))^n)^p\, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int (d\sec(e+fx))^m\, (a+b(c\tan(e+fx))^n)^p\, dx = \int (d\sec(e+fx))^m\, (a+b(c\tan(e+fx))^n)^p\, dx$$

Mathematica [A]
time = 1.75, size = 0, normalized size = 0.00

$$\int (d\sec(e+fx))^m\, (a+b(c\tan(e+fx))^n)^p\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \left(d\sec\left(fx+e\right)\right)^m\, \left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d \sec{(e + fx)})^m (a + b(c \tan{(e + fx)})^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((d*sec(e + f*x))**m*(a + b*(c*tan(e + f*x))**n)**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (a + b\,(c\tan(e + f\,x))^n)^p \left(\frac{d}{\cos{(e + f\,x)}}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p*(d/cos(e + f*x))^m,x)
[Out] int((a + b*(c*tan(e + f*x))^n)^p*(d/cos(e + f*x))^m, x)
```

3.487 $\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A]
time = 2.97, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(\sec^3(fx+e)\right)\left(a+b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)^p}{{\cos\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3,x)
[Out] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3, x)
```

3.488 $\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\left(c\tan\left(e + fx\right)\right)^{n}\right)^{p}\sec\left(e + fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*tan(e + f*x))**n)**p*sec(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x),x)
[Out] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x), x)
```

3.489 $\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A]
time = 1.78, size = 0, normalized size = 0.00

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+b\left(c\tan\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b(c\tan\left(e + fx\right))^{n}\right)^{p} \cos\left(e + fx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*tan(e + f*x))**n)**p*cos(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)\,{\left(a+b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int(cos(e + f*x)*(a + b*(c*tan(e + f*x))^n)^p, x)
```

3.490 $\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Mathematica [A]
time = 3.09, size = 0, normalized size = 0.00

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \left(\cos^3(fx+e)\right)\left(a+b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^3\,{\left(a+b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int(cos(e + f*x)^3*(a + b*(c*tan(e + f*x))^n)^p, x)
```

3.491 $\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=244

$$\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}}{f}+\frac{2\,{}_2F_1\left(\frac{3}{n},-p;\right.}{}$$

```
[Out] hypergeom([-p, 1/n],[1+1/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)*(a+b*(c*tan(f
*x+e))^n)^p/f/((1+b*(c*tan(f*x+e))^n/a)^p)+2/3*hypergeom([-p, 3/n],[(3+n)/n
],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p/f/((1+b*(c*t
an(f*x+e))^n/a)^p)+1/5*hypergeom([-p, 5/n],[(5+n)/n],-b*(c*tan(f*x+e))^n/a)
*tan(f*x+e)^5*(a+b*(c*tan(f*x+e))^n)^p/f/((1+b*(c*tan(f*x+e))^n/a)^p)
```

Rubi [A]

time = 0.14, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3756, 1907, 252, 251, 372, 371}

$$\frac{\tan^5(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{5f}+\frac{2\tan^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{3f}+\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
) + (2*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^n)/a)]*T
an[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e + f*x])^n
)/a)^p) + (Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*(c*Tan[e + f*x])^n)/a
)]*Tan[e + f*x]^5*(a + b*(c*Tan[e + f*x])^n)^p)/(5*f*(1 + (b*(c*Tan[e + f*x
])^n)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \frac{\text{Subst}\left(\int \left(c^2+x^2\right)^2\left(a+bx^n\right)^p\,dx, x, c\tan(e+fx)\right)}{c^5 f}$$

$$= \frac{\text{Subst}\left(\int \left(c^4(a+bx^n)^p+2c^2x^2(a+bx^n)^p+x^4(a+bx^n)^p\right)\,dx\right.}{c^5 f}$$

$$= \frac{\text{Subst}\left(\int x^4(a+bx^n)^p\,dx, x, c\tan(e+fx)\right)}{c^5 f}+\frac{2\text{Subst}\left(\int x^2(a\right.}{}$$

$$= \frac{\left(\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x\right.}{c^5 f}$$

$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e\right.}{f}$$

Mathematica [A]
time = 1.43, size = 165, normalized size = 0.68

$$\frac{\tan(e+fx)\left(15\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)+10\,{}_2F_1\left(\frac{3}{n},-p;\frac{3+n}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan^2(e+fx)+3\,{}_2F_1\left(\frac{5}{n},-p;\frac{5+n}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan^4(e+fx)\right)(a+b(c\tan(e+fx))^n)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(15*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e +
 f*x])^n)/a)] + 10*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*
x])^n)/a)]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*(c
*Tan[e + f*x])^n)/a)]*Tan[e + f*x]^4)*(a + b*(c*Tan[e + f*x])^n)^p)/(15*f*(
1 + (b*(c*Tan[e + f*x])^n)/a)^p)
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\sec^6(fx+e)\right)(a+b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^6, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.38Unable to divide
, perhaps due to rounding error%%%{1,[0,1,4,0,0]%%%}+%%%{2,[0,1,2,2,0]%%%}+
%%%{1,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p}{{\cos\left(e+f\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6,x)
```

```
[Out] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6, x)
```

3.492 $\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=160

$$\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}}{f}+\frac{{}_2F_1\left(\frac{3}{n},-p;\right.}{}$$

```
[Out] hypergeom([-p, 1/n],[1+1/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)*(a+b*(c*tan(f
*x+e))^n)^p/f/((1+b*(c*tan(f*x+e))^n/a)^p)+1/3*hypergeom([-p, 3/n],[(3+n)/n
],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p/f/((1+b*(c*t
an(f*x+e))^n/a)^p)
```

Rubi [A]

time = 0.09, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3756, 1907, 252, 251, 372, 371}

$$\frac{\tan^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{3f}+\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
) + (Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e + f*x])^n)/
a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\frac{\text{Subst}\left(\int \left(c^2+x^2\right)\left(a+bx^n\right)^p\,dx,x,c\tan(e+fx)\right)}{c^3f}$$

$$=\frac{\text{Subst}\left(\int \left(c^2(a+bx^n)^p+x^2(a+bx^n)^p\right)\,dx,x,c\tan(e+fx)\right)}{c^3f}$$

$$=\frac{\text{Subst}\left(\int x^2(a+bx^n)^p\,dx,x,c\tan(e+fx)\right)}{c^3f}+\frac{\text{Subst}\left(\int (a+\right.}{}$$

$$=\frac{\left(\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x\right.}{c^3f}$$

$$=\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e\right.}{f}$$

Mathematica [A]

time = 2.42, size = 122, normalized size = 0.76

$$\frac{\tan(e+fx)\left(3\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)+{}_2F_1\left(\frac{3}{n},-p;\frac{3+n}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan^2(e+fx)\right)(a+b(c\tan(e+fx))^n)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Tan[e + f*x]*(3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e +
f*x])^n)/a)] + Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^
n)/a)]*Tan[e + f*x]^2)*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan[e
+ f*x])^n)/a)^p)
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(\sec^4(fx+e)\right)(a+b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.8Unable to divide,
 perhaps due to rounding error%%%{1,[0,1,2,0,0]%%%}+%%%{1,[0,1,0,2,0]%%%} /
 %%%{1
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\left(c\tan(e+f\,x)\right)^n\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4,x)
```

```
[Out] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4, x)
```

3.493 $\int \sec^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=75

$$\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}}{f}$$

```
[Out] hypergeom([-p, 1/n],[1+1/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)*(a+b*(c*tan(f
*x+e))^n)^p/f/((1+b*(c*tan(f*x+e))^n/a)^p)
```

Rubi [A]

time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3756, 252, 251}

$$\frac{\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
```

```
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\frac{\text{Subst}\left(\int\left(a+bx^n\right)^p\,dx,x,c\tan(e+fx)\right)}{cf}$$

$$=\frac{\left(\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(\right.\right.}{cf}$$

$$=\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e\right.}{f}$$

Mathematica [**A**]
time = 0.09, size = 75, normalized size = 1.00

$$\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)^{-p}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Tan
[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p
)
```

Maple [**F**]
time = 0.30, size = 0, normalized size = 0.00

$$\int\left(\sec^2(fx+e)\right)\left(a+b(c\tan(fx+e))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\left(c \tan{\left(e + fx \right)}\right)^{n}\right)^{p} \sec^{2}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((a + b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 1.23Unable to divide
, perhaps due to rounding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Er
ror: B
```

Mupad [B]
time = 14.05, size = 76, normalized size = 1.01

$$\frac{\tan(e+f\,x)\,\left(a+b\,{\left(c\tan(e+f\,x)\right)}^n\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,{\left(c\tan(e+f\,x)\right)}^n}{a}\right)}{f\,{\left(\frac{b\,{\left(c\tan(e+f\,x)\right)}^n}{a}+1\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^2,x)
```

```
[Out] (tan(e + f*x)*(a + b*(c*tan(e + f*x))^n)^p*hypergeom([1/n, -p], 1/n + 1, -(
b*(c*tan(e + f*x))^n)/a))/(f*((b*(c*tan(e + f*x))^n)/a + 1)^p)
```

3.494 $\int (a+b(c\tan(e+fx))^n)^p \, dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left((a+b(c\tan(e+fx))^n)^p,x\right)$$

```
[Out] Unintegrable((a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b(c\tan(e+fx))^n)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Defer[Int][(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int (a+b(c\tan(e+fx))^n)^p \, dx = \int (a+b(c\tan(e+fx))^n)^p \, dx$$

Mathematica [A]
time = 0.57, size = 0, normalized size = 0.00

$$\int (a+b(c\tan(e+fx))^n)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*(c*Tan[e + f*x])^n)^p,x]
[Out] Integrate[(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int (a+b(c\tan(fx+e))^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b(c\tan{(e + fx)})^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Integral((a + b*(c*tan(e + f*x))**n)**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int {\left(a+b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int((a + b*(c*tan(e + f*x))^n)^p, x)
```

3.495 $\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx=\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx$$

Mathematica [A]
time = 2.64, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+b(c\tan(fx+e))^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*(c*tan(e + f*x))^n)^p,x)
[Out] int(cos(e + f*x)^2*(a + b*(c*tan(e + f*x))^n)^p, x)
```

3.496 $\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=98

$$\frac{\cos^2(e+fx)^{\frac{1}{2}+p}(d\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1-m+2p);\frac{1}{2}(3-m+2p);\sin^2(e+fx)\right)\tan(e+fx)\,(}{f(1-m+2p)}$$

[Out] (cos(f*x+e)^2)^(1/2+p)*(d*csc(f*x+e))^m*hypergeom([1/2+p, 1/2-1/2*m+p],[3/2-1/2*m+p],sin(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1-m+2*p)

Rubi [A]

time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3739, 2698, 2682, 2657}

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+1);\frac{1}{2}(-m+2p+3);\sin^2(e+fx)\right)}{f(-m+2p+1)}$$

Antiderivative was successfully verified.

[In] Int[(d*Csc[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]

[Out] ((Cos[e + f*x]^2)^(1/2 + p)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 + 2*p)/2, (1 - m + 2*p)/2, (3 - m + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 - m + 2*p))

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*FracPart[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPart[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx &= \left(\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\right)\int (d\csc(e+fx))^m\tan^{2p}(e+\\ &= \left((d\csc(e+fx))^m\left(\frac{\sin(e+fx)}{d}\right)^m\tan^{-2p}(e+fx)\left(b\tan^2(e+f\right.\right.\\ &= \frac{\left(\cos^{2p}(e+fx)(d\csc(e+fx))^{1+m}\sin(e+fx)\left(\frac{\sin(e+fx)}{d}\right)^{m-2p}(b\right.}{d}\\ &= \frac{\cos^2(e+fx)^{\frac{1}{2}+p}(d\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2}(1+2p),\frac{1}{2}(1-m+2p)\right.}{df(1}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.30, size = 299, normalized size = 3.05

$$-\frac{d(-3+m-2p)F_1\left(\frac{1}{2}-\frac{m}{2}+p;2p,1-m;\frac{3}{2}-\frac{m}{2}+p;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(d\csc(e+fx))^{-1+m}\left(b\tan^2(e+fx)\right)^p}{f(-1+m-2p)\left((-3+m-2p)F_1\left(\frac{1}{2}-\frac{m}{2}+p;2p,1-m;\frac{3}{2}-\frac{m}{2}+p;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\left(-\left((-1+m)F_1\left(\frac{3}{2}-\frac{m}{2}+p;2p,2-m;\frac{5}{2}-\frac{m}{2}+p;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)-2pF_1\left(\frac{3}{2}-\frac{m}{2}+p;1+2p,1-m;\frac{5}{2}-\frac{m}{2}+p;\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((d*(-3 + m - 2*p)*AppellF1[1/2 - m/2 + p, 2*p, 1 - m, 3/2 - m/2 + p, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(d*Csc[e + f*x])^(-1 + m)*(b*Tan[e + f
*x]^2)^p)/(f*(-1 + m - 2*p)*((-3 + m - 2*p)*AppellF1[1/2 - m/2 + p, 2*p, 1
- m, 3/2 - m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*(-((-1 + m
)*AppellF1[3/2 - m/2 + p, 2*p, 2 - m, 5/2 - m/2 + p, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]) - 2*p*AppellF1[3/2 - m/2 + p, 1 + 2*p, 1 - m, 5/2 - m/2
 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int (d\csc(fx+e))^m\left(b\left(\tan^2(fx+e)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

```
[Out] int((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{2}\left(e+fx\right)\right)^{p}\left(d\csc\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

```
[Out] Integral((b*tan(e + f*x)**2)**p*(d*csc(e + f*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan(e+f\,x)^2\right)^p \left(\frac{d}{\sin\left(e+f\,x\right)}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m,x)
[Out] int((b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m, x)
```

3.497 $\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=127

$$\frac{F_1\left(\frac{1-m}{2};1-\frac{m}{2},-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\csc(e+fx))^m \sec^2(e+fx)^{-m/2}\tan(e+fx)\,(a+}{f(1-m)}$$

```
[Out] AppellF1(1/2-1/2*m,1-1/2*m,-p,3/2-1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d
*csc(f*x+e))^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(1-m)/((sec(f*x+e)^2)^(1/2
*m))/((1+b*tan(f*x+e)^2/a)^p)
```

Rubi [A]

time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3762, 3748, 525, 524}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\csc(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1-m}{2};1-\frac{m}{2},-p;\frac{3-m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 - m)/2, 1 - m/2, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/a)]*(d*Csc[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1
 - m)*(Sec[e + f*x]^2)^(m/2)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3748

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff
```

```
*(d*Sin[e + f*x])^m*((Sec[e + f*x]^2)^(m/2)/(f*Tan[e + f*x]^m)), Subst[Int[
(ff*x)^m*((a + b*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]
/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]
```

Rule 3762

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(d*Csc[e + f*x])^FracPart[m]*(Sin
[e + f*x]/d)^FracPart[m], Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sin[e + f*x]/d)
^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \left((d\csc(e+fx))^m \left(\frac{\sin(e+fx)}{d}\right)^m\right) \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} (\ldots$$

$$= \frac{\left((d\csc(e+fx))^m \sec^2(e+fx)^{-m/2} \tan^m(e+fx)\right) \text{Subst}\left(\int x \ldots\right.}{f}$$

$$= \frac{\left((d\csc(e+fx))^m \sec^2(e+fx)^{-m/2} \tan^m(e+fx) \left(a+b\tan^2 \ldots\right.\right.}{\ldots}$$

$$= \frac{F_1\left(\frac{1-m}{2}; 1-\frac{m}{2}, -p; \frac{3-m}{2}; -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a}\right) (d\csc \ldots}{\ldots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(127) = 254.
time = 2.40, size = 292, normalized size = 2.30

$$-\frac{a(-3+m)F_1\left(\frac{1}{2}-\frac{m}{2};1-\frac{m}{2},-p;\frac{3}{2}-\frac{m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cos^2(e+fx)\cot(e+fx)(d\csc(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p}{f(-1+m)\left(-2bpF_1\left(\frac{3}{2}-\frac{m}{2};1-\frac{m}{2},1-p;\frac{5}{2}-\frac{m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)-a(-2+m)F_1\left(\frac{3}{2}-\frac{m}{2};2-\frac{m}{2},-p;\frac{5}{2}-\frac{m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)+a(-3+m)F_1\left(\frac{1}{2}-\frac{m}{2};1-\frac{m}{2},-p;\frac{3}{2}-\frac{m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^2(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

```
[Out] -((a*(-3 + m)*AppellF1[1/2 - m/2, 1 - m/2, -p, 3/2 - m/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/a)]*Cos[e + f*x]^2*Cot[e + f*x]*(d*Csc[e + f*x])^m*(a
+ b*Tan[e + f*x]^2)^p)/(f*(-1 + m)*(-2*b*p*AppellF1[3/2 - m/2, 1 - m/2, 1 -
 p, 5/2 - m/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(-2 + m)*Appel
lF1[3/2 - m/2, 2 - m/2, -p, 5/2 - m/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/a)] + a*(-3 + m)*AppellF1[1/2 - m/2, 1 - m/2, -p, 3/2 - m/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/a)]*Cot[e + f*x]^2)))
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int (d\csc (fx+e))^{m} \left(a+b\left(\tan^{2}(fx+e)\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
[Out] int((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan(e+f\,x)^2+a\right)^p\,\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m,x)
[Out] int((a + b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m, x)
```

3.498 $\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal. Leaf size=104

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}(d\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1-m+np);\frac{1}{2}(3-m+np);\sin^2(e+fx)\right)\tan(e+f}{f(1-m+np)}$$

```
[Out] (cos(f*x+e)^2)^(1/2*n*p+1/2)*(d*csc(f*x+e))^m*hypergeom([1/2*n*p+1/2, 1/2*n
*p-1/2*m+1/2],[1/2*n*p-1/2*m+3/2],sin(f*x+e)^2)*tan(f*x+e)*(b*(c*tan(f*x+e)
)^n)^p/f/(n*p-m+1)
```

Rubi [A]
time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3740, 2698, 2682, 2657}

$$\frac{\tan(e+fx)(d\csc(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(np+1)}\left(b(c\tan(e+fx))^n\right)^p\,{}_2F_1\left(\frac{1}{2}(np+1),\frac{1}{2}(-m+np+1);\frac{1}{2}(-m+np+3);\sin^2(e+fx)\right)}{f(-m+np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n*p)/2)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n
*p)/2, (1 - m + n*p)/2, (3 - m + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx = \left((c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\right)\int (d\csc(e+fx))^m(c$$

$$= \left((d\csc(e+fx))^m\left(\frac{\sin(e+fx)}{d}\right)^m(c\tan(e+fx))^{-np}\,(b(c\tan$$

$$= \frac{\left(\cos^{np}(e+fx)(d\csc(e+fx))^{1+m}\sin(e+fx)\left(\frac{\sin(e+fx)}{d}\right)^{m-np}\right.}{d}$$

$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}(d\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2}(1+np),\frac{1}{2}(1-m\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.46, size = 319, normalized size = 3.07

$$-\frac{d(-3+m-np)F_1\left(\frac{1}{2}(1-m+np);np,1-m;\frac{1}{2}(3-m+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(d\csc(e+fx))^{-1+m}\left(b(c\tan(e+fx))^n\right)^p}{f(-1+m-np)\left((-3+m-np)F_1\left(\frac{1}{2}(1-m+np);np,1-m;\frac{1}{2}(3-m+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((-1+m)F_1\left(\frac{1}{2}(3-m+np);np,2-m;\frac{1}{2}(5-m+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+npF_1\left(\frac{1}{2}(3-m+np);1+np,1-m;\frac{1}{2}(5-m+np);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Csc[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] -((d*(-3 + m - n*p)*AppellF1[(1 - m + n*p)/2, n*p, 1 - m, (3 - m + n*p)/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(d*Csc[e + f*x])^(-1 + m)*(b*(c*Ta
n[e + f*x])^n)^p)/(f*(-1 + m - n*p)*((-3 + m - n*p)*AppellF1[(1 - m + n*p)/
2, n*p, 1 - m, (3 - m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
2*((-1 + m)*AppellF1[(3 - m + n*p)/2, n*p, 2 - m, (5 - m + n*p)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1[(3 - m + n*p)/2, 1 + n*p, 1
 - m, (5 - m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)))
```

Maple [**F**]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{m} \left(b(c\tan\left(fx+e\right))^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

```
[Out] int((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b(c\tan\left(e+fx\right))^{n}\right)^{p} \left(d\csc\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

```
[Out] Integral((b*(c*tan(e + f*x))**n)**p*(d*csc(e + f*x))**m, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{d}{\sin\left(e+f\,x\right)}\right)^m\,{\left(b\,{\left(c\,\tan\left(e+f\,x\right)\right)}^n\right)}^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

```
[Out] int((d/sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

3.499 $\int (d\csc(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$

Optimal. Leaf size=57

$$(d\csc(e+fx))^m \left(\frac{\sin(e+fx)}{d}\right)^m \operatorname{Int}\left(\left(\frac{\sin(e+fx)}{d}\right)^{-m} (a+b(c\tan(e+fx))^n)^p, x\right)$$

```
[Out] (d*csc(f*x+e))^m*(sin(f*x+e)/d)^m*Unintegrable((a+b*(c*tan(f*x+e))^n)^p/((s
in(f*x+e)/d)^m),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d\csc(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] (d*Csc[e + f*x])^m*(Sin[e + f*x]/d)^m*Defer[Int][(a + b*(c*Tan[e + f*x])^n)
^p/(Sin[e + f*x]/d)^m, x]
```

Rubi steps

$$\int (d\csc(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \left((d\csc(e+fx))^m \left(\frac{\sin(e+fx)}{d}\right)^m\right) \int \left(\frac{\sin(e+fx)}{d}\right)$$

Mathematica [A]
time = 2.94, size = 0, normalized size = 0.00

$$\int (d\csc(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Maple [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int (d\csc(fx+e))^m (a+b(c\tan(fx+e))^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
[Out] int((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
[Out] integral(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
[Out] integrate(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a+b\,(c\tan(e+f\,x))^n)^p\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*tan(e + f*x))^n)^p*(d/sin(e + f*x))^m,x)
[Out] int((a + b*(c*tan(e + f*x))^n)^p*(d/sin(e + f*x))^m, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```